Sorting Algorithms & Convex Hull

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- Problem
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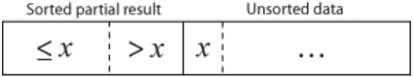
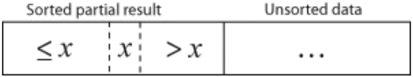
Insertion sort

Idee

- Bij elke iteratie een element nemen uit het niet gesorteerde gedeelte van de rij en het op de juiste plaats zetten in het gesorteerde gedeelte
- 

- deviens
- 

Insertion sort

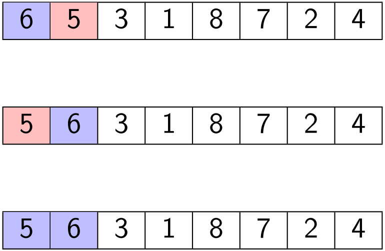
6	5	3	1	8	7	2	4

Insertion sort

6	5	3	1	8	7	2	4

6	5	3	1	8	7	2	4

Insertion sort

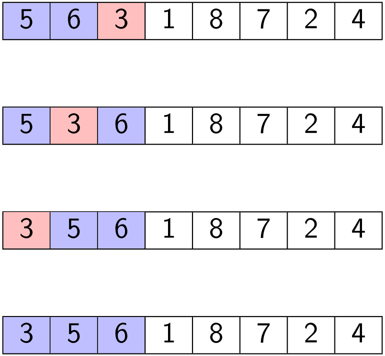


6	5	3	1	8	7	2	4

5	6	3	1	8	7	2	4

5	6	3	1	8	7	2	4

Insertion sort



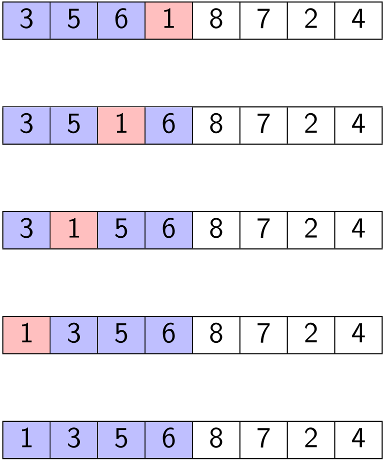
5	6	3	1	8	7	2	4

5	3	6	1	8	7	2	4

3	5	6	1	8	7	2	4

3	5	6	1	8	7	2	4

Insertion sort



3	5	6	1	8	7	2	4

3	5	1	6	8	7	2	4

3	1	5	6	8	7	2	4

1	3	5	6	8	7	2	4

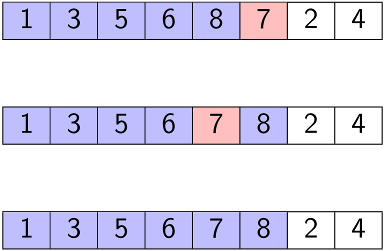
1	3	5	6	8	7	2	4

Insertion sort

1	3	5	6	8	7	2	4

1	3	5	6	8	7	2	4

Insertion sort

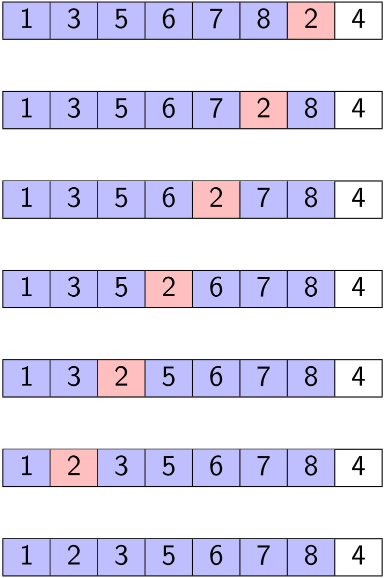


1	3	5	6	8	7	2	4

1	3	5	6	7	8	2	4

1	3	5	6	7	8	2	4

Insertion sort



1	3	5	6	7	8	2	4

1	3	5	6	7	2	8	4

1	3	5	6	2	7	8	4

1	3	5	2	6	7	8	4

1	3	2	5	6	7	8	4

1	2	3	5	6	7	8	4

1	2	3	5	6	7	8	4

Insertion sort

1	2	3	5	6	7	8	4

1	2	3	5	6	7	4	8

1	2	3	5	6	4	7	8

1	2	3	5	4	6	7	8

1	2	3	4	5	6	7	8

1	2	3	4	5	6	7	8

Insertion sort

Complexiteit

Gemiddeld	Best	Slechtst	Geheugen
$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$

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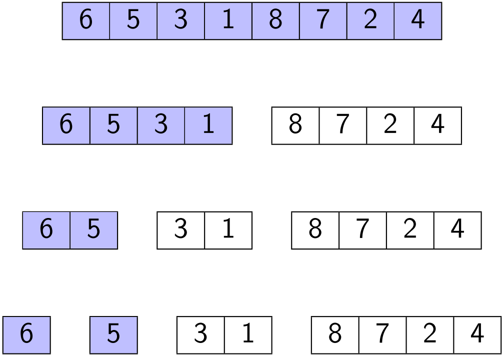
Merge Sort

Idee

- Recursief de rij opdelen in 2 delen tot een stuk lengte 1 heeft
- De twee gesorteerde delen samenvoegen tot 1 groot gesorteerd stuk

Merge Sort



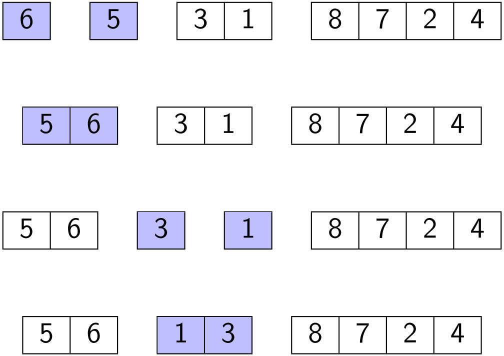
6 5 3 1 8 7 2 4

6 5 3 1   8 7 2 4

6 5   3 1   8 7 2 4

6   5   3 1   8 7 2 4

Merge Sort



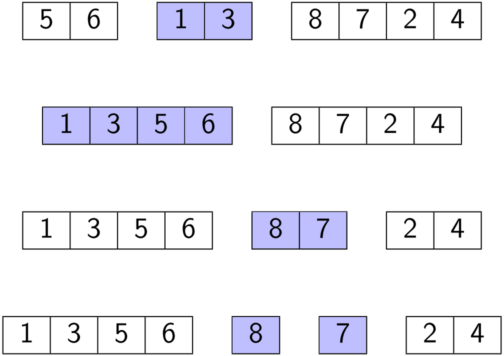
6 5 3 1 8 7 2 4

5 6 3 1 8 7 2 4

5 6 3 1 8 7 2 4

5 6 1 3 8 7 2 4

Merge Sort



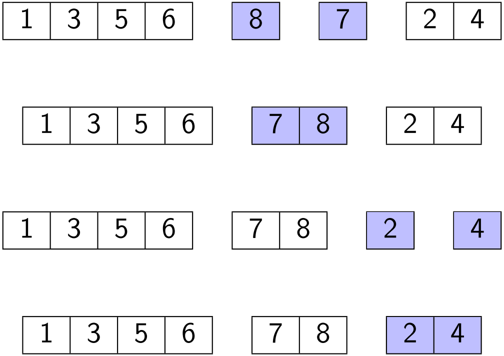
5 6 | 1 3 | 8 7 2 4

1 3 5 6 | 8 7 2 4

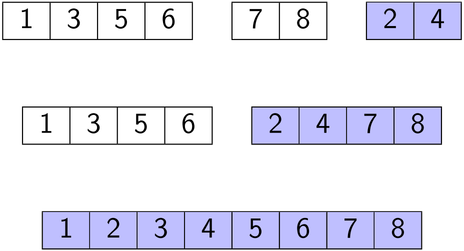
1 3 5 6 | 8 7 | 2 4

1 3 5 6 | 8 | 7 | 2 4

Merge Sort



Merge Sort



1 3 5 6   7 8   2 4

1 3 5 6   2 4 7 8

1 2 3 4 5 6 7 8

Merge Sort

Complexiteit

Gemiddeld	Best	Slechtst	Geheugen
O(n log(n))	O(n log(n))	O(n log(n))	O(n)

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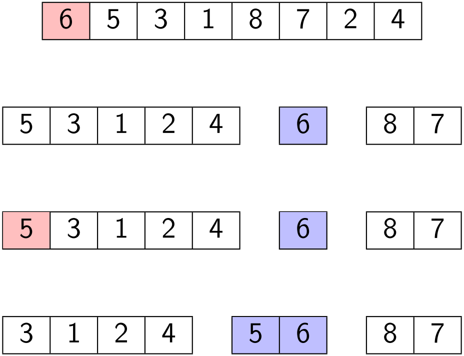
Quicksort

Idee

- Een element in de rij kiezen (de pivot)
- De elementen die kleiner of gelijk aan de pivot zijn links zetten, de andere rechts
- Recursief sorteren op het linker- en rechtergedeelte

Quicksort



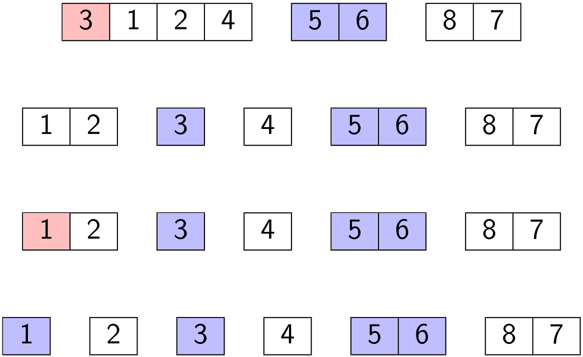
6 5 3 1 8 7 2 4

5 3 1 2 4   6   8 7

5 3 1 2 4   6   8 7

3 1 2 4   5 6   8 7

Quicksort



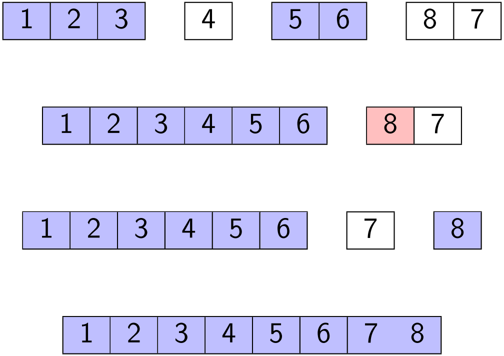
3 1 2 4　5 6　8 7

1 2　3　4　5 6　8 7

1 2　3　4　5 6　8 7

1　2　3　4　5 6　8 7

Quicksort



1 2 3 4 5 6 8 7

1 2 3 4 5 6 8 7

1 2 3 4 5 6 7 8

1 2 3 4 5 6 7 8

Quicksort

Complexiteit

Gemiddeld	Best	Slechtst	Geheugen
O(n log(n))	O(n log(n))	O(n^2)	O(1)

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Heap

Een heap is een datastructuur met 2 operaties:

- `push`: Een element toevoegen aan de heap. $O(\log n)$.
- `pop`: Het grootste element van de heap opvragen en verwijderen. $O(\log n)$.

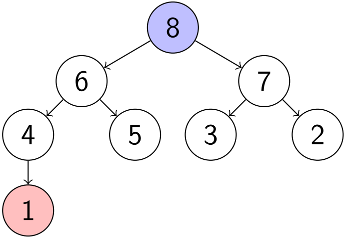
Heapsort

Idee

- Alle elementen op een heap plaatsen
- Alle elementen opvragen en van de heap verwijderen, één voor één

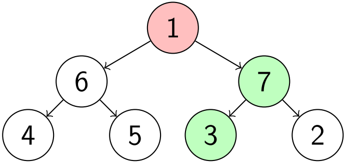
Heapsort

8	6	7	4	5	3	2	1



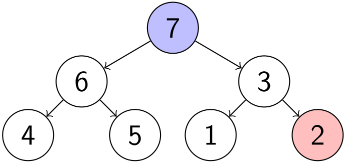
Heapsort

1	6	7	4	5	3	2	8



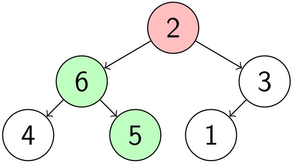
Heapsort

7	6	3	4	5	1	2	8



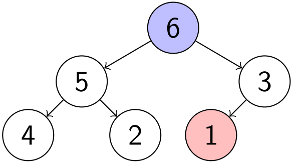
Heapsort

2	6	3	4	5	1	7	8



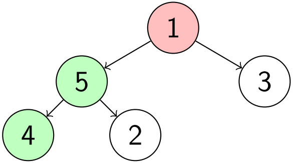
Heapsort

6	5	3	4	2	1	7	8



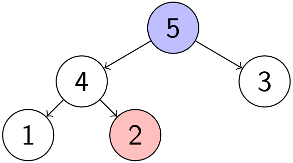
Heapsort

1	5	3	4	2	6	7	8



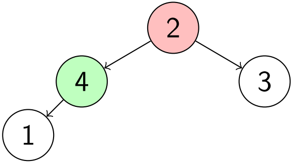
Heapsort

5	4	3	1	2	6	7	8



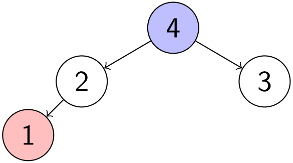
Heapsort

2	4	3	1	5	6	7	8



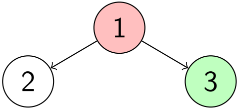
Heapsort

4	2	3	1	5	6	7	8



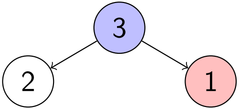
Heapsort

1	2	3	4	5	6	7	8



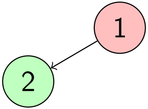
Heapsort

3	2	1	4	5	6	7	8



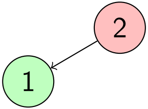
Heapsort

1	2	3	4	5	6	7	8



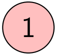
Heapsort

2	1	3	4	5	6	7	8



Heapsort

1	2	3	4	5	6	7	8



Heapsort

1	2	3	4	5	6	7	8

Heapsort

Complexiteit

Gemiddeld	Best	Slechtst	Geheugen
O(n log(n))	O(n log(n))	O(n log(n))	O(1)

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Convex Hull

Given a set of points in the plane, compute the smallest convex polygon in the plane that contains all the points.

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Graham Scan

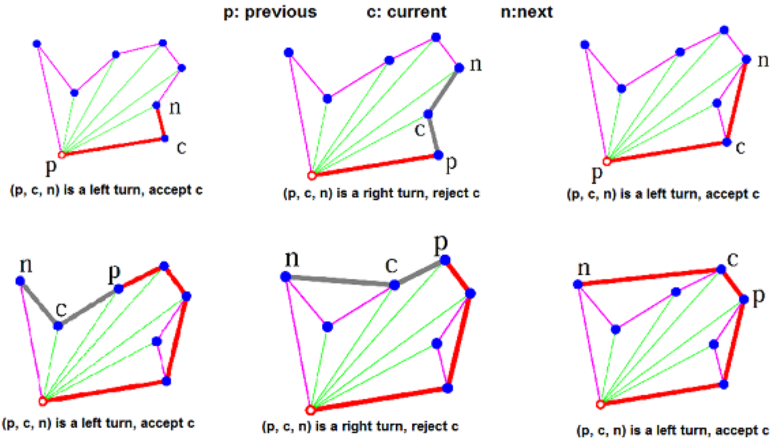
Method of computing the convex hull of a finite set of points in the plane with time complexity O(n log(n)). The algorithm finds all vertices of the convex hull ordered along its boundary.

Graham Scan

Idea

- find the point with the lowest y-coordinate. if there are multiple points with the lowest y-coordinate, the pick that one of them with the lowest x-coordinate. call this point P
- now sort all the points in increasing order of the angle they and point P make with the x-axis
- consider each point in the sorted array in sequence. For each point determine whether coming from the 2 previous points it makes a left of a right turn.
- if it makes a left turn, proceed with the next point
- if it makes a right turn, the second-to-last point is not part of the convex hull and should be removed from the convex hull, continue this removing for as long as the last 3 points make up a right turn

Graham Scan



p: previous c: current n:next

(p, c, n) is a left turn, accept c

(p, c, n) is a right turn, reject c

(p, c, n) is a left turn, accept c

(p, c, n) is a left turn, accept c

(p, c, n) is a right turn, reject c

(p, c, n) is a left turn, accept c

In the above algorithm and below code, a stack of points is used to store convex hull points. With reference to the code, p is next-to-top in stack, c is top of stack and n is points[i].

Graham Scan

Direction of the turn

To determine whether 3 points constitute a left or a right turn we do not have to compute the actual angles but we can use a cross product.
Consider the 3 points (x_1, y_1), (x_2, y_2) and (x_3, y_3), which we will call P_1, P_2 and P_3.
Now compute the z-component of the cross product of the vectors P_1P_2 and P_1P_3. Which is given by the expression $(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)$.
If the result is 0, the points are collinear. If the result is positive, the 3 points constitute a left turn. If the result is negative, the 3 points constitute a right turn.