Sorting Algorithms & Convex Hull

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March 7, 2016

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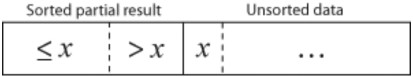
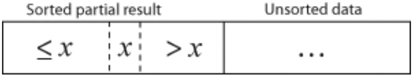
Tri par insertion

Idea

- A chaque itération, prendre une élement dans la partie non triée du tableau et le mettre au bon endroit dans la partie triée
- 

- deviens
- 

Tri par insertion

6	5	3	1	8	7	2	4

Tri par insertion

6	5	3	1	8	7	2	4

6	5	3	1	8	7	2	4

Tri par insertion

6	5	3	1	8	7	2	4

5	6	3	1	8	7	2	4

5	6	3	1	8	7	2	4

Tri par insertion

5	6	3	1	8	7	2	4

5	3	6	1	8	7	2	4

3	5	6	1	8	7	2	4

3	5	6	1	8	7	2	4

Tri par insertion

3	5	6	1	8	7	2	4

3	5	1	6	8	7	2	4

3	1	5	6	8	7	2	4

1	3	5	6	8	7	2	4

1	3	5	6	8	7	2	4

Tri par insertion

1	3	5	6	8	7	2	4

1	3	5	6	8	7	2	4

Tri par insertion

1	3	5	6	8	7	2	4

1	3	5	6	7	8	2	4

1	3	5	6	7	8	2	4

Tri par insertion

1	3	5	6	7	8	2	4

1	3	5	6	7	2	8	4

1	3	5	6	2	7	8	4

1	3	5	2	6	7	8	4

1	3	2	5	6	7	8	4

1	2	3	5	6	7	8	4

1	2	3	5	6	7	8	4

Tri par insertion

1	2	3	5	6	7	8	4

1	2	3	5	6	7	4	8

1	2	3	5	6	4	7	8

1	2	3	5	4	6	7	8

1	2	3	4	5	6	7	8

1	2	3	4	5	6	7	8

Tri par insertion

Complexité

Moyenne	Meilleure	Pire	Mémoire
$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$

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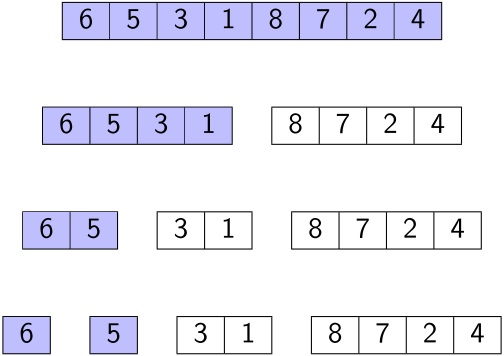
Merge Sort

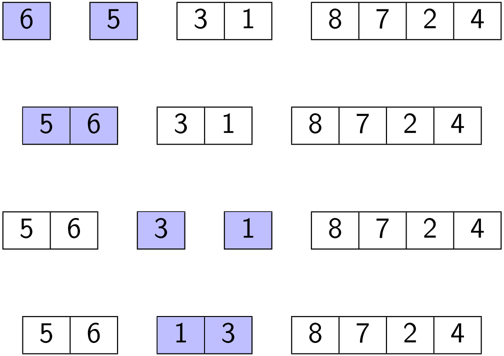
Idée

- Division récursive du tableau en "moitiés", jusqu'à atteindre des tableaux de taille 1
- Fusion des sous-tableaux pour produire des sous-tableau plus grand et triés

Merge Sort



Merge Sort



Merge Sort

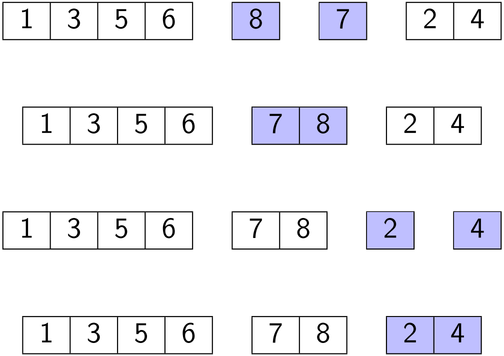
5	6		1	3		8	7	2	4

1	3	5	6		8	7	2	4

1	3	5	6		8	7		2	4

1	3	5	6		8		7		2	4

Merge Sort



Merge Sort

1	3	5	6

7	8

2	4

1	3	5	6

2	4	7	8

1	2	3	4	5	6	7	8

Moyenne	Meilleure	Pire	Mémoire
O(n log(n))	O(n log(n))	O(n log(n))	O(n)

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Quicksort

Idée

- Choisir un élement dans le tableau (le pivot)
- Partionnement: mettre les élements du tableau plus grand que le pivot à droite de celui-ci (et donc les éléments plus petits à gauche)
- Appliquer récursivement à gauche et à droite

Quicksort

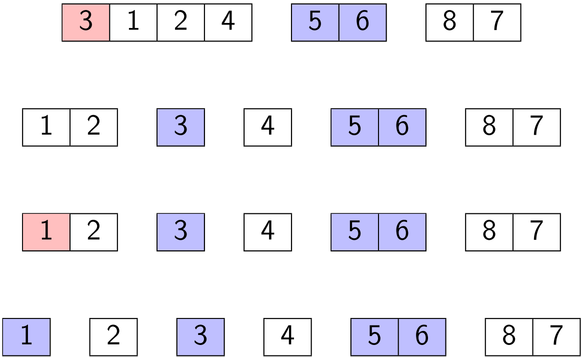
6	5	3	1	8	7	2	4

5	3	1	2	4		6		8	7

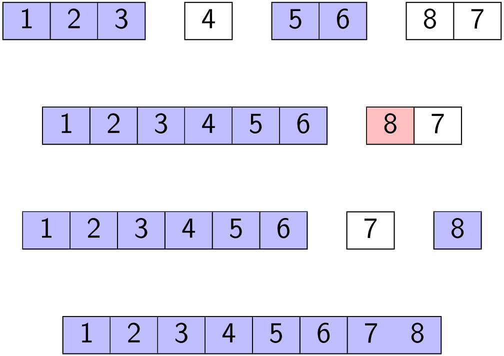
5	3	1	2	4		6		8	7

3	1	2	4		5	6		8	7

Quicksort



Quicksort



Quicksort

Complexité

Moyenne	Meilleure	Pire	Mémoire
$O(n \log(n))$	$O(n \log(n))$	$O(n^2)$	$O(1)$

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Tas (Heap)

Le tas est une structure de donnée qui a deux opérations:

- `push`: ajouter un élément au tas. $O(\log n)$.
- `pop`: retire l'élément le plus grand. $O(\log n)$.

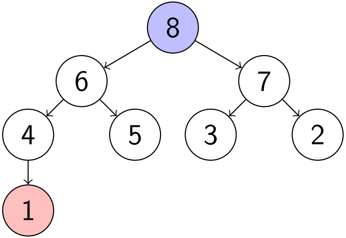
Tri par tas (Heap sort)

Idée

- Mettre tout les élements dans un tas
- Retirer les élements du tas, un à un

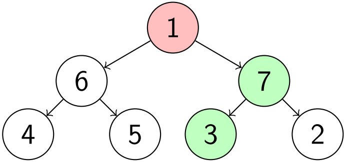
Heapsort

8	6	7	4	5	3	2	1

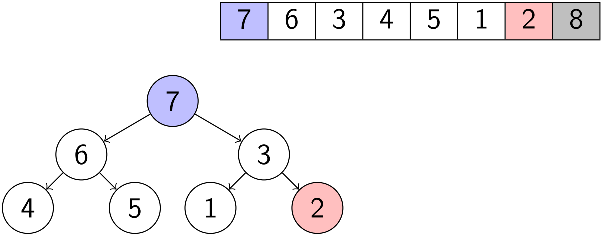


Heapsort

1	6	7	4	5	3	2	8

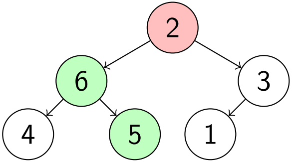


Heapsort



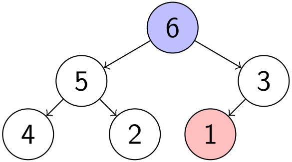
Heapsort

2	6	3	4	5	1	7	8



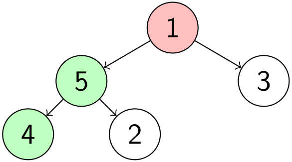
Heapsort

6	5	3	4	2	1	7	8



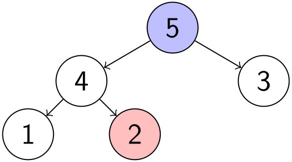
Heapsort

1	5	3	4	2	6	7	8



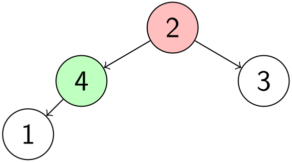
Heapsort

5	4	3	1	2	6	7	8



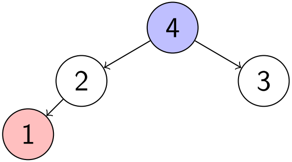
Heapsort

2	4	3	1	5	6	7	8

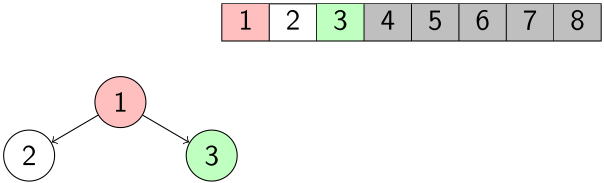


Heapsort

4	2	3	1	5	6	7	8

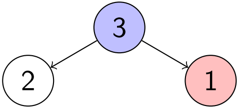


Heapsort



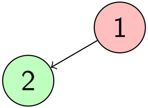
Heapsort

3	2	1	4	5	6	7	8



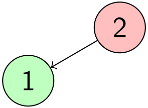
Heapsort

1	2	3	4	5	6	7	8



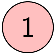
Heapsort

2	1	3	4	5	6	7	8



Heapsort

1	2	3	4	5	6	7	8



Heapsort

1	2	3	4	5	6	7	8

Heapsort

Complexité

Moyenne	Meilleure	Pire	Mémoire
O(n log(n))	O(n log(n))	O(n log(n))	O(1)

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Convex Hull

Given a set of points in the plane, compute the smallest convex polygon in the plane that contains all the points.

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Graham Scan

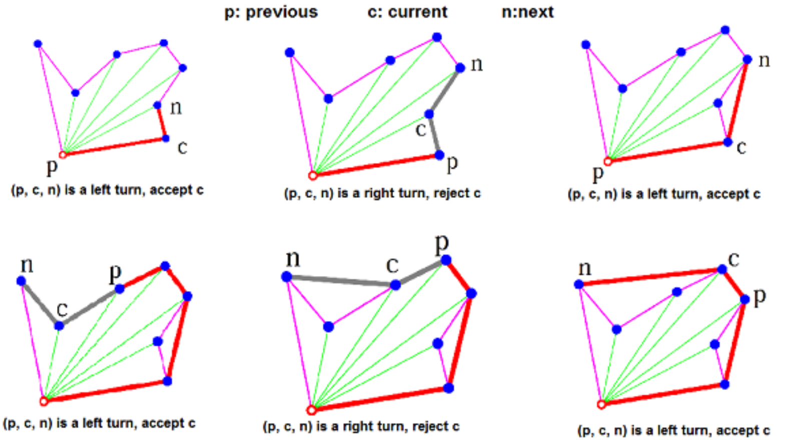
Method of computing the convex hull of a finite set of points in the plane with time complexity O(n log(n)). The algorithm finds all vertices of the convex hull ordered along its boundary.

Graham Scan

Idea

- find the point with the lowest y-coordinate. if there are multiple points with the lowest y-coordinate, the pick that one of them with the lowest x-coordinate. call this point P
- now sort all the points in increasing order of the angle they and point P make with the x-axis
- consider each point in the sorted array in sequence. For each point determine whether coming from the 2 previous points it makes a left of a right turn.
- if it makes a left turn, proceed with the next point
- if it makes a right turn, the second-to-last point is not part of the convex hull and should be removed from the convex hull, continue this removing for as long as the last 3 points make up a right turn

Graham Scan



In the above algorithm and below code, a stack of points is used to store convex hull points. With reference to the code, p is next-to-top in stack, c is top of stack and n is points[i].

Graham Scan

Direction of the turn

To determine whether 3 points constitute a left or a right turn we do not have to compute the actual angles but we can use a cross product.

Consider the 3 points (x_1, y_1), (x_2, y_2) and (x_3, y_3), which we will call P_1, P_2 and P_3.

Now compute the z-component of the cross product of the vectors P_1P_2 and P_1P_3. Which is given by the expression

$(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)$.

If the result is 0, the points are collinear. If the result is positive, the 3 points constitute a left turn. If the result is negative, the 3 points constitute a right turn.