Complejidad Computacional

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1er semestre 2018

Contents

Chapter 1

Turing Machines

Objective: Measure complexity of a problem.

That is: We want to measure computational resources needed to solve a problem.

- Time
- Space
- ...

But first, let us define a problem

Definition 1.0.1 (Alphabet Σ)**.** A finite set of symbols

Example: 1.0.1.

$$\Sigma = \{0, 1\}$$

Definition 1.0.2 (Word w)**.** A finite sequence of symbols from Σ

Definition 1.0.3 ($\Sigma^\star$)**.** The set of all words over the alphabet Σ

Definition 1.0.4 (Language L)**.** A set of words

Example: 1.0.2.

$$L = \{0^n 1^n | n \in \mathbb{N}$$

$$L = \{p | p \text{ has integer roots}$$

Decision problem associated to a language L: Given $w \in \Sigma^\star$, decide if $w \in L$ (or $w \notin L$)

The complexity of a language L is the complexity of a decision problem associated to L

When can we say that L can be solved efficiently?

How can we show that a problem is difficult?

What is an algorithm?

Turing machines: An intent to formalize this concept

Is it possible to show that Turing machines capture the notion of an algorithm?

No way. Algorithm is an intuitive concept.

Why do we believe that Turing machines are a good formalisation of the notion of an algorithm?

- All programs of a Turing machine can be implemented
- All known algorithms can be implemented on a Turing machine
- All other attempts to formalize the concept are reducible to Turing machines
  - The best attempts are equivalent to Turing machines
  - All "reasonable" formalisations are efficiently inter-reducible.

1.1 Turing machines: The idea

- An infinite tape to read from and write to
- A read/write head that can move along the tape
- Accept/Reject state to tell us if the word belongs to the language or not

Observation 1.1.1. The machine can go forever

Definition 1.1.1 (Deterministic Turing machine)**.**

$$Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}$$

- Q is a finite set of states
- Σ is the input alphabet, where $\vdash, B \notin \Sigma$
- Γ is the tape alphabet, where $\Sigma \cup \{\vdash, B\} \subset \Gamma$
- $q_0 \in Q$ initial state
- $q_{accept} \in Q$ accepted state
- $q_{reject} \in Q$ rejected state
- δ transition function