Индивидуальное домашнее задание №2

По дисциплине: «Математический анализ»

Курс 1. Семестр 1

Вариант №11

№1

$y = \sqrt[5]{(x-2)^6} - \frac{3}{7x^3-x^2-4}$

$y' = (x-2)^{\frac{6}{5}} - \frac{3}{7x^3-x^2-4} = \frac{6}{5}\sqrt[5]{x-2} -$

$- \frac{3'\cdot(7x^3-x^2-4) - 3\cdot(21x^2-2x)}{(7x^3-x^2-4)^2} =$

$= \frac{6\sqrt[5]{x-2}}{5} - \frac{-3\cdot(21x^2-2x)}{(7x^3-x^2-4)^2} =$

$= \frac{6\sqrt[5]{x-2}}{5} + \frac{3(21x^2-2x)}{(7x^3-x^2-4)^2}$

$y = ctg\frac{1}{x}\cdot arccos\, x^4$

$y' = -\frac{1}{\sin^2(\frac{1}{x})}\cdot arccos\, x^4 + ctg\frac{1}{x}\cdot$

$\cdot\left(-\frac{1}{\sqrt{1-x^8}}\right) = -\frac{arcos\, x^4}{\sin^2(\frac{1}{x})}\cdot\left(\frac{1}{x}\right)' -$

$- \frac{ctg(\frac{1}{x})}{\sqrt{1-x^8}}\cdot(x^4)' = \frac{arccos\, x^4}{\sin^2(\frac{1}{x})\cdot x^2} -$

$- \frac{4x^3 - ctg\frac{1}{x}}{\sqrt{1-x^8}}$

$y = lg(x+3)\cdot arcsin^2 5x$

$y' = \frac{1}{(x+3)\ln 10}\cdot arcsin^2 5x +$

$+ lg(x+3)\cdot 2 arcsin 5x\cdot\frac{1}{\sqrt{1-25x^2}}\cdot 5 =$

$= \frac{arcsin^2 5x}{(x+3)\ln 10} + \frac{lg(x+3)\cdot 10 arcsin 5x}{\sqrt{1-25x^2}}$

$y = \frac{\ln^3 x}{ctg(x-3)}$

$y' = \frac{3\ln^2 x\cdot\frac{1}{x}\cdot ctg(x-3) - \ln^3 x\cdot\left(-\frac{1}{\sin^2(x-3)}\right)}{ctg^2(x-3)} =$

$$= \frac{\frac{3\ln^2 x \cdot ctg(x-3)}{x} + \frac{\ln^3 x}{\sin^2(x-3)}}{ctg^2(x-3)} =$$

$$= \frac{\frac{3\ln^2 x \cdot \cos(x-3)}{x \sin(x-3)} + \frac{\ln^3 x}{\sin^2(x-3)}}{ctg^2(x-3)} =$$

$$= \frac{\frac{\sin(x-3) \cdot 3\ln^2 x \cdot \cos(x-3) + \ln^3 x \cdot x}{x \sin^2(x-3)}}{ctg^2(x-3)} =$$

$$= \frac{3\ln^2 x \cdot \cos(x-3) \cdot \sin(x-3) + \ln^3 x \cdot x}{x \cdot \sin^2(x-3) \cdot ctg^2(x-3)} =$$

$$= \frac{\frac{3}{2}\ln^2 x \cdot \sin(2x-6) + \ln^3 x \cdot x}{x \cdot \cos^2(x-3)} =$$

$$= \frac{\ln^2 x \cdot \sin(2x-6) + 2x\ln^3 x}{2x \cdot \cos^2(x-3)} =$$

$$= \frac{\ln^2 x(\sin(2x-6) + 2x \cdot \ln x)}{2x \cos^2(x-3)}$$

$$y = \frac{7\log_5(x^2+x)}{(x+3)^3}$$

$$y' = \frac{7 \cdot \frac{1}{\ln 5 \cdot (x^2+x)} \cdot (2x+1) \cdot (x+3)^3 - 7\log_5(x^2+x) \cdot 3(x+3)^2}{((x+3)^3)^2} = \frac{(x+3)^2\left(\frac{7 \cdot (2x+1)}{\ln 5 \cdot (x^2+x)} \cdot (x+3) - 21\log_5(x^2+x)\right)}{(x+3)^6} =$$

$$= \frac{\frac{(14x+7) \cdot (x+3)}{\ln 5 (x^2+x)} - 21\log_5(x^2+x)}{(x+3)^4} =$$

$$= \frac{\frac{14x^2+49x+21}{\ln 5\cdot(x^2+x)} - 21\log_5(x^2+x)}{(x+3)^4} =$$

$$= \frac{\frac{14x^2+49x+21 - 21\log(x^2+x)\cdot\ln 5\cdot}{\ln 5(x^2+x)}}{(x+3)^4}$$

$$\frac{\cdot(x^2+x)}{\cdot\ln 5\cdot(x^2+x)} = \frac{14x^2+49x+21-21\cdot\log(x^2+x)\cdot}{\ln 5(x^2+x)\cdot(x+3)^4}$$

$$y = \sqrt{\frac{2x+3}{2x-3}}\,ctg(3x^2+5)$$

$$y' = \frac{1}{2\sqrt{\frac{2x+3}{2x-3}}}\cdot\frac{2(2x-3)-(2x+3)\cdot 2}{(2x-3)^2}\cdot$$

$$\cdot ctg(3x^2+5) + \sqrt{\frac{2x+3}{2x-3}}\cdot\left(-\frac{1}{\sin^2(3x^2+5)}\right)\cdot$$

$$\cdot 6x = \frac{1}{2\sqrt{\frac{2x+3}{2x-3}}}\cdot\frac{-12}{(2x-3)^2}\cdot ctg(3x^2+5)$$

$$-\sqrt{\frac{2x+3}{2x-3}}\cdot\frac{6x}{\sin^2(3x^2+5)} = -\frac{\sqrt{2x-3}}{\sqrt{2x+3}}\cdot$$

$$\cdot\frac{6}{(2x-3)^2}\cdot(ctg(3x^2+5)) - \sqrt{\frac{2x+3}{2x-3}}\cdot$$

$$\cdot\frac{6x}{\sin^2(3x^2+5)} = -\frac{6\sqrt{2x-3}\cdot\cos(3x^2+5)}{\sqrt{2x+3}\,(2x-3)^2\cdot\sin(3x^2+5)}$$

$$-\sqrt{\frac{2x+3}{2x-3}}\cdot\frac{6x}{\sin^2(3x^2+5)^2}$$

$$y = (x\sin x)^{3\ln(x\sin x)}$$

$$y' = \left((x\sin x)^{3\ln(x\sin x)}\right)' = \left(e^{8\ln^2(x\sin x)}\right)' =$$

$$= e^{8\ln^2(x\sin x)}\cdot(8\ln^2(x\sin x))' =$$

$$= e^{8\ln^2(x\sin x)}\cdot 2\cdot\ln(x\sin x)\cdot\frac{1}{x\sin x}\cdot$$

$$\cdot(\sin x + x\cos x) = 16e^{8\ln^2(x\sin x)}\cdot$$

$$\cdot \ln(x\sin x) \cdot \frac{1}{x\sin x} \cdot (\sin x + x\cos x) =$$

$$= \frac{16(x\sin x)\cdot(\sin x + \cos x)\cdot \ln(x\sin x)}{x\sin x}$$

$$y = (\operatorname{arctg} 5x)^{\log_2(x+4)}$$

$$y' = e^{\ln(\operatorname{arctg}5x)\cdot\log_2(x+4)} =$$

$$= e^{\ln(\operatorname{arctg}5x)/\log_2(x+4)} \cdot \left(\frac{1}{\operatorname{arctg}5x}\cdot\frac{1}{1+5x}\cdot\right.$$

$$\left.\cdot 5\log_2(x+4) + \ln(\operatorname{arctg}5x)\cdot\frac{1}{\ln 2\cdot(x+4)}\right) =$$

$$= \frac{(5\operatorname{arctg}5x)^{\log_2(x+4)-1}\cdot(\ln(x+4)\cdot(x+4)+ + (\operatorname{arctg}5x)^{\log_2(x+4)}\cdot(1+25x^2)\cdot\ln(\operatorname{arctg}5x)}{\ln 2\cdot(25x^3+100x^2+x+4)}$$

$$x^4+y'-2y=0$$

$$f(x) = 4x^3$$

$$f(y) = 4y^3-2$$

$$y = \frac{4x^3}{4y^3-2} = -\frac{4x^3}{2(2y^3-1)} = -\frac{2x^3}{2y^3-1}$$

$$y = \frac{\sqrt[3]{x-3}\,(x+7)^5}{(x-4)^2} = \frac{(x-3)^{\frac13}(x+7)^5}{(x-4)^2}$$

$$y' = \frac{\left(\frac13\cdot(x-3)^{-\frac23}\cdot(x+7)^5 + (x-3)^{\frac13}\cdot 5(x+7)^4\right)\cdot(x-4)^2 - (x-3)^{\frac13}\cdot(x+7)^5\cdot 2(x-4)}{(x-4)^4} =$$

$$= \frac{\left((x+7)^5 + (15x-45)\cdot(x+7)^4\right)\cdot(x-4) + (-6x+18)\cdot(x+7)^5}{3\cdot\sqrt[3]{(x-3)^2}\cdot(x-4)^3}$$

№2

$$y = \frac{1}{x} \qquad y^{(n)} = (y^{(n-1)})' \qquad \left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$$

$$y' = \left(\frac{1}{t}\right)' = \frac{1' \cdot t - 0 \cdot 1}{t^2} = -\frac{1}{t^2}$$

$$y'' \left(-\frac{1}{t^2}\right) = \frac{0 \cdot t^2 - 1 \cdot 2t}{t^4} = \frac{0 - 2t}{t^4} =$$

$$= \frac{2}{t^3}$$

$$y''' \cdot \left(\frac{2}{t^3}\right) = \frac{2' \cdot t^3 - 2 \cdot 3t^2}{t^6} = \frac{0 - 6t^2}{t^6} = -\frac{6}{t^4}$$

$$y^{IV} \left(-\frac{6}{t^4}\right) = \frac{0 + 6 \cdot 4t^3}{t^8} = \frac{24t^3}{t^8} = \frac{24}{t^5}$$

$$\left[-\frac{1}{t^2}; \frac{2}{t^3}; -\frac{6}{t^4}; \frac{24}{t^5}\right] \quad \frac{\cdot 1 \cdot 2 \cdot 3 \cdot 4 \ldots n}{- + - + \ldots} \quad t^2 \; t^3 \; t^4 \ldots$$

$$y^{(n)} = n \cdot (n-1) \cdot t^{-(n+2)}$$

1) $-\frac{1}{t^2} = \frac{1}{t} \cdot \left(-\frac{1}{t}\right)$

2) $\frac{2}{t^3} = \frac{1}{t} \cdot \frac{2}{t^2}$

3) $-\frac{6}{t^4} = \frac{1}{t} \cdot \left(-\frac{6}{t^3}\right)$

4) $\frac{24}{t^5} = \frac{1}{t} \cdot \frac{24}{t^4}$

$$y^{(4)} = 4^2 - 4 \cdot t^{-(4+2)}$$

$$y^{(4)} = 16 - 4 \cdot t^{-6}$$

$$y^{(4)} = 12 \cdot \frac{1}{t^6}$$

№3

$$y' = \left(\frac{5t - 8}{2^t}\right)' = -\frac{5\ln 2 \cdot t - 8\ln(2) - 5}{2^t}$$

$$y'' = \left(-\frac{5\ln(2) \cdot t - 8\ln(2) - 5}{2^t}\right)' = \frac{5\ln^2(2) \cdot t -}{2^t}$$

$$\underline{- 8\ln^2(2) - 10\ln(2)}$$

$$y^{IV} = \left(\frac{5\ln^2(2)\cdot x - 8\ln^2(2) - 1\ln(2)}{2^x}\right)' =$$

$$= -\frac{5\cdot\ln^3(2)\cdot x - 8\cdot\ln^3(2) - 15\ln^2(2)}{2^x}$$

№4

$$\sqrt{x} + \sqrt{y} = \sqrt{7}$$

$$\sqrt{y} = \sqrt{7} - \sqrt{x}$$

$$\left(\sqrt{y} = \sqrt{7} - \sqrt{x}\right)' = \frac{1}{2}\cdot\frac{1}{\sqrt{y}}\cdot y' =$$

$$= -(\sqrt{x})' + (\sqrt{7})'$$

$$\frac{y'}{2\sqrt{y}} = -\frac{1}{2}\cdot\frac{1}{\sqrt{x}} + 0$$

$$\frac{y'}{2\sqrt{y}} = -\frac{1}{2\sqrt{x}}$$ ← первый порядок

$$\left(\frac{y'}{2\sqrt{y}} = -\frac{1}{2\sqrt{x}}\right)' = \frac{1}{2}\cdot\left(\frac{y'}{\sqrt{y}}\right)' = -\frac{1}{2}\left(-\frac{1}{\sqrt{x}}\right)'$$

$$\frac{1}{2}\cdot\frac{(y')'\cdot\sqrt{y} - (\sqrt{y})'\cdot y'}{y} = -\left(\frac{1}{4x^{\frac{3}{2}}}\right)$$

$$\frac{y''}{2\sqrt{y}} - \frac{y'^2}{4y^{\frac{3}{2}}} = \frac{1}{4x^{\frac{3}{2}}}$$

№5

$$\begin{cases} x = 5\cos t \\ y = 4\sin t \end{cases}$$

$$y'_x = \frac{y'_t}{x'_t} \qquad y''_x = \frac{(y'_x)'_t}{x'_t}$$

$$y'_x = \frac{4\cos(t)}{-5\sin(t)}$$

$$y''_x = \frac{\left(\frac{4\cos(t)}{-5\sin(t)}\right)'}{-5\sin(t)} =$$

$$= \frac{\frac{4}{5\sin(t)^2}}{-5\sin(t)} = -\frac{4}{\sin(t)}$$

$$y'_x = \frac{4\cos(t)}{-5\sin(t)}$$

$$y''_x = \frac{\left(\frac{4\cos(t)}{-5\sin(t)}\right)'}{-5\sin(t)} =$$

$$= \frac{\frac{4}{5\sin(t)^2}}{-5\sin(t)} = -\frac{4}{\sin(t)}$$

№6

$$\lim_{x\to 0} \frac{\frac{\pi}{x}}{ctg(\pi x/2)} = \lim_{x\to 0} \frac{-\frac{\pi}{x^2}}{-\frac{1}{\sin^2(\frac{\pi}{2x})}} =$$

$$= \lim_{x\to 0} -\frac{\pi}{x^2} \cdot \sin^2\left(\frac{\pi}{2x}\right) = \lim_{x\to 0} \frac{\sin^2(\frac{\pi}{2x})\pi}{x^2} =$$

$$= -\lim_{x\to 0} \left(\sin^2\left(\frac{\pi}{2}\cdot x\right)\cdot \pi \cdot \frac{1}{x^2}\right) = -\pi \cdot$$

$$\cdot \lim_{x\to 0} \left(\sin\left(\frac{\pi}{2}\cdot x\right)^2 - \frac{1}{x^2}\right) = -\pi \lim_{x\to 0} \left(\frac{\sin(\frac{\pi}{2}\cdot x)^2}{x^2}\right) = -\pi \lim_{x\to 0} \left(\frac{\frac{d}{dx}\left(\sin\left(\frac{\pi}{2}\cdot x\right)^2\right)}{\frac{d}{dx}(x^2)}\right) =$$

$$= -\pi \lim_{t\to 0}\left(\frac{\pi \cdot \sin\left(\frac{\pi t}{2}\right)\cos\left(\frac{\pi t}{2}\right)}{2t}\right) =$$

$$= -\pi \lim_{t\to 0}\left(\frac{\frac{d}{dt}\left(\pi \sin\left(\frac{\pi t}{2}\right)\cos\left(\frac{\pi t}{2}\right)\right)}{\frac{d}{dt}(2t)}\right) =$$

$$= -\pi \lim_{t\to 0}\left(\frac{\frac{\pi^2 \cos(\pi t)}{2}}{2}\right) =$$

$$= -\pi \cdot \lim_{t\to 0}\left(\frac{\pi^2 \cos(\pi t)}{4}\right) =$$

$$= -\pi \cdot \frac{\pi^2 \cos(\pi \cdot 0)}{4} = -\frac{\pi^3}{4}$$

$$\lim_{t\to 0}\frac{1-\cos at}{1-\cos bt} = \lim_{t\to 0}\frac{0+a\sin at}{0+b\sin bt} =$$

$$= \lim_{t\to 0}\frac{a\sin at}{b\sin bt} = \lim_{t\to 0}\frac{a^2-\sin t}{b^2-\sin t} =$$

$$= \frac{a^2}{b^2}$$

№7

$y = t^7 \quad t = 1{,}996$

$f(x) \approx f(t_0) + f'(t_0)\cdot(t - t_0)$

$f'(t) = 7t^6$

$t_0 = 1{,}9$

$f(x) \approx 89{,}38717 +$

$+ 329{,}321167 \cdot 0{,}096 \approx 121$

$y = x^2 \quad x = \lg 0{,}9$

$f(x) \approx f(x_0) + f'(x)(x - x_0)$

$f'(x) = 7x^6$

$x_0 = \lg 0{,}9 \cdot 7 = -7$

$f(x) \approx -1 + 7 \cdot 0{,}8 = 9{,}6$

№8

$y = (x-1)^2 (x-3)^2 \quad D(y) = (0; +\infty) \quad D(x) = \mathbb{R}$

~~D(x)=(-\infty;+\infty)~~

$f'(x) = ((x-1)^2 (x-3)^2)'$

$f'(x) = 4x^3 - 24x^2 + 44x - 24$

$4x^3 - 24x^2 + 44x - 24 = 0$

$4(x^3 - 6x^2 + 11x - 6) = 0$

$4(x^2(x-1) - 5x(x-1) +$

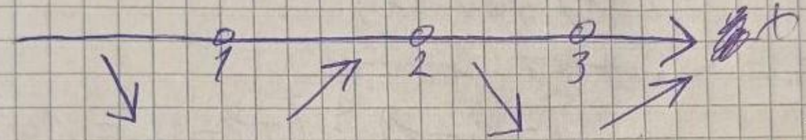
$+ 6(x-1)) = 0$

$4(x-1)(x^2 - 5x + 6) = 0$

$4(x-1)(x(x-2) - 3(x-2)) = 0$

$4(x-1)(x-2)(x-3) = 0$

$\left.\begin{matrix} x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{matrix}\right\}$ экстремумы



функц. ↓

$x=1 \quad (1-1)^2\cdot(1-3)^2=0$ ~~…~~ ~~…~~ $x \ngtr 0$

$x=2 \quad (2-1)^2\cdot(2-3)^2=1 \quad x>0 \Rightarrow$ функц. ↗

$x=3 \quad (3-1)^2\cdot(3-3)^2=0 \quad x \ngtr 0$ ~~функц.~~

функц. ↓

$y\downarrow \in (-\infty; 1) \cup (2; 3)$

$y\nearrow \in (1; 2) \cup (3; +\infty)$

~~$y=$~~ $0=(x-1)^2(x-3)^2$

$y=0 \quad x_1=1 \quad x_2=3$ — точки пересеч. с осью координат x

$x=0$

$(0-1)^2\cdot(0-3)^2=9$

$x=0 \quad y=9$ — точка пересечения с осью y

$f(-x) \neq -f(x)$

$x=-4$

$(-4-1)^2\cdot(-4-3)^2=1225$

$x=4$

$-((4-1)^2\cdot(4-3)^2)=-9$

$1225 \neq 9$

Функция ни чётная ни нечётная

~~$y=(1$~~ $y=(x-1)^2\cdot(x-3)^2$

$$y'' = \left((x-1)^2 \cdot (x-3)^2\right)''$$

$$y'' = 12x^2 - 48x + 4$$

$+ \quad 0 \quad - \quad 0 \quad +$

$\frac{6-\sqrt{33}}{3} \qquad \frac{6+\sqrt{33}}{3}$

$$12x^2 - 48x + 4 = 0$$

$$D = b^2 - 4ac$$

$$D = -48^2 - 4 \cdot 12 \cdot 4 = 2112$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{48 + \sqrt{2112}}{24} =$$

$$= \frac{6+\sqrt{33}}{3}$$

$$x_2 = \frac{6-\sqrt{33}}{3} \approx 0{,}085$$

$$12 \cdot 0{,}08^2 - 48 \cdot 0{,}08 + 4 > 0$$

$$12 \cdot 0{,}09^2 - 48 \cdot 0{,}09 + 4 < 0$$

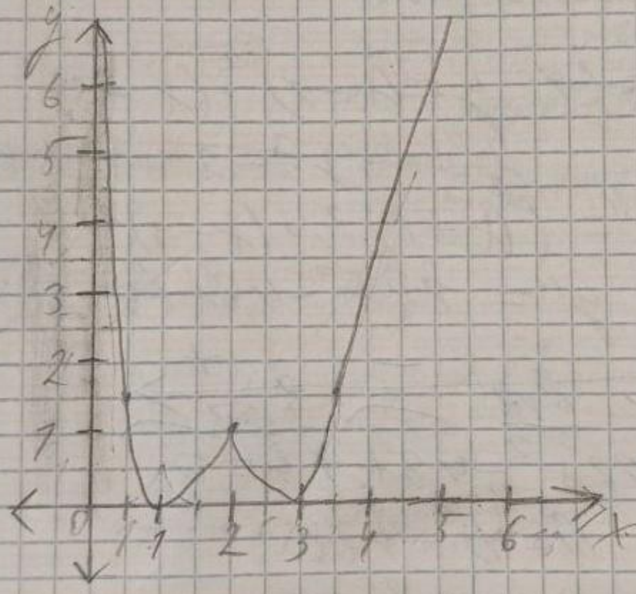
$+ \quad 0 \quad - \quad 0 \quad +$

$\frac{6-\sqrt{33}}{3} \qquad \frac{6+\sqrt{33}}{3}$

$$\lim_{x\to\infty} (x-1)^2(x-3)^2 = \lim_{x\to\infty} (\infty-1)^2(\infty-3)^2 = \infty$$

$$\lim_{x\to 1} (x-1)^2(x-3)^2 = \lim_{x\to 1} (1-1)^2(x-3)^2 = 0$$

Асимптот нет т.к. функция $\to \infty$ только если $x \to \infty$



$x_1 = 1 \quad y = 0$

$x_2 = 3 \quad y = 0$

$x = 2 \quad y = 1$

$x = 0,5 \quad y = 1,5$

$x = 3,5 \quad y = 1,5$

$y = \frac{2+x}{(x+1)^2}$ $\quad D(x) = (\mathbb{R}) \cup (x \neq -1)$

$D(y) = (\mathbb{R})$

$f(x) = -\frac{x\cancel{2}+3}{(x+1)^3}$

$-\frac{x+3}{(x+1)^3} = 0$

$\frac{x+3}{(x+1)^3} = 0$

$x \neq -1$

$x+3=0$

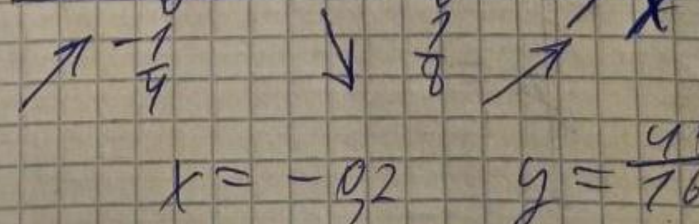
$x=-3$

$\frac{2-3}{(-3+1)^2} = -\frac{1}{4}$ - экстремум максимума

$y'' = \frac{2x+8}{(x+1)^4}$ $\quad x=-3$

$\cancel{y'}$ $\cancel{2x+}$

$\frac{2\cdot(-3)+8}{(-3+1)^4} = \frac{1}{8}$ - экстремум минимума



$x = -0{,}2$ $\quad y = \frac{45}{16} = 2\frac{13}{16}$

$x = 0{,}124$ $\quad y = 1\frac{53789}{78961}$

$0 = \frac{2+x}{(x+1)^2}$

$x \neq -1$

$2+x=0$

$y=0 \quad x=-2$ — точка пересеч. с y

$x=0 \quad y=2$ — точка пересеч. с x

$f(-x) \neq f(x)$

$-x = \cancel{} -3$

$\cancel{2-1} \quad \frac{2-3}{(-3+1)^2} = -\frac{1}{4}$

$\frac{2+3}{(3+1)^2} = \frac{5}{16}$

$-\frac{1}{4} \neq \frac{5}{16}$

$f(-x) \neq -f(x)$

$-x = -4$

$\frac{2-4}{(-4+1)^2} = -\frac{2}{9}$

$-\left(\frac{2+4}{(4+1)^2}\right) = \frac{6}{25}$

$-\frac{2}{9} \neq \frac{6}{25}$

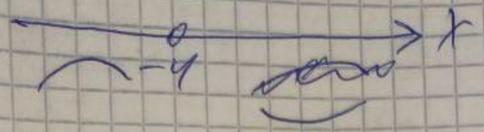
Функция ни чётная, ни нечётная

$y'' = \frac{2x+8}{(x+1)^4}$

$x \neq -1$

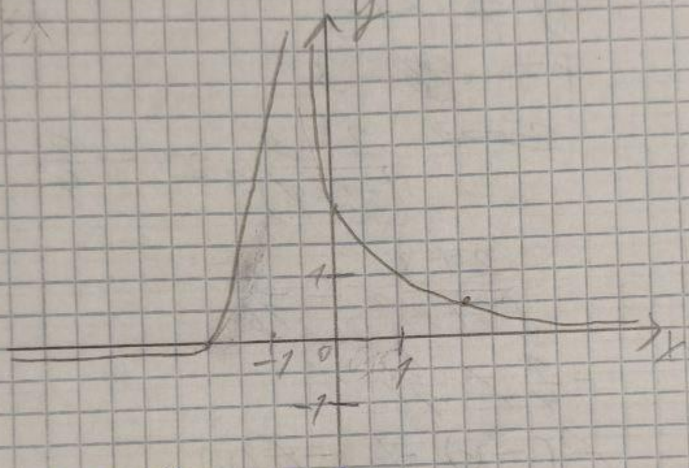
$2x+8=0$

$x=-4$ — точка перегиба



$$\lim_{x\to -1} \frac{2+x}{(x+1)^2} = \frac{2-1}{(-1+1)^2} = +\infty$$ — вертик. асимптота

$$\lim_{x\to -2} \frac{2+x}{(x+1)^2} = \frac{2-2}{(-2+1)^2} = 0$$ — гориз. асимптота



$x_1 = 0 \quad y_1 = 2$

$x_2 = 2 \quad y_2 = 0{,}5$

$x_3 = -2 \quad y_3 = 0$

№9

$u = \cos^2(xy) + 3z$

$M(\frac{\pi}{4}; 1; 2)$

$\vec{S} = \vec{i} - 3j + 4k$

$\frac{\partial u}{\partial x} = 2\cos(xy) \cdot (-\sin(xy)) \cdot y + 0 =$
$= 2\cos(xy) \cdot (-\sin(xy) \cdot y = -\sin(2xy) \cdot y$

$\frac{\partial u}{\partial y} = 2\cos(xy) \cdot (-\sin(xy)) \cdot x + 0 =$
$= -\sin(2xy) \cdot x$

$\frac{\partial u}{\partial z} = 0 + 3 = 3$

$\frac{\partial u}{\partial x}(M_0) = -\sin(2 \cdot \frac{\pi}{4} \cdot 1) \cdot 1 =$
$= -\sin\frac{\pi}{2}$

$\frac{\partial u}{\partial y}(M_0) = -\sin(2 \cdot \frac{\pi}{4} \cdot 1) \cdot \frac{\pi}{4} =$
$= -\sin\frac{\pi}{2} \cdot \frac{\pi}{4}$

$\frac{\partial u}{\partial z}(M_0) = 3$

$\mathrm{grad}\,\vec{u}\,(-\sin\frac{\pi}{2}; -\sin\frac{\pi}{2} \cdot \frac{\pi}{4}; 3)$

$\vec{S}\,(1; -3; 4)$

$|\vec{S}| = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{26}$

$\vec{l}\,(\frac{1}{\sqrt{26}}; \frac{-3}{\sqrt{26}}; \frac{4}{\sqrt{26}})$

$\frac{\partial u}{\partial l} = \frac{\partial u}{\partial x}(M_0) \cdot l_x + \frac{\partial u}{\partial y}(M_0) \cdot l_y +$
$+ \frac{\partial u}{\partial z}(M_0) \cdot l_z$

$\frac{\partial u}{\partial l} = \frac{1}{\sqrt{26}} \cdot (-\sin\frac{\pi}{2}) + \sin\frac{\pi}{2} \cdot \frac{\pi}{4} \cdot \frac{3}{\sqrt{26}} +$
$+ 3 \cdot \frac{4}{\sqrt{26}} = \frac{1}{\sqrt{26}} \cdot (-\sin\frac{\pi}{2}) + \frac{3}{\sqrt{26}} \cdot \frac{\pi}{4} \cdot$

$\cdot \sin\frac{\pi}{2} + \frac{4}{\sqrt{26}} \cdot 3$

№10

$$\frac{\partial z}{\partial x} = -\frac{1}{1+x^2y^4} \cdot y^2 = -\frac{y^2}{1+x^2y^4}$$

$$\frac{\partial z}{\partial y} = -\frac{1}{1+x^2y^4} \cdot 2xy = -\frac{2xy}{1+x^2y^4}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$dz = -\frac{y^2}{1+x^2y^4}dx - \frac{2xy}{1+x^2y^4}dy$$

№11

$$z = (x-2)^2 + 2y^2 - 10 = x^2 - 4x + 4 + 2y^2 - 10 =$$
$$= x^2 - 4x + 2y^2 - 6$$

$$\begin{cases} z'_x = 0 \\ z'_y = 0 \end{cases} \quad \begin{cases} z'_x = (x^2 - 4x + 2y^2 - 6)' \\ z'_y = (x^2 - 4x + 2y^2 - 6)' \end{cases}$$

$$\begin{cases} 2x - 4 = 0 \\ 4y = 0 \end{cases} \quad \begin{cases} 2x = 4 \\ 4 = -y \end{cases}$$

$$\begin{cases} x = 2 \\ y = -4 \end{cases}$$

$M(2; -4)$ – подозрен. на экстремум

$$\Delta = \begin{vmatrix} Z_{xx}(M) & Z_{xy}(M) \\ Z_{yx}(M) & Z_{yy}(M) \end{vmatrix} > 0$$

$Z_{xx} > 0 \Rightarrow \min$ $\quad Z_{xx}(M) < 0 \Rightarrow \max$

$Z = (2-2)^2 + 2 \cdot 16 - 10 = 32 - 10 = 22$

$\Delta < 0 \Rightarrow (M)$ – не экстремум

1) $Z''_{xx} = 2$ $\quad$ 3) $Z''_{xy} = 0$

2) $Z''_{xy} = 0$ $\quad$ 4) $Z''_{yy} = 4$ $\quad \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} =$

$= 8 - 0 = 8 \Rightarrow$ экстремум

$Z_{xx}(M) > 0 \Rightarrow \min$