Sorting Algorithms & Convex Hull

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April 15, 2015

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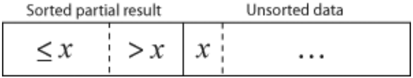
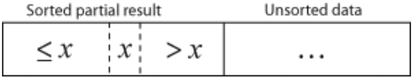
Insertion Sort

Idea

- each iteration, pick one element from the unsorted part of the data and put it in the right place in the sorted part of the data.
- 

- becomes
- 

Insertion Sort

6	5	3	1	8	7	2	4

Insertion Sort

6	5	3	1	8	7	2	4

6	5	3	1	8	7	2	4

Insertion Sort

6	5	3	1	8	7	2	4

5	6	3	1	8	7	2	4

5	6	3	1	8	7	2	4

Insertion Sort

5	6	3	1	8	7	2	4

5	3	6	1	8	7	2	4

3	5	6	1	8	7	2	4

3	5	6	1	8	7	2	4

Insertion Sort

3	5	6	1	8	7	2	4

3	5	1	6	8	7	2	4

3	1	5	6	8	7	2	4

1	3	5	6	8	7	2	4

1	3	5	6	8	7	2	4

Insertion Sort

1	3	5	6	8	7	2	4

1	3	5	6	8	7	2	4

Insertion Sort

1	3	5	6	8	7	2	4

1	3	5	6	7	8	2	4

1	3	5	6	7	8	2	4

Insertion Sort

1	3	5	6	7	8	2	4
1	3	5	6	7	2	8	4
1	3	5	6	2	7	8	4
1	3	5	2	6	7	8	4
1	3	2	5	6	7	8	4
1	2	3	5	6	7	8	4
1	2	3	5	6	7	8	4

Insertion Sort

1	2	3	5	6	7	8	4

1	2	3	5	6	7	4	8

1	2	3	5	6	4	7	8

1	2	3	5	4	6	7	8

1	2	3	4	5	6	7	8

1	2	3	4	5	6	7	8

Insertion Sort

Complexity

Average	Best	Worst	Memory
$O(n^2)$	$O(n)$	$O(n^2)$	$O(1)$

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Merge Sort

Idea

- recursively divide the unsorted list into sublists until all the sublists have length 1
- repeatedly merge sublists to produce new sorted sublists until there is only 1 list remaining
- this will be the sorted list

Merge Sort

6	5	3	1	8	7	2	4

6	5	3	1

8	7	2	4

6	5

3	1

8	7	2	4

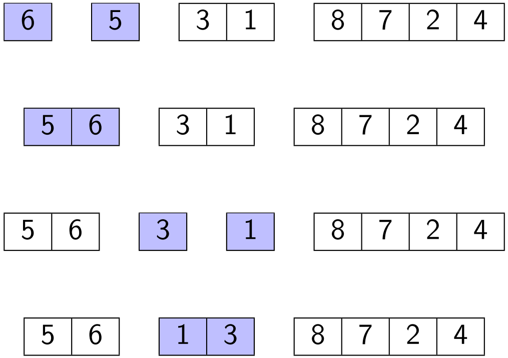
6

5

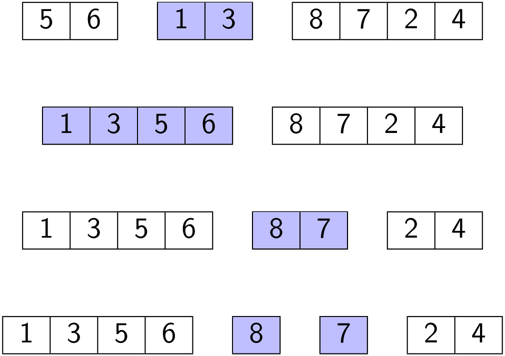
3	1

8	7	2	4

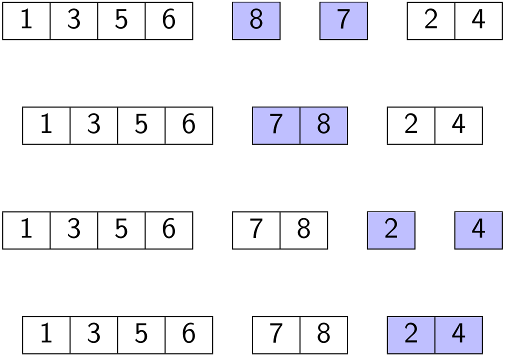
Merge Sort



Merge Sort



Merge Sort



Merge Sort

1	3	5	6		7	8		2	4

1	3	5	6		2	4	7	8

1	2	3	4	5	6	7	8

Merge Sort

Complexity

Average	Best	Worst	Memory
O(n log(n))	O(n log(n))	O(n log(n))	O(n)

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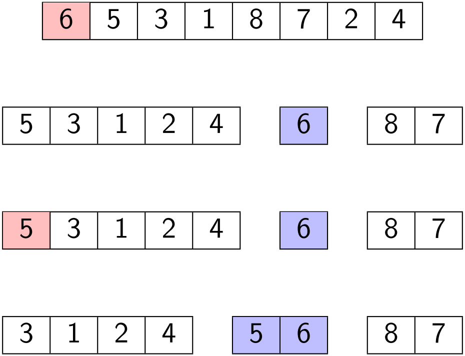
Quicksort

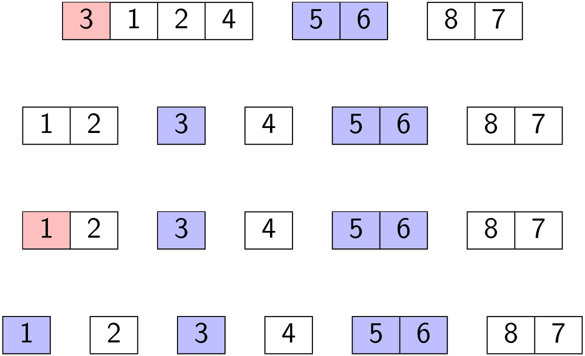
Idea

- pick an element from the array (the pivot)
- partitioning: reorder the array such that all elements smaller than the pivot come before the pivot and all the elements with a value greater than the pivot come after the pivot (so the pivot is already in its correct position)
- recursively apply these steps to the subarray of smaller elements and to the subarray of the bigger elements

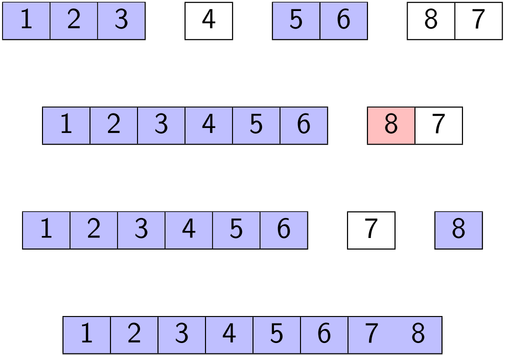
Quicksort



Quicksort



Quicksort



Quicksort

Complexity

Average	Best	Worst	Memory
$O(n \log(n))$	$O(n \log(n))$	$O(n^2)$	$O(1)$

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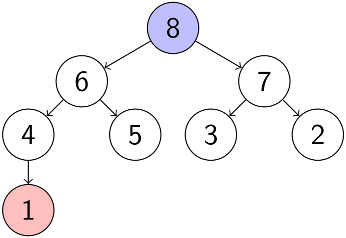
Heapsort

Idea

- put all (unsorted) elements in heap structure
- extract elements from the heap one by one (from high to low, or vice versa)
- is a variation of selection sort with extraction of elements in logarithmic rather than linear time

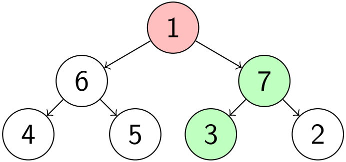
Heapsort

8	6	7	4	5	3	2	1



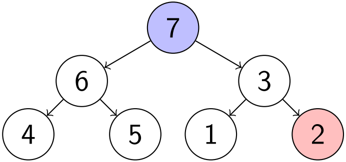
Heapsort

1	6	7	4	5	3	2	8



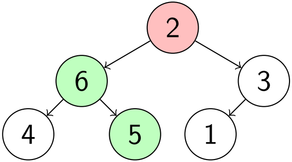
Heapsort

7	6	3	4	5	1	2	8



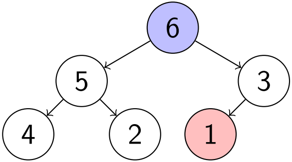
Heapsort

2	6	3	4	5	1	7	8



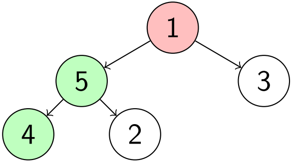
Heapsort

6	5	3	4	2	1	7	8



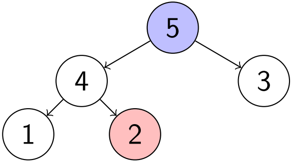
Heapsort

1	5	3	4	2	6	7	8



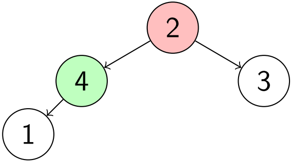
Heapsort

5	4	3	1	2	6	7	8



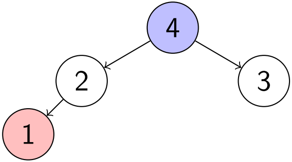
Heapsort

2	4	3	1	5	6	7	8



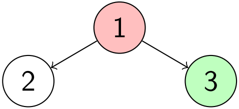
Heapsort

4	2	3	1	5	6	7	8



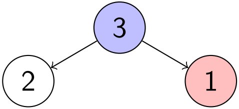
Heapsort

1	2	3	4	5	6	7	8



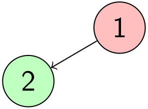
Heapsort

3	2	1	4	5	6	7	8



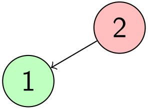
Heapsort

1	2	3	4	5	6	7	8



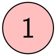
Heapsort

2	1	3	4	5	6	7	8



Heapsort

1	2	3	4	5	6	7	8



Heapsort

1	2	3	4	5	6	7	8

Heapsort

Complexity

Average	Best	Worst	Memory
O(n log(n))	O(n log(n))	O(n log(n))	O(1)

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Convex Hull

Given a set of points in the plane, compute the smallest convex polygon in the plane that contains all the points.

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Graham Scan

Method of computing the convex hull of a finite set of points in the plane with time complexity O(n log(n)). The algorithm finds all vertices of the convex hull ordered along its boundary.

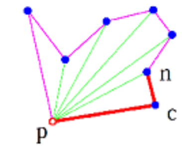
Graham Scan

Idea

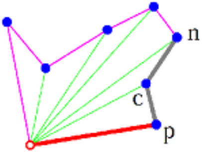
- find the point with the lowest y-coordinate. if there are multiple points with the lowest y-coordinate, the pick that one of them with the lowest x-coordinate. call this point P
- now sort all the points in increasing order of the angle they and point P make with the x-axis
- consider each point in the sorted array in sequence. For each point determine whether coming from the 2 previous points it makes a left of a right turn.
- if it makes a left turn, proceed with the next point
- if it makes a right turn, the second-to-last point is not part of the convex hull and should be removed from the convex hull, continue this removing for as long as the last 3 points make up a right turn

Graham Scan

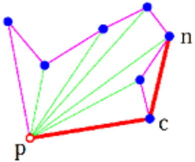
p: previous c: current n:next



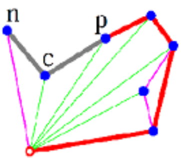
(p, c, n) is a left turn, accept c



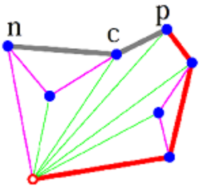
(p, c, n) is a right turn, reject c



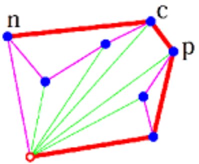
(p, c, n) is a left turn, accept c



(p, c, n) is a left turn, accept c



(p, c, n) is a right turn, reject c



(p, c, n) is a left turn, accept c

In the above algorithm and below code, a stack of points is used to store convex hull points. With reference to the code, p is next-to-top in stack, c is top of stack and n is points[i].

Graham Scan

Direction of the turn

To determine whether 3 points constitute a left or a right turn we do not have to compute the actual angles but we can use a cross product.
Consider the 3 points (x_1, y_1), (x_2, y_2) and (x_3, y_3), which we will call P_1, P_2 and P_3.
Now compute the z-component of the cross product of the vectors P_1P_2 and P_1P_3. Which is given by the expression $(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)$.
If the result is 0, the points are collinear. If the result is positive, the 3 points constitute a left turn. If the result is negative, the 3 points constitute a right turn.