String processing

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30 januari 2016

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Ad hoc

- Straightforward solution

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- See CP3, section 6.3 (pages 236 - 240)

Ad hoc

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- See CP3, section 6.3 (pages 236 - 240)
- If you know regular expressions, C++ 11 has those as well

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Trie

Properties

- <Re*trie*val (but can be pronounce as either *tree* or *try*)

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- insert/retrieve in $O(S)$, with S the length of the string

Trie

Properties

- $<$Re*trie*val (but can be pronounce as either *tree* or *try*)
- Store a set of words (with or without associated values)
- insert/retrieve in $O(S)$, with S the length of the string
- Allows for non-exact matches ($<>$ set/map)

Trie

Structure

- Tree structure

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- Stores the *path* for the string instead of the string

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- Edges labeled with single characters

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- Can vary in type of *character* (bits/ints/...)

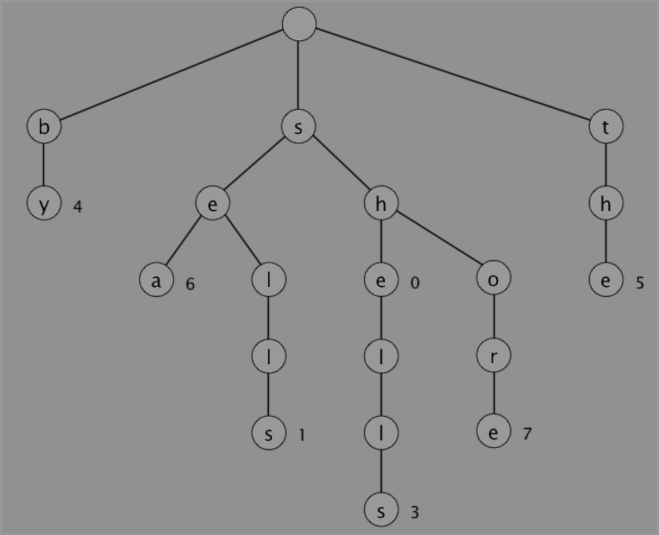
Trie

Structure

- Tree structure
- Stores the *path* for the string instead of the string
- Edges labeled with single characters
- If the last character of a stored word, marked (+ associated value)
- Can vary in type of *character* (bits/ints/...)
- Can be compressed by eliminating successive single-edge nodes

Trie

Structure



Trie

Usage

- Spelling suggestions

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- Autocompletion

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- Bioinformatics (DNA/RNA)

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- Spelling suggestions
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- Bioinformatics (DNA/RNA)
- Alphabetical ordering / sorting
- (Similar to structure for Aho-Corasick)
- (Basis for *suffix tree*)

Trie

Code

```
#include <map>
using namespace std;

struct Trie
{
    //Can be map/unordered_map/direct adressing table/implicit edge/...
    map<char, Trie*> children;
    bool marked;
};
```

Trie

Code

```
void insert(Trie* t, string s)
{
    for (int index = 0; index < s.length(); index++)
    {
        if (t->children.find(s[index]) == t->children.end())
        {
            t->children[s[index]] = new Trie();
        }
        t = t->children[s[index]];
    }
    t->marked = true;
}
```

Trie

Code

```
bool contains(Trie* t, string s)
{
    for (auto c : s)
    {
        if (t->children.find(c) == t->children.end())
            return false;
        t = t->children[c];
    }
    return t->marked;
}
```

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Naive matching

principle

- Straightforward

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- Check for match at each index
- Usually, use the one in the standard library, don't write your own
- $O(s * p)$ (s = length of string, p = length of pattern)

Naive matching

Code

```
#include <string>
using namespace std;

int match(string s, string pat)
{
    if (s.length() < pat.length())
        return -1;
    for (int i = 0; i <= s.length() - pat.length(); i++)
    {
        bool found = true;
        for (int j = 0; j < pat.length(); j++)
        {
            if (s[i+j] != pat[j])
            {
                found = false;
                break;
            }
        }
        if (found) return i;
    }
    return -1;
}
```

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Rabin-Karp

Idea

- Checking for a match with the pattern: $O(n)$

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- Checking for a match with the pattern: $O(n)$
- Faster possible?
- What about hashes, integer comparison $= O(n)$
- We still need a $O(1)$ way to generate the hashes.
- Useful for multiple same-length patterns (check all hashes)

Rabin-Karp

Polynomial hashing

- Generate successive hashes of the same length as the pattern (and hash the pattern)

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- $s_i, s_{i+1}, \ldots, s_{i+k-1} = s_i \times B^{k-1} + s_{i+1} \times B^{k-2} + \ldots + s_{i+k-1} \times B^0$

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- Too big $\Rightarrow$ modulo H (usually prime)
- Watch out for false positives

Rabin-Karp

Rolling hashes

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- $s_{i+1}, s_{i+2}, \ldots, s_{i+k} = ((s_i, \ldots, s_{i+k-1}) - s_i \times B^{k-1}) \times B) + s_{i+k}$

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- A rolling hash *frame*
- $O(1)$

Rabin-Karp

Collision strategies

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- Worst case, still $O(n^2)$
- *Gambling*: keep 2 hashes (with distinct B and H)
- Collision chance is low, if the two hashes match, guess it's correct
- $\Rightarrow$ triple hashing, ...

Rabin-Karp

code

```
const int B = 17;
const int H = 12632251;

int hash_pattern(string pat, int start, int end)
{
    int h = 0;
    for (int i = start; i <= end; i++)
    {
        h = ((h * B) % H + pat[i]) % H;
    }
    return h;
}
```

Rabin-Karp

code

```
bool check(string s, string pat, int start)
{
    for (int i = 0; i < pat.length(); i++)
    {
        if (s[start + i] != pat[i])
            return false;
    }
    return true;
}

int modpow(int exp) { //This can be done in O(log N)
  int result = 1;
  for (int i = 0; i < exp; i++)
  {
      result = (result * B) % H;
  }
  return result;
}
```

Rabin-Karp

code

```
int match(string s, string pat)
{
    if (pat.length() > s.length()) return -1;
    int k = pat.length();
    int Hp = hash_pattern(pat, 0, k - 1);
    int Hs = hash_pattern(s, 0, k - 1);
    int Bk = modpow(k-1);
    for (int i = 0; i <= s.length() - k; i++)
    {
        if (Hs == Hp && check(s, pat, i))
        {
            return i;
        }
        Hs = ((B * (Hs - (s[i] * Bk) % H)) % H + s[i+k]) % H;
        if (Hs < 0) Hs += H;
    }
    return -1;
}
```

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Z-algorithm

terminology

- Z-box = substring that matches with a prefix from the string

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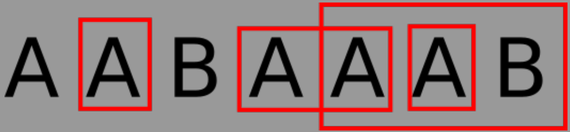

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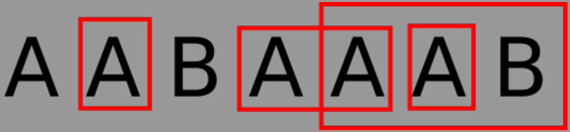

letter	A	A	B	A	A	A	B
Z-score	7	1	0	2	3	1	0

Z-algorithm

Matching

- $P =$ pattern

Z-algorithm

Matching

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- $S =$ search string

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Matching

- P = pattern
- S = search string
- $\$$ = sentinel (not part of alphabet)

Z-algorithm

Matching

- $P =$ pattern
- $S =$ search string
- $\$ =$ sentinel (not part of alphabet)
- return i for each $i > 0$ where $Z_i(P\$S) = |P|$

Z-algorithm

Calculating Z-scores

- Naive $\Rightarrow O(n^2)$, possible in $O(n)$

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- Keep track of the Z-box with right end furthest to the right (bounds: $[l, r]$)

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- if current character in $[l, r]$: look at corresponding character in prefix (computed previously)

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- else: calculate explicitly, update $[l, r]$

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Calculating Z-scores

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- Keep track of the Z-box with right end furthest to the right (bounds: $[l, r]$)
- if current character in $[l, r]$: look at corresponding character in prefix (computed previously)
- expand if grows beyond r, update $[l, r]$
- else: calculate explicitly, update $[l, r]$
- (Nicely illustrated: https://www.cs.umd.edu/class/fall2011/cmsc858s/Lec02-zalg.pdf)

Z-algorithm

code

```
int match(string s, string pat)
{
    string S = pat + "$" + s;
    vector<int> Z(S.length());
    int l = -1, r = -1;

    for (int i = 1; i < S.length(); i++)
    {
        if (i > r) //Outside furthest Z-box
        {
            int len = 0;
            for (int j = i; j < S.length() && S[j] == S[j-i]; j++)
            {
                len++;
            }
            Z[i] = len;
            if (len > 0)
            {
                l = i;
                r = i + len - 1;
            }
        }
```

Z-algorithm

code

```java
        else
        {
            int inside = r - i + 1;
            int corresponding = i - l;
            if (Z[corresponding] < inside)
            {
                Z[i] = Z[corresponding];
            }
            else //Need to grow beyond r
            {
                int len = 0;
                for (int j = r + 1; j < S.length() && S[j] == S[j - i]; j++)
                {
                    len++;
                }
                Z[i] = inside + len;
                l = i;
                r = i + len - 1;
            }
        }
    }
    for (int i = 1; i < S.length(); i++)
    {
        if (Z[i] == pat.length())
            return i - pat.length() - 1; //Don't forget to subtract the sentinel
    }
    return -1;
}
```

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Knuth-Morris-Pratt

Idea

- Don't inspect substring starting at each index

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- Fail smart

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- Don't inspect substring starting at each index
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- Jump ahead to the next possible index, given what we've seen so far

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Idea

- Don't inspect substring starting at each index
- Fail smart
- Jump ahead to the next possible index, given what we've seen so far
- Precompute the failure jumps

Knuth-Morris-Pratt

Idea

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Knuth-Morris-Pratt

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- Next possible partially matched pattern = longest proper suffix (of the partial match) that is a prefix

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- Precompute and keep the length of this suffix/prefix in an array (call this L)

Knuth-Morris-Pratt

Idea

- How to choose the jump?
- Next possible partially matched pattern = longest proper suffix (of the partial match) that is a prefix
- Precompute and keep the length of this suffix/prefix in an array (call this L)
- $L[i]$ = length of that prefix for $S[0..i-1]$ (inclusive)

Knuth-Morris-Pratt

Precomputation

- $L[0] = L[1] = 0$ (it should be a *proper* suffix)

Knuth-Morris-Pratt

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- $L[0] = L[1] = 0$ (it should be a *proper* suffix)
- Search for the next *parent* in L that can be expanded with the current character

Knuth-Morris-Pratt

Precomputation

- $L[0] = L[1] = 0$ (it should be a *proper* suffix)
- Search for the next *parent* in L that can be expanded with the current character
- $L[i] = j + 1$ (j is the length of the *parent*'s match)

Knuth-Morris-Pratt

Precomputation

- $L[0] = L[1] = 0$ (it should be a *proper* suffix)
- Search for the next *parent* in L that can be expanded with the current character
- $L[i] = j + 1$ (j is the length of the *parent*'s match)
- If none can be found: $L[i] = 0$

Knuth-Morris-Pratt

Matching

- Precompute the failure jumps (L) of the pattern

Knuth-Morris-Pratt

Matching

- Precompute the failure jumps (L) of the pattern
- Skip characters using L while matching

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- Very similar to the actual precomputation

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Matching

- Precompute the failure jumps (L) of the pattern
- Skip characters using L while matching
- Very similar to the actual precomputation
- $O(S + P)$

Knuth-Morris-Pratt

code

```
vector<int> precompute(string pat)
{
    vector<int> L(pat.length() + 1);
    L[0] = L[1] = 0;
    for (int i = 2; i <= pat.length(); i++)
    {
        for (int j = L[i-1]; ; j = L[j])
        {
            if (pat[j] == pat[i - 1])
            {
                L[i] = j + 1;
                break;
            }
            if (j == 0)
            {
                L[i] = 0;
                break;
            }
        }
    }
    return L;
}
```

Knuth-Morris-Pratt

code

```
int match(string s, string pat)
{
    vector<int> L = precompute(pat);
    int j = 0;  //The current index for L
    for (int i = 0; i < s.length(); i++)
    {
        while (j != 0 && s[i] != pat[j])
        {
            j = L[j];
        }
        j++;
        if (j == pat.length())
            //Found a match, reconstruct the beginning of the substring
            return i + 1 - j;
    }
    return -1;
}
```

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Exercises

Ad hoc

- UVa 10115 (*Automatic Editing*)
- UVa 10361 (*Automatic Poetry*)
- UVa 10082 (*WERTYU*)
- UVa 1368 (*DNA Consensus String*)

Exercises

Tries

- UVa 902 (*Password Search*)
- UVa 755 (*487–3279*)
- Codechef Remember the recipe (https://www.codechef.com/problems/TWSTR/)

Exercises

String matching

- UVa 363 (*Approximate matches*)
- UVa 455 (*Periodic strings*)
- UVa 1223 (*Editor*)
- Codeforces 126, problem B (http://codeforces.com/contest/126/problem/B)
- UVa 11151 (*Longest Palindrome*)
- UVa 11475 (*Extend to Palindrome*)