Tarea V

Nicholas Mc-Donnell

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Capítulo 10

10.5

1

Describe the ring obtained from $\mathbb{Z}$ by adjoining an elementα satisfying the two relations $2\alpha-6=0$ and $\alpha-10=0$

7

Analyze the ring obtained from $\mathbb{Z}$ by adjoining an element α which satisfies the pair of relations $\alpha^3+\alpha^2+1=0$ and $\alpha^2+\alpha=0$

9

Describe the ring obtained fro $\mathbb{Z}/12\mathbb{Z}$ by adjoining an inverse of 2

Demostración. Se sabe que adjuntar un inverso a un anillo es equivalente a cocientar de la siguiente forma:

$$R[a]=R[x]/(2x-1)$$

Donde a es el inverso del elemento en cuestión. Para este caso en especifico es el inverso de 2.

$$\implies \mathbb{Z}_{12}[a]=\mathbb{Z}_{12}[x]/(2x-1)$$

Usando las propiedades del anillo:

$$12x\equiv 0$$

$$x\equiv a$$

$$\therefore 12a\equiv 0$$

Pero notamos lo siguiente:

$$12=6\cdot 2 \implies 6\equiv 0$$

Mas aun:

$$3\equiv 0$$

Observamos que $2\cdot 2=4=3+1\equiv 1$

$$\implies 2\equiv a$$

Esto nos lleva a que $\mathbb{Z}_{12}[a]\simeq\mathbb{Z}_3$ $\square$

10.6 Dominio de Enteros y Cuerpos Fraccionarios

1

Prove that the subring of an integral domain is an integral domain.

Demostración. Sea $R' \subset R$ anillo, y R dominio.

$$a, b \in R' : ab = 0$$

$$\therefore a, b \in R \implies (a = 0 \vee b = 0)$$

Lo que implica que R' es dominio. $\square$

3

Let R be an integral domain. Prove that the polynomial ring $R[x]$ is an integral domain.

Demostración. Sean $a, b \in R[x]$

$$\therefore a = \sum_{i=0}^{n} \alpha_i x^i \quad \forall i : \alpha_i \in R$$

$$\therefore b = \sum_{j=0}^{k} \beta_j \text{Ç} x^j \quad \forall i : \beta_i \in R$$

$\square$

5

Is there an integral domain containing exactly 10 elements?

Demostración. Hay dos grupos de orden 10, el dihedral y $\mathbb{Z}_{10}$, notamos que solo $\mathbb{Z}_{10}$ es un grupo abeliano. Vemos los siguientes elementos de $\mathbb{Z}_{10}$:

$$2 \cdot 5 = 10 = 0$$

$$\implies \text{No hay dominio de orden 10}$$ $\square$

10.7 Ideales Máximos

1

Prove that the maximal ideals of the ring of integers are the principal ideals generated by prime integers.

3

Prove that the ideal () in $\mathbb{C}[x, y]$ is a maximal ideal

7

Prove that the ring $\mathbb{F}_2/$ is a field, but that $\mathbb{F}_3/$ is not a field.

10.8 Geometría Algebraica

1

Determine the following points of intersection of two complex plane curves in each of the following:

(a)

(b)

(c)

(d)

(e)

5

Let $f_1, ..., f_r; g_1, ..., g_r \in \mathbb{C}[x_1, ..., x_n]$, and let U, V be the zeros of $\{f_1, ..., f_r\}, \{g_1, ..., g_r\}$ respectively. Prove that if U and V do not meet, then $(f_1, ..., f_r; g_1, ..., g_r)$ is the unit ideal.

7

Prove that the variety defined by a set $\{f_1, ..., f_r\}$ of polynomials depends only on the ideal $(f_1, ..., f_r)$ that they generate.

Capítulo 11

11.1 Factorización de Enteros y Polinomios

3

Prove that if d is the greatest common divisor of $a_1, ..., a_n$ then the greatest common divisor of $a_1/d, ..., a_n/d$ is 1.

5

8

Factor the following polynomials into irreducible factors in $\mathbb{F}_p[x]$