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1 Graphs

1.1 Graph Traversal

1.1.1 Breadth First Search

```
1 #include "../../headers/headers.h"
    void bfs(graph &g, int start)
        int n = g.size();
        vi visited(n, 1);
        queue<int> q;
        q.emplace(start);
        visited[start] = 0;
10
        while (not q.empty())
11
12
            int u = q.front();
13
14
            q.pop();
15
            for (int v : g[u])
16
17
                if (visited[v])
18
19
                    q.emplace(v);
20
                     visited[v] = 0;
^{21}
22
23
24
25
```

1.1.2 Recursive Depth First Search

```
#include "../../headers/headers.h"
//Recursive (create visited filled with 1s)
void dfs_r(graph &g, vi &visited, int u)

{
    cout << u << '\n';
    visited[u] = 0;

    for (int v : g[u])
        if (visited[v])
        dfs_r(g, visited, v);
}</pre>
```

1.1.3 Iterative Depth First Search

```
#include "../../headers/headers.h"
//Iterative
void dfs_i(graph &g, int start)

int n = g.size();
vi visited(n, 1);
```

```
stack<int> s;
7
8
        s.emplace(start);
9
        visited[start] = 0;
10
11
        while (not s.empty())
12
13
            int u = s.top();
14
15
            s.pop();
16
17
            cout << u << '\n';
18
19
            for (int v : g[u])
                if (visited[v])
20
21
                    s.emplace(v);
22
23
                    visited[v] = 0;
24
25
26 }
```

1.2 Shortest Path Algorithms

1.2.1 Dijsktra

All edges have non-negative values

```
#include "../../headers/headers.h"
   //g has vectors of pairs of the form (w, index)
   int dijsktra(wgraph g, int start, int end)
   {
4
5
        int n = g.size();
        vi cost(n, 1e9); //~INT_MAX/2
7
        priority_queue<ii, greater<ii>> q;
8
9
        q.emplace(0, start);
        cost[start] = 0;
10
        while (not q.empty())
12
            int u = q.top().second, w = q.top().first;
13
            q.pop();
14
15
            // we skip all nodes in the q that we have discovered before at
16
                 a lower cost
            if (cost[u] < w) continue;</pre>
17
18
19
            for (auto v : g[u])
20
                if (cost[v.second] > v.first + w)
^{22}
                    cost[v.second] = v.first + w;
23
                    q.emplace(cost[v.second], v.second);
^{24}
25
26
       }
27
28
```

```
29     return cost[end];
30 }
```

1.2.2 Bellman Ford

Edges can be negative, and it detects negative cycles

```
1 #include "../../headers/headers.h"
   bool bellman_ford(wgraph &g, int start)
3
       int n = g.size();
       vector<int> dist(n, 1e9); //~INT_MAX/2
       dist[start] = 0;
       rep(i, n - 1) rep(u, n) for (ii p : g[u])
            int v = p.first, w = p.second;
            dist[v] = min(dist[v], dist[u] + w);
10
       }
11
12
       bool hayCicloNegativo = false;
13
       rep(u, n) for (ii p : g[u])
14
15
            int v = p.first, w = p.second;
16
            if (dist[v] > dist[u] + w)
17
                hayCicloNegativo = true;
18
       }
19
20
^{21}
       return hayCicloNegativo;
22 | }
```

1.2.3 Floyd Warshall

Shortest path from every node to every other node

```
#include "../../headers/headers.h"
2
   Floyd Warshall implemenation, note that g is using an adjacency matrix
         and not an
    adjacency list
    graph floydWarshall (const graph g)
8
       int n = g.size();
       graph dist(n, vi(n, -1));
10
11
12
       rep(i, n)
            rep(j, n)
13
                dist[i][j] = g[i][j];
14
15
       rep(k, n)
16
            rep(i, n)
17
                rep(j, n)
18
                    if (dist[i][k] + dist[k][j] < dist[i][j] &&
19
```

```
20 | dist[i][k] != INF &&&
21 | dist[k][j] != INF)
22 | dist[i][j] = dist[i][k] + dist[k][j];
23 |
24 | return dist;
25 |}
```

1.3 Minimum Spanning Tree (MST)

1 | #include "../../headers/headers.h"

1.3.1 Kruskal

```
struct edge
2
   {
3
4
        int u, v;
5
        edge(int u, int v, ll w) : u(u), v(v), w(w) {}
6
7
        bool operator<(const edge &o) const
8
9
            return w < o.w;
10
11
   };
12
13
    class Kruskal
14
15
     private:
16
17
        ll sum;
        vi p, rank;
18
19
      public:
20
      //Amount of Nodes n, and unordered vector of Edges E
^{21}
        Kruskal(int n, vector<edge> E)
22
23
            sum = 0;
^{24}
            p.resize(n);
25
            rank.assign(n, 0);
26
            rep(i, n) p[i] = i;
27
            sort(E.begin(), E.end());
28
            for (auto &e : E)
29
                UnionSet(e.u, e.v, e.w);
30
        }
31
        int findSet(int i)
32
33
            return (p[i] == i) ? i : (p[i] = findSet(p[i]));
34
35
36
        bool isSameSet(int i, int j)
37
38
            return findSet(i) == findSet(j);
39
        void UnionSet(int i, int j, ll w)
40
41
            if (not isSameSet(i, j))
42
43
            {
```

```
int x = findSet(i), y = findSet(j);
44
                if (rank[x] > rank[y])
45
                    p[y] = x;
46
                else
47
                    p[x] = y;
48
49
                if (rank[x] == rank[y])
50
                    rank[y]++;
51
52
                sum += w;
53
54
55
        11 mst_val()
56
57
            return sum;
59
60 };
```

1.4 Lowest Common Ancestor (LCA)

Supports multiple trees

```
1 | #include "../../headers/headers.h"
    class LcaForest
3
        int n;
        vi parent;
        vi level;
        vi root;
        graph P;
    public:
10
        LcaForest(int n)
11
12
            this->n = n;
13
            parent.assign(n, -1);
14
            level.assign(n, -1);
15
            P.assign(n, vi(lg(n) + 1, -1));
16
            root.assign(n, -1);
17
18
        void addLeaf(int index, int par)
19
20
            parent[index] = par;
21
            level[index] = level[par] + 1;
22
            P[index][0] = par;
23
            root[index] = root[par];
24
            for (int j = 1; (1 << j) < n; ++j)
25
26
                if (P[index][j - 1] != -1)
27
                    P[index][j] = P[P[index][j - 1]][j - 1];
28
29
       }
30
        void addRoot(int index)
31
32
            parent[index] = index;
33
            level[index] = 0:
34
```

```
35
            root[index] = index;
36
        int lca(int u, int v)
37
38
            if (root[u] != root[v] || root[u] == -1)
39
                return -1;
40
            if (level[u] < level[v])</pre>
41
42
                swap(u, v);
            int dist = level[u] - level[v];
43
            while (dist != 0)
44
45
                int raise = lg(dist);
46
                u = P[u][raise];
47
                dist -= (1 << raise);
48
49
            if (u == v)
50
51
                return u;
            for (int j = lg(n); j >= 0; --j)
52
53
                if (P[u][j] != -1 && P[u][j] != P[v][j])
54
55
                    u = P[u][j];
56
                    v = P[v][j];
57
58
59
            return parent[u];
60
61
62 };
```

1.5 Max Flow

```
#include "../../headers/headers.h"
   class Dinic
2
   {
3
        struct edge
4
5
            int to, rev;
6
7
            11 f, cap;
       };
8
9
10
        vector<vector<edge>> g;
        vector<ll> dist;
11
12
        vector<int> q, work;
13
        int n, sink;
14
        bool bfs(int start, int finish)
15
16
            dist.assign(n, -1);
17
18
            dist[start] = 0;
            int head = 0, tail = 0;
19
            q[tail++] = start;
20
            while (head < tail)
21
^{22}
23
                int u = q[head++];
```

```
24
                for (const edge &e : g[u])
25
                     int v = e.to;
26
                     if (dist[v] == -1 and e.f < e.cap)</pre>
27
                     {
28
                         dist[v] = dist[u] + 1;
29
                         q[tail++] = v;
30
31
                }
32
33
34
            return dist[finish] != -1;
        }
35
36
        11 dfs(int u, 11 f)
37
38
            if (u == sink)
39
40
                 return f;
            for (int &i = work[u]; i < (int)g[u].size(); ++i)</pre>
41
42
                 edge &e = g[u][i];
43
                 int v = e.to;
44
                if (e.cap <= e.f or dist[v] != dist[u] + 1)</pre>
45
                     continue:
46
                11 df = dfs(v, min(f, e.cap - e.f));
47
                if (df > 0)
48
49
                     e.f += df;
50
                     g[v][e.rev].f -= df;
51
                     return df;
52
53
            }
54
55
            return 0;
        }
56
57
      public:
58
59
        Dinic(int n)
        {
60
            this->n = n;
61
            g.resize(n);
62
            dist.resize(n);
63
            q.resize(n);
64
        }
65
66
        void add_edge(int u, int v, ll cap)
67
68
            edge a = {v, (int)g[v].size(), 0, cap};
69
            edge b = {u, (int)g[u].size(), 0, 0}; //Poner cap en vez de 0 si
70
                   la arista es bidireccional
71
            g[u].pb(a);
            g[v].pb(b);
72
        }
73
74
        11 max_flow(int source, int dest)
75
76
            sink = dest:
77
```

```
11 \text{ ans} = 0;
78
             while (bfs(source, dest))
79
80
                 work.assign(n, 0);
81
                 while (11 delta = dfs(source, LLONG MAX))
82
                     ans += delta;
83
            }
84
85
            return ans;
86
87 };
```

1.6 Others

1.6.1 Diameter of a tree

```
1 | #include "../../headers/headers.h"
2
   graph Tree;
3
   vi dist;
4
5
    // Finds a diameter node
6
    int bfs1()
7
   {
8
        int n = Tree.size();
9
10
        queue<int> q;
11
12
        q.emplace(0);
13
        dist[0] = 0;
14
        int u;
        while (not q.empty())
15
16
            u = q.front();
17
            q.pop();
18
19
            for (int v : Tree[u])
20
21
                if (dist[v] == -1)
22
23
                    q.emplace(v);
24
                    dist[v] = dist[u] + 1;
25
26
27
       }
28
29
        return u;
30
31
    // Fills the distances from one diameter node and finds another diameter
32
          node
   int bfs2()
33
   1
34
        int n = Tree.size();
        vi visited(n, 1);
36
        queue<int> q;
37
        int start = bfs1();
38
39
        q.emplace(start);
```

```
visited[start] = 0;
40
41
        int u;
        while (not q.empty())
42
43
           u = q.front();
44
            q.pop();
45
46
            for (int v : Tree[u])
47
48
                if (visited[v])
49
50
                   q.emplace(v);
51
                    visited[v] = 0;
52
                   dist[v] = max(dist[v], dist[u] + 1);
53
               }
54
           }
55
       }
56
       return u;
57
58
59
    // Finds the diameter
    int bfs3()
61
62
       int n = Tree.size();
63
       vi visited(n, 1);
64
        queue<int> q;
65
        int start = bfs2();
66
        q.emplace(start);
67
       visited[start] = 0;
68
       int u;
69
        while (not q.empty())
70
       {
71
           u = q.front();
72
            q.pop();
73
74
            for (int v : Tree[u])
75
76
77
                if (visited[v])
78
                   q.emplace(v);
                    visited[v] = 0;
80
                   dist[v] = max(dist[v], dist[u] + 1);
81
82
83
       }
84
       return dist[u];
85
86 }
```