Computer Vision 2

Summer Semester 2019, Homework Assignment: 2

Prof. Roth, Xiang Chen, Jochen Gast

TECHNISCHE UNIVERSITÄT DARMSTADT

Total points: 20+Programming problems

visual inference

Due date: 7.6.2019, 11:00

Group Number: 67 Yannik Phil Frisch, 2989417 Marius Kleboth, 2678333

Problem 2.1 Graphical Models [20 Points]

a) 1: [1 Points]

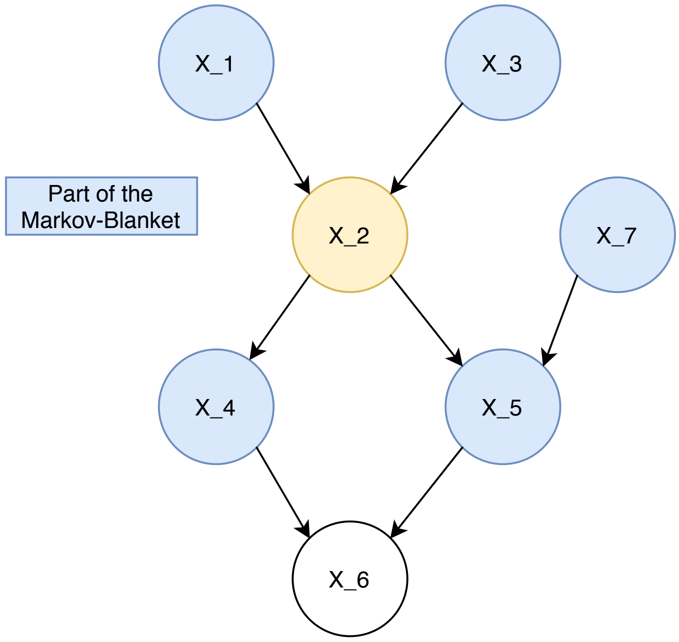
One needs to identify all random variables which are represented by the nodes of the graphical model. Additionally the dependencies between the random variables are represented by the edges connecting these nodes and the independence is encoded by the absence of an edge between two nodes .

b) 2: [1 Points]

Directed graphical models (e.g. Bayesian nets) are not capable of representing cyclical dependencies, while undirected graphical models (e.g. Markov random fields) can deal with that problem.

c) 3: [2 Points]

$x_1 \longrightarrow x_2$
$x_3 \longrightarrow x_2$
$x_2 \longrightarrow x_4, x_5$
$x_7 \longrightarrow x_5$
$x_4 \longrightarrow x_6$
$x_5 \longrightarrow x_6$
Markov blanket of x_2 : $[x_1, x_3, x_4, x_5, x_7]$



d) 4: [2 Points]

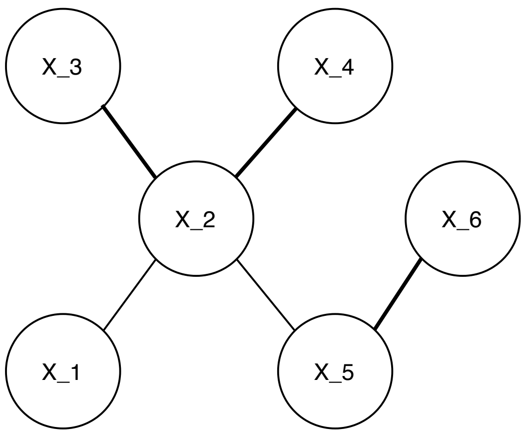
Directed model:

$$p\left(x_1,x_2,\ldots,x_{15}\right)=p\left(x_1\right)p\left(x_2\right)p\left(x_3\right)p\left(x_6\right)p\left(x_7\right)p\left(x_4|x_1,x_2\right)p\left(x_5|x_2,x_3\right)p\left(x_8|x_4,x_5\right)p\left(x_9|x_3,x_5,x_6\right)p\left(x_{10}|x_7\right)$$
$$p\left(x_{11}|x_{10},x_8\right)p\left(x_{12}|x_8,x_9\right)p\left(x_{13}|x_9\right)p\left(x_{14}|x_{11}\right)p\left(x_{15}|x_{12}\right)$$

Undirected model:

$$p\left(x_1,x_2,\ldots,x_{15}\right)=\frac{1}{Z}f_1\left(x_1,x_4\right)f_2\left(x_2,x_4,x_8\right)f_3\left(x_2,x_5,x_8\right)f_4\left(x_5,x_9\right)f_5\left(x_8,x_{12}\right)f_6\left(x_6,x_9\right)f_7\left(x_9,x_{12},x_{13},x_{15}\right)$$
$$f_8\left(x_8,x_{11}\right)f_9\left(x_7,x_{11}\right)f_{10}\left(x_7,x_{10}\right)f_{11}\left(x_{11},x_{14}\right)f_{12}\left(x_3\right)$$

e) 5: [1 Points]



f) 6: [2 Points]

$p\left(x_1, x_2, x_3, x_4\right) = \frac{1}{Z}16.0$
$p\left(x_1, x_2, \neg x_3, x_4\right) = \frac{1}{Z}0.04$
$p\left(x_1, x_2, x_3, \neg x_4\right) = \frac{1}{Z}0.04$
$p\left(x_1, x_2, \neg x_3, \neg x_4\right) = \frac{1}{Z}0.02$
$p\left(x_1, \neg x_2, x_3, x_4\right) = \frac{1}{Z}0.04$
$p\left(x_1, \neg x_2, \neg x_3, x_4\right) = \frac{1}{Z}0.02$
$p\left(x_1, \neg x_2, x_3, \neg x_4\right) = \frac{1}{Z}0.0001$
$p\left(x_1, \neg x_2, \neg x_3, \neg x_4\right) = \frac{1}{Z}0.01$
$p\left(\neg x_1, x_2, x_3, x_4\right) = \frac{1}{Z}0.04$
$p\left(\neg x_1, x_2, \neg x_3, x_4\right) = \frac{1}{Z}0.0001$
$p\left(\neg x_1, x_2, x_3, \neg x_4\right) = \frac{1}{Z}0.02$
$p\left(\neg x_1, x_2, \neg x_3, \neg x_4\right) = \frac{1}{Z}0.01$
$p\left(\neg x_1, \neg x_2, x_3, x_4\right) = \frac{1}{Z}0.02$
$p\left(\neg x_1, \neg x_2, \neg x_3, x_4\right) = \frac{1}{Z}0.01$
$p\left(\neg x_1, \neg x_2, x_3, \neg x_4\right) = \frac{1}{Z}0.01$
$p\left(\neg x_1, \neg x_2, \neg x_3, \neg x_4\right) = \frac{1}{Z}1.0$

g) 7: [2 Points]

$p(a,b) = \sum_c p(a=1,b=1,c) = 0.048 + 0.096 = 0.144$
$p(a) = \sum_{b,c} p(a=1,b,c) = 0.192 + 0.064 + 0.048 + 0.096 = 0.4$
$p(b) = \sum_{a,c} p(a,b=1,c) = 0.048 + 0.216 + 0.048 + 0.096 = 0.408$
$p(a) * p(b) = 0.4 * 0.408 = 0.163 \neq p(a,b)$
a and b are therefore independent.

$p(c=1) = \sum_{a,b} p(a,b,c=1) = 0.144 + 0.216 + 0.064 + 0.096 = 0.52$
$p(a|c) = \frac{p(a,c)}{p(c)} = \frac{\sum_b p(a=1,b,c=1)}{p(c=1)} = \frac{0.096+0.064}{0.52} = \frac{4}{13}$
$p(b|c) = \frac{\sum_a p(a,b,c)}{p(c)} = \frac{0.096+0.216}{0.52} = 0.6$
$p(a|c)\,p(b|c) = \frac{12}{65}$
$p(a=1,b=1|c=1) = \frac{p(a=1,b=1,c=1)}{p(c=1)} = \frac{0.096}{0.52} = \frac{12}{65}$
Hence for $c=1$ a and b are conditionally independent given c.

$p(c=0) = 1 - p(c=1) = 0.48$
$p(a|c=0) = \frac{p(a,c)}{p(c)} = \frac{\sum_b p(a=1,b,c=0)}{p(c=0)} = \frac{0.192+0.048}{0.48} = 0.5$
$p(b|c=0) = \frac{\sum_a p(a,b=1,c=0)}{p(c=0)} = \frac{0.048+0.048}{0.48} = 0.2$
$p(a=1|c=0)\,p(b=1|c=0) = 0.1$
$p(a=1,b=1|c=0) = \frac{p(a=1,b=1,c=0)}{p(c=0)} = \frac{0.048}{0.48} = 0.1$
Hence a and b are also independent given $c=1$.

h) 8: [3 Points]

$p(a=1) = \sum_{b,c} p(a=1,b,c) = 0.192 + 0.064 + 0.048 + 0.096 = 0.4$
$p(c|a) = \frac{\sum_b p(a=1,b,c=1)}{p(a=1)} = \frac{0.064+0.096}{0.4} = 0.4$ From P7:
$p(b=1|c=1) = 0.6$
$p(a=1,b=1,c=1) = 0.096 = 0.4 \cdot 0.4 \cdot 0.6 = p(a)\,p(c|a)\,p(b|c)$
Hence $p(a,b,c)$ factorizes to $p(a)\,p(b|c)\,p(c|a)$

i) 9: [2 Points]

Directed Model:

Markov blanket of x_1 : $[x_2, x_3]$
Markov blanket of x_2 : $[x_1, x_3, x_4]$
Markov blanket of x_3 : $[x_1, x_2, x_4]$
Markov blanket of x_4 : $[x_2, x_3]$

Undirected Model:

Markov blanket of x_1 : $[x_2, x_3]$
Markov blanket of x_2 : $[x_3, x_4]$
Markov blanket of x_3 : $[x_1, x_4]$
Markov blanket of x_4 : $[x_2, x_3]$

j) 10: [4 Points]

Directed model:

1.) Given the markov blanket of x_1, which is $[x_2, x_3]$, we can assume conditional independence to all other nodes, including x_4: $x_1 \perp\!\!\!\perp x_4 | x_2, x_3$
2.) We can use d-separation (tail-to-tail) to assume conditional independence for x_2 and x_3 given x_1: $x_2 \perp\!\!\!\perp x_3 | x_1$

Undirected model:

1.) Due to the definitions of a markov blanket in exercise 3, we can say that x_1 is independent from x_4 given its markov blanket (x_2 and x_3) :$x_1 \perp\!\!\!\perp x_3 | x_2, x_3$
2.) We can again use the definitions of a markov blanket, which states that x_2 is independent from x_3 given its markov blanket (x_1 and x_4) :$x_2 \perp\!\!\!\perp x_3 | x_1, x_4$

Problem 2.2 Stereo Likelihood [0 Points]

a) P2, last bullet point [0 Points]

The MRF prior favors small (or rather no) differences between neighbouring pixels. Because for 0 difference between two pixels the log of the student-t function evaluates to 0 the log-prior density of a "constant map" is 0.
The more noise the map has the smaller the likelihood becomes. As in the "gt map" neighbouring pixels only have differing values at edges this too produces a rather high prior value.
Since the neighbouring pixel values of the "noise map" are not correlated at all the student-t prior produces a very small value.
The prior captures the flat surfaces in the world quite well but it doesn't account for edges and specifically the MRF setup does not capture recurring edges or symmetries which isn't accounted for in the pairwise MRF.

b) P3 [0 Points]

The algorithm easily gets stuck in local optima.

c) P4, last bullet point [0 Points]

Not sure if the function is not continuously differentiable wrt e.g. the shape parameter α .