Odyssey Final Report - RLVR For Reasoning LLMs

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Abstract

This final report summarizes the work done throughout the past weeks on Reinforcement Learning (RL), Large Language Models (LLMs), and RL with Verifiable Rewards (RLVR) in LLMs, culminating in a report which demonstrates how RLVR can be used to train LLMs for performing reasoning tasks. This report presents most of the key results and findings from all weeks of the project, and it is hoped that the report serves as a good roadmap for understanding RLVR's potential to elicit reasoning capabilities in LLMs.

1 Introduction

Large Language Models (LLMs) have long been demonstrated capable of performing a staggering variety of tasks in the Natural Language Processing (NLP) domain, ranging from text generation, sentiment analysis, to machine translation, question answering (Brown et al. (2020)), and in recent years, even complex reasoning tasks (Lambert et al. (2025); Shao et al. (2024); DeepSeek-AI et al. (2025)).

By gaining deeper insights and practical experience in training LLMs with RLVR, one can become better equipped to understand why LLMs are capable of reasoning, how they fall short, and to what extent RLVR can be used to improve their reasoning capabilities.

2 Background

This section informally summarizes the key concepts for understanding RLVR. More details can be found in the Appendices (where applicable) as well as in previous weeks' works.

2.1 LLMs

As their name partly suggests, LLMs are a class of neural networks which attempt to learn the statistical distribution of natural language (Zhao et al. (2025); Karpathy (2025)). Especially in the modern context, LLMs are **by and large** [1] characterized by the following properties (Beh (2025e)):

- LLMs model language *causally* (Jurafsky & Martin (2024); Karpathy (2025)).
- LLMs adopt *some variant of the Transformer architecture* (Vaswani et al. (2017)).
- LLMs require a lot of data (often in the tens of terabytes (Karpathy (2025))) to train.
- LLMs typically have *billions or even trillions* of parameters (Zhao et al. (2025)).
- Modern LLMs typically have *diverse capabilities*, (Brown et al. (2020)) such as generating coherent text, following instructions, and even engaging in conversations.

2.1.1 How are LLMs trained?

Per Karpathy (2025), LLMs typically undergo multiple stages of training, developing different capabilities at each stage.

[1] Exceptions exist and are mentioned in Beh (2025e). They are omitted here for brevity.

Firstly, during *pre-training*, LLMs are trained on a large corpus of text data to learn general language patterns and structures by optimizing the objective in Equation (1). This stage is typically *self-supervised* in that the corpus provides both the conditioning context and the target output, eliding the need for human-curated labels. [2] [3]

Next, some LLMs (Ouyang et al. (2022)) undergo *supervised fine-tuning* on more specialized corpora to adapt their capabilities to specific tasks or domains (Radford et al. (2019); Brown et al. (2020)). While the objective is identical to Equation (1) (Beh (2025e)), the corpus *consists of human-annotated examples* and the objective *does not consider the log-likelihoods of the prompt tokens* (since strictly speaking, the prompt tokens are not part of the target output).

Finally, most modern LLMs (Lambert et al. (2025); DeepSeek-AI et al. (2025)) undergo *reinforcement learning post-training*, such as RL with Human Feedback (RLHF) (Christiano et al. (2017); Ouyang et al. (2022)) or RLVR (Lambert et al. (2025); Shao et al. (2024); DeepSeek-AI et al. (2025)) to refine their output quality and alignment with human preferences. We defer discussion of RLVR to Section 3, as it is the main focus of this report.

2.2 RL

Generally, RL is a machine learning paradigm where an agent learns to take a sequence of *actions* in an *environment* to maximize cumulative reward (Sutton & Barto (2018)).

2.2.1 RL Problems as Markov Decision Processes (MDPs)

Most commonly, RL problems are formulated as MDPs by *assuming the Markov property*, where environment dynamics only depend on the current state and action (Achiam (2018); Levine et al. (2023); Sutton & Barto (2018)) [4] This *Markov assumption* is **key** to enabling the theoretical performance guarantees of many RL algorithms (Sutton & Barto (2018)).

Typically, an agent interacts with the environment via a *parameterized, stochastic* policy to collect experiences known as *trajectories* or *episodes*. The agent then uses these trajectories to update its policy parameters in a way that maximizes the expected cumulative reward (Sutton & Barto (2018); Achiam (2018); Levine et al. (2023)).

2.2.2 Policy Gradient Methods

Where RLVR is concerned, we are primarily interested in *policy gradient methods* (Beh (2025c); Achiam (2018); Weng (2018)), whose direct optimization on the RL objective in Equation (2) is **uses the observation that the expected cumulative reward is differentiable with respect to the policy parameters** θ (Achiam (2018); Levine et al. (2023); Weng (2018)).

This gives rise to a simple gradient ascent algorithm (Beh (2025c)) which a wide variety of algorithms like REINFORCE (Williams (1992)), Proximal Policy Optimization (PPO) (Schulman et al. (2017)), and Group Relative Policy Optimization (GRPO) (Shao et al. (2024)) are ultimately based on, the details of which we relegate to Algorithm 1 in Appendix C.

2.2.3 Special Mention: GRPO

A significant algorithm distinguished for its use in RLVR is the Group Relative Policy Optimization (GRPO) algorithm (Shao et al. (2024)).[5] It is best described as a *computationally-motivated variant* of PPO (Schulman et al. (2017)) without a neural network baseline, which allows it to optimize LLMs on a tight memory budget, and has been shown to be effective in training LLMs with RLVR (Shao et al. (2024); DeepSeek-AI et al. (2025)).

[2] See Appendix B.2 and (Beh (2025f)) for an experiment on pre-training a small LLM on the TinyShakespeare corpus (Karpathy (2015a;b)).

[3] We provide

[4] A formal treatment of the following paragraph is given in Appendix A.2.

[5] A summary of the GRPO Objective may be found in Appendix A.3.

3 RLVR

At the intersection of RL and LLMs lies RLVR, which is characterized by its use of RL to steer LLMs' outputs towards *verifiably correct* ones, according to task-specific criteria. (Lambert et al. (2025); Shao et al. (2024); DeepSeek-AI et al. (2025)).

This method is not only very simple to implement and able to extend the frontier of LLM capabilities (Lambert et al. (2025); DeepSeek-AI et al. (2025)), but it also escapes many of the usual pitfalls associated with RL on LLMs. For example, models trained on RLHF (Ouyang et al. (2022)) can provide *incorrect, but convincing* answers to questions, (Wen et al. (2024)), or learn mismatched preferences due to biases mentioned in Pitis (2023).

Formally speaking, RLVR is typically formulated as a contextual bandit problem (Sutton & Barto (2018); Langford & Zhang (2008)) (as with other RL methods on LLMs), where each model output $(o_1, \ldots, o_n)$ denotes a *single action*. A sample objective for RLVR may be obtained by considering Section A.4.

4 Experiments

Equipped with the requisite background knowledge on RLVR, we can now proceed to explore how LLMs can be effectively trained to perform reasoning tasks with this technique.

While Beh (2025b) has already explored RLVR on the GSM8K (Cobbe et al. (2021)) and Countdown-3-to-4 (Pan (2025)) datasets, this report additionally contains new experiments and findings contributing to our knowledge of reward function design in RLVR.

Hence, building on the results from Beh (2025b), **we will be varying the reward function in order to *also minimize* the length of models' responses, in addition to maximizing the correctness of their answers.**

4.1 Countdown-3-to-4 (Extracted from Beh (2025b))

Countdown-3-to-4 (Pan (2025)) is a simple task where the model is given three or four numbers, as well as a target number, and must yield a mathematical expression in +, -, *, / that evaluates to the target, using each number at most once.

Though in a vacuum, this task can be generated by a simple program, Pan (2025) presents the task as a dataset of 490,364 such problems, all of whose answers are programmatically verified using Python's `eval` function.

4.2 GSM8K

GSM8K (Cobbe et al. (2021)) is a grade school level math problem dataset consisting of 8,792 word problems. All demand a single integer as the answer, and all were deliberately designed to be solvable by elementary school students.

Due to the natural language format, this task is difficult to solve algorithmically. Instead, it concurrently assesses models' ability to parse diverse problem statements, reason about numbers, and perform arithmetic operations accurately (Cobbe et al. (2021)).

4.3 Experimental Configurations

For all experiments, we relied on an adapted version of the nano-aha-moment repository (Kazemnejad et al. (2025)), which provides a faithful single-file implementation of the GRPO algorithm (Shao et al. (2024)) and a simple training pipeline for arbitrary LLMs on arbitrary datasets (as long as they are supported by HuggingFace Transformers (Wolf et al. (2020))) (Beh (2025b)).

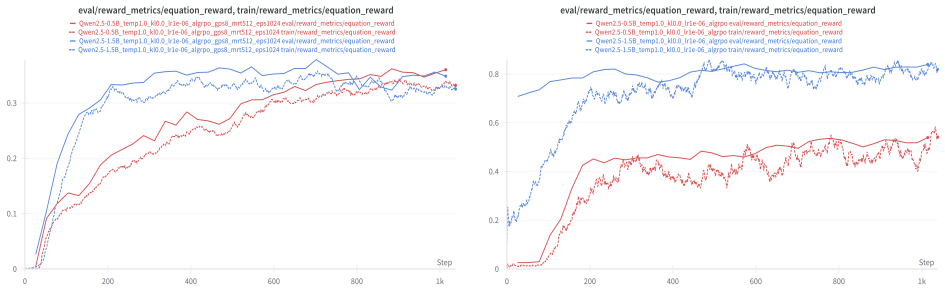

Figure 1: Performance of RLVR on Countdown-3-to-4 and GSM8K tasks. Dotted lines are training curves, solid lines are Pass@1 scores on the test set. (Red) Qwen2.5-0.5B, (Blue) Qwen2.5-1.5B. Curves are smoothed using Time-Weighted Exponential Moving Average (TWEMA) with $\alpha = 0.95$.

Training was alternately conducted on 1x NVIDIA 4090 GPU with 48GB VRAM, and 1x NVIDIA 3090 GPU with 24GB VRAM [6]. All runs, including those in Beh (2025b), took a combined $\sim$ 60 hours of wall-clock time to complete. As in Beh (2025b), *we only performed one trial for each reward function due to compute restrictions*.

4.4 Hyperparameters

For the experiments in Beh (2025b), the hyperparameters can be found in Table D.1. They were not chosen through hyperparameter search (due to compute restrictions), but rather tuned with expert advice and a desire to hold the training epoch count equal (Beh (2025b)). For parity, we used the same hyperparameters while varying the reward function in this report. More details found in Appendix 2.

4.5 Reward Modelling

For brevity we detail the reward function for Beh (2025b) in Appendix D.3.1, and the custom reward functions in Appendix D.3.2.

In attempting to minimize the length of the model's responses, we also considered the following reward functions:

- a Linear Penalty on the response length,
- an Exponential Moving Average Penalty on the Batch average response length,
- the ShorterBetter reward function by Yi et al. (2025)

5 Results & Discussion

Figure 1 shows the Pass@1 performance of the models trained with RLVR on the Countdown-3-to-4 and GSM8K tasks, while Figure 2 shows the Pass@1 performance of the models trained with different reward functions on the GSM8K dataset, as well as their response lengths.

5.1 Stronger Evidence That RLVR Elicits Reasoning From Base Models

We observe that the models trained in Beh (2025b) were unable to achieve much success on the Countdown-3-to-4 task (Pan (2025)), but were able to perform significantly better on the GSM8K dataset.

Given that the Qwen2.5 (Qwen et al. (2025)) family of models are suspected to have dataset contamination specifically pertaining to the math domain (Wu et al. (2025); Liu et al. (2025)),

[6] Thanks Chann.

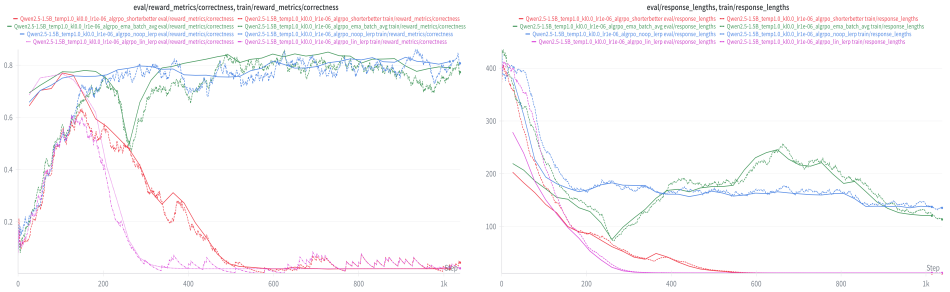

Figure 2: Pass@1 performance of the models trained with different reward functions on the GSM8K dataset. Dotted lines represent the train set performance, (computed with TWEMA and $\alpha = 0.95$), while solid lines represent the test set performance. (Blue) Control (Beh (2025b)), (Purple) Linear Penalty, (Green) Batch Exponential Moving Average Penalty, (Red) ShorterBetter (Yi et al. (2025)).

it is reasonable that their performance on the GSM8K dataset could have been buoyed by having seen similar problems during pre-training.

This effect could be exacerbated by RLVR, whose effect on language models has long been described as *simply increasing the likelihood that the model outputs the correct answer* (Shao et al. (2024); DeepSeek-AI et al. (2025); Yue et al. (2025)). Hence, we believe that this experiment provides some evidence of RLVR eliciting reasoning capabilities from base models' pre-trained knowledge, and it remains unclear whether RLVR can lead to the actual *development* of new reasoning abilities.

5.2 The Difficulty of Managing Multiple Objectives

With the exception of the control, other reward functions induced the model to negotiate the two objectives of "answering correctly" and "answering concisely". It is clear by jointly considering Figures 2 that the two objectives may be anti-correlated, since with increasing performance comes poorer (i.e. longer) responses.

Hence, it is possible some kind of tradeoff exists between the two objectives which must be negotiated with careful hyperparameter selection. Otherwise, as evidenced by Batch Exponential Moving Average Penalty, the model may simply learn to ignore the length penalty altogether, resulting in longer responses with little improvement in correctness. Vice versa, the Linear Penalty and ShorterBetter (Yi et al. (2025)) reward functions appear to have successfully induced the model to produce significantly shorter responses, but at the cost of driving Pass@1 down to nearly 0% on the test set due to mode collapse.

5.3 The Surprising Inefficacy of ShorterBetter

Taking the last point further, the ShorterBetter reward function's performance tradeoff is surprising, given that the authors claim it can significantly shorten responses at little to no cost to correctness.

Certainly, it is possible that our one run used a poor penalty hyperparameter ($\beta = 0.001$) for the GSM8K dataset (Cobbe et al. (2021)) on Qwen2.5-1.5B (Qwen et al. (2025)), which provides a much worse initialization than the DeepSeek-R1-Distill- Qwen-1.5B (DeepSeek-AI et al. (2025)) used in Yi et al. (2025).

However, this finding *opens doors* to downstream research questions in concise response generation, such as **whether concision training requires extremely high-quality priors to work**, or **whether concision training naturally comes with tradeoffs for correctness**. making it extremely worthy to note.

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A Objective Functions

A.1 LLM Pre-training Objective

Where θ^* are the model parameters and N is the model's context length, an optimal language model $LM_{(\theta^*,N)}$ maximizes the likelihood of the next token given all previous tokens, across all training examples $\left\{\{x_{j+i}\}_{i=0}^{N-1}\right\}_{j=1}^{M-N}$ in a corpus of size M:

$$\theta^* = \arg\min_{\theta} \sum_{j=1}^{M-N} \left[\sum_{i=0}^{N-1} -\log P_\theta(x_{j+i}|x_j, \cdots, x_{<(j+i)}) \right] \tag{1}$$

A.2 General RL Objective

Formally speaking, an MDP is defined as a tuple $(\mathcal{S}, \mathcal{A}, \tau, r, \gamma)$, where $\mathcal{S}$ is the set of *states* in the environment, $\mathcal{A}$ is the set of *actions* the agent can take, $\tau : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0,1]$ is the *transition function*, $r : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ is the *reward function*, and $\gamma \in [0,1]$ is (an optional) *discount factor*.

Typically, an agent interacts with the environment via a *parameterized*, *stochastic* policy $\pi_\theta : \mathcal{S} \times \mathcal{A} \to [0,1]$, in an *episodic* manner, where $s_0 \sim \rho_0$ (the initial state distribution), and $\forall_{0 \le k < n} \left(a_k \sim \pi_\theta(s_k), s_{k+1} \sim \tau(s_k, a_k)\right)$ (Beh (2025a)). The agent thus induces a sequence of interactions $T = (s_0, a_0, s_1, a_1, \ldots, s_n)$ (a *trajectory*), which should be regarded analogously to one of many games of Tic-Tac-Toe, or a single playthrough of a video game.

Simply put, the optimal policy π_{θ^*} is the one that maximizes the *expected cumulative reward* (Beh (2025c)):

$$\pi_{\theta^*} = \arg\max_{\pi_\theta} \mathop{\mathbb{E}}_{T \sim \pi_\theta} \left[\sum_{t=0}^{n} \gamma^t r(s_t, a_t) \right] \tag{2}$$

A.3 GRPO Objective

Consider a policy network π_θ and critic network V_ϕ in a single-state, single-action armed bandit environment (Sutton & Barto (2018); Langford & Zhang (2008)) with state s and action $a = (o_1, \ldots, o_n)$, where o_k is the k-th output token of the language model. Then the GRPO objective is defined as follows (Shao et al. (2024); Beh (2025b)):

$$\mathcal{J}(\theta) \quad = \mathbb{E}_{(s,o_1,\ldots,o_n)\sim\pi_\theta} \left[\frac{1}{n} \sum_{k=1}^{n} \min \left\{ \frac{\pi_\theta(o_k|s,o_{<k})}{\pi_{\theta_{old}}(o_k|s,o_{<k})} A(s,a), g(s,a) \right\} \right] - \lambda D_{KL}(\pi_\theta || \pi_{\theta_{old}}) \tag{3}$$

where $g(s,a) = clip\left(\frac{\pi_\theta(o_k|s,o_{<k})}{\pi_{\theta_{old}}(o_k|s,o_{<k})}, 1-\epsilon, 1+\epsilon\right) A(s,a)$, and $\pi_{\theta_{old}}$ is the policy network from the previous GRPO step.

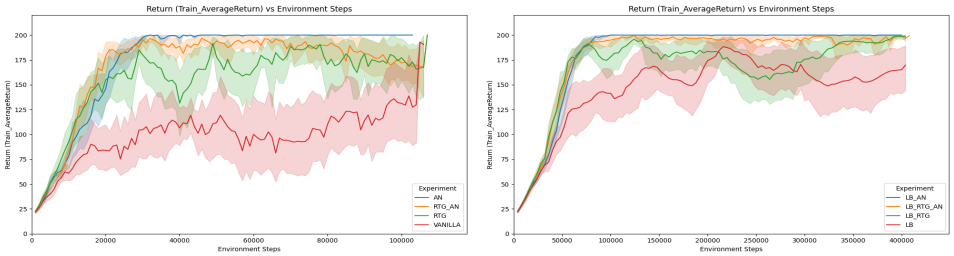

Figure 3: The training loss curves of the REINFORCE algorithm on the CartPole environment.

A.4 RLVR Objective

GRPO (Shao et al. (2024)) has historically been the algorithm of choice for RLVR (DeepSeek-AI et al. (2025); Shao et al. (2024)), hence an objective function for RLVR can simply be 3.

B Miscellaneous Case Studies

B.1 Case Summary: REINFORCE

In Beh (2025d), we experimented with a simple RL training pipeline by Levine et al. (2023) to train a simple network on the CartPole environment (Towers et al. (2024)) using the REINFORCE algorithm (Williams (1992)).

Holding all other variables constant, we sequentially varied the policy gradient batch size, whether to use advantage normalization, and whether to use rewards-to-go, or the full return in computing the policy gradient. (More details on the experiment, such as hyperparameters and compute requirements, can be found in Beh (2025d))

The key findings of the experiment, as corroborated by the results in Figure 3, are as follows:

- REINFORCE benefits separately from advantage normalization and rewards-to-go as variance reduction techniques. **However**, both techniques combined yield a smaller performance improvement than either technique alone.
- RL training runs are very sensitive to hyperparameters and choice of random seed.
- Increasing batch size can stabilize performance (in particular, it reduces the variations in performance across seeds), although more samples are required to achieve peak performance.
- We occasionally observe *extended periods of performance degradation*, where the average return drops appreciably before recovering.
  - This is possibly due to *catastrophic forgetting* (Goodfellow et al. (2015)), which occurs as a consequence of the online nature of the algorithm, where collected data is always discarded after each update.
  - Notably, since we can even see it in the averaged returns in Figure **??**, this proves to be a relatively common occurrence.

B.2 Case Summary: nanoGPT

In Beh (2025f), experiments in pre-training a small LLM on the TinyShakespeare corpus (Karpathy (2015a;b)) were conducted with nanoGPT (Karpathy (2022)). While the details of the experiment can be found in Beh (2025f) and Figure 4, we summarize the key findings here:

- Despite being tokenized on a character level, the model could generate valid whole words with few to no errors.

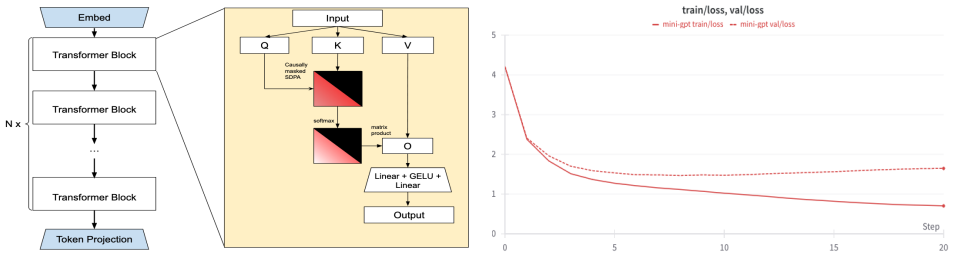

Figure 4: (Left) The architecture of nanoGPT, illustrated. (Right) The training loss curves of the pre-trained nanoGPT model.

- The model could generate text that (superficially) resembled the training corpus.
- The model often produced nonsensical outputs.

Given the experiment was conducted on a relatively small model of known-good source code and architecture (Karpathy (2022)), with a small corpus of size $\sim$ 1.5M tokens (Karpathy (2015a;b)), it is possible that the model's success would predicate on having more parameters, a more diverse corpus, or both.

C Policy Gradient Algorithm

Algorithm 1 Improved Policy Gradient Algorithm

Input: Policy π_θ, learning rate α, baseline of choice $b(s_t)$
Output: Updated policy π_θ
while not converged **do**
    Sample a batch of trajectories T from the policy π_θ
    for each trajectory T in the batch **do**
        Compute the rewards-to-go R_t for each time step t in the trajectory
        Compute the baseline $b(s_t)$ for each time step t in the trajectory
        Estimate $\nabla_\theta J(\pi_\theta) \approx \frac{1}{m} \left(\sum_{t=0}^{n} R_t - b(s_t)\right) \left(\sum_{t=0}^{n} \nabla_\theta \log(\pi_\theta(a_t|s_t))\right)$
    end for
    $\theta \leftarrow \theta + \alpha \nabla_\theta J(\pi_\theta)$
end while

D Hyperparameters

D.1 Hyperparameters used in Beh (2025b)

Hyperparameter	Countdown-3-to-4		GSM8K	
Model Family	Qwen2.5 (Qwen et al. (2025))			
Model Scale	0.5B	1.5B	0.5B	1.5B
Batch Size	16			
Learning Rate	1e-6			
Max. Response Length	512			
Completions per Task	8			
Episodes per GRPO Iteration	1024	1024	16	16
KL Divergence Coefficient λ	0			
Random Seed	42			

Table 1: Hyperparameters used in our experiments.

D.2 Numerical Hyperparameters

As in Beh (2025b), we used the following hyperparameters for all runs of the GSM8K task:

Hyperparameter	**GSM8K**
Model Family	Qwen2.5 (Qwen et al. (2025))
Model Scale	1.5B
Batch Size	16
Learning Rate	1e-6
Max. Response Length (L)	512
Completions per Task	8
Episodes per GRPO Iteration	16
KL Divergence Coefficient λ	0
Random Seed	42

Table 2: Universal Hyperparameters across our experiments.

D.3 Reward Functions

D.3.1 Control Reward Function

Owing to its central role as an independent variable in our experiment, the reward function used in Beh (2025b) demands a closer look.

It is in fact a variant of Kazemnejad et al. (2025)'s, which is itself a more permissive version of that described by DeepSeek-AI et al. (2025).

$$\begin{aligned} R(s,a) &= C(s,a) + F(s,a) \\ &= \begin{cases} 1 & \text{if } a \text{ contains the correct answer} \\ 0 & \text{otherwise} \end{cases} \\ &+ \begin{cases} 1 & \text{if } a \text{ contains thinking, } \backslash n\text{, and a single integer answer.} \\ 0.5 & \text{if } a \text{ contains thinking, } \backslash n\text{, and any answer.} \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{4}$$

We speculate the additional half-reward clause encourages the model to produce valid answers even if they do not obey the strict response format (Beh (2025b)).

D.3.2 Other Reward Functions

Reward Function	Description	Function Name
Control (Beh (2025b))	$r(s,a) = R(s,a)$ (Equation 4)	`noop_decay`
Linear Penalty	$r(s,a) = \left(1 - 0.5 clip\left(\frac{len(a)}{L}, 1\right)\right) R(s,a)$	`linear_length_decay`
Batch Exponential Moving Average Penalty	$r(s,a) = \beta R(s,a)$ $\beta = \begin{cases} 1.2 & \text{if } a < \texttt{avg\_response\_length} - 10 \\ 0.8 & \text{if } a > \texttt{avg\_response\_length} + 10 \\ 1.0 & \text{otherwise} \end{cases}$	`batch_exp_avg_decay`
ShorterBetter (Yi et al. (2025))	$r(s,a) = R(s,a) - 0.001\lvert len(a) - l_{SOL}\rvert$ $l_{SOL} = \begin{cases} min(len(a_1), \ldots, len(a_n)) & \text{if any } a_i \text{ is correct} \\ \frac{1}{n}\sum_{i=1}^{n} len(a_i) & \text{otherwise} \end{cases}$	None (See `nano_r1_gsm8k_shorterbetter.py`)

Table 3: Reward Functions used in our experiments.