MAD101 - Assignment 3

Deadline: July 21, 2023

1. For which values of n are these graphs bipartite?
   (a) K_n
   (b) C_n
   (c) W_n
   (d) Q_n
2. What is the sum of the entries in a row of the adjacency matrix for an undirected graph? For a directed graph?
3. Find the number of paths of length n between any two adjacent vertices in $K_{3,3}$ for the following values of n:
   (a) n = sum of the digits in your Student ID
   (for example, if your student ID is HE102359, then $n = 1 + 0 + 2 + 3 + 5 + 9 = 20$)
   (b) n = sum of the digits in your Student ID + 1
   (for example, if your student ID is HE102359, then $n = 1 + 0 + 2 + 3 + 5 + 9 + 1 = 21$)
4. For which values of n do these graphs have an Euler circuit?
   (a) K_n
   (b) C_n
   (c) W_n
   (d) Q_n
5. How many non-isomorphic simple graphs are there with five vertices and three edges?
6. Using alphabetical order, construct a binary search tree for the words in the sentence "[Your full name] is a student at FPT University."
   For example, if your full name is "Nguyen Van A", then the words in the sentence are "Nguyen","Van", "A", "is", "a", "student","at", "FPT", "University".

7. Use Huffman coding to encode the symbols $\{a, b, c, d, e, f\}$ where the frequencies are given by the following rule:

   Take your Student ID and add 1 to each number so that they are all non-zero. The results are then labeled A, B, C, D, E, and F. The frequencies of the symbols x are then given by

   $$\frac{X}{A+B+C+D+E+F}$$

   For example, if your ID is HE102359, then

   $A = 1+1 = 2, B = 0+1 = 1, C = 3, D = 4, E = 6, F = 10$.

   The frequency of the symbol a is $\frac{2}{2+1+3+4+6+10} = \frac{2}{26}$.

   What is the encoding, and what is the average number of bits required to encode a character?

8. Describe the trees produced by breadth-first search and depth-first search of the following graphs:

   (a) K_n

   (b) W_n, starting at the center point.

9. Represent the expression $((x+2) \uparrow 3) \times (y - (3+x)) - 5$ using a binary tree. Write this expression in prefix and postfix notations.

10. Which connected simple graphs have exactly one spanning tree?