Game theory

Nim game, Minimax, Alpha-Beta pruning

beOI Training

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What is game theory?

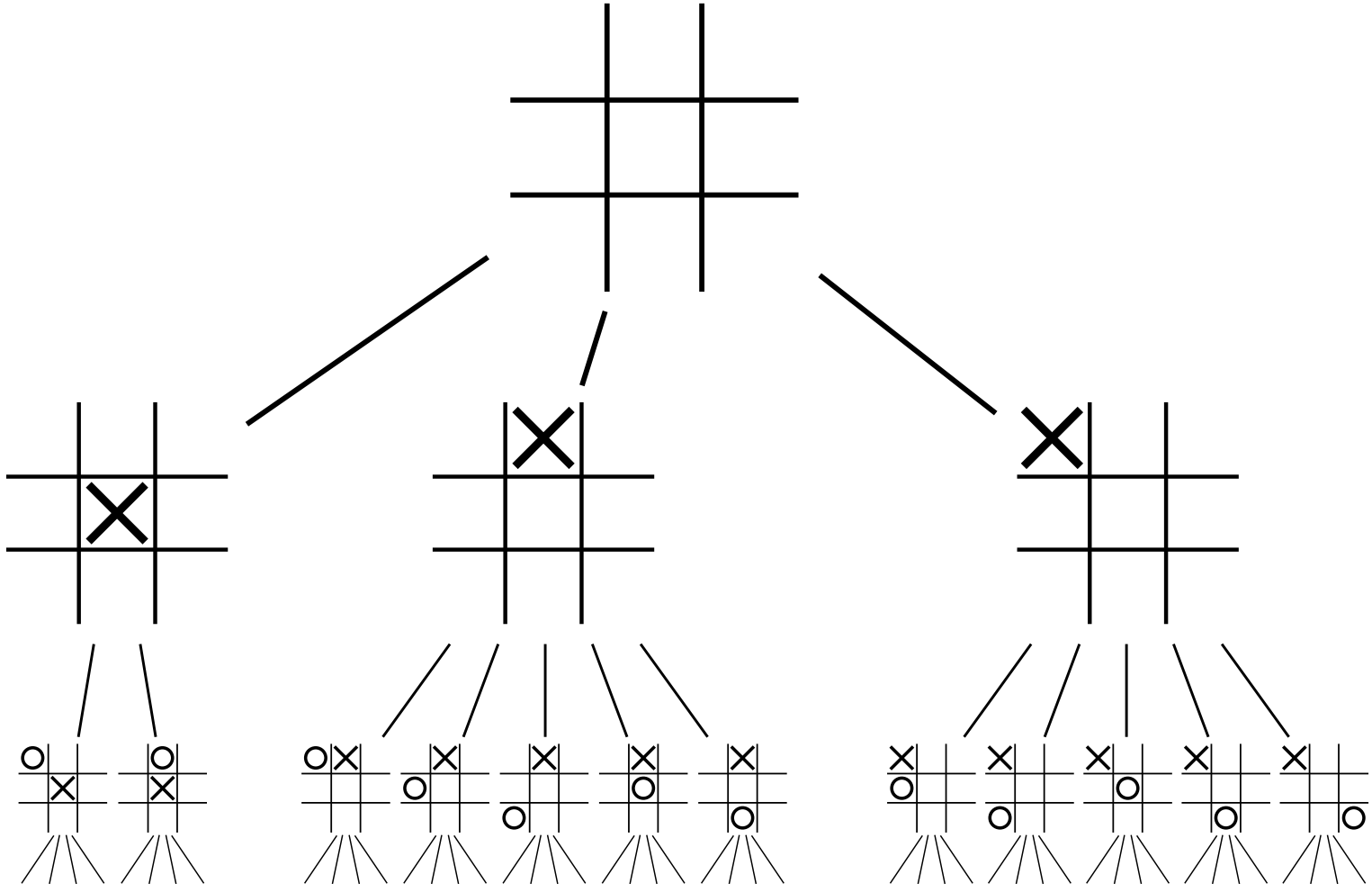
Modelling strategic situations:

- with conflict or cooperation
  - chess, football, pictionary, ...
- decisions based on personal goals
  - beating the opponent, maximizing points, ...
- influenced by the choice patterns of other players
  - if someone plays predictably, you can use it against them
- might involve randomness
  - dice rolls, card draws, ...

Goal: compute choices, optimal strategies, expected gains

Decision trees

A useful tool to examine decisions and their consequences.



Utility vectors and zero-sum games

Utility vectors:

- define the "gains" of an end state
- one entry per player: $(2, 3, -2)$, $(0, 7)$, ...
- determine the choices of players
  - player 1 will choose $(\mathbf{3}, 5)$ over $(\mathbf{2}, 3)$
- but not always!
  - should player 2 choose $(0, \mathbf{6}, 5)$ or $(2, \mathbf{6}, 3)$?

Main focus: two-player zero-sum games:

- only two players: no complicated interactions
- zero-sum: our benefits are the opponent's losses
  - $(5, -5)$, $(-3, 3)$, $(0, 0)$, ... (or just 5, -3, 0)
- the value of a choice is always well-defined

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Subtraction game: rules

Subtraction game: decision tree

Subtraction game: winning states

Subtraction game: optimal play

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Nim game: rules

Nim game: two stacks

Nim game: key invariant

Nim game: losing situation

Nim game: winning situation

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Minimax principles

Minimax example

Minimax implementation

Minimax with DP

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Very large search spaces

Alpha-beta definition

Alpha-beta example

Alpha-beta implementation

Sources of figures

- https://commons.wikimedia.org/wiki/File:Tic-tac-toe-game-tree.svg