# String processing

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30 januari 2016

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# Ad hoc

- Straightforward solution

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- See CP3, section 6.3 (pages 236 - 240)

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- If you know regular expressions, C++ 11 has those as well

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# Trie

## Properties

- <Re*trie*val (but can be pronounce as either *tree* or *try*)

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# Trie

## Properties

- <Re*trie*val (but can be pronounce as either *tree* or *try*)
- Store a set of words (with or without associated values)
- insert/retrieve in $O(S)$, with S the length of the string
- Allows for non-exact matches (<> set/map)

# Trie

## Structure

- Tree structure

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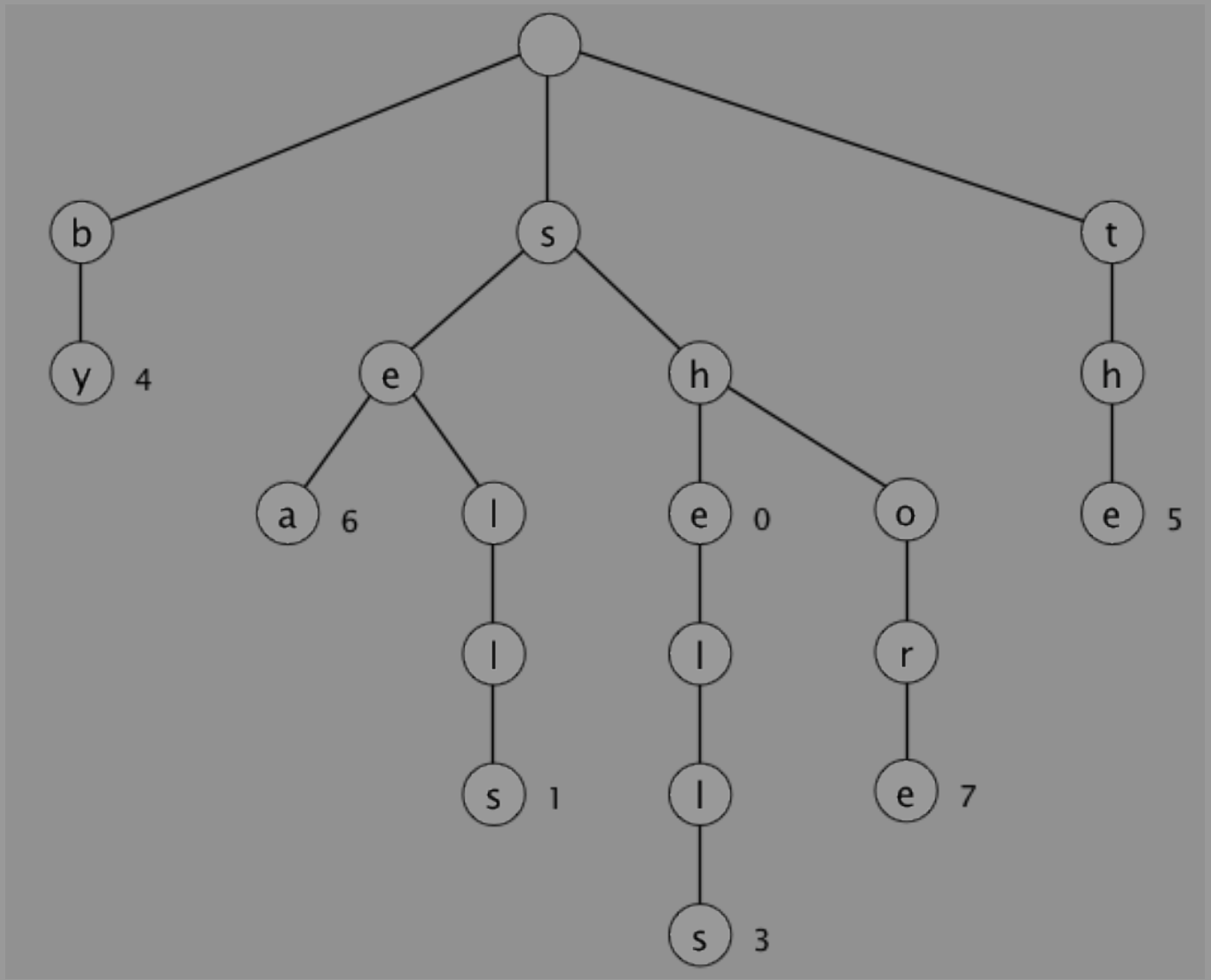
- Tree structure
- Stores the *path* for the string instead of the string
- Edges labeled with single characters
- If the last character of a stored word, marked (+ associated value)
- Can vary in type of *character* (bits/ints/...)
- Can be compressed by eliminating successive single-edge nodes

# Trie

## Structure



# Trie

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- Spelling suggestions

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- Spelling suggestions
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- Bioinformatics (DNA/RNA)
- Alphabetical ordering / sorting
- (Similar to structure for Aho-Corasick)
- (Basis for *suffix tree*)

# Trie

## Code

```cpp
#include <map>
using namespace std;

struct Trie
{
    //Can be map/unordered_map/direct adressing table/implicit edge/...
    map<char, Trie*> children;
    bool marked;
};
```

# Trie

## Code

```
void insert(Trie* t, string s)
{
    for (int index = 0; index < s.length(); index++)
    {
        if (t->children.find(s[index]) == t->children.end())
        {
            t->children[s[index]] = new Trie();
        }
        t = t->children[s[index]];
    }
    t->marked = true;
}
```

# Trie

## Code

```
bool contains(Trie* t, string s)
{
    for (auto c : s)
    {
        if (t->children.find(c) == t->children.end())
            return false;
        t = t->children[c];
    }
    return t->marked;
}
```

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## principle

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- Check for match at each index
- Usually, use the one in the standard library, don't write your own
- $O(s * p)$ (s = length of string, p = length of pattern)

# Naive matching

## Code

```
#include <string>
using namespace std;

int match(string s, string pat)
{
    if (s.length() < pat.length())
        return -1;
    for (int i = 0; i <= s.length() - pat.length(); i++)
    {
        bool found = true;
        for (int j = 0; j < pat.length(); j++)
        {
            if (s[i+j] != pat[j])
            {
                found = false;
                break;
            }
        }
        if (found) return i;
    }
    return -1;
}
```

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# Rabin-Karp

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- Checking for a match with the pattern: $O(n)$

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## Idea

- Checking for a match with the pattern: $O(n)$
- Faster possible?
- What about hashes, integer comparison $= O(n)$
- We still need a $O(1)$ way to generate the hashes.
- Useful for multiple same-length patterns (check all hashes)

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## Polynomial hashing

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- $s_i, s_{i+1}, \ldots, s_{i+k-1} = s_i \times B^{k-1} + s_{i+1} \times B^{k-2} + \ldots + s_{i+k-1} \times B^0$

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- Too big $\Rightarrow$ modulo $H$ (usually prime)
- Watch out for false positives

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- $s_{i+1}, s_{i+2}, \ldots, s_{i+k} = ((s_i, \ldots, s_{i+k-1}) - s_i \times B^{k-1}) \times B) + s_{i+k}$

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- A rolling hash *frame*
- $O(1)$

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- $\Rightarrow$ triple hashing, ...

# Rabin-Karp

code

```
const int B = 17;
const int H = 12632251;

int hash_pattern(string pat, int start, int end)
{
    int h = 0;
    for (int i = start; i <= end; i++)
    {
        h = ((h * B) % H + pat[i]) % H;
    }
    return h;
}
```

# Rabin-Karp

code

```
bool check(string s, string pat, int start)
{
    for (int i = 0; i < pat.length(); i++)
    {
        if (s[start + i] != pat[i])
            return false;
    }
    return true;
}

int modpow(int exp) { //This can be done in O(log N)
  int result = 1;
  for (int i = 0; i < exp; i++)
  {
      result = (result * B) % H;
  }
  return result;
}
```

# Rabin-Karp

code

```
int match(string s, string pat)
{
    if (pat.length() > s.length()) return -1;
    int k = pat.length();
    int Hp = hash_pattern(pat, 0, k - 1);
    int Hs = hash_pattern(s, 0, k - 1);
    int Bk = modpow(k-1);
    for (int i = 0; i <= s.length() - k; i++)
    {
        if (Hs == Hp && check(s, pat, i))
        {
            return i;
        }
        Hs = ((B * (Hs - (s[i] * Bk) % H)) % H + s[i+k]) % H;
        if (Hs < 0) Hs += H;
    }
    return -1;
}
```

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# Z-algorithm

## terminology

- Z-box = substring that matches with a prefix from the string

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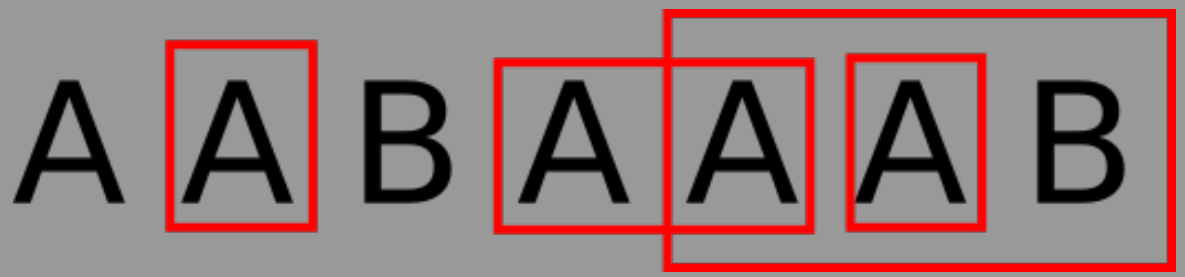

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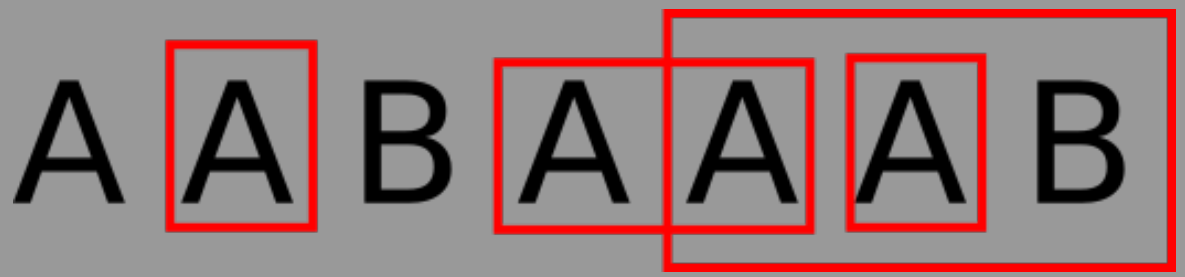


| letter | A | A | B | A | A | A | B |
|---|---|---|---|---|---|---|---|
| Z-score | 7 | 1 | 0 | 2 | 3 | 1 | 0 |

# Z-algorithm

## Matching

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## Matching

- $P =$ pattern
- $S =$ search string
- \$ = sentinel (not part of alphabet)
- return $i$ for each $i > 0$ where $Z_i(P\$S) = |P|$

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- if current character in $[l, r]$: look at corresponding character in prefix (computed previously)
- expand if grows beyond $r$, update $[l, r]$
- else: calculate explicitely, update $[l, r]$
- (Nicely illustrated: https://www.cs.umd.edu/class/fall2011/cmsc858s/Lec02-zalg.pdf)

# Z-algorithm

code

```
int match(string& s, string& pat)
{
    string S = pat + "$" + s;
    vector<int> Z(S.length());
    int l = -1, r = -1;

    for (int i = 1; i < S.length(); i++)
        if (i > r) //Outside furthest Z-box
        {
            int len = 0;
            for (int j = i; j < S.length() && S[j] == S[j-i]; j++)
            {
                len++;
            }
            Z[i] = len;
            if (len > 0)
            {
                l = i;
                r = i + len - 1;
            }
        }
        else
```

# Z-algorithm

code

```
        {
            int inside = r - i + 1;
            int corresponding = i - l;
            if (Z[corresponding] < inside)
            {
                Z[i] = Z[corresponding];
            }
            else //Need to grow beyond r
            {
                int len = 0;
                for (int j = r + 1; j < S.length() && S[j] == S[j - i]; j++)
                {
                    len++;
                }
                Z[i] = inside + len;
                if (i + len - 1 > r)  //Only update if better
                {
                    l = i;
                    r = i + len - 1;
                }
            }
        }
    for (int i = 1; i < S.length(); i++)
        if (Z[i] == pat.length())
            return i - pat.length() - 1; //Don't forget to subtract the sentinel
    return -1;
}
```

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# Knuth-Morris-Pratt

## Idea

- Don't restart a match every time

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- Re-use previous (partial) match information

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## Idea

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- Fail smart
- Re-use previous (partial) match information
- Precompute possible *sub*matches

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- Next possible partially matched pattern = longest proper suffix (of the partial match) that is a prefix

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- Precompute and keep the length of this suffix/prefix in an array (call this $L$)

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## Idea

- How to choose the next possible match?
- Next possible partially matched pattern = longest proper suffix (of the partial match) that is a prefix
- (What is this in terms of Z-boxes?)
- Precompute and keep the length of this suffix/prefix in an array (call this $L$)
- $L[i]$ = length of that prefix for $S[0..i-1]$ (inclusive)

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## Precomputation

- $L[0] = -1$ (could be 0, but this eliminates a few checks)

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- $L[i] = j + 1$ ($j$ is the length of the *parent*'s match)

# Knuth-Morris-Pratt

## Precomputation

- $L[0] = -1$ (could be 0, but this eliminates a few checks)
- $L[1] = 0$ (it should be a *proper* suffix)
- Search for the next *parent* in $L$ that can be expanded with the current character
- $L[i] = j + 1$ ($j$ is the length of the *parent*'s match)
- If none can be found: $L[i] = 0$

# Knuth-Morris-Pratt

## Matching

- Precompute the suffix lengths ($L$) of the pattern

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- Very similar to the actual precomputation
- $O(S + P)$

# Knuth-Morris-Pratt

code

```
vector<int> precompute(string pat)
{
    vector<int> L(pat.length() + 1);
    L[0] = -1; L[1] = 0;
    for (int i = 2; i <= pat.length(); i++)
    {
        int j = L[i-1];
        while (j >= 0 && pat[j] != pat[i-1])
            j = L[j];
        L[i] = j + 1;
    }
    return L;
}
```

# Knuth-Morris-Pratt

code

```
int match(string s, string pat)
{
    vector<int> L = precompute(pat);
    int j = 0;  //The current index for L
    for (int i = 0; i < s.length(); i++)
    {
        while (j >= 0 && s[i] != pat[j])
            j = L[j];
        j++;
        if (j == pat.length())
            //Found a match, reconstruct the beginning of the substring
            return i + 1 - j;
    }
    return -1;
}
```

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# Exercises

Ad hoc

- UVa 10115 (*Automatic Editing*)
- UVa 10361 (*Automatic Poetry*)
- UVa 10082 (*WERTYU*)
- UVa 1368 (*DNA Consensus String*)

# Exercises

Tries

- UVa 902 (*Password Search*)
- UVa 755 (*487–3279*)
- Codechef Remember the recipe (https://www.codechef.com/problems/TWSTR/)

# Exercises

String matching

- UVa 363 (*Approximate matches*)
- UVa 455 (*Periodic strings*)
- UVa 1223 (*Editor*)
- Codeforces 126, problem B (http://codeforces.com/contest/126/problem/B)
- UVa 11151 (*Longest Palindrome*)
- UVa 11475 (*Extend to Palindrome*)