Single source Shortest path

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Breath First Search

- Unweighted graphs

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- $O(|V| + |E|)$

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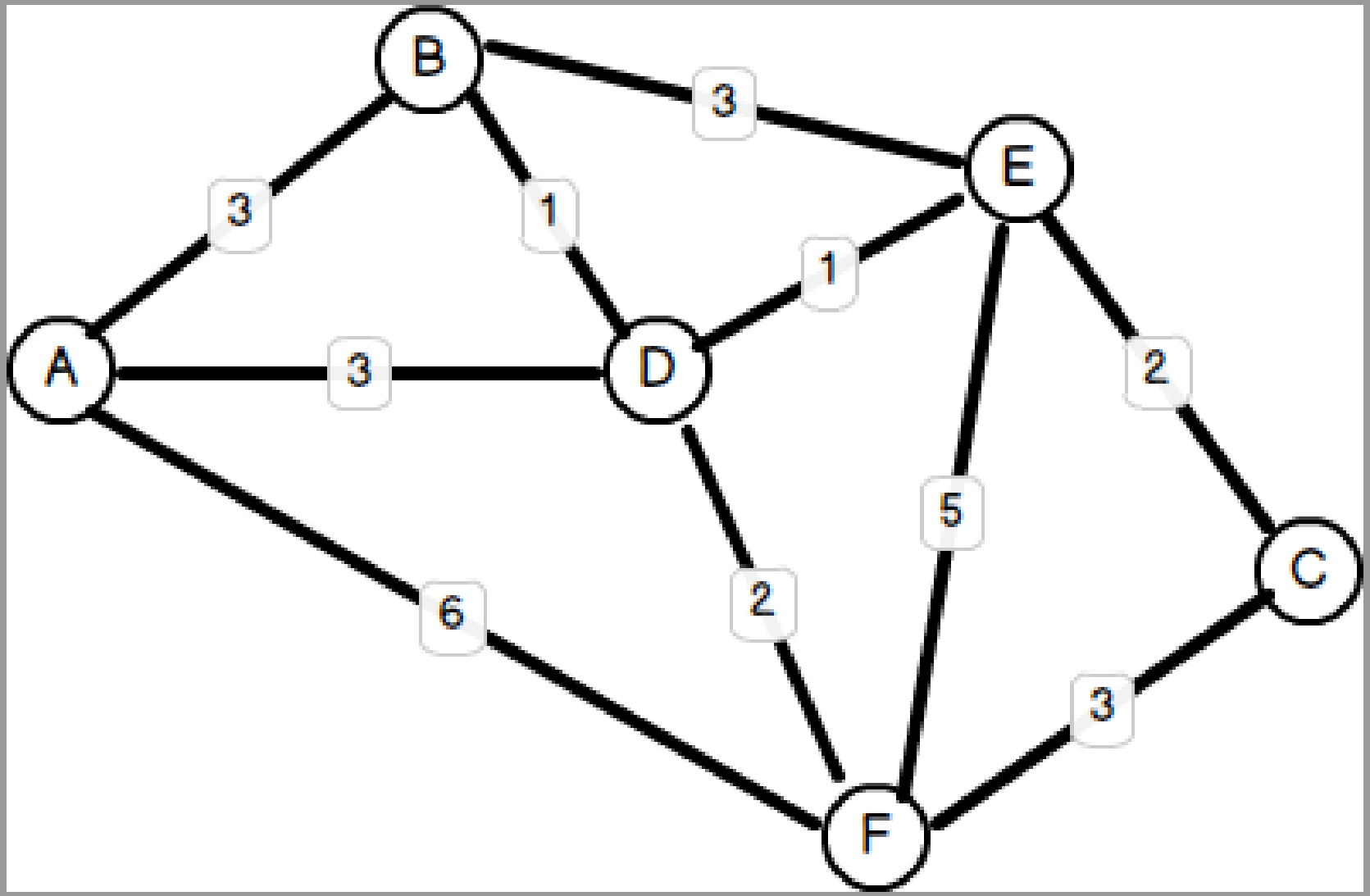
- Only add to the heap if shorter than current shortest = a *relax* operation
- Don't process again (cf. BFS)

Dijkstra

Example



Dijkstra

Extra remarks

- Degrades to BFS on unweighted graphs

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- Why do negative weights fail?

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  $dist[v] = min(dist[v], dist[u] + weight[u, v])$

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- $O(|V||E|)$

Bellman-Ford

Negative weight cycle detection

- Run the normal Bellman-Ford algorithm

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Bellman-Ford

Negative weight cycle detection

- Run the normal Bellman-Ford algorithm
- Do one more iteration
- If the iteration finds a shorter path $\Rightarrow$ a negative weight cycle was found