[redacted]

Functions

*) A Function may be defined by a formula that tells how to calculate the output for a given Input.

$$\text{Input } x \Longrightarrow \boxed{\text{function}} \Longrightarrow \text{Output } f(x)$$

Eg. $f(x) = x - 1$

(I) Definition of a function

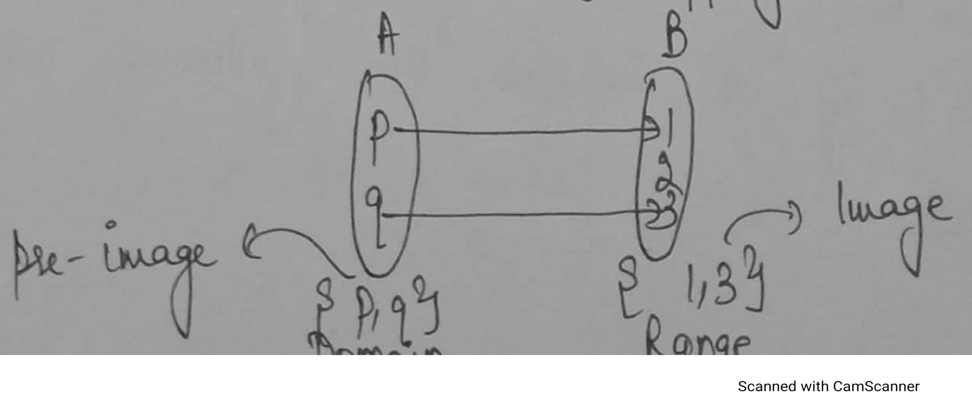
*) It is a special type of Relation, with the following properties ⟶

a) $\forall x \in$ Domain, there is a mapping.

b) Unique Image $\forall x \in$ Domain

Eg. $\underbrace{A = \{p, q\}}_{\text{Domain}}$ $\underbrace{B = \{1, 2, 3\}}_{\text{Co-domain}}$

$f : A \to B$ } Mapping



*) Let X and Y be two non-empty sets. ②

*) A function $f: X \to Y$ where,

Set $X \to$ Domain

Set $Y \to$ Co-Domain

*) f maps every element $x \in X$ to the element $y \in Y$ and can be written as $\to$

$$y = f(x)$$

*) The element $y \in Y$ is called <u>Image</u> of $x \in X$

*) The element $x \in X$ is called <u>Pre-Image</u> of $y \in Y$

*) The set of all image values $\{f(x): x \in X\}$ is called the <u>Range</u> of f

<u>Range</u> is always a <u>Subset of Co-domain</u>

ⓐ Relation Vs Function

A function $f: X \to Y$ is a special kind of Relation $R: X \to Y$, if it satisfies the following additional properties :$\to$

1) Every element $x \in X$ has an image $y \in Y$ [Domain $= X$]
2) One element of X can have only one image, that if $(x, y) \in f$ and $(x, z) \in f$, then $y = z$. [Unique Image]

[Every function is a Relation, but not vice-versa]

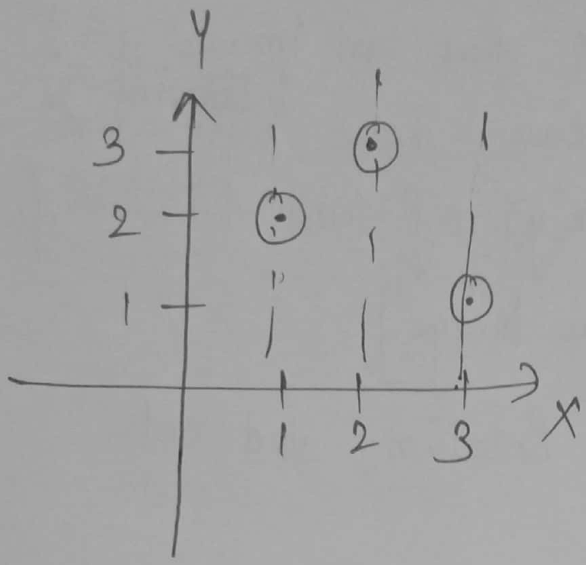
(4) Graphical Determination of a Relation as a function

*) Use Vertical Line Test
*) If the line intersects the graph of the relation at more than one point, then not a function, else a function.

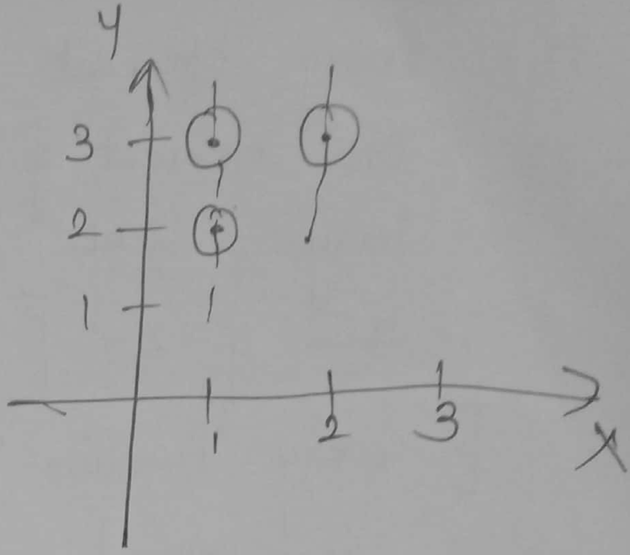
Eg. $X = \{1, 2, 3\}$

$R_1 = \{ (1,2), (2,3), (3,1) \}$

$R_2 = \{ (1,2), (2,3), (1,3) \}$



$R_1 \to$ is a function



$R_2 \to$ Not a function

(C) Why do we need functions?

ADD (2, 3) ⟶ Guaranteed Result

Unambigous Output

↓

5 Ans

and not

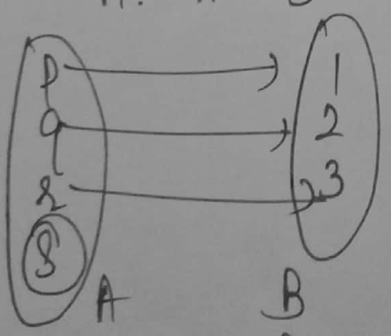
1, 10, 20 etc ✗

Examples of functions

$A = \{p, q, r, s\}$ $B = \{1, 2, 3\}$

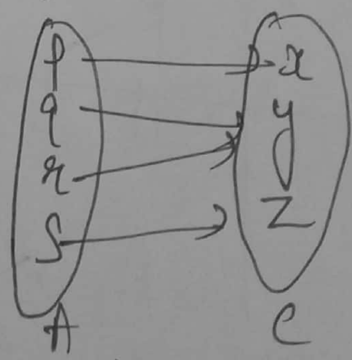
$C = \{x, y, z\}$

$f_1 : A \to B$



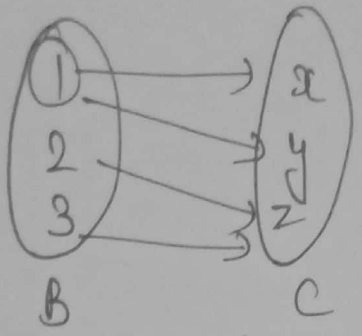
Not a function

$f_2 : A \to C$



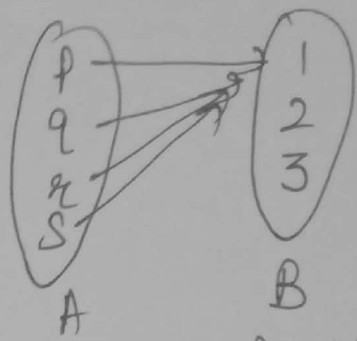
Yes a function

$F_3: B \to C$



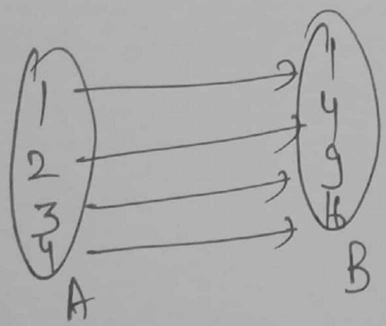
Not a function

$f_4: A \to B$



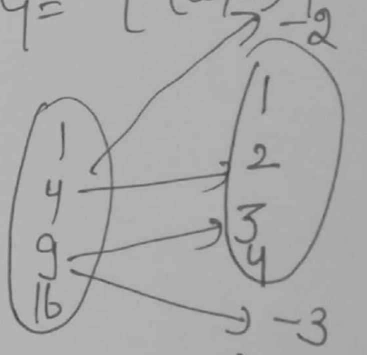
Yes, a function, Called a Constant function

$f = \{(x, x^2) \mid x \in \mathbb{Z}\}$



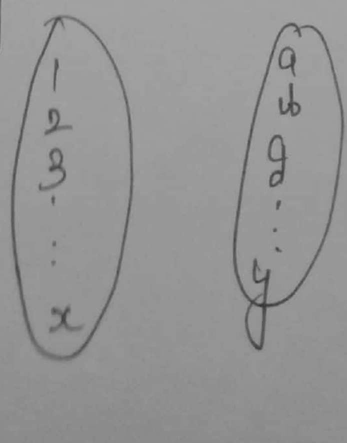
Yes a function

$g = \{(x^2, x) \mid x \in \mathbb{Z}\}$



Not a function

(d) How many functions are possible from a set of 'x' elements to a set of 'y' elements?



$n(f) = \underbrace{y \times y \times y \ldots}_{(x)} \; x \text{ times} = y^x$

$n(f) = \boxed{y^x}$

(II) Types of functions

(a) One - One function

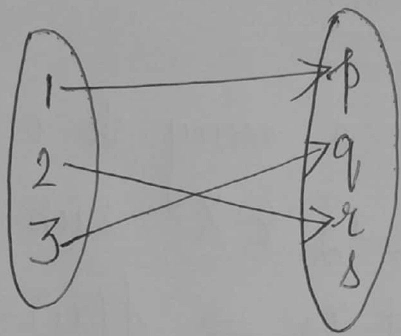
$f : X \to Y$ is one to one if,

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

or

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

Eg



one-one
(Injection)
[Unique Image]

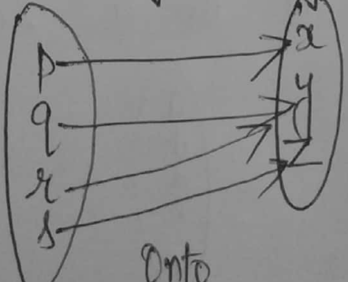
(b) Onto function

$f : X \to Y$ is an Onto function if,

$Ran(f) = Y$ i.e for each $y \in Y$, there is an $x \in X$ such that,

$$f(x) = y$$

Eg.



Onto
(Surjection)

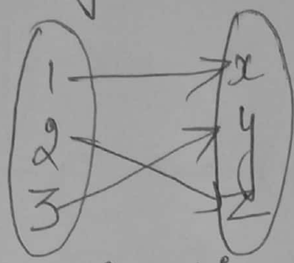
$\boxed{\text{Range} = \text{Codomain}}$

(c) One-One Onto function

→ It is both one-one as well as Onto

→ Also called Bijection.

Eg.

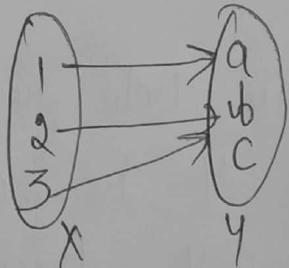


Bijection

(d) Many-One function

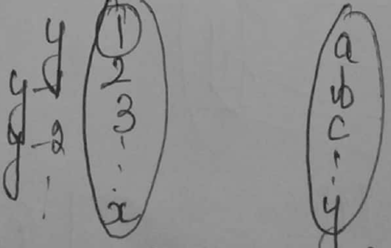
$f: X \to Y$ is a many to one if,
$\exists x_1, x_2 \in X$ such that
$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2)$

Example



Many-One

Q How many One-One functions are possible from a set of 'n' elements to a set of 'y' elements?



$${}^nP_r = \frac{n!}{(n-r)!}$$

$$n(f) = y \times (y-1) \cdots y-(x-1))$$

$$= {}^yP_x = \frac{y!}{(y-x)!}$$

Q How many one - one functions are there from a set A with 'n' elements onto itself.

$$f : X \to Y$$
$$n \to n$$

$$n(f) = {}^{y}P_{x}$$

$$n(f) = {}^{n}P_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$