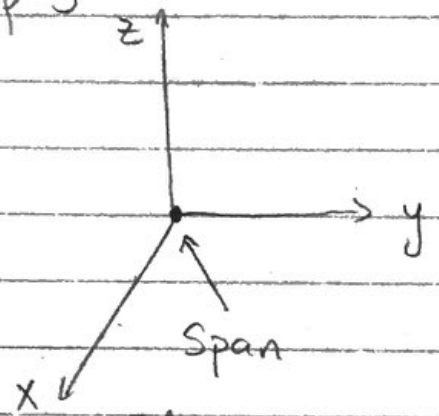
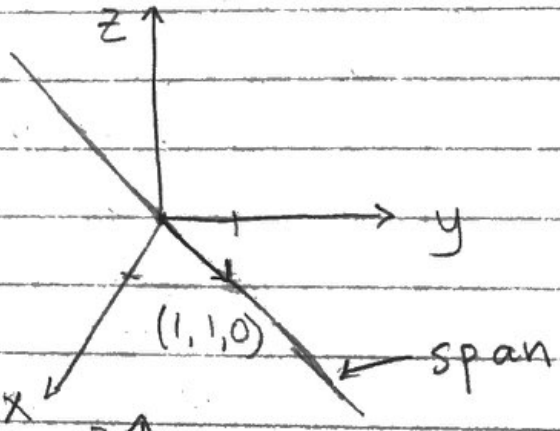
Workshop 3

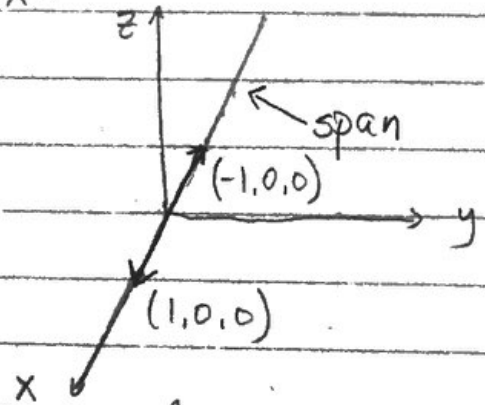
1a.



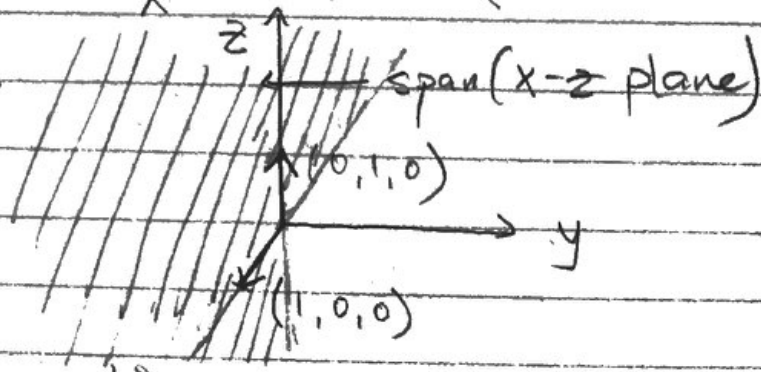

b.



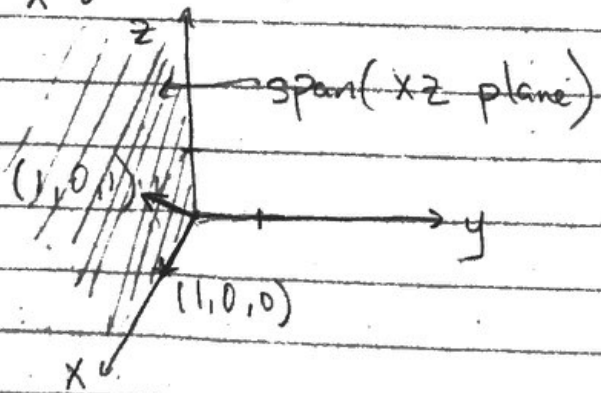

c.



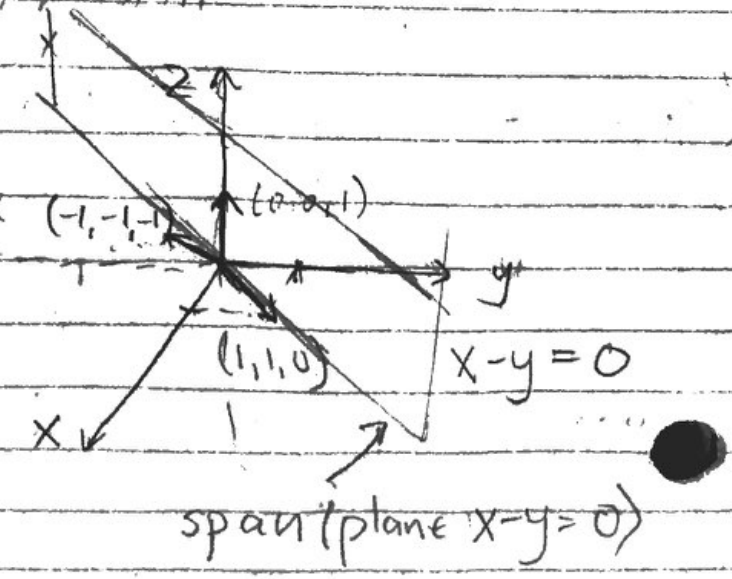

d.

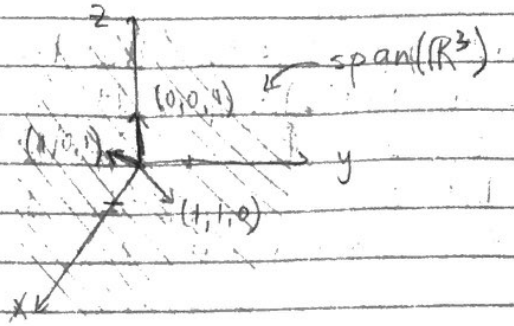



e.




f.






2a. The span of $\{0\}$ is the zero vector because all linear combinations of the zero vector result in the zero vector; $c_1 \cdot [0] = 0$

b. The span of $\{1+x\}$ is $c_1 \cdot [1+x]$ for all values of $c_1$ because the span is all of the linear combinations of of vector $\{1+x\}$

c. The span of $\{1, -1\}$ is $c_3 \cdot [1]$ for all values of $c_1$. The linear combinations of $\{1, -1\}$ are $c_1 \cdot [1] + c_2 \cdot [-1]$, which can be simplified to $(c_1 - c_2) \cdot [1]$. Since $(c_1 - c_2)$ is an arbitrary constant, the span can be represented as $c_3 \cdot [1]$ for all multiples of $c_3$

d. The span of $\{1, x^2\}$ is $c_1[1] + c_2[x^2]$. The linear combinations of $\{1, x^2\}$ are $c_1 \cdot [1] + c_2[x^2]$, so that is the span of this set of vectors.

e. The span of $\{1, 1+x^2\}$ is $c_3[1] + c_2[x^2]$ for all values of $c_1$ and $c_2$. The linear combinations of $\{1, 1+x^2\}$ are $c_1[1] + c_2[1+x^2]$, which can be simplied to $c_1 + c_2 + c_2x^2$ and rewritten as $(c_1 + c_2) + c_2x^2$. Since $(c_1 + c_2)$ is an arbitrary constant, the span is equavalent to $c_3 + c_2x^2$, where $c_3, c_2$ can be any value.

f. The span of $\{1+x, x^2, -1-x-x^2\}$ is $c_4[1+x] + c_5[x^2]$. The linear combinations of $\{1+x, x^2, -1-x-x^2\}$ can be written as $c_1[1+x] + c_2[x^2] + c_3[-1-x-x^2]$. This can be further simplified to $(c_1 - c_3) + (c_1 - c_3)x + (c_2 - c_3)x^2$. Since $(c_1 - c_3)$ and $(c_2 - c_3)$ are arbitrary constants, the span can be rewritten as $c_4[1] + c_4[x] + c_5[x^2]$

g. The span of $\{1+x, x^2, 1+x^2\}$ is $P_2$. The linear combinations of this set of vectors is $c_1[1+x]+c_2[x^2]+c_3[1+x^2]$, which can be simplified to $(c_1+c_3)[1]+c_1[x]+(c_2+c_3)[x^2]$. Since the coefficients are arbitrary, this can be rewritten to $c_4+c_5x+c_6x^2$, and this is the same form as $P_2$

3a. From Q1

$$c_1\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\rightarrow\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

a) Vector $\{(0,0,0)\}$ is linearly dependent a'

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad c_1=0$$

b) Vector $\{(1,1,0)\}$ is linearly independent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}-1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1-c_2=0$

$c_1=c_2$

c) Vectors $\{(1,0,0),(-1,0,0)\}$ are linearly dependent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1=0 \qquad c_2=0$

d) Vectors $\{(1,0,0),(0,0,1)\}$ are linearly independent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1+c_2=0 \qquad c_1=0$

$c_2=0$

e) Vectors $\{(1,0,0),(1,0,1)\}$ are linearly independent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}+c_2\begin{bmatrix}0\\0\\1\end{bmatrix}+c_3\begin{bmatrix}-1\\-1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 - c_3 = 0 \qquad c_1 = c_3$

$c_1 - c_3 = 0$

$c_2 - c_3 = 0 \qquad c_2 = c_3$

$c_1 = c_2 = c_3$

f) Vectors $\{(1,1,0),(0,0,1),(-1,-1,-1)\}$ are linearly dependent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}+c_2\begin{bmatrix}0\\0\\1\end{bmatrix}+c_3\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 + c_3 = 0 \qquad c_3 = 0$

$c_1 = 0$

$c_2 + c_3 = 0 \qquad c_2 = 0$

g) Vectors $\{(1,1,0),(0,0,1),(1,0,1)\}$ are linearly independent

b. A set of three vectors is linearly dependent in $\mathbb{R}^3$ if the third vector lies on the span of the first two vectors. For example, if the span of the first two vectors is a plane in space, then the set would be linearly dependent if the third vector was on that plane. If that is case, then the third vector is a linear combination of the first two vectors, so it is redundant. However, if the third vector is not on the plane (eg $\perp$ to the plane), then the set is linearly independent since the third vector adds to the span (the span becomes $\mathbb{R}^3$ instead of the plane)

From Q2 ↘

c. $$c_1\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$ $\vec{v}$ is a vector of coefficients $(c_1\vec{v}=\vec{0})$

a) Vector $\{0\}$ is linearly dependent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad c_1 = 0$$

Vector $\{1+x\}$ is linearly independent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}-1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 - c_2 = 0$

$c_1 = c_2$

Vectors $\{1, -1\}$ are linearly dependent

d. $$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 = 0$

$c_2 = 0$

Vectors $\{1, x^2\}$ are linearly independent

e. $$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 + c_2 = 0 \qquad c_1 = 0$

$c_2 = 0$

Vectors $\{1, 1+x^2\}$ are linearly independent

f. $$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\0\\1\end{bmatrix} + c_3\begin{bmatrix}-1\\-1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 - c_3 = 0 \qquad c_1 = c_3$

$c_1 - c_3 = 0$

$c_2 - c_3 = 0 \qquad c_2 = c_3$

$c_1 = c_2 = c_3$

Vectors $\{1+x, x^2, -1-x-x^2\}$ are linearly dependent

g. $$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\0\\1\end{bmatrix} + c_3\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 + c_3 = 0 \qquad c_1 = 0 \qquad c_2 + c_3 = 0 \qquad c_3 = 0 \qquad c_2 = 0$

Vectors $\{1+x, x^2, 1+x^2\}$ are linearly independent

4. The results from 1 and 2 are equivalent because the vectors created from the polynomial coefficients are the same as the vectors in 1.