Индивидуальное домашнее задание №2

По дисциплине: «Алгебра»

Курс 1. Семестр 1

Вариант №11

№1

$$\begin{vmatrix} -1 & -2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & -2 & 1 & 4 \\ 3 & 1 & -2 & -1 \end{vmatrix}$$

а) $-3\begin{vmatrix} -2 & 4 & 1 \\ 3 & 0 & 6 \\ -2 & 1 & 4 \end{vmatrix} + 1\begin{vmatrix} -1 & 4 & 1 \\ 2 & 0 & 6 \\ 2 & 1 & 4 \end{vmatrix} +$

$+ 2\begin{vmatrix} -1 & -2 & 1 \\ 2 & 3 & 6 \\ 2 & -2 & 4 \end{vmatrix} - 1\begin{vmatrix} -1 & -2 & 4 \\ 2 & 3 & 0 \\ 2 & -2 & 1 \end{vmatrix} =$

$= 6\begin{vmatrix} 0 & 6 \\ 1 & 4 \end{vmatrix} + 12\begin{vmatrix} 3 & 6 \\ -2 & 4 \end{vmatrix} - 3\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} -$

$- 1\begin{vmatrix} 0 & 6 \\ 1 & 4 \end{vmatrix} - 4\begin{vmatrix} 2 & 6 \\ 2 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} -$

$- 2\begin{vmatrix} 3 & 6 \\ -2 & 4 \end{vmatrix} + 4\begin{vmatrix} 2 & 6 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ 2 & -2 \end{vmatrix} +$

$+ 1\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 2 & -2 \end{vmatrix} =$

$= -36 + 288 - 9 + 6 + 16 + 2 - 48 - 16 -$

$- 20 + 3 - 4 + 40 = 222$

б) $4\begin{vmatrix} 2 & 3 & 6 \\ 2 & -2 & 4 \\ 3 & 1 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 1 \\ 2 & 3 & 6 \\ 3 & 1 & -1 \end{vmatrix} +$

$$+2\begin{vmatrix} -1 & -2 & 1 \\ 2 & 3 & 6 \\ 2 & -2 & 4 \end{vmatrix} = 8\begin{vmatrix} -2 & 4 \\ 1 & -1 \end{vmatrix} - 8\cdot\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} + 12\begin{vmatrix} 3 & 6 \\ -2 & 4 \end{vmatrix} - 1\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} -$$

$$-2\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} -2 & 1 \\ 3 & 6 \end{vmatrix} - 2\begin{vmatrix} 3 & 6 \\ -2 & 4 \end{vmatrix} - 4\cdot$$

$$\cdot\begin{vmatrix} -2 & 1 \\ -2 & 4 \end{vmatrix} + 4\begin{vmatrix} -2 & 1 \\ 3 & 6 \end{vmatrix} =$$

$$= -16+72+288+9-2-45-48+$$

$$+24-60=222$$

№2

$$A=\begin{pmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{pmatrix} \quad B=\begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 0 & 5 & 2 \end{pmatrix}$$

a) $$AB=\begin{pmatrix} 6\cdot1+9\cdot3+4\cdot0 & 6\cdot1+9\cdot4+4\cdot5 & 6\cdot1+9\cdot3+4\cdot2 \\ -1\cdot1+(-1)\cdot3+1\cdot0 & -1\cdot1+(-1)\cdot4+1\cdot5 & -1\cdot1+(-1)\cdot3+1\cdot2 \\ 10\cdot1+1\cdot3+7\cdot0 & 10\cdot1+1\cdot4+7\cdot5 & 10\cdot1+1\cdot3+7\cdot2 \end{pmatrix}$$

$$AB=\begin{pmatrix} 33 & 62 & 41 \\ -4 & 0 & -2 \\ 13 & 49 & 27 \end{pmatrix}$$

б) $BA = \begin{pmatrix} 1\cdot 6+1\cdot(-1)+1\cdot 0 & 1\cdot 9+1\cdot(-1)+1\cdot 1 & 1\cdot 4+1\cdot 1+1\cdot 7 \\ 3\cdot 6+4\cdot(-1)+3\cdot 10 & 3\cdot 9+4\cdot(-1)+3\cdot 1 & 3\cdot 4+4\cdot 1+3\cdot 7 \\ 0\cdot 6+5\cdot(-1)+2\cdot 10 & 0\cdot 9+5\cdot(-1)+2\cdot 1 & 0\cdot 4+5\cdot 1+2\cdot 7 \end{pmatrix}$

$BA = \begin{pmatrix} 15 & 9 & 12 \\ 44 & 26 & 37 \\ 15 & -3 & 19 \end{pmatrix}$

в) A^{-1}

$\begin{vmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{vmatrix} = -42+90-4+40-$

$-6+63=141$

$\det A = 141$

$A_{11}\begin{vmatrix} -1 & 1 \\ 1 & 7 \end{vmatrix} = -8$ $\quad A_{23}\begin{vmatrix} 6 & 9 \\ 10 & 1 \end{vmatrix} = 84$

$A_{12}\begin{vmatrix} -1 & 1 \\ 10 & 7 \end{vmatrix} = 17$ $\quad A_{31}\begin{vmatrix} 9 & 4 \\ -1 & 1 \end{vmatrix} = 13$

$A_{13}\begin{vmatrix} -1 & -1 \\ 10 & 1 \end{vmatrix} = 9$ $\quad A_{32}\begin{vmatrix} 6 & 4 \\ -1 & 1 \end{vmatrix} = -10$

$A_{21}\begin{vmatrix} 9 & 4 \\ 1 & 7 \end{vmatrix} = -53$ $\quad A_{33}\begin{vmatrix} 6 & 9 \\ -1 & -1 \end{vmatrix} = 3$

$A_{22}\begin{vmatrix} 6 & 4 \\ 10 & 7 \end{vmatrix} = 2$

$$A^{-1}=\frac{1}{141}\begin{pmatrix}-8 & -59 & 13\\ 17 & 2 & -10\\ 9 & 84 & 3\end{pmatrix}=$$

$$=\begin{pmatrix}-\frac{8}{141} & -\frac{59}{141} & \frac{1}{141}\\ \frac{17}{141} & \frac{2}{141} & -\frac{10}{141}\\ \frac{3}{47} & \frac{28}{47} & \frac{1}{47}\end{pmatrix}$$

г) AA^{-1}

$$\begin{pmatrix}6 & 9 & 4\\ -1 & -1 & 1\\ 10 & 1 & 7\end{pmatrix}\cdot\begin{pmatrix}-\frac{8}{141} & -\frac{59}{141} & \frac{1}{141}\\ \frac{17}{141} & \frac{2}{141} & -\frac{10}{141}\\ \frac{3}{47} & \frac{28}{47} & \frac{1}{47}\end{pmatrix}=$$

$$=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

в) $A^{-1}A$ $$\begin{pmatrix}-\frac{8}{141} & -\frac{59}{141} & \frac{1}{141}\\ \frac{17}{141} & \frac{2}{141} & \frac{-10}{141}\\ \frac{3}{47} & \frac{28}{47} & \frac{1}{47}\end{pmatrix}\cdot\begin{pmatrix}6 & 9 & 4\\ -1 & -1 & 1\\ 10 & 1 & 7\end{pmatrix}=$$

$$=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

№3

$$\begin{cases} 5x_1 + 2x_2 - 4x_3 = -16 \\ x_1 + 3x_3 = -6 \\ 2x_1 - 3x_2 + x_3 = 9 \end{cases}$$

a) $\Delta = \begin{vmatrix} 5 & 2 & -4 \\ 1 & 0 & 3 \\ 2 & -3 & 1 \end{vmatrix} = 0 + 12 + 12 - 0 +$

$+ 45 - 2 = 67$

$\Delta_1 = \begin{vmatrix} -16 & 2 & -4 \\ -6 & 0 & 3 \\ 9 & -3 & 1 \end{vmatrix} = 0 + 54 - 72 - 0 - 144 + 12 = -150$

$\Delta_2 = \begin{vmatrix} 5 & -16 & -4 \\ 1 & -6 & 3 \\ 2 & 9 & 1 \end{vmatrix} = -30 - 96 - 36 - 48 - 135 + 16 = -329$

$\Delta_3 = \begin{vmatrix} 5 & 2 & -16 \\ 1 & 0 & -6 \\ 2 & -3 & 9 \end{vmatrix} = 0 - 24 + 48 - 0 -$

$-90-18=-84$

$$x_1=\frac{\Delta_1}{\Delta}=-\frac{150}{67}$$

$$x_2=\frac{\Delta_2}{\Delta}=-\frac{329}{67}$$

$$x_3=\frac{\Delta_3}{\Delta}=-\frac{84}{67}$$

б) $A=\begin{pmatrix} 5 & 2 & -4 \\ 1 & 0 & 3 \\ 2 & -3 & 1 \end{pmatrix}$

$\det A = 67$

$A_{11}=\begin{vmatrix} 0 & 3 \\ -3 & 1 \end{vmatrix}=9$

$A_{12}=-\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}=5$

$A_{13}=\begin{vmatrix} 1 & 0 \\ 2 & -3 \end{vmatrix}=-3$

$A_{21}=\begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix}=10$

$A_{22}=\begin{vmatrix} 5 & -4 \\ 2 & 1 \end{vmatrix}=13$

$A_{23}=-\begin{vmatrix} 5 & 2 \\ 2 & -3 \end{vmatrix}=19$

$A_{31}=\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}=6$

$A_{32}=-\begin{vmatrix} 5 & -4 \\ 1 & 3 \end{vmatrix}=-19$

$$A_{33} = \begin{vmatrix} 5 & 2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A^{-1} = \frac{1}{67} \begin{pmatrix} 9 & 10 & 6 \\ 5 & 13 & -19 \\ -3 & 19 & -2 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{9}{67} & \frac{10}{67} & \frac{6}{67} \\ \frac{5}{67} & \frac{13}{67} & -\frac{19}{67} \\ -\frac{3}{67} & \frac{19}{67} & -\frac{2}{67} \end{pmatrix}$$

$$\begin{pmatrix} \frac{9}{67} & \frac{10}{67} & \frac{6}{67} \\ \frac{5}{67} & \frac{13}{67} & -\frac{19}{67} \\ -\frac{3}{67} & \frac{19}{67} & -\frac{2}{67} \end{pmatrix} \cdot \begin{pmatrix} -16 \\ -6 \\ 9 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{150}{67} \\ -\frac{329}{67} \\ -\frac{84}{67} \end{pmatrix}$$

$$x_1 = -\frac{150}{67} \quad x_2 = -\frac{329}{67}$$

$$x_3 = -\frac{84}{67}$$

№4

$$x = \{x_1; x_2; x_3\}$$

$$Ax = \{x_2 - x_3; x_1; x_1 + x_3\}$$

$$Bx = \{x_2; 2x_3; x_1\}$$

$$2(B - A + B^2)x - ?$$

$$A \cdot \{x_1; x_2; x_3\} = \{x_2 - x_3;$$
$$x_1; x_1 + x_3\}$$

$$\begin{cases} A_x = \frac{x_2 - x_3}{x_1} \\ A_y = \frac{x_1}{x_2} \\ A_z = \frac{x_1 + x_3}{x_3} \end{cases}$$

$$B \cdot \{x_1; x_2; x_3\} = \{x_2; 2x_3; x_1\}$$

$$\begin{cases} B_x = \frac{x_2}{x_1} \\ B_y = \frac{2x_3}{x_2} \\ B_z = \frac{x_1}{x_3} \end{cases}$$

$$2\left(\left(\frac{t_2}{t_1}-\frac{t_2-t_3}{t_1}+\left(\frac{t_2}{t_1}\right)^2\right)\cdot\right.$$
$$\cdot\left(\frac{2t_3}{t_2}-\frac{t_1}{t_2}+\left(\frac{2t_3}{t_2}\right)^2\right)\cdot\left(\frac{t_1}{t_3}-\right.$$
$$\left.\left.-\frac{t_1+t_3}{t_3}+\left(\frac{t_1}{t_3}\right)^2\right)\right)\cdot(t_1;t_2;t_3)$$

№5

$A(-2;-3;-2)$ $B(1;4;2)$

$C(1;-3;3)$

а) $\vec{a}=2AC-4BC$

$\vec{a}=(2\cdot(1+2;-3+3;3+$
$+2))-(4(1-1;-3-4;3-2))=$
$=2\cdot(3;0;5)-4\cdot(0;-7;1)=$
$=(6;0;10)-(0;-28;4)=$
$=(6;28;6)$

$|a|=\sqrt{x^2+y^2+z^2}$

$|a|=\sqrt{6^2+28^2+6^2}=2\sqrt{214}$

б) $\vec{a}=2AC-4BC$

$\vec{b}=AB$

$\vec{a}=(6;28;6)$

$\vec{b}=(1+2;4+3;2+2)=$

$= (3; 7; 4)$

$(\vec{a}; \vec{b}) = (8 \cdot 3 + 28 \cdot 7 + 6 \cdot 4) =$
$= 238$

в) $\alpha = 3 \quad \beta = 1 \quad L = BC$

$\lambda = \frac{\alpha}{\beta} \quad \lambda = \frac{3}{1}$

$$\begin{cases} x_M = \dfrac{x_\beta + \lambda \cdot x_\alpha}{1+\lambda} \\ y_M = \dfrac{y_\beta + \lambda y_\alpha}{1+\lambda} \\ z_M = \dfrac{z_\beta + \lambda z_\alpha}{1+\lambda} \end{cases}$$

$x_M = \dfrac{1 + \frac{3}{1} \cdot 1}{\frac{3}{1} + 1} = 1$

$y_M = \dfrac{4 + \frac{3}{1}(-3)}{\frac{3}{1} + 1} = -\frac{5}{4} =$
$= -1\frac{1}{4}$

$z_M = \dfrac{2 + \frac{3}{1} \cdot 3}{1 + \frac{3}{1}} = \frac{9}{4} = 2\frac{1}{4}$

$$\begin{cases} x_M = 1 \\ y_M = -1\frac{1}{4} \end{cases}$$

$\left[z_4 = 2\frac{1}{4} \right.$

№6

$\vec{a} = -7\vec{i} + 2\vec{k}$

$\vec{b} = 2\vec{i} - 6\vec{j} + 4\vec{k}$

$\vec{c} = \vec{i} - 3\vec{j} + 4\vec{k}$

a) $\vec{a} = \{-7; 0; 2\}$

$\vec{b} = \{2; -6; 4\}$

$\vec{c} = \{1; -3; 4\}$

$(2\vec{a}, -2\vec{b}, -7\vec{c})$

$2\vec{a} = \{-14; 0; 4\}$

$-2\vec{b} = \{-4; 12; -8\}$

$-7\vec{c} = \{-7; 21; -28\}$

$$\begin{vmatrix} -14 & 0 & 4 \\ -4 & 12 & -8 \\ -7 & 21 & -28 \end{vmatrix} = 14\begin{vmatrix} 12 & -8 \\ 21 & -28 \end{vmatrix} -$$

$$- 0 \cdot \begin{vmatrix} -4 & -8 \\ -7 & -28 \end{vmatrix} + 4\begin{vmatrix} -4 & 12 \\ -7 & 21 \end{vmatrix} =$$

$$= -14(-168) + 4(-84 - (-84)) =$$

$= 7056 + (-672) = 2352$

б) $4\vec{b}$ $3\vec{c}$

$4\vec{b} = (8; -24; 16)$

$3\vec{c} = (3; -9; 12)$

$$\Delta = \begin{vmatrix} i & j & k \\ 8 & -24 & 16 \\ 3 & -9 & 12 \end{vmatrix} = i \begin{vmatrix} -24 & 16 \\ -9 & 12 \end{vmatrix} -$$

$$- j \begin{vmatrix} 8 & 16 \\ 3 & 12 \end{vmatrix} + k \begin{vmatrix} 8 & -24 \\ 3 & -9 \end{vmatrix} =$$

$= i(-288 + 144) - j(96 - 48) + k(-72 +$
$+ 72) = -144i - 48j + 0 \cdot k$

$|\Delta| = \sqrt{(-144)^2 + (-48)^2 + (0)^2} =$
$= 48\sqrt{10}$

в) $2\vec{a}$ $-7\vec{c}$

$2a = (-14; 0; 4)$

$-7c = (-7; 21; -28)$

$(2a; -7c) = (-14 \cdot (-7)) + (0 \cdot 21) +$
$+ (4 \cdot (-28)) = -14$

г) $\vec{b}$ $\vec{c}$

$\frac{2}{1} \neq \frac{-6}{-3} \neq \frac{4}{4} \Rightarrow$ вектора не коллинеарны

$(2\cdot 1)+(-6\cdot(-3))+(4\cdot 4)=$

$=36 \Rightarrow$ не ортогональны т. к. $\neq 0$

д) $2\vec{a}$ $4\vec{b}$ $3\vec{c}$

$2\vec{a}=(-14; 0; 2)$

$4\vec{b}=(8; -24; 16)$

$3\vec{c}=(3; -9; 12)$

$$\begin{vmatrix} -14 & 0 & 2 \\ 8 & -24 & 16 \\ 3 & -9 & 12 \end{vmatrix} = -14\begin{vmatrix} -24 & 16 \\ -9 & 12 \end{vmatrix} - 0\cdot\begin{vmatrix} 8 & 16 \\ 3 & 12 \end{vmatrix} + 2\begin{vmatrix} 8 & -24 \\ 3 & -9 \end{vmatrix} =$$

$= -14((-24\cdot 12)-(16\cdot(-9))+$

$+2((8\cdot(-9))-(3\cdot(-24)))=$

$=2016 \Rightarrow$ не компланарны