Tarea VI

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Índice

2. Dominios de factorización única, Dominios de Ideales Principales y Dominios Euclidianos

1

Prove or disprove the following.

(a) The polynomial ring $\mathbb{R}[x, y]$ in two variables is a Euclidean domain.

(b) The ring $\mathbb{Z}[x]$ is a principal ideal domain.

3

Give an example showing that division with remainder need not be unique in a Euclidean domain.

9

(a) Prove that $2, 3, 1 \pm \sqrt{-5}$ are irreducible elements of the ring $R = \mathbb{Z}[\sqrt{-5}]$ and that the units of this ring are ± 1.

(b) Prove that the existence of factorization is true for this ring.

13

If a, b are integers and if a divides b in the ring of Gauss integers, then a divides b in $\mathbb{Z}$

3. Lema de Gauss

1

Let a, b be elements of a field F, with $a \neq 0$. Prove that the polynomial $f(x) \in F[x]$ is irreducible if and only if $f(ax + b)$ is irreducible.

3

Let f be an irreducible polynomial in $\mathbb{C}[x, y]$, and let g be another polynomial. Prove that if the variety of zeros of g in $\mathbb{C}^2$ contains the variety of zeros of f, then f divides g.

9

Prove that the kernel of the homomorphism $\mathbb{Z}[x] \to \mathbb{R}$ sending $x \mapsto 1 + \sqrt{2}$ is a principal ideal, and find a generator for this ideal.

4. Factorización explicita de polinomios

1

Prove that the following polynomials are irreducible in $\mathbb{Q}[x]$.

(a) $x^2 + 27x + 213$

(b) $x^3 + 6x + 12$

(c) $8x^3 - 6x + 1$

(d) $x^3 + 6x^2 + 7$

(e) $x^5 - 3x^4 + 3$

3

Factor $x^3 + x + 1$ in $\mathbb{F}_p[x]$, when $p = 2, 3, 5$.

7

Factor the following polynomials into irreducible factors in $\mathbb{Q}[x]$.

(a) $x^3 - 3x - 2$

(b) $x^3 - 3x + 2$

(c) $x^9 - 6x^6 + 9x^3 - 3$

5. Primos en el anillo de Enteros de Gauss

1

Prove that every Gauss prime divides exactly one integer prime.

3

Factor the following into Gauss primes.

(a) $1 - 3i$

(b) 10

(c) $6 + 9i$

7

Describe the residue ring $\mathbb{Z}[i]/(p)$ in each case.

(a) $p = 2$

(b) $p \equiv 1 \pmod{4}$

(c) $p \equiv 3 \pmod{4}$