# String processing

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## Ad hoc

- Straightforward solution
- See CP3, section 6.3 (pages 236 - 240)
- If you know regular expressions, C++ 11 has those as well

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## Trie

### Properties

- <Re*trie*val (but can be pronounce as either *tree* or *try*)
- Store a set of words (with or without associated values)
- insert/retrieve in $O(S)$, with S the length of the string
- Allows for non-exact matches ($<>$ set/map)

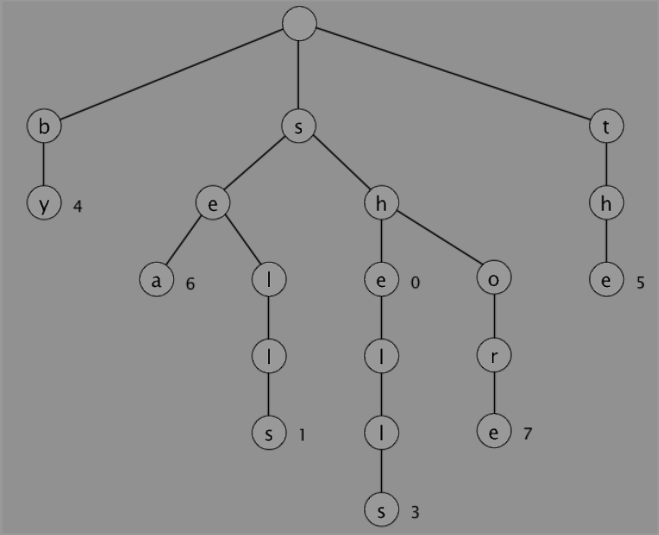
## Trie

### Structure

- Tree structure
- Stores the *path* for the string instead of the string
- Edges labeled with single characters
- If the last character of a stored word, marked (+ associated value)
- Can vary in type of *character* (bits/ints/...)
- Can be compressed by eliminating successive single-edge nodes

## Trie

### Structure



## Trie

### Usage

- Spelling suggestions
- Autocompletion
- Bioinformatics (DNA/RNA)
- Alphabetical ordering / sorting
- (Similar to structure for Aho-Corasick)
- (Basis for *suffix tree*)

## Trie

### Code

```
#include <map>
using namespace std;

struct Trie
{
    //Can be map/unordered_map/direct adressing table/implicit edge/...
    map<char, Trie*> children;
    bool marked;
};
```

# Trie

## Code

```
void insert(Trie* t, string s)
{
    for (int index = 0; index < s.length(); index++)
    {
        if (t->children.find(s[index]) == t->children.end())
        {
            t->children[s[index]] = new Trie();
        }
        t = t->children[s[index]];
    }
    t->marked = true;
}
```

## Trie

### Code

```
bool contains(Trie* t, string s)
{
    for (auto c : s)
    {
        if (t->children.find(c) == t->children.end())
            return false;
        t = t->children[c];
    }
    return t->marked;
}
```

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## Naive matching

principle

- Straightforward
- Check for match at each index
- Usually, use the one in the standard library, don't write your own
- $O(s * p)$ (s = length of string, p = length of pattern)

## Naive matching

### Code

```
#include <string>
using namespace std;

int match(string s, string pat)
{
    if (s.length() < pat.length())
        return -1;
    for (int i = 0; i <= s.length() - pat.length(); i++)
    {
        bool found = true;
        for (int j = 0; j < pat.length(); j++)
        {
            if (s[i+j] != pat[j])
            {
                found = false;
                break;
            }
        }
        if (found) return i;
    }
    return -1;
}
```

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## Rabin-Karp

Idea

- Checking for a match with the pattern: $O(n)$
- Faster possible?
- What about hashes, integer comparison $= O(n)$
- We still need a $O(1)$ way to generate the hashes.
- Useful for multiple same-length patterns (check all hashes)

## Rabin-Karp

### Polynomial hashing

- Generate successive hashes of the same length as the pattern (and hash the pattern)
- Polynomial hashing: the string is an integer in some base $B$ (usually prime)
- $s_i, s_{i+1}, \ldots, s_{i+k-1} = s_i \times B^{k-1} + s_{i+1} \times B^{k-2} + \ldots + s_{i+k-1} \times B^0$
- Too big $\Rightarrow$ modulo $H$ (usually prime)
- Watch out for false positives

## Rabin-Karp

Rolling hashes

- Once we have a hash, it's easy to compute the next
- $s_{i+1}, s_{i+2}, \ldots, s_{i+k} = ((s_i, \ldots, s_{i+k-1}) - s_i \times B^{k-1}) \times B) + s_{i+k}$
- A rolling hash *frame*
- $O(1)$

## Rabin-Karp

Collision strategies

- If equal hashes $\Rightarrow$ compare the strings explicitely
- Worst case, still $O(n^2)$
- *Gambling*: keep 2 hashes (with distinct $B$ and $H$)
- Collision chance is low, if the two hashes match, guess it's correct
- $\Rightarrow$ triple hashing, ...

## Rabin-Karp

code

```
const int B = 17;
const int H = 12632251;

int hash_pattern(string pat, int start, int end)
{
    int h = 0;
    for (int i = start; i <= end; i++)
    {
        h = ((h * B) % H + pat[i]) % H;
    }
    return h;
}
```

# Rabin-Karp

## code

```
bool check(string s, string pat, int start)
{
    for (int i = 0; i < pat.length(); i++)
    {
        if (s[start + i] != pat[i])
            return false;
    }
    return true;
}

int modpow(int exp) { //This can be done in O(log N)
  int result = 1;
  for (int i = 0; i < exp; i++)
  {
      result = (result * B) % H;
  }
  return result;
}
```

## Rabin-Karp

code

```
int match(string s, string pat)
{
    if (pat.length() > s.length()) return -1;
    int k = pat.length();
    int Hp = hash_pattern(pat, 0, k - 1);
    int Hs = hash_pattern(s, 0, k - 1);
    int Bk = modpow(k-1);
    for (int i = 0; i <= s.length() - k; i++)
    {
        if (Hs == Hp && check(s, pat, i))
        {
            return i;
        }
        Hs = ((B * (Hs - (s[i] * Bk) % H)) % H + s[i+k]) % H;
        if (Hs < 0) Hs += H;
    }
    return -1;
}
```

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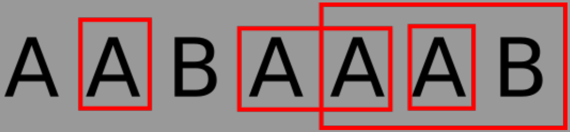
# Z-algorithm

terminology

- Z-box = substring that matches with a prefix from the string
- Z-score $Z_i(S)$ = length of Z-box starting at index $i$



| letter | A | A | B | A | A | A | B |
|---|---|---|---|---|---|---|---|
| Z-score | 7 | 1 | 0 | 2 | 3 | 1 | 0 |

## Z-algorithm

Matching

- $P$ = pattern
- $S$ = search string
- \$ = sentinel (not part of alphabet)
- return $i$ for each $i > 0$ where $Z_i(P\$S) = |P|$

## Z-algorithm

Calculating Z-scores

- Naive $\Rightarrow O(n^2)$, possible in $O(n)$
- Keep track of the Z-box with right end furthest to the right (bounds: $[l, r]$)
- if current character in $[l, r]$: look at corresponding character in prefix (computed previously)
- expand if grows beyond $r$, update $[l, r]$
- else: calculate explicitely, update $[l, r]$
- (Nicely illustrated: https://www.cs.umd.edu/class/fall2011/cmsc858s/Lec02-zalg.pdf)

## Z-algorithm

### code

```
int match(const string& s, const string& pat)
{
    string S = pat + "$" + s;
    int n = S.length();
    vector<int> Z(n);
    int l = -1, r = -1;

    for (int i = 1; i < n; i++)
    {
        if (i > r) //Outside furthest Z-box
        {
            int j;
            for (j = i; j < n && S[j] == S[j-i]; j++);
            Z[i] = j - i;
            l = i;
            r = j - 1;
        }
        else
        {
            int inside = r - i + 1;
            int corresponding = i - l;
            if (Z[corresponding] < inside)
            {
                Z[i] = Z[corresponding];
            }
```

## Z-algorithm

code

```
            else //Need to grow beyond r
            {
                int j;
                for (j = r + 1; j < n && S[j] == S[j - i]; j++);
                Z[i] = j - i;
                l = i;
                r = j - 1;
            }
        }
    }

    for (int i = 1; i < n; i++)
        if (Z[i] == pat.length())
            return i - pat.length() - 1; //Don't forget to subtract the sentinel
    return -1;
}
```

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## Knuth-Morris-Pratt

### Idea

- Don't restart a match every time
- Fail smart
- Re-use previous (partial) match information
- Precompute possible *sub*matches

## Knuth-Morris-Pratt

Idea

- How to choose the next possible match?
- Next possible partially matched pattern = longest proper suffix (of the partial match) that is a prefix
- (What is this in terms of Z-boxes?)
- Precompute and keep the length of this suffix/prefix in an array (call this $L$)
- $L[i]$ = length of that prefix for $S[0..i-1]$ (inclusive)

## Knuth-Morris-Pratt

Precomputation

- $L[0] = -1$ (could be 0, but this eliminates a few checks)
- $L[1] = 0$ (it should be a *proper* suffix)
- Search for the next *parent* in $L$ that can be expanded with the current character
- $L[i] = j + 1$ ($j$ is the length of the *parent*'s match)
- If none can be found: $L[i] = 0$

## Knuth-Morris-Pratt

### Matching

- Precompute the suffix lengths ($L$) of the pattern
- Re-use partial matches using $L$ while matching
- Very similar to the actual precomputation
- $O(S + P)$

## Knuth-Morris-Pratt

code

```
vector<int> precompute(string pat)
{
    vector<int> L(pat.length() + 1);
    L[0] = -1; L[1] = 0;
    for (int i = 2; i <= pat.length(); i++)
    {
        int j = L[i-1];
        while (j >= 0 && pat[j] != pat[i-1])
            j = L[j];
        L[i] = j + 1;
    }
    return L;
}
```

## Knuth-Morris-Pratt

code

```
int match(string s, string pat)
{
    vector<int> L = precompute(pat);
    int j = 0;  //The current index for L
    for (int i = 0; i < s.length(); i++)
    {
        while (j >= 0 && s[i] != pat[j])
            j = L[j];
        j++;
        if (j == pat.length())
            //Found a match, reconstruct the beginning of the substring
            return i + 1 - j;
    }
    return -1;
}
```