Equivalence of CFG and PDA

Pushdown Automata

Transition function

Input

- δ takes argument as a triple where δ = (q,a,x)
  - q is a state in Q
  - a is either an input symbol Σ or ϵ
  - x is a stack symbol, that is a member of Γ

Output

- Output of δ is a finite set of pairs (p, γ) where
- P is a new state
- γ is a string of stack symbols that replaces x at the top of the stack
- If
  - γ = ϵ then the stack is popped
  - γ = x then the stack is unchanged
  - γ = yz then x is replaced by z and y is pushed onto the stack.

Equivalence of CFG and PDA

- CFG to PDA
- PDA to CFG

CFG to PDA Conversion

- S → 0BB
- B → 0S| 1S| 0

We will generate the transition function of PDA from the given CFG

CFG = {V, Σ, S, P}

Two types of rules:

1. δ (q, ϵ, S) = {(q, α) | A → α in P}
2. δ (q, a, a) = {(q, ϵ) | for every a ∈ Σ}

CFG to PDA Conversion

- S $\rightarrow$ 0BB
- B $\rightarrow$ 0S| 1S| 0

We will generate the transition function of PDA from the given CFG

R1 = δ (q, ϵ, S) = δ (q, 0BB)

R2 = δ (q, ϵ, B) = δ (q, 0S)

R3 = δ (q, ϵ, B) = δ (q, 1S)

R4 = δ (q, ϵ, B) = δ (q, 0)

2 types of rules:

1. δ (q, ϵ, S) = {(q, α) | A $\rightarrow$ α in P}
2. δ (q, a, a) = {(q, ϵ) | for every a $\in \Sigma$}

CFG to PDA Conversion

- S $\rightarrow$ 0BB
- B $\rightarrow$ 0S| 1S| 0

We will generate the transition function of PDA from the given CFG

R5 = δ (q, 0, 0) = δ (q, ϵ)

R6 = δ (q, 1, 1) = δ (q, ϵ)

2 types of rules:

1. δ (q, ϵ, S) = {(q, α) | A $\rightarrow$ α in P}
2. δ (q, a, a) = {(q, ϵ) | for every a $\in \Sigma$}

CFG to PDA Conversion

- $S \rightarrow 0BB$
- $B \rightarrow 0S \mid 1S \mid 0$

We will generate the transition function of PDA from the given CFG

R1 = δ (q, ϵ, S) = δ (q, 0BB)
R2 = δ (q, ϵ, B) = δ (q, 0S)
R3 = δ (q, ϵ, B) = δ (q, 1S)
R4 = δ (q, ϵ, B) = δ (q, 0)
R5 = δ (q, 0, 0) = δ (q, ϵ)
R6 = δ (q, 1, 1) = δ (q, ϵ)

2 types of rules:

1. δ (q, ϵ, S) = {(q, α) | A $\rightarrow \alpha$ in P}
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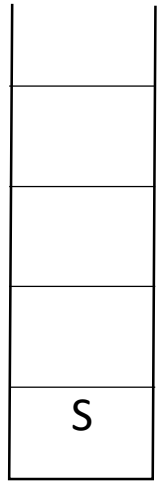
CFG to PDA Conversion

- O10000

(q, 010000, S)



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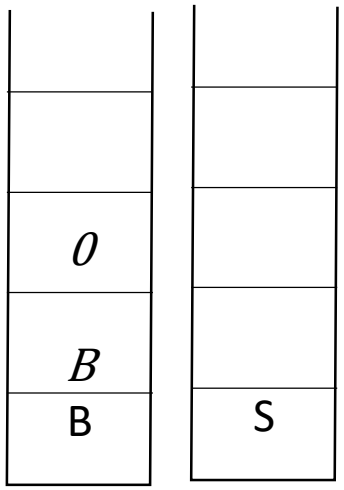
R5 = δ (q, 0, 0) = δ (q, ϵ)
R6 = δ (q, 1, 1) = δ (q, ϵ)

CFG to PDA Conversion

- O10000

(q, 010000, S) R1

(q, 010000, 0BB)



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R6 = δ (q, 1, 1) = δ (q, ϵ)

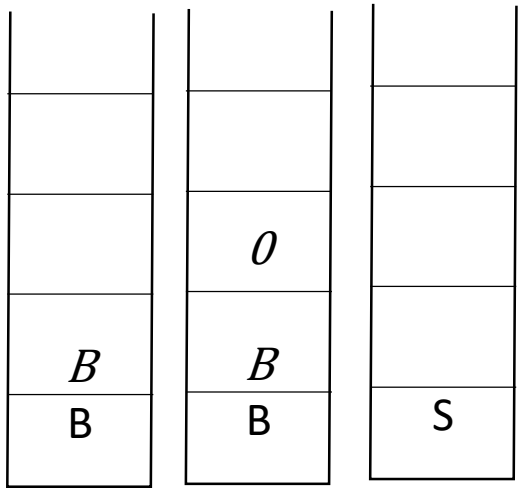
CFG to PDA Conversion

- O10000

(q, 010000, S) R1

(q, 010000, 0BB) R5

(q, 10000, BB)



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R6 = δ (q, 1, 1) = δ (q, ϵ)

CFG to PDA Conversion

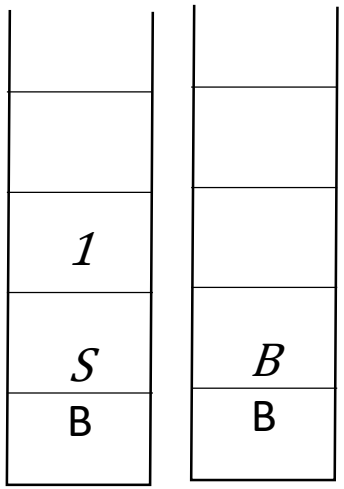
- O10000

(q, 010000, S) R1

(q, 010000, 0BB) R5

(q, 10000, BB) R3

(q, 10000, 1SB)



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CFG to PDA Conversion

- O10000

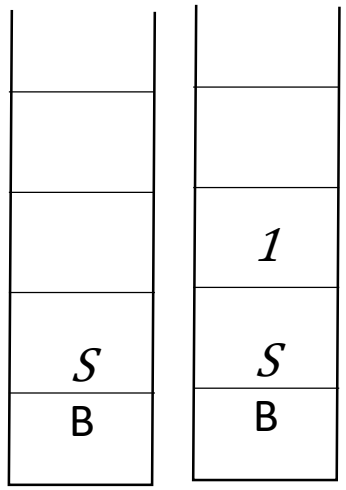
(q, 010000, S) R1

(q, 010000, 0BB) R5

(q, 10000, BB) R3

(q, 10000, 1SB) R6

(q, 0000, SB)



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R5 = δ (q, 0, 0) = δ (q, ϵ)
R6 = δ (q, 1, 1) = δ (q, ϵ)

CFG to PDA Conversion

- O10000

(q, 010000, S) R1

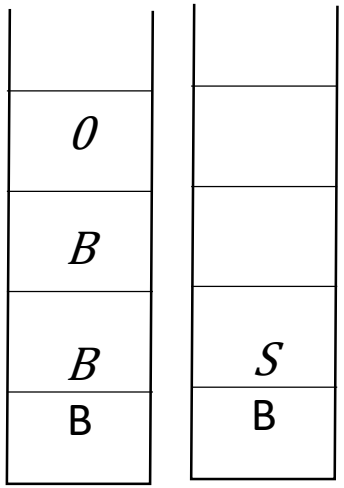
(q, 010000, 0BB) R5

(q, 10000, BB) R3

(q, 10000, 1SB) R6

(q, 0000, SB) R1

(q, 0000, 0BBB)



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CFG to PDA Conversion

- O10000

(q, 010000, S) R1

(q, 010000, 0BB) R5

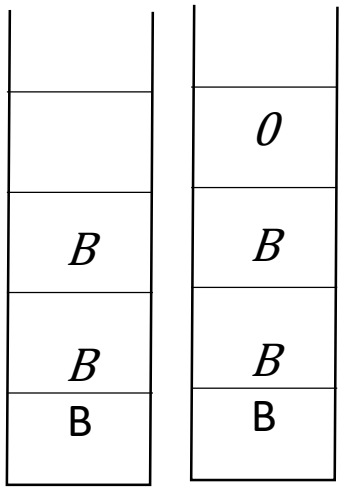
(q, 10000, BB) R3

(q, 10000, 1SB) R6

(q, 0000, SB) R1

(q, 0000, 0BBB) R5

(q, 000, BBB)



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CFG to PDA Conversion

- O10000

(q, 010000, S) R1

(q, 010000, 0BB) R5

(q, 10000, BB) R3

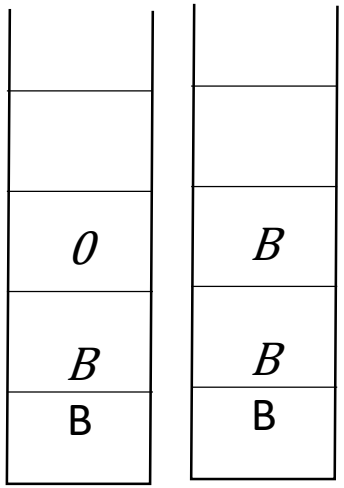
(q, 10000, 1SB) R6

(q, 0000, SB) R1

(q, 0000, 0BBB) R5

(q, 000, BBB) R4

(q, 000, 0BB)



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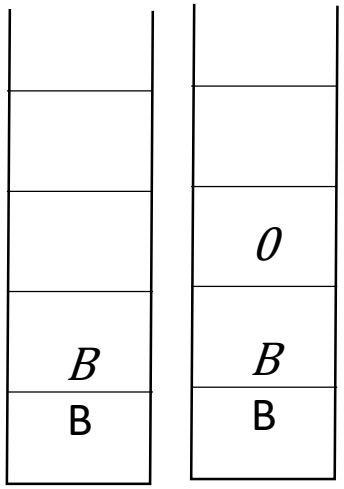
CFG to PDA Conversion

- O10000

(q, 010000, S) R1
(q, 010000, 0BB) R5
(q, 10000, BB) R3
(q, 10000, 1SB) R6
(q, 0000, SB) R1
(q, 0000, 0BBB) R5
(q, 000, BBB) R4
(q, 000, 0BB) R5
(q, 00, BB) R4
(q, 00, 0B)



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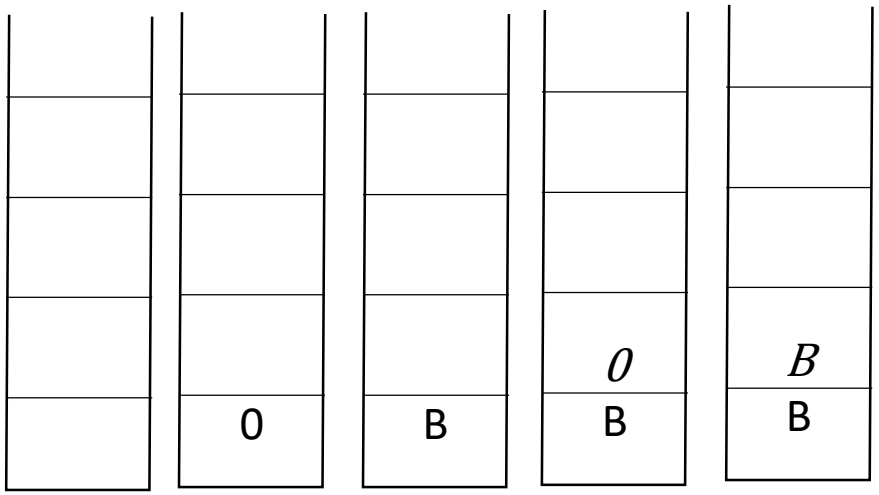
CFG to PDA Conversion

- O10000

(q, 010000, S) R1
(q, 010000, 0BB) R5
(q, 10000, BB) R3
(q, 10000, 1SB) R6
(q, 0000, SB) R1
(q, 0000, 0BBB) R5
(q, 000, BBB) R4
(q, 000, 0BB) R5
(q, 00, BB) R4
(q, 00, 0B) R5
(q, 0, B) R4
(q, 0, B) R5
(q, 0, 0) R4
(q, ϵ, ϵ)



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R5 = δ (q, 0, 0) = δ (q, ϵ)
R6 = δ (q, 1, 1) = δ (q, ϵ)

PDA to CFG Conversion

$A = \{(q_0, q_1), (a,b),(z_0, z), \delta, q_0, z_0, F\}$

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$
2. $\delta\ (q_0, \epsilon, z_0) = (q_0, \epsilon)$
3. $\delta\ (q_0, \text{b}, \text{z}) = (q_0, \text{zz})$
4. $\delta\ (q_0, \text{a}, \text{z}) = (q_1, \text{z})$
5. $\delta\ (q_1, \text{b}, \text{z}) = (q_1, \epsilon)$
6. $\delta\ (q_1, \text{a}, z_0) = (q_0, z_0)$

Three rules for creating production from the transition of PDA

1. For start symbol S → $[q_0, z_0, \text{q}]$ for every q ϵ Q
2. For $\delta\ (q_0, \text{a}, \text{z}) = (q', \epsilon)$
   $[\text{q}, \text{z}, \text{q}'] \rightarrow \text{a}$
3. For δ (q, a, z) = (q', zz….)
   $[\text{q}, \text{z}, \text{q}''] \rightarrow \text{a}\ [\text{q}'\ ,\ \text{z}\ ,\ \text{q}'''\ ]\ [\ \text{q}'''\ ,\ \text{z},\ \text{q}''\ ]$ ……..

q'' = for every q ϵ Q

q''' = for every q ϵ Q

PDA to CFG Conversion

- S $\rightarrow [q_0, z_0, q_0]$ or A
- S $\rightarrow [q_0, z_0, q_1]$ or B
- S $\rightarrow$ A|B

1. For start symbol S $\rightarrow [q_0, z_0, \text{q}]$ for every q ϵ Q

PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [\quad, \text{z}\ ,\quad]\ [\quad, z_0, \quad]$

3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

   q''' = for every q ϵ Q

PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [\quad, \text{z}\ , \quad]\ [\quad, z_0, \quad]$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [q_0\ , \text{z}\ , \quad]\ [\quad, z_0, \quad]$

3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

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PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [\quad, \text{z}\ , \quad]\ [\quad, z_0, \quad]$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [q_0\ , \text{z}\ , \quad]\ [\quad, z_0, \quad]$

   $[q_0, z_0\ , q_0] \rightarrow \text{b}\ [q_0\ , \text{z}\ , \quad]\ [\quad, z_0, q_0\ ]$

3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

   q''' = for every q ϵ Q

PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [\quad, \text{z}\ , \quad]\ [\quad, z_0, \quad]$

   $[q_0, z_0, \quad] \rightarrow \text{b}\ [q_0\ , \text{z}\ , \quad]\ [\quad, z_0, \quad]$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0\ , \text{z}\ , \quad]\ [\quad, z_0, q_0\ ]$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_0]\ [q_0, z_0, q_0]$

3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

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PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$
   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_0]\ [q_0\ , z_0\ , q_0]$

3. For δ (q, a, z) = (q', zz….)
   [q, z , q'’] → a [q' , z , q'’’] [q'’’ , z, q'’] ……..
   q'’ = for every q ϵ Q
   q'’’ = for every q ϵ Q

PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_0]\ [q_0\ , z_0\ , q_0]$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ ,\quad]\ [\quad , z_0\ , q_0]$

3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

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PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_0]\ [q_0\ , z_0\ , q_0]$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_1]\ [q_1, z_0, q_0]$

3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

   q''' = for every q ϵ Q

PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_0]\ [q_0\ , z_0\ , q_0]$

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   $[q_0, z_0, q_1] \rightarrow \text{b}\ [q_0, \text{z}\ ,\ \ ]\ [\ \ ,\ z_0\ , q_1]$

3. For δ (q, a, z) = (q', zz….)

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   q'' = for every q ϵ Q

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PDA to CFG Conversion

1. $\delta\ (q_0, \text{b}, z_0) = (q_0, \text{z}z_0)$

   $[q_0, z_0, q_0] \rightarrow \text{b}\ [q_0, \text{z}\ , q_0]\ [q_0, z_0, q_0]$

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3. For δ (q, a, z) = (q', zz….)

   [q, z , q''] → a [q' , z , q'''] [q''' , z, q''] ……..

   q'' = for every q ϵ Q

   q''' = for every q ϵ Q

PDA to CFG Conversion

2. $\delta\ (q_0, \epsilon, z_0) = (q_0, \epsilon)$

   $[q_0, z_0, q_0] \rightarrow \epsilon$

2. For δ (q, a , z) = (q', ϵ)

   [q, z , q’] → a

PDA to CFG Conversion

3. δ (q_0, b, z) = (q_0, zz)

$[q_0$, z ,] → b [, z ,] [, z ,]

$[q_0$, z ,] → b [, z ,] [, z ,]

$[q_0$, z ,] → b [, z ,] [, z ,]

$[q_0$, z ,] → b [, z ,] [, z ,]

PDA to CFG Conversion

3. $\delta\ (q_0$, b, z) = (q_0, zz)

[q_0, z ,] → b [q_0 , z ,] [, z ,]

[q_0, z ,] → b [q_0 , z ,] [, z ,]

[q_0, z ,] → b [q_0 , z ,] [, z ,]

[q_0, z ,] → b [q_0 , z ,] [, z ,]

PDA to CFG Conversion

3. $\delta\ (q_0, \text{b},\ \text{z}\ ) = (q_0, \text{zz})$

$[q_0, \text{z}, q_0] \rightarrow \text{b}\ [q_0, \text{z}, \ ]\ [\ , \text{z}, q_0]$

$[q_0, \text{z}, q_0] \rightarrow \text{b}\ [q_0, \text{z}, \ ]\ [\ , \text{z}, q_0]$

$[q_0, \text{z}, q_1] \rightarrow \text{b}\ [q_0, \text{z}, \ ]\ [\ , \text{z}, q_1]$

$[q_0, \text{z}, q_1] \rightarrow \text{b}\ [q_0, \text{z}, \ ]\ [\ , \text{z}, q_1]$

PDA to CFG Conversion

3. $\delta\ (q_0, \text{b},\ \text{z}\ ) = (q_0, \text{zz})$

$[q_0, \text{z}\ , q_0\ ] \rightarrow \text{b}\ [q_0\ , \text{z}\ , q_0\ ]\ [q_0\ , \text{z}\ , q_0\ ]$

$[q_0, \text{z}\ , q_0\ ] \rightarrow \text{b}\ [q_0\ , \text{z}\ , q_1\ ]\ [q_1\ , \text{z}\ , q_0\ ]$

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PDA to CFG Conversion

4. $\delta\ (q_0, \text{a},\ \text{z}\ ) = (q_1, \text{z})$

   $[q_0, \text{z}\ , q_0\ ] \rightarrow \text{a}\ [q_1\ , \text{z}\ , q_0\ \ ]$

   $[q_0, \text{z}\ , q_1\ ] \rightarrow \text{a}\ [q_1\ , \text{z}\ , q_1\ \ ]$

PDA to CFG Conversion

5. $\delta\ (q_1, \text{b}, \text{ z}) = (q_1, \epsilon)$

   $[q_1, \text{z}, q_1] \rightarrow \text{b}$

6. $\delta\ (q_1, \text{a}, z_0) = (q_0, z_0)$

   $[q_1, z_0, q_0] \rightarrow \text{a}\ [q_0, z_0, q_0]$

   $[q_1, z_0, q_1] \rightarrow \text{a}\ [q_0, z_0, q_1]$

PDA to CFG Conversion

S → A|B

$[q_0, z_0, q_0]$ → b $[q_0, z, q_0]$ $[q_0, z_0, q_0]$ or A → bCA

$[q_0, z_0, q_0]$ → b $[q_0, z, q_1]$ $[q_1, z_0, q_0]$ or A → bDF

$[q_0, z_0, q_1]$ → b $[q_0, z, q_0]$ $[q_0, z_0, q_1]$ or B → bCB

$[q_0, z_0, q_1]$ → b $[q_0, z, q_1]$ $[q_1, z_0, q_1]$ or B → bDG

$[q_0, z_0, q_0]$ → ϵ or A → ϵ

$[q_0, z, q_0]$ → b $[q_0, z, q_0]$ $[q_0, z, q_0]$ or C → bCC

$[q_0, z, q_0]$ → b $[q_0, z, q_1]$ $[q_1, z, q_0]$ or C → bDH

$[q_0, z, q_1]$ → b $[q_0, z, q_0]$ $[q_0, z, q_1]$ or D → bCD

$[q_0, z, q_1]$ → b $[q_0, z, q_1]$ $[q_1, z, q_1]$ or D → bDE

$[q_0, z, q_0]$ → a $[q_1, z, q_0]$ or A → aH

$[q_0, z, q_1]$ → a $[q_1, z, q_1]$ or B → aE

$[q_1, z, q_1]$ → b or E → b

$[q_1, z_0, q_0]$ → a $[q_0, z_0, q_0]$ or F → aA

$[q_1, z_0, q_1]$ → a $[q_0, z_0, q_1]$ or G → aB

$[q_0, z_0, q_0]$ = A

$[q_0, z_0, q_1]$ = B

$[q_0, z, q_0]$ = C

$[q_0, z, q_1]$ = D

$[q_1, z, q_1]$ = E

$[q_1, z_0, q_0]$ = F

$[q_1, z_0, q_1]$ = G

$[q_1, z, q_0]$ = H