Recursive backtracking

Robin Jadoul

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Try all the answers

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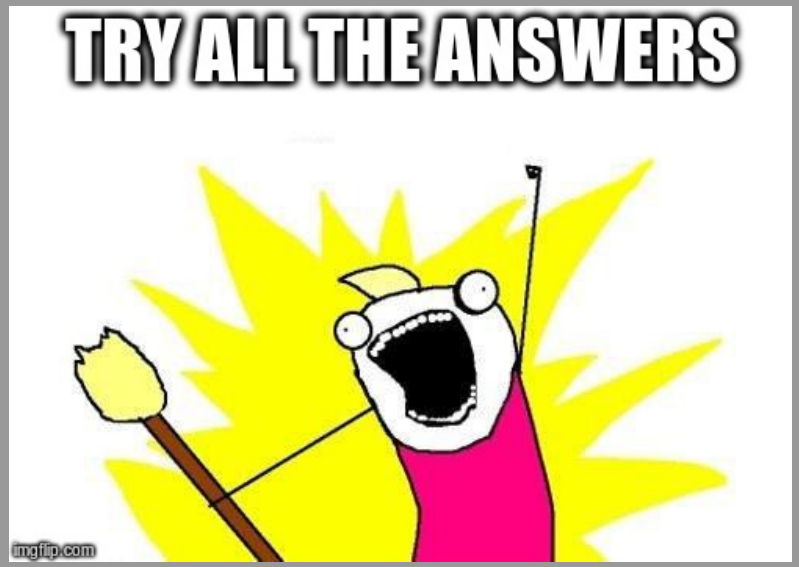

Complete Search

Approaches

- Generating

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  - Usually iteratively

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- Generating
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  - (Sometimes heavily nested) loops
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  - Usually recursively
  - Recursive function(s)
  - Can *prune* along the way (see later)

Complete Search

Easy iterative example

```
// count the number of pairs (a, b) (natural numbers)
// such that a + b == c
unsigned countSumPairs(unsigned c) {
    unsigned num = 0;
    for (unsigned a = 0; a <= c; a++) {
        for (unsigned b = 0; b <= c; b++) {
            if (a + b == c)
                ++num;
        }
    }
    return num;
}
```

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(Simplified) sudoku solving

- Running example: Solving a sudoku puzzle

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(Simplified) sudoku solving

- Running example: Solving a sudoku puzzle
- Simplified: without 3×3 box constraint
- Note: there are other ways to solve these (constraint satisfaction)
- Representation: a 9×9 grid of integers (0 is an unfilled cell)

Checking a filled sudoku

```
typedef int Grid[9][9];
bool correct(Grid& grid) {
    //Simple, but can be faster
    //Does not consider 3x3 boxes
    for (int i = 0; i < 9; i++)
        for (int j = 0; j < 9; j++)
            for (int k = j + 1; k < 9; k++)
                if (grid[i][j] <= 0 || grid[i][j] > 9
                        || grid[i][k] <= 0 || grid[i][k] > 9
                        || grid[j][i] <= 0 || grid[j][i] > 9
                        || grid[k][i] <= 0 || grid[k][i] > 9
                        || grid[i][j] == grid[i][k]
                        || grid[j][i] == grid[k][i])
                    return false;
    return true;
}
```

Recursive Complete Search

Filling the sudoku

```
bool solve_slow(Grid& grid, int i=0, int j=0) {
    if (i >= 9)
        return correct(grid);

    int nextj = (j + 1) % 9;
    int nexti = i + ((j + 1) / 9);

    if (grid[i][j] == 0) { //Not yet filled
        for (int val = 1; val <= 9; val++) {
            grid[i][j] = val;
            if (solve_slow(grid, nexti, nextj))
                return true;
        }
        grid[i][j] = 0;
        return false;
    } else {
        return solve_slow(grid, nexti, nextj);
    }
}
```

Recursive Complete Search

Filling the sudoku

Some (really big) problems with this version

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- We can still improve the correct (but this will not be necessary, as we will see)
- We keep going, even if we can see that the solution already fails!
- Even this grid is generated:

1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1
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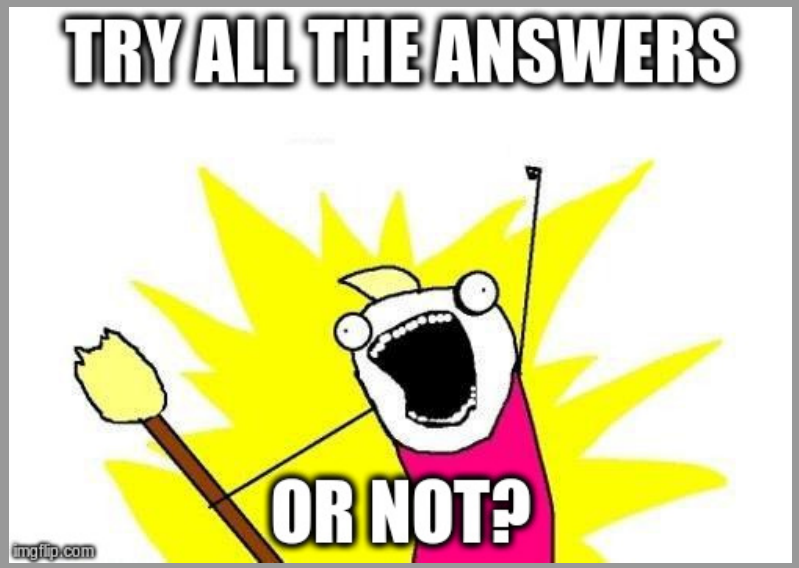

Pruning

Sudoku: check a single cell

```
bool good(Grid& grid, int r, int c) {
    for (int i = 0; i < 9; i++) {
        if (i != r && grid[i][c] == grid[r][c])
            return false;
        if (i != c && grid[r][i] == grid[r][c])
            return false;
    }
    return true;
}
```

Pruning

Sudoku: solve it

```
bool solve(Grid& grid, int i=0, int j=0) {
    if (i >= 9)
        //Check happens before going to the next cell
        //So we have found a solution
        return true;

    int nextj = (j + 1) % 9;
    int nexti = i + ((j + 1) / 9);

    if (grid[i][j] == 0) { //Not yet filled
        for (int val = 1; val <= 9; val++) {
            grid[i][j] = val;
            //Short circuiting to the rescue
            if (good(grid, i, j) && solve(grid, nexti, nextj))
                return true;
        }
        grid[i][j] = 0;
        return false;
    } else {
        return solve(grid, nexti, nextj);
    }
}
```

Pruning

Sudoku: wrap up

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Pruning

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- Just with pruning this time

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Sudoku: wrap up

- Very similar
- Just with pruning this time
- Filling an empty maze: instant ($0.002s$)

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- Better: `bitset<N>`, a statically sized boolean collection
- If the size if small enough ($size <= 32$ or $size <= 64$), store it in an integer

bitmasks

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Using ints

- Needed operations: `<<`, `&`, `|`
- set at index *i*: `bitmask |= 1 << i`
- test at index *i*: `bitmask & (1 << i)`
- less than 32 bools: use `unsigned int`
- less than 64 bools: use `unsigned long long`