# Single-source shortest path

## Directed Graph

A directed graph G, also known as a digraph, is a graph in which every edge has a direction assigned to it. An edge of a directed graph is given as an ordered pair (u, v) of nodes in G. For an edge (u, v),

- The edge begins at u and terminates at v.
- u is known as the origin or initial point of edge e. Correspondingly, v is known as the destination or terminal point of edge e.
- u is the predecessor of v. Correspondingly, v is the successor of u.
- Nodes u and v are adjacent to each other.

## Terminologies

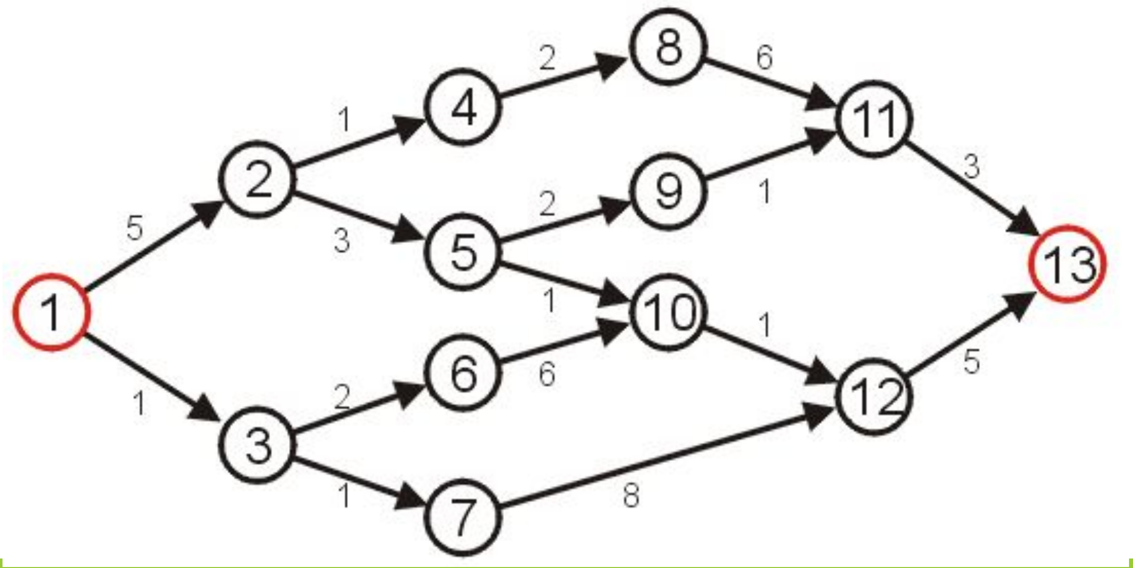
- **Out-degree of a node:** The out-degree of a node u, written as outdeg(u), is the number of edges that originate at u.
- ~~**In-degree of a node:** The in-degree of a node u, written as indeg(u), is the number of~~ edges that terminate at u.
- **Degree of a node:** The degree of a node, written as deg(u), is equal to the sum of in-degree and out-degree of that node. Therefore, deg(u) = indeg(u) + outdeg(u).
- **Isolated vertex:** A vertex with degree zero.
- **Pendant vertex:** (also known as leaf vertex) A vertex with degree one.
- **Cut vertex:** A vertex which when deleted would disconnect the remaining graph.
- **Source:** A node u is known as a source if it has a positive out-degree but a zero in-degree.
- **Sink:** A node u is known as a sink if it has a positive in-degree but a zero out-degree.
- **Reachability:** A node v is said to be reachable from node u, if and only if there exists a (directed) path from node u to node v.
- **Strongly connected directed graph:** A digraph is said to be strongly connected if and only if there exists a path between every pair of nodes in G. That is, if there is a path from node u to v, then there must be a path from node v to u.

## Shortest Path

Given a weighted directed graph, one common problem is finding the shortest path between two given vertices

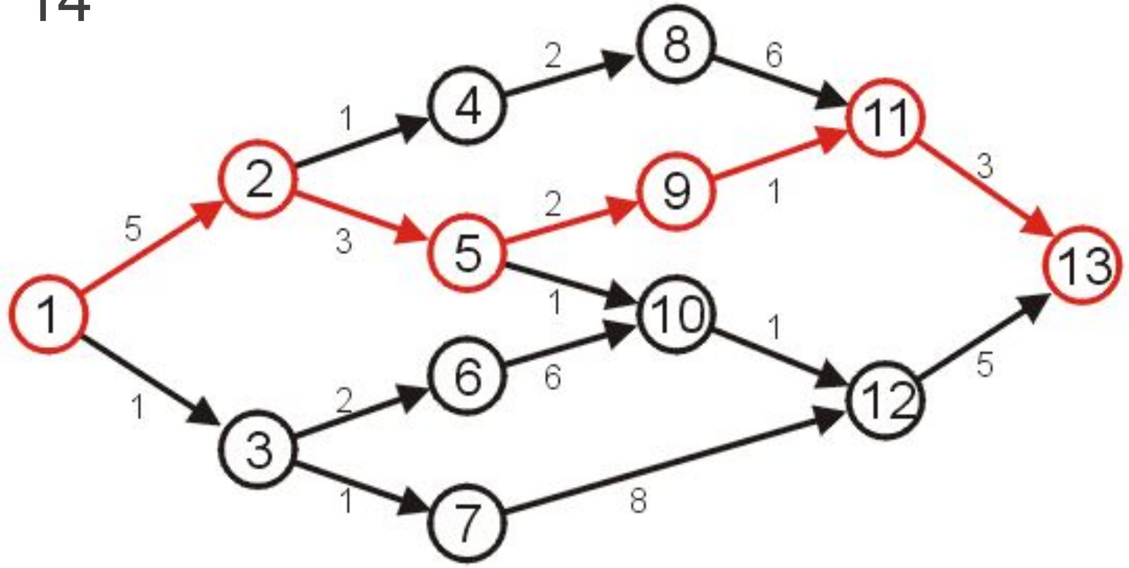
- Recall that in a weighted graph, the *length* of a path is the sum of the weights of each of the edges in that path

Given the graph, suppose we wish to find the shortest path from vertex 1 to vertex 13



## Shortest Path

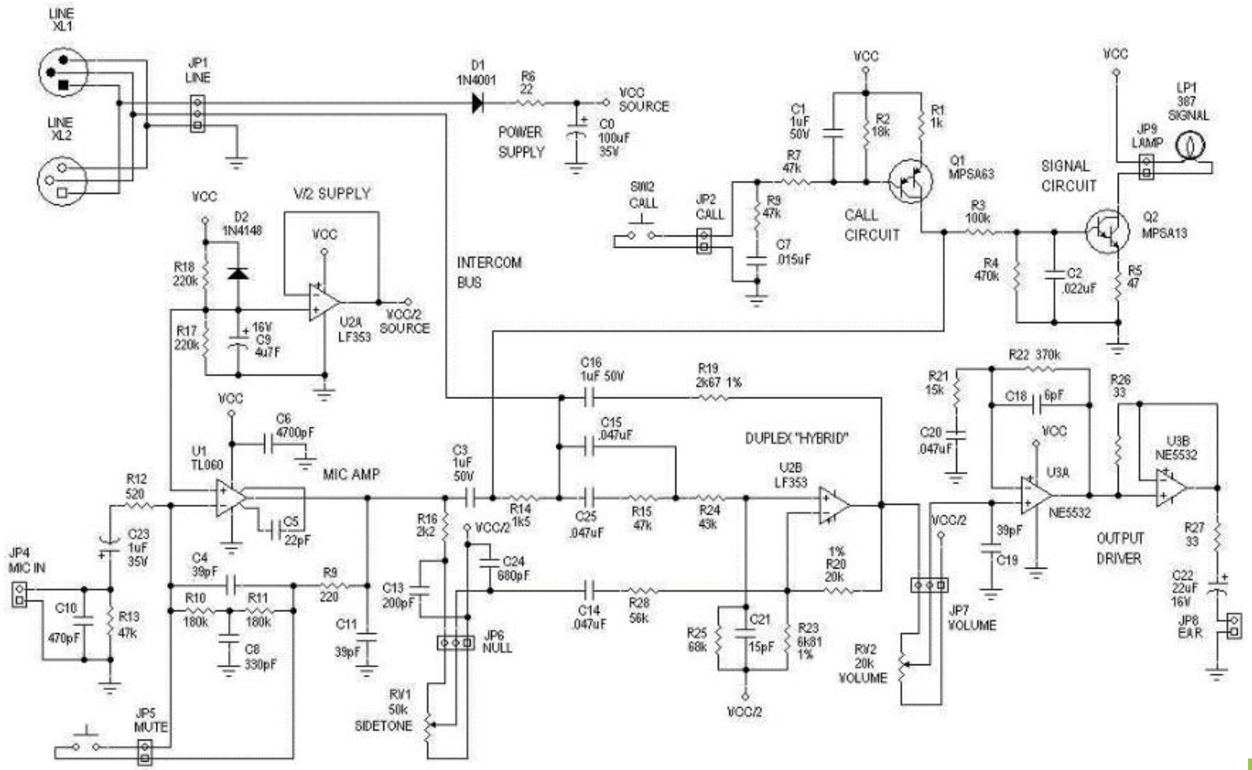
After some consideration, we may determine that the shortest path is as follows, with length 14



Other paths exists, but they are longer

## Applications

one application is circuit design: the time it takes for a change in input to affect an output depends on the shortest path



## Applications

The Internet is a collection of interconnected computer networks

- Information is passed through *packets*

Packets are passed from the source, through routers, to their destination

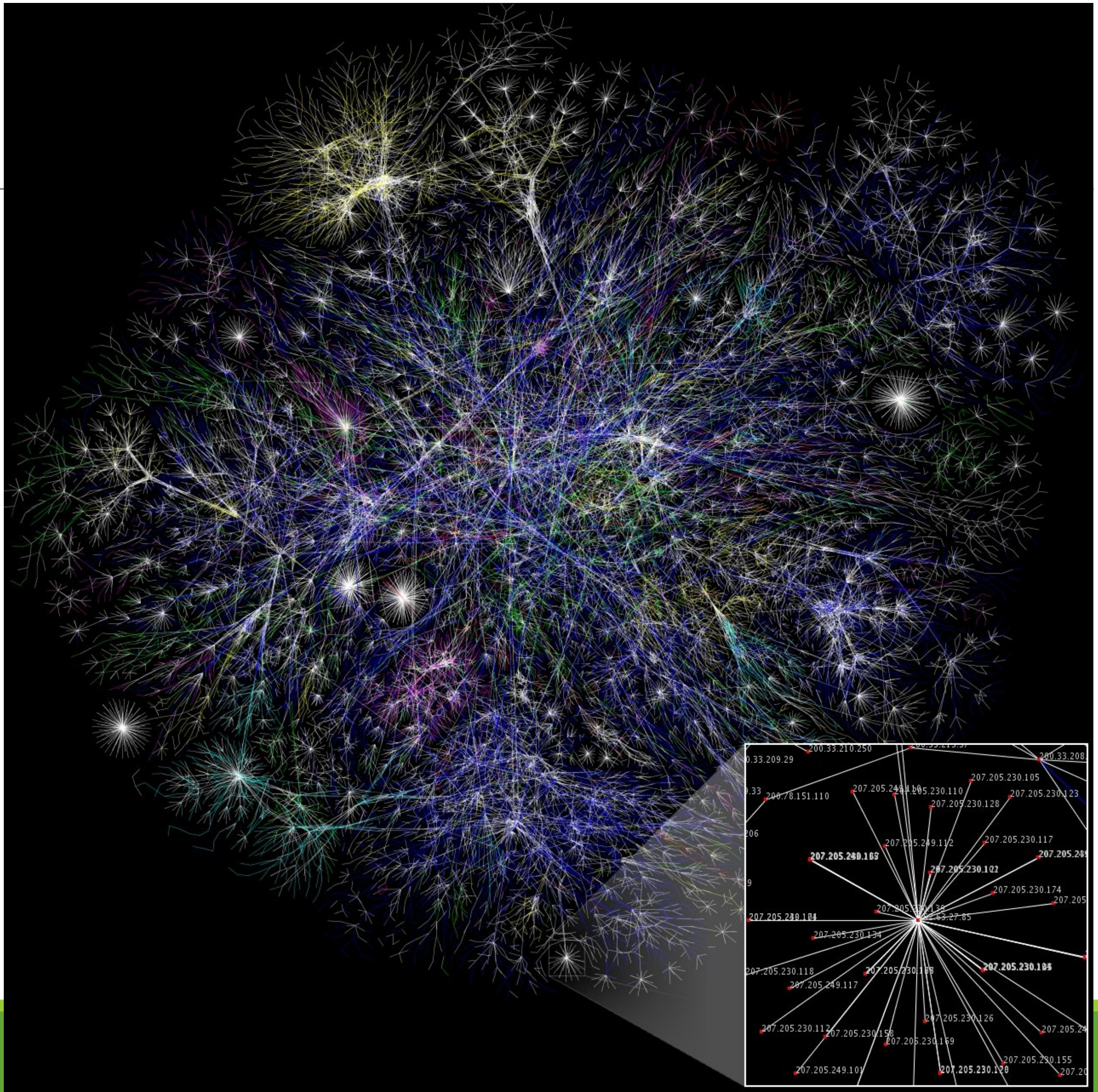
Routers are connected to either:

- individual computers, or
- other routers

These may be represented as graphs

## Applications

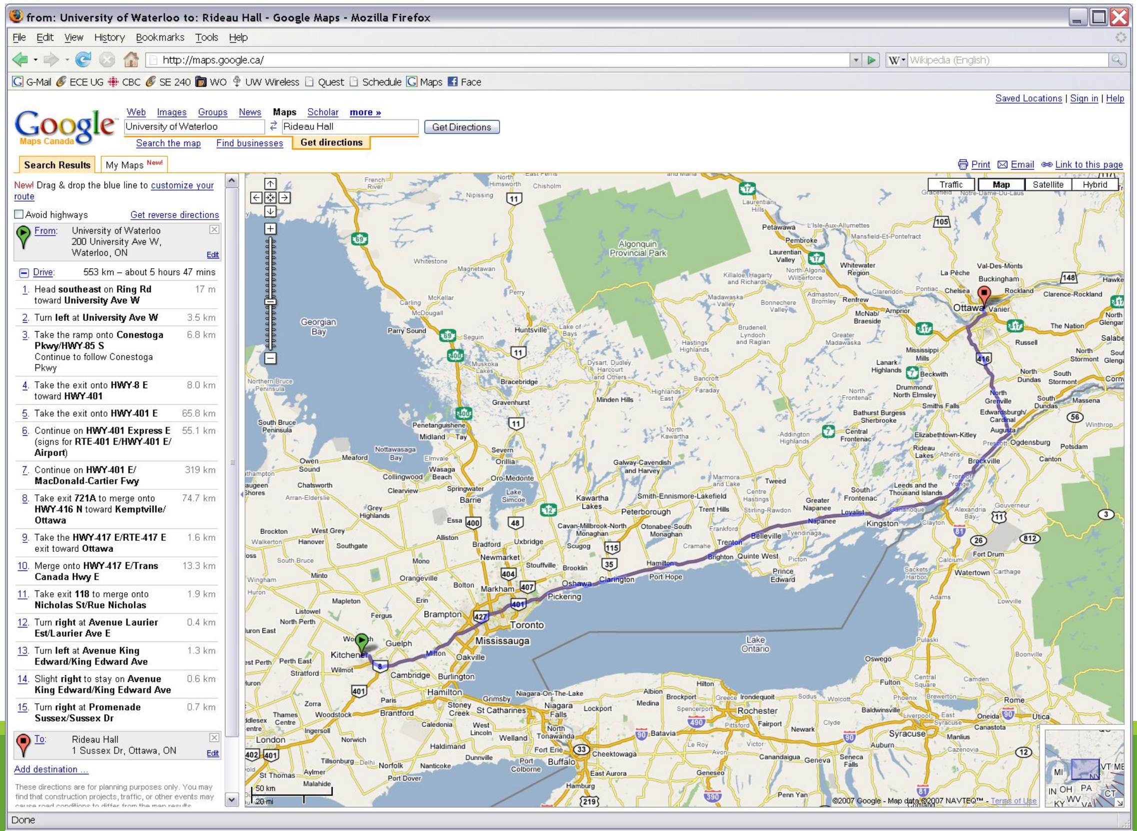
A visualization of the graph of the routers and their various connections through a portion of the Internet



# Applications

In software engineering, one obvious problem is finding the shortest route between to points on a map

- Shortest path, however, need not refer to distance...


from: University of Waterloo to: Rideau Hall - Google Maps - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://maps.google.ca/
G-Mail ECE UG CBC SE 240 WO UW Wireless Quest Schedule Maps Face
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University of Waterloo
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Get Directions
Search the map Find businesses Get directions
Search Results
My Maps New!
Print Email Link to this page
Traffic Map Satellite Hybrid
Ottawa
Kingston
Toronto
Mississauga
Kitchener
Lake Ontario
Georgian Bay
Algonquin Provincial Park
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Done

New! Drag & drop the blue line to customize your route

Avoid highways — Get reverse directions

From: University of Waterloo
200 University Ave W,
Waterloo, ON

Drive: 553 km – about 5 hours 47 mins

| | | |
|---|---|---|
| 1. | Head **southeast** on **Ring Rd** toward **University Ave W** | 17 m |
| 2. | Turn **left** at **University Ave W** | 3.5 km |
| 3. | Take the ramp onto **Conestoga Pkwy/HWY-85 S** Continue to follow Conestoga Pkwy | 6.8 km |
| 4. | Take the exit onto **HWY-8 E** toward **HWY-401** | 8.0 km |
| 5. | Take the exit onto **HWY-401 E** | 65.8 km |
| 6. | Continue on **HWY-401 Express E** (signs for **RTE-401 E/HWY-401 E/ Airport**) | 55.1 km |
| 7. | Continue on **HWY-401 E/ MacDonald-Cartier Fwy** | 319 km |
| 8. | Take exit **721A** to merge onto **HWY-416 N** toward **Kemptville/ Ottawa** | 74.7 km |
| 9. | Take the **HWY-417 E/RTE-417 E** exit toward **Ottawa** | 1.6 km |
| 10. | Merge onto **HWY-417 E/Trans Canada Hwy E** | 13.3 km |
| 11. | Take exit **118** to merge onto **Nicholas St/Rue Nicholas** | 1.9 km |
| 12. | Turn **right** at **Avenue Laurier Est/Laurier Ave E** | 0.4 km |
| 13. | Turn **left** at **Avenue King Edward/King Edward Ave** | 1.3 km |
| 14. | Slight **right** to stay on **Avenue King Edward/King Edward Ave** | 0.6 km |
| 15. | Turn **right** at **Promenade Sussex/Sussex Dr** | 0.7 km |

To: Rideau Hall
1 Sussex Dr, Ottawa, ON

Add destination ...

These directions are for planning purposes only. You may find that construction projects, traffic, or other events may cause road conditions to differ from the map results

## Applications

The *shortest path* using distance as a metric is obvious, however, a driver may be more interested in minimizing time

For example, using the 407 may be preferable to using the 401 during rush hour, however, there is an added cost

A company will be interested in minimizing the cost which includes the following factors:

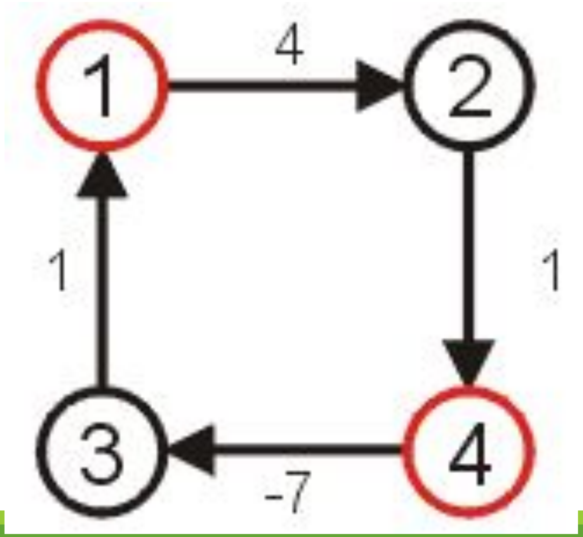
- salary of the truck driver (overtime?)
- possible tolls and administrative costs
- bonuses for being early
- penalties for being late
- cost of fuel

## Shortest Path

The goal of this algorithm will be to find the shortest path and its length

We will make the assumption that the weights on all edges is a positive number

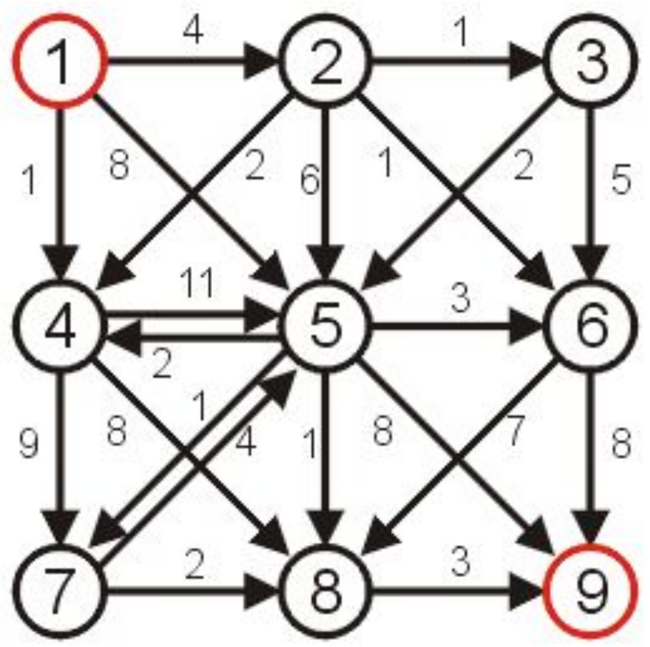
- Clearly, if we have negative vertices, it may be possible to end up in a cycle whereby each pass through the cycle decreases the total *length*
- Thus, a shortest length would be undefined for such a graph
- Consider the shortest path from vertex 1 to 4...



## Shortest Path

Consider the following graph

- All edges have positive weight
- There exists cycles—it is not a DAG



## Algorithms

Algorithms for finding the shortest path include:

- Dijkstra’s algorithm
- A* search algorithm
- Bellman-Ford algorithm
- BFS

# Breadth-First Search

Breadth-first search (BFS) is a general technique for traversing a graph

A BFS traversal of a graph G

- Visits all the vertices and edges of G
- Determines whether G is connected
- Computes the connected components of G

BFS on a graph with $n$ vertices and $m$ edges takes $O(n + m)$ time

BFS can be further extended to solve other graph problems

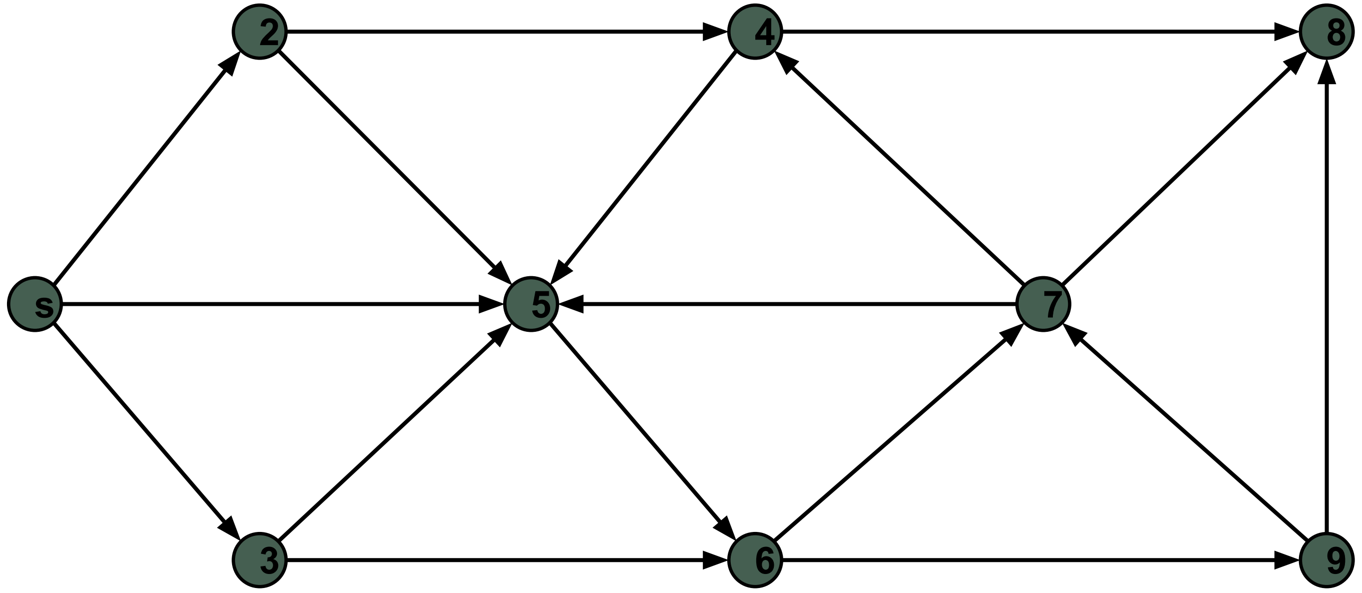
- Find and report a path with the minimum number of edges between two given vertices
- Find a simple cycle, if there is one

## BFS Algorithm

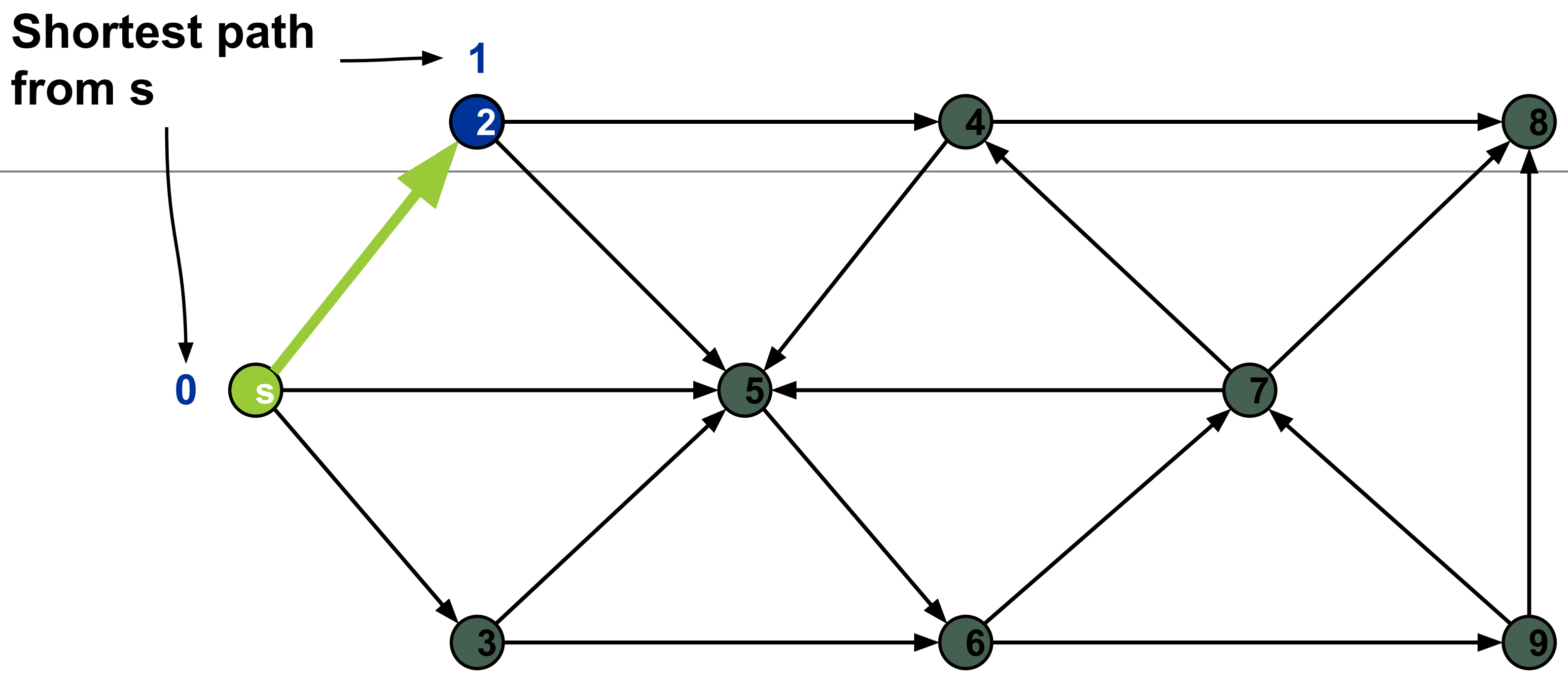
The algorithm uses a mechanism for setting and getting “labels” of vertices and edges

```
Algorithm $BFS(G, s)$
    $L_0 \leftarrow$ new empty sequence
    $L_0.insertLast(s)$
    $setLabel(s, VISITED)$
    $i \leftarrow 0$
    while $\neg L_i.isEmpty()$
        $L_{i+1} \leftarrow$ new empty sequence
        for all $v \in L_i.elements()$
            for all $e \in G.incidentEdges(v)$
                if $getLabel(e) = UNEXPLORED$
                    $w \leftarrow opposite(v,e)$
                    if $getLabel(w) = UNEXPLORED$
                        $setLabel(e, DISCOVERY)$
                        $setLabel(w, VISITED)$
                        $L_{i+1}.insertLast(w)$
                    else
                        $setLabel(e, CROSS)$
        $i \leftarrow i+1$
```

# Breadth First Search



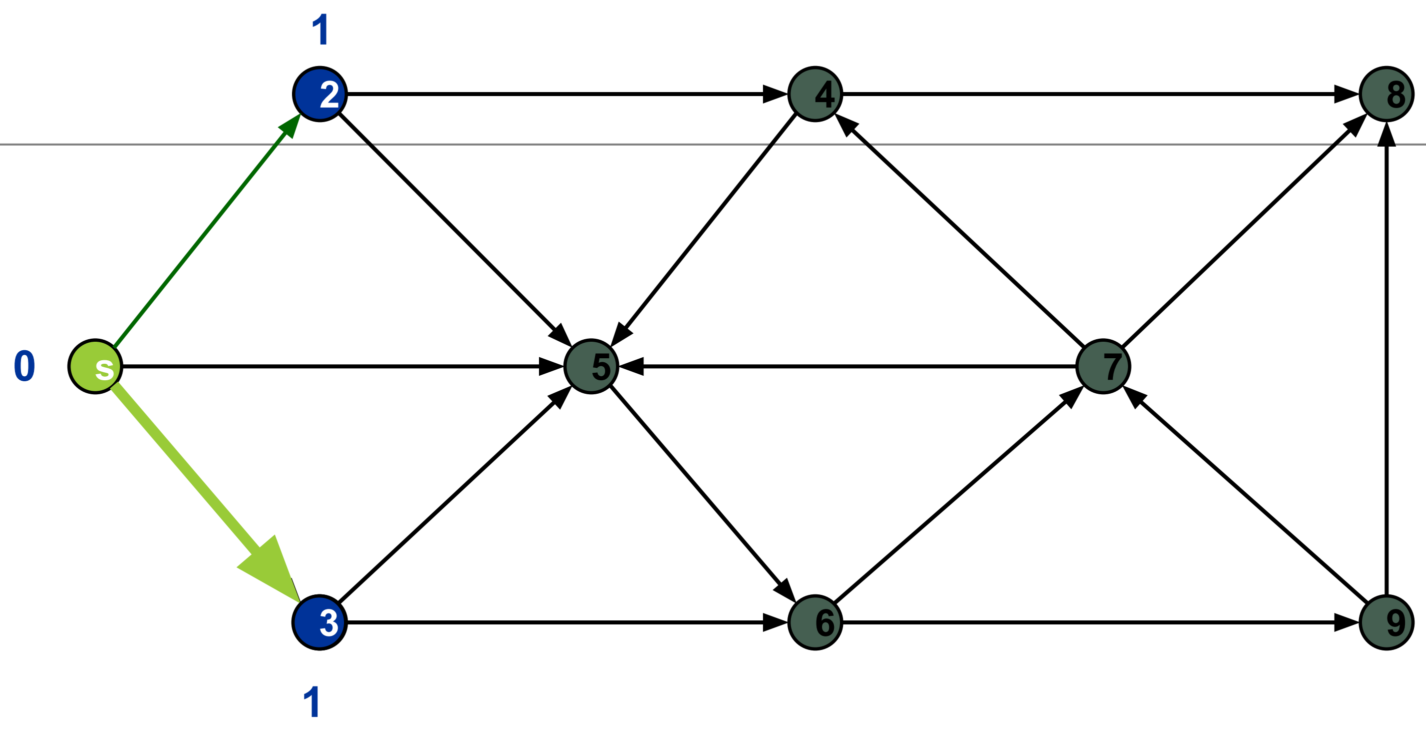
# Breadth First Search



| |
|---|
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: s

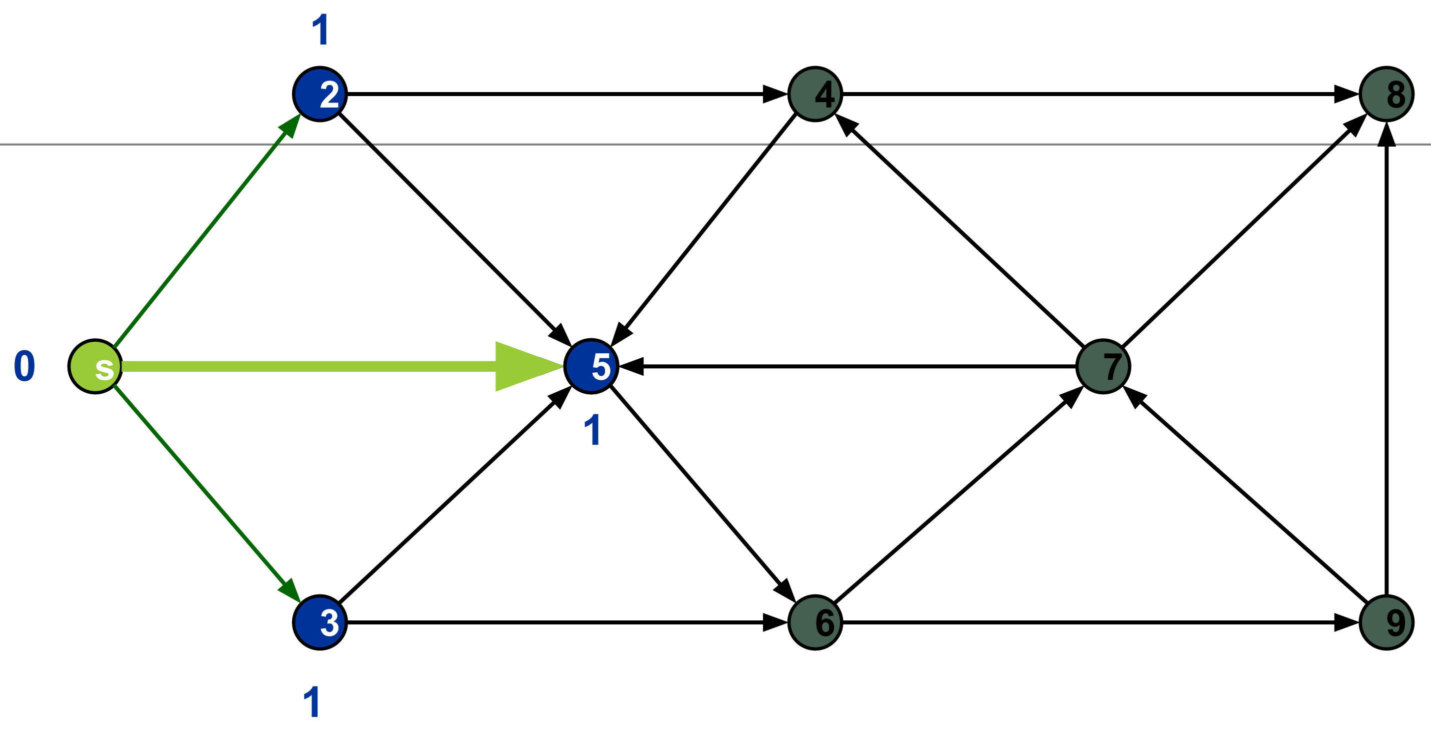
# Breadth First Search



| |
|---|
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: s 2

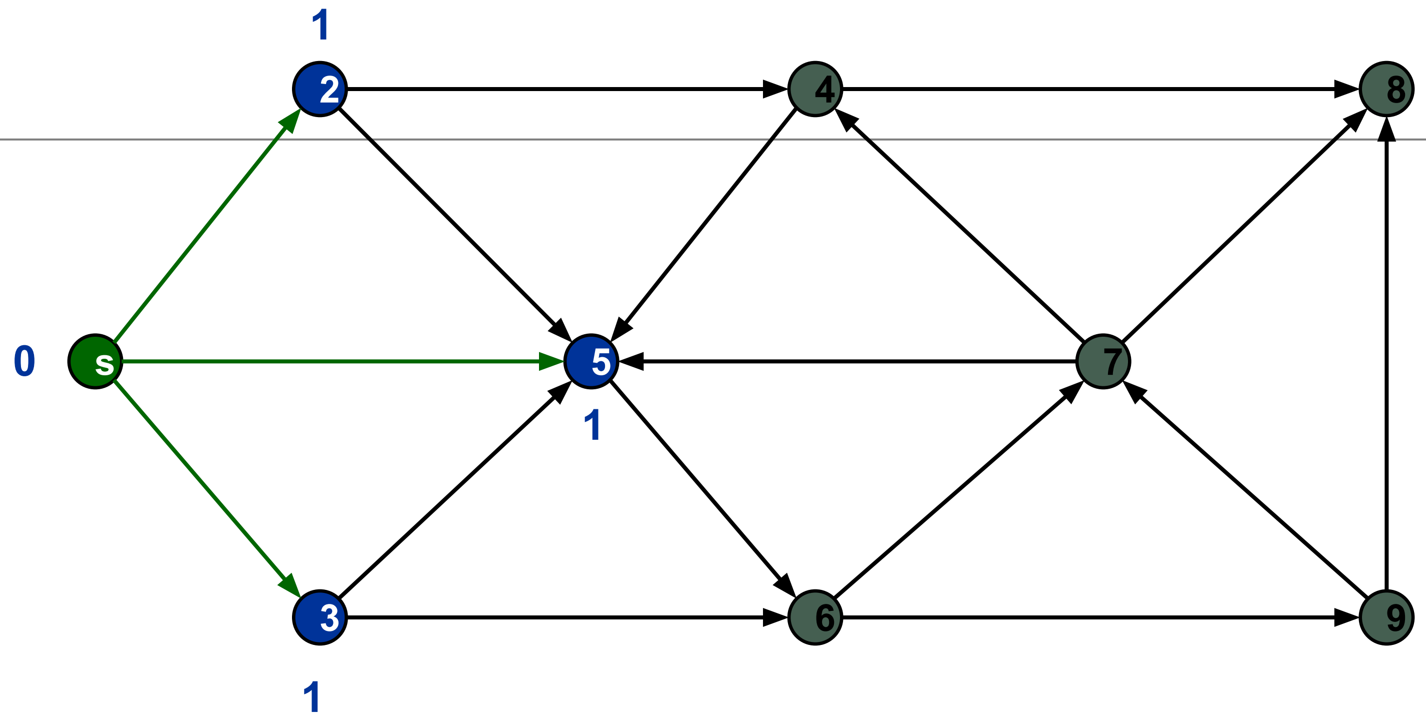
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: s 2 3

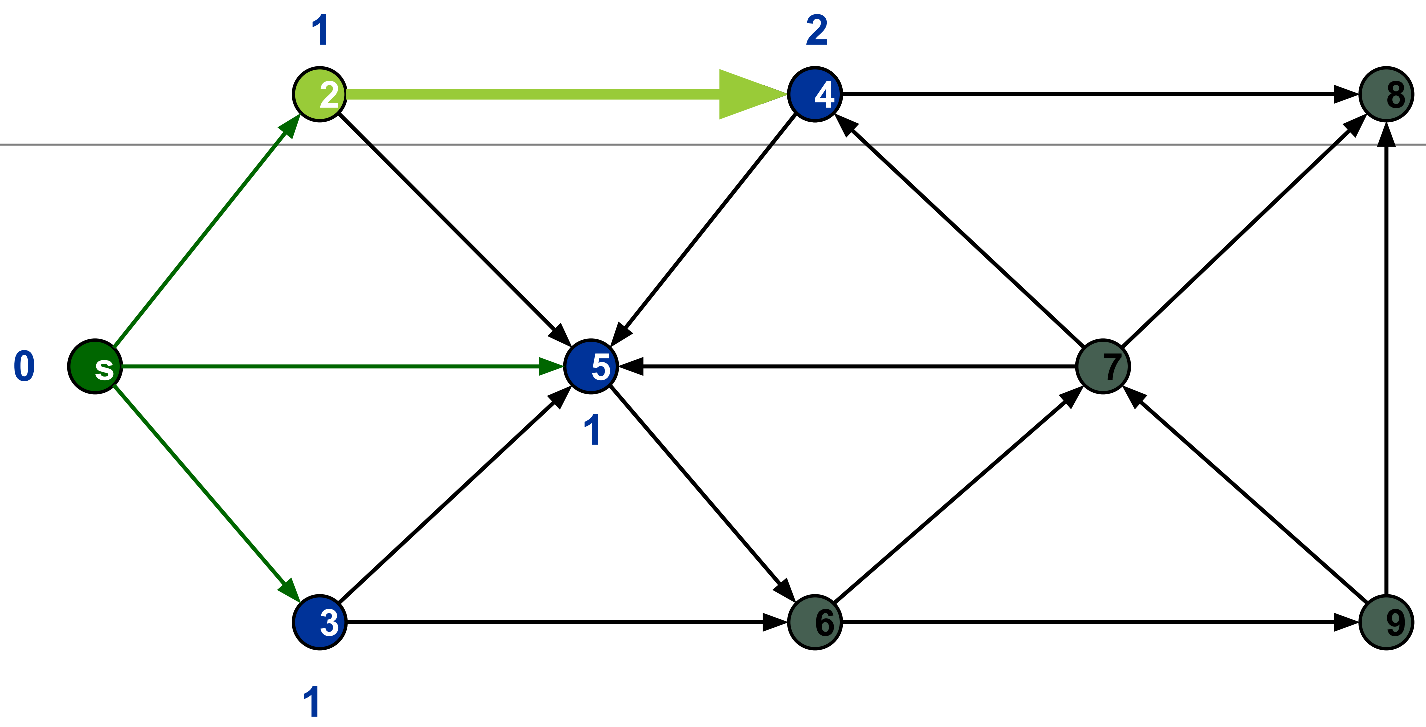
# Breadth First Search



| |
|---|
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 2 3 5

# Breadth First Search



| Undiscovered |
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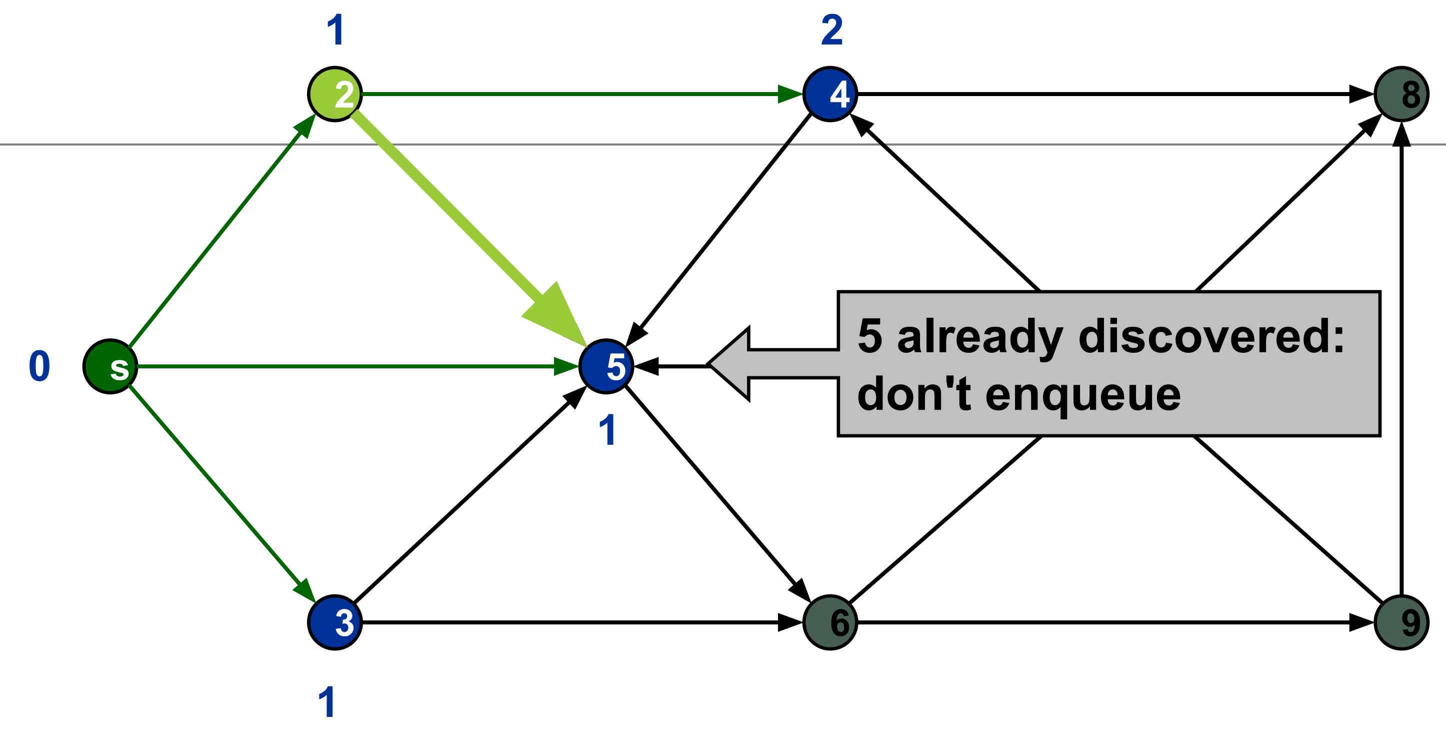
| Discovered |
| Top of queue |
| Finished |

Queue: 2 3 5

# Breadth First Search



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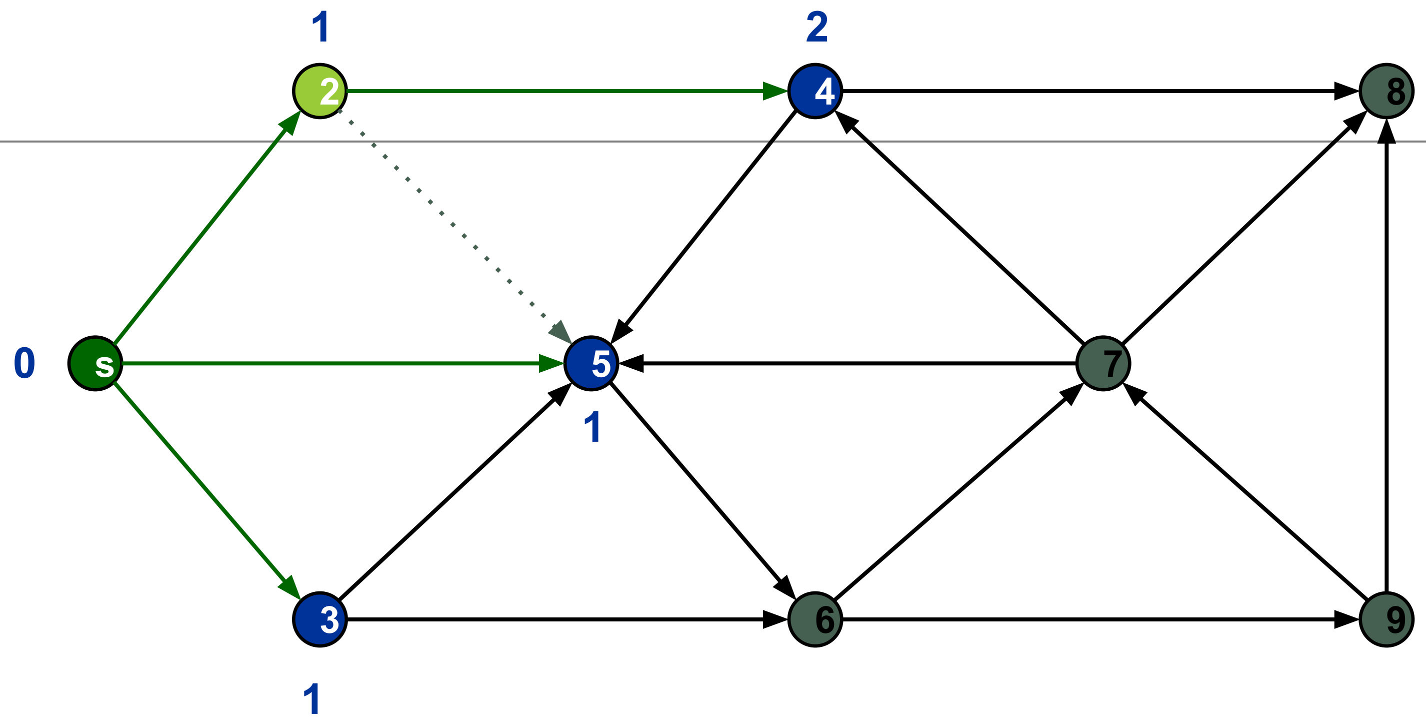
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 2 3 5 4

# Breadth First Search



| Undiscovered |
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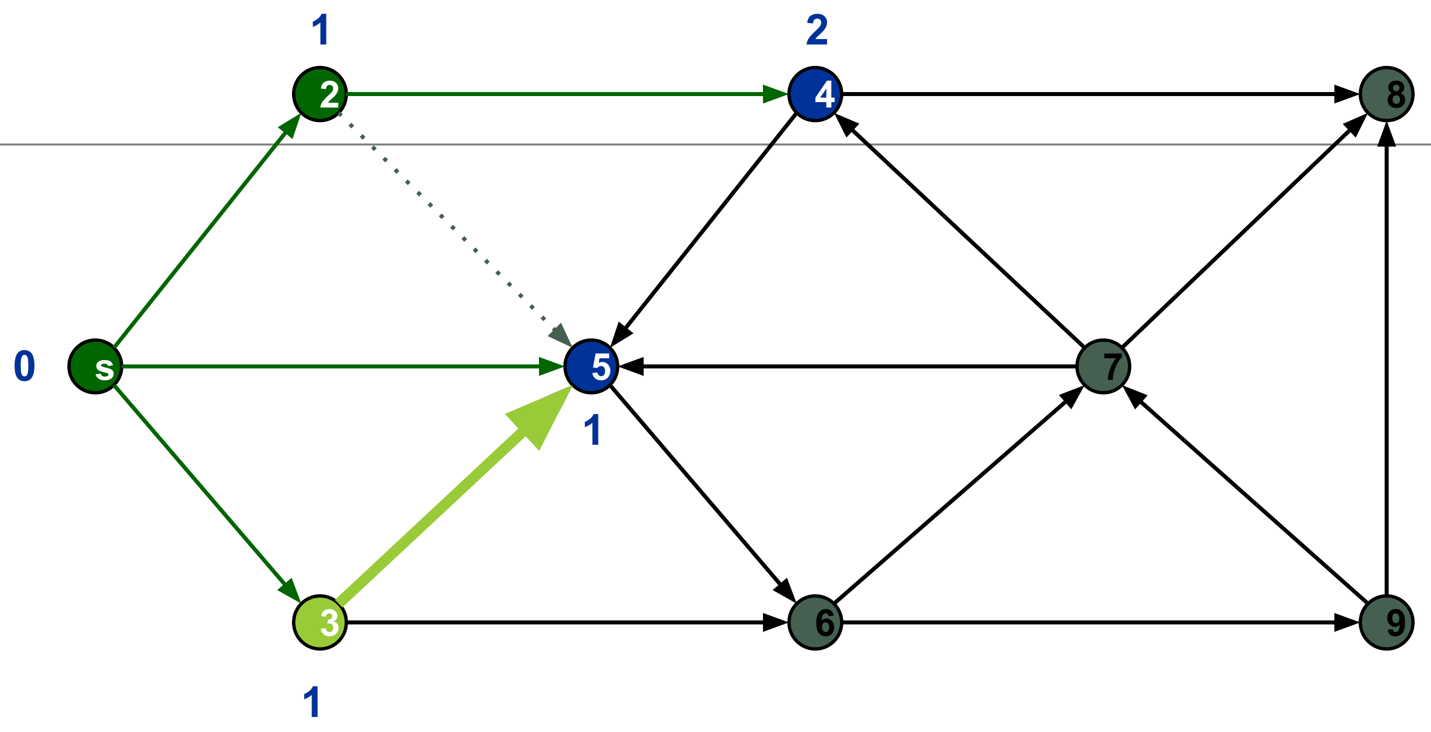
| Discovered |
| Top of queue |
| Finished |

Queue: 2 3 5 4

# Breadth First Search



| Undiscovered |
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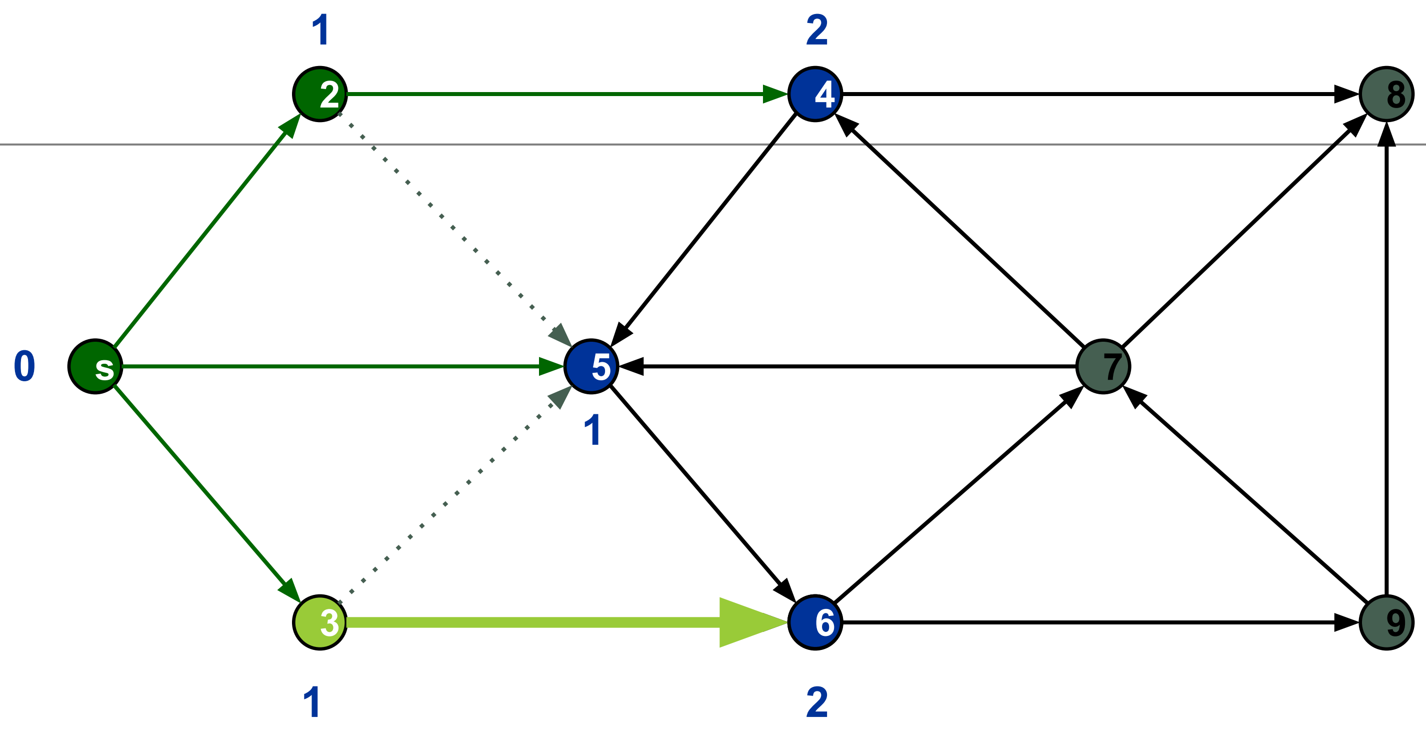
| Discovered |
| Top of queue |
| Finished |

Queue: 3 5 4

# Breadth First Search



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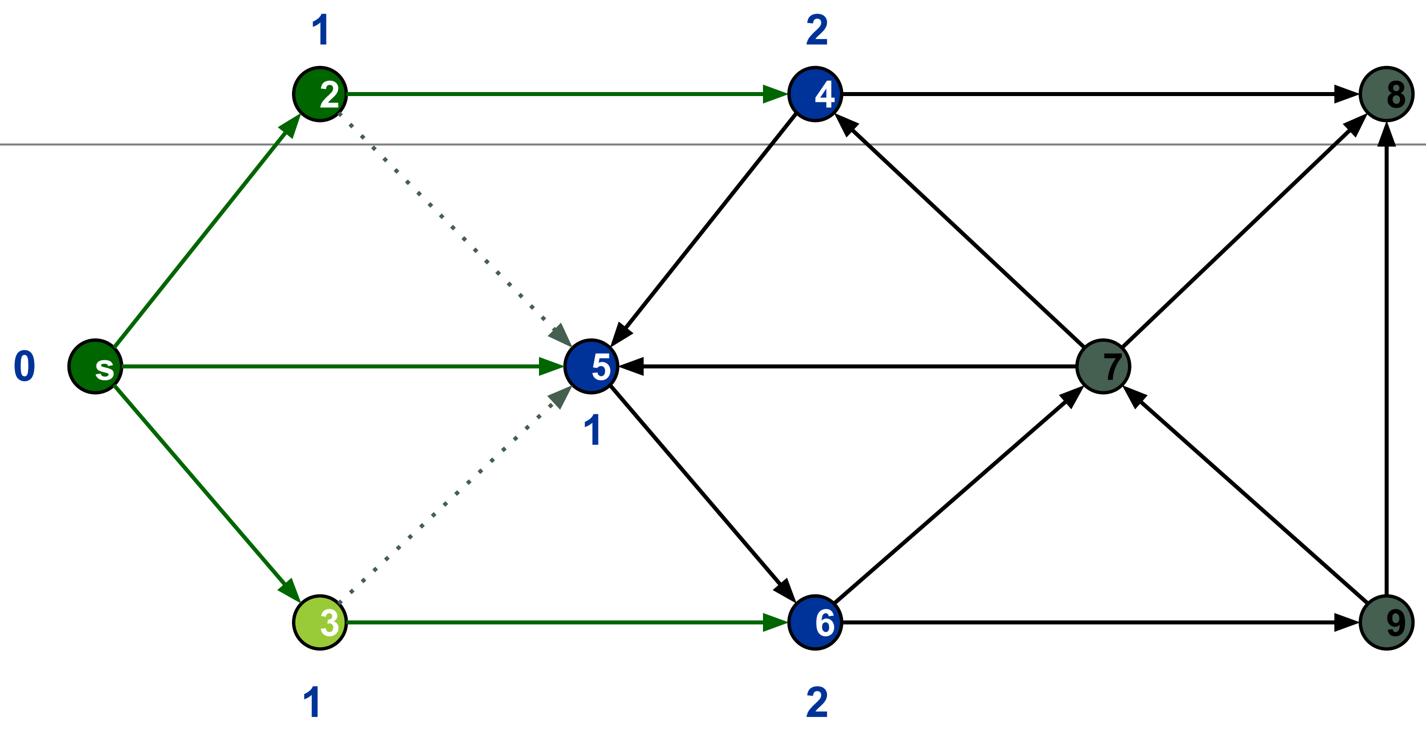
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 3 5 4

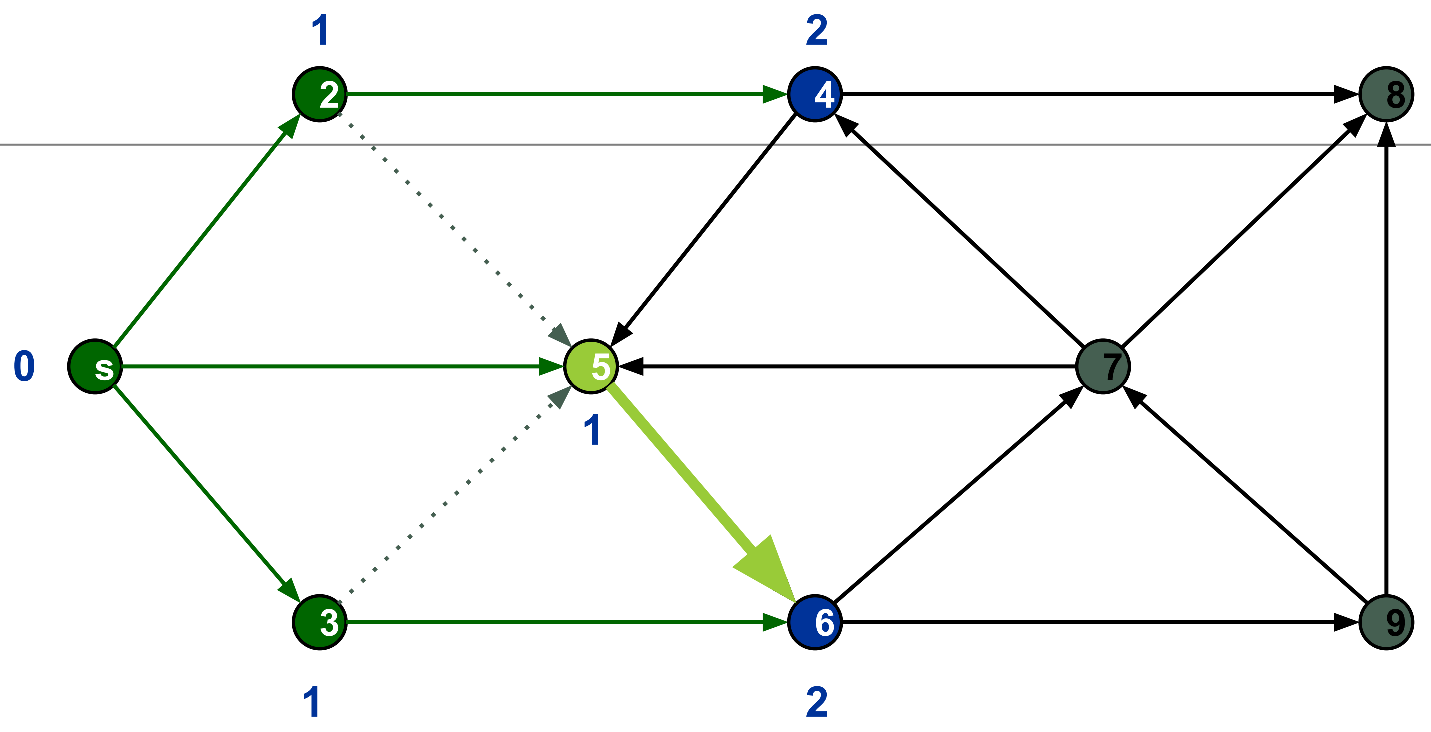
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: 3 5 4 6

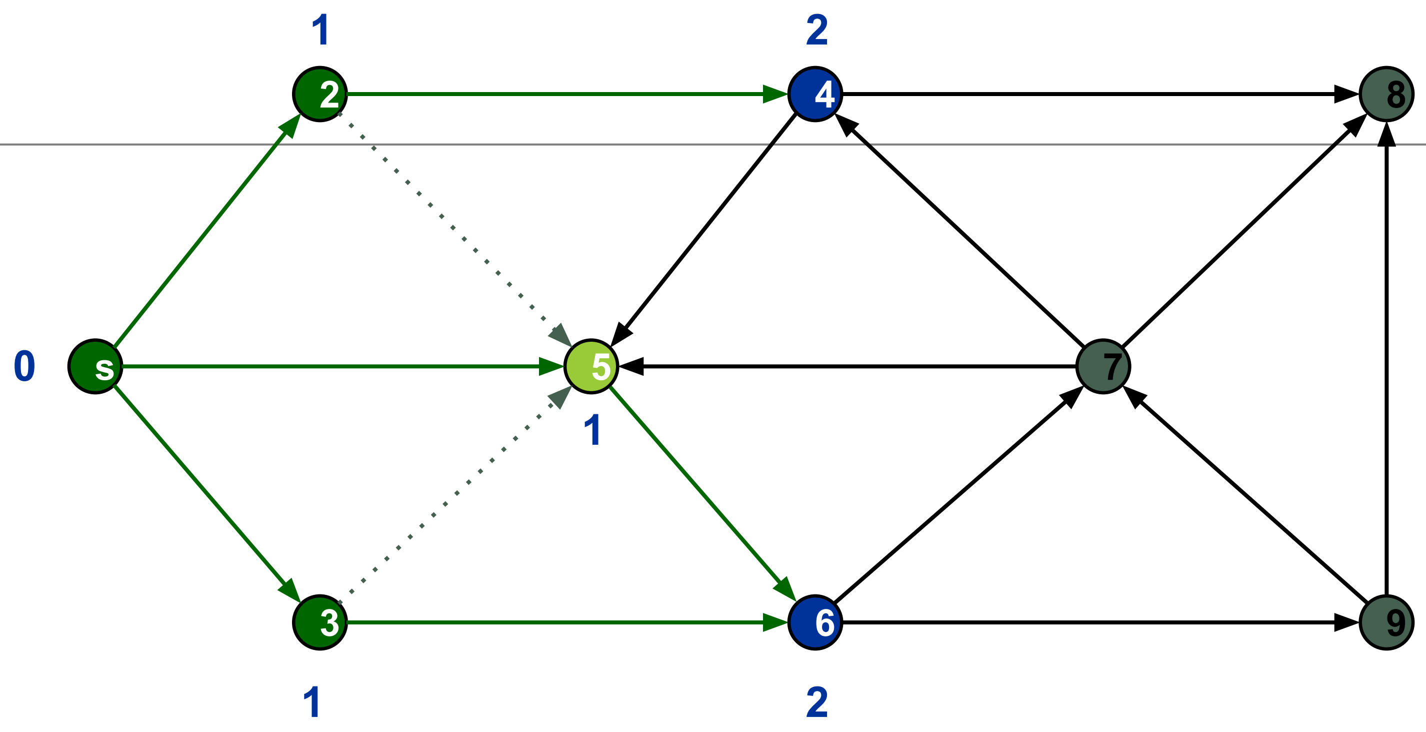
# Breadth First Search



Undiscovered
Discovered
Top of queue
Finished

Queue: 5 4 6

# Breadth First Search



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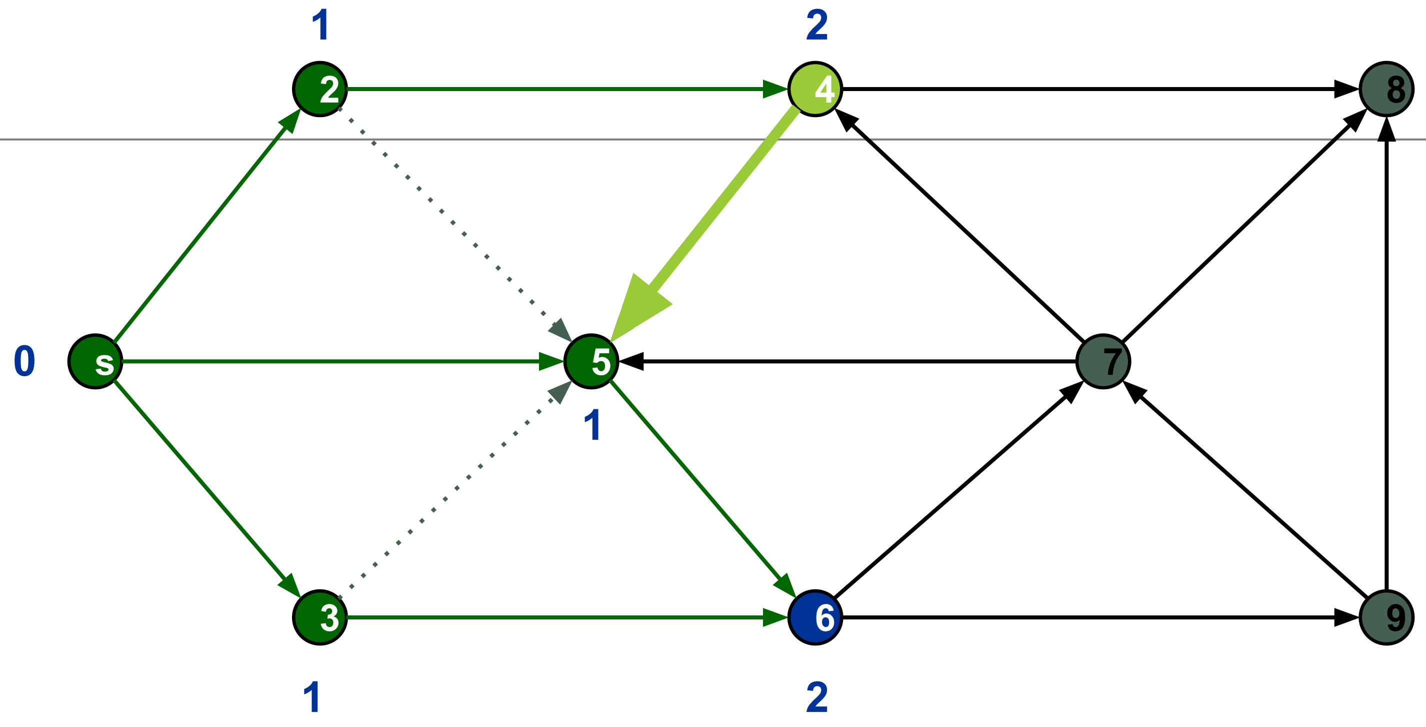
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 5 4 6

# Breadth First Search



Undiscovered

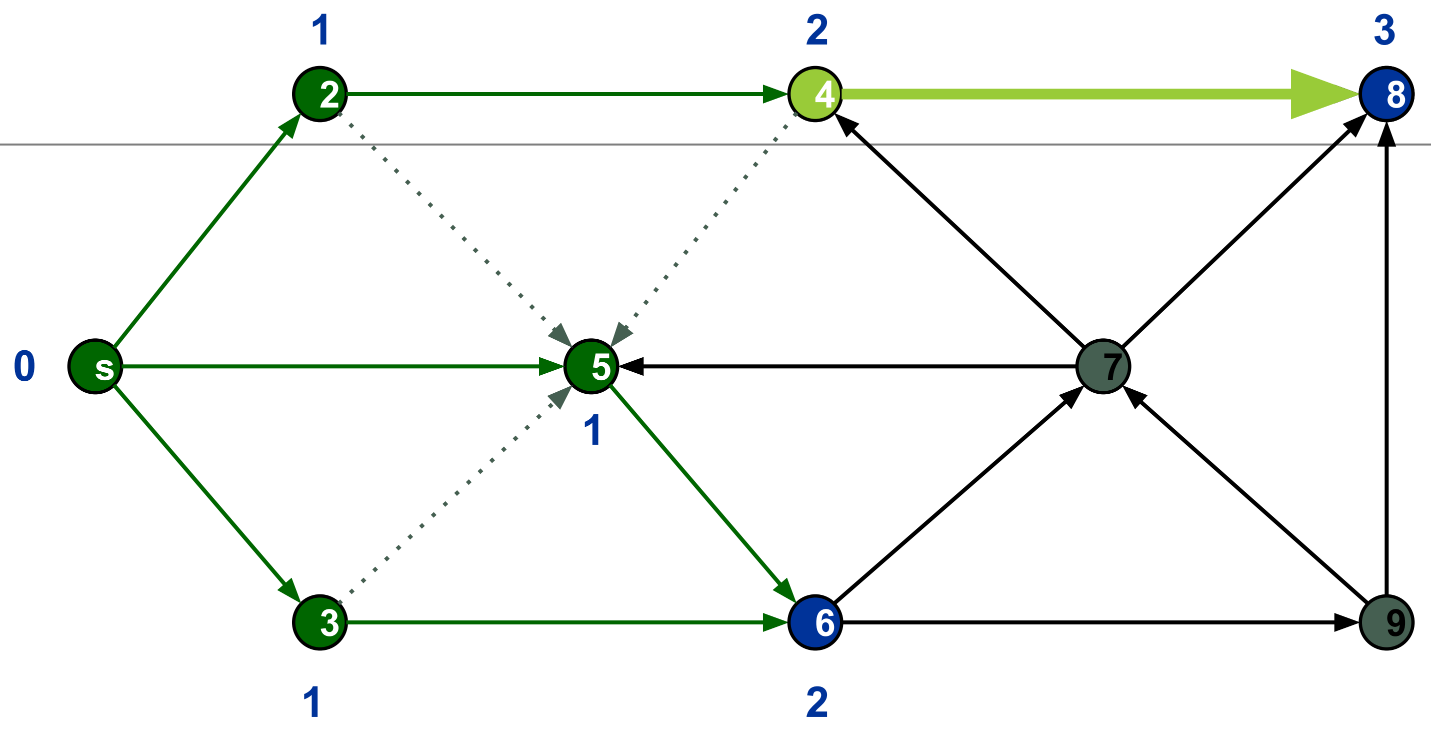
Discovered

Top of queue

Finished

Queue: 4 6

# Breadth First Search



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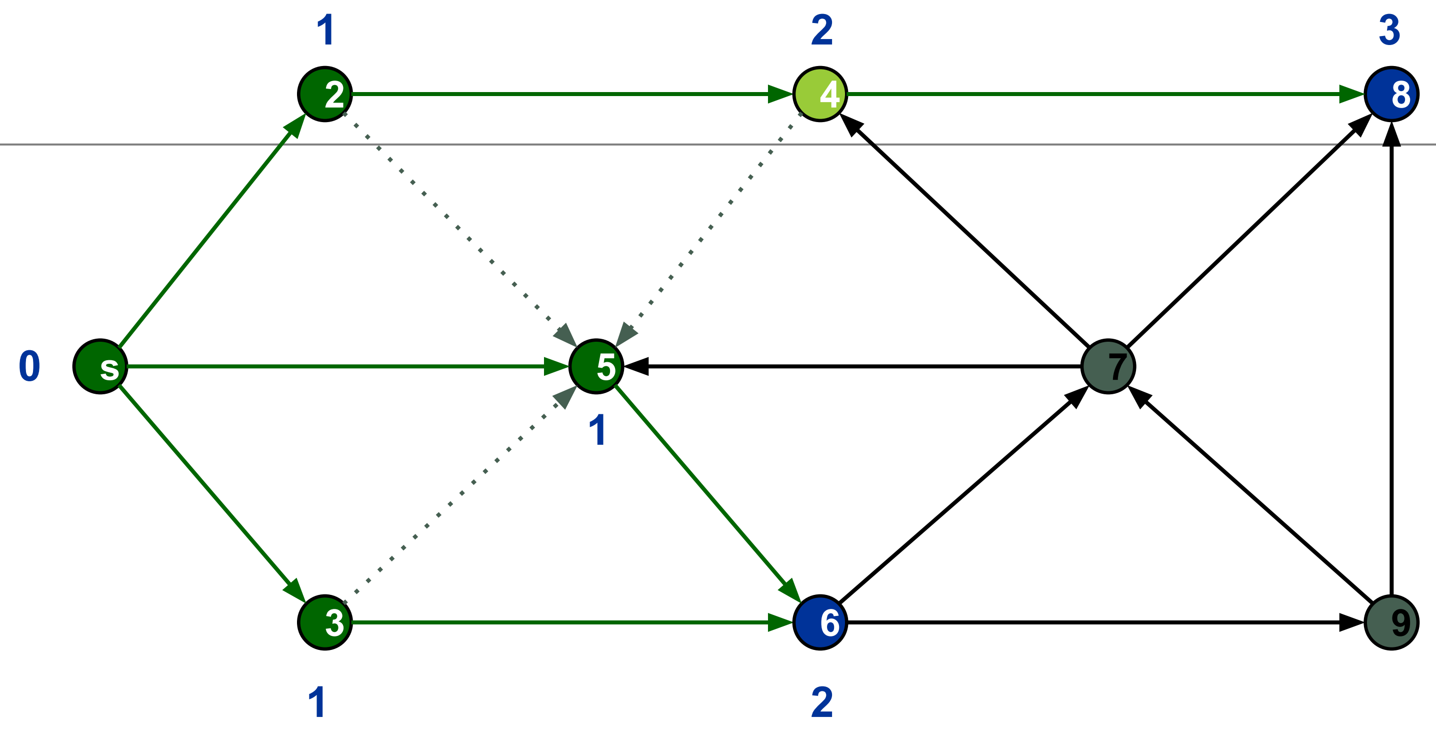
| Discovered |
| Top of queue |
| Finished |

Queue: 4 6

# Breadth First Search



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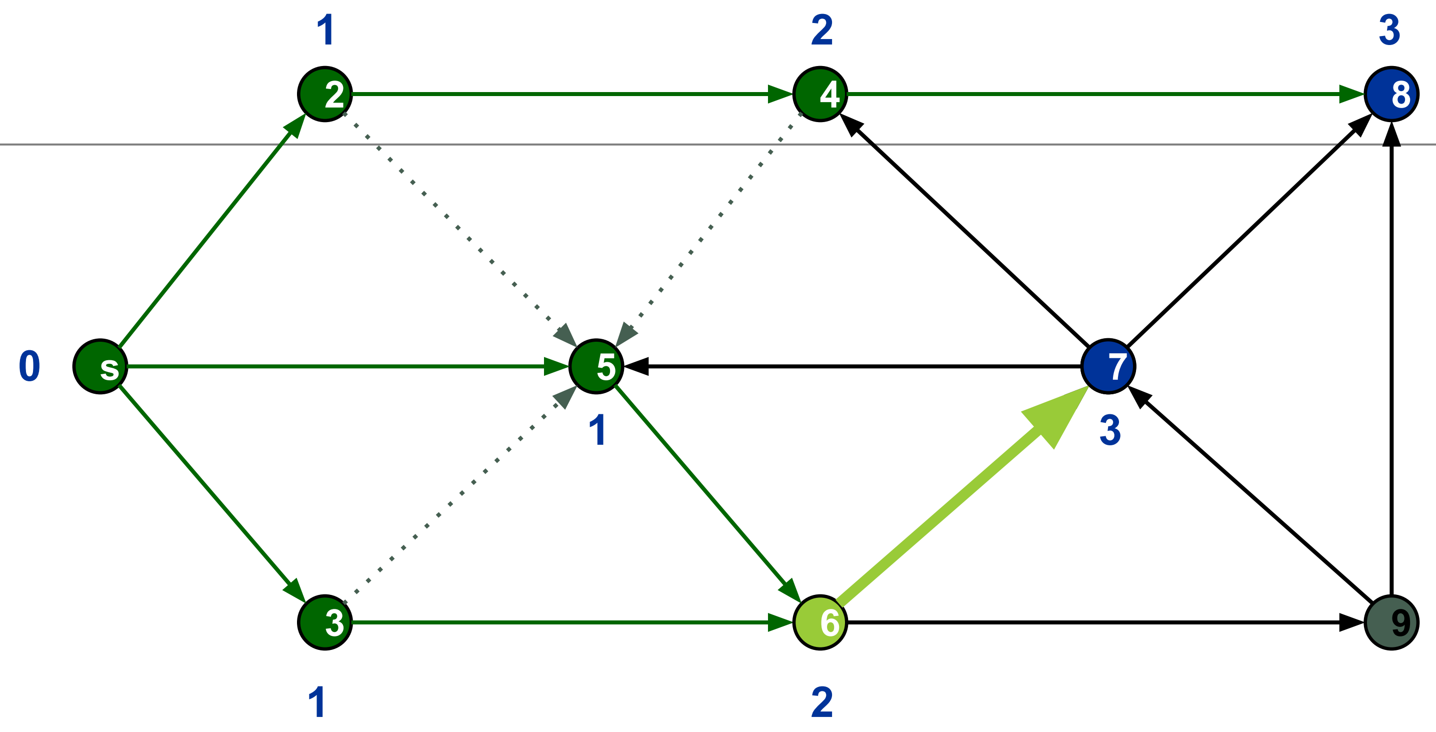
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

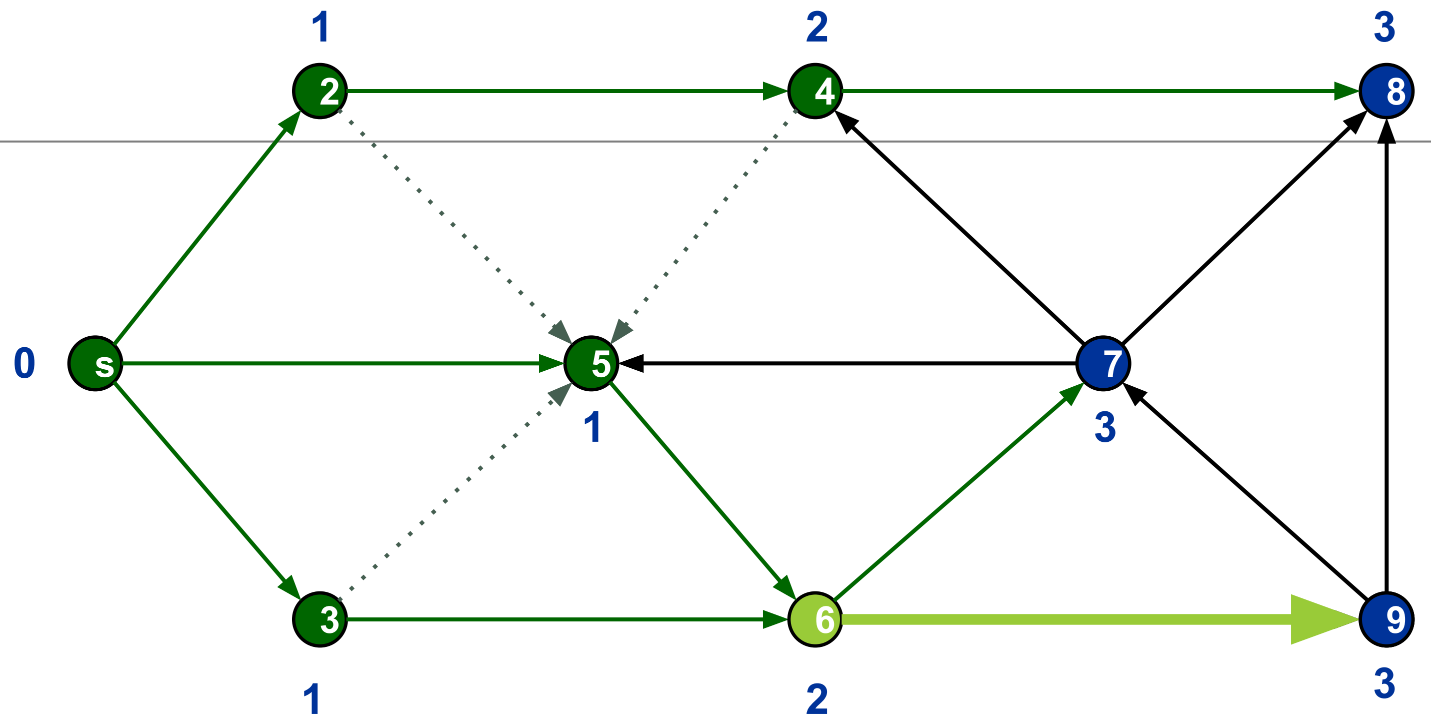
Queue: 4 6 8

# Breadth First Search



Queue: 6 8

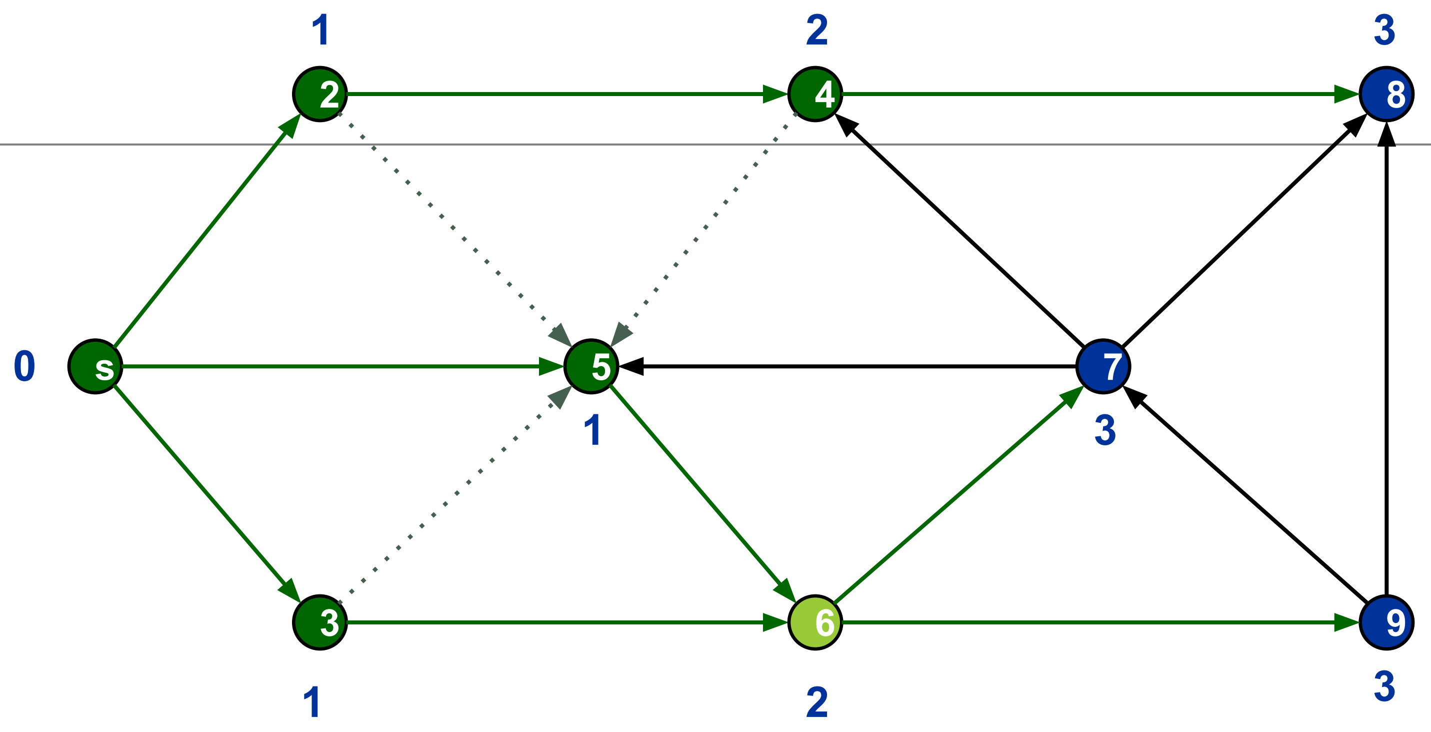
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: 6 8 7

# Breadth First Search



| Undiscovered |
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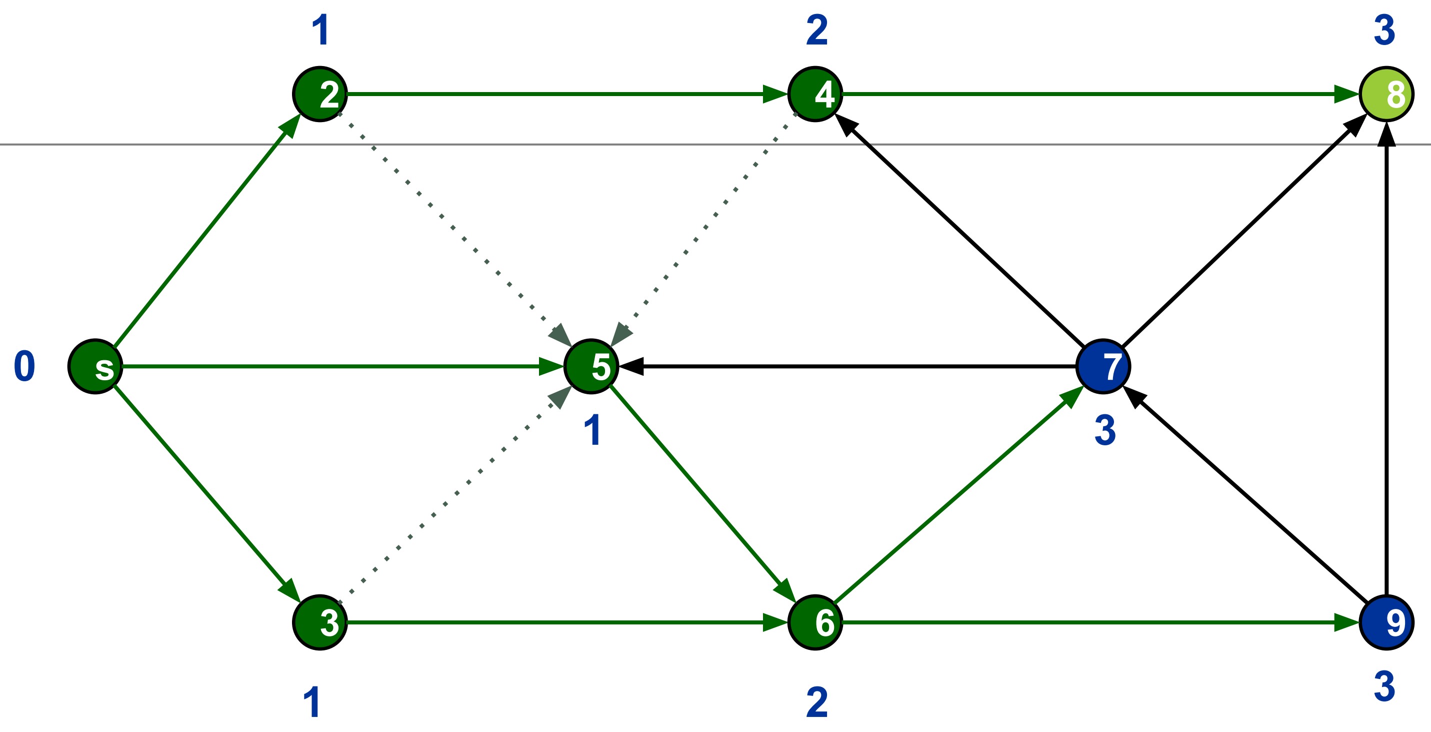
| Discovered |
| Top of queue |
| Finished |

Queue: 6 8 7 9

# Breadth First Search



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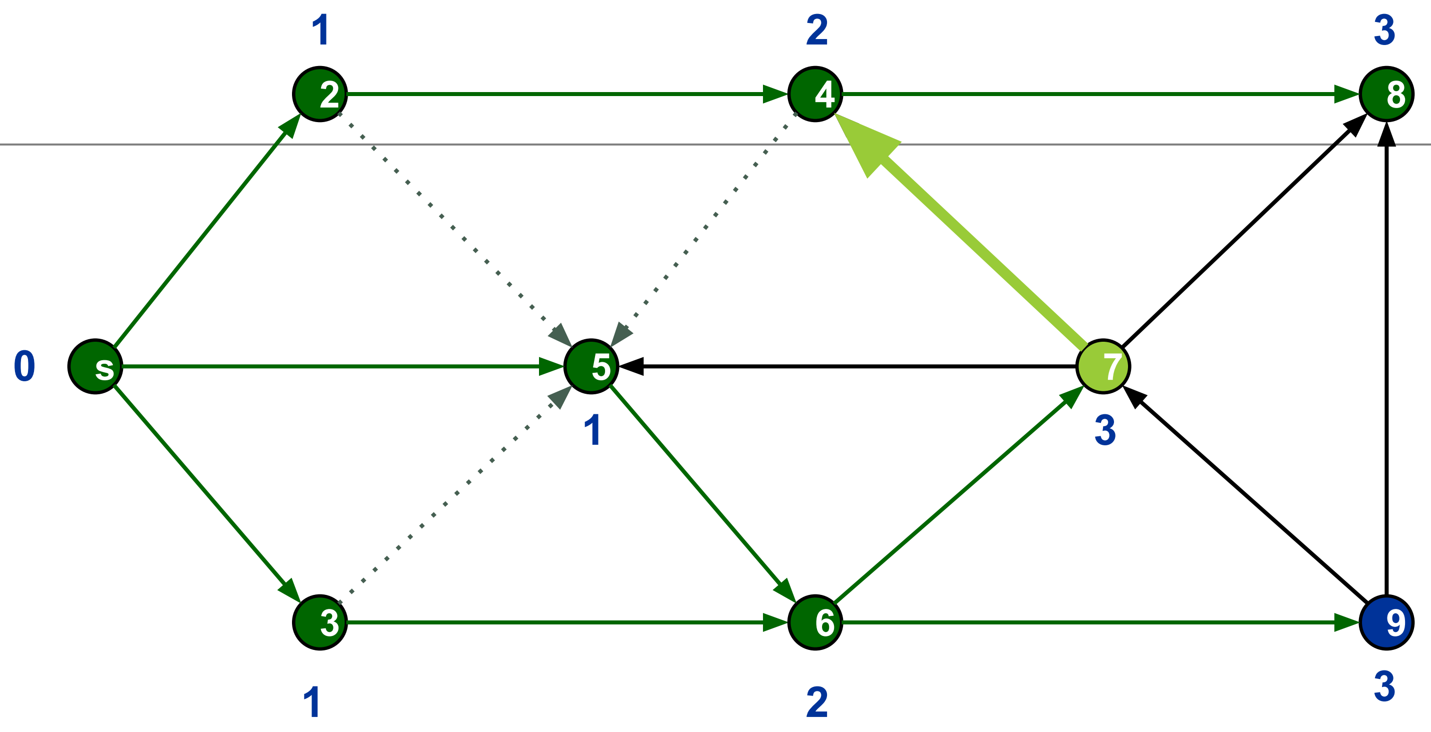
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 8 7 9

# Breadth First Search



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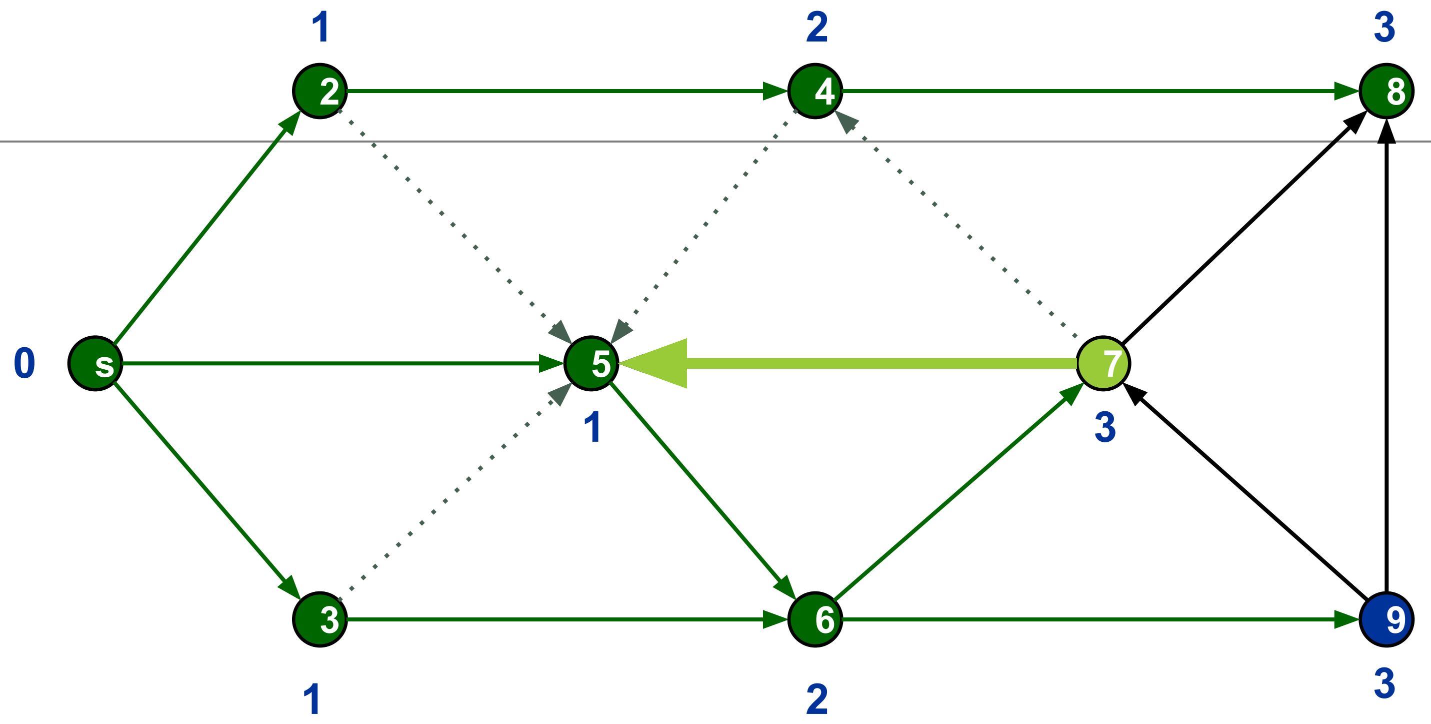
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 7 9

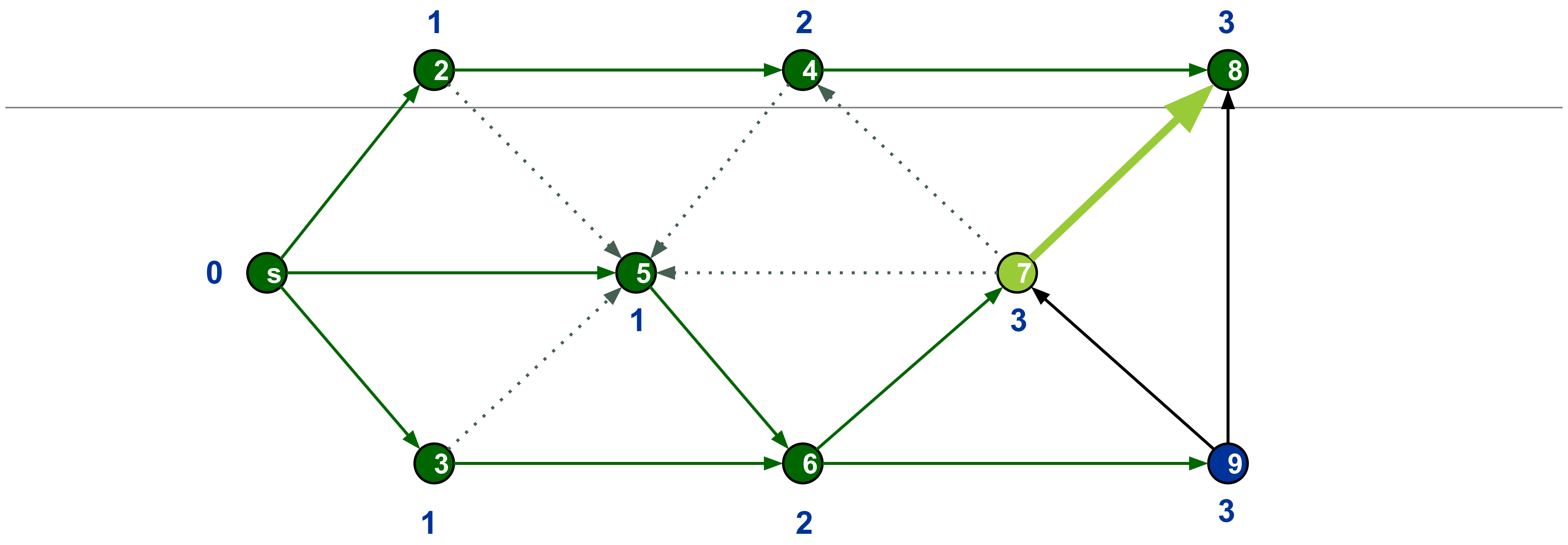
# Breadth First Search



| |
|---|
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue: 7 9

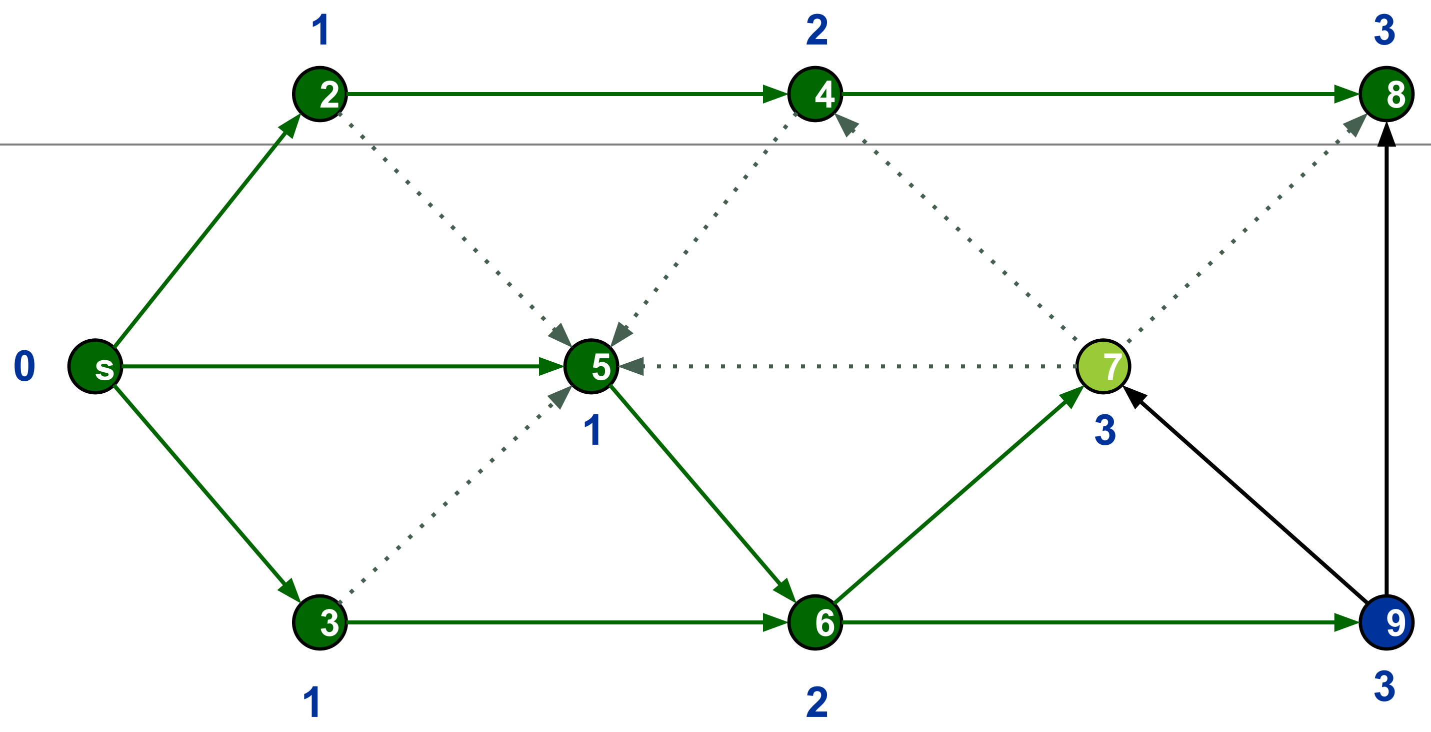
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: 7 9

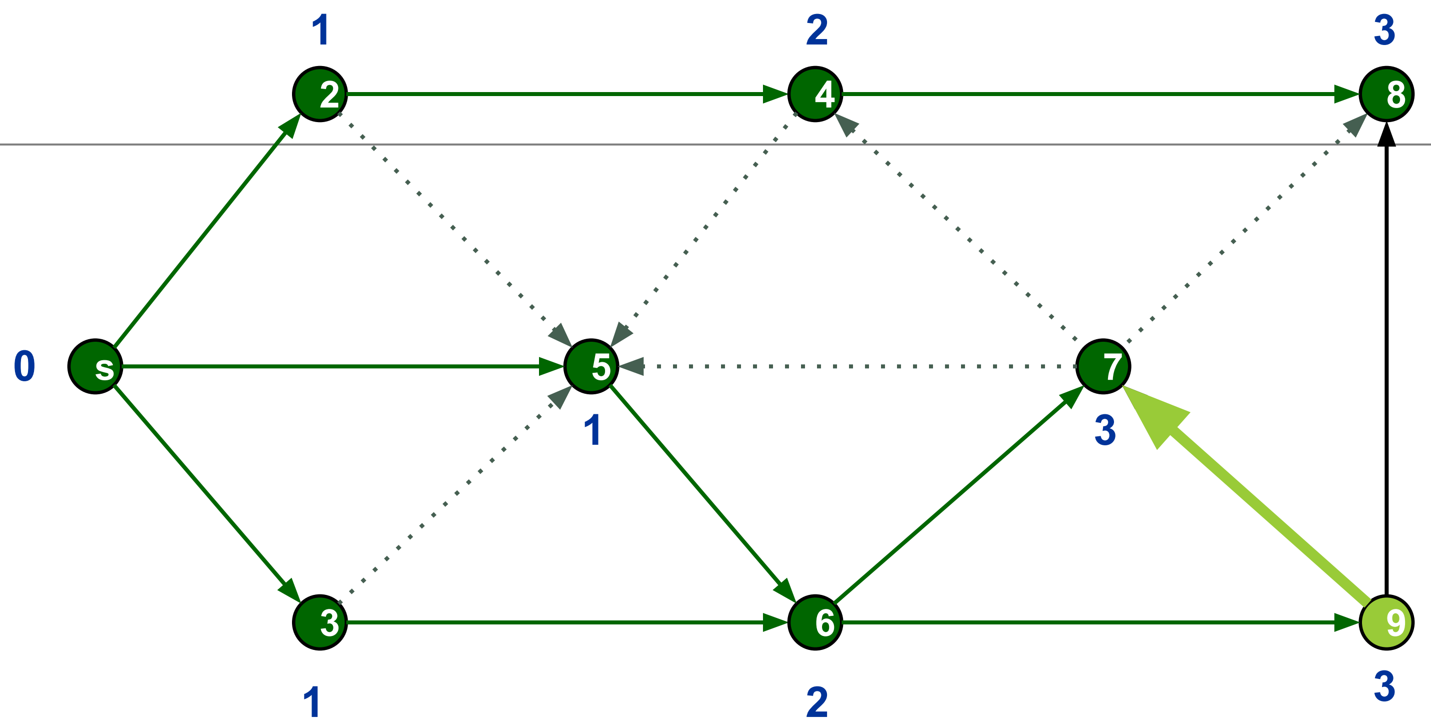
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: 7 9

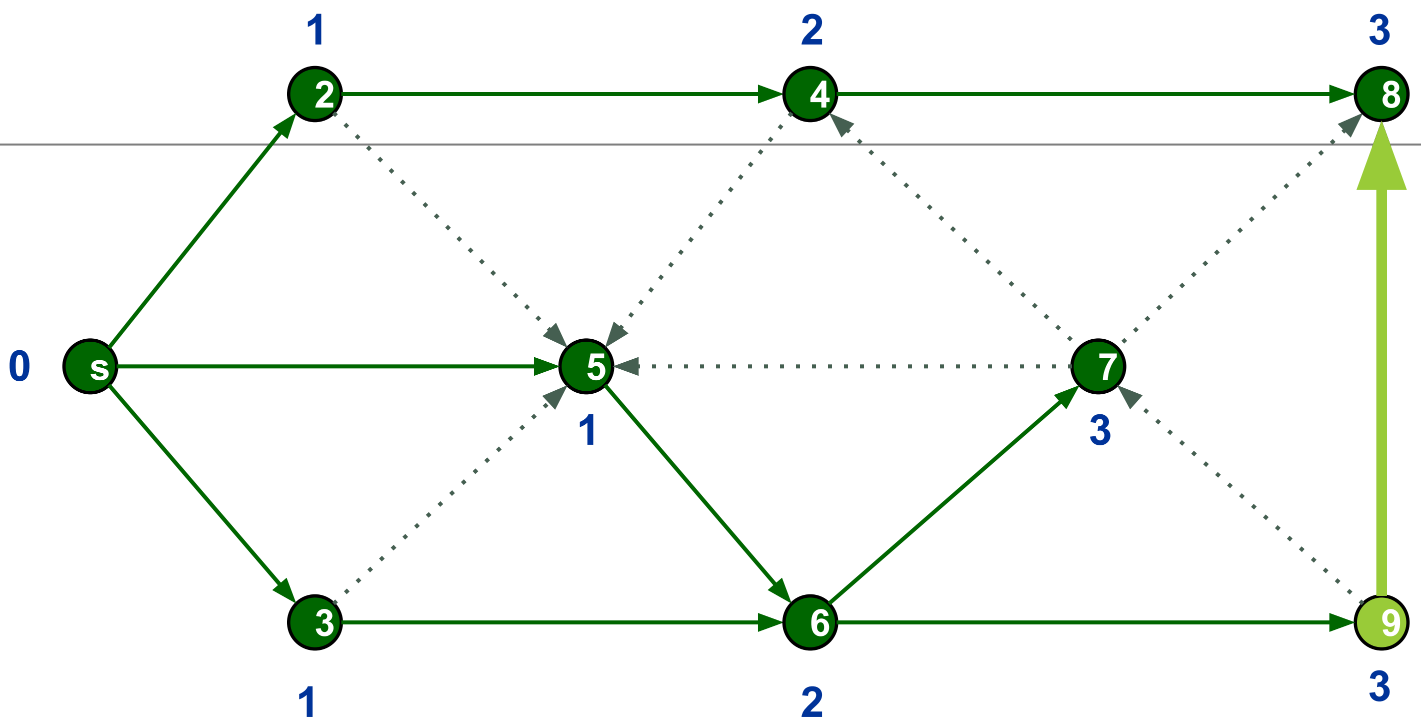
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: 9

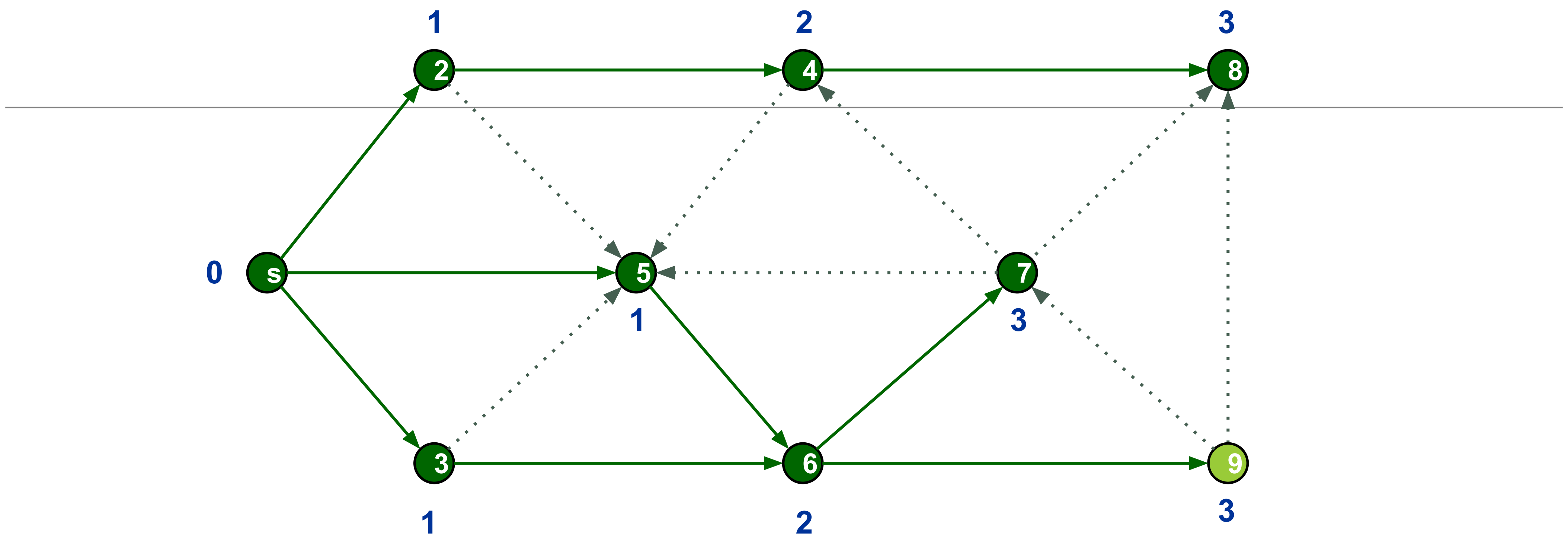
# Breadth First Search



| Undiscovered |
|---|
| Discovered |
| Top of queue |
| Finished |

Queue: 9

# Breadth First Search



Undiscovered

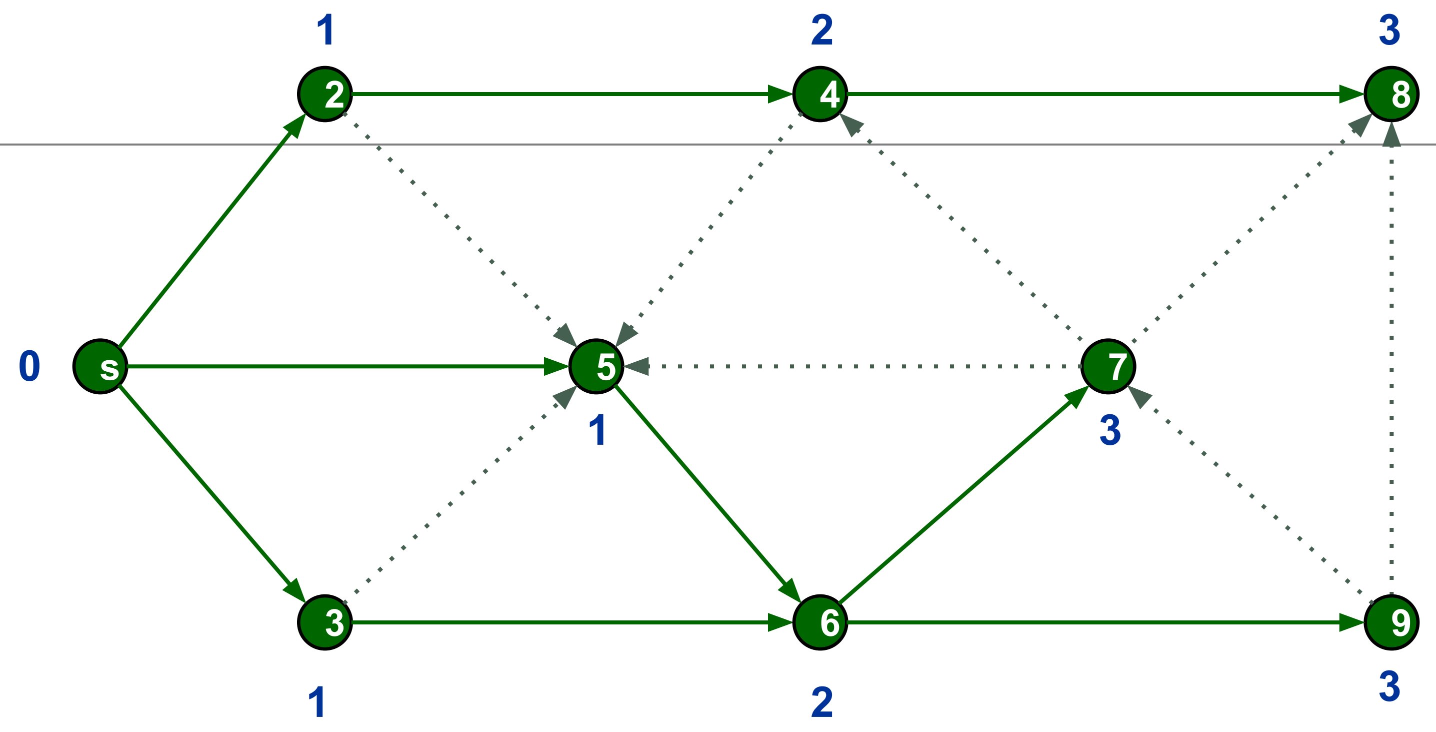
Discovered

Top of queue

Finished

Queue: 9

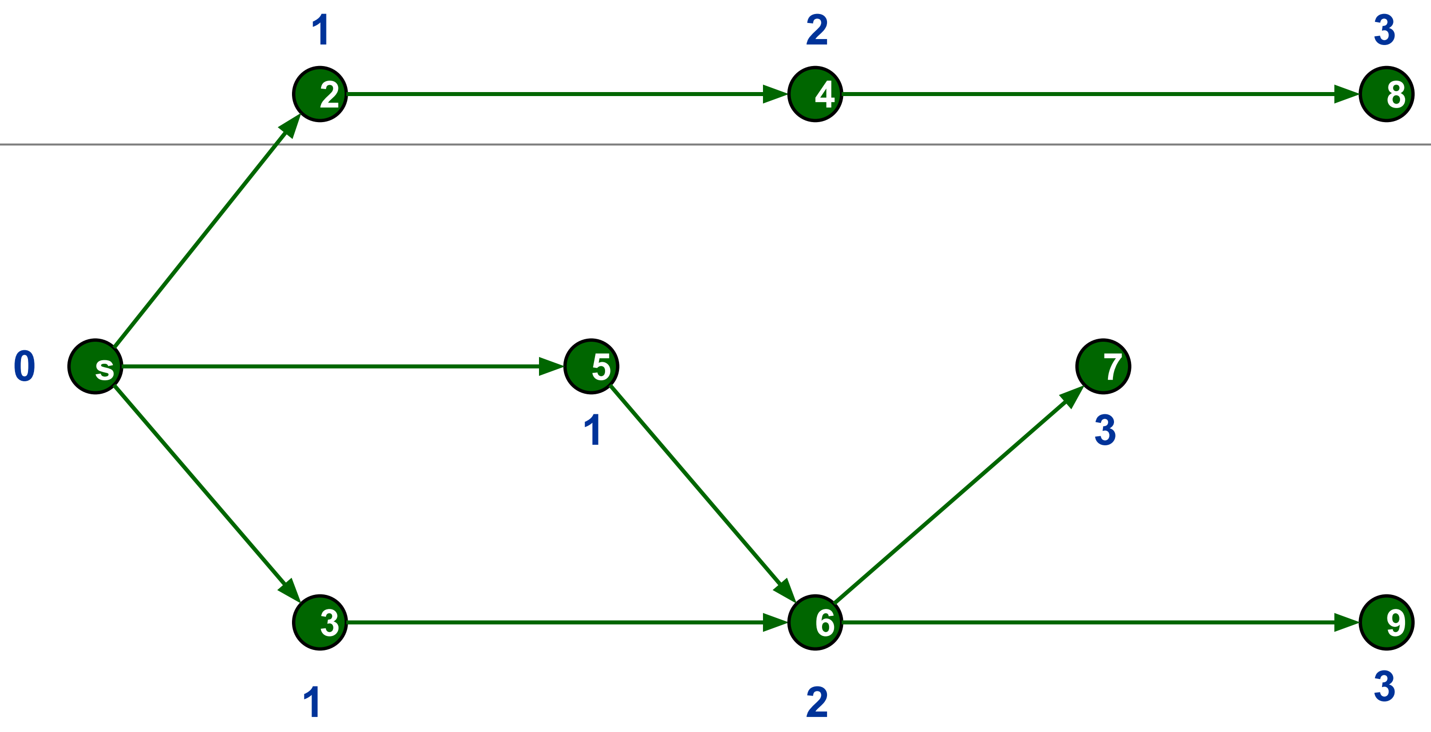
# Breadth First Search



| |
|---|
| Undiscovered |
| Discovered |
| Top of queue |
| Finished |

Queue:

# Breadth First Search



Level Graph

## Depth-First Search

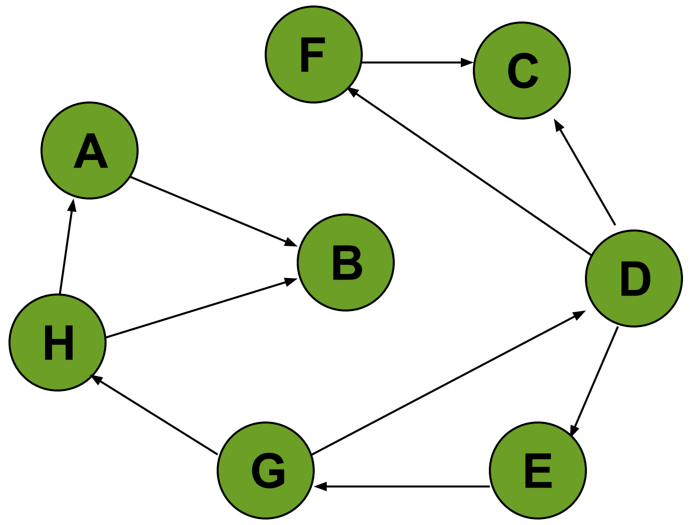
Can be used to attempt to visit all nodes of a graph in a systematic manner

Works with directed and undirected graphs

Works with weighted and unweighted graphs

```
depth-first-search
  mark vertex as visited
  for each adjacent vertex
    if unvisited
      do a depth-first search on adjacent vertex
```

## Walk-Through



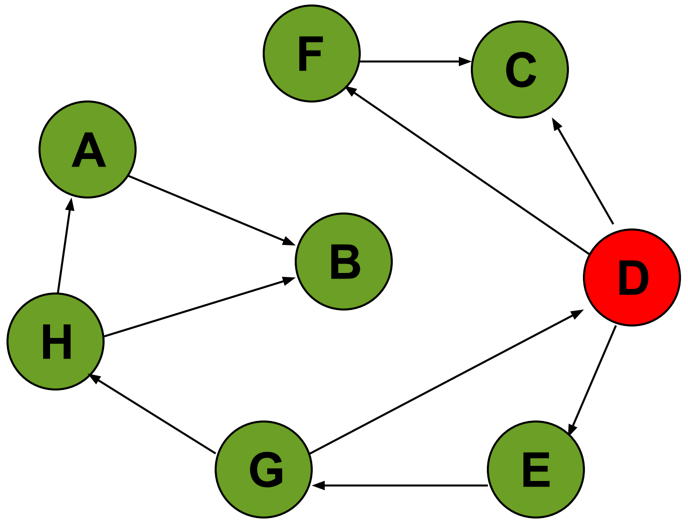
Visited Array

| | |
|---|---|
| A | |
| B | |
| C | |
| D | |
| E | |
| F | |
| G | |
| H | |

Stack

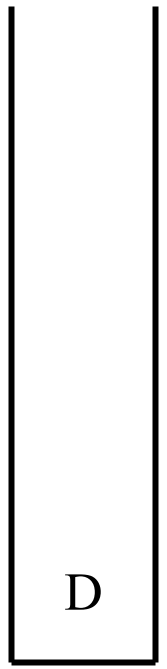
**Task: Conduct a depth-first search of the graph starting with node D**

## Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | |
| D | √ |
| E | |
| F | |
| G | |
| H | |

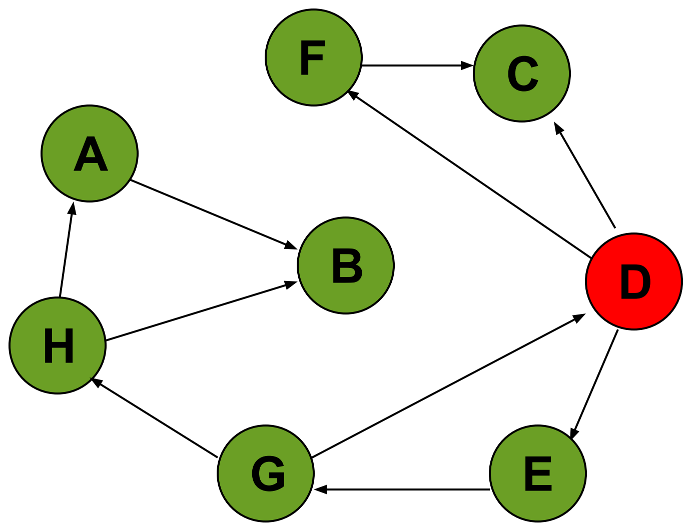


The order nodes are visited:

D

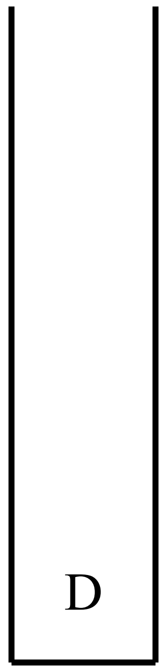
**Visit D**

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | |
| D | √ |
| E | |
| F | |
| G | |
| H | |

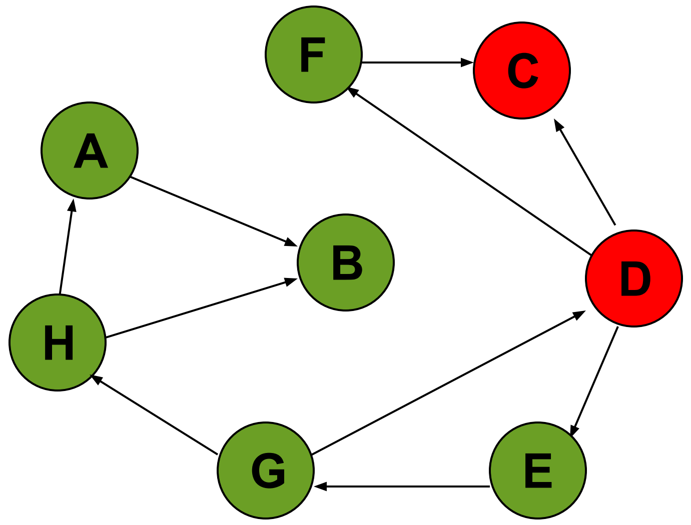


The order nodes are visited:

D

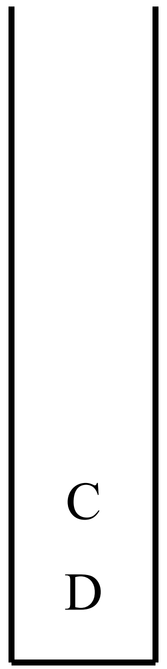
**Consider nodes adjacent to D, decide to visit C first (Rule: visit adjacent nodes in alphabetical order)**

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | |
| F | |
| G | |
| H | |

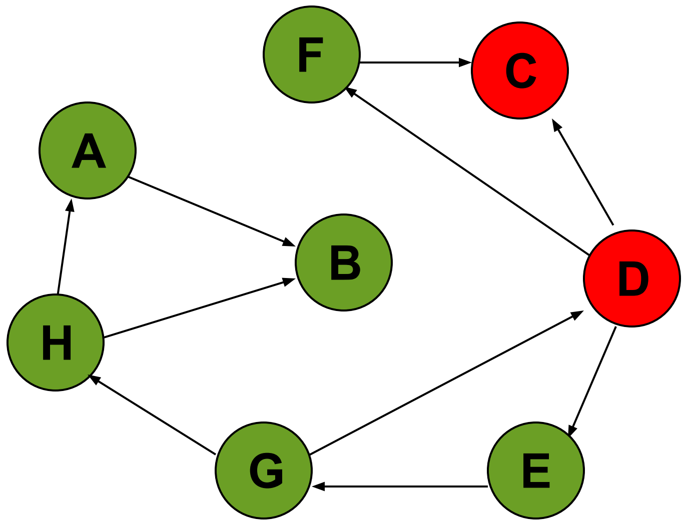


**Visit C**

The order nodes are visited:

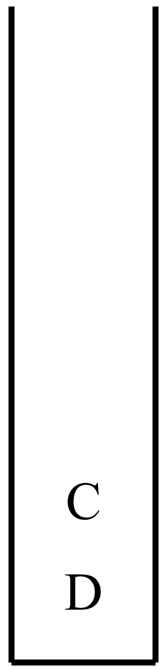
D, C

# Walk-Through



Visited Array

|   |   |
|---|---|
| A |   |
| B |   |
| C | √ |
| D | √ |
| E |   |
| F |   |
| G |   |
| H |   |

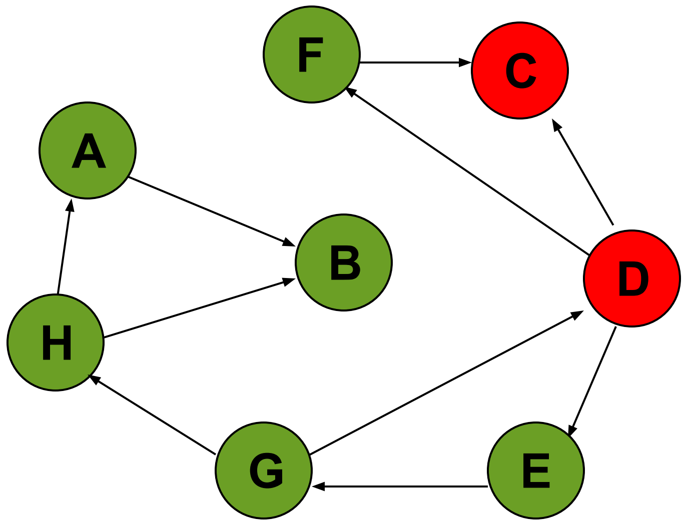


The order nodes are visited:

D, C

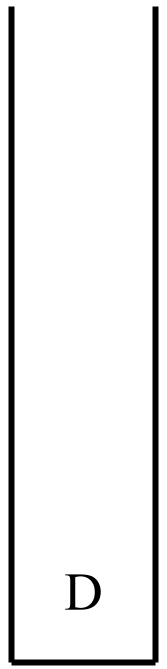
**No nodes adjacent to C; cannot continue  *backtrack*, i.e., pop stack and restore previous state**

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | |
| F | |
| G | |
| H | |

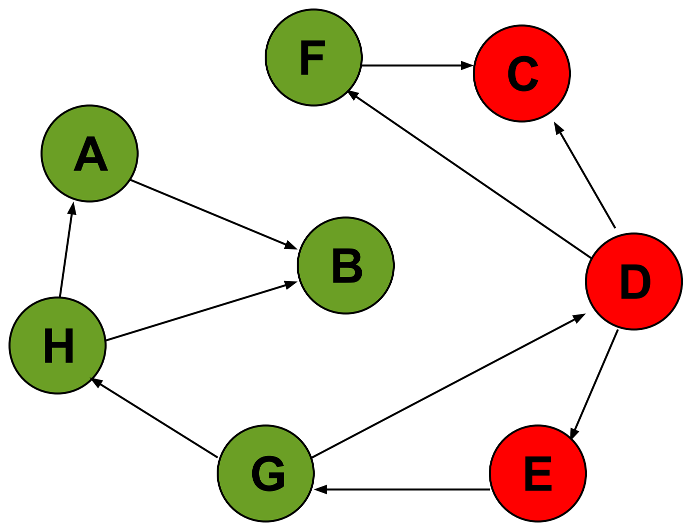


The order nodes are visited:

D, C

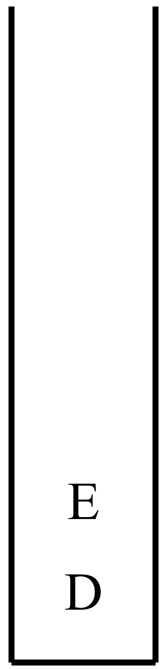
**Back to D – C has been visited, decide to visit E next**

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | |
| H | |

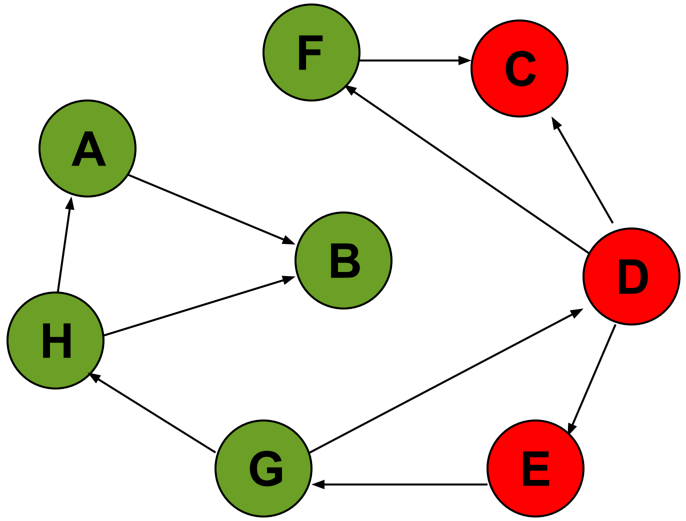


The order nodes are visited:

D, C, E

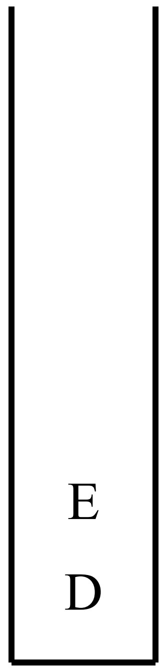
**Back to D – C has been visited, decide to visit E next**

## Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | |
| H | |

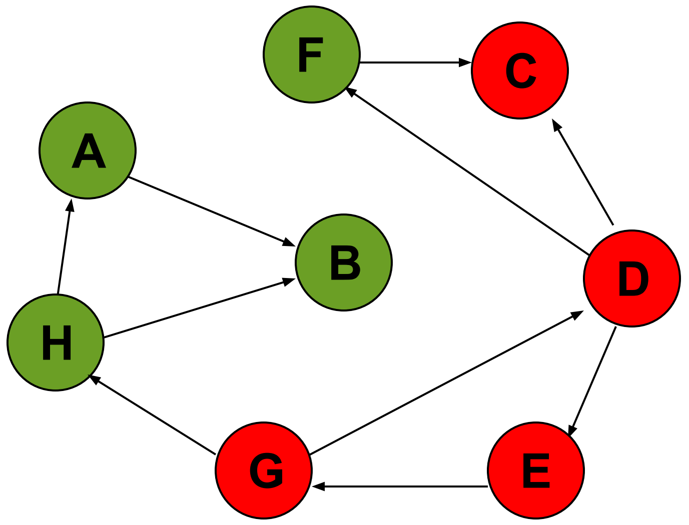


The order nodes are visited:

D, C, E

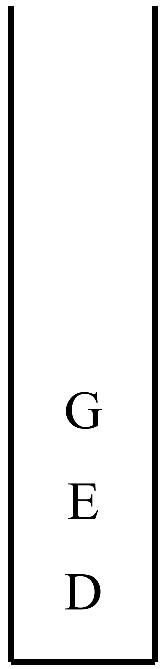
**Only G is adjacent to E**

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | |

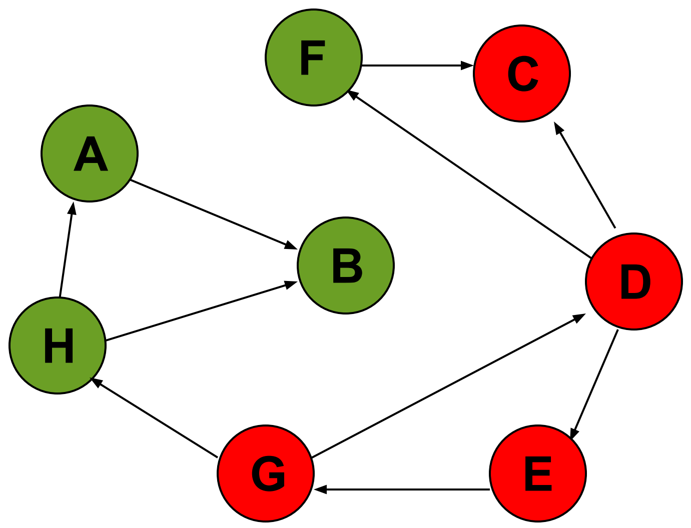


**Visit G**

The order nodes are visited:

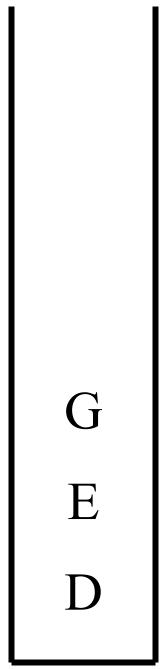
D, C, E, G

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | |

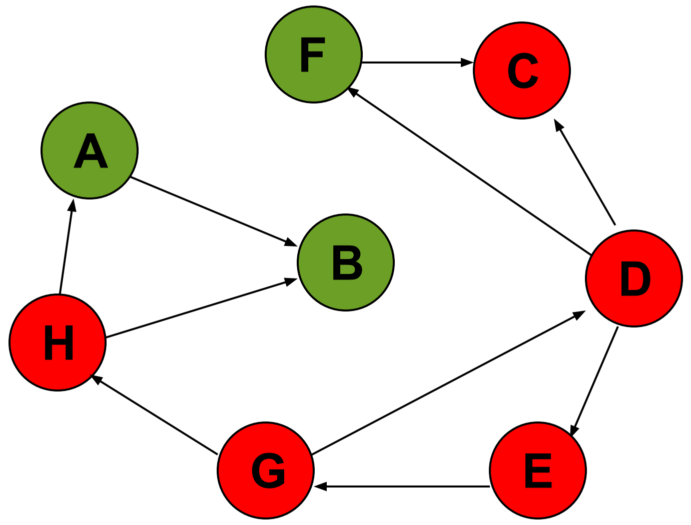


The order nodes are visited:

D, C, E, G

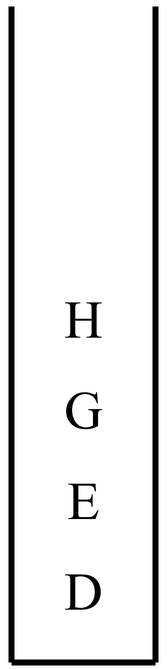
**Nodes D and H are adjacent to G. D has already been visited. Decide to visit H.**

# Walk-Through



Visited Array

| | |
|---|---|
| A | |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |

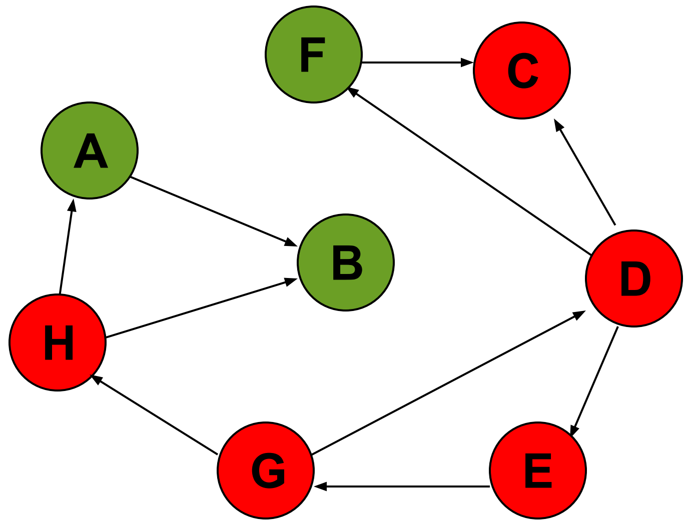


**Visit H**

The order nodes are visited:

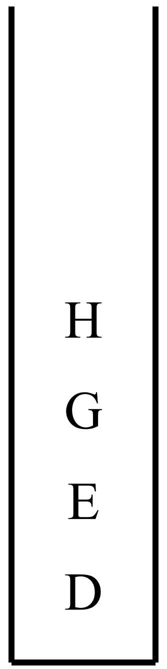
D, C, E, G, H

# Walk-Through



Visited Array

|  |  |
|---|---|
| A |  |
| B |  |
| C | √ |
| D | √ |
| E | √ |
| F |  |
| G | √ |
| H | √ |

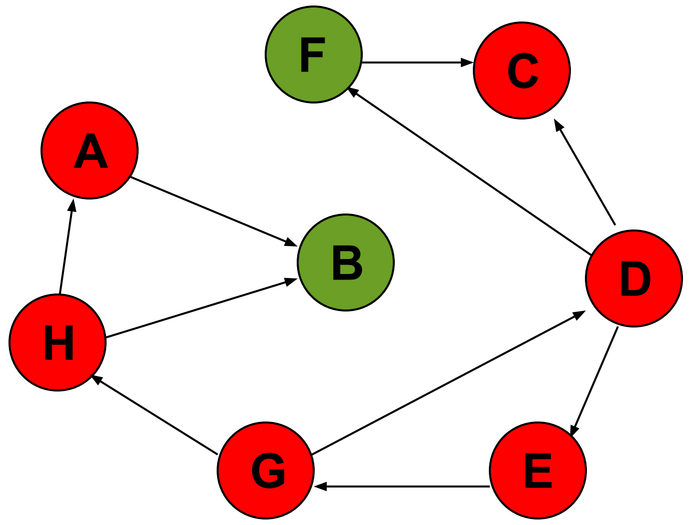


The order nodes are visited:

D, C, E, G, H

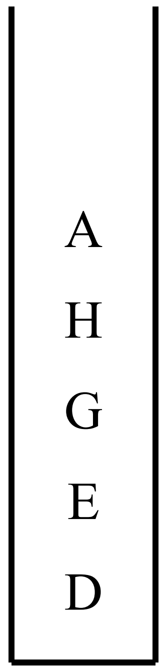
**Nodes A and B are adjacent to H. Decide to visit A next.**

# Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |

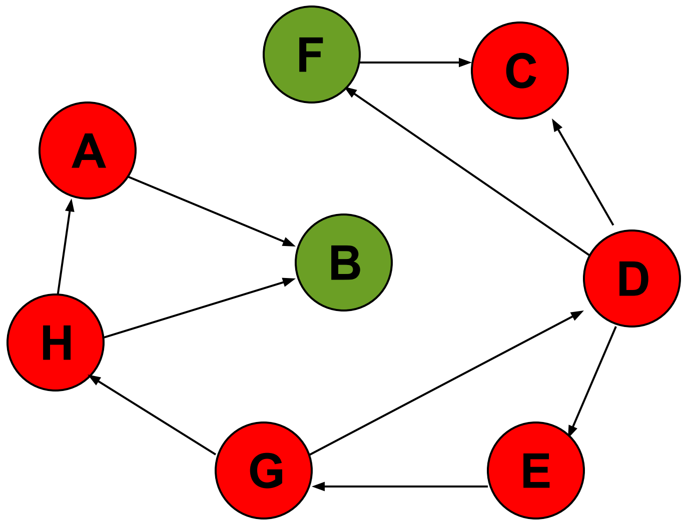


**Visit A**

The order nodes are visited:

D, C, E, G, H, A

# Walk-Through

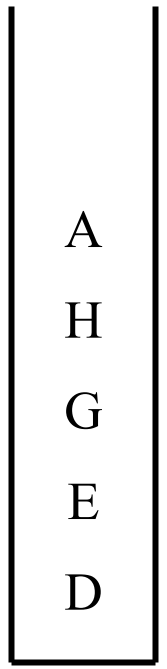


The order nodes are visited:

D, C, E, G, H, A

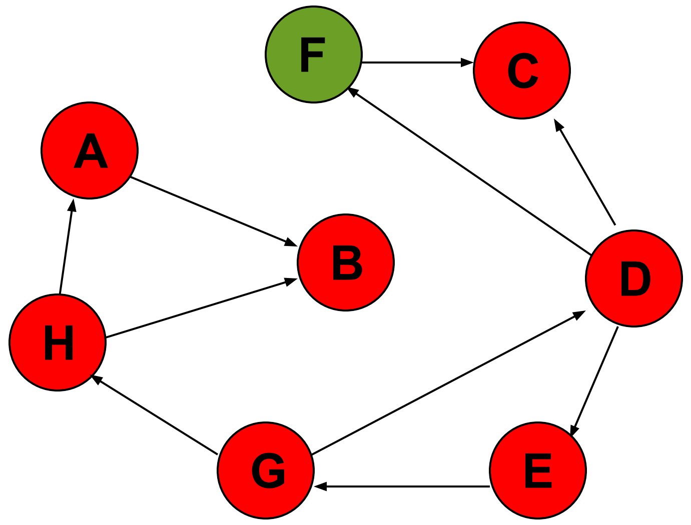
Visited Array

| | |
|---|---|
| A | √ |
| B | |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |



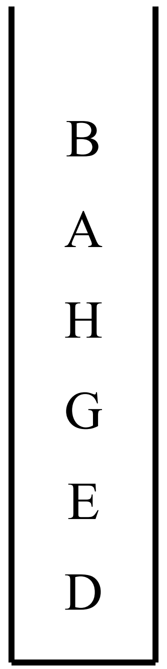
**Only Node B is adjacent to A.**
**Decide to visit B next.**

# Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |

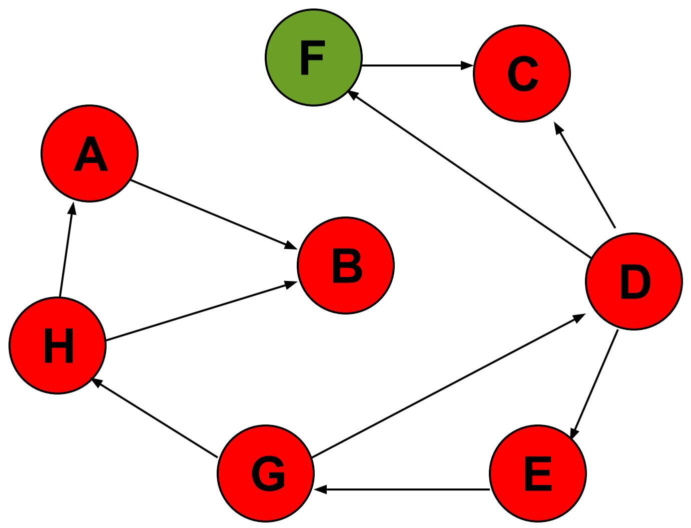


**Visit B**

The order nodes are visited:

D, C, E, G, H, A, B

## Walk-Through

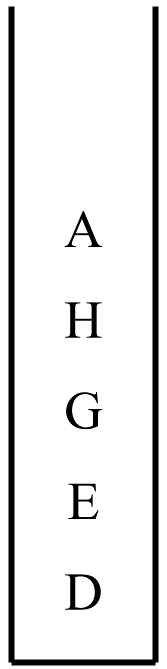


The order nodes are visited:

D, C, E, G, H, A, B

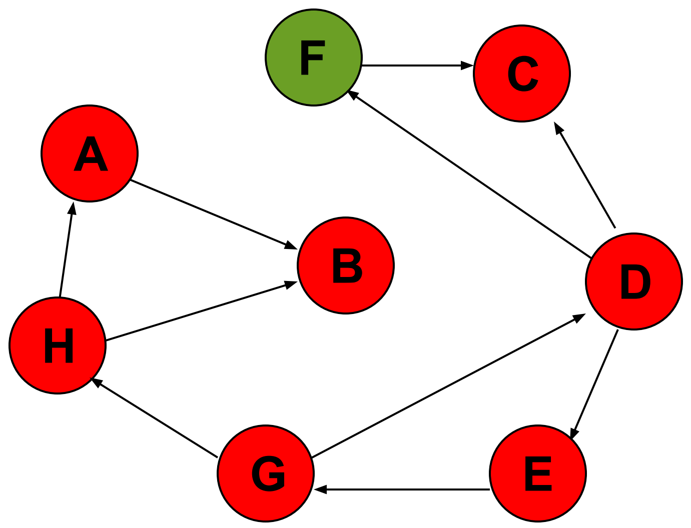
Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |



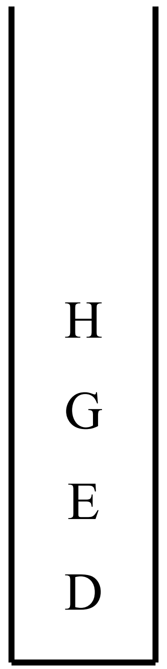
**No unvisited nodes adjacent to B. Backtrack (pop the stack).**

# Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |

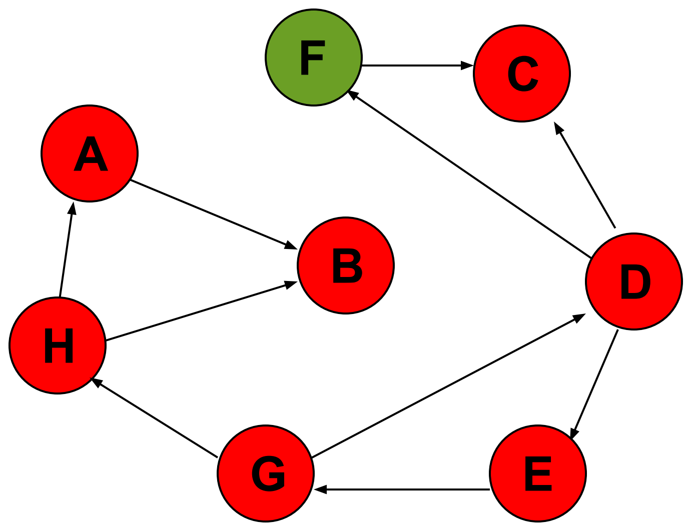


The order nodes are visited:

D, C, E, G, H, A, B

**No unvisited nodes adjacent to A. Backtrack (pop the stack).**

# Walk-Through

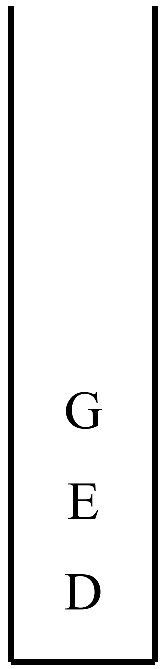


The order nodes are visited:

D, C, E, G, H, A, B

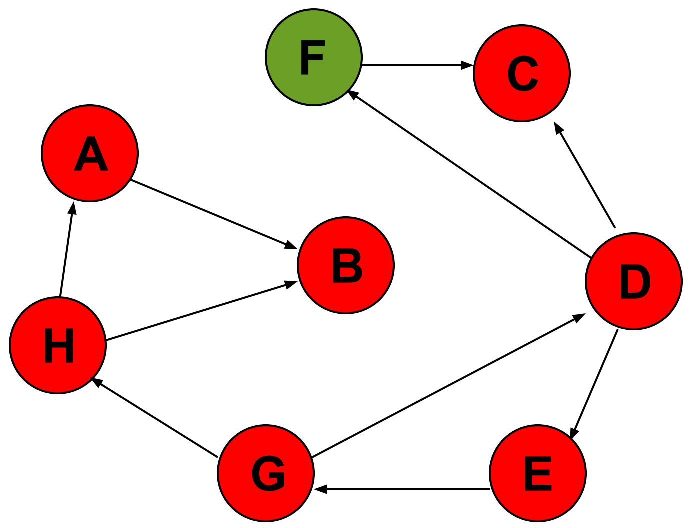
Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |



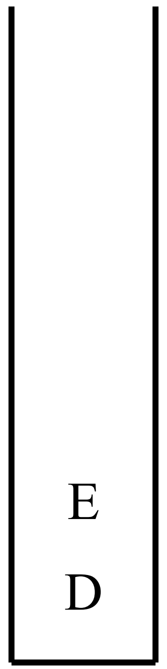
**No unvisited nodes adjacent to H. Backtrack (pop the stack).**

## Walk-Through



Visited Array

|   |   |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F |   |
| G | √ |
| H | √ |

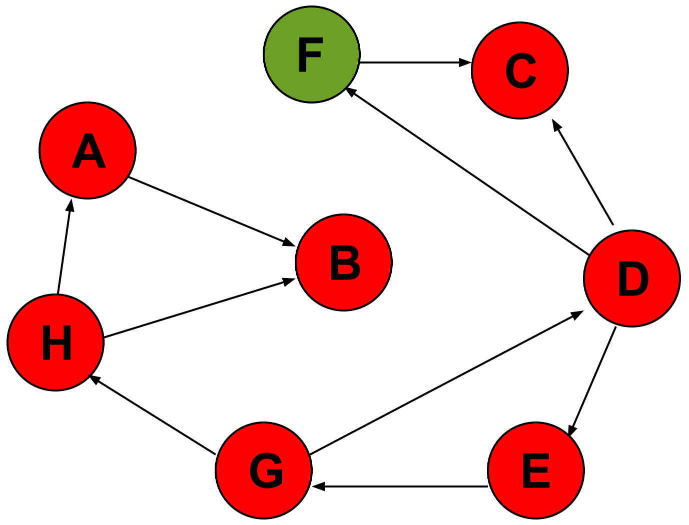


The order nodes are visited:

D, C, E, G, H, A, B

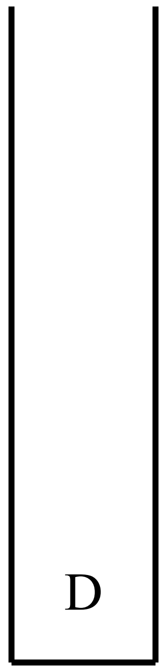
**No unvisited nodes adjacent to G. Backtrack (pop the stack).**

## Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | |
| G | √ |
| H | √ |

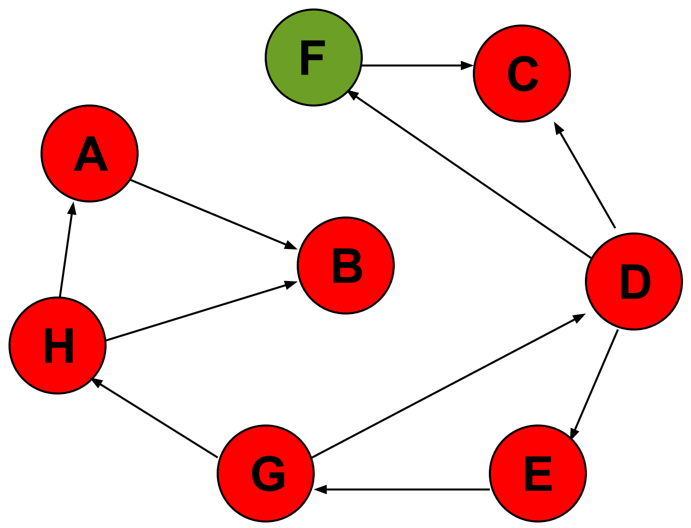


The order nodes are visited:

D, C, E, G, H, A, B

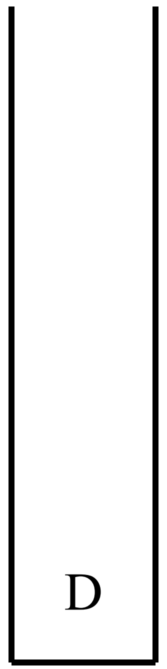
**No unvisited nodes adjacent to E. Backtrack (pop the stack).**

## Walk-Through



Visited Array

|  |  |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F |  |
| G | √ |
| H | √ |

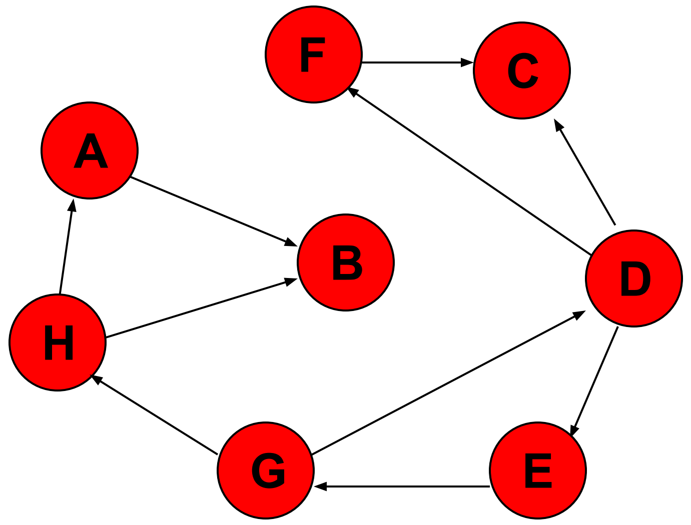


The order nodes are visited:

D, C, E, G, H, A, B

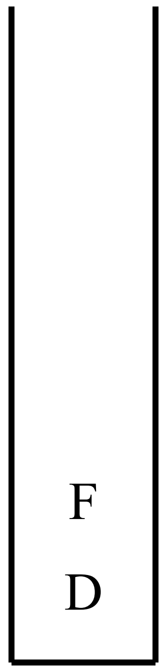
**F is unvisited and is adjacent to D. Decide to visit F next.**

# Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | √ |
| G | √ |
| H | √ |

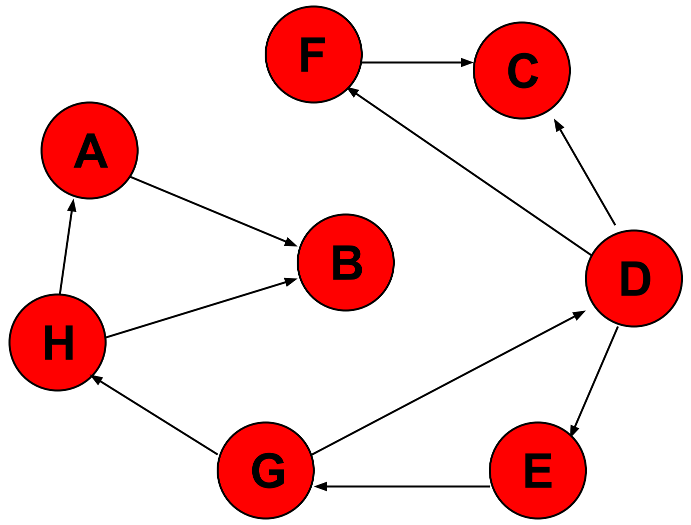


**Visit F**

The order nodes are visited:

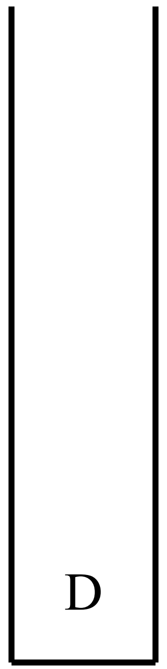
D, C, E, G, H, A, B, F

## Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | √ |
| G | √ |
| H | √ |

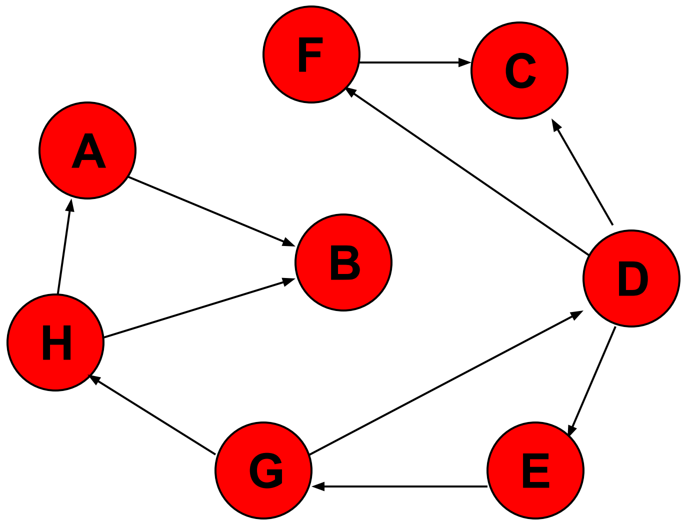


The order nodes are visited:

D, C, E, G, H, A, B, F

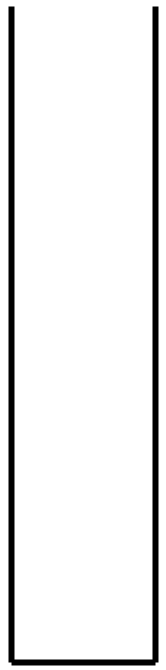
**No unvisited nodes adjacent to F. Backtrack.**

## Walk-Through



Visited Array

| | |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | √ |
| G | √ |
| H | √ |

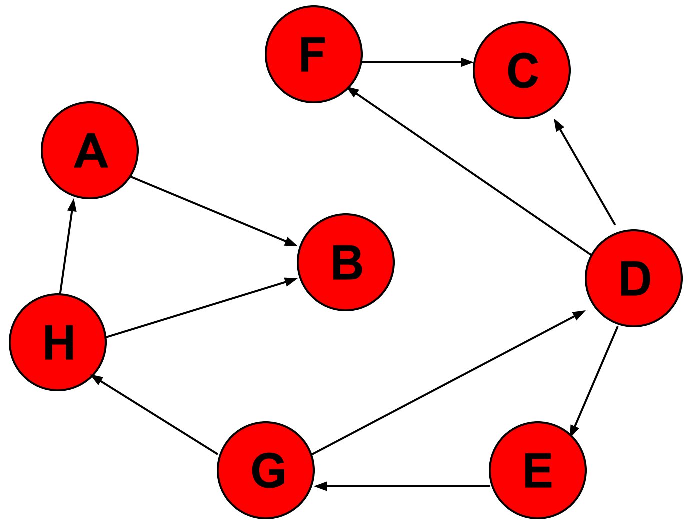


The order nodes are visited:

D, C, E, G, H, A, B, F

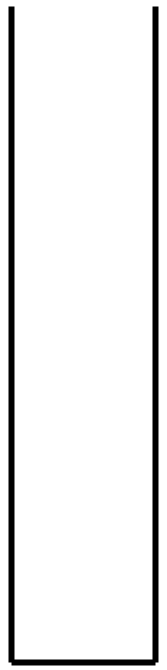
**No unvisited nodes adjacent to D. Backtrack.**

## Walk-Through



Visited Array

|   |   |
|---|---|
| A | √ |
| B | √ |
| C | √ |
| D | √ |
| E | √ |
| F | √ |
| G | √ |
| H | √ |

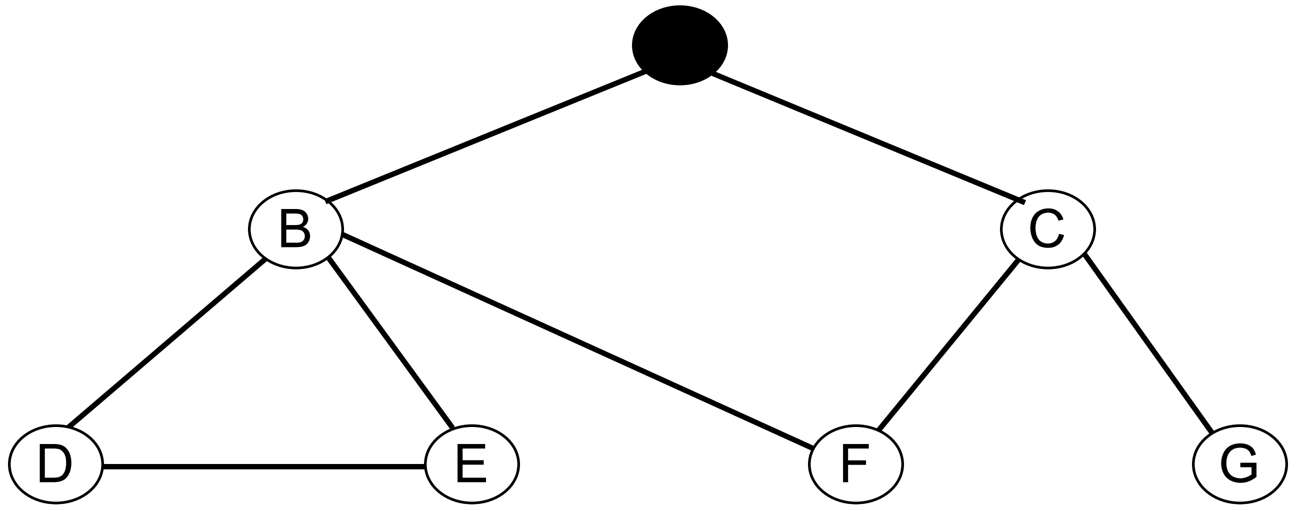


The order nodes are visited:

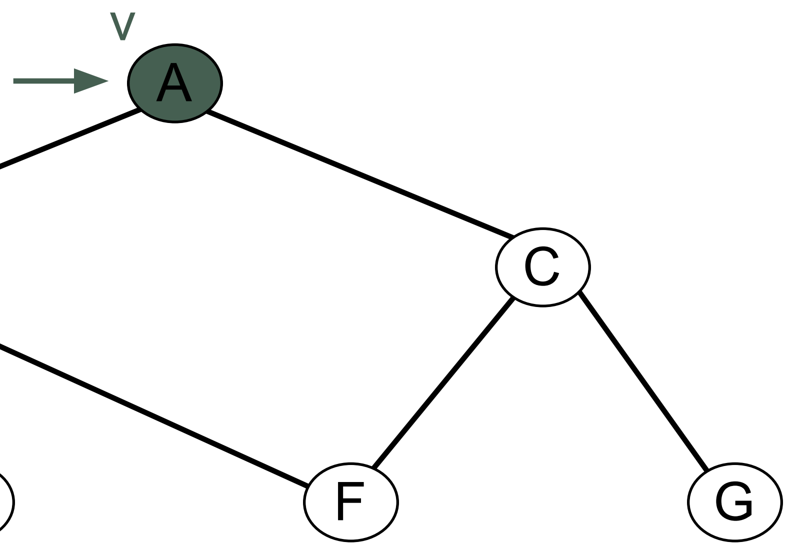
D, C, E, G, H, A, B, F

**Stack is empty. Depth-first traversal is done.**

## Depth-First Search

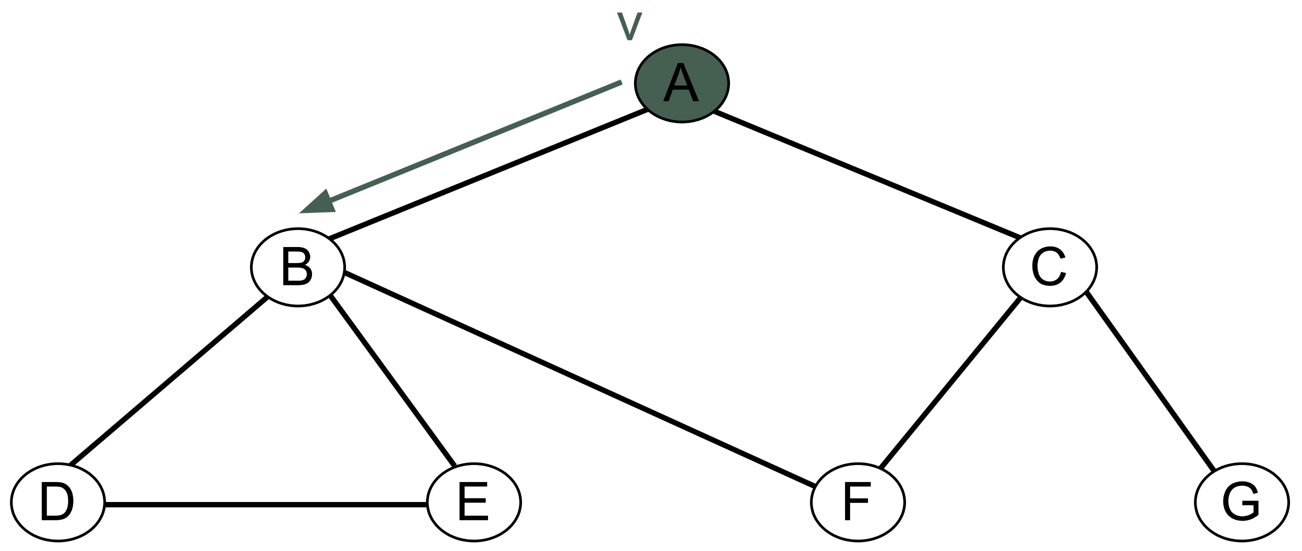


## Depth-First Search



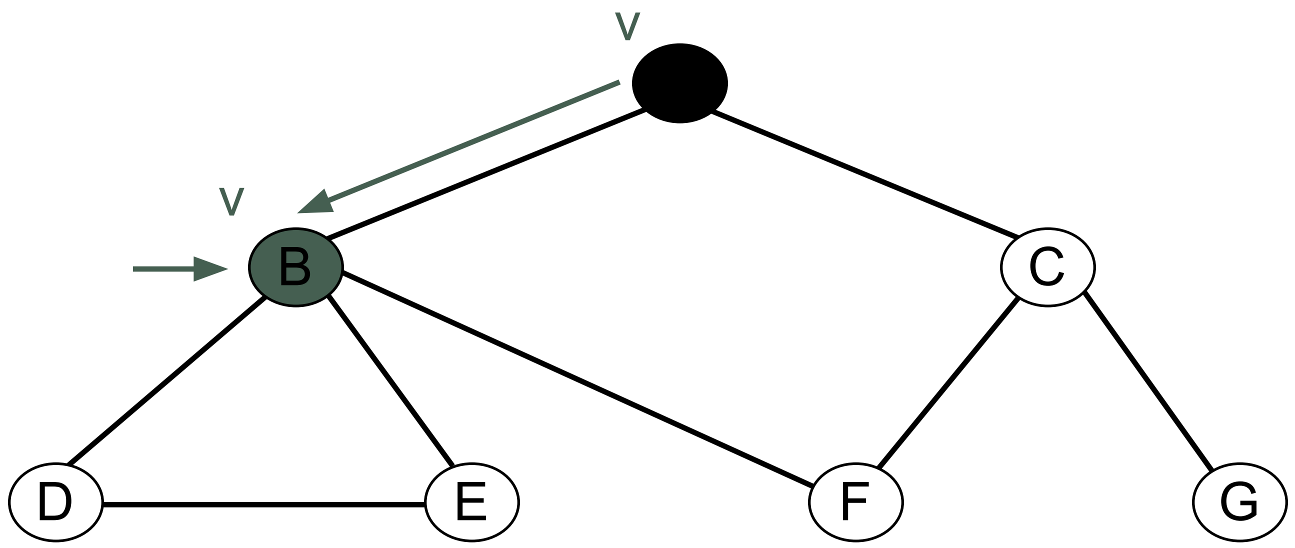
A

## Depth-First Search



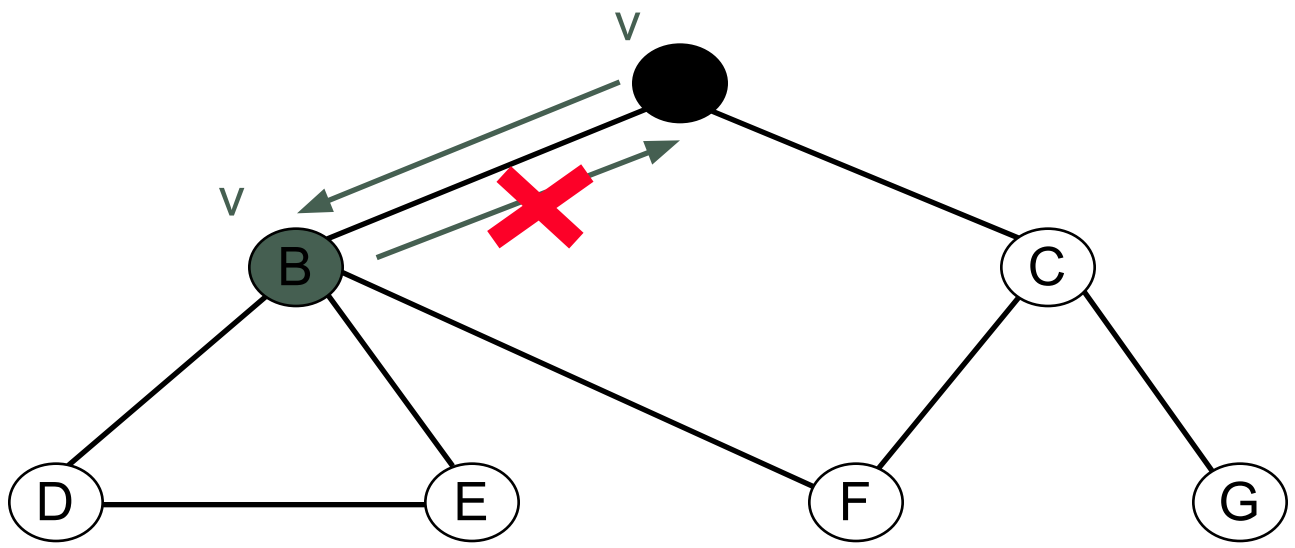
A

## Depth-First Search



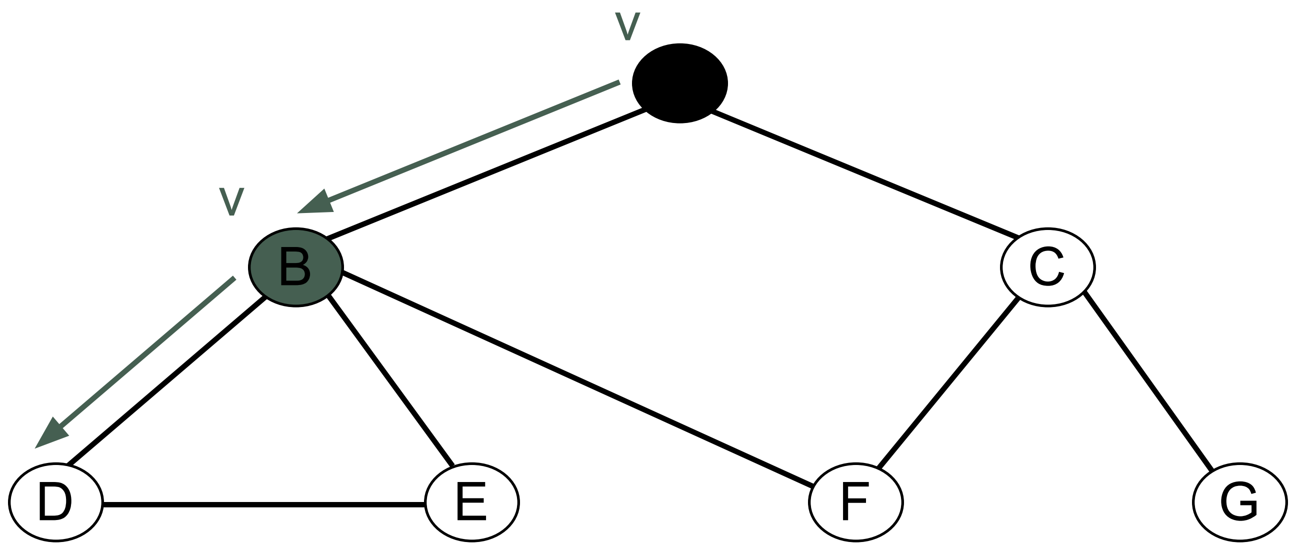
A B

## Depth-First Search



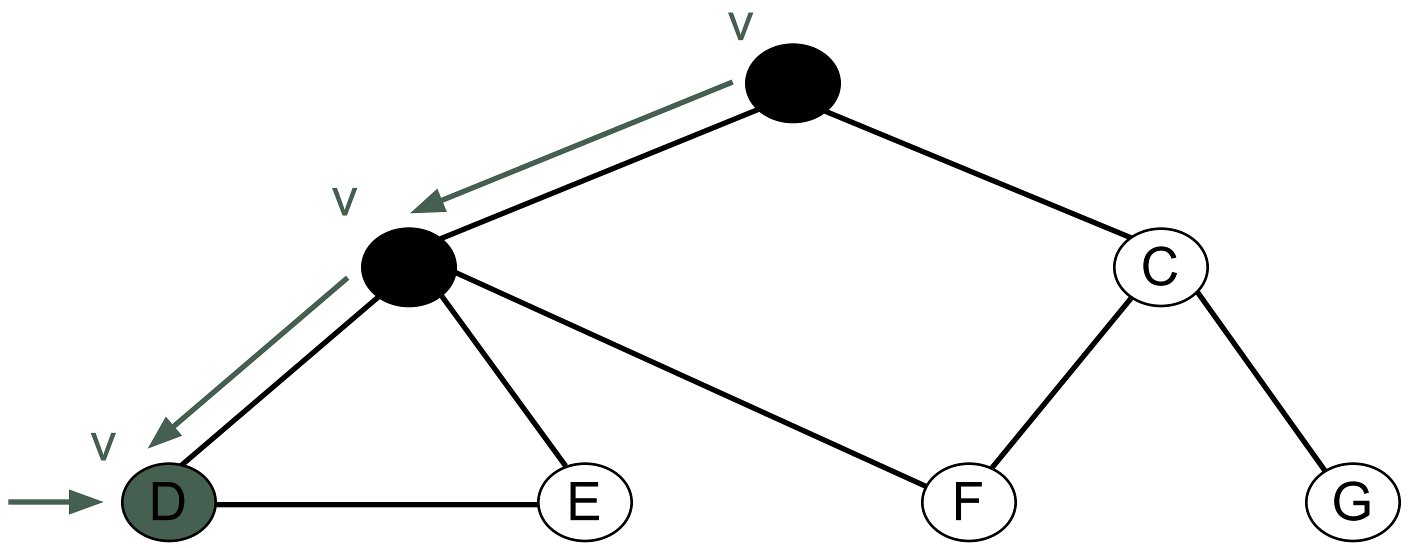
A B

## Depth-First Search



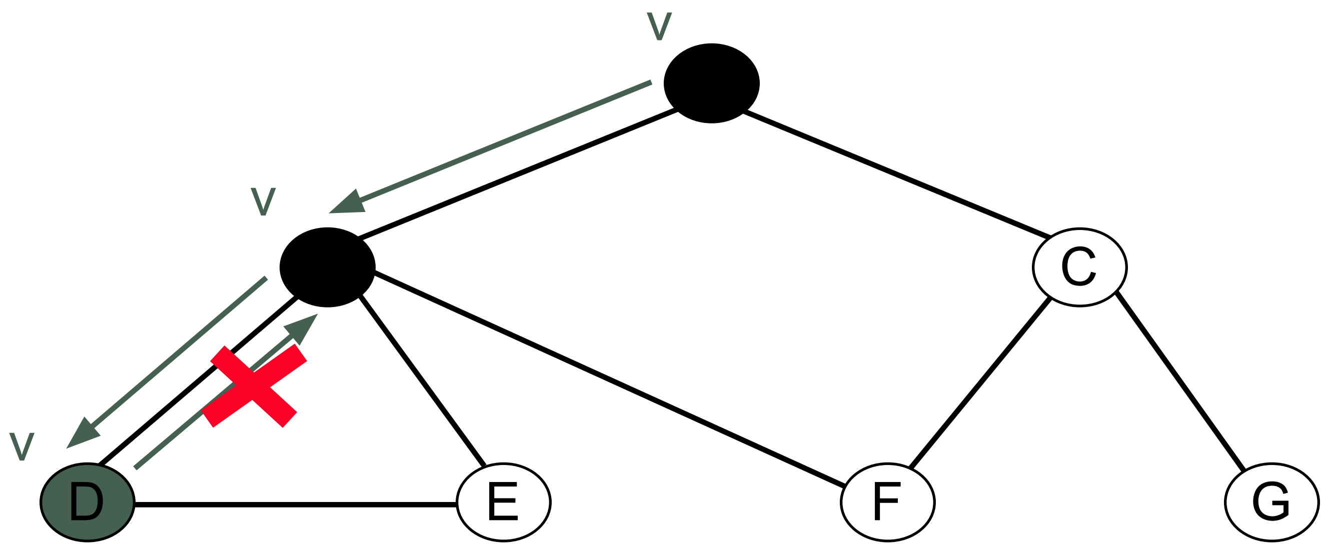
A B

## Depth-First Search



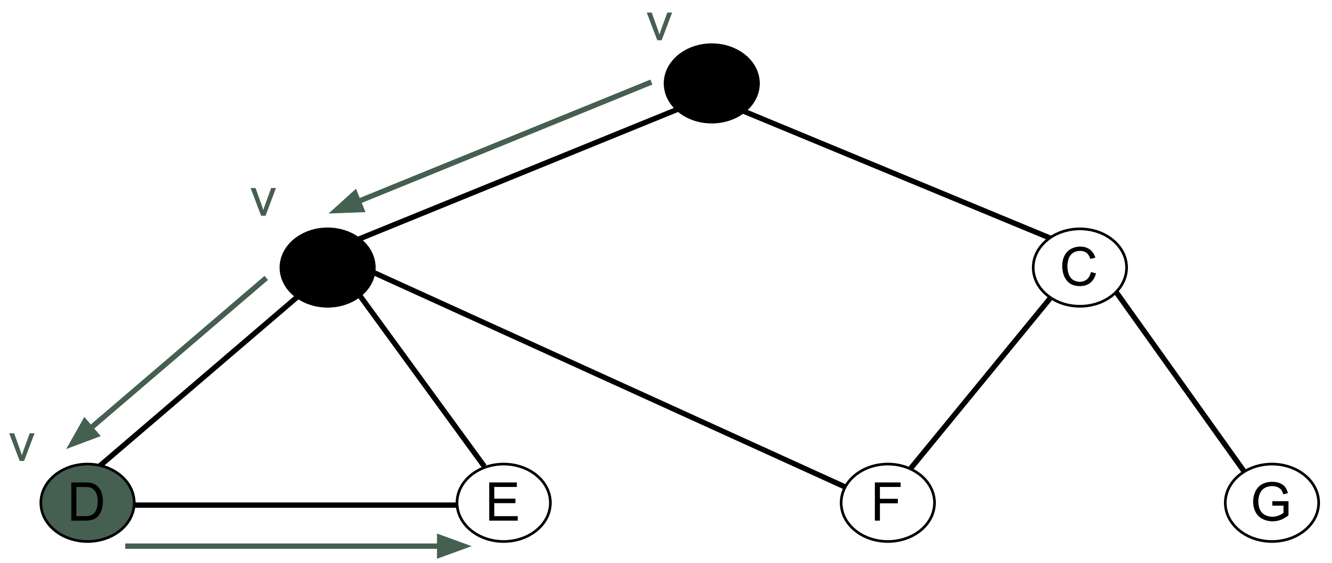
A B D

## Depth-First Search



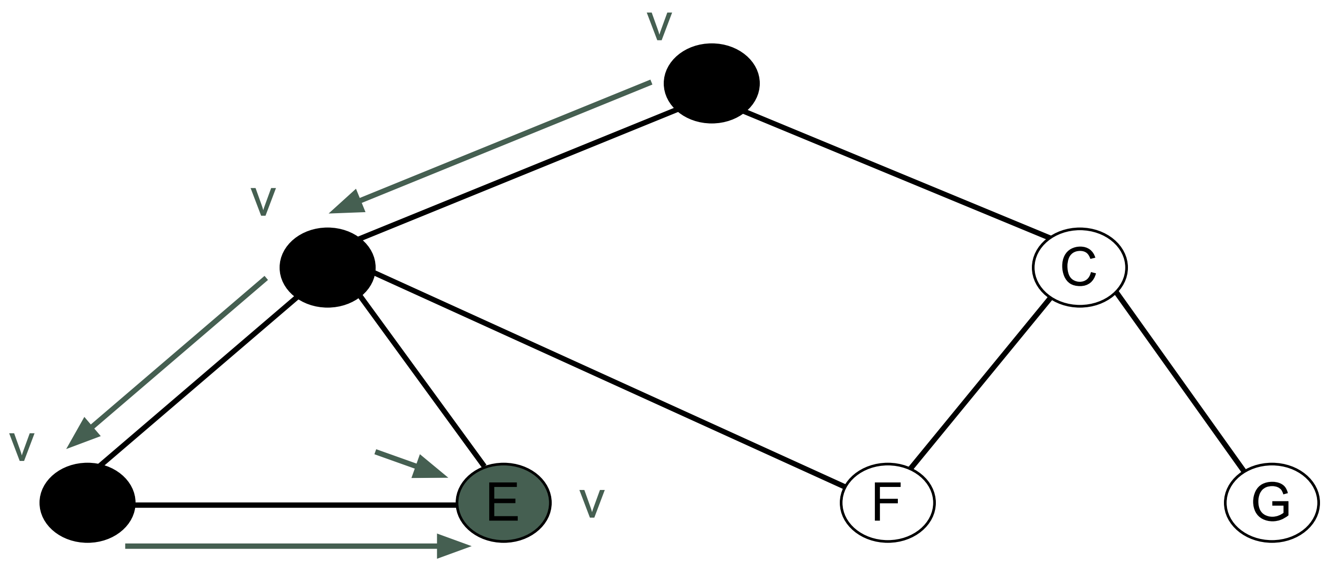
A B D

# Depth-First Search



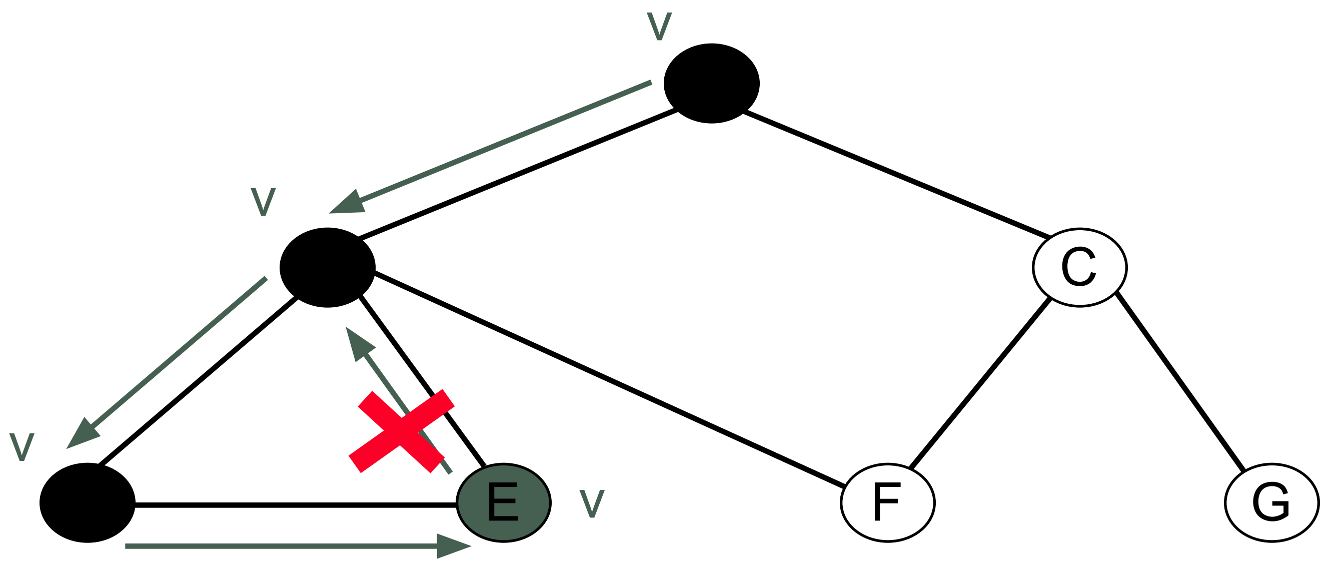
A B D

## Depth-First Search



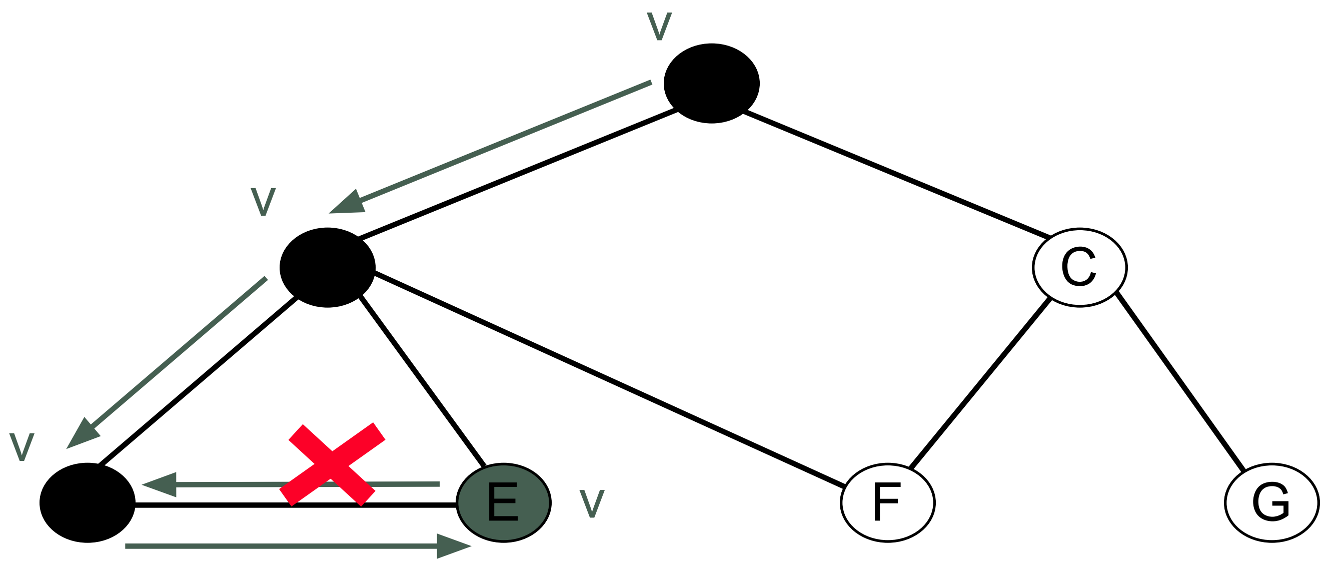
A B D E

# Depth-First Search



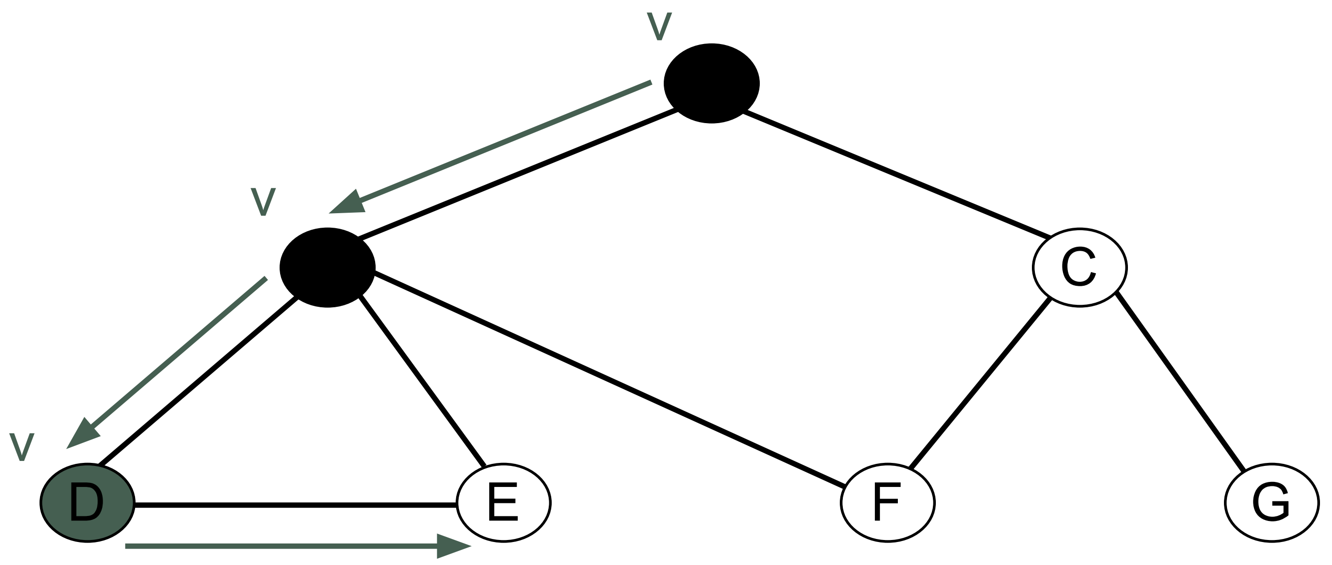
A B D E

## Depth-First Search



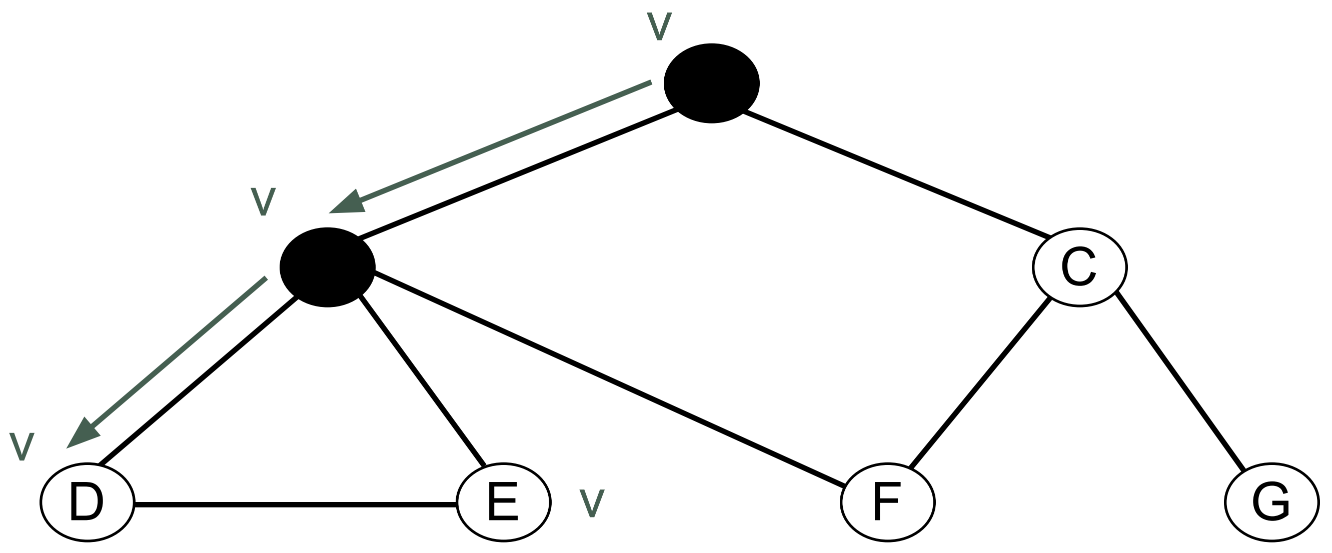
A B D E

## Depth-First Search



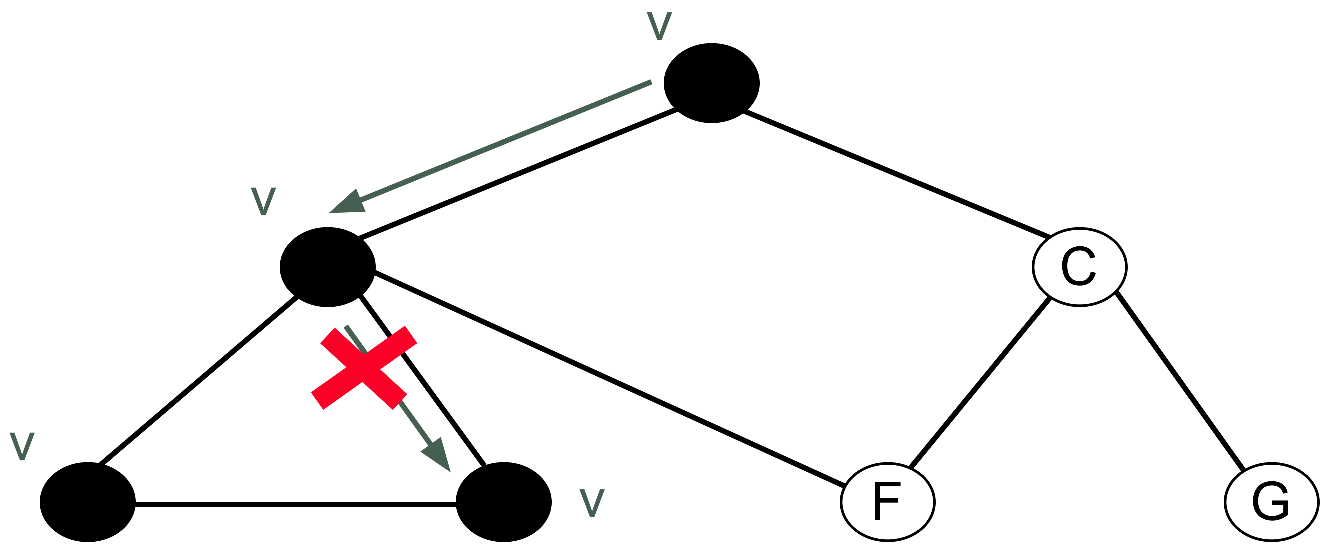
A B D E

## Depth-First Search



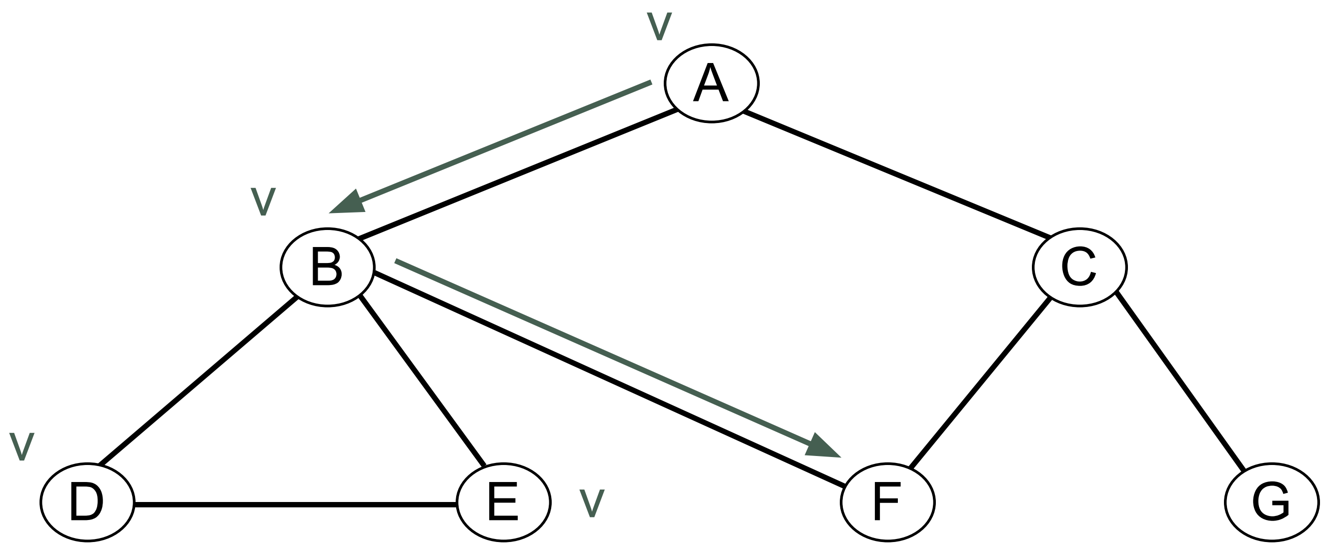
A B D E

## Depth-First Search



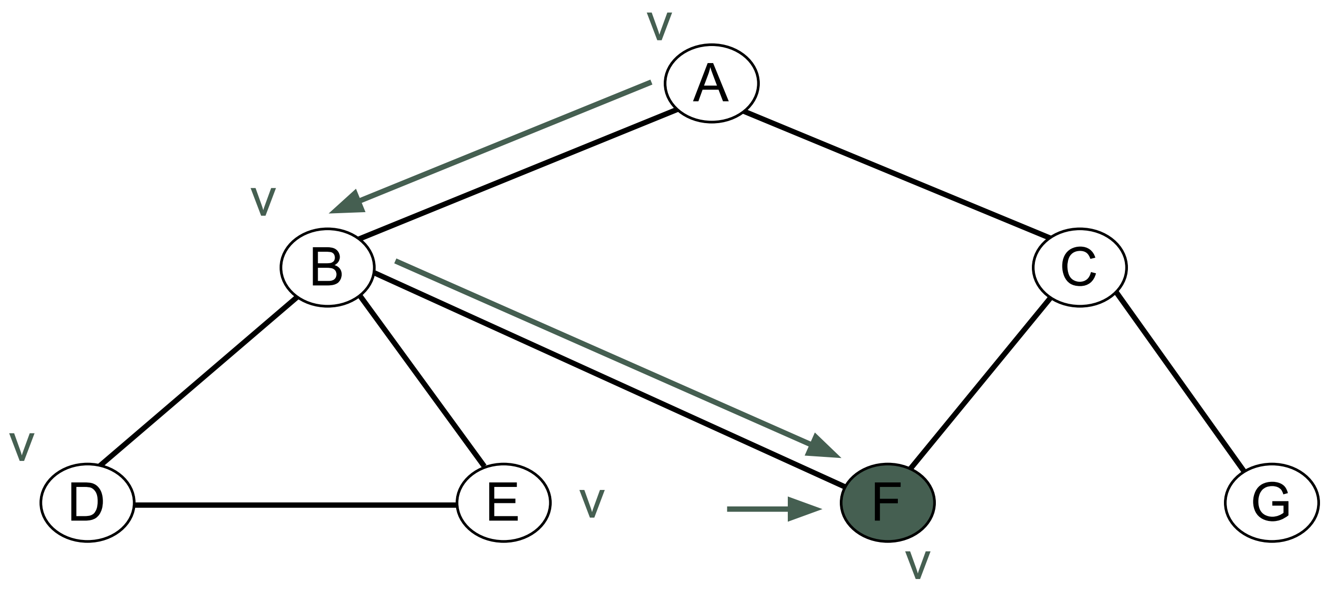
A B D E

## Depth-First Search



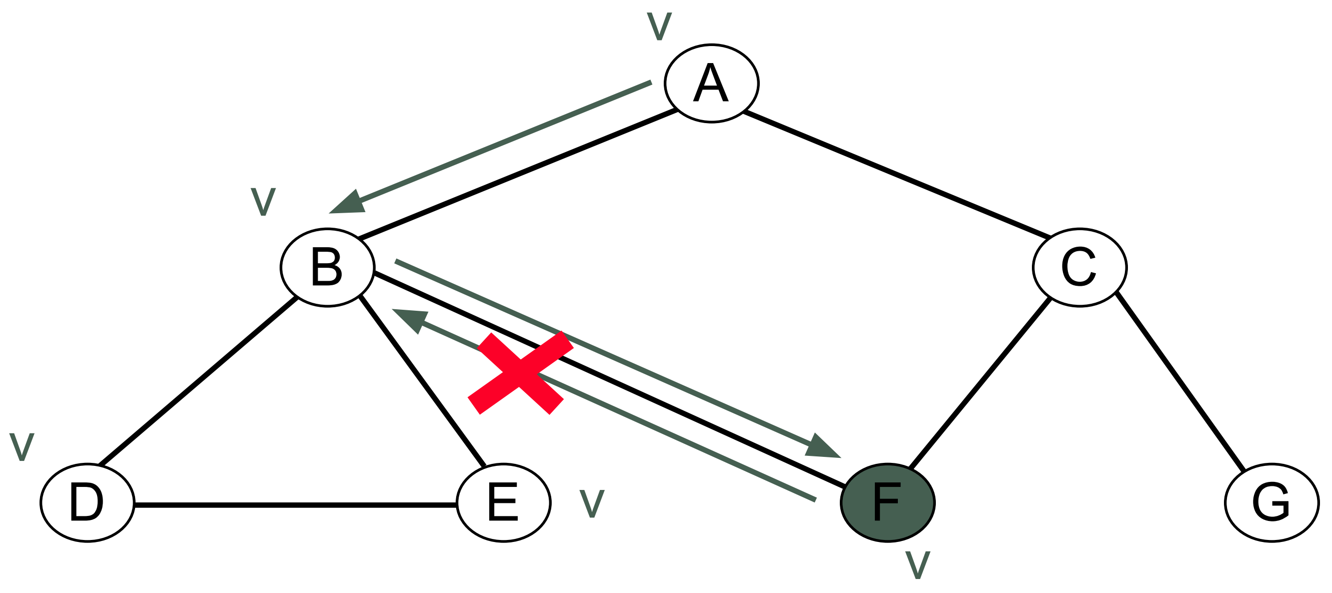
A B D E

## Depth-First Search



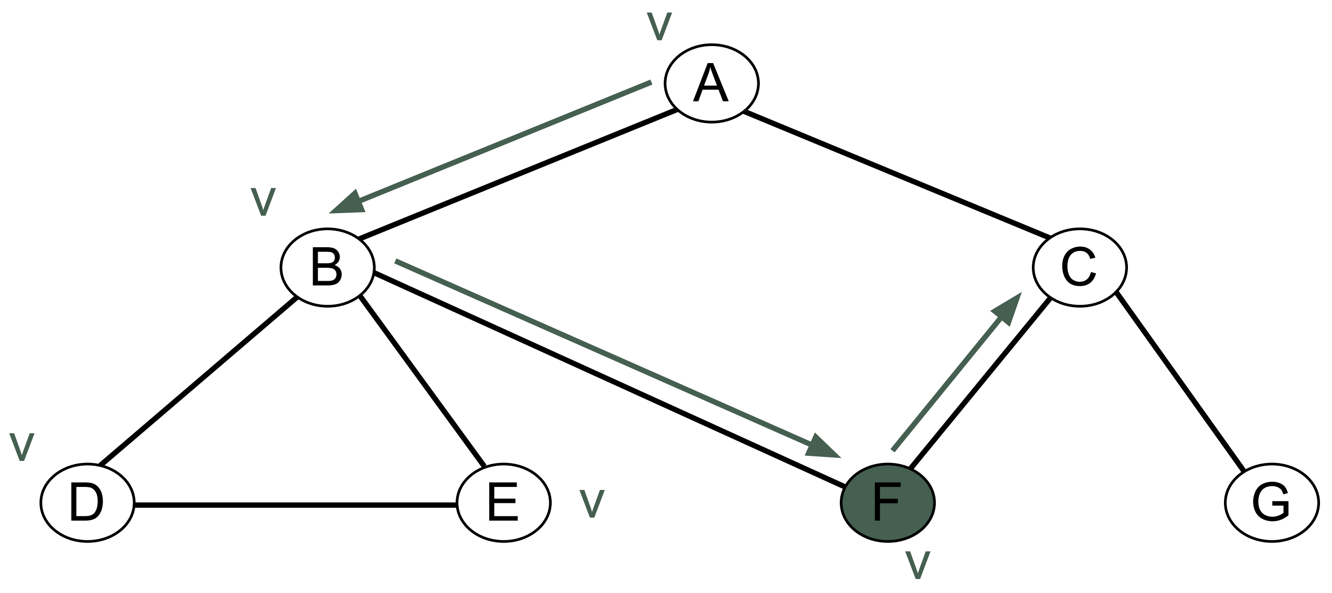
A B D E F

# Depth-First Search



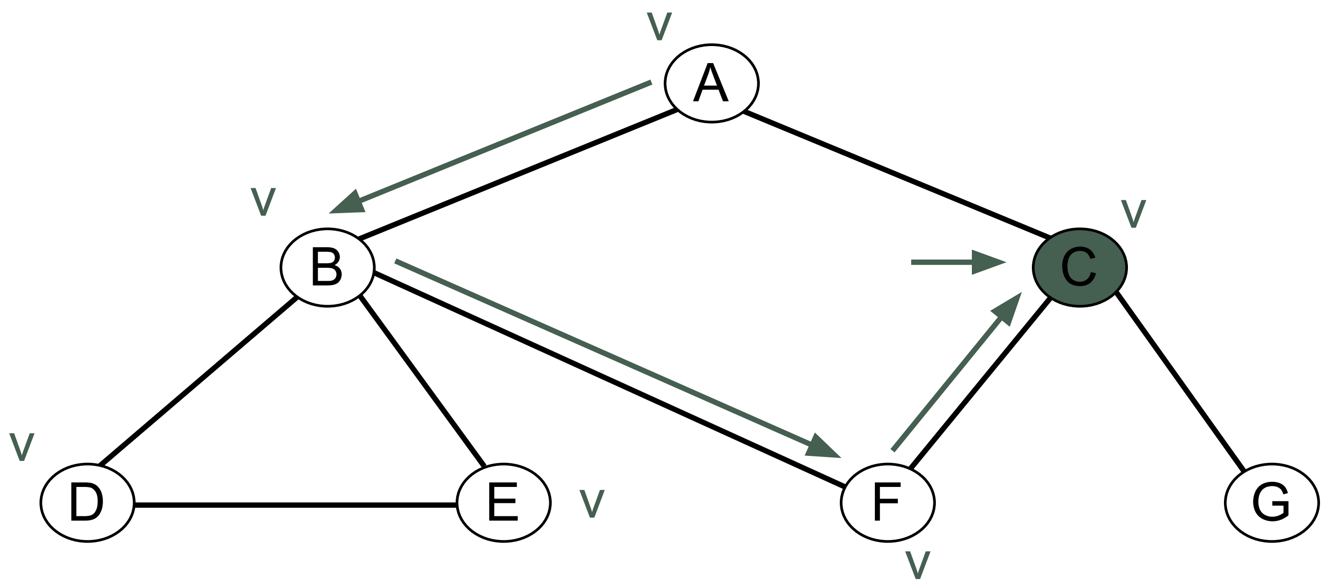
A B D E F

## Depth-First Search



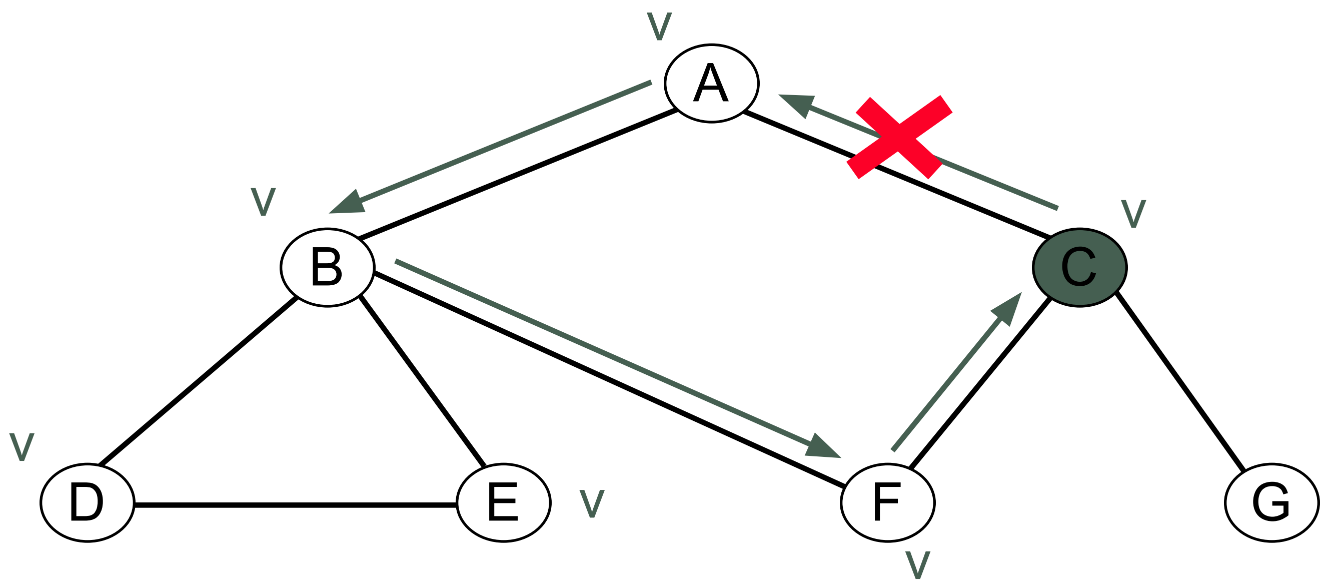
A B D E F

## Depth-First Search



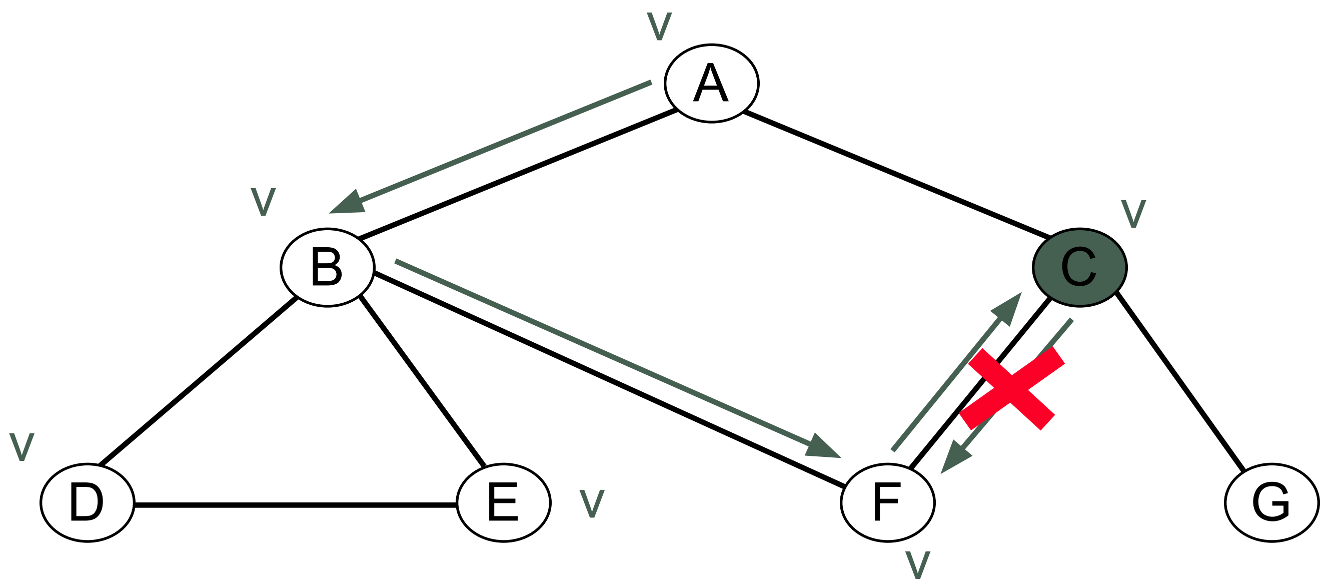
A B D E F C

## Depth-First Search



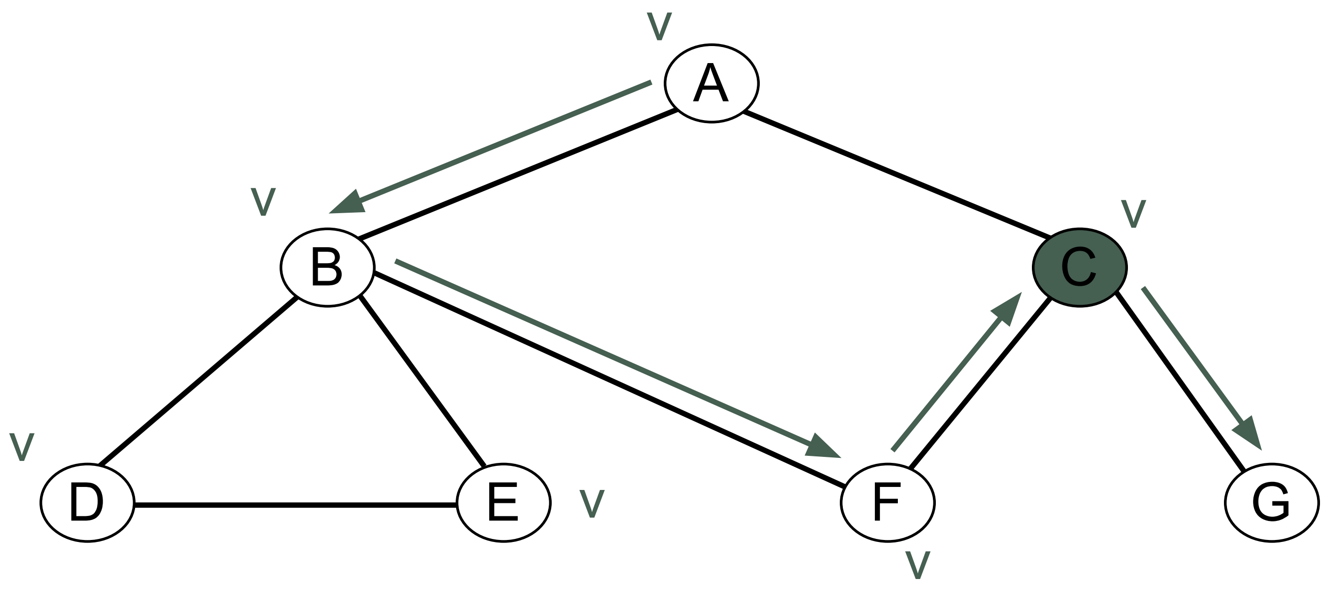
A B D E F C

## Depth-First Search



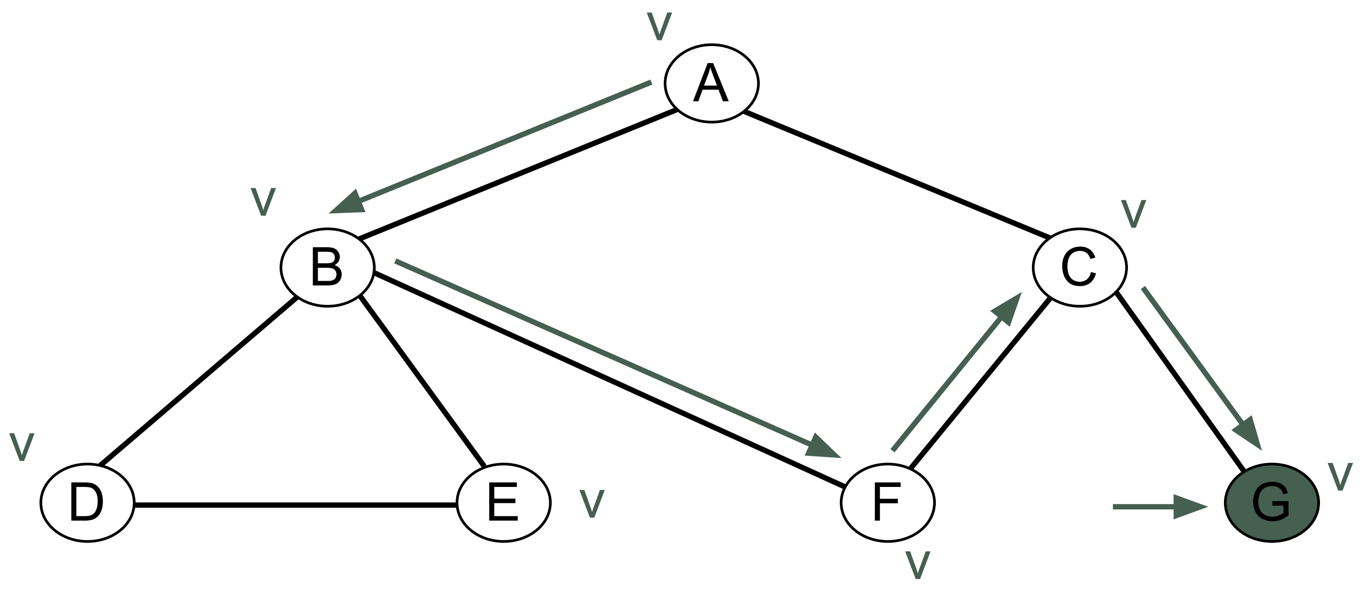
A B D E F C

## Depth-First Search



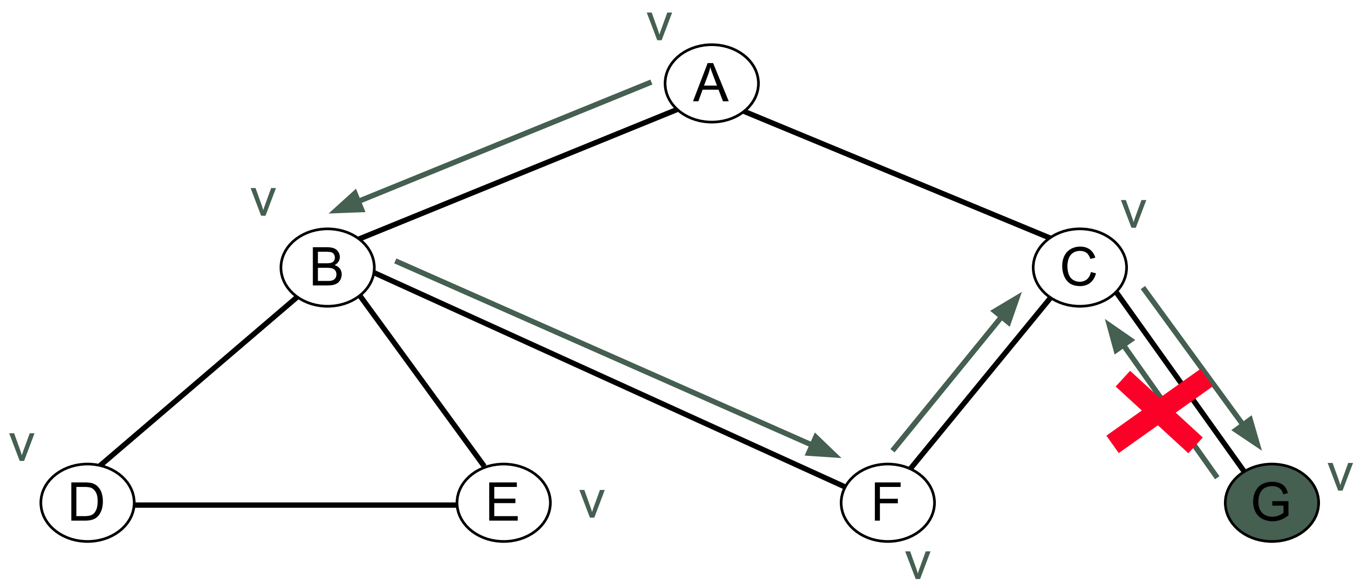
A B D E F C

## Depth-First Search



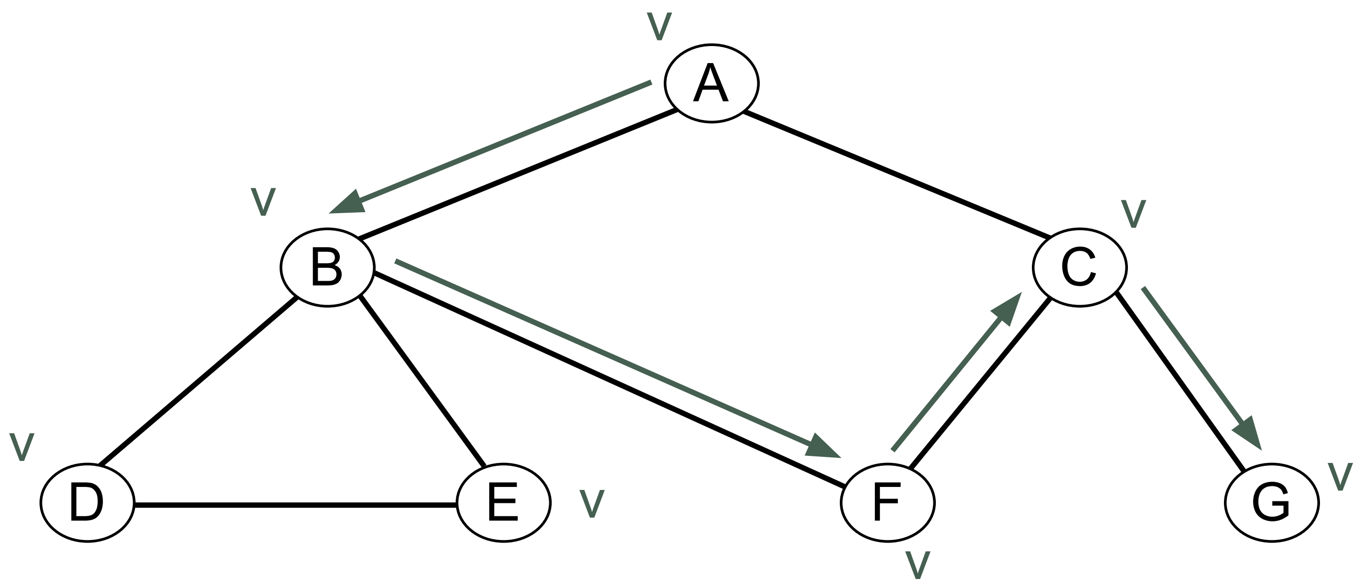
A B D E F C G

## Depth-First Search



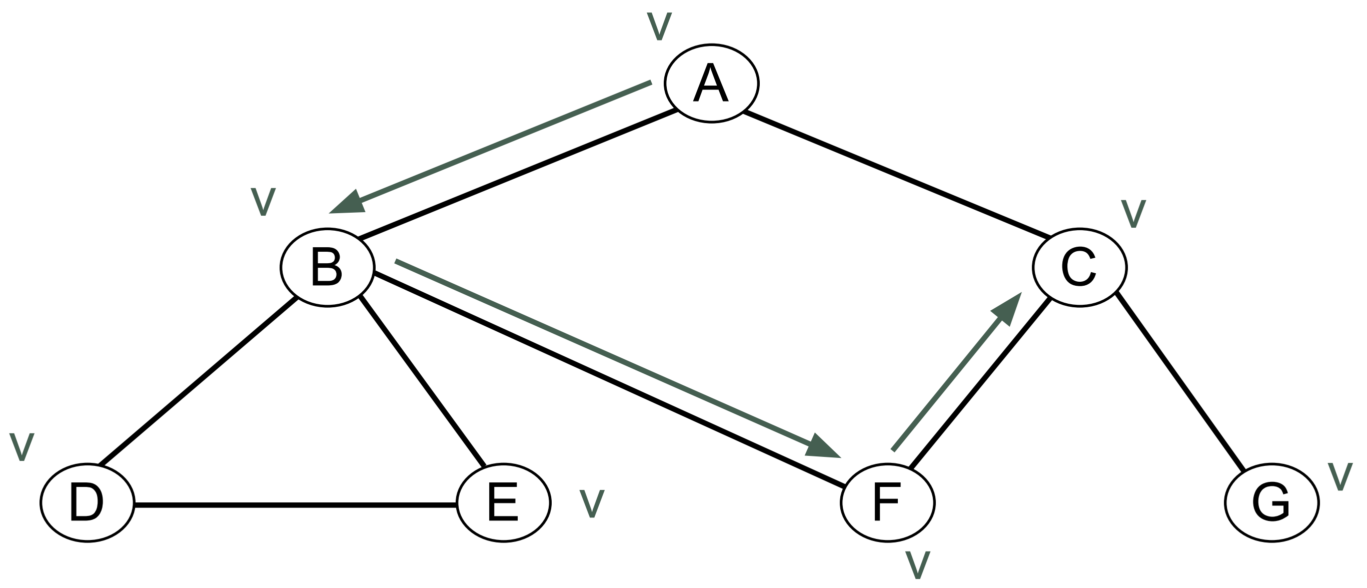
A B D E F C G

## Depth-First Search



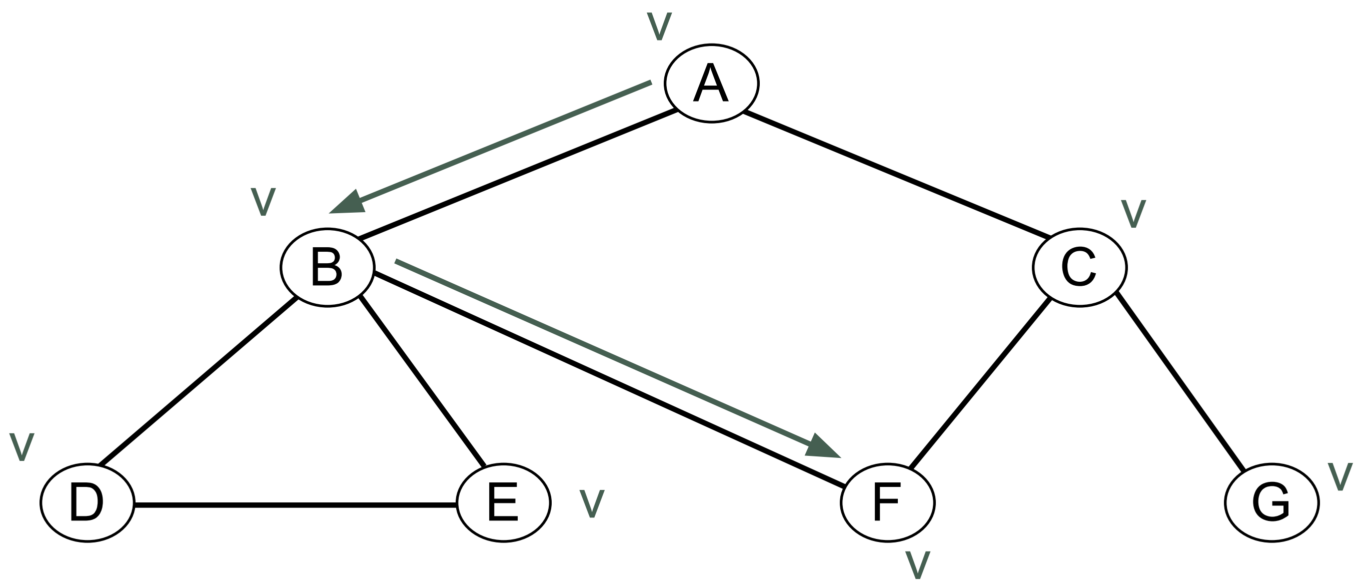
A B D E F C G

## Depth-First Search



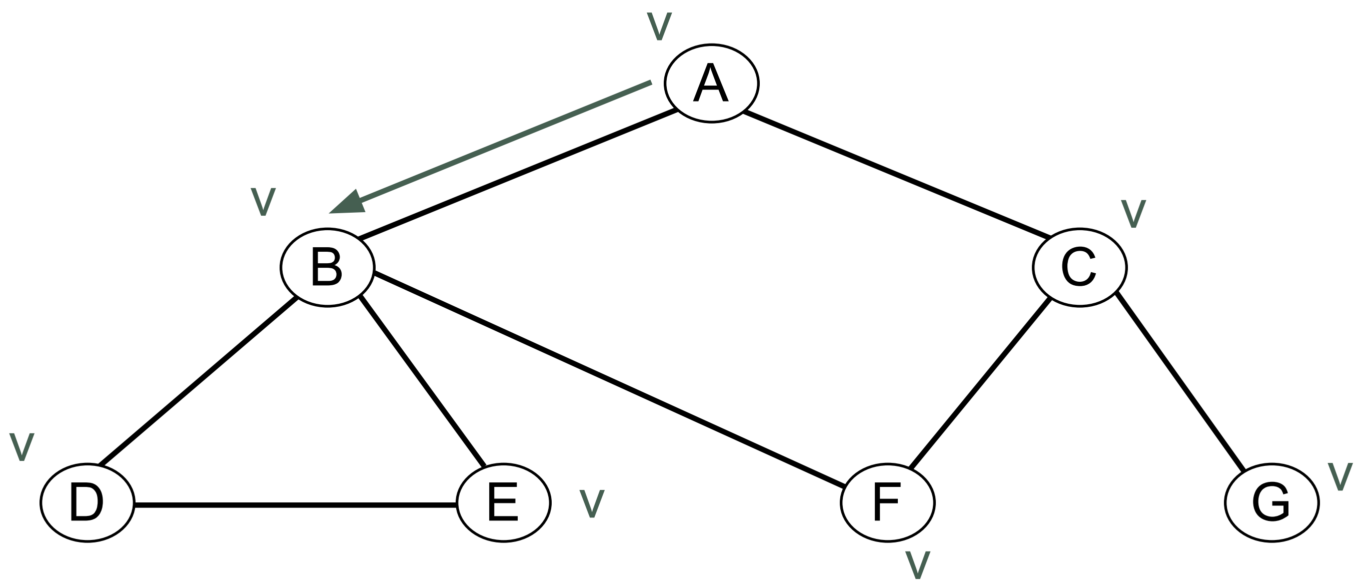
A B D E F C G

## Depth-First Search



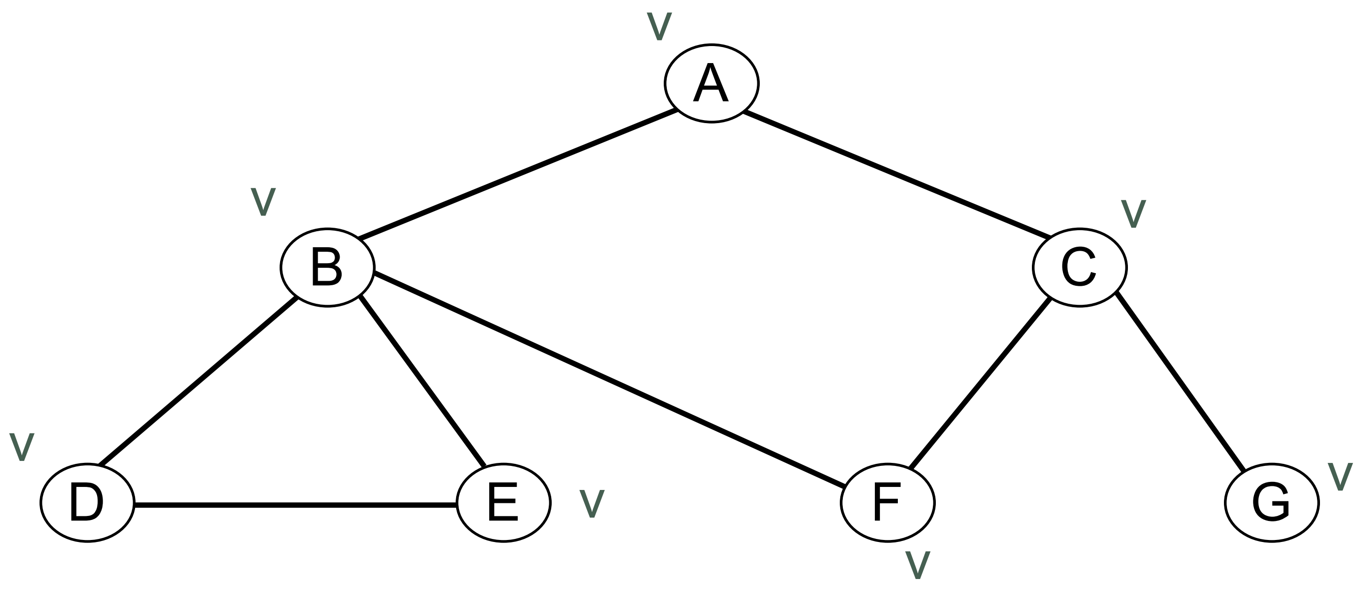
A B D E F C G

## Depth-First Search



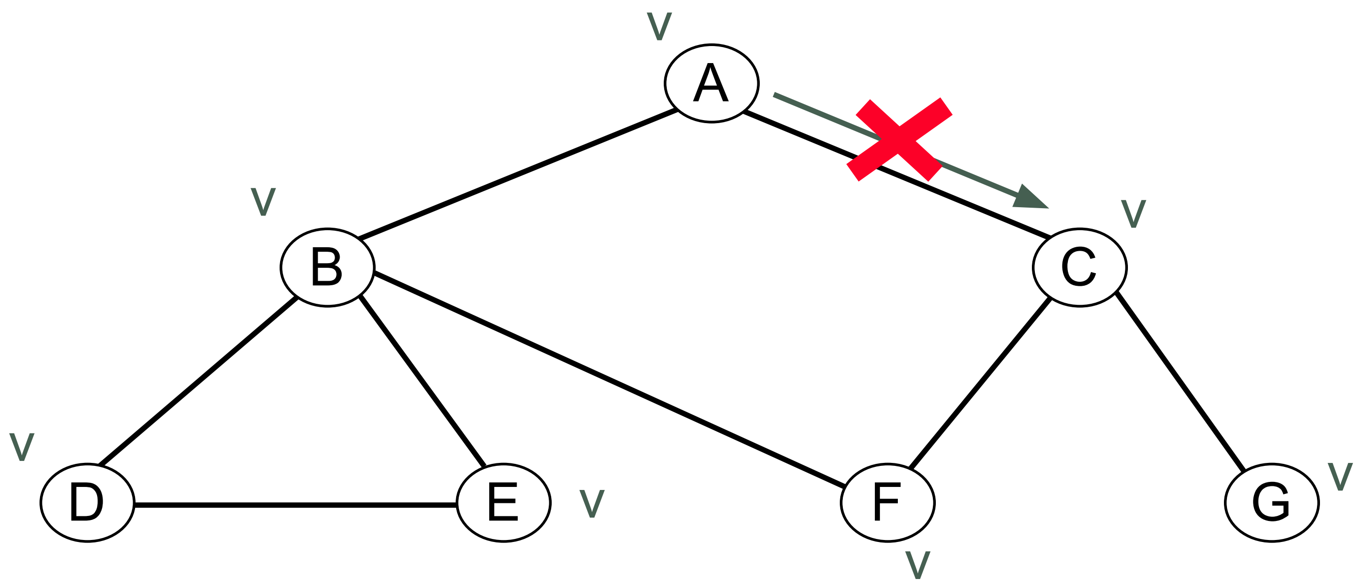
A B D E F C G

## Depth-First Search



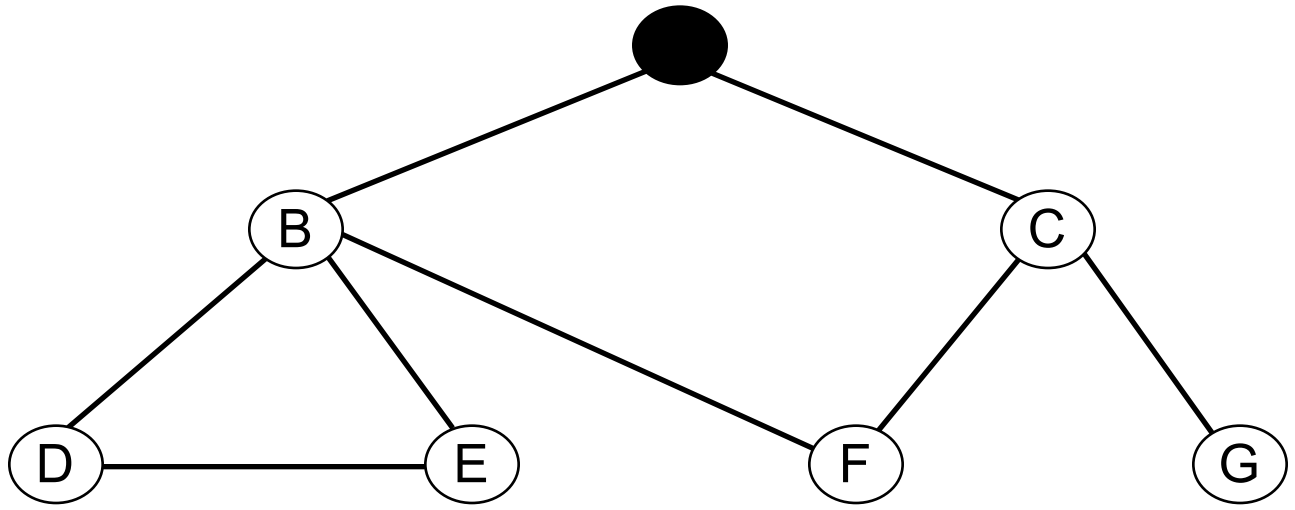
A B D E F C G

## Depth-First Search



A B D E F C G

## Depth-First Search



A B D E F C G

# Time and Space Complexity for Depth-First Search

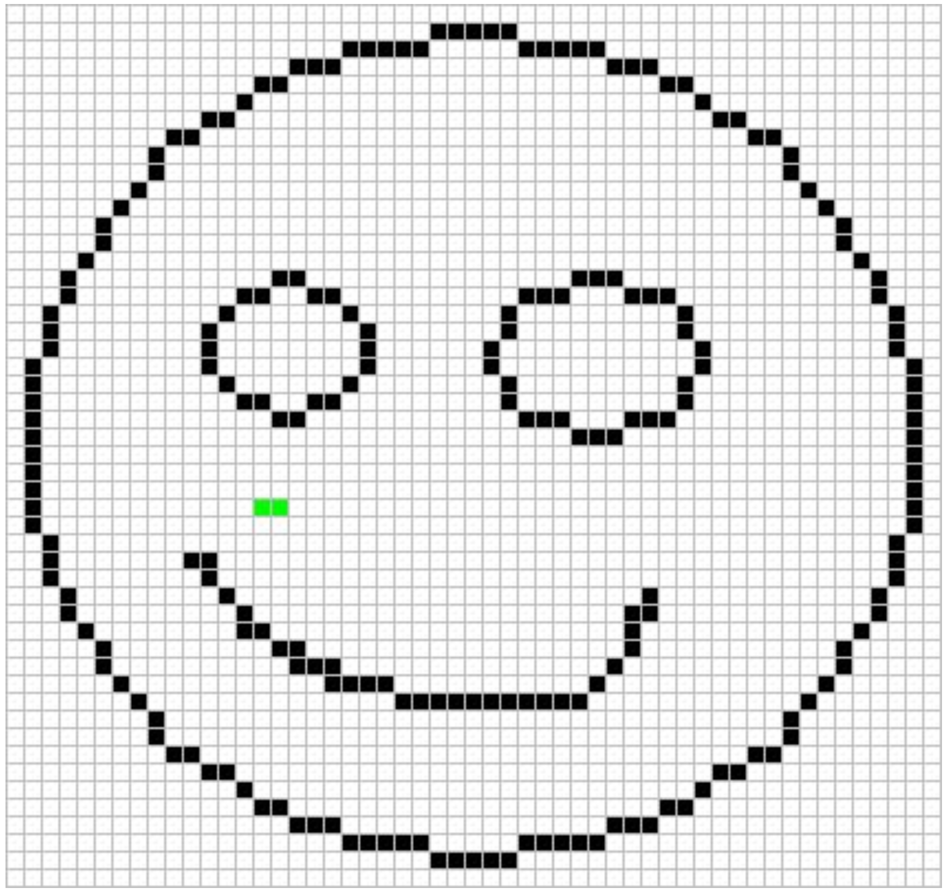
## Time Complexity

- Adjacency Lists
  - Each node is marked visited once
  - Each node is checked for each incoming edge
  - O (v + e)
- Adjacency Matrix
  - Have to check all entries in matrix: $O(n^2)$

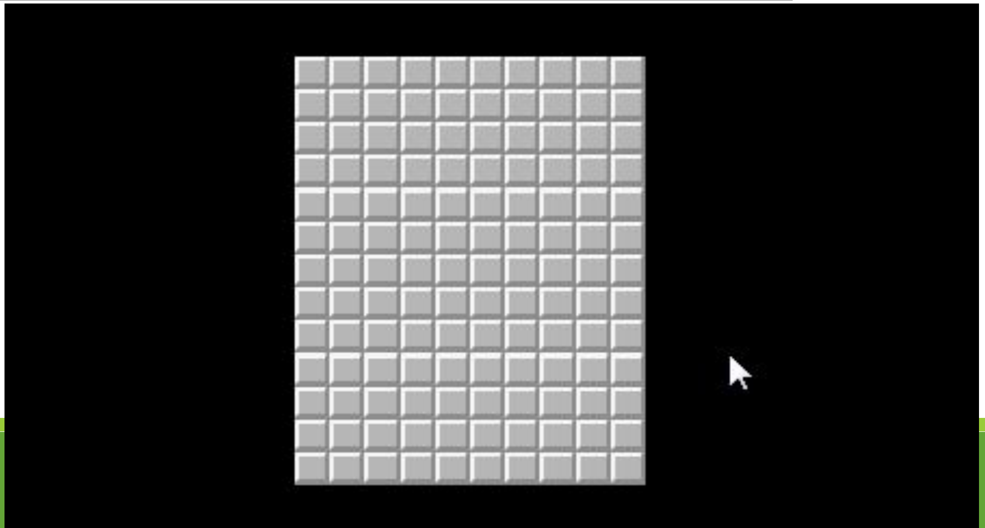
## Space Complexity

- Stack to handle nodes as they are explored
  - Worst case: all nodes put on stack (if graph is linear)
  - O(n)

## The “Flood Fill” Algorithm

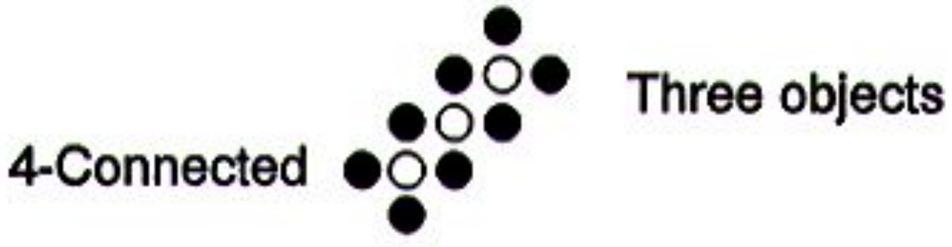


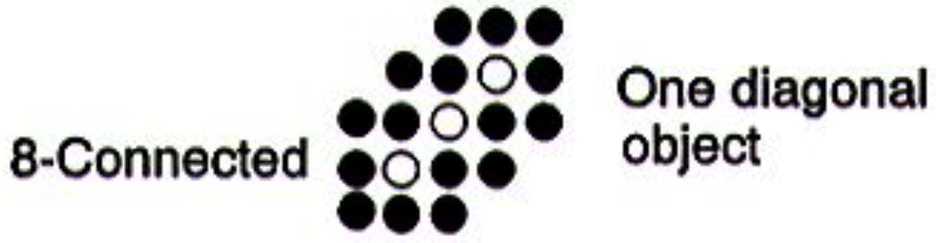
A RECURSIVE GRAPHICS ALGORITHM USED TO FILL IN IRREGULAR-SHAPED REGIONS WITH A SOLID COLOR

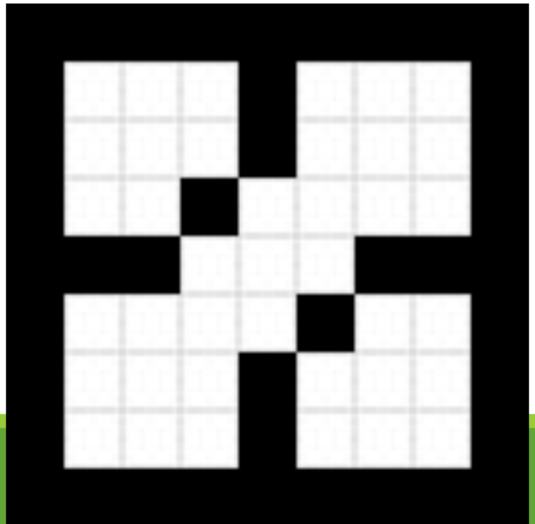


# A flood-fill algorithm

```
void fill( int x, int y, int interiorcolor, int newcolor )
{       //Check boundary crossing
        if ( get_pixel( x, y ) == interiorcolor )
                {
                put_pixel( x, y, newcolor );
                fill( x -1, y, interiorcolor, newcolor );
                fill( x+1, y, interiorcolor, newcolor );
                fill( x, y -1, interiorcolor, newcolor );
                fill( x, y+1, interiorcolor, newcolor );
                }
}
```







## A flood-fill algorithm

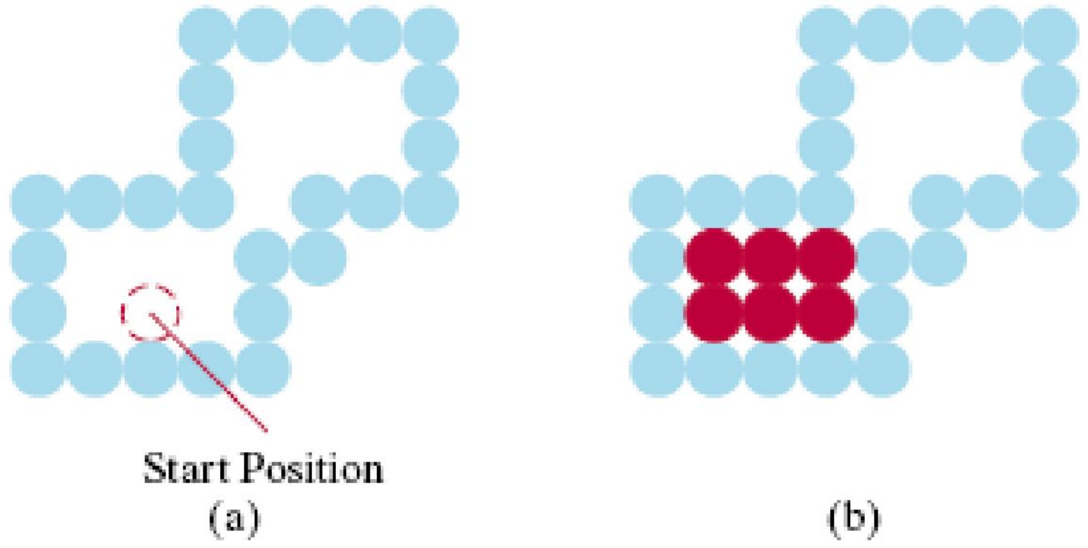
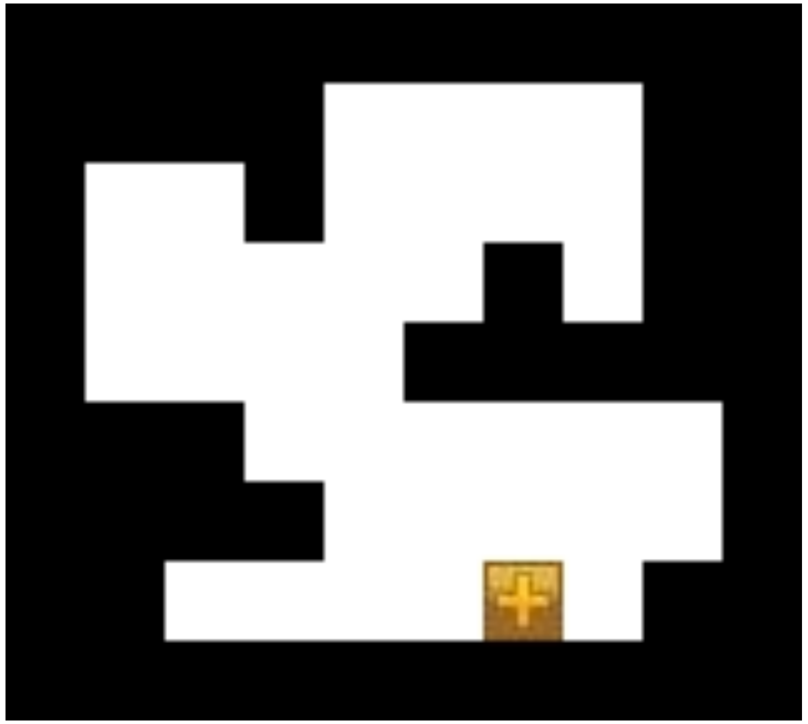


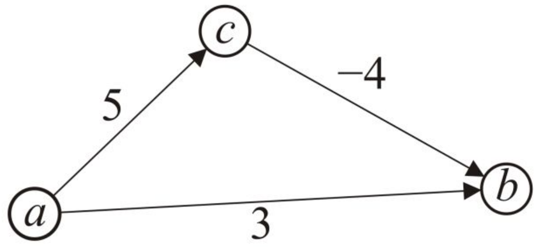
FIGURE 4–28 The area defined within the color boundary (a) is only partially filled in (b) using a 4-connected boundary-fill algorithm.



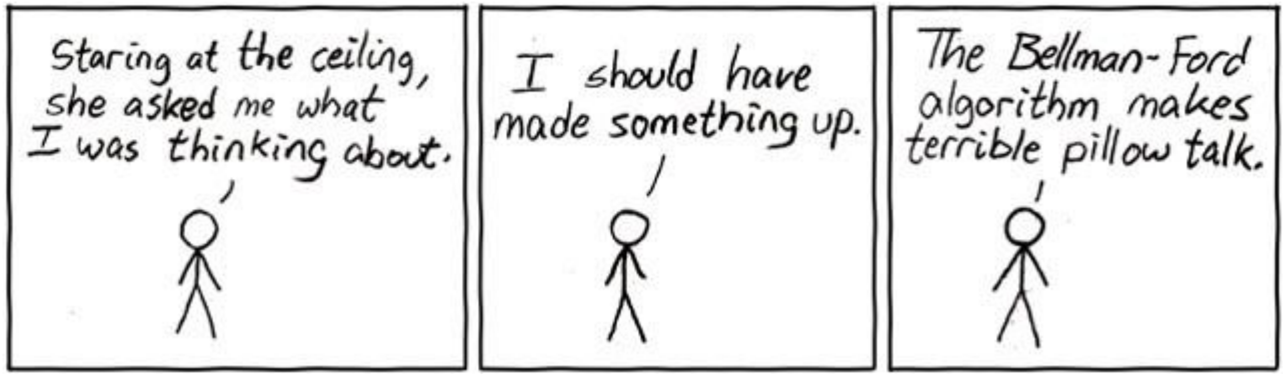
## Negative Weights

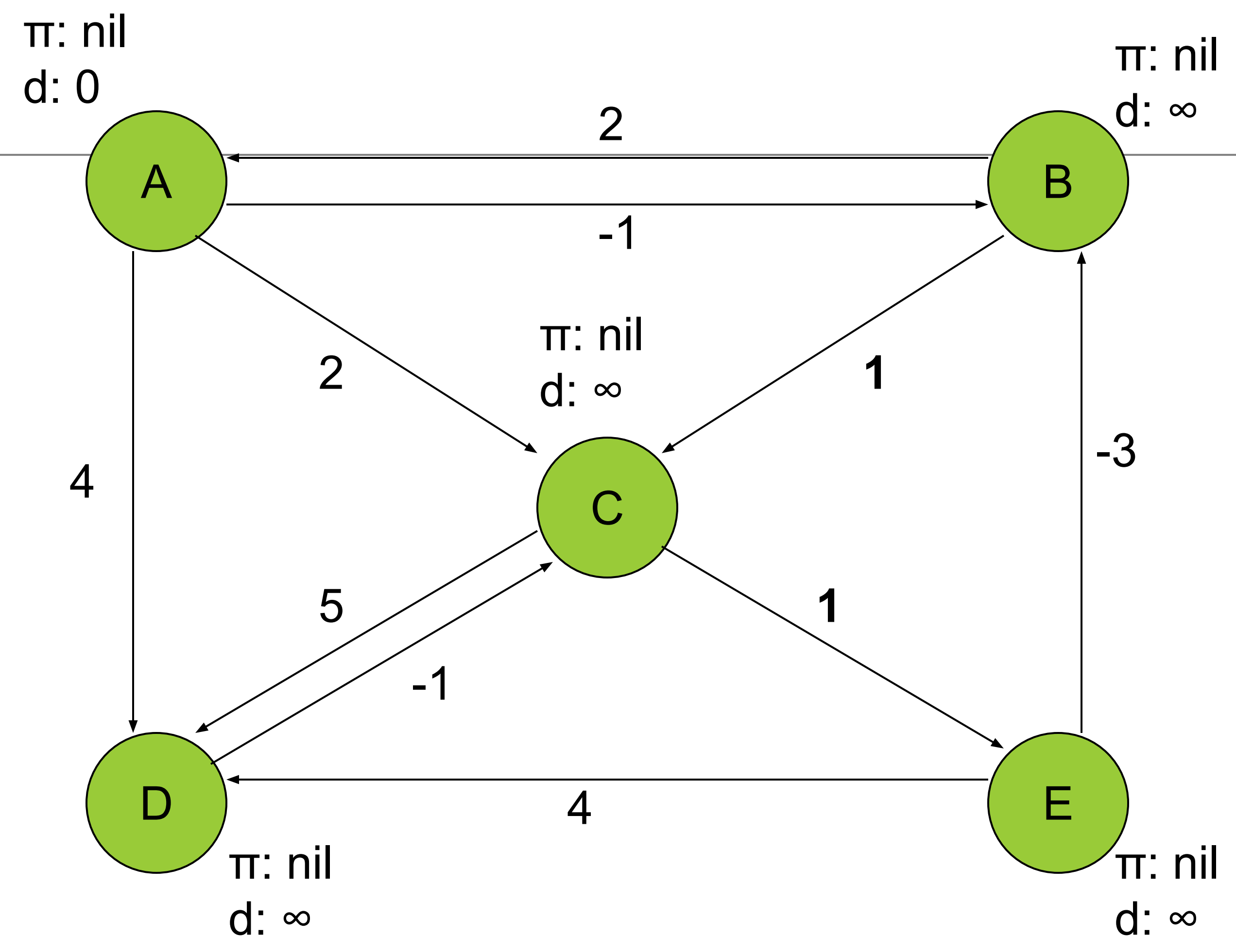
If some of the edges have negative weight, so long as there are no cycles with negative weight, the Bellman-Ford algorithm will find the minimum distance

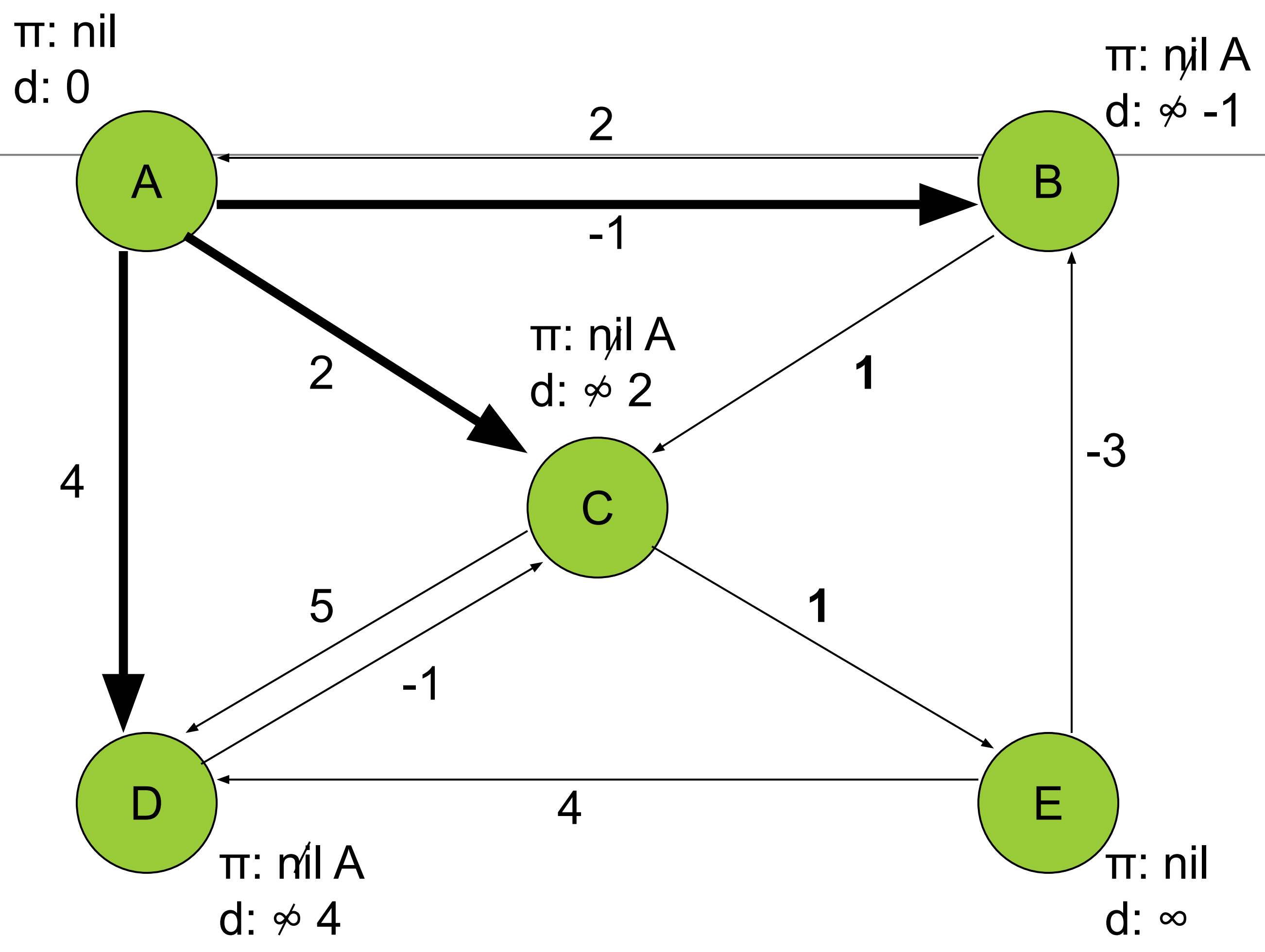
- It is slower than Dijkstra's algorithm

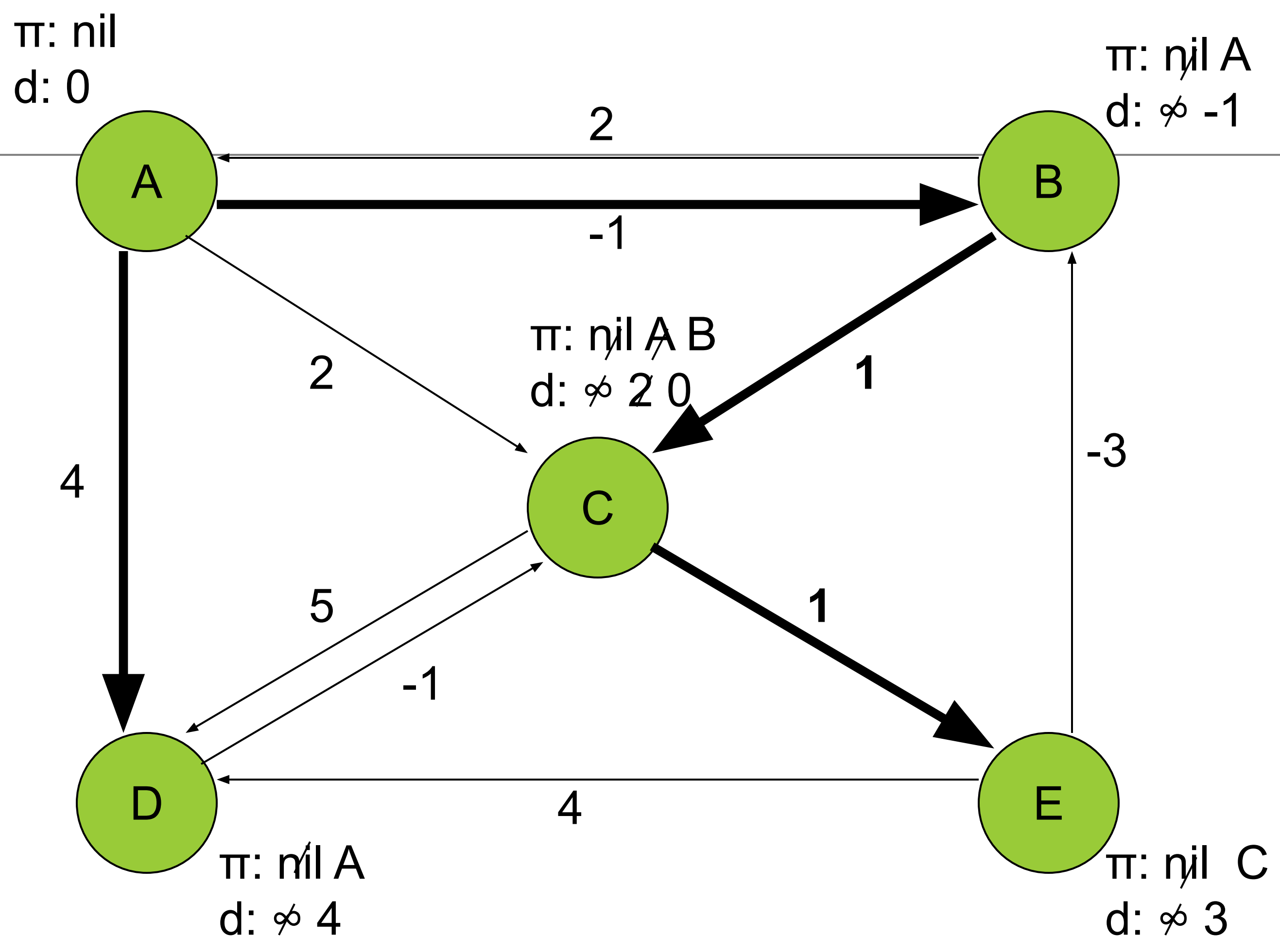


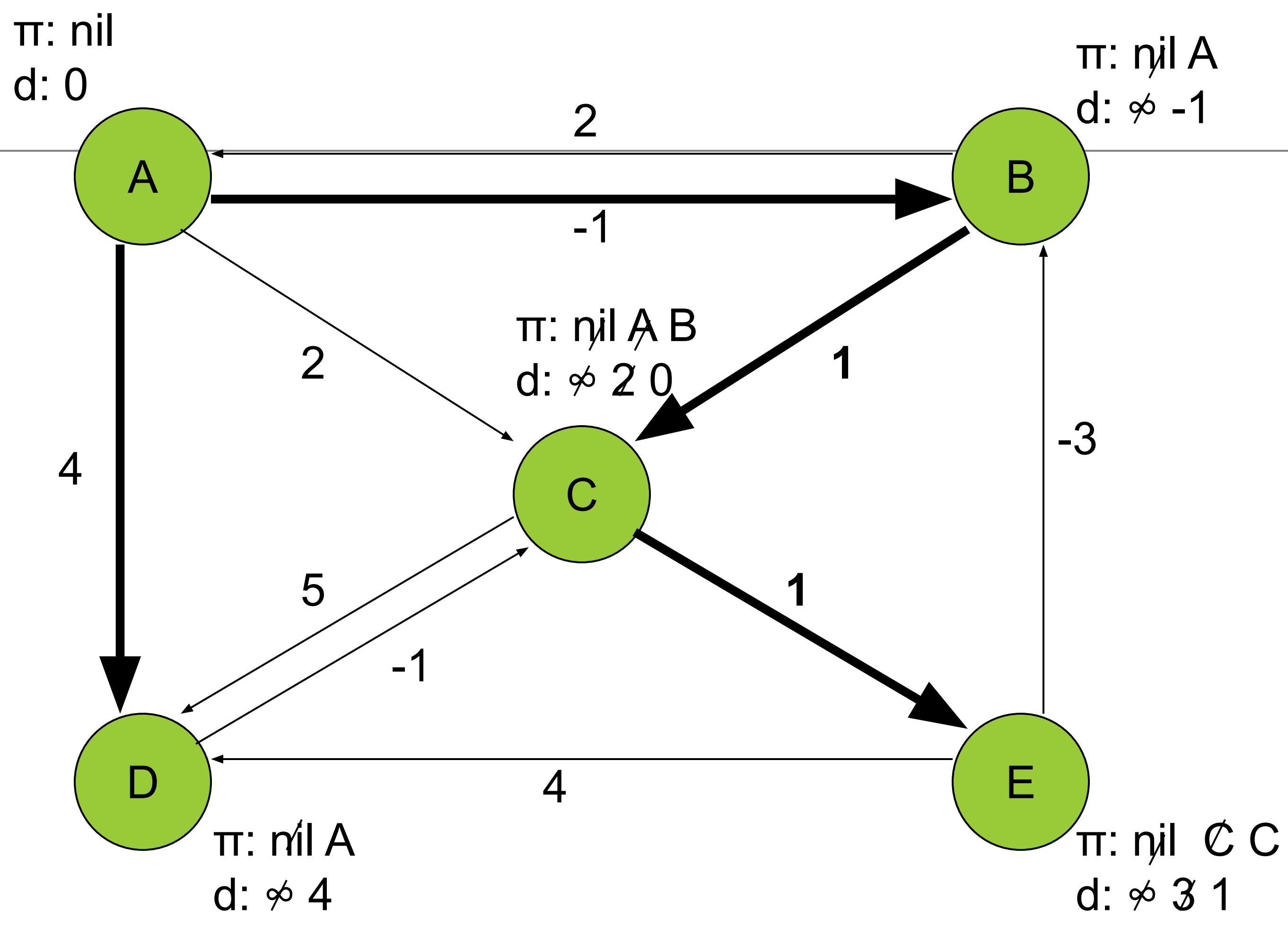
$$\begin{tikzcd} & c \arrow[dr, "-4"] & \\ a \arrow[ur, "5"] \arrow[rr, "3"'] & & b \end{tikzcd}$$

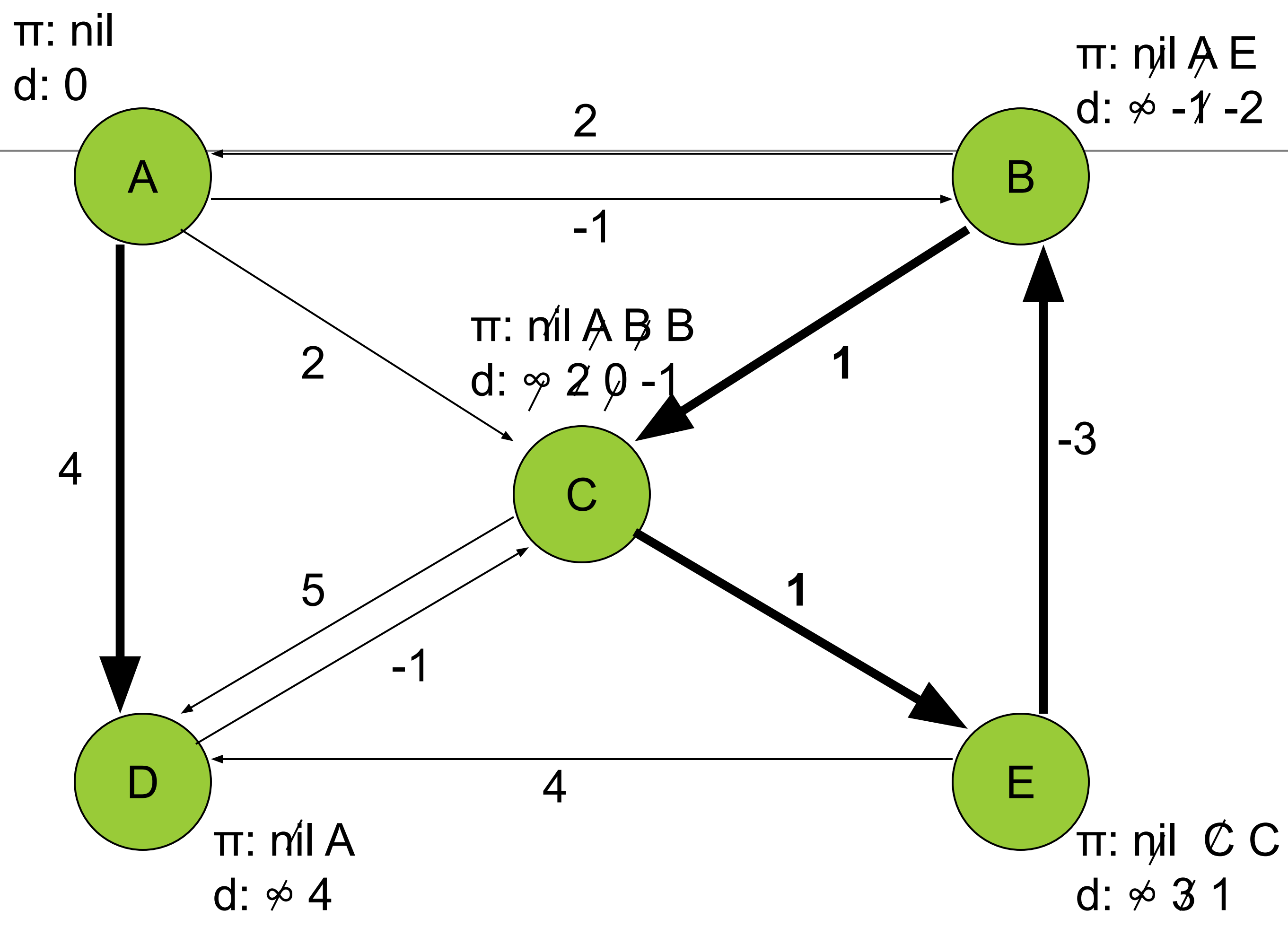

Staring at the ceiling, she asked me what I was thinking about.
I should have made something up.
The Bellman-Ford algorithm makes terrible pillow talk.


π: nil
d: 0
A
π: nil
d: ∞
B
π: nil
d: ∞
C
π: nil
d: ∞
D
π: nil
d: ∞
E
2
-1
2
1
4
-3
5
-1
1
4


π: nil
d: 0
π: ~~nil~~ A
d: ~~∞~~ -1
π: ~~nil~~ A
d: ~~∞~~ 2
π: ~~nil~~ A
d: ~~∞~~ 4
π: nil
d: ∞
A
B
C
D
E
2
-1
2
4
1
-3
5
-1
1
4


π: nil
d: 0
A
2
-1
π: ~~nil~~ A
d: ~~∞~~ -1
B
π: ~~nil~~ ~~A~~ B
d: ~~∞~~ ~~2~~ 0
2
1
C
4
-3
5
-1
1
D
4
E
π: ~~nil~~ A
d: ~~∞~~ 4
π: ~~nil~~ C
d: ~~∞~~ 3


π: nil
d: 0
π: ~~nil~~ A
d: ~~∞~~ -1
π: ~~nil~~ ~~A~~ B
d: ~~∞~~ ~~2~~ 0
π: ~~nil~~ A
d: ~~∞~~ 4
π: ~~nil~~ ~~C~~ C
d: ~~∞~~ ~~3~~ 1
A
B
C
D
E
2
-1
2
1
4
-3
5
-1
1
4


π: nil
d: 0
A
2
-1
B
π: ~~nil~~ ~~A~~ E
d: ~~∞~~ ~~-1~~ -2
2
1
π: ~~nil~~ ~~A~~ ~~B~~ B
d: ~~∞~~ ~~2~~ ~~0~~ -1
4
C
-3
5
1
-1
D
4
E
π: ~~nil~~ A
d: ~~∞~~ 4
π: ~~nil~~ ~~C~~ C
d: ~~∞~~ ~~3~~ 1

## The Bellman-Ford Algorithm

```
Bellman-Ford(G, w, s)
1.  Initialize-Single-Source(G, s)
2.  for i := 1 to |V| - 1 do
3.      for each edge (u, v) ∈ E do
4.          Relax(u, v, w)
5.  for each vertex v ∈ u.adj do
6.      if d[v] > d[u] + w(u, v)
7.          then return False    // there is a negative cycle
8.  return True
```

```
Relax(u, v, w)
if d[v] > d[u] + w(u, v)
   then  d[v] := d[u] + w(u, v)
         parent[v] := u
```

## Time Complexity

Bellman-Ford(G, w, s)

```
1.  Initialize-Single-Source(G, s)
2.  for i := 1 to |V| - 1 do                  ——————→ O(|V|)
3.      for each edge (u, v) ∈ E do
4.          Relax(u, v, w)                    ——————→ O(|V||E|)
5.  for each vertex v ∈ u.adj do              ——————→ O(|E|)
6.      if d[v] > d[u] + w(u, v)
7.          then return False    // there is a negative cycle
8.  return True
```

Time complexity: O(|V||E|)