## Instructions

**1.** To receive full credit you must explain how you got your answer.

**2.** While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

**3.** Workshop must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Late submission will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. 1 point out of 10 may be taken off if your solutions are hard to read or poorly presented.

## Workshop 2 & 3

**1.** Let $\mathcal{P}_n$ denote the set of all polynomials of the form $a_0 + a_1x + ... + a_nx^n$, where n is a non-negative integer and $a_0, a_1, ..., a_n$ are real numbers.

a. Define appropriate addition and scalar multiplication on $\mathcal{P}_n$ and show that it is a vector space over $\mathbb{R}$.

b. Show that $\mathcal{P}_1$ is a subspace of $\mathcal{P}_2$. Can you generalize this statement?

c. Show that you can do the same over $\mathbb{C}$.

**2.** Draw the span of the following sets of vectors in $\mathbb{R}^3$.

a. $\{(0, 0, 0)\}$.
b. $\{(1, 1, 0)\}$.
c. $\{(1, 0, 0), (-1, 0, 0)\}$
d. $\{(1, 0, 0), (0, 0, 1)\}$.
e. $\{(1, 0, 0), (1, 0, 1)\}$.
f. $\{(1, 1, 0), (0, 0, 1), (-1, -1, -1)\}$
g. $\{(1, 1, 0), (0, 0, 1), (1, 0, 1)\}$.

**3.** Let $\mathcal{P}_2$ be as in 1. Describe the span of the following sets of vectors in the simplest possible terms.

a. $\{0\}$.
b. $\{1 + x\}$.
c. $\{1, -1\}$.
d. $\{1, x^2\}$.
e. $\{1, 1 + x^2\}$.
f. $\{1 + x, x^2, -1 - x - x^2\}$.
g. $\{1 + x, x^2, 1 + x^2\}$.

**4.** a. In 2, which sets of vectors are linearly dependent and which sets are linearly independent?

b. What do linear dependence and linear independence in $\mathbb{R}^3$ mean geometrically?

c. In 3, which sets of vectors are linearly dependent and which sets are linearly independent?

**5.** Do you see connections between 2 and 3? Try to describe it. (We will talk more about it soon.)

**6.** a.What sized matrices can be multiplied to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ on the left and right, respectively? Find ALL allowed sizes.

b. What do you get when you do such multiplications? Computing a few examples might help you draw the general conclusion.

c. In general, the **square matrices** (*i.e. matrices with the same number of rows and columns*) with 1's on the diagonal (*when we speak of the diagonal in this class, we always mean the upper-left to lower-right diagonal*) and 0's elsewhere are called **Identity matrices**. Can you generalize your conclusion in b to all identity matrices?

**7.** In the videos we saw that for general matrices A and B, when the product AB is defined, the product BA may not be defined. Give examples to show:

a. Even if AB and BA are both defined, they may have different sizes.

b. Even if AB and BA are both defined and have the same size, they may not be equal.