



# Iterative calculation of the heat transfer coefficient

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## Aim

The plate temperature of a cooling heat sink is an important parameter that has to be determined with accuracy. The estimated value depends on its geometrical shape, on the total amount of energy to be dispersed, and on the air flow. The heat transfer coefficient,  $h$ , is the most difficult parameter to be settled. In this report it is shown a fast and easy iterative method to calculate the  $h$  value and later, the temperature of cooling for heat sink.

## Introduction

The heat transfer coefficient or convective coefficient ( $h$ ), is used in thermodynamics to calculate the heat transfer typically occurring by convection. A simple way to calculate  $h$  is to define it through the classical formula for convection, and compare it with a different definition of  $h$ , through dimensionless parameters. Unfortunately, even if defined by means of different parameters, both the environment and the heat sink temperature are important to estimate  $h$ . An iterative method is then required, by setting an initial value of the  $T_p$ .

## Convection heat transfer coefficient

The formula for heat transfer is:

$$Q = h * S * (T_p - T_a) \quad (1)$$

Where:

- $Q$  = heat transferred, J/s = W
- $h$  = heat transfer coefficient, W/(m<sup>2</sup> K)
- $S$  = transfer surface, m<sup>2</sup>
- $T_p$  = Plate temperature, K
- $T_a$  = Air temperature, K

For convection we use the convection heat transfer coefficient  $h_c$ , W/(m<sup>2</sup> K). A different approach is to define  $h$  through the Nusselt number  $Nu$ , which is the ratio between the convective and the conductive heat transfer:

$$Nu = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}} = (h_c * L)/k \quad (2)$$

Where:

- $Nu$  = Nusselt number
- $h_c$  = convective heat transfer coefficient
- $k$  = thermal conductivity, W/mK
- $L$  = characteristic length, m



The convection heat transfer coefficient is then defined as following:

$$h_c = \frac{Nu \cdot k}{L} \quad (3)$$

The **Nusselt number** depends on the geometrical shape of the heat sink and on the air flow. For natural convection on flat isothermal plate the formula of Na is given in table 1.

Table 1: Nusselt number formula.

	Vertical fins		Horizontal fins
Laminar flow	$Nu = 0.59 * Ra^{0.25}$	Upward laminar flow	$Nu = 0.54 * Ra^{0.25}$
Turbulent flow	$Nu = 0.14 * Ra^{0.33}$	Downward laminar flow	$Nu = 0.27 * Ra^{0.25}$
		Turbulent flow	$Nu = 0.14 * Ra^{0.33}$

Where:

$$Ra = Gr * Pr \quad (4)$$

is the **Rayleigh number** defined in terms of Prandtl number (Pr) and Grashof number (Gr). If  $Ra < 10^9$  the heat flow is laminar, while if  $Ra > 10^9$  the flow is turbulent.

The **Grashof number**, Gr is defined as following:

$$Gr = \frac{g * L^3 * \beta * (T_p - T_a)}{\eta^2} \quad (5)$$

Where:

- g = acceleration of gravity = 9.81, m/s<sup>2</sup>
- L = longer side of the fin, m
- $\beta$  = air thermal expansion coefficient. For gases, is the reciprocal of the temperature in Kelvin:  
 $\beta = \frac{1}{T_a}$ , 1/K
- Tp = Plate temperature, °C.
- Ta = Air temperature, °C
- $\eta$  = air kinematic viscosity, is 1.5<sup>5</sup> at 20 °C. 1.6<sup>5</sup> at 30 °C.

For plate temperature, Tp, set a expected value. Finally, the **Prandtl number**, Pr is defined as:

$$Pr = \frac{\mu * c_p}{k} \quad (6)$$

Where:

- $\mu$  = air dynamic viscosity, is 1.81<sup>5</sup> at 20 °C. 1.86<sup>5</sup> at 30 °C.
- cp = air specific heat = 1005 J/(Kg\*K) for dry air
- k = air thermal conductivity = 0.026 W/(m\*K) at 27 °C



## Radiant heat transfer coefficient

The total amount of energy leaving a surface as radiant heat depends on the absolute temperature and on the nature of the surface. A non-blackbody, emits radiant energy from its surface at a rate that is given by:

$$q = \sigma * \varepsilon * S * \Delta T^4 = \sigma * \varepsilon * S * (T_p^4 - T_a^4) \quad (7)$$

Where  $\sigma = 5.67 * 10^{-8} \frac{W}{m^2 * K^4}$  is the Stefan-Boltzmann constant and  $\varepsilon$  is the emissivity. Emissivity varies from 0 (for non emitting body) to 1 (for blackbody). Values of emissivities for aluminum is presented in Table 2. Replacing q of Eq. 7 in Eq.1 we obtain:

$$\sigma * \varepsilon * S * (T_p - T_a)^4 = h * S * (T_p - T_a) \quad (8)$$

And solving for h obtain the formula for radiant heat transfer coefficient,  $h_r$ :

$$h_r = \frac{\sigma * \varepsilon * (T_p^4 - T_a^4)}{(T_p - T_a)} \quad (9)$$

Table 2. Emissivities of different aluminum surface treatment

Aluminum	310 K	530 K	800 K
Polished	0.04	0.05	0.08
Oxidized	0.11	0.12	0.18
24-ST weathered	0.40	0.32	0.27
Anodized (at 1000°F)	0.94	0.42	0.60

For not negligible radiation the overall heat transfer coefficient is:

$$h = h_c + h_r \quad (10)$$

In Figure 1 are reported the values of  $h_r$  as a function of  $T_p$  for emissivity of 0.8 and  $T_a = 20^\circ C$ .

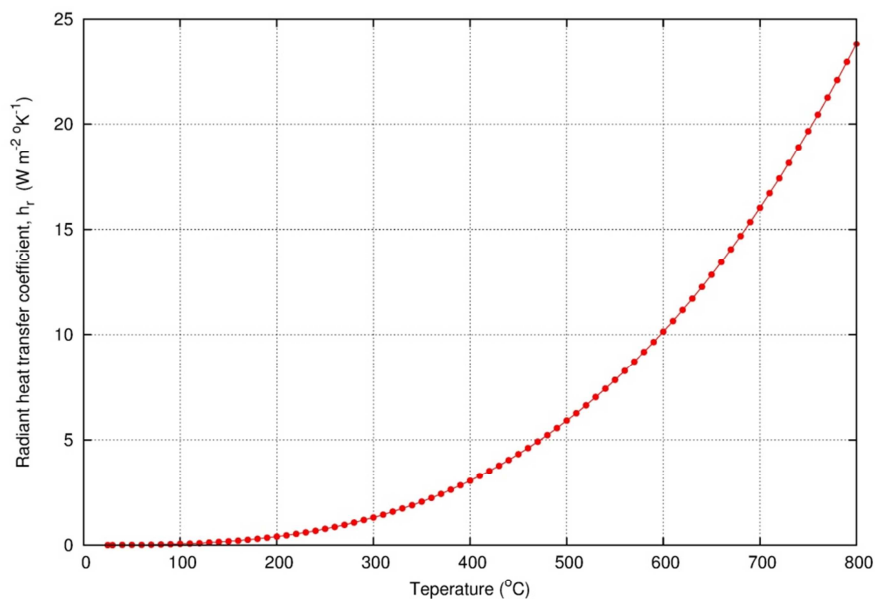


Fig. 1: Radiant heat transfer coefficient for varies temperature plate with ambient temperature 20 °C and emissivity 0.8.

## Iterative method

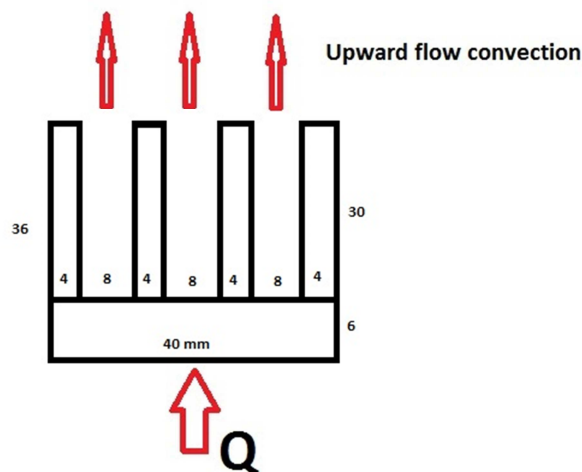
An initial guess value for  $T_p$  is set in Eq. 5 to determine  $Gr$ . Once calculated the dimensionless parameters  $Gr$ ,  $Pr$ ,  $Ra$  and  $Nu$ , the value of  $h$  is obtained through the Eq. 3. By introducing the value of  $h$  in Eq. 1, a new value of  $T_p$  is given by:

$$T_p = T_a + \frac{Q}{h \cdot S} \quad (7)$$

and the obtained  $T_p$  can be used to estimate iteratively the proper value of  $Gr$  for the particular experimental set-up by using it in the Eq. 5. The obtained value of  $h$  is then used again to estimate the  $Gr$  parameters, in an iterative way, that rapidly converges to the correct estimation of  $h$ .

## Example 1.

A LED is assembled on a 40 x 40 x 6 mm aluminum heat sink with four fins 30 mm long and 4 mm large. The LED generate 3 W and the room temperature is 20 °C. Which is the value of the heat sink temperature?



**Fig. 2** scheme showing the assembled heat sink and energy flow.

1. Define Grashof number with  $L = 40$  mm, the greater side of fins and assuming a plate temperature of 100 °C:

$$Gr = \frac{9,81 * 0,04^3 * \frac{1}{273 + 20} * (100 - 20)}{(1,5 * 10^{-5})^2} = 7.62 * 10^5$$

2. Define Prandtl number:

$$Pr = \frac{1,81 * 10^5 * 1005}{0,026} = 0.7$$

3. Define the Rayleigh number:  $Ra = 7.62 * 10^5 * 0.7 = 5.33 * 10^5 < 1 * 10^9$  so is laminar flow.

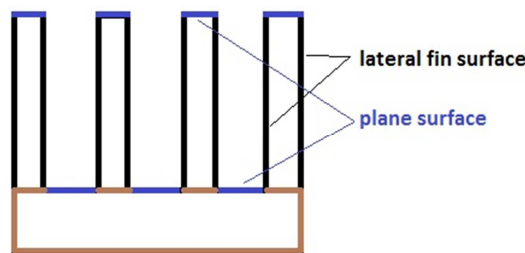
For laminar flow, horizontal fins (flat and isothermal with good approximation) and heat upward flow, from Table 1 the value of Nusselt number  $Nu$  is determined by using the relation:

$$Nu = 0.54 * Ra^{0.25} = 0.54 * (5.33 * 10^5)^{0.25} = 14.59$$

4. Finally, the heat transfer coefficient turns out to be:

$$h_c = \frac{14.59 * 0.026}{0.04} = 9.5 \frac{W}{m^2 * K}$$

5. The cooling area is the sum of lateral fins surface and the plane surface, as shown in Fig. 3.



**Fig. 3.** Explanation of the dispersion surfaces

The total lateral fins surface is:  $40 * 30 * 8 = 9600 \text{ mm}^2 = 0.0096 \text{ m}^2$ , the plan surface is  $40 * 40 = 1600 \text{ mm}^2 = 0.0016 \text{ m}^2$  for a total cooling surface of  $0.0112 \text{ m}^2$ .

By using the value of the cooling surface, it is possible to check the correctness of the  $T_p$  that was initially set to calculate a starting value for  $Gr$  (Eq. 5).

$$T_p = 20 \text{ } ^\circ\text{C} + \frac{3 \text{ W}}{9.5 \frac{W}{m^2 * K} * 0.0112 \text{ m}^2} = 48 \text{ } ^\circ\text{C}$$

As evident, the set and the estimated values of  $T_p$  are inconsistent. The calculated value of  $T_p$  is used in Eq. 5 to estimate a more appropriate value of  $Gr$ . With  $T_p = 48 \text{ } ^\circ\text{C}$ , the new parameters are:

$$Gr = 2.67 * 10^5,$$

$$Ra = 1.87 * 10^5,$$

$$Nu = 11.22,$$

$$h_c = 7.3 \frac{W}{m^2 * K}.$$

$Pr$  is always 0.7. Now with a check the plate temperature result to be  $T_p = 57 \text{ } ^\circ\text{C}$ .

Further iterations gets the following values:

$$1. h_c = 7.8 \frac{W}{m^2 * K} \text{ and } T_p = 54 \text{ } ^\circ\text{C}.$$

$$2. h_c = 7.7 \frac{W}{m^2 * K} \text{ and } T_p = 55 \text{ } ^\circ\text{C}.$$

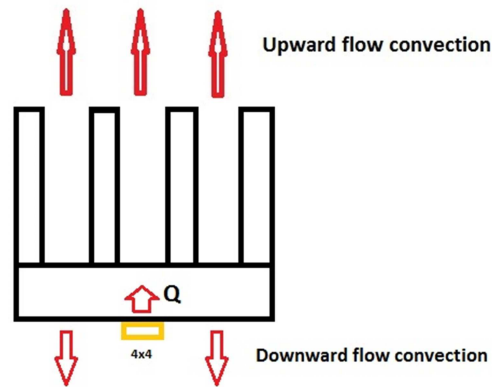
The last iteration give a values of  $T_p$  for both convection heat formula and via Nusselt number. The value of

$$h_c = 7.7 \frac{W}{m^2 * K} \text{ can be considered.}$$

If also the flat side of the heat sink is in free air we must add this other convection which has a different  $h$  value. In the Example 2 such effect is also considered.

## Example 2.

Consider the heat sink in example 1 with heat generated by a 4 x 4 mm LED die set in the middle of the plane as show in figure 4. Define the heat sink temperature,  $T_p$ , considering the two different heat transfer coefficients.



**Fig.4. Example with upward and downward convection**

The flat surface is  $40 * 40 = 1600 \text{ mm}^2$ . For the cooling area we must subtract the die area:  $1600 - 16 = 1584 \text{ mm}^2 = 0.001584 \text{ m}^2$ .

The Nusselt formula for horizontal flat plate in natural air flow and downward heat flow is:

$$Nu = 0.27 * Ra^{0.25}$$

We have already defined the value for  $Gr$ , and  $Ra$  with  $L = 40 \text{ mm}$  and  $T_p = 55 \text{ °C}$  and can calculate  $Nu$  and  $h$ :

$$Gr = 3.33 * 10^5; \quad Ra = 2.33 * 10^5,$$

$$Nu = 5.93; \quad h_c = 3.9 \frac{W}{m^2 * K}.$$

With two different convection coefficient the Eq.1 becomes:

$$Q = (h_1 * S_1 + h_2 * S_2) * (T_p - T_a) \quad (8)$$

Where the subscript 1 is for upward convection and 2 for downward. Solving for  $T_p$  the eq.7 becomes:

$$T_p = T_a + \frac{Q}{h_1 * S_1 + h_2 * S_2} \quad (9)$$

Substituting the values we get:

$$T_p = 20 + \frac{3}{7.7 * 0.011200 + 3.9 * 0.001584} = 52 \text{ °C}$$

For the iterative method we set  $T_p = 52 \text{ °C}$  in eq.5 to re-calculate  $Nu$  and  $h_c$  for upward and downward convection and set the new  $Nu$  and  $h$  value in eq.9.

$$1. h_1 = 7.5 \frac{W}{m^2 * K}, h_2 = 3.8 \frac{W}{m^2 * K}, T_p = 53 \text{ °C}$$

$$2. h_1 = 7.6 \frac{W}{m^2 \cdot K}, h_2 = 3.8 \frac{W}{m^2 \cdot K}, T_p = 53 \text{ } ^\circ\text{C}$$

The last iteration give a values of  $T_p$  for both convection heat formula and via Nusselt number. The value of  $h_1 = 7.6 \frac{W}{m^2 \cdot K}$  and  $h_2 = 3.8 \frac{W}{m^2 \cdot K}$  can be considered.

## Validation of the method

The iterative method was applied to define the heat sink temperature of the LED floodlight SIRIO24 (a new product developed by Progettazione Ottica Roncati and Paolo Colombani Design, see Figure 5). The results of the iterative method were compare with both the thermal simulation (performed with the finite element analysis software LISA 8.0.0) and the laboratory tests.



**Fig.5. LED floodlight SIRIO24**

The SIRIO24 heat sink is a black anodized aluminum 198 x 132 x6 mm plate with 20 fins 132 x 34 x 3 mm. The heat flux transferred from LEDs to the heat sink is 16 Watt. The laboratory temperature is 15 °C and the projector is placed with the LEDs on the downside. Input parameters for the iterative method are:

- Heat power,  $Q = 16 \text{ W}$
- Ambient temperature,  $t_a = 15 \text{ } ^\circ\text{C}$
- Greater fin dimension,  $L = 132 \text{ mm}$
- air kinematic viscosity,  $\eta = 1.45 \cdot 10^{-5}$
- air dynamic viscosity,  $\mu = 1.78 \cdot 10^{-5}$

As show in Figure 3 the cooling surface,  $S$  is equal to lateral fin surface ( $S_f$ ) plus plane surface ( $S_p$ ).

$$S_f = 132 \cdot 34 \cdot 20 \cdot 2 = 179520 \text{ mm}^2 = 0.17952 \text{ m}^2$$

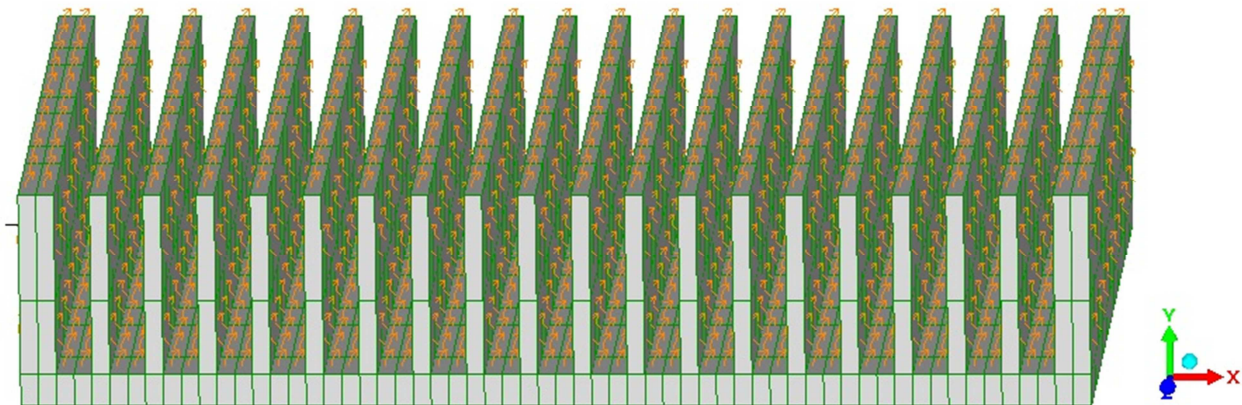
$$S_p = 198 \cdot 132 = 26136 \text{ mm}^2 = 0.026136 \text{ m}^2$$

$$S = S_f + S_p = 0.205656 \text{ m}^2$$

Applying the method for horizontal fins, laminar air flow and upward heat flow we find a heat transfer convection coefficient,  $h_c = 4.8 \frac{W}{m^2 \cdot K}$  and a plate temperature,  $T_p = 31.20 \text{ } ^\circ\text{C}$ .

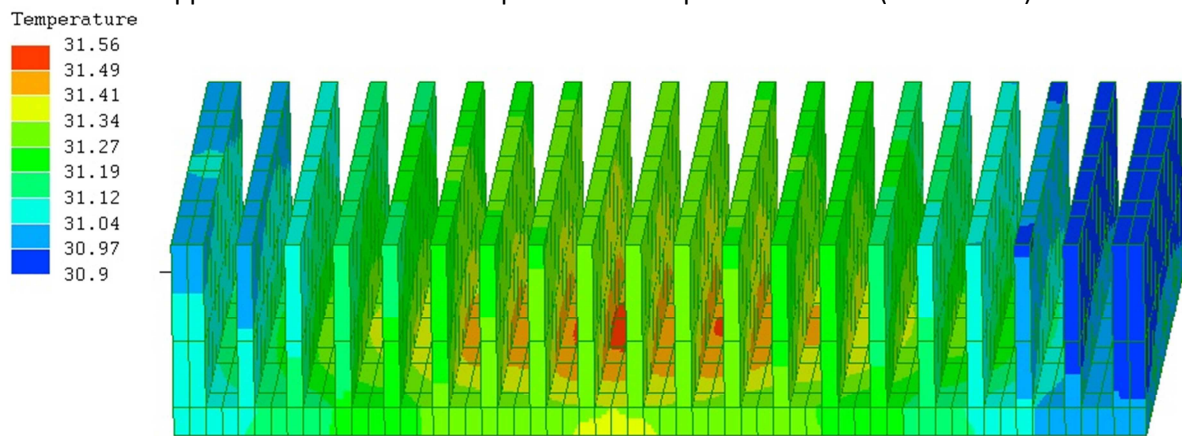
At 31.20 °C radiation heat transfer is negligible.





**Fig.6. LED floodlight SIRIO24 3D model for thermal analysis created with LISA 8.0.0**

At first, a 3D model of the heat sink was created with LISA 8.0.0. A 16 W flow rate was then applied on the plane surfaces as well as a convection for cooling the surfaces with a coefficient  $h_c = 4.8 \frac{W}{m^2 \cdot K}$ . The result of the simulation are shown in Fig. 7 in which the estimated temperature varies from 30.90 °C to 31.56 °C while the average temperature is 31.23 °C. The analytic/iterative method based on the Nusselt's formula for isothermal fin approximation was also compared with the previous results (See Table 3).



**Fig.7. LED floodlight SIRIO24 thermal analysis done with LISA 8.0.0**

The previous results were compared with a laboratory test of heat sink temperature (15 °C ambient temperature, horizontal fins with upward heat flow) using an infrared thermal gun with precision 5%, resolution 0.1 °C. The experimental results agree with the simulations within the accuracy of the testing device (see Table 3).

**Table 3.** SIRIO24 heat sink temperature results.

Iterative method	FEA with LISA 8.0.0	Laboratory
31.2 °C	31.2 °C	32.1 °C





## Conclusions.

Iterative method and thermal analysis done with finite element software LISA given values that match with difference of 0.1% with a heat sink 3D model precision of 5%. The difference of 0.9 °C (2.8%) with the laboratory run can be explained in terms of uncertainty in the determination of Q. Indeed, the heat power LED generated is calculated as difference between power absorption and radiant power. The latter is estimated from complex computation.

The iterative method can be used to evaluate accurately the temperature value for heat sink or to define with better precision the heat transfer convection coefficient for finite elements software for thermal analysis like LISA. It is worth noting that the Nusselt formula tabulated are for rectangular plate and natural air flow (free convection) and is not valid for other cases like for heat sink with different shape or for forced convection.

## About the Author



Dario Roncati is an Optical Designer specialized in LED systems. He got his Master degree in Physics and Advanced Physical Technologies at the Department of Physics and Earth Science of the University of Ferrara. Worked for different company as Optical Designer in railway and airfield light sectors. Now as owner of Progettazione Ottica Roncati realize optical analysis and design for LED and solar concentrator applications.