① $\frac{dy}{dx} = (x+y-5)^2$ dif. denklemin genel çözümü?

$x+y-5=u \Rightarrow y = u-x+5$

$\frac{dy}{dx} = \frac{du}{dx} - 1$

$\frac{du}{dx} - 1 = u^2 \Rightarrow \frac{du}{dx} = u^2+1$

$\frac{du}{u^2+1} = dx$

$\arctan u = x + c$

$u = \tan(x+c)$

$x+y-5 = \tan(x+c) \Rightarrow y = \tan(x+c) - x + 5$

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② $x^4y' + 2x^3y = e^{\frac{-1}{x}}$ $y(1)=0$ dif. denk. genel çözümü?

(x^4'e böl) $y' + \frac{2}{x}y = \frac{e^{\frac{-1}{x}}}{x^4}$ (Lineer D.D $y' + P(x)y = Q(x)$)

$P(x) = \frac{2}{x}$ $Q(x) = \frac{e^{\frac{-1}{x}}}{x^4}$ $M(x) = e^{\int \frac{2}{x}} = e^{2\ln x} = x^2$

x^2 ile çarptık

$x^2y' + 2xy = \frac{e^{-1/x}}{x^2} \Rightarrow \int (x^2 \cdot y)' = \int \frac{e^{-1/x}}{x^2}$

$\frac{-1}{x} = u$ $\int e^u du = e^u + c$

$\frac{1}{x^2}dx = du$

$x^2 \cdot y = \int \frac{e^{-1/x}}{x^2}$

$x^2 \cdot y = e^{\frac{-1}{x}} + c$

$y = \frac{e^{-1/x}}{x^2} + \frac{c}{x^2}$

---

③ $(x^4+y^4)dx - xy^3dy = 0$ dif. denk. genel çözümü?

$\frac{x^4+y^4}{xy^3} = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \left(\frac{x}{y}\right)^3 + \frac{y}{x}$ (Homojen $y=ux$ $y'=u'x+u$)

$u'x + \cancel{u} = \left(\frac{1}{u}\right)^3 + \cancel{u} \Rightarrow \frac{du}{dx}x = \frac{1}{u^3}$

$\int u^3 du = \int \frac{1}{x}dx$

$\frac{u^4}{4} = \ln|x| + c \Rightarrow u^4 = 4\ln|x| + c$

$\frac{y^4}{x^4} = 4\ln|x| + c$

$\Rightarrow y^4 = x^4\, 4\ln|x| + c$

---

④ $y' + \frac{2}{x}y = xy^3$ dif. denk. genel çözümü?

($y' + P(x)y = Q(x)y^n$ = Bernoulli Dif. Denk)

$P(x) = \frac{2}{x}$ $Q(x) = x$ $n=3$ $z = y^{1-n} = y^{-2}$

$z' = -2y^{-3}y'$

($-2y^{-3}$ ile çarptık)

$\underbrace{-2y^{-3} \cdot y'}_{z'} - \frac{4}{x}y^{-2} = -2x$

$z' - \frac{4}{x}z = -2x$ (Lineer D.D) ($z' + P(x)z = Q(x)$)

$M(x) = e^{\int \frac{-4}{x}} = \frac{1}{x^4}$

$1/x^4$ ile çarptık

$\int \frac{1}{x^4}z' - \frac{4}{x^5}z = \int \frac{-2}{x^3}$

$\int \left(z\frac{1}{x^4}\right)' = \int \frac{-2}{x^3}$

$z\frac{1}{x^4} = \underbrace{\int \frac{-2}{x^3}}_{\frac{1}{x^2}+c}$

$z \cdot \frac{1}{x^4} = \frac{1}{x^2} + c$

$z = x^2 + x^4c$

$\frac{1}{y^2} = x^2 + x^4c$

$y^2 = \frac{1}{x^2 + x^4c}$

→ ⑤ $\underbrace{(x^2+y^2+3)}_{M(x,y)}dx + \underbrace{(2xy+y+1)}_{N(x,y)}dy = 0$ dif. denk. genel çözümü?

$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = 2y$ ikisi de eşit old. için tam. d. denklemdir

$F(x,y) = \int M(x,y)\,dx + h(y) \Rightarrow \int (x^2+y^2+3)\,dx + h(y)$

$F(x,y) = \frac{x^3}{3} + xy^2 + 3x + h(y)$

$\frac{\partial F(x,y)}{\partial y} = 2xy + h'(y) = N(x,y) = 2xy + y + 1$

$\int h'(y) = \int y+1 \Rightarrow h(y) = \frac{y^2}{2} + y + c$

$F(x,y) = \frac{x^3}{3} + xy^2 + 3x + \frac{y^2}{2} + y + c$

→ ⑥ $(x^2+1)\cot y\, y' = 2x$, $y(1) = \pi/2$ başlangıç değer koşulu ile verilen d. denk. çözümü?

$(x^2+1)\cot y \frac{dy}{dx} = 2x \Rightarrow \int \cot y\, dy = \int \frac{2x}{x^2+1}\,dx$ (Değişkenlere ayrıl

$\int \cot y\, dy \qquad \Rightarrow \ln|\sin y| = \ln|x^2+1| + \ln c$

$\int \frac{\cos y}{\sin y}\,dy = \ln|\sin y| + c \Rightarrow \ln|\sin y| = \ln|(x^2+1).c|$

$|\sin y| = (x^2+1).c$ c'de pot olmalı.

$\oplus \qquad \oplus$

$-\sin y = (x^2+1).c \qquad \sin y = (x^2+1)\,c$

$y_1 = -\arcsin(cx^2+c) \qquad * \; y_2 = +\arcsin(cx^2+c)$

$\frac{\pi}{2} = -\arcsin(2c) \qquad \therefore \frac{\pi}{2} = +\arcsin(2c)$

$c = -1/2 \qquad c = 1/2$

→ ⑦ $\underbrace{x\frac{dy}{dx}} = y\ln(xy)$ dif. denk. genel çözümü nedir?

$u = xy \qquad \frac{du}{dx} = y + x\frac{dy}{dx} \Rightarrow \frac{du}{dx} - y = x\frac{dy}{dx}$, $y = \frac{u}{x}$

$x\frac{dy}{dx} = y\ln(x.y)$

$\frac{du}{dx} - \frac{u}{x} = \frac{u}{x}\ln u \Rightarrow \frac{du}{dx} = \frac{u}{x}\ln u + \frac{u}{x} \Rightarrow \frac{du}{dx} = \frac{u}{x}(\ln u + 1)$

Ayrılabilir dif. denklem-

$\int \frac{du}{u.(\ln u+1)} = \int \frac{dx}{x}$

$\int \frac{du}{u.\ln(u+1)} \qquad \ln u + 1 = w$

$\frac{1}{u}du = dw$

$\int \frac{dw}{w} = \ln|w| + c$

$\ln(|\ln(u+1)| + 1) = \ln x + \ln c$

$\ln u + 1 = x + c$

$\ln u = cx - 1 \Rightarrow u = \frac{e^{cx}}{e} \Rightarrow xy = \frac{e^{cx}}{e} \Rightarrow y = \frac{e^{cx}}{e.x}$

→ ⑧ $\frac{dy}{dx} + \frac{4}{x}y = \frac{\cos x}{x^4}$ dif. denk. genel çözümü nedir?

$\frac{dy}{dx} + \frac{4}{x}y = \frac{\cos x}{x^4}$ ($y' + P(x)y = Q(x)$ Lineer Dif. Denk.)

$M(x) = e^{\int \frac{4}{x}dx} = e^{4\ln x} = e^{\ln x^4} = x^4$

x^4 ile çarptık

$x^4\frac{dy}{dx} + 4x^3y = \cos x \Rightarrow \int (x^4 y)' = \int \cos x$

$x^4 y = \sin x + c$

$y = \frac{\sin x}{x^4} + \frac{c}{x^4}$

→ (9) $\frac{dy}{dx} + P(x) \cdot y = Q(x)\, y^n$ Bernoulli d.d. $u = y^{1-n}$ dönüşümü ile bir lineer d.d. dönüştüğünü göster.

$u = y^{1-n}$ $\quad \frac{d(u)}{dx} = (1-n)\, y^{-n} \frac{dy}{dx}$

$\frac{dy}{dx} + P(x) y = Q(x)\, y^n$ denklemi $(1-n)\, y^{-n}$ ile çarp $\quad (1-n)\, y^{-n} \frac{dy}{dx} + P(x)(1-n)$ ---------

$\Rightarrow \underbrace{(1-n)\, y^{-n} \frac{dy}{dx}}_{\frac{du}{dx}} + P(x) \underbrace{(1-n)}\, y^{1-n} = Q(x)(1-n)$

$\frac{du}{dx} + P(x) \cdot u = Q(x) \rightarrow$ Lineer

∇ P(x) ve Q(x) bir katsayı ile çarpmak karakterlerini değiştirmez.

→ (10) $x^2 \frac{dy}{dx} + 2xy = y^3$ dif. denk genel çöz. bulun?

$\frac{dy}{dx} + P(x)\, y = Q(x)\, y^n$
↳ Katsayı 1 olduğundan x^2'ye böl

$\frac{dy}{dx} + \frac{2x}{x^2} y = \frac{1}{x^2} y^3$

$u = y^{1-n} = y^{-2}$ $\quad \frac{du}{dx} = \underbrace{-2 y^{-3}}\frac{dy}{dx}$

Denklemde çarp

$\underbrace{-2 y^{-3} \frac{dy}{dx}} + \frac{2}{x}(-2) y^{-3} \cdot y = \frac{1}{x^2}(-2)\, y^{-3} y^3$

$\frac{du}{dx} - \frac{4}{x} u = \frac{-2}{x^2}$ (Lineer D.D.)

$M(x) = e^{\int \frac{-4}{x}} = \frac{1}{x^4}$ $\Rightarrow$ $\frac{1}{x^4}$ ile çarp $\quad \frac{1}{x^4}\frac{du}{dx} - \frac{4}{x^5} u = \frac{-2}{x^6}$

$\Rightarrow \int \frac{d}{dx}\left(\frac{1}{x^4} \cdot u\right) = \int -2 x^{-6}$

$\Rightarrow \frac{1}{x^4} \cdot u = \frac{2}{5} x^{-5} + c$

$\boxed{u = y^{-2}}$

$\Rightarrow y^2 = \frac{5x}{2 + 5cx^5}$

→ (11) $\left(\ln (y/x)^y + y\right) dx - x\, dy = 0$ dif. denk. genel çözümü?

$\Rightarrow \left(y \ln (y/x) + y\right) dx = x\, dy \Rightarrow \frac{y \cdot \ln \frac{y}{x} + y}{x} = \frac{dy}{dx}$

$\Rightarrow \frac{y}{x} \ln \frac{y}{x} + \frac{y}{x} = \frac{dy}{dx}$ (Homojen Dif. D. $y = ux$)

$y = ux$, $\frac{dy}{dx} = \frac{du}{dx} x + u$, $\frac{y}{x} = u$

$u \cdot \ln u + u = \frac{du}{dx} \cdot x + u \Rightarrow \int \frac{dx}{x} = \int \frac{du}{u \cdot \ln u}$ (Ayrılabilir D.D.)

$\ln x + \ln c = \ln(\ln u)$

$x \cdot c = \ln u$

$u = e^{xc}$

$\frac{y}{x} = e^{xc} \Rightarrow y = x \cdot e^{cx}$

→ (12) $\frac{dy}{dx} = \frac{xy^2 + y^2}{-2xy}$ dif. denk. genel çözümü nedir?

$\underbrace{(xy^2 + y^2)}_{M(x,y)} dx + \underbrace{2xy}_{N(x,y)}\, dy = 0 \Rightarrow$ $\frac{\partial M(x,y)}{\partial y} = 2xy + 2y$, $\frac{\partial N(x,y)}{\partial x} = 2y$ ↗ Eşit değil, Tam değil

$\boxed{\frac{M_y - N_x}{N}} = \frac{2xy + 2y - 2y}{2xy} = 1 = f(x) \rightarrow$ Sadece x'e bağlı

$\lambda(x) = e^{\int 1\, dx} = e^x$

$\underbrace{e^x (xy^2 + y^2)}_{M(x,y)} dx + \underbrace{e^x\, 2xy}_{N(x,y)}\, dy = 0 \Rightarrow$ $\frac{\partial M(x,y)}{\partial y} = 2x e^x y + 2 e^x y$, $\frac{\partial N(x,y)}{\partial x} = 2y e^x + 2y x e^x$ Tam Dif. Denk.

$F(x,y) = \int (2x e^x y)\, dy + k(x)$

$F(x,y) \Rightarrow x \cdot e^x y^2 + k(x)$ $\xrightarrow{}$ x'e göre kısmi türev $\quad y^2 (1 \cdot e^x + x \cdot e^x) + k'(x) = e^x x y^2 + e^x y^2$ $\quad k'(x) = 0$, $k(x) = c$

$\boxed{F(x,y) = x e^x y^2 + c}$

→ 13) $\frac{dy}{dx} - \tan^2 x \sec y - \sec y = 0$ dif. denk. genel çözümü?

$$\frac{dy}{dx} = \sec y(\tan^2 x + 1) \Rightarrow \frac{dy}{\sec y} = (\tan^2 x + 1)\,dx$$

$$\int \cos y \, dy = \int (\tan^2 x + 1)\, dx$$ (Ayrılabilir D.D)

$\sin y = \tan x + c$ (arcsin alma) $y = \arcsin(\tan x + c)$

→ 14) $(2 + x\cos y)\,dx = x^2 \sin y \, dy = 0$ dif. denk. genel çözümü?

($\cos y$'nin türevi soruda var)

$\cos y = u \Rightarrow -\sin y = \frac{du}{dy} \Rightarrow -\sin y \, dy = du$

$(2 + xu)\,dx + x^2\,du = 0 \Rightarrow (2 + xu)\,dx = -x^2\,du$

$$\Rightarrow \frac{2+xu}{-x^2} = \frac{du}{dx} \Rightarrow \frac{-2}{x^2} - \frac{u}{x} = \frac{du}{dx}$$

$$\Rightarrow \frac{du}{dx} + \frac{1}{x}u = \frac{-2}{x^2}$$ (Lineer D.D.) $(u' + P(x)u = Q(x))$

$$\mu(x) = e^{\int \frac{1}{x}} = e^{\ln x} = x$$

$$x\frac{du}{dx} + u = \frac{-2}{x}$$

$$\int (x.u)' = \int \frac{-2}{x} \Rightarrow x.u = -2\ln|x| + c$$

$$u = \ln\left|\frac{1}{x^2}\right| + c$$

$$\cos y = \frac{\ln 1/x^2 + c}{x}$$

$$y = \arccos\left(\frac{\ln 1/x^2 + c}{x}\right)$$

→ 15) $y^2\,dx + (-x^2 + xy)\,dy = 0$

$$y^2\,dx = (x^2 - xy)\,dy \Rightarrow \frac{y^2}{x^2 - xy} = \frac{dy}{dx}$$ (Her tarafı x^2'ye böl)

$$\Rightarrow \frac{\frac{y^2}{x^2}}{\frac{x^2-xy}{x^2}} = \frac{dy}{dx} \Rightarrow \frac{\left(\frac{y}{x}\right)^2}{1 - \frac{y}{x}} = y'$$ (Homojen D.D.) $y = ux$

$$\frac{x^2 - xy}{y^2} = \frac{dx}{dy} \Rightarrow \left(\frac{x}{y}\right)^2 - \frac{x}{y} = \frac{dx}{dy}$$ (Homojen D.D. $x = uy$, $x' = u'y + u$)

$$\Rightarrow u^2 - u = u'y + u$$

$$\Rightarrow \int \frac{du}{u^2 - 2u} = \int \frac{dy}{y}$$

$$\frac{1}{u^2 - 2u} = \frac{A}{u} + \frac{B}{(u-2)}$$

$1 = A(u-2) + B(u)$

$1 = u(A+B) - 2A$

$A + B = 0$ $\quad A = -1/2$

$-2A = 1$ $\quad B = 1/2$

$$\int \left(\frac{-1/2}{u} + \frac{1/2}{u-2}\right) du = \int \frac{dy}{y}$$

$$-1/2 \ln|u| + 1/2 \ln|u-2| = \ln|y| + \ln c$$

$$\Rightarrow \ln\left|\frac{1}{\sqrt{u}}\right| + \ln\left|\sqrt{u-2}\right| = \ln y + \ln c$$

$$\Rightarrow \ln\left|\frac{1}{\sqrt{u}} \cdot \sqrt{u-2}\right| = \ln|cy| \Rightarrow \sqrt{\frac{u-2}{u}} = cy$$

$$\Rightarrow \frac{\sqrt{\frac{x}{y} - 2}}{\sqrt{x/y}} = c.y$$

$$\Rightarrow \sqrt{1 - \frac{2y}{x}} = c.y$$

→ 16 Dif. denklemlerin lineer olup olmadığını incele.

lineer için
→ y'nin üssü olmayacak y^2, y^3 —
→ y'nin türev üssü $(y')^2, (y')^4$ —
→ $y \cdot y'$, $y \cdot y''$ → e^y, $\ln y$ bozar.

a) $\dfrac{d^2y}{dx^2} + y\dfrac{dy}{dx} + \sin x = 0 \quad y'' + yy' + \sin x = 0$ lineer değil

b) $\dfrac{d^3y}{dx^3}\sin^3 x + \dfrac{dy}{dx}\cos x + y\sin x = 0 \quad y''' \sin^3 x + y'\cos x + y\sin x = 0$ ✓

c) $\dfrac{d^2y}{dx^2} = x\left(\dfrac{dy}{dx}\right)^3 \quad y'' = x(y')^3$ lineer değil

→ 17 $y' + y\cot x = 4\sin x \quad y\left(\dfrac{-\pi}{2}\right) = 0$ başlangıç değer prb. çöz?

$y' + P(x)y = Q(x)$

$\mu(x) = e^{\int \cot x\, dx} = e^{\int \cot x\, dx = \ln|\sin x|} = |\sin x|$

$\sin x\,(y' + y\cot x) = 4\sin x \sin x$

$\int (\sin x\, y)' = \int 4\sin^2 x \Rightarrow \sin x\, y = \int 4\left(\dfrac{1-\cos 2x}{2}\right)dx$

$\Rightarrow \sin x\, y = \int (2 - 2\cos 2x)\, dx$

$\Rightarrow \sin x\, y = 2x - \sin 2x + c \qquad c = \pi$

$\Rightarrow y = \dfrac{2x}{\sin x} - \dfrac{\sin 2x}{\sin x} + \dfrac{c}{\sin x}$

$y\left(\dfrac{-\pi}{2}\right) = 0$ (x, y)

$\Rightarrow 0 = \dfrac{-\pi}{-1} - \dfrac{0}{-1} + \dfrac{c}{-1} \qquad c = \pi$

→ 18 $y'(x^2+1) - xy = x^3y^3$ dif. denk. çöz?

bernoulli

$y' + P(x)y = q(x)y^n$

$V = y^{1-n}$

(x^2+1)'e böl $\quad y' - \dfrac{x}{x^2+1}y = \dfrac{x^3}{x^2+1}y^3$

$V = y^{1-n} = y^{-2} \quad V' = -2y^{-3}y' \quad y' = \dfrac{V'y^3}{-2}$

$\dfrac{V'y^{\cancel{3}2}}{-2} - \dfrac{x}{x^2+1}\cancel{y} = \dfrac{x^3}{x^2+1}y^{\cancel{3}2}$ (her tarafı y'e böl)

$\dfrac{V'}{-2V} - \dfrac{x}{x^2+1} = \dfrac{x^3}{x^2+1} \cdot \dfrac{1}{V}$ ($-2V$ ile çarpıp lineer yapılacak)

$V' + \left(+\dfrac{2x}{x^2+1}\right)V = -\dfrac{2x^3}{x^2+1} \Rightarrow \mu(x) = e^{\int \frac{2x}{x^2+1}dx}$

$u = x^2+1$
$du = 2x\,dx$

$\mu(x) = e^{\int \frac{1}{u}du} \Rightarrow e^{\ln|u|} \Rightarrow x^2+1$

$\int (x^2+1)\left(V' + \dfrac{2x}{x^2+1}V\right) = \int (x^2+1)\dfrac{-2x^3}{x^2+1}$

$(x^2+1) \cdot V = \int -2x^3\, dx$

$(x^2+1)V = \dfrac{-x^4}{2} + c \Rightarrow V = \dfrac{\dfrac{-x^4}{2} + c}{x^2+1}$

$V = y^{-2}$ ise

$y^{-2} = \dfrac{\dfrac{-x^4}{2} + c}{x^2+1}$

19) $(x^n+y^n)\,y' - x^{n-1}\,y = 0$ $\quad x,y>0$ biçiminde verilen dif. denklem için n'nin hangi değeri için tam olur?

$$(x^n+y^n)\frac{dy}{dx} - x^{n-1}\,y = 0 \Rightarrow (x^n+y^n)\,dy - x^{n-1}\,y\,dx = 0$$

$$\underset{M}{(-x^{n-1}y)}\,dx + \underset{N}{(x^n+y^n)}\,dy = 0 \qquad M_y = -x^{n-1} \qquad N_x = n.\,x^{n-1} \qquad \boxed{n=-1}$$

$$(-x^{-2}y)\,dx + (x^{-1}+y^{-1})\,dy = 0$$

$$\int -x^{-2}y\,dx = \frac{-x^{-1}}{-1}y + c(y) = \frac{y}{x} + c(y)$$

$$=$$

$$\int (x^{-1}+y^{-1})\,dy = x^{-1}y + \ln|y| + c(x) = \frac{y}{x} + \ln|y| + c(x) = 0$$

$$c(y) = \ln(y)$$
$$c(x) = 0$$

$$\frac{y}{x} + \ln|y| = c$$