№1

а)
$$\begin{pmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -2 & 0 & 7 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{pmatrix} =$$

$$= 2 \begin{vmatrix} 0 & -2 & 3 \\ 3 & 1 & 2 \\ 4 & 1 & -2 \end{vmatrix} - 7 \begin{vmatrix} 0 & 2 & -2 \\ 3 & 4 & 1 \\ 4 & 1 & 1 \end{vmatrix} =$$

$$= 2 \cdot (-16 + 9 - 12 - 12) - 7(8 - 6 + 32 - 6) = -196 - 62 =$$

$$= -258$$

б)
$$\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{vmatrix} = -1 \begin{pmatrix} 0 & -2 & 3 \\ 3 & 1 & 2 \\ 4 & 1 & -2 \end{pmatrix} =$$

$$= -2 \begin{pmatrix} 4 & 1 & 5 \\ 3 & 1 & 2 \\ 4 & 1 & -2 \end{pmatrix} + 4 \begin{pmatrix} 4 & 1 & 5 \\ 0 & -2 & 3 \\ 4 & 1 & -2 \end{pmatrix} - 1 \begin{pmatrix} 4 & 1 & 5 \\ 0 & -2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$= -1(-16 + 9 - 12 - 12) - 2(-8 + 8 + 15 - 20 + 6 - 8) +$$

$$+ 4(16 + 12 + 40 - 12) - (-16 + 9 + 30 - 12) = -258$$

№2

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -4 & 1 \\ 4 & -3 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & -4 \\ 2 & 5 & -3 \\ 4 & -3 & 2 \end{pmatrix}$$

а) $A \cdot B =$

$$= \begin{pmatrix} 1\cdot1 + 1\cdot2 + (-1)\cdot4 & 1\cdot0 + 1\cdot5 + (-1)\cdot(-3) & 1\cdot(-4)+1\cdot(-3)+(-1)\cdot2 \\ 2\cdot1 + (-4)\cdot2 + 1\cdot4 & 2\cdot0 + (-4)\cdot5 + 1\cdot(-3) & 2\cdot(-4)+(-4)\cdot(-3)+1\cdot2 \\ 4\cdot1 + (-3)\cdot2 + 1\cdot4 & 4\cdot0 + (-3)\cdot5 + 1\cdot(-3) & 4\cdot(-4)+(-3)\cdot(2)-1\cdot2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 8 & -9 \\ -2 & -23 & 6 \\ 2 & -18 & -5 \end{pmatrix}$$

б) $B \cdot A = \begin{pmatrix} 1\cdot1 + 0\cdot2 + (-4)\cdot4 \\ 2\cdot1 + 5\cdot2 + (-3)\cdot4 \\ 4\cdot1 + (-3)\cdot2 + 2\cdot4 \end{pmatrix}$

$$\begin{pmatrix} 1\cdot1 + 0\cdot(-4) + (-4)\cdot(-3) & 1\cdot(-1) + 0\cdot1 + (-4)\cdot1 \\ 2\cdot1 + 5\cdot(-4) + (-3)\cdot(-3) & 2\cdot(-1) + 5\cdot1 + (-3)\cdot1 \\ 4\cdot1 + (-3)\cdot(-4) + 2\cdot(-3) & 4\cdot(-1) + (-3)\cdot1 + 2\cdot1 \end{pmatrix} =$$

$$= \begin{pmatrix} -15 & 13 & -5 \\ 0 & -9 & 0 \\ 6 & 10 & -5 \end{pmatrix}$$

в) A^{-1}

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -4 & 1 \\ -3 & 1 \end{vmatrix} = -1$

$A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} = 2$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -4 \\ 4 & -3 \end{vmatrix} = 10$

$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix} = 2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} = 5$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix} = 7$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & -1 \\ -4 & 1 \end{vmatrix} = -3$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = -3$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} = -6$

$\begin{vmatrix} 1 & 1 & -1 \\ 2 & -4 & 1 \\ 4 & -3 & 1 \end{vmatrix} = (-4 + 4 + 6 - (16 + 2 + (-3))) = $

$= -15 + 6 = \underline{-9}$

$$A^{-1} = \frac{1}{9}\begin{pmatrix} -1 & 2 & -3 \\ 2 & 5 & -3 \\ 10 & 7 & -6 \end{pmatrix}^T$$

$$= \begin{pmatrix} -\frac{1}{9} & -\frac{2}{9} & \frac{1}{3} \\ -\frac{2}{9} & -\frac{5}{9} & \frac{1}{3} \\ -\frac{10}{9} & -\frac{7}{9} & \frac{2}{3} \end{pmatrix}$$

2) AA^{-1}

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -4 & -1 \\ 4 & -3 & 1 \end{pmatrix}\begin{pmatrix} -\frac{1}{9} & -\frac{2}{9} & \frac{1}{3} \\ -\frac{2}{9} & -\frac{5}{9} & \frac{1}{3} \\ -\frac{10}{9} & -\frac{7}{9} & \frac{2}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{9} + \left(-\frac{2}{9}\right) + \left(-1\right)\cdot\left(-\frac{10}{9}\right) \\ 2\cdot\frac{1}{9} + \left(-4\right)\cdot\left(-\frac{2}{9}\right) + \left(-\frac{10}{9}\right) \\ 4\cdot\frac{1}{9} + \left(-3\right)\cdot\left(-\frac{2}{9}\right) + \left(-\frac{10}{9}\right) \end{pmatrix}$$

$$-\frac{2}{9} + \left(-\frac{5}{9}\right) + \left(-1\right)\cdot\left(-\frac{7}{9}\right) \qquad -\frac{1}{3} + \frac{1}{3} + \left(-1\right)\cdot\frac{2}{3}$$

$$2\cdot\left(-\frac{2}{9}\right) + \left(-4\right)\left(-\frac{5}{9}\right) + \left(-\frac{7}{9}\right) \quad \text{✳} \quad 2\cdot\frac{1}{3} + \left(-4\right)\cdot\frac{1}{3} + \frac{2}{3} \qquad =$$

$$4\cdot\left(-\frac{2}{9}\right) + \left(-3\right)\cdot\left(-\frac{5}{9}\right) + \left(-\frac{7}{9}\right) \quad 4\cdot\frac{1}{3} + \left(-3\right)\cdot\frac{1}{3} + \frac{2}{3}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$g) \ A \cdot A^{-1} =$

$$= \begin{pmatrix} \frac{1}{9} + \left(-\frac{7}{9}\right) \cdot 2 + \frac{2}{3} \cdot 4 & \frac{1}{9} + \left(-\frac{2}{9}\right) \cdot (-4) + \frac{1}{3} \cdot (-3) \\ -\frac{4}{9} + \left(\frac{5}{9}\right) \cdot 2 + \frac{1}{3} \cdot 4 & -\frac{4}{9} + \left(-\frac{5}{9}\right) \cdot (-4) + \frac{1}{3} \cdot (-3) \\ -\frac{10}{9} + \left(-\frac{7}{9}\right) \cdot 2 + \frac{2}{3} \cdot 4 & -\frac{10}{9} + \left(-\frac{7}{9}\right) \cdot (-4) + \frac{2}{3} \cdot (-3) \end{pmatrix}$$

$$\left. \begin{matrix} \frac{1}{9} \cdot (-1) + \left(-\frac{2}{9}\right) + \frac{1}{3} \\ -\frac{4}{9} \cdot (-1) + \left(-\frac{5}{9}\right) + \frac{1}{3} \\ -\frac{10}{9} \cdot (-1) + \left(-\frac{7}{9}\right) + \frac{2}{3} \end{matrix} \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

№3

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 14 \\ 2x_1 + 3x_2 - 4x_3 = -16 \\ 3x_1 - 2x_2 - 5x_3 = -8 \end{cases}$$

$a) \quad \Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -5 \end{vmatrix} = 1 \cdot 3 \cdot (-5) + (-2) \cdot (-4) \cdot 3 +$

$+ 3 \cdot 2 \cdot (-2) - 3 \cdot 3 \cdot 3 - 1 \cdot (-4) \cdot (-2) - (-2) \cdot 2 \cdot (-5) =$

$= -15 + 24 - 12 - 27 = -58$

$$A_1 = \begin{vmatrix} 14 & -2 & 3 \\ -16 & 3 & -4 \\ -8 & -2 & -5 \end{vmatrix} = 14 \cdot 3 \cdot (-5) + (-2) \cdot (-4) \cdot (-8) +$$

$$+ 3 \cdot (-16) \cdot (-2) - 3 \cdot 3 \cdot (-8) - 14 \cdot (-4) \cdot (-2) - (-2) \cdot (-16) \cdot$$

$$\cdot (-5) = -210 - 64 + 96 + 72 - 112 + 160 = -58$$

$$\Delta_2 = \begin{vmatrix} 1 & 14 & 3 \\ 2 & -16 & -4 \\ 3 & -8 & -5 \end{vmatrix} = 1 \cdot (-16) \cdot (-5) + 14 \cdot (-4) \cdot 3 +$$

$$+ 3 \cdot 2 \cdot (-8) - 3 \cdot (-16) \cdot 3 - 1 \cdot (-4) \cdot (-8) - 14 \cdot 2 \cdot (-5) =$$

$$= 80 - 168 - 48 + 144 - 32 + 140 = 116$$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 14 \\ 2 & 3 & -16 \\ 3 & -2 & -8 \end{vmatrix} = 1 \cdot 3 \cdot (-8) + (-2) \cdot (-16) \cdot 3 +$$

$$+ 14 \cdot 2 \cdot (-2) - 14 \cdot 3 \cdot 3 - 1 \cdot (-16) \cdot (-2) - (-2) \cdot 2 \cdot$$

$$\cdot (-8) = -24 + 96 - 36 - 126 - 32 - 32 = -174$$

$$X_1 = \frac{\Delta_1}{\Delta} = 1$$

$$X_2 = \frac{\Delta_2}{\Delta} = \frac{116}{-58} = -2$$

$$X_3 = \frac{\Delta_3}{\Delta} = \frac{-174}{-58} = 3$$

б)

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -5 \end{pmatrix} \quad B = \begin{pmatrix} 14 \\ -16 \\ -8 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

$$A \cdot X = B \Rightarrow X = A^{-1} \cdot B$$

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & -5 \end{vmatrix} = -58$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & -4 \\ -2 & -5 \end{vmatrix} = -23$$

$$A_{12} = (-1)^{1+2} = \begin{vmatrix} 2 & -4 \\ 3 & -5 \end{vmatrix} = -2$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -13$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -2 & -3 \end{vmatrix} = -16$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix} = -14$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -2 \\ 3 & -2 \end{vmatrix} = -4$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & -4 \end{vmatrix} = -1$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = 10$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 7$$

$$A^{-1} = \frac{1}{-58} \cdot \begin{pmatrix} -23 & -16 & -1 \\ -2 & -19 & 10 \\ -13 & -4 & 5 \end{pmatrix} = \begin{pmatrix} \frac{23}{58} & \frac{8}{29} & \frac{1}{58} \\ \frac{1}{29} & \frac{9}{29} & -\frac{5}{29} \\ \frac{13}{58} & \frac{2}{29} & -\frac{7}{58} \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} \frac{23}{58} & \frac{8}{29} & \frac{1}{58} \\ \frac{1}{29} & \frac{7}{29} & -\frac{5}{29} \\ \frac{13}{58} & \frac{2}{29} & -\frac{7}{58} \end{pmatrix} \cdot \begin{pmatrix} 14 \\ -16 \\ -8 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{23}{58} \cdot 14 + \frac{8}{29} \cdot (-16) + \frac{1}{58} \cdot (-8) \\ \frac{1}{29} \cdot 14 + \frac{7}{29} \cdot (-16) + \left(-\frac{5}{29}\right) \cdot (-8) \\ \frac{13}{58} \cdot 14 + \frac{9}{29} \cdot (-16) + \left(-\frac{7}{58}\right) \cdot (-8) \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} X_1 = 1 \\ X_2 = -2 \\ X_3 = 3 \end{cases}$$

№ 4

$X = \{ X_1, X_2, X_3 \}$

$AX = \{ X_2 - X_3, \ X_1; \ X_1 + X_3 \}$

$Bx = \{ X_2, \ 2X_3, \ X_1 \}$

$(2B - A^2)X - ?$

$2B = (2X_1; 4X_3; -2X_1)$

$A^2 = \{X_2^2 - 2X_2X_3 + X_3^2; X_1^2; X_1^2 + 2X_1X_3 + X_3^2\}$

$(2B - A^2) = \{2X_2 - (X_3^2 - 2X_2X_3 + X_3^2);$

$4X_3 - X_1^2;$

$2X_1 - (X_1^2 + 2X_1X_3 + X_3^2)\} =$

$= \{2X_2 - X_2^2 + 2X_2X_3 - X_3^2; 4X_3 - X_1^2; 2X_1 - X_1^2 -$

$- 2X_1X_3 - X_3^2\}$

$(2B - A^2)X = \{2X_2X_1 - X_2^2X_1 + 2X_2X_3X_1 - X_3^2X_1;$

$4X_3X_2 - X_1^2X_2; 2X_1X_3 - X_1^2X_3 - 2X_1X_3^2 - X_3^3\}$

№ 5

$A(5,4,4)$ $B(-5,2,3)$ $C(4,2;-5)$

2) $a = 11AC - 6AB$

$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2; a_3b_1 - a_1b_3; a_1b_2 - a_2b_1)$

$AC = (4-5; 2-4; -5-4) = (-1; -2; -9)$

$AB = (-5-5; 2-4; 3-4) = (-10; -2; -1)$

$11AC = (-11; -22; -99)$

$6AB = (-60; -12; -6)$

$a = 11AC - 6AB = (-11 + 60_6, -22 + 12, -99 + 6) =$

$= (49, -10, -93)$

$|11AC - 6AB| = \sqrt{49^2 + 10^2 + 93^2} = 5\sqrt{446}$

б) $b = BC = (4 + 5, 2 - 2, -5 - 3) = (9, 0, -8)$

$(a, b) = 49 \cdot 9 - 10 \cdot 0 - 93 \cdot (-8) =$

$= 441 + 744 = 1185$

в) $M = ?$ $\alpha = 3; \beta = 1 \quad \dfrac{\alpha}{\beta} = 3$

$B(-5, 2, 3)$

$C(4, 2, -5)$

$X_m = \dfrac{X_B + 3 \cdot X_C}{1 + 3} = \dfrac{-5 + 3 \cdot 4}{4} = 1\dfrac{3}{4}$

$Y_m = \dfrac{Y_0 + 3 \cdot Y_C}{1 + 3} = \dfrac{2 + 3 \cdot 2}{4} = 2$

$Z_m = \dfrac{Z_B + 3 \cdot Z_C}{1 + 3} = \dfrac{3 + 3 \cdot (-5)}{1 + 3} = -3$

$M\left(1\dfrac{3}{4}, 2, -3\right)$

№6

$$\vec{a} = -4\vec{i} - 6\vec{j} + 2\vec{k}$$
$$\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$$
$$\vec{c} = \vec{i} + 5\vec{j} - 3\vec{k}$$

а) $5\vec{a}$, $7\vec{b}$, $2\vec{c}$

$$3\vec{a} = 20\vec{i} - 30\vec{j} + 10\vec{k}$$
$$7\vec{b} = 14\vec{i} + 21\vec{j} - 7\vec{k}$$
$$2\vec{c} = 2\vec{i} + 10\vec{j} - 6\vec{k}$$

$$(5\vec{a}, 7\vec{b}, 2\vec{c}) = \begin{vmatrix} -20i & -30j & 10k \\ 14i & 21j & -7k \\ 2i & 10j & -6k \end{vmatrix} =$$

$$= -30i \cdot 21j \cdot (-6k) + (-30j)(-7k) \cdot (2i) + 14i \cdot 10j \cdot 10k -$$
$$- (2i \cdot 21j \cdot 10k + 14i \cdot (-30j) \cdot (-6k) + 10j \cdot (-7k) \cdot (20i)) =$$
$$= 2 \cdot 520 ijk + 420 ijk + 1400 ijk - (420 ijk + 2520 ijk +$$
$$+ 1400 ijk) = 0$$

д) $-4\vec{b}$; $11\vec{a}$

$$-4\vec{b} = -8\vec{i} - 12\vec{j} + 4\vec{k}$$
$$11\vec{a} = -44\vec{i} - 66\vec{j} + 22\vec{k}$$

$$-4\vec{b} \times 11\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & -12 & 4 \\ -44 & -11 & 22 \end{vmatrix} =$$

$$= \vec{i}(-12 \cdot 24 - (-66) \cdot 4) - \vec{j}(-8 \cdot 22 - 4 \cdot (-44) +$$

$$+ \vec{k}(-8 \cdot (-11) - (-12) \cdot (-44)) = -24\vec{i}$$

$$|-4\vec{b} \cdot 11\vec{a}| = \sqrt{(-24)^2} = \sqrt{576}^2 = 24\vec{i}$$

б) $3\vec{a}$; $-7\vec{c}$

$$3\vec{a} = -12\vec{i} - 18\vec{j} + 6\vec{k}$$

$$-7\vec{c} = -7\vec{i} - 35\vec{j} + 21\vec{k}$$

$$3\vec{a} \cdot (-7\vec{c}) = -12\vec{i} \cdot (-7\vec{i}) + 18\vec{j} \cdot 35\vec{j} + 6\vec{k} \cdot 21\vec{k} = 84\vec{i}^2 +$$

$$+ 630\vec{j}^2 + 126\vec{k}^2$$

г) $\vec{a}$, $\vec{b}$

$$\vec{a} = u \cdot \vec{b}$$

$$\vec{a} = -4\vec{i} - 6\vec{j} + 2\vec{k}$$

$$\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$$

$$u = -2$$

$$2\vec{b} = -4\vec{i} - 6\vec{j} + 2\vec{k} \Rightarrow \vec{a} \text{ и } \vec{b} =$$

$$= \vec{a} \cdot \vec{b} = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z = 0$$

$$\vec{a} \cdot \vec{b} = -4\vec{i} \cdot 2\vec{i} - 6\vec{j} \cdot 3\vec{j} - 2\vec{k} \cdot \vec{k} = -8\vec{i}^2 - 18\vec{j}^2 - 2\vec{k}^2 \Rightarrow$$

$$\Rightarrow \text{ векторы не ортогональны}$$

g) $3\vec{a}$, $7\vec{b}$, $-2\vec{c}$

$3\vec{a} = -12\vec{i} - 18\vec{j} + 6\vec{k}$

$7\vec{b} = 14\vec{i} + 21\vec{j} - 7\vec{k}$

$-2\vec{c} = -2\vec{i} - 10\vec{j} + 6\vec{k}$

$$(3\vec{a} \cdot 7\vec{b} \cdot (-2\vec{c})) = \begin{pmatrix} -12i & -18j & 6k \\ +14i & 21j & -7k \\ 2i & -10j & 6k \end{pmatrix} =$$

$= -12i \cdot 21j \cdot 6k + (-18j) \cdot (-7k) \cdot (-2i + 14i \cdot (-10j)$
$\cdot 6k - (-2i \cdot 21j \cdot 6k + 14i \cdot -18j) \cdot 6k + (-10j)$
$\cdot (-7k) \cdot (-12j) = -1512 - 252 - 840 - (-252$
$- 1512 - 840) < 0 \Longrightarrow$ векторп

компланарн.