Single source Shortest path

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March 13, 2016

Table of Contents

BFS

Breath First Search

- Unweighted graphs

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- Works with a queue (*First in - First out*)

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- $O(|V| + |E|)$

Table of Contents

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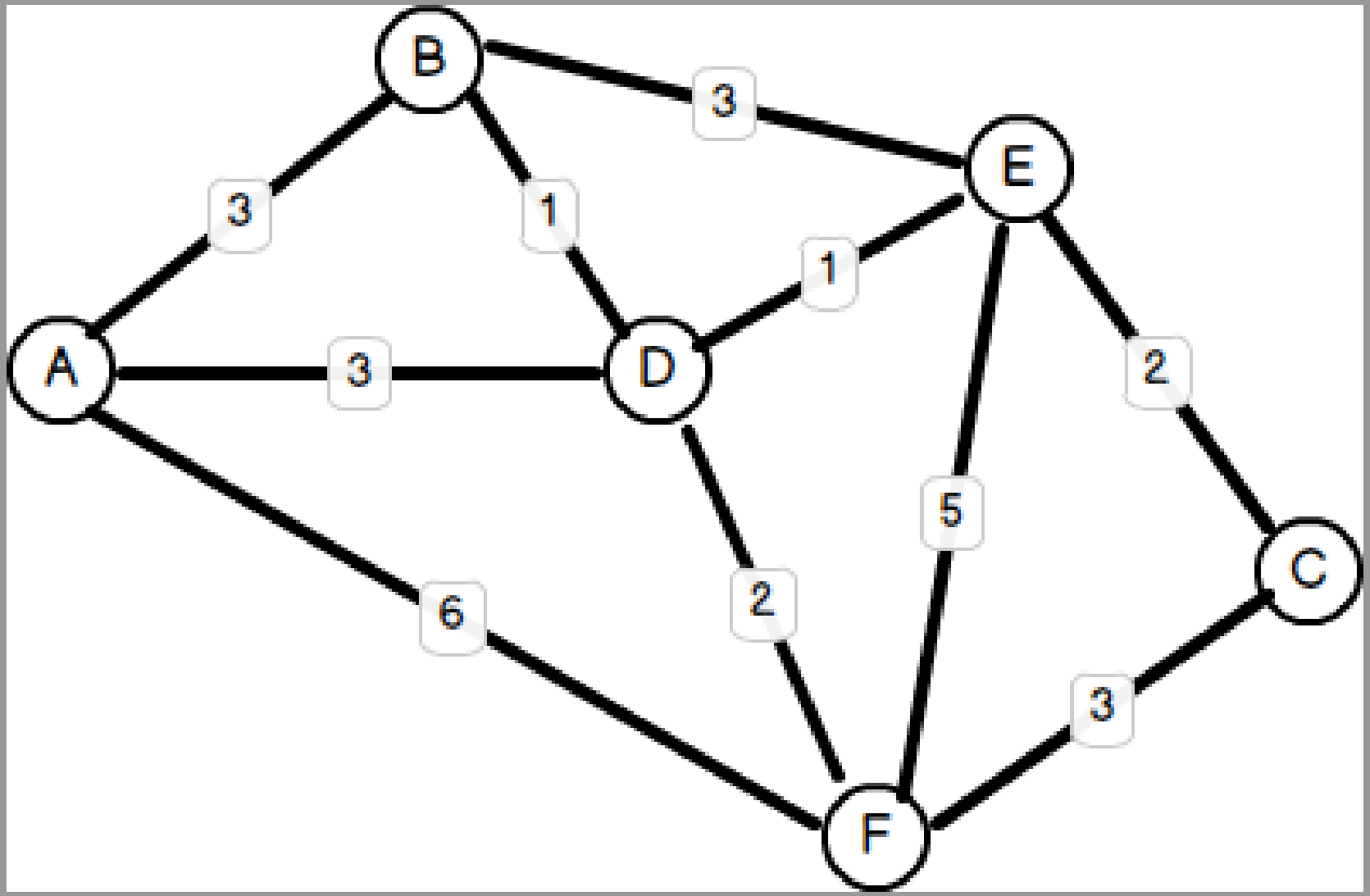
- Only add to the heap if shorter than current shortest = a *relax* operation
- Don't process again (cf. BFS)

Dijkstra

Example



Dijkstra

Extra remarks

- Degrades to BFS on unweighted graphs

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- Why do negative weights fail?

Table of Contents

Bellman-Ford

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Bellman-Ford

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- $|V| - 1$ times:
- Relax every edge
  $dist[v] = min(dist[v], dist[u] + weight[u, v])$

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- $O(|V||E|)$

Bellman-Ford

Negative weight cycle detection

- Run the normal Bellman-Ford algorithm

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- Do one more iteration

Bellman-Ford

Negative weight cycle detection

- Run the normal Bellman-Ford algorithm
- Do one more iteration
- If the iteration finds a shorter path $\Rightarrow$ a negative weight cycle was found

Table of Contents

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- For all these algorithms

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- Keep mapping *parent*

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Path reconstruction

Breath First Search

- For all these algorithms
- Keep mapping *parent*
- When expanding/relaxing to node v:
- $parent[v] =$ the node from where you came
- Goal found $\Rightarrow$ move up the *parent chain* until the starting node