Investigation of Energy Quantization and the Properties of Light

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Three experiments were conducted which tested light-emitting diodes, single slit diffraction patterns, and black-body radiation; these produced values for Planck's constant, confirmed theory for single slit diffraction, reconstructed the black-body curve of a lightbulb and predicted its temperature, and allowed for a derivation of Heisenberg's uncertainty principle. Planck's constant was experimentally found to be (6.10±.04) E-34 m^2 kg s^{-1}, with an error of 7.88% from the theoretical value. A mean squared error analysis revealed a strong correlation of 207 for the periodicity of the single slit diffraction, but a weak correlation of 11 for the intensity amplitudes.

I Introduction

The importance of quantum mechanics is felt in everyday life; the field is the basis of all subatomic interactions. These experiments aim to investigate the behaviour of light and further understand its properties.

II Theory

The theory for the diode circuit was based upon the work of Godkhindi et al in Transients and Semi Conductors (2016).

A photon's energy is:

$$E = \frac{hc}{\lambda} \qquad [1]$$

(Halliday, 2011)

Where:

E = energy of a photon (J)
h = Plancks constant (m^2 kg s^{-1})
c = speed of light (m s^{-1})
λ = wavelength (m)

The de Broglie hypothesis describes wave behaviour of matter as:

$$p = \frac{h}{\lambda} \qquad [2]$$

(Halliday, 2011)

Where:

p = momentum of particle (kg m s^{-1})

For an LED in a DC circuit:

$$E = eV_f \qquad [3]$$

(Halliday, 2011)

Where:

e = elementary charge (C)
V_f = forward potential of diode (V)

Planck's constant from Halliday (2011):

$$h = 6.62607004 \text{ E-34 m}^2 \text{ kg s}^{-1}$$

The angle of deflection from a diffraction grating is described as:

$$\theta = sin^{-1}(M\lambda d^{-1}) \qquad [4]$$

(Halliday, 2011)

Where:

d = grating spacing (m)
θ = angle of diffraction (rad)
M = order number of maxima line

The intensity versus position on a plane perpendicular to the path of the light for single slit diffraction is:

$$I = \frac{sin^2(\pi ay\lambda^{-1}D^{-1})}{(\pi ay\lambda^{-1}D^{-1})^2} \qquad [5]$$

(Halliday, 2011)

Where:

a = width of slit (m)
y = position on plane (m)
D = distance between slit and surface (m)

The speed of light from Halliday (2011):

$$c = 299792458 \text{ m s}^{-1}$$

The intensity minima of the light will be located approximately at angles, α, such that,

$$sin(\alpha) = M\lambda a^{-1} \qquad [6]$$

(Halliday, 2011)

Where:

α = angle of diffraction (rad)

Planck's law, which relates intensity, temperature, and wavelength of light emitted from a black body, reads as follows:

$$E = 2hc^2\lambda^{-5}(e^{(\frac{hc}{\lambda kT})} - 1)^{-1} \qquad [7]$$

(Britannica, 2011)

Where:

T = Temperature (K)

Wien's Law relates a black body's temperature and peak wavelength as follows:

$$\lambda_{max} = 0.028983\ T^{-1} \quad [8]$$

(Halliday, 2011)

Where: λ_{max} = highest intensity wavelength (m)

Uncertainty calculations and conventions were followed as outlined by van Bemmel (2016).

III Method

To measure Planck's constant, a (997±1) Ω resistor was connected in series with an LED. The resistance was measured with a Digitek 890D multimeter. LEDs with clear casings each emitted six different wavelengths of light. Two Vernier voltmeters were connected across the diode and the resistor. A BK Precision 1506 40-watt power supply was connected to the circuit, and the potential increased from 0 V until the LED lit up. The power supply, multimeter, and voltmeter used were confirmed to be reliable by Godkhindi et al (2016).

To measure the wavelength of the LEDs, a diffraction grating was placed parallel to and (35.8±.2) cm distant from a flat board. A ML 820 Helium Neon Laser projected light of wavelength (632.8±.5) nm through the grating with a perpendicular beam onto the board. The distance between the original projection and the closest bright fringe was measured. Then, each of the six LEDs were placed on the board and viewed through the diffraction grating. The distances between the maximas of the formed images were measured.

A single slit diffraction pattern was arranged using a (632.8±.5) nm wavelength laser shone through a $(87.864 \pm .005)\ \mu$m slit (1.211±.003) m away from a flat, vertical, white sheet of paper. A Canon 550D with a DIGIC 4 image processor was used to capture the diffraction pattern in RAW format. A black image was taken and subtracted from all the observation images. Numeric values were extracted from the images through MATLAB by correcting for distortion created by the standard lens, cropping the images to include only the diffraction patterns, averaging the pixel intensities down each column, and then normalizing these values. Uncertainty was computed through analysis of the focus of the lens versus the size of the pixels.

To obtain a white light spectrum, an Alden Industries Ltd Model 401 1003 Lightbulb with an advertised temperature of 2700K encased in a ray box was shone through a diffraction grating at an angle of incidence of -(3.1±.1)°. The grating was held parallel to a white surface (1.481±.002) m away, and a picture of the resulting spectrum was taken with the Canon camera.

Using MATLAB, the spectrum data was extracted from the images. The images were straightened. Then, noise was reduced, the image was lens-corrected, and a black image was subtracted before each column of pixels was averaged. The RGB pixel values were converted to relative luminance using the spectral response of the Canon 550D. The scale of the image was determined to be 1 pixel = (0.75±.02) mm using a ruler in the image.

IV Data

A total of 30 images were taken for both the single slit experiment and the white light spectrum experiment. The raw images are shown below. Clearly, the images show much noise and unwanted interference that distorts the signal. Through the use of multiple images, image processing, and image correction, these distortions were mitigated.

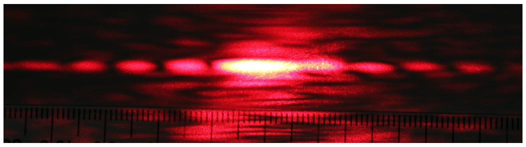

Fig. 1 - A photo of a single slit diffraction pattern: A laser was shone through a 87.864 μm slit onto a white surface.

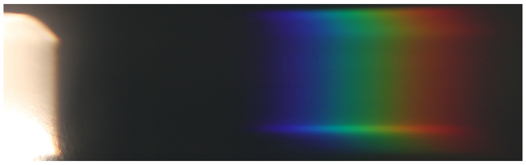

Fig. 2 - Image of white spectrum: Light from a ray box was shone through a diffraction grating against a white surface. A ruler was used to scale pixels in the image.

V Analysis

Using Ohm's law, the potential of the resistor was used to find the current through the circuit. Current was plotted against potential and an exponential trend was found, matching theory. The portion of the graph before the rapid increase in current was approximated as linear. A linear regression was performed for resistor potential values ranging from 20 μA to 0 μA (Figure 3). These values were chosen because theoretical wavelengths, [1], and [3] theorized that the knees of the graphs occurred when the current was greater than 20 μA. From this, the x-intercept of each graph was found. This was taken to be the potential at which current was no longer zero: the forward potential of the diode.

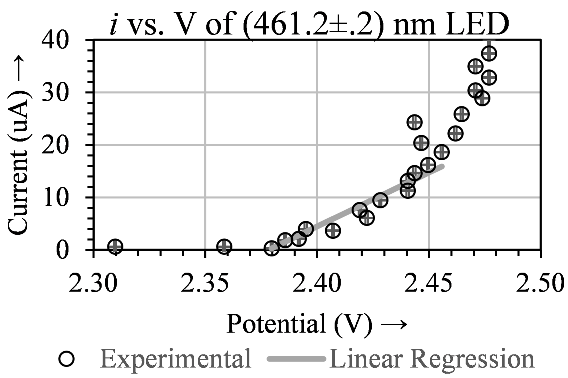

Fig. 3 - Current vs. Potential of (461.2±.2) nm LED: A small portion of the LEDs $i-V$ graph was regressed as an approximate linear function. The x-intercept of this function was found to determine the V_f of each diode.

The distance between the diffraction grating and flat board, and that between the ML 820 Helium Neon Lasers projection and its closest maxima line, were used in conjunction with trigonometry to calculate a grating spacing of $(2.05 \pm .06)$ μm. This value was used alongside similar measurements made for the LEDs and [4] to find the angle of diffraction and, subsequently, the wavelength of each LED.

Using the diffraction grating and data, [1] and [3] were combined to find that:

$$eV_f = E = \frac{hc}{\lambda} \quad [9]$$

The energies of the light emitted by the LED were graphed against the reciprocals of the six wavelengths in Figure 4. These same energies were also graphed against sourced wavelength values.

Since [9] shows a linear relationship between E and λ^{-1} with a slope of hc, values of h could be calculated from the regressed slopes of Figure 4.

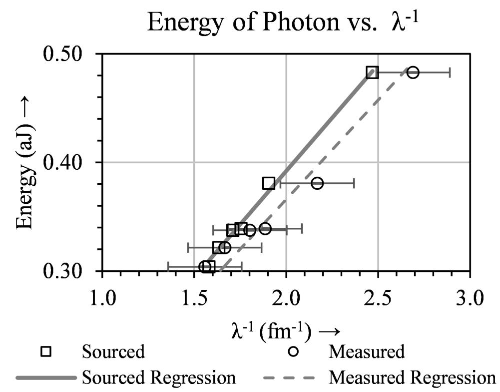

Fig. 4 - Energy of Photons vs. λ^{-1}: The sourced and measured values for λ^{-1} were linearly related with the photons' energy by a slope of *hc*, as graphed above.

From the slopes, the calculated value for Planck's constant from the measured regression was found to be (6.10±.04) E-34 m^2 kg s^{-1}, while the sourced wavelength regression yielded (6.54±.03) E-34 m^2 kg s^{-1}. These resulted in errors of 7.88% and 1.31% respectively from the theoretical Planck's constant.

Due to the accuracy of the sourced wavelengths' regression, it was concluded that measuring the wavelengths created the majority of the experiment's error. From Figure 4, the experimental λ^{-1} values were lower than the sourced ones, meaning the λ values found with the diffraction grating were too great. It is likely that parallax caused erroneous readings when using the diffraction grating with the LEDs. Furthermore, human error while measuring the distance between the LED maxima lines was inevitable.

The theoretical diffraction patterns given by [5] were plotted with the normalized experimental values:

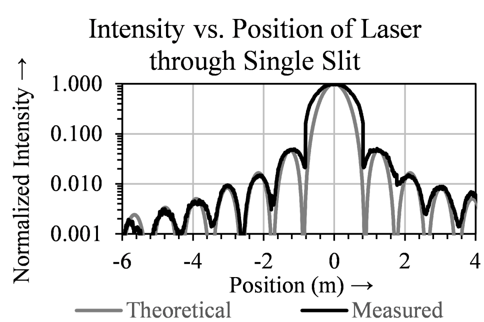

Fig. 5 - Intensity vs. Position of Laser through Single Slit: A line was chosen to represent the measured values due to the high number of data points. The intensity's uncertainty was ±.001, and the positional uncertainty, ±.002 m. A logarithmic scale was chosen to improve clarity.

The measured data conformed with theory, specifically the locations of the extrema of the diffraction pattern as predicted by [5] and [6]. A mean squared error (MSE) analysis of the minima and maxima positions revealed an average signal to MSE ratio of 207. Since this is above 100, it shows a strong correlation.

The intensities, however, yielded a MSE of 0.00849, resulting in an average signal to MSE ratio of 11. This divergence was attributed to the spread in the lasers beam, as a laser is not perfectly collimated. Furthermore, non-uniform air movement and density along with ambient light further impeded the measurements taken of the single slit diffraction pattern. Finally, the sensitivity of the camera to light, although corrected for, was imperfect and caused variance.

The wavelength of each pixel in Figure 2 was found by using [6] and isolating for wavelength. The angle of deflection was found using d, D, and basic trigonometry. The intensity of the pixels, a regression based on Planck's Law, as well as a theoretical black body curve using the listed filament temperature of 2700 K were graphed against wavelength. The experimental data has a domain of (390±1) nm to (646±1) nm, which encapsulates a deflection angle of (25.8±.2)° to (43.2±2)°.

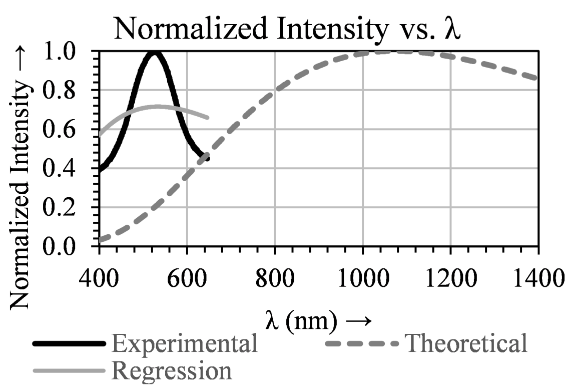

Fig. 6 - Normalized Intensity vs. Experimental, Regressed, and Theoretical λ: The intensities of the experimental and theoretical data are not equivalent, as no absolute intensity could be obtained from the camera; thus, the normalized scale was shared to show the peak wavelengths. The intensities of each graph were scaled to the highest value.

The equation for the regression is:

$$I = 2hc^2\lambda^{-5}(e^{\frac{hc}{(5430\pm9)k\lambda}} - 1)^{-1} \quad [10]$$

The discrepancy between the listed filament temperature of 2700 K and the measured value of 5500 K was likely due to the method of measurement. The camera is designed to capture light in a similar fashion to the human eye. Even after correction factors were applied, the design of the Bayer filter colour system, along with the innate sensitivities of the DIGIC 4 sensor, ensured that wavelengths between 480 and 560 nm prevailed. Thus, a peak was artificially created at (527±1) nm.

Additionally, in Figure 2, the light closest to the colour green covers much less space than that of red or blue. Based on [6], and given that the differences in wavelength from violet to green and green to red are similar, one would expect the green-yellow sections of the band to cover a distance smaller than that of the orange-red section, and larger than that of the violet-blue section. Presuming that the emission of green light was consistent with the black body curve, this would cause the green values to have an artificially high intensity, as a similar amount of light as the other colours would hit a much smaller area. This would result in artificially high intensities of green light.

The experimental temperature of the bulb was obtained both by using Wien's Law, and with the regression in Figure 6. The peak wavelength was found to be (527±1) nm, and using [8], the temperature of the filament was found to be (5500.24±.01) K. By regressing the experimental data based on Plancks Law, the temperature of the filament was found to be (5430±9) K, supporting the results from Wien's Law.

Using [6], which was supported by the results of the single slit diffraction experiment, as well as [2], Heisenberg's uncertainty principle was derived.

Only the central bright fringe, with $M = 1$, was considered in the derivation, so [6] became,

$$sin(\alpha) = \frac{\lambda}{a} \quad [11]$$

[2] may be used to calculate the momentum of a photon. The momentum may be substituted with $p = mc$ such that,

$$\lambda = \frac{h}{mc} \quad [12]$$

When a photon is sent through the slit, it has a velocity vector in an unknown direction. While the x-component of this vector does not contribute to the light distribution, the y-component does. The y-component has an uncertainty, σ_{vy}, that is equal to the y-component of the velocity vector. Thus,

$$\sigma_{vy} = csin(\alpha) \tag{13}$$

Using [13] to find σ_{py}, uncertainty of momentum in the y-direction:

$$\sigma_{py} = mcsin(\alpha) \tag{14}$$

Isolating for mc in [12] and then substituting into [14] yields:

$$\sigma_{py}(\frac{\lambda}{h}) = sin(\alpha) \tag{15}$$

Substitute [11] into the right side of [15] to obtain:

$$\sigma_{py}(\frac{\lambda}{h}) = (\frac{\lambda}{a}) \tag{16}$$

Simplifying,

$$a\sigma_{py} = h \tag{17}$$

Since a is the width of the slit, it is equal to the photon's uncertainty in the y-position, σ_y. Thus,

$$\sigma_y\sigma_{py} = h \tag{18}$$

Because $h \geq \frac{1}{2}\hbar$, where $\hbar = \frac{1}{2\pi}h$, [18] is rewritten as:

$$\sigma_y\sigma_{py} \geq \frac{1}{2}\hbar \tag{19}$$

[19] is Heisenberg's uncertainty principle; it dictates that the more precisely the position of a particle is known, the less precisely known is its momentum, and vice versa. Within an experiment such as the single slit, Heisenberg's principle limits the observations of the photons, and is applicable because of the wave-particle duality of all matter; it is not caused by inadequacies in technology, or the act of observation. Due to these limitations, it is only possible to calculate probabilities for where particles are and how they will behave.

VI Sources of Error

When determining Planck's constant, there was error due to the impreciseness of the human eye. Moreover, errors may have been introduced by the laser being knocked over by the instructor prior to use.

When taking the image of the white light spectrum, it is possible that the ruler used for scaling was not parallel to the surface the image was taken on, which would make the conversion between pixel and metres inaccurate. However, efforts were made to ensure the ruler was as parallel as possible.

The camera also posed many potential sources of error. Due to the age of the device, the DIGIC 4 image processor's performance may have diverged from reported statistics. Debris and dust on the lens, and perhaps the sensor, would have also caused image variances. The long exposure times required to photograph the single slit and black body phenomena left the camera vulnerable to movement, and therefore, blur.

VII Conclusion

This experiment tested light-emitting diodes, single slit diffraction patterns, and a white light spectrum. A Planck's constant value of $(6.10 \pm .04)$ E-34 m^2 kg s^{-1} was found, within an error of 7.88% from the theoretical value. In the single slit experiment, the signal to mean squared error between the theorized and experimental periodicity was 207—a strong correlation. Uncontrollable variables in the experiment caused a weak signal to mean squared error of 11 for the intensities of the single slit experiment. Using a reconstructed black-body and Wien's Law, a light bulb's temperature was estimated to be 7500 K. Furthermore, de Broglie's hypothesis was used to mathematically derive Heisenberg's uncertainty principle.

VIII Sources

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