

Competitive Programming Notes

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1 Notas Útiles

$O(f(n))$	Limite
$O(n!)$	10...11
$O(2^n n^2)$	15...18
$O(2^n n)$	18...21
$O(n^4)$	100
$O(n^3)$	500 ¹
$O(n^2 \log^2 n)$	1000
$O(n^2 \log n)$	2000
$O(n^2)$	1e4 ²
$O(n \log^2 n)$	3e5
$O(n \log n)$	1e6
$O(n)$	1e8 ³

Primos hasta	
1e2	25
1e3	168
1e4	1229
1e5	9592
1e6	78.498
1e7	664.579
1e8	5.761.455
1e9	50.487.534

El mayor HCN⁴ menor a 1e9 tiene 1344 divisores.

El “prime gap” hasta 1e9 es menor a 288

¹Este caso esta justo en el limite de tiempo, además en 256 MB cabe a los una matriz de 400³ ints

²En general solo funciona hasta 6e3

³En general solo funciona hasta 4e7

⁴Highly Compositve Number, números que tienen más divisores que cualquier número más pequeño

2 ./headers/headers/headers.h

```
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;
typedef unsigned long long ull;
typedef pair<int, int> ii;
typedef vector<int> vi;
typedef vector<ll> vll;
typedef vector<ii> vii;

typedef vector<vi> graph;
typedef vector<vii> wgraph;

#ifdef declaraciones_h
#define declaraciones_h

#define rep(i, n) for (int i = 0; i < (int)n; i++)
#define repx(i, a, b) for (int i = a; i < (int)b; i++)
#define invrep(i, a, b) for (int i = b; i-- > (int)a;)

#define pb push_back
#define eb emplace_back
#define ppb pop_back

#define lg(x) (31 - __builtin_clz(x))
#define lgg(x) (63 - __builtin_clzll(x))
#define gcd __gcd

#define umap unordered_map
#define uset unordered_set

//ios::sync_with_stdio(0); cin.tie(0);
//cout.setf(ios::fixed); cout.precision(4);

#define debugx(x) cerr << #x << ": " << x << endl
#define debugv(v) \
    cerr << #v << ":\n"; \
    for (auto e : v) \
    { \
        cerr << " " << e; \
    } \
    cerr << endl
#define debugm(m) \
```

```

cerr << #m << endl;
rep(i, (int)m.size())
{
    cerr << i << ":";
    rep(j, (int)m[i].size()) cerr << " " << m[i][j];
    cerr << endl;
}
#define debugmp(m)
    cerr << #m << endl;
    rep(i, (int)m.size())
    {
        cerr << i << ":";
        rep(j, (int)m[i].size())
        {
            cerr << " {" << m[i][j].first << "," << m[i][j].second << "}";
        }
        cerr << endl;
    }
#define print(x) copy(x.begin(), x.end(), ostream_iterator<int>(cout, "\n")), cout << endl
#endif

```

3 ./strings/trie/trie.cpp

```
#include "../headers/headers/headers.h"

/* Implementation from: https://pastebin.com/fyqsH65k */
struct TrieNode
{
    int leaf; // number of words that end on a TrieNode (allows for duplicate words)
    int height; // height of a TrieNode, root starts at height = 1, can be changed with the default value
    // number of words that pass through this node,
    // ask root node for this count to find the number of entries on the whole Trie
    // all nodes have 1 as they count the words than end on themselves (ie leaf nodes count themselves)
    int count;
    TrieNode *parent; // pointer to parent TrieNode, used on erasing entries
    map<char, TrieNode *> child;
    TrieNode(TrieNode *parent = NULL, int height = 1):
        parent(parent),
        leaf(0),
        height(height),
        count(0), // change to -1 if leaf nodes are to have count 0 instead of 1
        child()
    {}
};

/**
 * Complexity:  $O(|key| * \log(k))$ 
 */
TrieNode *trie_find(TrieNode *root, const string &str)
{
    TrieNode *pNode = root;
    for (string::const_iterator key = str.begin(); key != str.end(); key++)
    {
        if (pNode->child.find(*key) == pNode->child.end())
            return NULL;
        pNode = pNode->child[*key];
    }
    return (pNode->leaf) ? pNode : NULL; // returns only whole word
    // return pNode; // allows to search for a suffix
}

/**
 * Complexity:  $O(|key| * \log(k))$ 
 */
void trie_insert(TrieNode *root, const string &str)
{
    TrieNode *pNode = root;
```

```

    root->count += 1;
    for (string::const_iterator key = str.begin(); key != str.end(); key++)
    {
        if (pNode->child.find(*key) == pNode->child.end())
            pNode->child[*key] = new TrieNode(pNode, pNode->height + 1);
        pNode = pNode->child[*key];
        pNode->count += 1;
    }
    pNode->leaf += 1;
}

/**
 * Complexity:  $O(|key| * \log(k))$ 
 */
void trie_erase(TrieNode *root, const string &str)
{
    TrieNode *pNode = root;
    string::const_iterator key = str.begin();
    for (; key != str.end(); key++)
    {
        if (pNode->child.find(*key) == pNode->child.end())
            return;
        pNode = pNode->child[*key];
    }
    pNode->leaf -= 1;
    pNode->count -= 1;
    while (pNode->parent != NULL)
    {
        if (pNode->child.size() > 0 || pNode->leaf)
            break;
        pNode = pNode->parent, key--;
        pNode->child.erase(*key);
        pNode->count -= 1;
    }
}

```

4 ./estructuras/segmentTree/lazySegmentTree.cpp

```
#include "../headers/headers/headers.h"

struct RSQ // Range sum query
{
    static ll const neutro = 0;
    static ll op(ll x, ll y)
    {
        return x + y;
    }
    static ll
    lazy_op(int i, int j, ll x)
    {
        return (j - i + 1) * x;
    }
};

struct RMinQ // Range minimun query
{
    static ll const neutro = 1e18;
    static ll op(ll x, ll y)
    {
        return min(x, y);
    }
    static ll
    lazy_op(int i, int j, ll x)
    {
        return x;
    }
};

template <class t>
class SegTreeLazy
{
    vector<ll> arr, st, lazy;
    int n;

    void build(int u, int i, int j)
    {
        if (i == j)
        {
            st[u] = arr[i];
            return;
        }
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
```

```

    build(l, i, m);
    build(r, m + 1, j);
    st[u] = t::op(st[l], st[r]);
}

void propagate(int u, int i, int j, ll x)
{
    // nota, las operaciones pueden ser un and, or, ..., etc.
    st[u] += t::lazy_op(i, j, x); // incrementar el valor (+)
    // st[u] = t::lazy_op(i, j, x); // setear el valor
    if (i != j)
    {
        // incrementar el valor
        lazy[u * 2 + 1] += x;
        lazy[u * 2 + 2] += x;
        // setear el valor
        // lazy[u * 2 + 1] = x;
        // lazy[u * 2 + 2] = x;
    }
    lazy[u] = 0;
}

ll query(int a, int b, int u, int i, int j)
{
    if (j < a or b < i)
        return t::neutro;
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    if (lazy[u])
        propagate(u, i, j, lazy[u]);
    if (a <= i and j <= b)
        return st[u];
    ll x = query(a, b, l, i, m);
    ll y = query(a, b, r, m + 1, j);
    return t::op(x, y);
}

void update(int a, int b, ll value,
            int u, int i, int j)
{
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    if (lazy[u])
        propagate(u, i, j, lazy[u]);
    if (a <= i and j <= b)
        propagate(u, i, j, value);
    else if (j < a or b < i)
        return;
}

```



```

        else
        {
            update(a, b, value, l, i, m);
            update(a, b, value, r, m + 1, j);
            st[u] = t::op(st[l], st[r]);
        }
    }

public:
    SegTreeLazy(vector<ll> &v)
    {
        arr = v;
        n = v.size();
        st.resize(n * 4 + 5);
        lazy.assign(n * 4 + 5, 0);
        build(0, 0, n - 1);
    }

    ll query(int a, int b)
    {
        return query(a, b, 0, 0, n - 1);
    }

    void update(int a, int b, ll value)
    {
        update(a, b, value, 0, 0, n - 1);
    }
};

```

5 ./estructuras/segmentTree/segmentTree.cpp

```
#include "../headers/headers/headers.h"

// Se requiere un struct para el nodo (ej: prodsgn).
// Un nodo debe tener tres constructores:
//     Aridad 0: Construye el neutro de la operación
//     Aridad 1: Construye un nodo hoja a partir del input
//     Aridad 2: Construye un nodo según sus dos hijos
//
// Construcción del segment tree:
//     Hacer un arreglo de nodos (usar ctor de aridad 1).
//     ST<miStructNodo> miSegmentTree(arregloDeNodos);
// Update:
//     miSegmentTree.set_point(indice, miStructNodo(input));
// Query:
//     miSegmentTree.query(l, r) es inclusivo exclusivo y da un nodo. Usar la info del nodo para obtener l

// Logic And Query
struct ANDQ
{
    intt value;
    ANDQ() { value = -111; }
    ANDQ(intt x) { value = x; }
    ANDQ(const ANDQ &a,
          const ANDQ &b)
    {
        value = a.value & b.value;
    }
};

// Interval Product (LiveArchive)
struct prodsgn {
    int sgn;
    prodsgn() {sgn = 1;}
    prodsgn(int x) {
        sgn = (x > 0) - (x < 0);
    }
    prodsgn(const prodsgn &a,
            const prodsgn &b) {
        sgn = a.sgn*b.sgn;
    }
};

// Maximum Sum (SPOJ)
```

```

struct maxsum {
    int first, second;
    maxsum() {first = second = -1;}
    maxsum(int x) {
        first = x; second = -1;
    }
    maxsum(const maxsum &a,
           const maxsum &b) {
        if (a.first > b.first) {
            first = a.first;
            second = max(a.second,
                        b.first);
        } else {
            first = b.first;
            second = max(a.first,
                        b.second);
        }
    }
    int answer() {
        return first + second;
    }
};

```

```

// Range Minimum Query
struct rminq {
    int value;
    rminq() {value = INT_MAX;}
    rminq(int x) {value = x;}
    rminq(const rminq &a,
          const rminq &b) {
        value = min(a.value,
                    b.value);
    }
};

```

```

template <class node>
class ST
{
    vector<node> t;
    int n;

public:
    ST(vector<node> &arr)
    {
        n = arr.size();
        t.resize(n * 2);
    }
};

```

```

        copy(arr.begin(), arr.end(), t.begin() + n);
        for (int i = n - 1; i > 0; --i)
            t[i] = node(t[i << 1], t[i << 1 | 1]);
    }

    // 0-indexed
    void set_point(int p, const node &value)
    {
        for (t[p += n] = value; p > 1; p >>= 1)
            t[p >> 1] = node(t[p], t[p ^ 1]);
    }

    // inclusive exclusive, 0-indexed
    node query(int l, int r)
    {
        node ans1, ansr;
        for (l += n, r += n; l < r; l >>= 1, r >>= 1)
        {
            if (l & 1)
                ans1 = node(ans1, t[l++]);
            if (r & 1)
                ansr = node(t[--r], ansr);
        }
        return node(ans1, ansr);
    }
};

```

6 ./estructuras/fenwickTree/fenwickTree2D.cpp

```
#include "../headers/headers/headers.h"
//Numeración en [0,n-1] y [0,m-1]
template <class T>
class FenwickTree2D
{
    vector<vector<T>> t;
    int n, m;

public:
    FenwickTree2D() {}

    FenwickTree2D(int n, int m)
    {
        t.assign(n, vector<T>(m, 0));
        this->n = n;
        this->m = m;
    }

    void add(int r, int c, T value)
    {
        for (int i = r; i < n; i |= i + 1)
            for (int j = c; j < m; j |= j + 1)
                t[i][j] += value;
    }

    // sum[(0, 0), (r, c)]
    T sum(int r, int c)
    {
        T res = 0;
        for (int i = r; i >= 0; i = (i & (i + 1)) - 1)
            for (int j = c; j >= 0; j = (j & (j + 1)) - 1)
                res += t[i][j];
        return res;
    }

    // sum[(r1, c1), (r2, c2)]
    T sum(int r1, int c1, int r2, int c2)
    {
        return sum(r2, c2) - sum(r1 - 1, c2) - sum(r2, c1 - 1) + sum(r1 - 1, c1 - 1);
    }

    T get(int r, int c)
    {
        return sum(r, c, r, c);
    }
}
```

```
    }  
  
    void set(int r, int c, T value)  
    {  
        add(r, c, -get(t, r, c) + value);  
    }  
};
```

7 ./estructuras/fenwickTree/fenwickTree.cpp

```
#include "../headers/headers/headers.h"

struct FenwickTree
{
    vector<int> FT;
    FenwickTree(int N)
    {
        FT.resize(N + 1, 0);
    }

    int query(int i)
    {
        int ans = 0;
        for (; i; i -= i & (-i))
            ans += FT[i];
        return ans;
    }

    int query(int i, int j)
    {
        return query(j) - query(i - 1);
    }

    void update(int i, int v)
    {
        int s = query(i, i);
        for (; i < FT.size(); i += i & (-i))
            FT[i] += v - s;
    }

    //Queries puntuales, Updates por rango
    void update(int i, int j, int v)
    {
        update(i, v);
        update(j + 1, -v);
    }
};
```

8 `./math/simpsonsMethod/simpsonsMethod.cpp`

```
#include "../headers.h"
//Numerical Integration of f in interval [a,b]

double simpsons_rule(function<double(double)> f, double a, double b)
{
    double c = (a + b) / 2;
    double h3 = abs(b - a) / 6;
    return h3 * (f(a) + 4 * f(c) + f(b));
}

//Integrate f between a and b, using intervals of length (b-a)/n
double simpsons_rule(function<double(double)> f, double a, double b, int n)
{
    //n sets the precision for the result
    double ans = 0;
    double step = 0, h = (b - a) / n;
    rep(i, n)
    {
        ans += simpsons_rule(f, step, step + h);
        step += h;
    }
    return ans;
}
```


9 ./graphs/dinic/dinic.cpp

```
#include "../headers/headers/headers.h"
class Dinic
{
    struct edge
    {
        int to, rev;
        ll f, cap;
    };

    vector<vector<edge>> g;
    vector<ll> dist;
    vector<int> q, work;
    int n, sink;

    bool bfs(int start, int finish)
    {
        dist.assign(n, -1);
        dist[start] = 0;
        int head = 0, tail = 0;
        q[tail++] = start;
        while (head < tail)
        {
            int u = q[head++];
            for (const edge &e : g[u])
            {
                int v = e.to;
                if (dist[v] == -1 and e.f < e.cap)
                {
                    dist[v] = dist[u] + 1;
                    q[tail++] = v;
                }
            }
        }
        return dist[finish] != -1;
    }

    ll dfs(int u, ll f)
    {
        if (u == sink)
            return f;
        for (int &i = work[u]; i < (int)g[u].size(); ++i)
        {
            edge &e = g[u][i];
            int v = e.to;
```

```

        if (e.cap <= e.f or dist[v] != dist[u] + 1)
            continue;
        ll df = dfs(v, min(f, e.cap - e.f));
        if (df > 0)
        {
            e.f += df;
            g[v][e.rev].f -= df;
            return df;
        }
    }
    return 0;
}

public:
    Dinic(int n)
    {
        this->n = n;
        g.resize(n);
        dist.resize(n);
        q.resize(n);
    }

    void add_edge(int u, int v, ll cap)
    {
        edge a = {v, (int)g[v].size(), 0, cap};
        edge b = {u, (int)g[u].size(), 0, 0}; //Poner cap en vez de 0 si la arista es bidireccional
        g[u].pb(a);
        g[v].pb(b);
    }

    ll max_flow(int source, int dest)
    {
        sink = dest;
        ll ans = 0;
        while (bfs(source, dest))
        {
            work.assign(n, 0);
            while (ll delta = dfs(source, LLONG_MAX))
                ans += delta;
        }
        return ans;
    }
};

```

10 ./graphs/dijkstra/dijkstra.cpp

```
#include "../headers/headers/headers.h"
//g has vectors of pairs of the form (w, index)
int dijkstra(wgraph g, int start, int end)
{
    int n = g.size();
    vi cost(n, 1e9); //~INT_MAX/2
    priority_queue<ii, greater<ii>> q;

    q.emplace(0, start);
    cost[start] = 0;
    while (not q.empty())
    {
        int u = q.top().second, w = q.top().first;
        q.pop();

        for (auto v : g[u])
        {
            if (cost[v.second] > v.first + w)
            {
                cost[v.second] = v.first + w;
                q.emplace(cost[v.second], v.second);
            }
        }
    }

    return cost[end];
}
```

11 ./graphs/dfs/dfsRecursive.cpp

```
#include "../headers/headers/headers.h"
//Recursive (create visited filled with 1s)
void dfs_r(graph &g, vi &visited, int u)
{
    cout << u << '\n';
    visited[u] = 0;

    for (int v : g[u])
        if (visited[v])
            dfs_r(g, visited, v);
}
```

12 ./graphs/dfs/dfsIterative.cpp

```
#include "../headers/headers/headers.h"
//Iterative
void dfs_i(graph &g, int start)
{
    int n = g.size();
    vi visited(n, 1);
    stack<int> s;

    s.emplace(start);
    visited[start] = 0;

    while (not s.empty())
    {
        int u = s.top();
        s.pop();

        cout << u << '\n';

        for (int v : g[u])
            if (visited[v])
            {
                s.emplace(v);
                visited[v] = 0;
            }
    }
}
```

13 ./graphs/lca/lca.cpp

```
#include "../headers/headers/headers.h"
class LcaTree
{
    int n;
    vi parent;
    vi level;
    vi root;
    graph P;
public:
    LcaTree(int n){
        this->n = n;
        parent.assign(n,-1);
        level.assign(n,-1);
        P.assign(n,vi(lg(n)+1,-1));
        root.assign(n,-1);
    }
    void addLeaf(int index, int par){
        parent[index] = par;
        level[index] = level[par] + 1;
        P[index][0] = par;
        root[index] = root[par];
        for(int j=1; (1<<j) < n; ++j){
            if(P[index][j-1] != -1)
                P[index][j] = P[P[index][j-1]][j-1];
        }
    }
    void addRoot(int index){
        parent[index] = index;
        level[index] = 0;
        root[index] = index;
    }
    int lca(int u, int v){
        if(root[u] != root[v] || root[u] == -1)
            return -1;
        if(level[u] < level[v])
            swap(u,v);
        int dist = level[u] - level[v];
        while(dist != 0){
            int raise = lg(dist);
            u = P[u][raise];
            dist -= (1<<raise);
        }
        if(u == v)
            return u;
    }
}
```

```

    for(int j = lg(n); j>=0; --j){
        if(P[u][j] != -1 && P[u][j] != P[v][j]){
            u=P[u][j];
            v=P[v][j];
        }
    }
    return parent[u];
}
};

```

14 ./graphs/kruskal/kruskal.cpp

```
#include "../headers/headers/headers.h"
struct edge
{
    int u, v;
    ll w;
    edge(int u, int v, ll w) : u(u), v(v), w(w) {}

    bool operator<(const edge &o) const
    {
        return w < o.w;
    }
};

class Kruskal
{
private:
    ll sum;
    vi p, rank;

public:
    //Amount of Nodes n, and unordered vector of Edges E
    Kruskal(int n, vector<edge> E)
    {
        sum = 0;
        p.resize(n);
        rank.assign(n, 0);
        rep(i, n) p[i] = i;
        sort(E.begin(), E.end());
        for (auto &e : E)
            UnionSet(e.u, e.v, e.w);
    }
    int findSet(int i)
    {
        return (p[i] == i) ? i : (p[i] = findSet(p[i]));
    }
    bool isSameSet(int i, int j)
    {
        return findSet(i) == findSet(j);
    }
    void UnionSet(int i, int j, ll w)
    {
        if (not isSameSet(i, j))
        {
            int x = findSet(i), y = findSet(j);
```



```

        if (rank[x] > rank[y])
            p[y] = x;
        else
            p[x] = y;

        if (rank[x] == rank[y])
            rank[y]++;

        sum += w;
    }
}
ll mst_val()
{
    return sum;
}
};

```

15 ./graphs/unionFind/unionFind.cpp

```
#include "../headers/headers/headers.h"
class UnionFind
{
private:
    int numSets;
    vi p, rank, setSize;

public:
    UnionFind(int n)
    {
        numSets = n;
        rank.assign(n, 0);
        setSize.assign(n, 1);
        p.resize(n);
        rep(i, n) p[i] = i;
    }
    int findSet(int i)
    {
        return (p[i] == i) ? i : (p[i] = findSet(p[i]));
    }
    bool isSameSet(int i, int j)
    {
        return findSet(i) == findSet(j);
    }
    void UnionSet(int i, int j)
    {
        if (not isSameSet(i, j))
        {
            numSets--;
            int x = findSet(i), y = findSet(j);
            if (rank[x] > rank[y])
            {
                p[y] = x;
                setSize[x] += setSize[y];
            }
            else
            {
                p[x] = y;
                setSize[y] += setSize[x];
                if (rank[x] == rank[y])
                    rank[y]++;
            }
        }
    }
}
```

```
int numSets()
{
    return numSets;
}
int setOfSize(int i)
{
    return setSize[i];
}
};
```

16 ./graphs/bfs/bfs.cpp

```
#include "../headers/headers/headers.h"

void bfs(graph &g, int start)
{
    int n = g.size();
    vi visited(n, 1);
    queue<int> q;

    q.emplace(start);
    visited[start] = 0;
    while (not q.empty())
    {
        int u = q.front();
        q.pop();

        for (int v : g[u])
        {
            if (visited[v])
            {
                q.emplace(v);
                visited[v] = 0;
            }
        }
    }
}
```

17 ./graphs/bellmanFord/bellmanFord.cpp

```
#include "../headers/headers/headers.h"
bool bellman_ford(wgraph &g, int start)
{
    int n = g.size();
    vector<int> dist(n, 1e9); //~INT_MAX/2
    dist[start] = 0;
    rep(i, n - 1) rep(u, n) for (ii p : g[u])
    {
        int v = p.first, w = p.second;
        dist[v] = min(dist[v], dist[u] + w);
    }

    bool hayCicloNegativo = false;
    rep(u, n) for (ii p : g[u])
    {
        int v = p.first, w = p.second;
        if (dist[v] > dist[u] + w)
            hayCicloNegativo = true;
    }

    return hayCicloNegativo;
}
```