All pairs Shortest path

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Floyd-Warshall

All pairs Shortest path

- Adjacency matrix
- For every node k:
-   For every pair of nodes (u, v):
-   Can the distance between these pairs be improved by going to k first?
-   $g[u][v] = min(g[u][v], g[u][k] + g[k][v])$
- Very easy implementation (4 lines)
- $O(|V|^3) \Rightarrow$ feasible for up to approx. 400 nodes

Floyd-Warshall
All pairs Shortest path

- ▸ Reconstructing the path:
- ▸ Use another $|V| \times |V|$ matrix *parent*
- ▸ *parent*$[u][v] = $ The last node before v on the shortest path from u to v
- ▸ Update when improving a path (*parent*$[u][v] = $ *parent*$[k][v]$)
- ▸ Traverse the *parent chain* upwards

Floyd-Warshall

Alternative

- ► Repeated execution of Dijkstra's algorithm
- ► $O(|V| \times |E| \times \log |V|)$
- ► Better on sparse graphs ($|E| << |V|^2$)

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Applications

- Compute the transitive closure
  $close[u][v] = close[u][v] | (close[u][k] \& close[k][v])$

- Minimax of the graph: The path with minimal highest cost along the path
  $g[u][v] = min(g[u][v], max(g[u][k], g[k][v]))$

- Maximin of the graph: The path with maximal lowest cost along the path
  $g[u][v] = max(g[u][v], min(g[u][k], g[k][v]))$

- Safest path: the path with highest survival probability (weights are probs along the edge)
  $g[u][v] = max(g[u][v], g[u][] * g[k][v])$

- Most dangerous path: the path with lowest survival probability (weights are probs along the edge)
  $g[u][v] = min(g[u][v], g[u][] * g[k][v])$

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Applications

- Detecting negative weight cycles
- Run the normal *Floyd-Warshall* algorithm
- If any element on the diagonal becomes negative $\Rightarrow$ negative weight cycle found