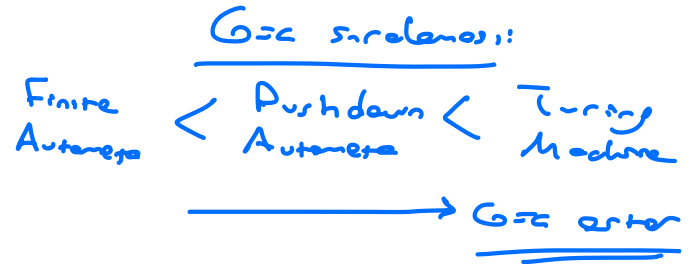# Genel Bilgi:

① Finite Automata (Sonlu Otomata)

* DFA, NFA
* Hafıza yok.
* Dil: Reguler Diller

② Pushdown Automata

* PDA
* Hafıza: Stack (yığın)
* Dil: Context-Free

③ Turing Machines (Turing Makinesi)

* Hafıza: Random Access Memory (RAM)
* Dil: Turing Acceptable

Güç sıralaması:

$$\text{Finite Automata} < \text{Pushdown Automata} < \text{Turing Machine}$$

Güç artar →

Dil Kuramı:

$$\text{Alfabe} \longrightarrow \text{String} \longrightarrow \text{Dil}$$

Binary sayılar kümesi: $\Sigma = \{0, 1\} \rightarrow$ Bu kümenin alfabesidir.

$\underline{1011}$ $\underline{1010}$ $\underline{120}$
String   String   String
(falet dilde tanımlı değildir!)

String işlemleri:

$w = abba$ } $wv = abbabbab$
$v = bbab$ } $vw = bbababba$

Reverse: $w^R \rightarrow$ üss: "R" ile gösterilir.

$w = babb$ $w^R = bbab$

Uzunluk: $|w| \rightarrow$ Mutlak değer ile gösterilir.

$w = aba \longrightarrow |w| = 3$, $|abba| = 4$

$|wv| = |w| + |v| = |v| + |w|$

Boş String:

"λ" veya "ε" ile gösterilir. $|\lambda| = 0$ $w\lambda = \lambda w = w$,

$abob\lambda = ab\lambda ob = obob$

Substring:

ab boobb
⟶ ab

boob
⋮

Üs Alma:

$$w^n = \underbrace{w.w.w.w \dots w}_{n \text{ adet}}$$

$(aba)^2 = aba\,aba$

$w^0 = \lambda$, $(aba)^0 = \lambda$

Prefix - Suffix

$$w = \underset{\downarrow}{v}\, u \longrightarrow \text{suffix}$$
$$\text{prefix}$$

$w = ababa$

Prefix	Suffix
λ	ababa
a	baba
⋮	⋮
ababa	λ

Yıldız (*) Operasyon:

$\Sigma = \{a, b\}$

$\Sigma^* = \{\lambda, a, b, aa, bb, ab, \dots\}$

Artı (+) Operasyonu

$\Sigma = \{a, b\}$

$\Sigma^+ = \Sigma^* - \lambda = \{a, b, aa, bb, ab, \dots\}$

Dil tanımlamaları:

$\Sigma = \{a, b\}$

$L = \{a^n b^n : n \geq 0\}$

$$\underbrace{a\,aa\dots aaa}_{n}\underbrace{bbb\dots bbb}_{n}$$

$\lambda \in L$ $aaabbb \in L$
$ab \in L$ $aaabb \notin L$ (n ham 3 ham 2 olamaz)
$aabb \in L$

- $|\{\}| = |\emptyset| = 0$, $|\lambda| = 0$
- $|\{\lambda\}| = 1$
- Peki: w ve w^R biliyoruz ama L (yani dili) ve L^R biliyor muyuz?

  $L^R = \{ab, aob, abob\}^R = \{bo, baa, baba\}$
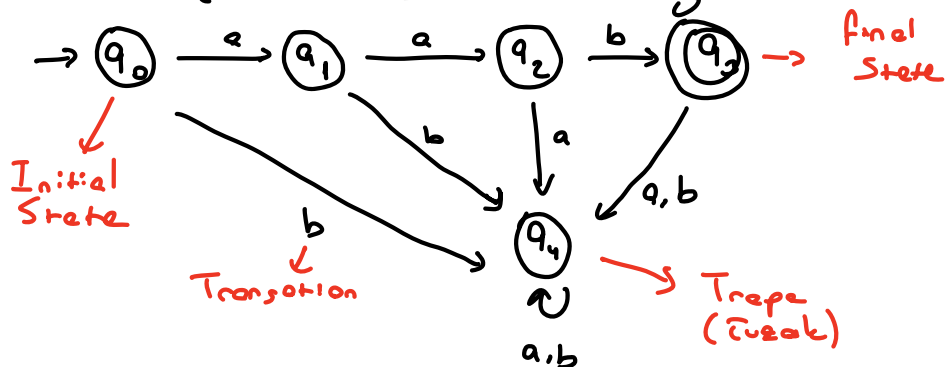
- $u = a$ $w = uv = ab$
  $v = b$

↳ Peki dillerde?

  $L_1 = \{ab, bo\}$, $L_2 = \{a, baa\}$

  $L_1 L_2 = \{aba, ab baa, baa, babaa\}$
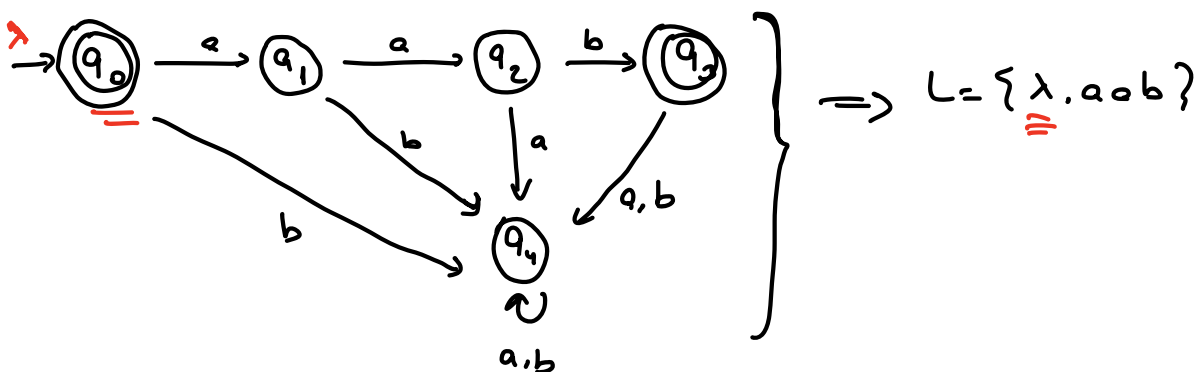
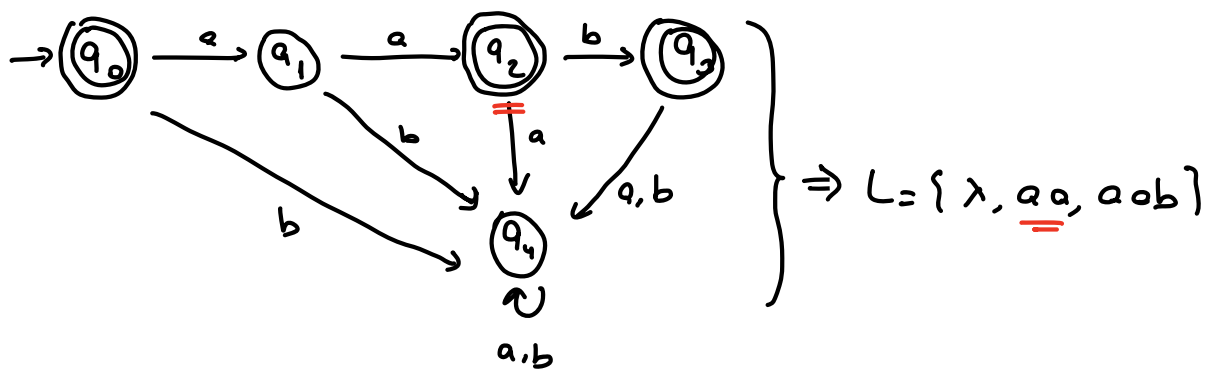- $L = \{a, b\}$, $L^0 = x$, $L^3 = \{a, b\}\{a, b\}\{ab\}$

  DFA (Deterministic Finite Automate):
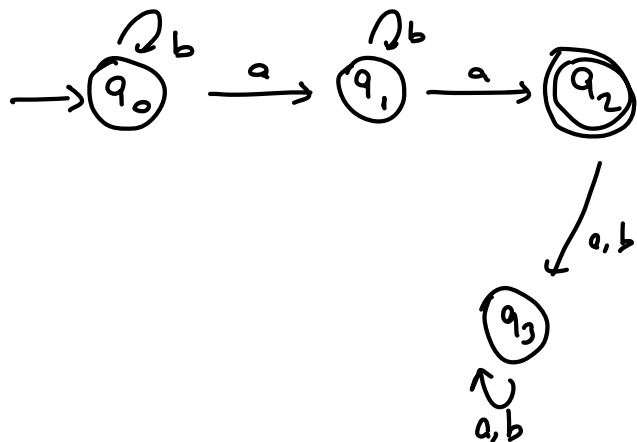
  Kararlıdır ve tek state'e gider.

  Initial State

  Transition

  Final State

  Trepe (Tuzak)

  aaab ✗   aob ✓   aba ✗

  $L = \{aab\}$

  $\Rightarrow L = \{\lambda, aob\}$

$$\Rightarrow L = \{\lambda, aa, aob\}$$

Örnek:

bob   ✗
boba   ✓
bba bba   ✓
babbba   ✓

$$L = \{ boba, bbabba, babbba, - \}$$

$$L = \{ b^n a b^m a, \; n \geq 0, \; m \geq 0 \}$$

formal Definition

$$M = (\Theta, \Sigma, \delta, q_0, F)$$

$\Theta = \{ q_0, q_1, q_2, q_3 \}$, $\Sigma = \{ a, b \}$, $\delta(q_0, a) = q_1$

↳ Stateler kümesi: ↳ Alfabe kümesi: ↳ Transation kümesi!
                                                    q_0'a a verilince q_1'e git.

$q_0 = q_0$, $F = q_2$

↳ Başlangıç state   ↳ final state
(Initial State)

Extended Transation

$$\delta^* (q_0, abba) = q_3 \rightarrow q_0\text{'dan sonra ``abba'' yazınca gidilen state}$$

$$\delta^* (q_1, bbaba) = q_4 \rightarrow q_1\text{'den sonra ``bbaba'' ile } q_4\text{'e gider.}$$

NFA (Non-Deterministic Finite Automata)
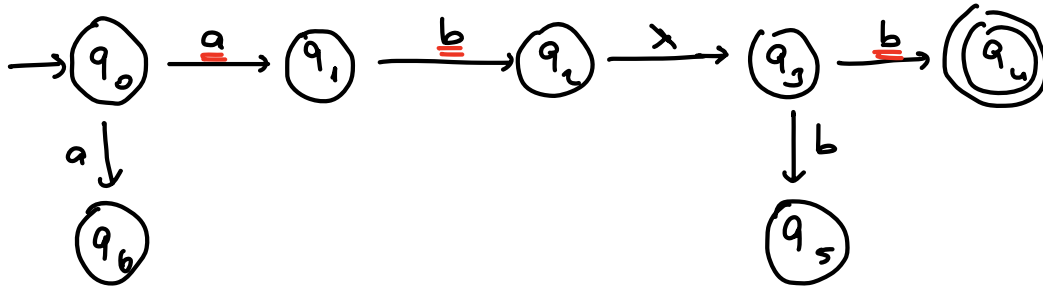
abb ✓   a ba ✗   abbb ✗

↳ **Formal Definition**

$M = (\mathcal{O}, \Sigma, \delta, q_0, f)$

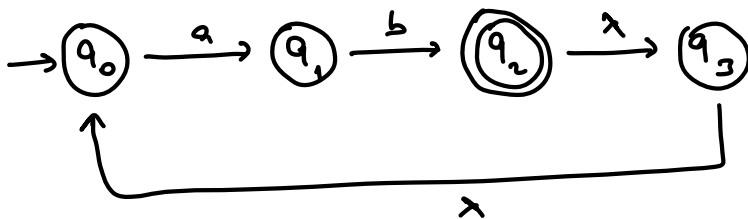$\mathcal{O} = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \}$ $\Sigma = \{ a, b \}$

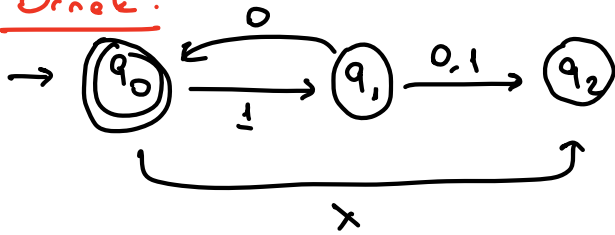$\delta(q_0, a) = \{ q_1, q_6 \}$ → Deterministic Finite Automata'dan tek farkı budur.

$\vdots$

$q_0 : q_0$ ve $f = q_4$

Örnek:

ab ✓
abab ✓   } $L = \{ ab, abab, ababab, \ldots \} = (ab)^+$
ababab ✓

$$x \checkmark$$
$$1 0 \checkmark$$
$$1 0 1 0 \checkmark$$
$$1 0 1 0 1 0 \checkmark$$

$$\left. \right\} \quad L = \{\lambda, 10, 1010, \ldots\} = (10)^*$$
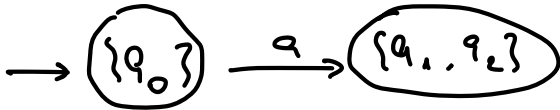
NFA'yı DFA'ya çevirme
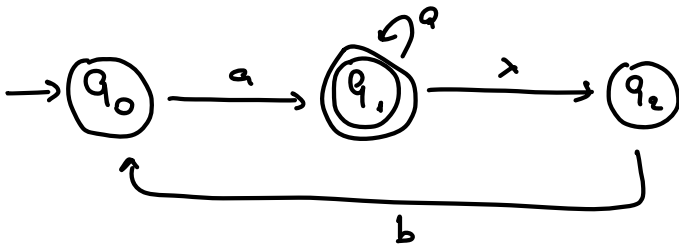
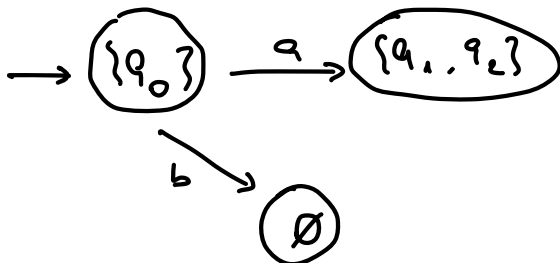- Öncelik Initial State bulmak $\longrightarrow \{q_0\}$ (Bu tek başına olmayabilirdi!)
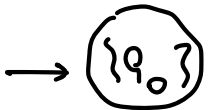
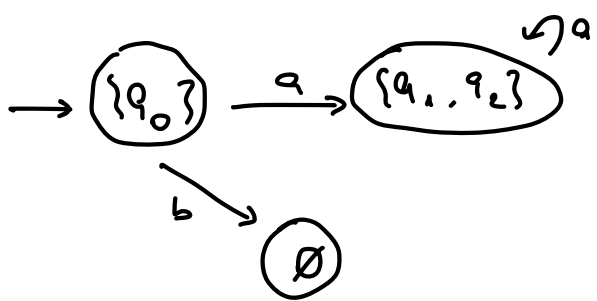- Sonra $\int_{\{q_0\}, a}$ transation yapısına bak.

$$\int(\{q_0\}, a) = \{q_1, q_2\}$$

- Sonra $\int_{\{q_0\}, b}$ transation yapısına bak.

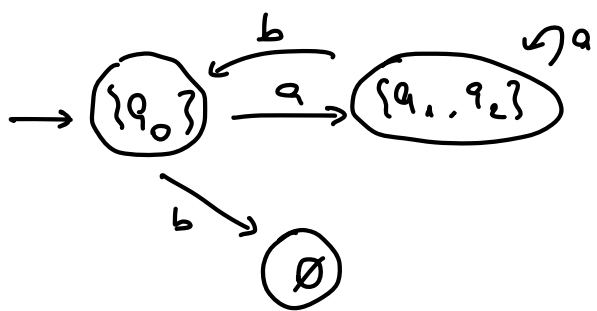$$\int(\{q_0\}, b) = \emptyset \rightarrow \text{Genellikle trap state oluyor.}$$

$$\delta(\{q_1,q_2\},a) = \underbrace{\delta(q_1,a)=q_1 \quad ve \quad \delta(q_2,a)=\emptyset}_{=\{q_1,q_2\}}$$

• Sonra $\delta_{\{q_1,q_2\},b}$ transation yapısını bul.

$$\delta(\{q_1,q_2\},b) = \underbrace{\delta(q_1,b)=\emptyset \quad ve \quad \delta(q_2,b)=q_0}_{=\{q_0\}}$$

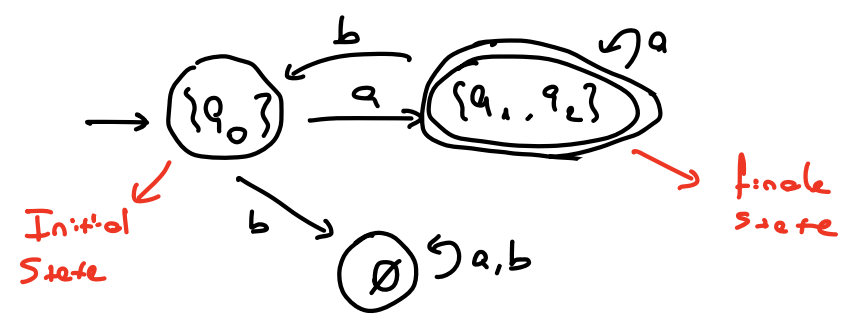• Sonra trap parcellenir.
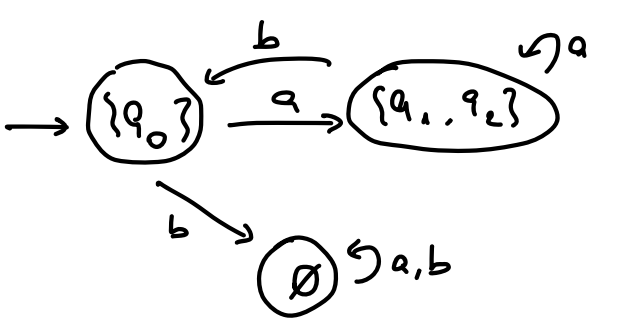
/ • En son finale state/lar belirlenir.

NFA'da finale Stareler: $\{q_1\}$ olup q_1 : içeren bütün stareler finale state olarak kabul edilir.

<u>Gencel Hali:</u>

Initial State

finale state

Örnek 2:

① **Initial State:** $\{a, b\}$
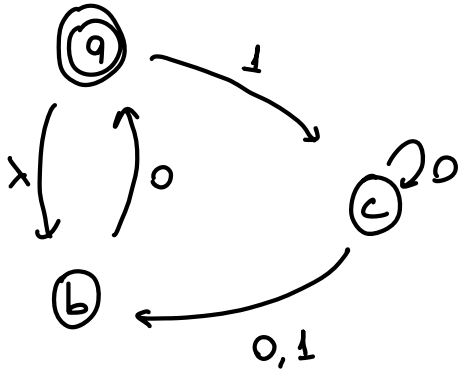
② $\{a,b\} \xrightarrow{1} \{\underline{c}\}$

$\{a,b\} \xrightarrow{0} \{a,b\}$

②·1 $\{c\} \xrightarrow{0} \{\underline{c,b}\}$

$\{c\} \xrightarrow{1} \{\underline{b}\}$

②·3 — 2. yeni state

$\{b\} \xrightarrow{0} \{a,b\}$

$\{b\} \xrightarrow{1} \emptyset$

②·4

$\{a,b,c\} \xrightarrow{0} \{a,b,c\}$

$\{a,b,c\} \xrightarrow{1} \{c,b\}$

②·2

$\{c,b\} \xrightarrow{0} \{\underline{a,b,c}\}$

$\{c,b\} \xrightarrow{1} \{b\}$

②·5 **DFA çizim:**

Initial State

Trap
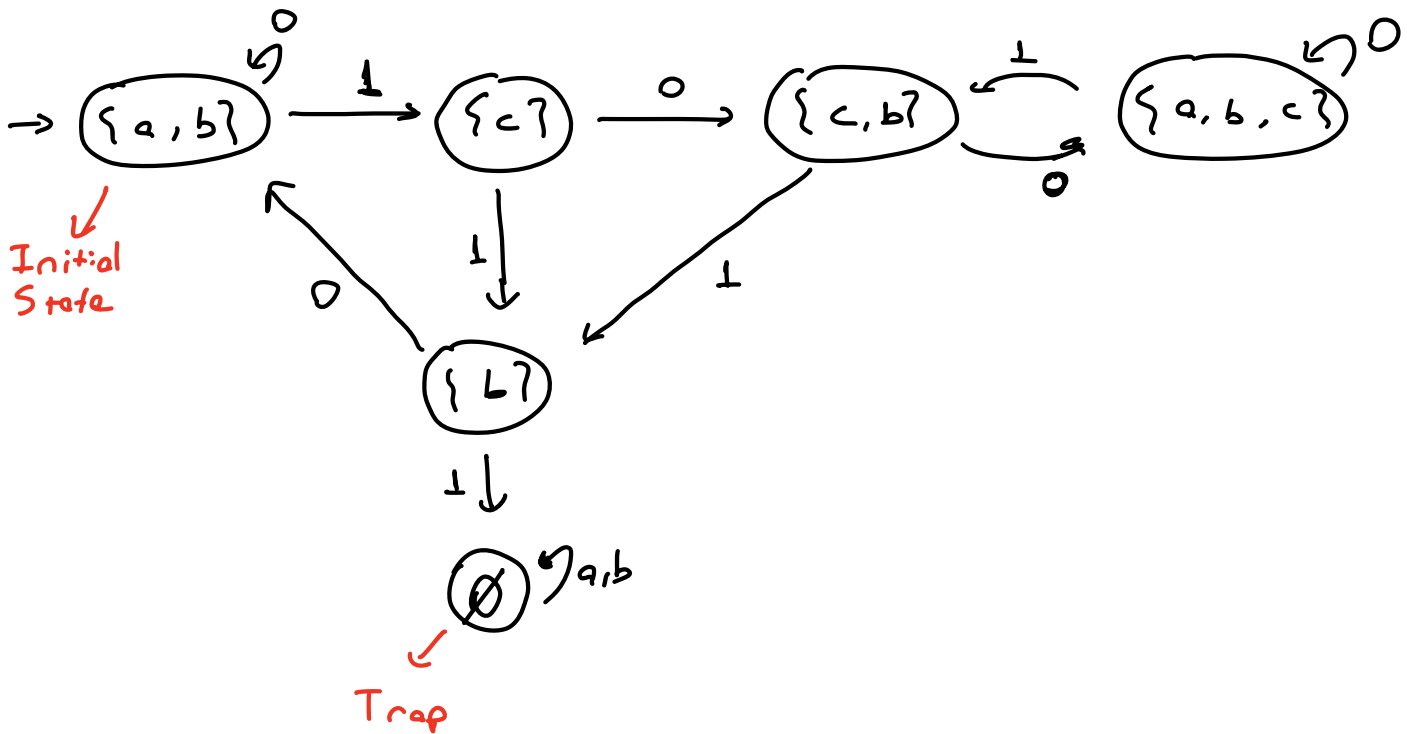
③ Finite State bulma (a'nın olduğu bütün durumlar)
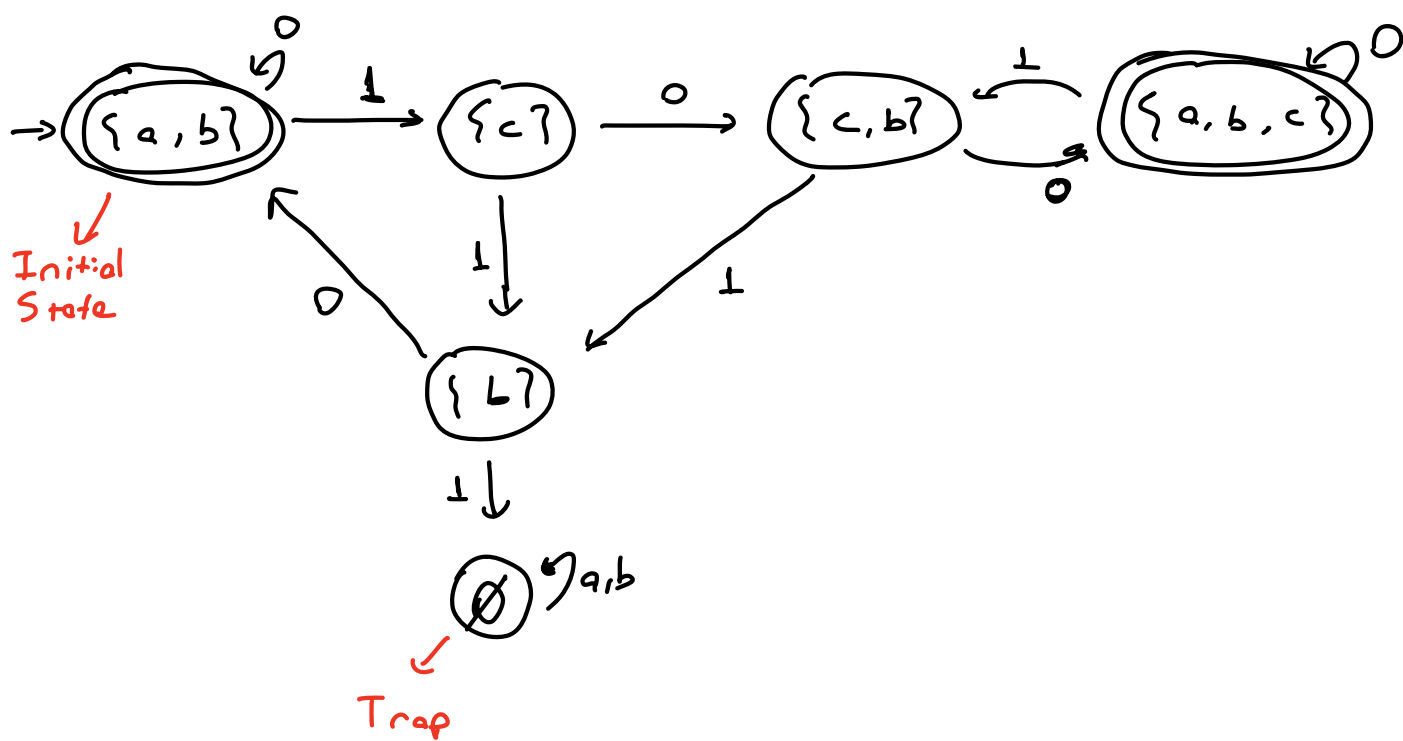

Initial State

Trap

PDA (Push Down Automata)

String (input) Okunan

Stack Okuduğu (Sildiği)

Stack yazılan

q_0 $a, b \longrightarrow c$ q_1

$\$ \rightarrow z$

Örnek:

$$L(M) = \{ a^n b^n : n \geq 0 \}$$

aa bb

q_0 $\xrightarrow{\lambda, \lambda \to \lambda}$ q_1 $\xrightarrow{b, a \to \lambda}$ q_2 $\xrightarrow{\lambda, \$ \to \$}$ q_3 $\implies$

$a, \lambda \to a$ (loop on q_1)

$b, a \to \lambda$ (loop on q_2)

aa bb için
$\downarrow$

Örnek 2:

q_0 $\xrightarrow{a, bc \to def}$ q_1

b
c
$

$\implies$

d
e
f
$
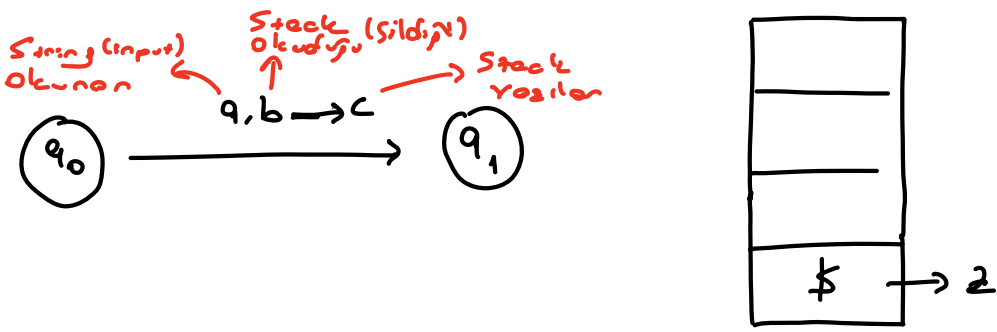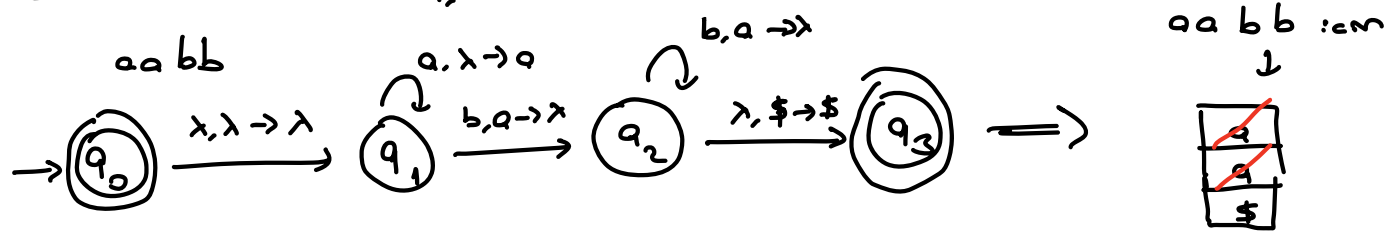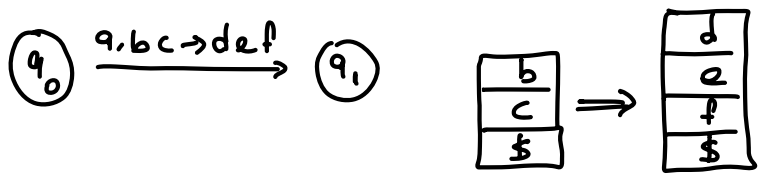
PDA'nın formal definition yapısı:

$$M = (\Theta, \Sigma, \Gamma, \int, q_0, \square, f)$$

Stateler ↓
Input Alfabe ↓
Stack Alfabe ↓
Transation ↓
Initial State ↓
Stack Başlangıç ↓
Final State ↓

Turing Makinesi:

Oku ↰   Yaz ↱

q_0 $a \rightarrow b, R$ q_1

Yön (Right)
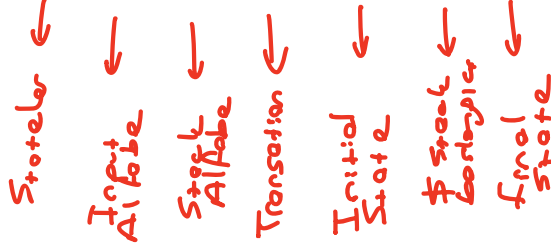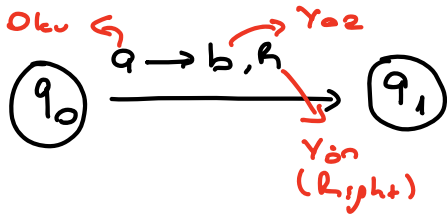
↳ Turing makinesinde sağa/sola hareket edebilmeliye!
Soplar.

| | | $\Diamond$ | $\Diamond$ | a | b | c | $\Diamond$ | $\Diamond$ | | |

↰ Tape   Blank

Örnek: *Yukarıdaki tape'e göre;

q_0 $a \rightarrow c, R$ q_1 $b \rightarrow a, L$ q_2

| ... | $\Diamond$ | $\Diamond$ | a | b | c | $\Diamond$ | $\Diamond$ | ... |
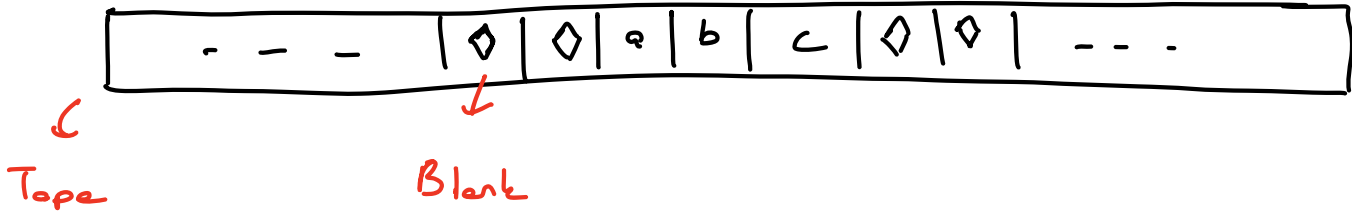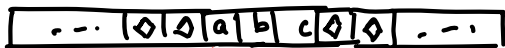
1. a gördüğün yere "c" yaz "R" sağa koy

| ... | $\Diamond$ | $\Diamond$ | a | b | c | $\Diamond$ | $\Diamond$ | ... | → | ... | $\Diamond$ | $\Diamond$ | c | b | c | $\Diamond$ | $\Diamond$ | ... |

2. b gördüğün yere "a" yaz "R" sağa koy

| ... | $\Diamond$ | $\Diamond$ | c | b | c | $\Diamond$ | $\Diamond$ | ... | → | ... | $\Diamond$ | $\Diamond$ | c | a | c | $\Diamond$ | $\Diamond$ | ... |

Halt (İlerlemenin Durması)

Bu durum ya bütün state yapılarının sonuna gelinilmiş ya da bir state durum ile eşleşmiş ve ilerlenilemez duruma sokulmuştur.
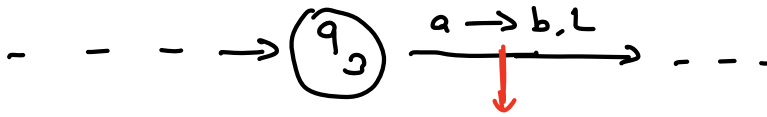
①

$- \quad - \quad - \quad \longrightarrow \; (q_s)$

Bu durum "Halt" olmuştur.

②

$- \quad - \quad - \longrightarrow (q_3) \xrightarrow{\; a \to b, L \;} - - -$

Bu durum "Halt" olmuştur.
"a" değeri yoktur. Bu durum sola/sağa kaymada da yaşanabilir.

Örnek:

$f(x,y) = x+y$ (Bunu 1'ler ile yap) Yani: $\left. \begin{array}{l} 2 = 11 \\ 3 = 111 \end{array} \right\}$ gibi

$2 + 3 = 5 = 11111 \to$ olacak

$1 \to 1, R$ (self loop on q_0)

$\to (q_0) \xrightarrow{\; 0 \to 1, R \;} (q_1) \xrightarrow{\; 0 \to 0, L \;} (q_2) \xrightarrow{\; 1 \to 0, L \;} (q_3)$

$1 \to 1, R$ (self loop on q_1)

$1 \to 1, L$ (self loop on q_3)

$(q_3) \xrightarrow{\; 0 \to \triangle, R \;} (q_u)$