Graph basics

Definitions, representations

beOI Training

January 28, 2016

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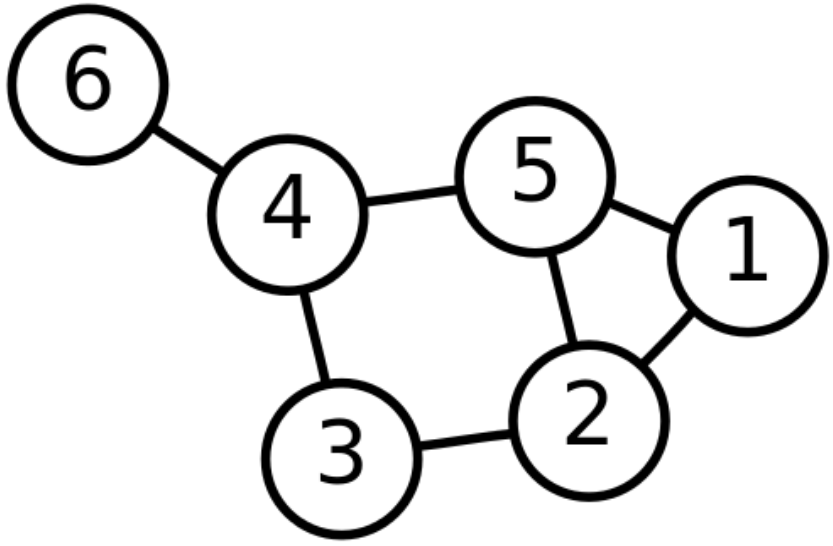
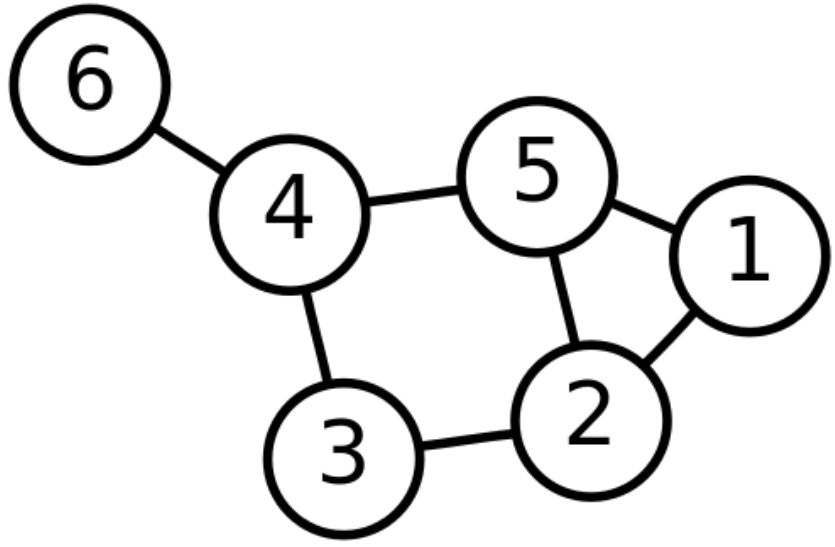
Graphs, intuitively

- Points (vertex, nodes)
- Lines between the points (edges, links)



Graphs, mathematically

$$\textbf{G}\text{raph} = (\textbf{V}\text{ertices}, \textbf{E}\text{dges})$$

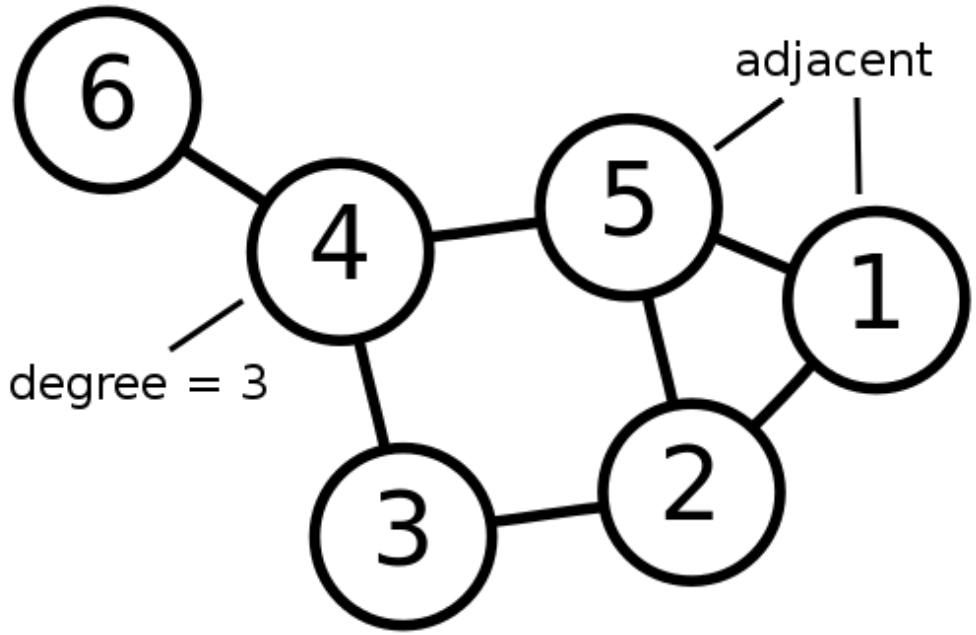


$$V = \{1, 2, 3, 4, 5, 6\}$$

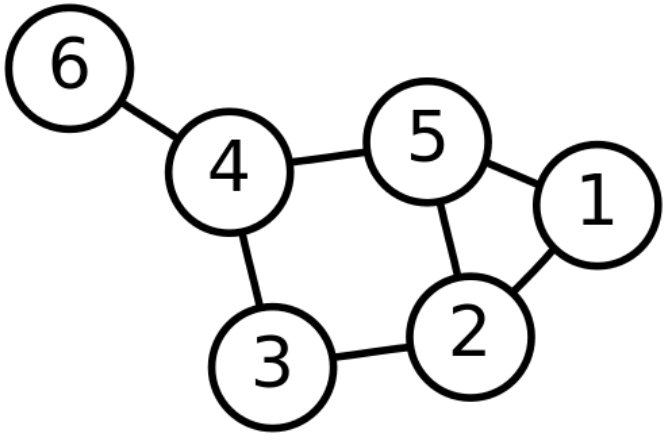
$$E = \{(1, 2), (1, 5), (2, 3), (2, 5), (3, 4), (4, 5), (4, 6)\}$$

Terminology

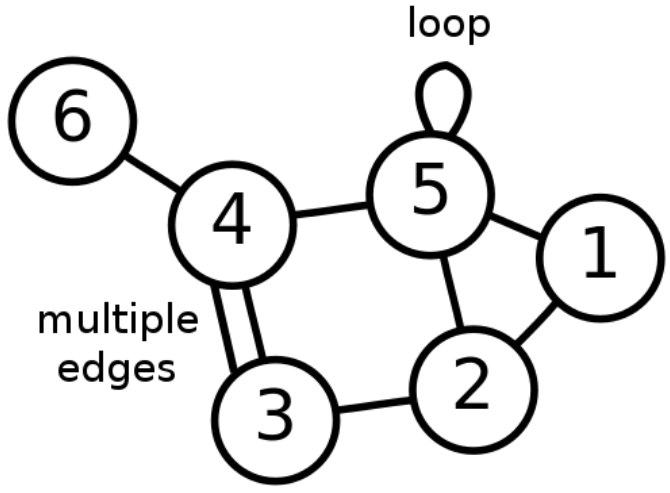
- u and v adjacent $\Leftrightarrow (u, v) \in E$
- Degree of $u = \#\{\text{edges from } u\}$



Simple graph vs multigraph

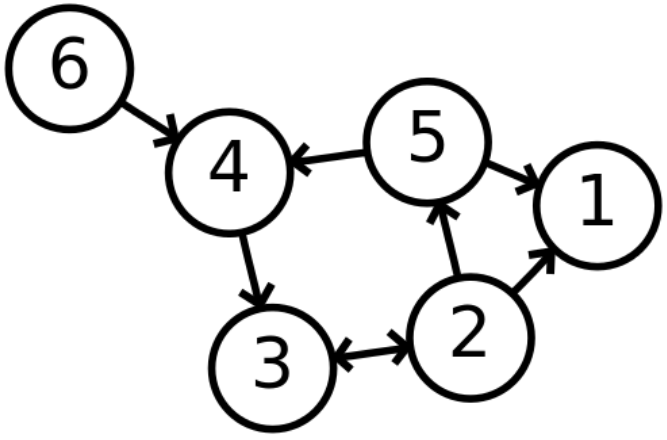


simple graph

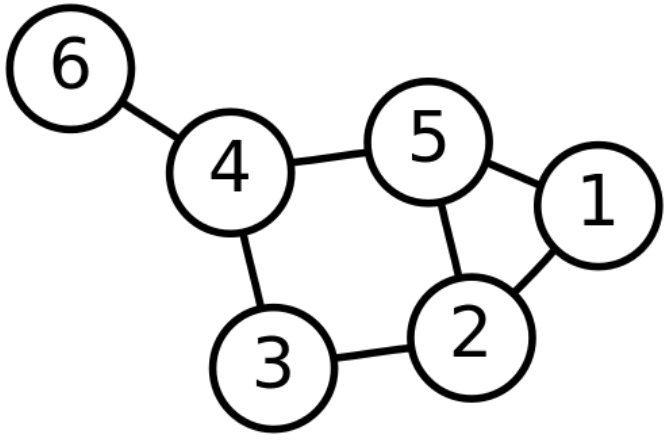


multigraph

Directed vs undirected



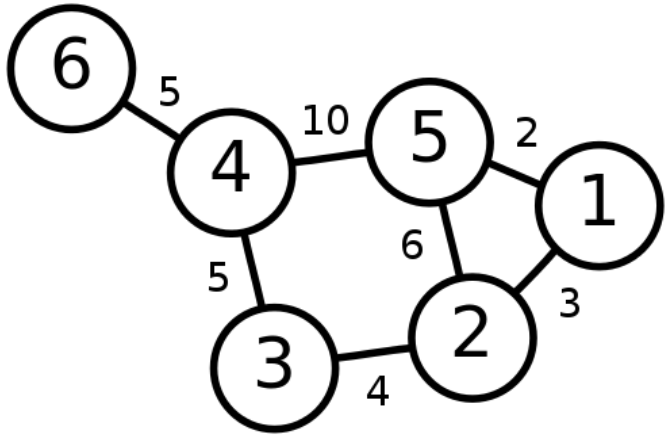
directed



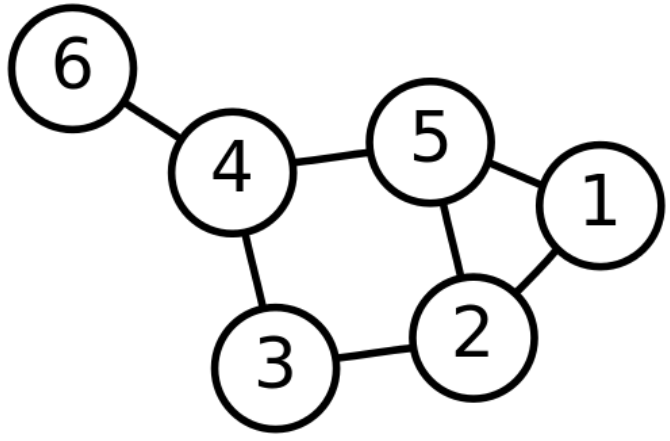
undirected

Weighted vs unweighted

- Represent costs, times, lengths, capacities



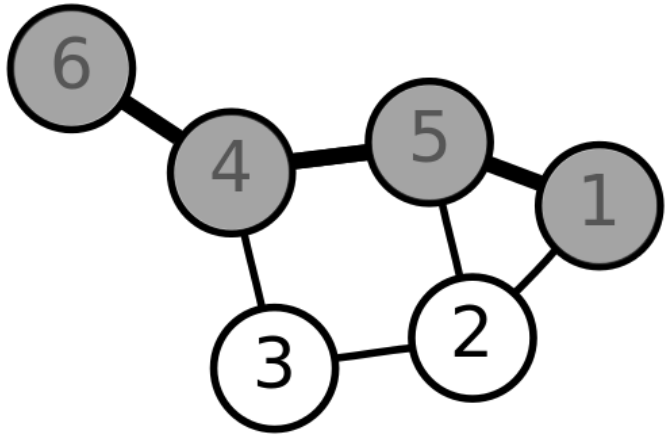
weighted



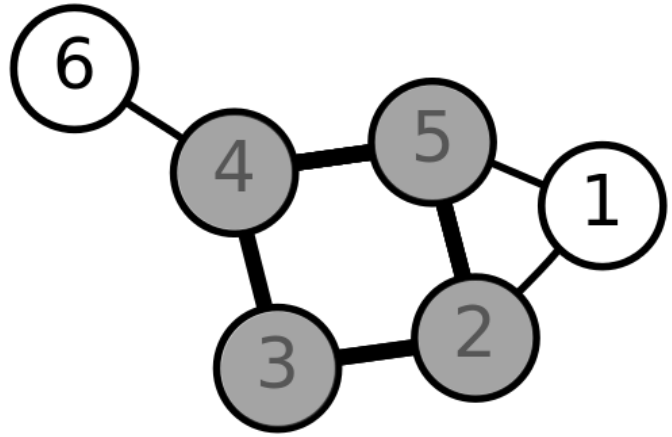
unweighted

Paths and cycles

- u and v connected $\Leftrightarrow$ path $u \rightarrow v$
- G connected $\Leftrightarrow$ all pairs connected



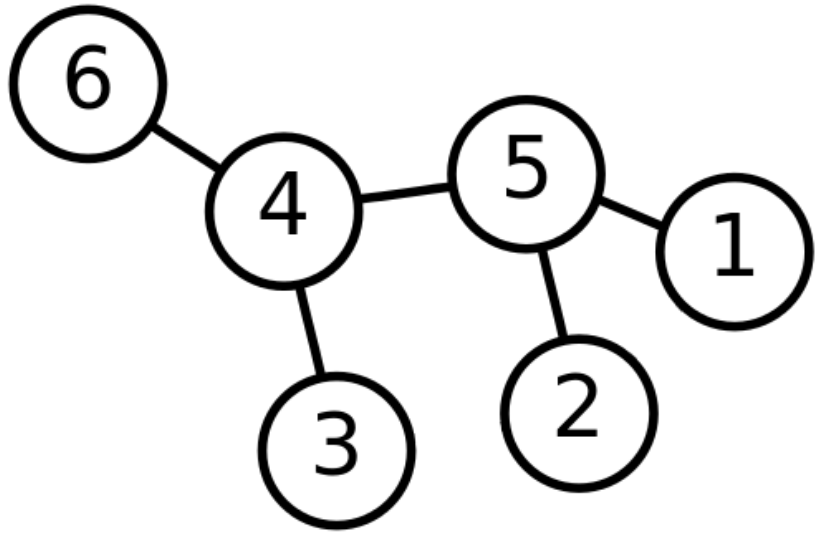
path



cycle

Trees

- Connected
- Acyclic
- $|E| = |V| - 1$



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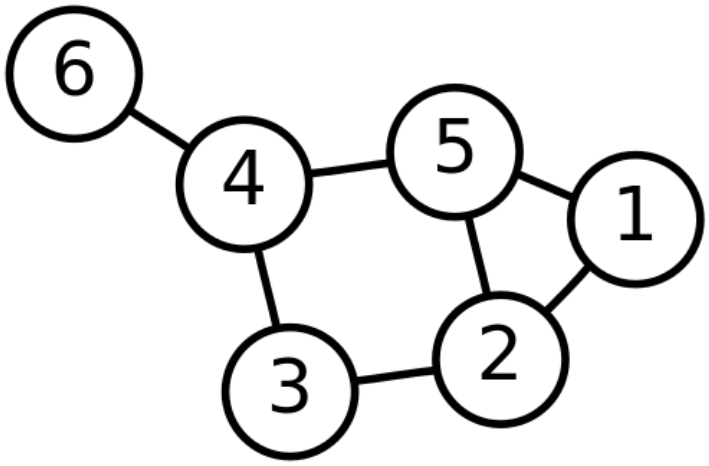
Adjacency matrix on undirected graph

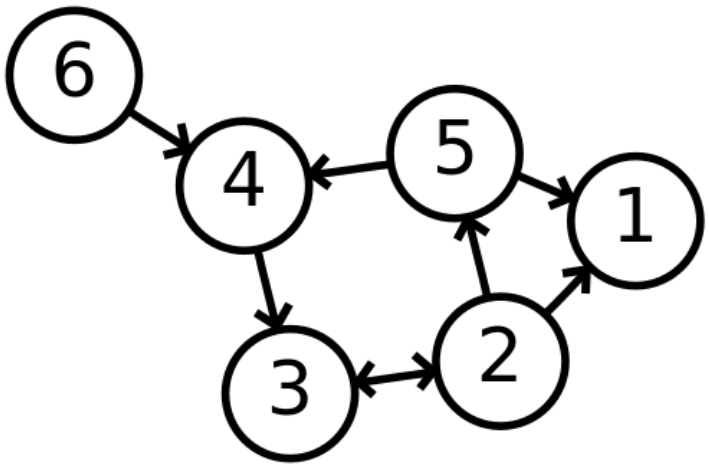
- Static two-dimensional array: `bool adj[MAXN][MAXN]`
- `adj[i][j] == true` if edge $i \to j$
- Symmetric



$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Adjacency matrix on directed graph

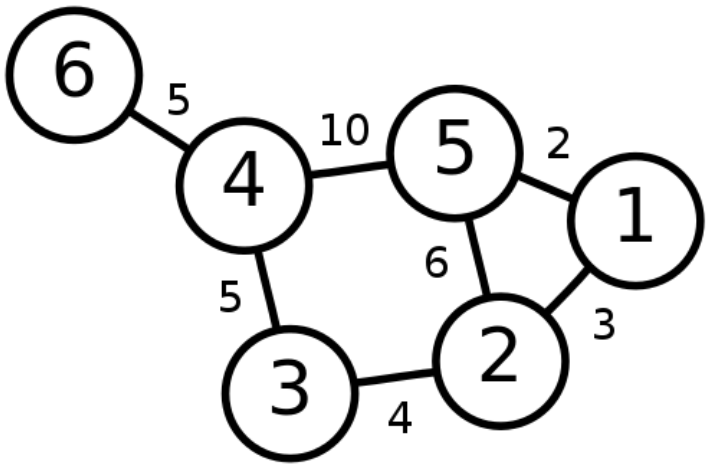
- Only put `true` in one direction
- Not symmetric



$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Adjacency matrix on weighted graph

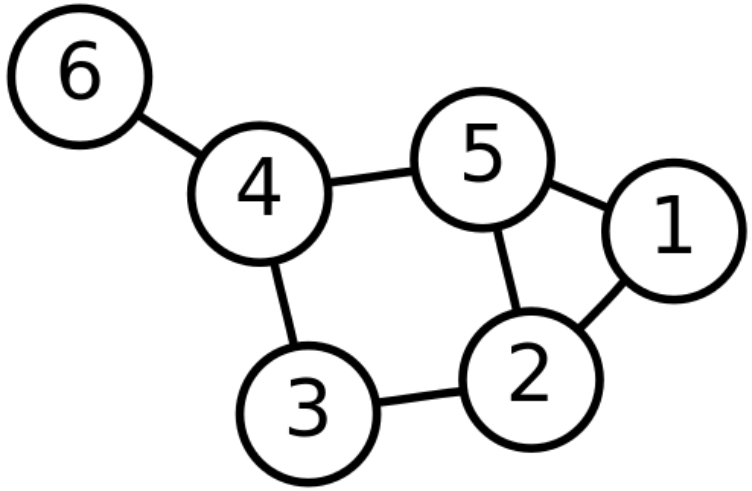
- Change the type: `int adj[MAXN][MAXN]`
- Instead of `true` put the weights



$$\begin{pmatrix} 0 & 3 & 0 & 0 & 2 & 0 \\ 3 & 0 & 4 & 0 & 6 & 0 \\ 0 & 4 & 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 & 10 & 5 \\ 2 & 6 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \end{pmatrix}$$

Adjacency list on undirected graph

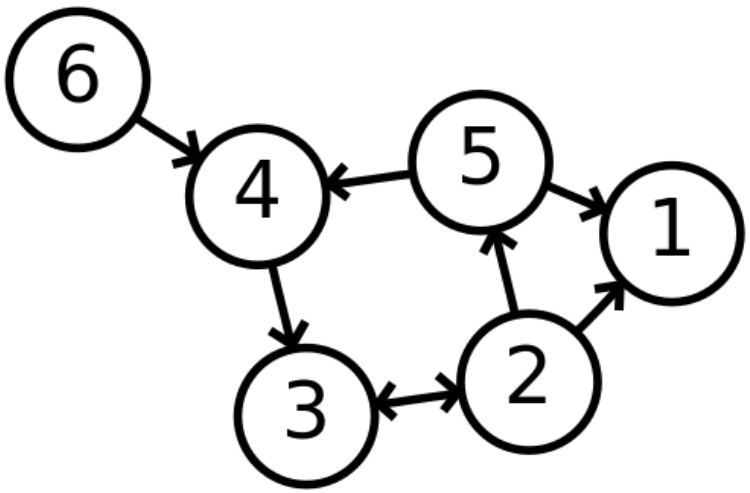
- Array of vectors: `vector<int> neigh[MAXN]`
- For each vertex, list the neighbors



1. $\{2, 5\}$
2. $\{1, 3, 5\}$
3. $\{2, 4\}$
4. $\{3, 5, 6\}$
5. $\{1, 2, 4\}$
6. $\{4\}$

Adjacency list on directed graph

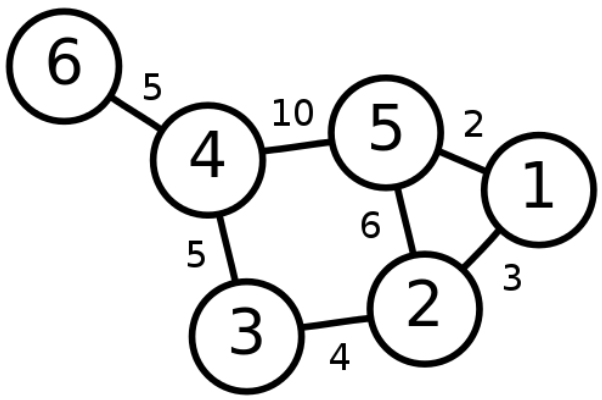
- Only list in one direction



1. $\{\}$
2. $\{1, 3, 5\}$
3. $\{2\}$
4. $\{3\}$
5. $\{1, 4\}$
6. $\{4\}$

Adjacency list on weighted graph

- Add the weight lists: `vector<int> weight[MAXN]`



`neigh[]`

1. $\{2, 5\}$
2. $\{1, 3, 5\}$
3. $\{2, 4\}$
4. $\{3, 5, 6\}$
5. $\{1, 2, 4\}$
6. $\{4\}$

`weight[]`

1. $\{3, 2\}$
2. $\{3, 4, 6\}$
3. $\{4, 5\}$
4. $\{5, 10, 5\}$
5. $\{2, 6, 10\}$
6. $\{5\}$

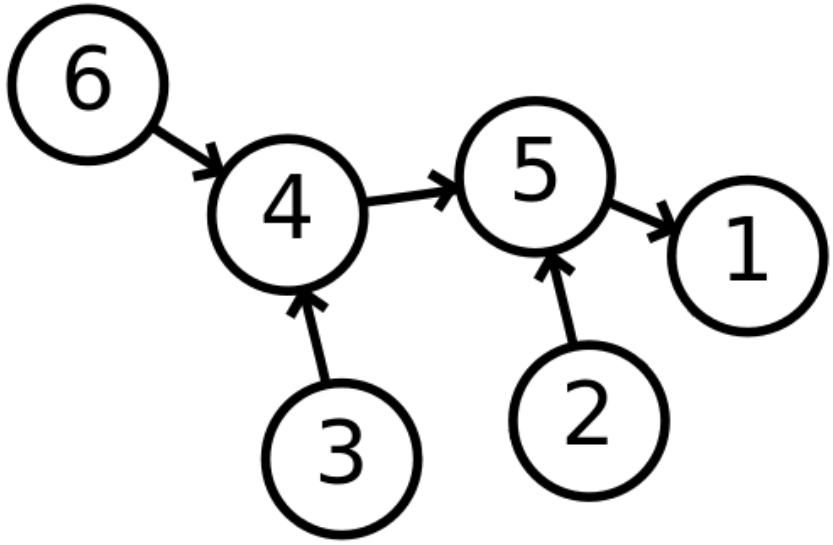
Comparison

- Use adjacency list (most of the time)
- Adjacency matrix only used for fast edge lookup

	Size	List all edges	Edge lookup
Matrix	$O(V^2)$	$O(V^2)$	$O(1)$
List	$O(V+E)$	$O(V+E)$	$O(\deg(u))$

Tree, parent representation

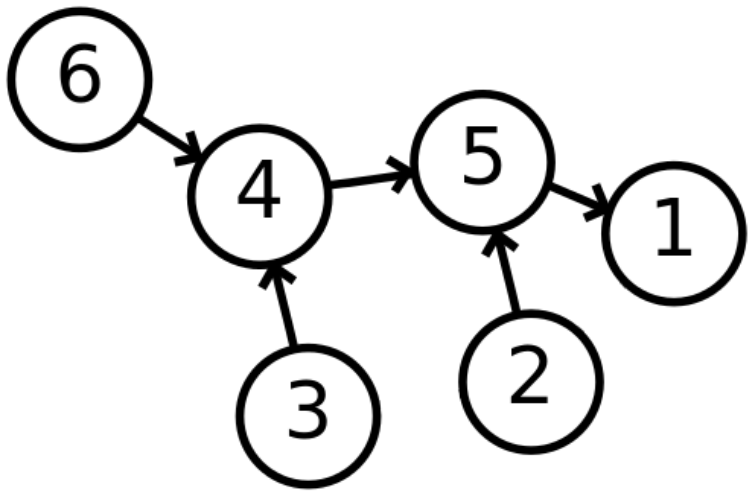
- Choose a root
- Every node has a parent (except the root)
- Parents lead to the root



$$\texttt{parent[]} = \{-, 5, 4, 5, 1, 4\}$$

Tree, child representation

- For each node, list the children
- Nodes 2, 3, 6 are leaves (no children)



1. $\{5\}$
2. $\{\}$
3. $\{\}$
4. $\{3, 6\}$
5. $\{2, 4\}$
6. $\{\}$

Source of figures

- https://en.wikipedia.org/wiki/File:6n-graf.svg