

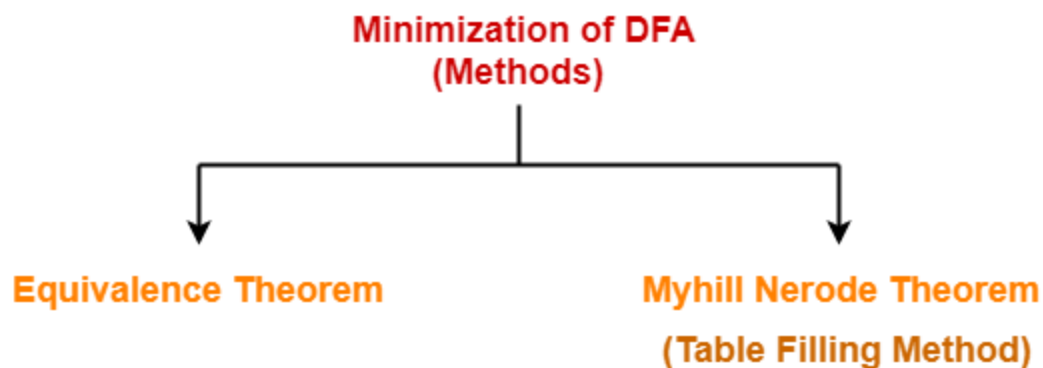
Minimization of DFA-

The process of reducing a given DFA to its minimal form is called as minimization of DFA.

- It contains the minimum number of states.
- The DFA in its minimal form is called as a **Minimal DFA**.

How To Minimize DFA?

The two popular methods for minimizing a DFA are-



In this article, we will discuss Minimization of DFA Using Equivalence Theorem.

Minimization of DFA Using Equivalence Theorem-

Step-01:

- Eliminate all the dead states and inaccessible states from the given DFA (if any).

Dead State

All those non-final states which transit to itself for all input symbols in Σ are called as dead states.

Inaccessible State

All those states which can never be reached from the initial state are called as inaccessible states.

Step-02:

- Draw a state transition table for the given DFA.
- Transition table shows the transition of all states on all input symbols in Σ .

Step-03:

Now, start applying equivalence theorem.

- Take a counter variable k and initialize it with value 0.
- Divide Q (set of states) into two sets such that one set contains all the non-final states and other set contains all the final states.
- This partition is called P_0 .

Step-04:

- Increment k by 1.
- Find P_k by partitioning the different sets of P_{k-1} .
- In each set of P_{k-1} , consider all the possible pair of states within each set and if the two states are distinguishable, partition the set into different sets in P_k .

Two states q_1 and q_2 are distinguishable in partition P_k for any input symbol 'a',
if $\delta(q_1, a)$ and $\delta(q_2, a)$ are in different sets in partition P_{k-1} .

Step-05:

- Repeat step-04 until no change in partition occurs.
- In other words, when you find $P_k = P_{k-1}$, stop.

Step-06:

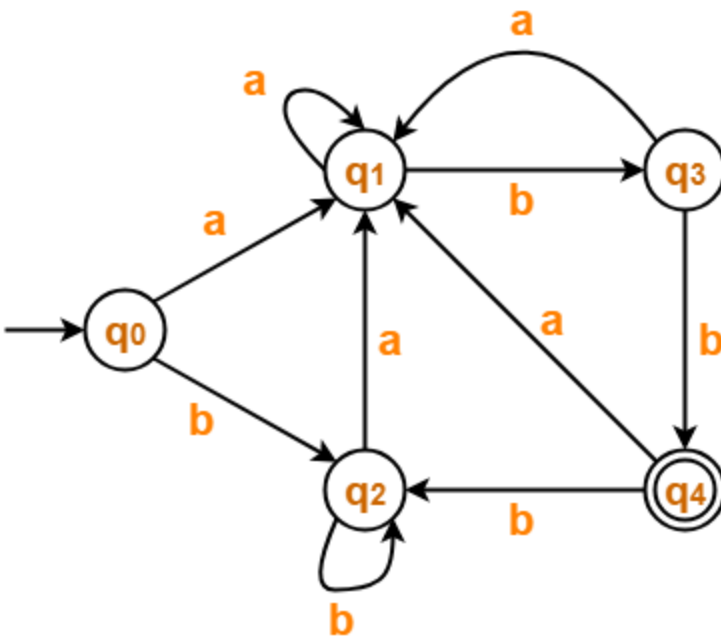
- All those states which belong to the same set are equivalent.
- The equivalent states are merged to form a single state in the minimal DFA.

Number of states in Minimal DFA
= Number of sets in P_k

PRACTICE PROBLEMS BASED ON MINIMIZATION OF DFA-

Problem-01:

Minimize the given DFA-



Solution-

Step-01:

The given DFA contains no dead states and inaccessible states.

Step-02:

Draw a state transition table-

	a	b
→q0	q1	q2
q1	q1	q3
q2	q1	q2
q3	q1	*q4
*q4	q1	q2

Step-03:

Now using Equivalence Theorem, we have-

$$P_0 = \{ q_0, q_1, q_2, q_3 \} \{ q_4 \}$$

$$P_1 = \{ q_0, q_1, q_2 \} \{ q_3 \} \{ q_4 \}$$

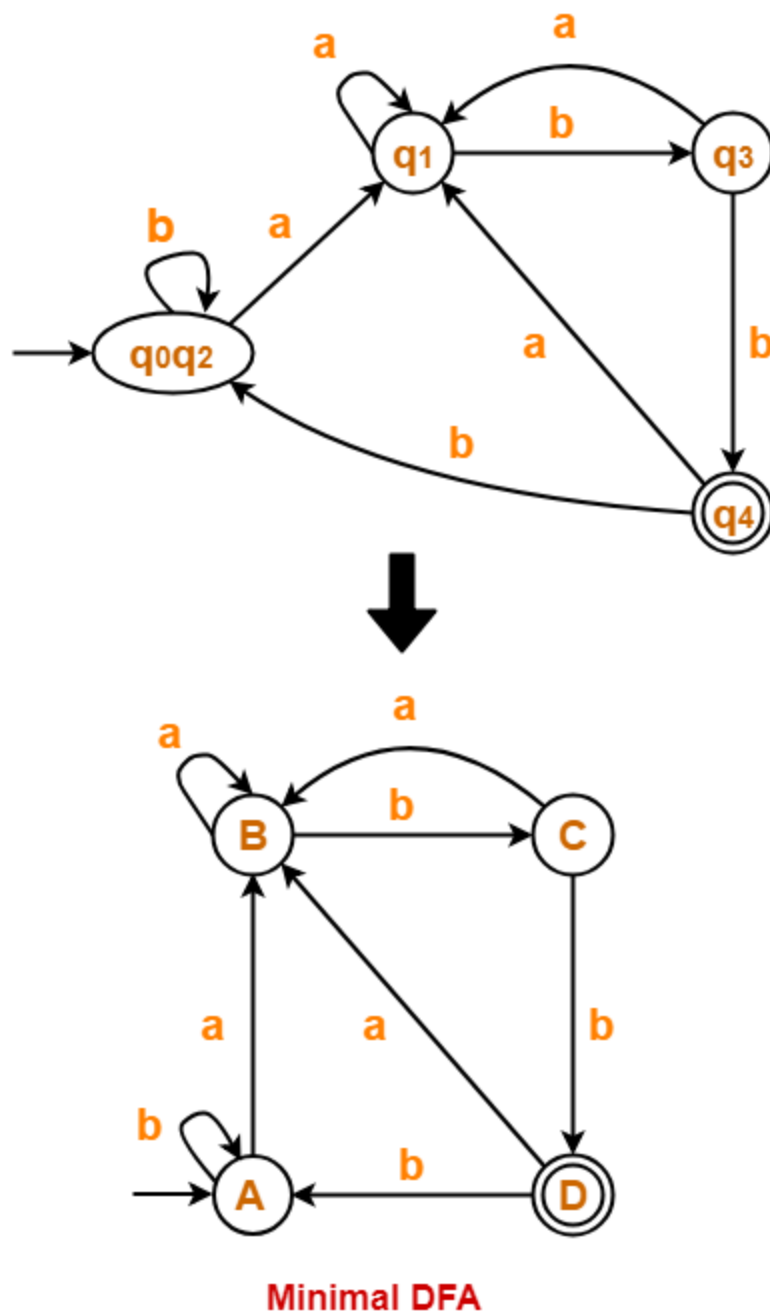
$$P_2 = \{ q_0, q_2 \} \{ q_1 \} \{ q_3 \} \{ q_4 \}$$

$$P_3 = \{ q_0, q_2 \} \{ q_1 \} \{ q_3 \} \{ q_4 \}$$

Since $P_3 = P_2$, so we stop.

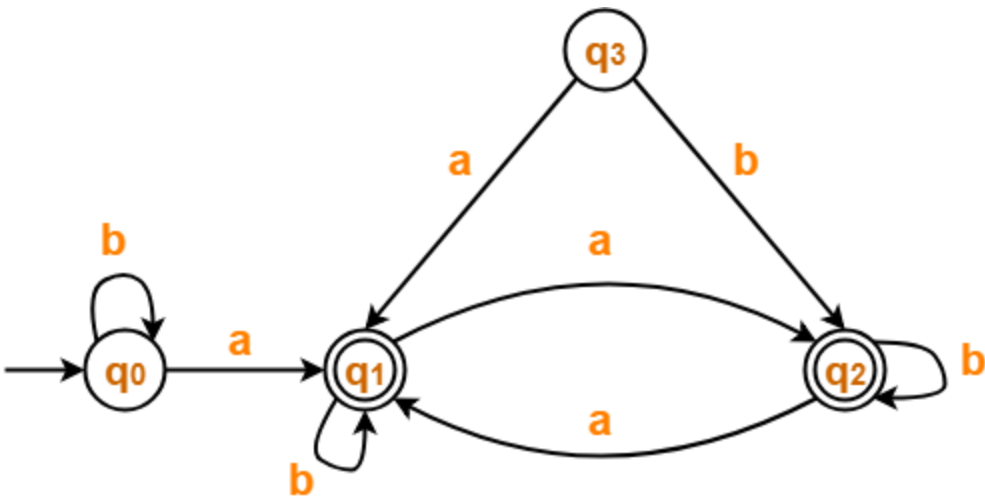
From P_3 , we infer that states q_0 and q_2 are equivalent and can be merged together.

So, Our minimal DFA is-



Problem-02:

Minimize the given DFA-

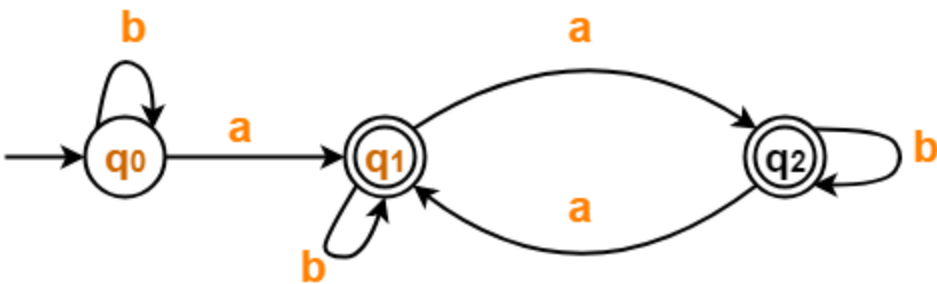


Solution-

Step-01:

- State q_3 is inaccessible from the initial state.
- So, we eliminate it and its associated edges from the DFA.

The resulting DFA is-



Step-02:

Draw a state transition table-

	a	b
→q0	*q1	q0
*q1	*q2	*q1
*q2	*q1	*q2

Step-03:

Now using Equivalence Theorem, we have-

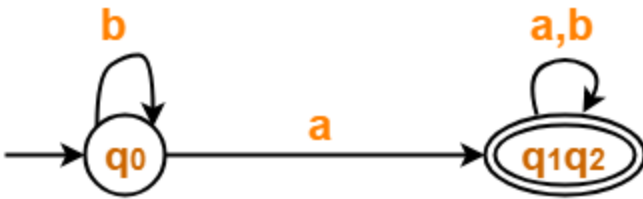
$$P_0 = \{ q_0 \} \{ q_1, q_2 \}$$

$$P_1 = \{ q_0 \} \{ q_1, q_2 \}$$

Since $P_1 = P_0$, so we stop.

From P_1 , we infer that states q_1 and q_2 are equivalent and can be merged together.

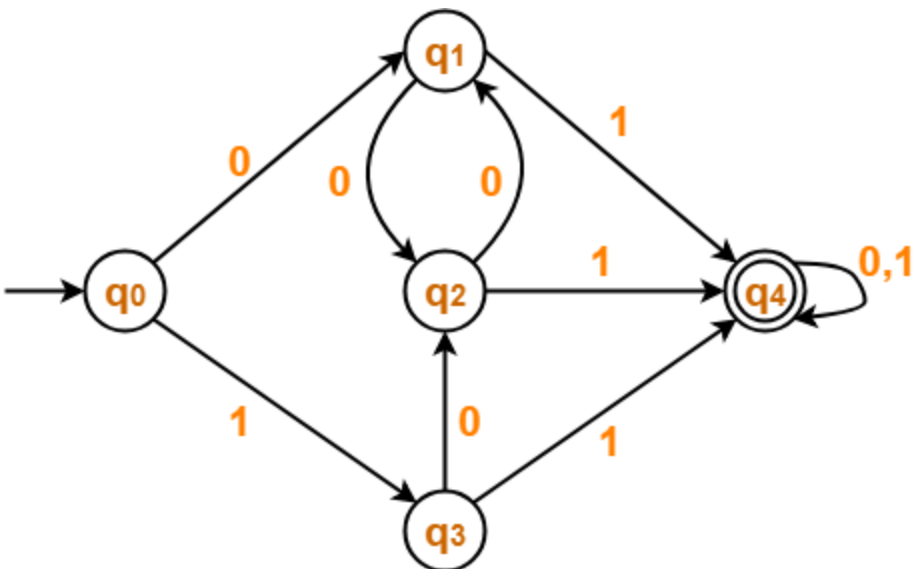
So, Our minimal DFA is-



Minimal DFA

Problem-03:

Minimize the given DFA-



Solution-

Step-01:

The given DFA contains no dead states and inaccessible states.

Step-02:

Draw a state transition table-

	0	1
→q0	q1	q3
q1	q2	*q4
q2	q1	*q4
q3	q2	*q4
*q4	*q4	*q4

Step-03:

Now using Equivalence Theorem, we have-

$$P_0 = \{ q_0, q_1, q_2, q_3 \} \{ q_4 \}$$

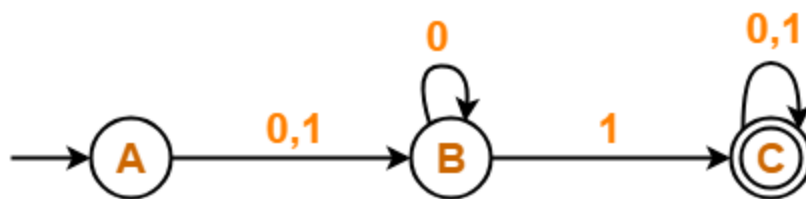
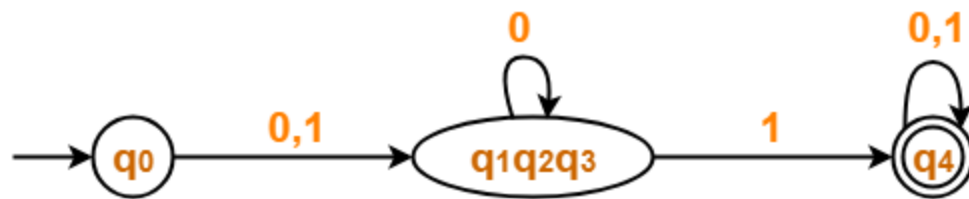
$$P_1 = \{ q_0 \} \{ q_1, q_2, q_3 \} \{ q_4 \}$$

$$P_2 = \{ q_0 \} \{ q_1, q_2, q_3 \} \{ q_4 \}$$

Since $P_2 = P_1$, so we stop.

From P_2 , we infer that states q_1 , q_2 and q_3 are equivalent and can be merged together.

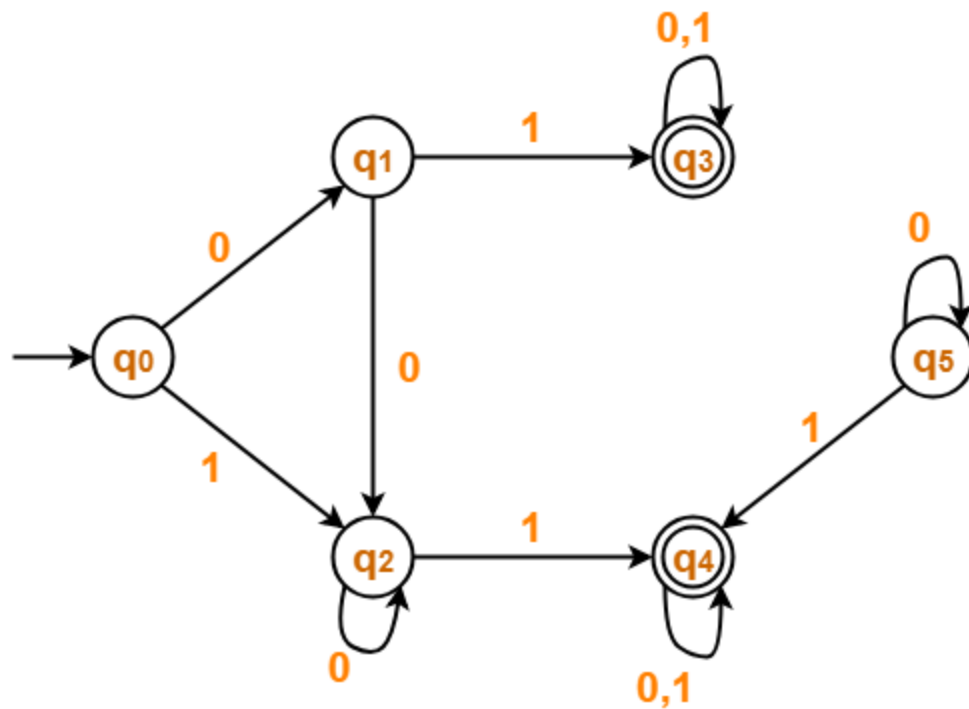
So, Our minimal DFA is-



Minimal DFA

Problem-04:

Minimize the given DFA-

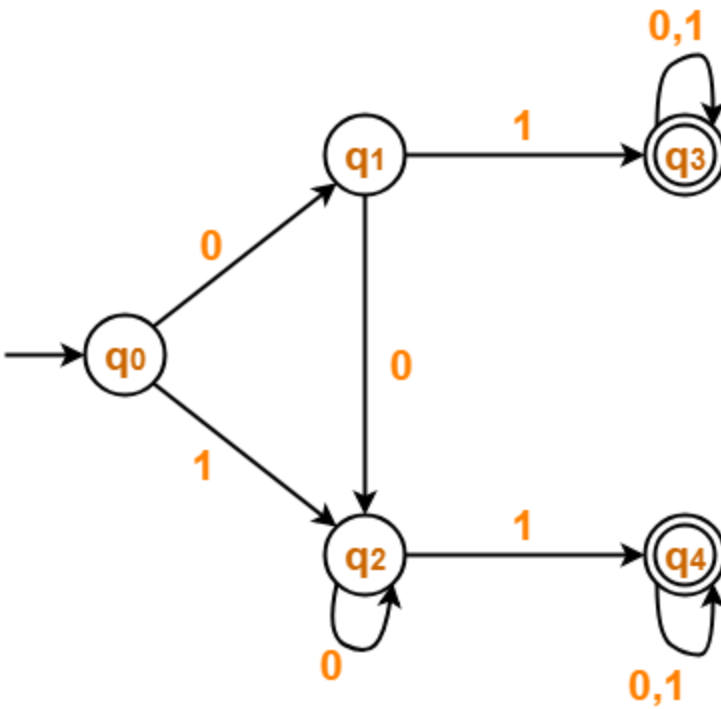


Solution-

Step-01:

- State q_5 is inaccessible from the initial state.
- So, we eliminate it and its associated edges from the DFA.

The resulting DFA is-



Step-02:

Draw a state transition table-

	0	1
→q0	q1	q2
q1	q2	*q3
q2	q2	*q4
*q3	*q3	*q3
*q4	*q4	*q4

Step-03:

Now using Equivalence Theorem, we have-

$$P_0 = \{ q_0, q_1, q_2 \} \{ q_3, q_4 \}$$

$$P_1 = \{ q_0 \} \{ q_1, q_2 \} \{ q_3, q_4 \}$$

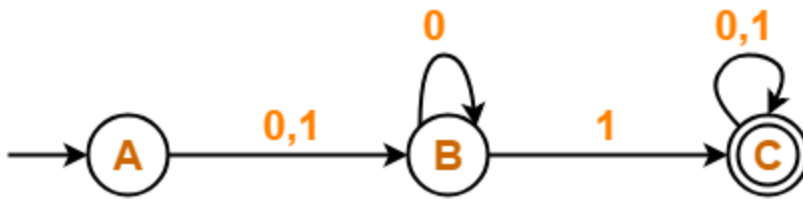
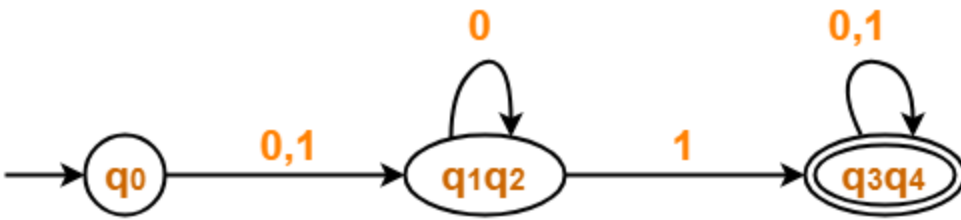
$$P_2 = \{ q_0 \} \{ q_1, q_2 \} \{ q_3, q_4 \}$$

Since $P_2 = P_1$, so we stop.

From P_2 , we infer-

- States q_1 and q_2 are equivalent and can be merged together.
- States q_3 and q_4 are equivalent and can be merged together.

So, Our minimal DFA is-



Minimal DFA