## Bir Matrisin Rankı

Bir A matrisin basamak matrise dönüşüp 0'dan farklı satırdan sayısı.

$$\begin{matrix} 1 & 2 & -1 & 0 & - \\ 0 & 2 & 1 & 2 & - \\ 0 & 0 & 5/2 & 2 & - \\ 0 & 0 & 0 & 0 \end{matrix} \quad \text{Rank}=3$$

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \rightarrow 3$$

## Bir Matrisin Tersi

$$A = \begin{matrix} 3 & 1 & -1 \\ 1 & 0 & 2 \\ 0 & 2 & 3 \end{matrix} \Rightarrow \begin{matrix} A_{11}\ -4 & A_{12}\ -3 & A_{13}\ 2 \\ A_{21}\ -5 & A_{22}\ 9 & A_{23}\ 6 \\ A_{31}\ 2 & A_{32}\ -7 & A_{33}\ -1 \end{matrix} \quad \underbrace{\begin{matrix} -4 & -5 & 2 \\ -3 & 9 & -7 \\ 2 & 6 & -1 \end{matrix}}_{\text{Ek Mat.}}$$

Transpot.

Matris Tersi $A^{-1} = \dfrac{\text{Ek } A}{\det A}$

Soru: $A \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 3 & -2 & 4 \end{vmatrix}$ $\det(A) = -19$

$A_{11}^{(+)} \begin{matrix} 1 & 0 \\ -2 & 4 \end{matrix}$ $A_{12}^{(-)} \begin{matrix} 2 & 0 \\ 3 & 4 \end{matrix}$ $A_{13}^{(+)} \begin{matrix} 2 & 1 \\ 3 & -2 \end{matrix}$ $A_{21}^{(-)} \begin{matrix} 2 & 1 \\ -2 & 4 \end{matrix}$ $A_{22}^{(+)} \begin{matrix} 1 & 1 \\ 3 & 4 \end{matrix}$

$A_{23}^{(-)} \begin{matrix} 1 & 2 \\ 3 & -2 \end{matrix}$ $A_{31}^{(+)} \begin{matrix} 2 & 1 \\ 1 & 0 \end{matrix}$ $A_{32}^{(-)} \begin{matrix} 1 & 1 \\ 2 & 0 \end{matrix}$ $A_{33}^{(+)} \begin{matrix} 1 & 2 \\ 2 & 1 \end{matrix}$

$$\begin{matrix} 4 & -8 & -7 \\ -10 & 1 & 8 \\ -1 & 2 & -3 \end{matrix} \longrightarrow \underbrace{\begin{matrix} 4 & -10 & -1 \\ -8 & 1 & 2 \\ -7 & 8 & -3 \end{matrix}}_{\text{EK } A} \Rightarrow A^{-1} = \frac{1}{-19} \cdot \begin{matrix} 4 & -10 & -1 \\ -8 & 1 & 2 \\ -7 & 8 & -3 \end{matrix}$$

# Matrisler



Skaler Matris ⇒ Asıl Köşegen elemanları birbirine eşit.

Üst Üçgen matris

$$\begin{matrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{matrix}$$

Alt Üçgen Matris

$$\begin{matrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{matrix}$$

Matris izi ⇒ Kare matristeki esas köşegen üzerindeki elemanların toplamı.

Periyodik Matris ⇒ A Kare matris $A^{x>1} = A$ ise

İdempotent Matris ⇒ " " " $A^2 = A$ ise

Nilpotent Matris ⇒ " " " $A^k = 0$ ise

İnvolut Matris ⇒ " " " $A^2 = I$ ise

Hermitian Matris ⇒ " " " $(\bar{A})^T = A$ ise

! İki matrisin eşit olabilmesi için aynı tipten matris olmalı)

! " " (+ −) " " " " " "

Matris Transpozu

$$\begin{matrix} a & b & c \\ d & e & f \end{matrix} \Rightarrow \begin{matrix} a & d \\ b & e \\ c & f \end{matrix}$$

Simetrik

$$\begin{matrix} x & a & b \\ a & y & c \\ b & c & z \end{matrix}$$

Ters Simetrik Matris

$$\begin{matrix} x & -a & c \\ a & y & -b \\ -c & b & z \end{matrix}$$

# Matris Çarpımı



$$\begin{pmatrix} 3 & 2 \\ 1 & 1 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 3.2+2.1 & 3(-1)+2.2 & 3.0+2.3 & 3.1+2.1 \\ 1.2+1.1 & 1(-1)+1.2 & 1.0+1.3 & 1.1+1.1 \\ 0.2+4.1 & 0(-1)+4.2 & 0.0+4.3 & 0.1+4.1 \end{pmatrix}$$

A 3x2    B 2x4

$$\begin{pmatrix} 8 & 1 & 6 & 5 \\ 3 & 1 & 3 & 2 \\ 4 & 8 & 12 & 4 \end{pmatrix}$$

! Matrislerde Çarpma işleminde değişme özelliği yok. $(M.N \neq N.M)$

## Hadamard Çarpım

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 7 & 9 & (-6) \end{pmatrix}\begin{pmatrix} 2 & -4 & -3 \\ 5 & -8 & 10 \\ -1 & -2 & -8 \end{pmatrix} \Rightarrow \begin{matrix} 1.2 & 2(-4) & 5.(-3) \\ 0.5 & 1(-8) & 2.10 \\ 7.(-1) & 9(-2) & (-6)(-8) \end{matrix} \Rightarrow \begin{matrix} 2 & -8 & 15 \\ 0 & -8 & 20 \\ -7 & -18 & 48 \end{matrix}$$

A    B    $A \circ B$

## Kronecker Çarpım

$$A = \begin{matrix} -1 & 2 \\ 3 & 5 \end{matrix} \quad B = \begin{matrix} 3 & 4 \\ -5 & 7 \end{matrix} \Rightarrow \begin{matrix} (-1)\begin{matrix} 3 & 4 \\ -5 & 7 \end{matrix} & (2)\begin{matrix} 3 & 4 \\ -5 & 7 \end{matrix} \\ (3)\begin{matrix} 3 & 4 \\ -5 & 7 \end{matrix} & (5)\begin{matrix} 3 & 4 \\ -5 & 7 \end{matrix} \end{matrix} \Rightarrow \begin{matrix} -3 & -4 & 6 & 8 \\ 5 & -7 & -10 & 14 \\ 9 & 12 & 15 & 20 \\ -15 & 21 & -25 & 35 \end{matrix}$$

$A \otimes B$

## LU ve LDU

$$A = \begin{matrix} 2 & 1 & -1 \\ -3 & -1 & 2 \\ -2 & 1 & 2 \end{matrix} \quad L \begin{matrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -1 & 5/3 & 1 \end{matrix} \quad D \begin{matrix} 2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 2 \end{matrix} \quad U \begin{matrix} 1 & 1/2 & -1/2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{matrix}$$

**Örnek** $A = \begin{matrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ -1 & 3 & 0 \end{matrix}$ LU ayrışımını bul?

$$A = \begin{matrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ -1 & 3 & 0 \end{matrix} \xRightarrow[R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \begin{matrix} 1 & 2 & 1 \\ 0 & -5 & -1 \\ 0 & 5 & 1 \end{matrix} \xRightarrow{R_2+R_3 \to R_3} \begin{matrix} 1 & 2 & 1 \\ 0 & -5 & -1 \\ 0 & 0 & 0 \end{matrix} \Rightarrow U$$

$$I = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \Rightarrow \xrightarrow{-2R_1+R_2 \to R_2} \begin{matrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \xrightarrow{R_1+R_3 \to R_3} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix} \xrightarrow{R_2+R_3 \to R_3} \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}$$

$$A = \underbrace{E_1^{-1} E_2^{-1} E_3^{-1} U}_{LU}$$

$$\begin{matrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} . \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{matrix} . \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{matrix} = \begin{matrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -1 & 1 \end{matrix} = L$$

# Elemanter Satır Sütun işlemleri



$c(i,j)$ i. sütun ile j. sütunun yer değişimi

$c(k.i)$ i. " k ile çarpılması

$c(i,j,k)$ i. sütunun " " çarpılıp j. sütuna eklenmesi

**Basamak Matris:**

$$\begin{matrix} 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 1 & 2 & 0 & 3 & 5 \\ 0 & 3 & 1 & 4 & 2 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 3 & 0 \end{matrix}$$

**Eşelon Matris:** → Tamamı 0 dan oluşan satır varsa en aşağıda
→ Her satırın ilk elemanı 1 olmalı
→ 1 ler basamak matris olmalı.

$$A \begin{matrix} 1 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \quad B \begin{matrix} 1 & 5 & -3 \\ 0 & 1 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \quad C \begin{matrix} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{matrix}$$

A: X    B: ✓    C: X

→ 1 lere pilot ismi verilir 1 lerin üstündeki rakamda 0 olursa indirgenmiş eşelon matris olur

**Normal Form:** $I_K \; 0$, $\begin{matrix} I_K \\ 0 \end{matrix}$, $\begin{matrix} I_K & 0 \\ 0 & 0 \end{matrix}$, $I_K$

## Determinant

1x1 det $A = |a| \Rightarrow a$    2x2 det $\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$ $a_{11}.a_{22} - a_{12}a_{21}$

→ $|A| = |A^T|$
→ $|AB| = |A||B|$
→ $|A^{-1}| = \dfrac{1}{|A|}$

3x3 Sarrus Kuralı
$$\begin{matrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{matrix} \qquad + + + \quad - - -$$

→ Satır veya sütunun hepsi 0 ise det 0 dır
→ " " " k ile çarpılırsa det.k
→ " " " yer değiştirirse −det
→ 2 satır " " eşitse det=0
→ Satır veya sütun orantılı ise det=0

→ $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{vmatrix} = 30$

Soru: A matrisi nxn ve det A = 2

det(3A) → $3^n.2$
det(−A) $(-1)^n.2$
det($A^2$) $2^2 = 4$
det($A^{-1}$) $1/2$

# Öz Değerler ve Öz Vektörler

- $\Delta_A(\lambda) = \det(\lambda I - A)$ polinoma A matrisin Karekteristik polinom denir.

- $\Delta_A(\lambda) = 0$ denklemine A matrisin Karakteristik denklemi denir

- $\Delta_A(\lambda) = 0$ denkleminin köklerine A matrisin öz değerleri denir.

Örnek $\begin{matrix} 2 & 5 & -5 \\ 3 & 1 & -4 \\ 5 & 6 & -9 \end{matrix}$ matrisin öz değer ve öz vektörlerini bul.

$\begin{matrix} \lambda-2 & -5 & 5 \\ -3 & \lambda-1 & 4 \\ -5 & -6 & \lambda+9 \end{matrix}$

$\lambda^3 + 6\lambda^2 + 9\lambda = 0$

$\lambda(\lambda^2 + 6\lambda + 9) = 0$

Öz değerler $\underset{\lambda_1}{0}, \underset{\lambda_2, \lambda_3}{(-3)}$

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$\lambda_1$ için
$$\begin{aligned} -2x_1 - 5x_2 + 5x_3 &= 0 \\ -3x_1 - x_2 + 4x_3 &= 0 \\ -5x_1 - 6x_2 + 9x_3 &= 0 \end{aligned} \Rightarrow \begin{aligned} 2x_1 + 5x_2 - 5x_3 &= 0 \\ 3x_1 + x_2 - 4x_3 &= 0 \end{aligned}$$

$x_3 = a$ ise

$2x_1 + 5x_2 = 5a$

$3x_1 + x_2 = 4a$

$x_1 = 15/13\,a$

$x_2 = 7/13\,a$

$a = 15$ için $V_1\ [15, 7, 13]$

$\lambda_2$ ve $\lambda_3$ için
$$\begin{aligned} -5x_1 - 5x_2 + 5x_3 &= 0 \\ -3x_1 - 4x_2 + 4x_3 &= 0 \\ -5x_1 - 6x_2 + 6x_3 &= 0 \end{aligned}$$

$V_2\ [0, 1, 1]$

Örnek $\begin{matrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{matrix}$ özdeğerleri ve özvektörleri bul.

Karakteristik polinom

$\begin{matrix} 1-\lambda & -1 & -1 \\ 1 & 3-\lambda & 1 \\ -3 & 1 & -1-\lambda \end{matrix} = -\lambda^3 + 3\lambda^2 + 4\lambda - 12 = 0$

↳ Bu denklemin kökleri özdeğerler

$\lambda = 2 \quad \lambda = 3 \quad \lambda = -2$

$\lambda = 2$ için

$$\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 1 \\ -3 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} -x_1 - x_2 - x_3 &= 0 \\ x_1 + x_2 + x_3 &= 0 \\ -3x_1 + x_2 - 3x_3 &= 0 \end{aligned}$$

$\left.\begin{matrix} -x_1 - x_2 - x_3 = 0 \\ x_1 + x_2 + x_3 = 0 \end{matrix}\right\}$ aynı

$x_1 = k$

$x_3 = -k$

$[-1, 0, -1]$

$\lambda = 3$ için

$-2x_1 - x_2 - x_3 = 0$

$x_1 + x_3 = 0$

$-3x_1 + x_2 - 4x_3 = 0$

$x_2 = -x_1 \quad k$

$x_3 = -x_1 \quad -k$

$-k$

$[1, -1, -1]$

$\lambda = -2$ için

$3x_1 - x_2 - x_3 = 0$

$x_1 + 5x_2 + x_3 = 0$

$-3x_1 + x_2 + x_3 = 0$

$x_2 = -x_1$

$x_3 = 4x_1$

$[1, -1, 4]$

matris inversi

Örnek
$$A = \begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{matrix}$$

matrisin tersini Karakteristik Polinomdan faydalanarak hesaplan

$$\begin{matrix} \lambda-1 & -2 & -3 \\ -2 & \lambda-3 & -4 \\ -3 & -4 & \lambda-2 \end{matrix}$$

$$\lambda^3 - 6\lambda^2 - 18\lambda - 3$$

$\lambda$ yerine $A$

$$\Delta_A(\lambda) = A^3 - 6A^2 - 18A - 3I = 0$$

$$\Delta_A(\lambda) = \cancel{A}(A^2 - 6A - 18) - 3I$$

$$A^{-1} \quad \frac{(A^2 - 6A - 18I)}{3}$$

$$\frac{1}{3}\left( \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{bmatrix}^2 - 6 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \end{bmatrix} - 18 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$\frac{1}{3}\left( \begin{bmatrix} 14 & 20 & 17 \\ 20 & 29 & 26 \\ 17 & 26 & 29 \end{bmatrix} + \begin{bmatrix} -6 & -12 & -18 \\ -12 & -18 & -24 \\ -18 & -24 & -12 \end{bmatrix} + \begin{bmatrix} -18 & 0 & 0 \\ 0 & -18 & 0 \\ 0 & 0 & -18 \end{bmatrix} \right)$$

$$\frac{1}{3} \begin{bmatrix} -10 & 8 & -1 \\ 8 & -7 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

Örnek $x+y+2z=1$
$2y+7z=4$
$3x+3y+6z=3$
denklem sis. çöz?

$[A:B]\ \begin{matrix}1&1&2&1\\0&2&7&4\\3&3&6&3\end{matrix} \rightarrow \begin{matrix}1&1&2&1\\0&2&7&4\\0&0&0&0\end{matrix} \rightarrow \begin{matrix}1&1&2&1\\0&2/2&7/2&4/2\\0&0&0&0\end{matrix} \rightarrow \begin{matrix}1&0&-3/2&-1\\0&1&7/2&2\\0&0&0&0\end{matrix}$

$r_A = r_{[A:B]} = 2 \quad n=3$

$3-2=1 \Rightarrow$ Sis. sonsuz çöz. var

Örnek $3x+2y=7$
$17x+y=0$
$6x+4y=3$
denk. sis. çöz?

$[A:B]\ \begin{matrix}3&2&7\\17&1&0\\6&4&3\end{matrix} \quad \begin{matrix}3&2&7\\-1&-11&-42\\0&0&-11\end{matrix} \quad \begin{matrix}0&-31&-119\\-1&-11&-42\\0&0&1\end{matrix}$

$\begin{matrix}-1&-11&-42\\0&-31&-119\\0&0&1\end{matrix} \quad \begin{matrix}1&11&42\\0&1&119/31\\0&0&1\end{matrix} \quad \begin{matrix}1&0&-7/31\\0&1&119/31\\0&0&1\end{matrix}$

$r_A=2 \quad r_{A:B}=3$ denk. sis. çözümü yok.

Örnek $x_1 - x_2 + x_3 + 2x_4 - 2x_5 = 0$
$3x_1 + 2x_2 - x_3 - x_4 + 3x_5 = 1$
$2x_1 - 3x_2 - 2x_3 + x_4 - x_5 = -1$
lineer denk. sistemin çözümü olup olmadığını araştır.

$\begin{matrix}1&-1&+1&+2&-2&0\\3&2&-1&-1&3&1\\2&-3&-2&1&-1&-1\end{matrix}$ 1.str (-3) 2.satıra, 1.str (-2) 3.satıra $\begin{matrix}1&-1&1&2&-2&0\\0&5&-4&-7&9&1\\0&-1&-4&-3&3&-1\end{matrix}$

3.str 5 ile çarp $\begin{matrix}1&-1&1&2&-2&0\\0&5&-4&-7&9&1\\0&-5&-20&-15&15&5\end{matrix}$ 2. satırı 3. satıra ekle $\begin{matrix}1&-1&1&2&-2&0\\0&5&-4&-7&9&1\\0&0&-24&-22&24&-4\end{matrix}$

2.str 1/5, 3.str -1/24 $\begin{matrix}1&-1&1&2&-2&0\\0&1&-4/5&-7/5&9/5&1/5\\0&0&1&11/12&-1&1/6\end{matrix}$

rank A = rank [A:B] = 3
Sistem uyumlu ve tek bir çözüme sahip

$x_1 - x_2 + x_3 + 2s - 2t = 0$

$x_2 - 4/5\, x_3 - 7/5\, s + 9/5\, t = 1/5$

$x_3 + 11/12\, s - t = 1/6$

$x_1 \Rightarrow 1/6 - 5/12\, s$

$x_2 \Rightarrow 1/3 + 2/3\, s - t$

$x_3 \Rightarrow 1/6 - 11/12\, s + t$

## Homojen Olmayan Lineer Denklem Çözümü

### a) Ters Matris

A n kare matris olmak üzere $AX=b$ sis. verilsin. Sis. katsayılar matrisinin determinantı sıfırdan farklı ise tek çözümü vardır.

Örnek $x_1 + 3x_2 + 4x_3 = -2$
$3x_1 + 4x_2 + 5x_3 = 6$
$4x_1 + 5x_2 + 6x_3 = 4$
denklemi ters matris yöntemiyle çöz.

$\begin{matrix}1&3&4&1&0&0\\3&4&5&0&1&0\\4&5&6&0&0&1\end{matrix} \rightarrow \begin{matrix}1&3&4&1&0&0\\0&-5&-7&-3&1&0\\0&-7&-10&-4&0&1\end{matrix} \rightarrow \begin{matrix}1&3&4&1&0&0\\0&-5&-7&-3&1&0\\0&0&-1/5&1/5&-7/5&1\end{matrix}$

$\rightarrow \begin{matrix}1&3&4&1&0&0\\0&-5&-7&-3&1&0\\0&0&1&-1&7&-5\end{matrix} \rightarrow \begin{matrix}1&3&0&5&-28&20\\0&-5&0&-10&50&-35\\0&0&1&-1&7&-5\end{matrix} \rightarrow \begin{matrix}1&3&0&5&-28&20\\0&1&0&2&-10&7\\0&0&1&-1&7&-5\end{matrix}$

$\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix} \quad \underbrace{\begin{matrix}-1&2&-1\\2&-10&7\\-1&7&-5\end{matrix}} \Rightarrow \begin{matrix}-1&2&-1\\2&-10&7\\-1&7&-5\end{matrix} \begin{matrix}-2\\6\\4\end{matrix} \quad \begin{matrix}10\\-36\\24\end{matrix}$

### b) Cramer Metodu

Determinant 0 ise cramer metodu yapılamaz.

Örnek $x+2y+z=5$
$2x+2y+z=6$
$x+2y+3z=9$
Cramer metoduyla çöz.

$\Delta = |A| \begin{matrix}1&2&1\\2&2&1\\1&2&3\end{matrix} = -4 \quad \Delta_1 \begin{matrix}5&2&1\\6&2&1\\9&2&3\end{matrix} -4 \quad \Delta_2 \begin{matrix}1&5&1\\2&6&1\\1&9&3\end{matrix} = -4$

$\Delta_3 \begin{matrix}1&2&5\\2&2&6\\1&2&9\end{matrix} -8 \quad x\ \dfrac{\Delta_1}{\Delta} = 1 \quad y\ \dfrac{\Delta_2}{\Delta} = 1 \quad z\ \dfrac{\Delta_3}{\Delta} = 2$

## C) Gauss Eliminasyon Metodu

Matriste ters üçgen yaparak çözüme

**Örnek**

$$\begin{aligned}
x_1 + 3x_2 + 2x_3 + x_4 + x_5 &= 1 \\
3x_1 + 2x_2 + x_3 + 5x_4 + x_5 &= 2 \\
-x_1 + 2x_2 + x_3 + 6x_4 + x_5 &= 1 \\
3x_1 + x_2 + 2x_3 + 4x_5 &= 1
\end{aligned}$$

Gauss eliminasyon metodu ile çözelim.

$$\begin{array}{cccccc}
1 & 3 & 2 & 1 & 1 & 1 \\
3 & 2 & 1 & 5 & 1 & 2 \\
-1 & 2 & 1 & 6 & 1 & 1 \\
3 & 1 & 2 & 0 & 4 & 1
\end{array}
\qquad
\begin{array}{cccccc}
1 & 3 & 2 & 1 & 1 & 1 \\
0 & -7 & -5 & 2 & -2 & -1 \\
0 & 5 & 3 & 7 & 2 & 2 \\
0 & \cancel{-8} & \cancel{-4} & \cancel{-3} & \cancel{1} & \cancel{-2}
\end{array}$$

$$\begin{array}{cccccc}
1 & 3 & 2 & 1 & 1 & 1 \\
0 & -1 & 1 & -5 & 3 & -1 \\
0 & 0 & 8 & -18 & 17 & -3 \\
0 & 0 & -12 & 37 & -23 & 6
\end{array}
\qquad
\begin{array}{cccccc}
1 & 3 & 2 & 1 & 1 & 1 \\
0 & -1 & 1 & -5 & 3 & -1 \\
0 & 0 & 8 & -18 & 17 & -3 \\
0 & 0 & 0 & 20 & 5 & 3
\end{array}$$

$$\begin{aligned}
x_1 + 3x_2 + 2x_3 + x_4 + x_5 &= 1 \\
x_2 - x_3 + 5x_4 - 3x_5 &= 1 \\
8x_3 - 18x_4 + 17x_5 &= -3 \\
20x_4 + 5x_5 &= 3
\end{aligned}$$

$x_5 = a$ olsun

$$x_1 = \frac{23}{80} - \frac{a}{16}$$

$$x_2 = \frac{17}{80} + \frac{25}{16}a$$

$$x_3 = \frac{-3}{80} - \frac{43}{16}a$$

$$x_4 = \frac{3}{20} - \frac{a}{4}$$

## ⇒ Lineer Denklem Sistemleri

Genel olarak $x_1\ x_2\ x_3\ ---\ x_n$ bilinmeyenler $b_1 b_2 -- b_m$ sayılar olmak üzere $n$ bilinmeyenli $m$ adet denklemden oluşan sistemdir.

**Örnek**

$$\begin{aligned}
x_1 + x_2 - x_3 + x_4 &= 2 \\
2x_2 + x_3 - x_4 &= 5 \\
x_1 - x_3 + x_4 &= 0 \\
-x_1 - x_2 + x_3 &= -4
\end{aligned}$$

lineer denklem sistemini çöz.

1. denklemi -1 ile çarp 3'e ekle
1 " 4'e ekle

$$\begin{aligned}
x_1 + x_2 - x_3 + x_4 &= 2 \\
2x_2 + x_3 - x_4 &= 5 \\
-x_2 &= -2 \\
x_4 &= 2
\end{aligned}$$

2 ile 3'ü yer değiştir

$$\begin{aligned}
x_1 + x_2 - x_3 + x_4 &= 2 \\
-x_2 &= -2 \\
2x_2 + x_3 - x_4 &= 5 \\
x_4 &= 2
\end{aligned}$$

2. denklemin 2 katını 3. denkleme ekle

$$\begin{aligned}
x_1 + x_2 - x_3 + x_4 &= 2 \\
-x_2 &= -2 \\
x_3 - x_4 &= 1 \\
x_4 &= -2
\end{aligned}$$

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| 1 | 2 | -1 | -2 |

◁ Gauss Yok Etme Yöntemi

## Homojen Lineer Denklem Sistemi

$$\boxed{\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \qquad\qquad & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{aligned}}$$

$b_1, b_2, ---, b_m = 0$ ise bu sisteme homojen lineer denklem sistemi denir.

$(x_1, x_2\ ---\ x_n) = (0, 0\ ---\ 0)$ her zaman çözüm buna aşikar (sıfır) çözümdür.

## Gauss Jordan (Denk Matrisler) Yöntemi

$$[A:b] = \left[\begin{array}{cccc|c}
a_{11} & a_{12} & --- & a_{1n} & b_1 \\
a_{21} & a_{22} & --- & a_{2n} & b_2 \\
\vdots & & & \vdots & \vdots \\
a_{m1} & a_{m2} & --- & a_{mn} & b_m
\end{array}\right]$$

- $r_A \neq r_{[A:B]}$ sis. çözümü yok
- $r_A = r_{[A:B]} = r$ sis. çözümü var
  - ↳ $r = n$ tek çözüm
  - ↳ $r < n$ sonsuz çözüm

## Gauss Eliminasyon Methodu

$x_1 - 2x_2 + 3x_3 + x_4 = 1$
$x_1 + x_2 + 3x_3 + 4x_4 = -2$
$2x_1 + 3x_2 - x_3 - 2x_4 = 5$
$4x_1 - 5x_2 + 4x_3 + 2x_4 = 7$

$$\begin{matrix} 1 & -2 & 3 & 1 & 1 \\ 1 & 1 & 3 & 4 & -2 \\ 2 & 3 & -1 & -2 & 5 \\ 4 & -5 & 4 & 2 & 7 \end{matrix} \rightarrow \begin{matrix} 1 & -2 & 3 & 1 & 1 \\ 0 & 3 & 0 & 3 & -3 \\ 0 & 7 & -7 & -4 & 3 \\ 0 & 3 & -8 & -2 & 3 \end{matrix} \; - - - .$$

$$\begin{matrix} 1 & -2 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & -6 & 4 \\ 0 & 0 & 0 & -53 & 38 \end{matrix}$$

$x_1 = \frac{109}{53} \quad x_2 = \frac{-15}{53} \quad x_3 = \frac{-16}{53} \quad x_4 = \frac{-38}{53}$

## Lineer Homojen Denk. sis.

$r_A = r_{A:B} = r$ olsun   m denklem   n bilinmiyen

$r = n$ ise   $X = 0$ ise çözümü

$r < n$ ise   sıfırdan farklı sonsuz çözümü

$m < n$ ise   " " " "

## Öz değer Öz vektörler $(Ax_1 = \lambda x_1)$

$$\begin{matrix} 2 & 5 & -5 \\ 3 & 1 & -4 \\ 5 & 6 & -9 \end{matrix} \quad \begin{bmatrix} \lambda-2 & -5 & +5 \\ -3 & \lambda-1 & 4 \\ -5 & -6 & \lambda+9 \end{bmatrix} \rightarrow \Delta(\lambda)\ (\text{det.})$$

Karakteristik denk.

$\lambda^3 + 6\lambda^2 + 9\lambda = 0$

$\lambda_1 = 0$

$\lambda_2 = \lambda_3 = -3$

$\lambda_1 = 0$ için
$-2x_1 - 5x_2 + 5x_3 = 0$
$-3x_1 - x_2 + 4x_3 = 0 \Rightarrow$
$-5x_1 - 6x_2 + 9x_3 = 0$

$2x_1 + 5x_2 - 5x_3 = 0$
$3x_1 + x_2 - 4x_3 = 0$

$x_1 = \frac{15}{13}a \quad x_2 = \frac{7}{13}a$

$V_1\ [15, 7, 13]$

## Skaler Çarpım

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos a$

$\vec{i}\,\vec{i} = \vec{j}\,\vec{j} = \vec{k}\,\vec{k} = 1$

$\vec{i}\,\vec{j} = \vec{i}\,\vec{k} = \vec{j}\,\vec{k} = 0$

### Özellikler

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$

$|\vec{a} \cdot \vec{b}| \leq |\vec{a}||\vec{b}|$

$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

## Vektörel Çarpım

$\vec{a}$ ve $\vec{b}$ vektörlerin çarpımı $\vec{c}$ ise

$\vec{c} = \vec{a} \wedge \vec{b}$ veya $\vec{a} \times \vec{b}$ $(\vec{c} \perp \vec{a}$ ve $\vec{c} \perp \vec{b})$

$\vec{c} = \vec{a} \wedge \vec{b} = |\vec{a}||\vec{b}| \sin a\ \vec{u} \rightarrow$ ( $\vec{c}$ vektörü doğrultusunda birim vek.)

### Özellikler

$\vec{a} \wedge \vec{b} = -(\vec{b} \wedge \vec{a})$

$\vec{k}(\vec{a} \wedge \vec{b}) = (\vec{k}\vec{a}) \wedge \vec{b}$

$\vec{i}.\vec{i} = \vec{j}.\vec{j} = \vec{k}.\vec{k} = 0$

$\vec{i}\,\vec{j} = \vec{k} \quad \vec{k}.\vec{i} = \vec{j} \quad \vec{j}\,\vec{k} = \vec{i}$

$\vec{a} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$

$\vec{b} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$

$$\vec{c} = \vec{a} \wedge \vec{b} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

* Paralel yüzlünün hacmi

$$\vec{a}(\vec{b} \wedge \vec{c}) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

* $\vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a}\,\bar{b}) \cdot \bar{c}$

* Özuzay Özvektörler + 0 matris   * $(A - \lambda I)x = 0$

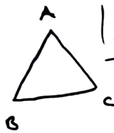
* Özvektörde 0 olmayacak

* $|\vec{a}| = \sqrt{a^2 + b^2 + c^2}$

* $\vec{u}$ vek. $\vec{v}$ üzerine dik izdüşümü

$$\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \cdot \frac{\vec{v}}{|\vec{v}|}$$

* üçgen alanı




$$\frac{|\vec{AB} \times \vec{AC}|}{2}$$

## Lineer Denklem Sis.

$$x+y+2z=1$$
$$2y+7z=4$$
$$3x+3y+6z=3$$

denk. sis. çöz.

$$\left(\begin{array}{cccc} 1 & 1 & 2 & 1 \\ 0 & 2 & 7 & 4 \\ 3 & 3 & 6 & 3 \end{array}\right) -3 \qquad \begin{array}{cccc} 1 & 1 & 2 & 1 \\ 0 & 2 & 7 & 4 \\ 0 & 0 & 0 & 0 \end{array} -1/2 \qquad \begin{array}{cccc} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/2 & 2 \\ 0 & 0 & 0 & 0 \end{array} - \qquad \begin{array}{cccc} 1 & 0 & -3/2 & -1 \\ 0 & 1 & 7/2 & 2 \\ 0 & 0 & 0 & 0 \end{array}$$

$$x-\frac{3}{2}z=-1 \qquad y+\frac{7}{2}z=2$$

$x=3k-1$
$y=2-7k$
$z=2k$

$r_A=r_{AB}=2$
$n=3$
$n>r$ olduğundan keyfi bilinmeyen

▽ rank [A:B] > rank [A] sis. uyumsuz.

▽ rank [A:B] = rank (A) ≤ r=min sonsuz çözüm.

$$x_1-x_2+x_3+2x_4-2x_5=0$$
$$3x_1+2x_2-x_3-x_4+3x_5=1$$
$$2x_1-3x_2-2x_3+x_4-x_5=-1$$

denk. sis. çöz.

$$\left(\begin{array}{ccccc|c} 1 & -1 & 1 & 2 & -2 & 0 \\ 3 & 2 & -1 & -1 & 3 & 1 \\ 2 & -3 & -2 & 1 & -1 & -1 \end{array}\right) \begin{array}{c} -3 \\ -2 \end{array} \quad \left|\begin{array}{ccccc|c} 1 & -1 & 1 & 2 & -2 & 0 \\ 0 & 5 & -4 & -7 & 9 & 1 \\ 0 & -1 & -4 & -3 & 3 & -1 \end{array}\right. /5 \quad \left|\begin{array}{cccccc} 1 & -1 & 1 & 2 & -2 & 0 \\ 0 & 5 & -4 & -7 & 9 & 1 \\ 0 & -5 & -20 & -15 & 15 & -5 \end{array}\right.$$

$$\begin{array}{cccccc} 1 & -1 & 1 & 2 & -2 & 0 \\ 0 & 5 & -4 & -7 & 9 & 1 \\ 0 & 0 & -24 & -22 & 24 & -4 \end{array} \begin{array}{c} \\ 1/5 \\ -1/24 \end{array} \quad \left\|\left(\begin{array}{cccccc} 1 & -1 & 1 & 2 & -2 & 0 \\ 0 & 1 & -4/5 & -7/5 & 9/5 & 1/5 \\ 0 & 0 & 1 & 11/12 & -1 & -1/6 \end{array}\right)\right.$$

rank A = rank AB = 3
n = bilinmeyen = 5
k = denklem = 3
$x_4=s$
$x_5=t$

$$x_1-x_2+x_3+2s-2t=0$$
$$x_2-\frac{4}{5}x_3-\frac{7}{5}s+\frac{9}{5}t=\frac{1}{5}$$
$$x_3+\frac{11}{12}s-t=\frac{1}{6}$$

$$x_3=\frac{1}{6}-\frac{11}{12}s+t$$
$$x_2=\frac{1}{3}+\frac{2}{3}s-t$$
$$x_1=\frac{1}{6}-\frac{5}{12}s$$

## Ters Matris Metodu

$$x+3y+4z=1$$
$$2x+y+5z=-1$$
$$3x+y+2z=1$$

denk. sis. ters matris ile çöz.

$$\begin{array}{ccc} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 1 & 2 \end{array} \quad \begin{array}{c} x \\ y \\ z \end{array} = \begin{array}{c} 1 \\ -1 \\ 1 \end{array} \Rightarrow \begin{array}{c} x \\ y \\ z \end{array} = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 1 & 2 \end{bmatrix}^{-1} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \\ 3 & 1 & 2 \end{vmatrix} = 26$$

minör
$$\begin{array}{ccc} -3 & 11 & -1 \\ -2 & -10 & 8 \\ 11 & 3 & -5 \end{array} \qquad \frac{1}{|\Delta|} \begin{array}{ccc} -3 & -2 & 11 \\ 11 & -10 & 3 \\ -1 & 8 & -5 \end{array}$$

$$\begin{array}{c} x \\ y \\ z \end{array} \begin{pmatrix} \left(\frac{5}{13}\right) \\ \left(\frac{12}{13}\right) \\ \left(\frac{-7}{13}\right) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{-3}{26} & \frac{-1}{13} & \frac{11}{26} \\ \frac{11}{26} & \frac{-10}{26} & \frac{+3}{26} \\ \frac{-1}{26} & \frac{8}{26} & \frac{-5}{26} \end{pmatrix}$$

## Cramer Methodu

$$x_1+3x_2+4x_3=-2$$
$$3x_1+4x_2+5x_3=6$$
$$4x_1+5x_2+6x_3=4$$

$$A=\begin{vmatrix} 1 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}=1$$

$$A_1\begin{vmatrix} -2 & 3 & 4 \\ 6 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}=10 \qquad A_2\begin{vmatrix} 1 & -2 & 4 \\ 3 & 6 & 5 \\ 4 & 4 & 6 \end{vmatrix}=-36 \qquad A_3\begin{vmatrix} 1 & 3 & -2 \\ 3 & 4 & 6 \\ 4 & 5 & 4 \end{vmatrix}=24$$

$$x_1=\frac{10}{1} \qquad x_2=\frac{-36}{1} \qquad x_3=\frac{24}{1}$$