

# The Furuta Pendulum

## Technical Report

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**Abstract** The Furuta Pendulum is an example of a complex non-linear system and therefore of big interest in control system theory. It consists of one controllable arm rotating in the horizontal plane and one pendulum uncontrollably moving in the vertical plane, which is attached to the end of this arm. The non-linearities result from an interplay between gravitational, Coriolis and centripetal forces.

XX We present an overview over it's technical details and proposed algorithms to solve the control problem. XX

**Keywords** First keyword · Second keyword · More

## 1 Introduction

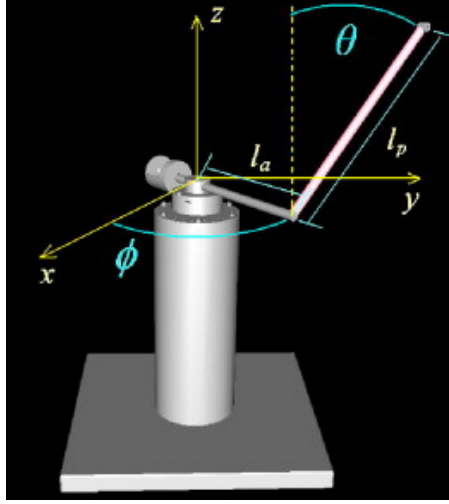
Many examples of the field of control engineering like aircraft landing, aircraft stabilizing and many more can be very well modelled by an inverted pendulum [1]. As a reaction to problems with the limited movement of the cart from the inverted pendulum, the furuta pendulum (also called rotary inverted pendulum) has been developed by Furuta et al.. The advantages of the furuta pendulum are that it needs less space and one moving arm is directly linked to the motor, therefore, the dynamics is less unmodeled thanks to a power transmission mechanism [2]. The furuta pendulum is an underactuated problem, this means that there two degrees of freedom ( $\phi$ ,  $\theta$ ), but only one arm is directly controlled by the motor. This is the one which changes the

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angle  $\phi$  in the horizontal plane. The other arm called pendulum is attached to the end of the controlled arm and therefore is moved indirectly by it in the vertical plane with angle  $\theta$  [4, 5]. The whole system is highly non-linear as a result of an interplay between gravitational, coriolis and centripetal forces. Finally, there are two different types of control problems. First, bringing the



**Fig. 1** The furuta pendulum, figure from [3]

flexible arm from a hanging position to a position which is nearly upright, further called "swing-up". Second, if the arm is nearly upright with low enough speed, balancing it to hold this state stable, called "stabilization".

### 1.1 Definitions

The system consists of an arm with length  $L_r$  mounted to a DC motor, which is able to apply a torque of  $\tau_1$  to it. It has a mass of  $m_1$  which is located at  $l_1$  alongside the arm. Another arm with length  $L_p$  and mass  $m_2$  located at  $\frac{L_p}{2}$  is attached to the remaining side of the first arm. Both arms have a moment of inertia  $J_r$  and  $J_p$  respectively. The angle  $\theta$  is zero if the pendulum is in an perfectly upright position.

## 2 Mathematical Modelling

### References

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