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1 Info About Memory and Time Limits

O(f(n))	Limite
O(n!)	$10,\ldots,11$
$O(2^n n^2)$	$15,\ldots,18$
$O(2^n n)$	$ 18,\ldots,21 $
$O(n^4)$	100
$O(n^3)$	500^{1}
$O(n^2 \log^2 n)$	1000
$O(n^2 \log n)$	2000
$O(n^2)$	$1e4^{2}$
$O(n\log^2 n)$	3e5
$O(n \log n)$	1e6
O(n)	$1e8^{3}$

2 C++ Cheat Sheet

2.1 Headers

```
1 #pragma GCC optimize("Ofast")
  #include <bits/stdc++.h>
3
    using namespace std;
4
   typedef long long 11;
    typedef unsigned long long ull;
    typedef pair<int, int> ii;
    typedef tuple<int, int, int> iii;
    typedef vector<int> vi;
    typedef vector<ll> vll;
   typedef vector<ii> vii;
    typedef vector<vi> graph;
    typedef vector<vii> wgraph;
   #ifndef declaraciones_h
17
   #define declaraciones_h
   #define rep(i, n) for (int i = 0; i < (int)n; i++)
21 #define repx(i, a, b) for (int i = a; i < (int)b; i++)
#define invrep(i, a, b) for (int i = b; i-- > (int)a;)
23
24 #define pb push_back
25 #define eb emplace_back
26 #define ppb pop_back
```

 $^{^1\}mathrm{Este}$ caso esta justo en el limite de tiempo, además en 256 MB cabe a los una matriz de 400^3 ints

²En general solo funciona hasta 6e3

³En general solo funciona hasta 4e7

```
27
    #define lg(x) (31 - __builtin_clz(x))
28
    #define lgg(x) (63 - __buitlin_clzll(x))
29
    #define gcd __gcd
30
31
    #define INF INT_MAX
32
33
    #define umap unordered_map
34
    #define uset unordered_set
35
36
    #define debugx(x) cerr << #x << ": " << x << endl
37
    #define debugv(v)
38
        cerr << #v << ":";
39
        for (auto e : v)
40
41
            cerr << " " << e; \
42
43
        }
        cerr << endl
44
    #define debugm(m)
45
        cerr << #m << endl;</pre>
46
        rep(i, (int)m.size())
47
        {
48
            cerr << i << ":":
49
            rep(j, (int)m[i].size()) cerr << " " << m[i][j]; \</pre>
50
             cerr << endl;
51
        }
52
    #define debugmp(m)
53
        cerr << #m << endl;</pre>
54
        rep(i, (int)m.size())
55
        {
56
             cerr << i << ":":
57
            rep(j, (int)m[i].size())
58
59
                 cerr << " {" << m[i][j].first << "," << m[i][j].second << "}</pre>
60
                      "; \
61
             cerr << endl;</pre>
62
63
    #define print(x) copy(x.begin(), x.end(), ostream_iterator<int>(cout,
         "")), cout << endl
65
    template <typename T1, typename T2>
    ostream &operator<<(ostream &os, const pair<T1, T2> &p)
```

2.2 Cheat Sheet

```
#include "../headers/headers.h"
2
    // Note: This Cheat Sheet is by no means complete
    // If you want a thorough documentation of the Standard C++ Library
    // please refer to this link: http://www.cplusplus.com/reference/
5
6
    /* ======= */
7
   /* Reading from stdin */
8
    /* ======= */
    // With scanf
    scanf("%d", &a);
                               //int
11
   scanf("%x", &a);
                               // int in hexadecimal
12
   scanf("%llx", &a);
                               // long long in hexadecimal
13
   scanf("%lld", &a);
                               // long long int
   scanf("%c", &c);
                               // char
   scanf("%s", buffer);
                              // string without whitespaces
    scanf("%f", &f);
                              // float
   scanf("%lf", &d);
                              // double
    scanf("%d %*s %d", &a, &b); //* = consume but skip
19
20
   // read until EOL
21
22
   // - EOL not included in buffer
    // - EOL is not consumed
   // - nothing is written into buffer if EOF is found
   scanf(" %[^\n]", buffer);
26
   //reading until EOL or EOF
27
    // - EOL not included in buffer
    // - EOL is consumed
    // - works with EOF
30
    char *output = gets(buffer);
   if (feof(stind))
32
33
   {
   } // EOF file found
   if (output == buffer)
36
   } // succesful read
37
   if (output == NULL)
39
   } // EOF found without previous chars found
40
    while (gets(buffer) != NULL)
43
44
        puts(buffer):
       if (feof(stdin))
45
46
```

```
/* ======= */
47
          break;
      }
                                                                              bool peekAndCheck(char c)
48
                                                                          100
                                                                          101 {
49
                                                                                  char c2 = getchar();
                                                                          102
50
   // read single char
                                                                                  ungetc(c2, stdin); // return char to stdin
                                                                          103
   getchar();
                                                                                  return c == c2;
                                                                          104
   while (true)
                                                                          105
                                                                          106
54
       c = getchar();
                                                                              /* ======= */
55
                                                                              /* Reading from cin */
       if (c == EOF || c == '\n')
                                                                          108
56
                                                                              /* ======= */
57
          break;
                                                                          109
                                                                              // reading a line of unknown length
58
                                                                          111 string line;
59
   /* ======= */
                                                                             getline(cin, line);
60
                                                                              while (getline(cin, line))
   /* Printing to stdout */
  /* ======== */
                                                                          114
63
  // With printf
                                                                          115
                                                                              }
64 printf("%d", a);
                            // int
                                                                          116
65 printf("%u", a);
                           // unsigned int
                                                                              // Optimizations with cin/cout
                                                                          117
   printf("%lld", a);
                            // long long int
                                                                              ios::sync_with_stdio(0);
                                                                          118
  printf("%llu", a);
                           // unsigned long long int
                                                                              cin.tie(0);
                                                                          119
  printf("%c", c);
                           // char
                                                                              cout.tie(0);
                                                                          120
  printf("%s", buffer);
                           // string until \0
                                                                          121
70 printf("%f", f);
                           // float
                                                                              // Fix precision on cout
                                                                          122
  printf("%lf", d);
                          // double
                                                                              cout.setf(ios::fixed);
                                                                          123
  printf("%0*.*f", x, y, f); // padding = 0, width = x, decimals = y
                                                                              cout.precision(4); // e.g. 1.000
                                                                          124
   printf("(%.5s)\n", buffer); // print at most the first five characters
                                                                          125
        (safe to use on short strings)
                                                                              /* ======= */
                                                                          126
                                                                              /* USING PAIRS AND TUPLES */
                                                                          127
74
   // print at most first n characters (safe)
                                                                              /* ======= */
                                                                          128
   printf("(%.*s)\n", n, buffer); // make sure that n is integer (with long
                                                                              // ii = pair<int,int>
        long I had problems)
                                                                          130
                                                                              ii p(5, 5);
   //string + \n
                                                                              ii p = make_pair(5, 5)
                                                                          131
77
   puts(buffer);
                                                                              ii p = \{5, 5\};
                                                                              int x = p.first, y = p.second;
   /* ====== */
                                                                              // iii = tuple<int,int,int>
80
                                                                          135 iii t(5, 5, 5);
   /* Reading from c string */
   /* ======= */
                                                                              tie(x, y, z) = t;
82
                                                                              tie(x, y, z) = make_tuple(5, 5, 5);
83
                                                                             get<0>(t)++;
   // same as scanf but reading from s
   int sscanf(const char *s, const char *format, ...);
                                                                              get<1>(t)--;
                                                                          139
86
                                                                          140
   /* ======= */
                                                                              /* ======= *,
                                                                          141
   /* Printing to c string */
                                                                          142
                                                                              /* CONVERTING FROM STRING TO NUMBERS */
   /* ======= */
                                                                              /* ========= */
                                                                          143
89
                                                                          144 //-----
   // Same as printf but writing into str, the number of characters is
                                                                          145 // string to int
   // or negative if there is failure
                                                                              // option #1:
                                                                              int atoi(const char *str);
   int sprintf(char *str, const char *format, ...);
                                                                          148 // option #2:
  int n = sprintf(buffer, "%d plus %d is %d", a, b, a + b);
                                                                              sscanf(string, "%d", &i);
   printf("[%s] is a string %d chars long\n", buffer, n);
                                                                          150
                                                                          151 // string to long int:
   /* ======= */
                                                                              // option #1:
                                                                          152
   /* Peek last char of stdin */
                                                                          long int strtol(const char *str, char **endptr, int base);
```

```
// it only works skipping whitespaces, so make sure your numbers
    // are surrounded by whitespaces only
                                                                               209
                                                                                   // find index of string or char within string
156
                                                                              210
    char szNumbers[] = "2001 60c0c0 -1101110100110100100000 0x6ffffff";
                                                                                   string str = "random";
157
                                                                                   std::size_t pos = str.find("ra");
    char *pEnd;
158
    long int li1, li2, li3, li4;
                                                                                   std::size_t pos = str.find('m');
159
    li1 = strtol(szNumbers, &pEnd, 10);
                                                                                   if (pos == string::npos) // not found
    li2 = strtol(pEnd, &pEnd, 16);
                                                                              215
161
    li3 = strtol(pEnd, &pEnd, 2);
                                                                                       // substrings
162
                                                                               216
    li4 = strtol(pEnd, NULL, 0);
                                                                                       string subs = str.substr(pos, length);
                                                                              217
163
    printf("The decimal equivalents are: %ld, %ld, %ld and %ld.\n", li1, li2
                                                                                   string subs = str.substr(pos); // default: to the end of the string
164
                                                                               218
    // option #2:
                                                                               220
                                                                                   // std::string from cstring's substring
165
    long int atol(const char *str);
                                                                                   const char *s = "bla1 bla2";
                                                                               221
166
                                                                                   int offset = 5, len = 4;
    // option #3:
    sscanf(string, "%ld", &1);
                                                                                   string subs(s + offset, len); // bla2
                                                                              223
168
169
    //-----
                                                                               224
                                                                                   // -----
    // string to long long int:
                                                                               225
                                                                                   // string comparisons
   // option #1:
                                                                              226
171
    long long int strtoll(const char *str, char **endptr, int base);
                                                                                   int compare(const string &str) const;
172
                                                                               227
    // option #2:
                                                                                   int compare(size_t pos, size_t len, const string &str) const;
                                                                               228
    sscanf(string, "%lld", &1);
                                                                                   int compare(size_t pos, size_t len, const string &str,
174
                                                                              229
                                                                                              size_t subpos, size_t sublen) const;
                                                                               230
    // string to double:
                                                                                   int compare(const char *s) const;
                                                                               231
                                                                                   int compare(size_t pos, size_t len, const char *s) const;
                                                                              232
177
    double strtod(const char *str, char **endptr); //similar to strtol
178
                                                                               233
    // option #2:
                                                                               234
                                                                                   // examples
179
    double atof(const char *str);
                                                                                   // 1) check string begins with another string
                                                                               235
180
                                                                                   string prefix = "prefix";
    // option #3:
    sscanf(string, "%lf", &d);
                                                                                   string word = "prefix suffix";
182
                                                                                   word.compare(0, prefix.size(), prefix);
183
    /* ======= */
184
                                                                               239
    /* C STRING UTILITY FUNCTIONS */
                                                                                   /* ======= */
                                                                               240
185
     /* ======= */
                                                                                   /* OPERATOR OVERLOADING */
186
    int strcmp(const char *str1, const char *str2);
                                                                                   /* ======= */
187
                                                                              242
    int memcmp(const void *ptr1, const void *ptr2, size_t num); // (-1,0,1)
188
                                                                               243
                                                                                   //-----
    void *memcpy(void *destination, const void *source, size_t num);
189
                                                                               244
                                                                                   // method #1: inside struct
190
                                                                              245
     /* ======= */
                                                                               246
                                                                                   struct Point
191
    /* C++ STRING UTILITY FUNCTIONS */
                                                                               247
    /* ======= */
193
                                                                              248
    // read tokens from string
                                                                               249
                                                                                       bool operator<(const Point &p) const
194
    string s = "tok1 tok2 tok3";
                                                                               250
    string tok;
                                                                               251
                                                                                           if (x != p.x)
196
    stringstream ss(s);
                                                                               252
                                                                                               return x < p.x;
197
    while (getline(ss, tok, ' '))
                                                                               253
                                                                                           return y < p.y;</pre>
        printf("tok = %s\n", tok.c_str());
199
                                                                               254
                                                                                       bool operator>(const Point &p) const
                                                                               255
200
    // split a string by a single char delimiter
201
                                                                               256
    void split(const string &s, char delim, vector<string> &elems)
                                                                                           if (x != p.x)
202
                                                                              257
203
                                                                               258
                                                                                              return x > p.x;
        stringstream ss(s);
                                                                               259
                                                                                           return y > p.y;
204
        string item;
205
                                                                               260
        while (getline(ss, item, delim))
                                                                                       bool operator == (const Point &p) const
206
                                                                               261
            elems.push_back(item);
                                                                              262
207
```

```
if (a.x != b.x)
263
            return x == p.x && y == p.y;
        }
                                                                                                 return a.x < b.x:
264
                                                                                 319
    }:
                                                                                             return a.y < b.y;
265
                                                                                 320
                                                                                         }
266
                                                                                 321
                                                                                 322
                                                                                     |};
267
                                                                                      // without operator overloading, you would have to use
     // method #2: outside struct
268
                                                                                      // an explicit comparison method when using library
269
    struct Point
     {
                                                                                      // functions or data structures that require sorting
270
                                                                                     priority_queue<Point, vector<Point>, cmp> pq;
271
        int x, y;
                                                                                     vector<Point> pts;
                                                                                 327
272
273
     bool operator (const Point &a, const Point &b)
                                                                                 328
                                                                                      sort(pts.begin(), pts.end(), cmp);
                                                                                      lower_bound(pts.begin(), pts.end(), {1, 2}, cmp);
274
        if (a.x != b.x)
                                                                                      upper_bound(pts.begin(), pts.end(), {1, 2}, cmp);
275
            return a.x < b.x;
                                                                                 331
                                                                                      set<Point, cmp> pt_set;
276
                                                                                      map<Point, int, cmp> pt_map;
277
        return a.y < b.y;
                                                                                 333
278
     bool operator>(const Point &a, const Point &b)
                                                                                 334
                                                                                      /* ======= */
279
                                                                                      /* VECTOR UTILITY FUNCTIONS */
280
                                                                                 335
        if (a.x != b.x)
                                                                                      /* ======= */
281
                                                                                 336
                                                                                      vector<int> myvector;
            return a.x > b.x;
282
                                                                                 337
        return a.y > b.y;
                                                                                     myvector.push_back(100);
283
                                                                                 338
                                                                                     myvector.pop_back(); // remove last element
284
                                                                                 339
    bool operator == (const Point &a, const Point &b)
                                                                                     myvector.back();  // peek reference to last element
285
                                                                                     myvector.front();
                                                                                                       // peek reference to first element
286
        return a.x == b.x && a.y == b.y;
                                                                                 myvector.clear(); // remove all elements
287
                                                                                      // sorting a vector
288
                                                                                      vector<int> foo;
                                                                                 344
289
     // Note: if you overload the < operator for a custom struct,
                                                                                      sort(foo.begin(), foo.end());
                                                                                 345
290
                                                                                      sort(foo.begin(), foo.end(), std::less<int>()); // increasing
     // then you can use that struct with any library function
                                                                                      sort(foo.begin(), foo.end(), std::greater<int>()); // decreasing
292
     // or data structure that requires the < operator
                                                                                 347
    // Examples:
293
                                                                                 348
                                                                                      /* ======= */
294
    priority_queue<Point> pq;
                                                                                 349
                                                                                      /* SET UTILITY FUNCTIONS */
    vector<Point> pts;
                                                                                 350
295
                                                                                      /* ======= */
    sort(pts.begin(), pts.end());
    lower_bound(pts.begin(), pts.end(), {1, 2});
                                                                                      set<int> myset;
297
                                                                                 352
                                                                                      myset.begin(); // iterator to first elemnt
298
     upper_bound(pts.begin(), pts.end(), {1, 2});
                                                                                     myset.end();  // iterator to after last element
     set<Point> pt_set;
299
    map<Point, int> pt_map;
                                                                                      myset.rbegin(); // iterator to last element
300
                                                                                      myset.rend(); // iterator to before first element
301
     /* ======= */
                                                                                      for (auto it = myset.begin(); it != myset.end(); ++it)
     /* CUSTOM COMPARISONS */
                                                                                     {
                                                                                 358
303
     /* ======= */
                                                                                          do something(*it):
                                                                                 359
304
     // method #1: operator overloading
                                                                                     } // left -> right
     // method #2: custom comparison function
                                                                                     for (auto it = myset.rbegin(); it != myset.rend(); ++it)
                                                                                 361
306
    bool cmp(const Point &a, const Point &b)
307
                                                                                 362
                                                                                         do_something(*it);
                                                                                 363
308
        if (a.x != b.x)
                                                                                     } // right -> left
309
                                                                                 364
                                                                                      for (auto &i : myset)
            return a.x < b.x;
310
                                                                                     {
311
        return a.y < b.y;
                                                                                 366
                                                                                          do_something(i);
312
                                                                                 367
    // method #3: functor
                                                                                     } // left->right shortcut
313
                                                                                      auto ret = myset.insert(5); // ret.first = iterator, ret.second =
    struct cmp
                                                                                 369
314
                                                                                          boolean (inserted / not inserted)
315
        bool operator()(const Point &a, const Point &b)
                                                                                      int count = mysert.erase(5); // count = how many items were erased
316
                                                                                 371 if (!myset.empty())
317
        {
```

```
372
                                                                                      /* BITSET UTILITY FUNCTIONS */
373
                                                                                 428
    // custom comparator 1: functor
                                                                                      /* ======= */
374
                                                                                 429
                                                                                     bitset<4> foo; // 0000
375
                                                                                     foo.size(); // 4
     {
376
                                                                                 431
        bool operator()(int i, int j) { return i > j; }
                                                                                      foo.set(); // 1111
377
                                                                                      foo.set(1, 0); // 1011
378
    set<int, cmp> myset;
                                                                                      foo.test(1); // false
                                                                                 434
379
     // custom comparator 2: function
                                                                                      foo.set(1); // 1111
380
    bool cmp(int i, int j) { return i > j; }
                                                                                     foo.test(1); // true
                                                                                 436
     set<int, bool (*)(int, int)> myset(cmp);
                                                                                 437
382
                                                                                      /* ======= */
383
     /* ======= */
                                                                                 439
                                                                                      /* RANDOM INTEGERS */
384
                                                                                      /* ======= */
     /* MAP UTILITY FUNCTIONS */
                                                                                 440
385
     /* ====== */
                                                                                      #include <cstdlib>
                                                                                     #include <ctime>
    struct Point
                                                                                 442
387
                                                                                 443
                                                                                      srand(time(NULL));
388
                                                                                      int x = rand() \% 100; // 0-99
        int x, y;
    };
                                                                                     int randBetween(int a, int b)
390
    bool operator<(const Point &a, const Point &b)
391
                                                                                 446
                                                                                         return a + (rand() % (1 + b - a));
392
                                                                                 447
        return a.x < b.x \mid \mid (a.x == b.x && a.y < b.y);
                                                                                     1
393
                                                                                 448
                                                                                 449
394
                                                                                      /* ====== */
    map<Point, int> ptcounts;
395
                                                                                 450
                                                                                     /* CLIMITS */
                                                                                 451
396
                                                                                     /* ====== */
397
     // inserting into map
                                                                                      #include <climits>
                                                                                 453
398
                                                                                      INT_MIN
                                                                                 454
399
     // method #1: operator[]
                                                                                      INT_MAX
     // it overwrites the value if the key already exists
                                                                                 456
                                                                                     UINT_MAX
401
    ptcounts[{1, 2}] = 1;
402
                                                                                      LONG_MIN
403
                                                                                      LONG_MAX
     // method #2: .insert(pair<key, value>)
                                                                                      ULONG_MAX
                                                                                 459
404
     // it returns a pair { iterator(key, value) , bool }
                                                                                      LLONG_MIN
405
     // if the key already exists, it doesn't overwrite the value
                                                                                      LLONG_MAX
                                                                                 461
     void update_count(Point &p)
407
                                                                                 462
                                                                                     ULLONG_MAX
408
                                                                                 463
        auto ret = ptcounts.emplace(p, 1);
                                                                                      /* ======= */
409
                                                                                 464
        // auto ret = ptcounts.insert(make_pair(p, 1)); //
                                                                                      /* Bitwise Tricks */
                                                                                 465
410
        if (!ret.second)
                                                                                      /* ======= */
411
                                                                                 466
            ret.first->second++;
412
                                                                                 467
                                                                                      // amount of one-bits in number
413
                                                                                 468
                                                                                      int __builtin_popcount(int x);
414
     // -----
                                                                                      int __builtin_popcountl(long x);
415
     // generating ids with map
                                                                                      int __builtin_popcountll(long long x);
416
                                                                                 471
     int get_id(string &name)
417
                                                                                 472
                                                                                      // amount of leading zeros in number
418
                                                                                 473
                                                                                      int __builtin_clz(int x);
        static int id = 0;
419
                                                                                      int __builtin_clzl(long x);
420
        static map<string, int> name2id;
        auto it = name2id.find(name);
                                                                                      int __builtin_clzll(ll x);
421
                                                                                 476
        if (it == name2id.end())
422
                                                                                 477
            return name2id[name] = id++;
                                                                                      // binary length of non-negative number
423
                                                                                 478
                                                                                      int bitlen(int x) { return sizeof(x) * 8 - __builtin_clz(x); }
424
        return it->second;
                                                                                      int bitlen(ll x) { return sizeof(x) * 8 - __builtin_clzll(x); }
425
                                                                                 480
426
                                                                                 481
```

```
// index of most significant bit
     int log2(int x) { return sizeof(x) * 8 - __builtin_clz(x) - 1; }
     int log2(11 x) { return sizeof(x) * 8 - __builtin_clzll(x) - 1; }
484
     // reverse the bits of an integer
486
     int reverse_bits(int x)
487
488
         int v = 0:
489
490
         while (x)
            v \iff 1, v \mid = x \& 1, x \implies 1;
491
492
         return v;
493
494
     // get string binary representation of an integer
495
     string bitstring(int x)
496
497
         int len = sizeof(x) * 8 - __builtin_clz(x);
498
         if (len == 0)
499
            return "0";
500
501
         char buff[len + 1];
502
         buff[len] = '\0';
503
         for (int i = len - 1; i \ge 0; --i, x >>= 1)
504
             buff[i] = (char)('0' + (x & 1));
505
         return string(buff);
506
507
508
     /* ====== */
509
     /* Hexadecimal Tricks */
510
     /* ======= */
511
512
     // get string hex representation of an integer
     string to_hex(int num)
514
515
         static char buff[100];
516
517
         static const char *hexdigits = "0123456789abcdef";
         buff[99] = '\0';
518
         int i = 98;
519
520
521
             buff[i--] = hexdigits[num & Oxf];
522
             num >>= 4:
523
         } while (num);
524
         return string(buff + i + 1);
525
526
527
     // ['0'-'9' 'a'-'f'] -> [0 - 15]
528
     int char_to_digit(char c)
529
530
         if ('0' \le c \&\& c \le '9')
531
532
            return c - '0';
         return 10 + c - 'a';
533
534
535
     /* ======= */
536
```

```
/* Other Tricks */
    /* ====== */
538
    // swap stuff
539
    int x = 1, y = 2;
    swap(x, y);
541
542
543
    /* TIPS
544
    // 1) do not use .emplace(x, y) if your struct doesn't have an explicit
546
        instead you can use .push({x, y})
    // 2) be careful while mixing scanf() with getline(), scanf will not
         consume \n unless
        you explicitly tell it to do so (e.g scanf("%d\n", &x)) )
549 //
```

3 General Algorithms

3.1 Search

4 DATA STRUCTURES -

- 3.2 Brute Force
- 4 Data Structures
- 4.1 Segment Tree
- 4.1.1 Lazy

```
#include "../../headers/headers.h"
2
   struct RSQ // Range sum query
3
4
        static ll const neutro = 0;
5
        static ll op(ll x, ll y)
6
7
8
            return x + y;
9
        static 11
10
        lazy_op(int i, int j, ll x)
11
12
            return (j - i + 1) * x;
13
14
15
    };
16
    struct RMinQ // Range minimum query
17
18
        static ll const neutro = 1e18;
19
20
        static ll op(ll x, ll y)
        {
            return min(x, y);
^{22}
23
^{24}
        static 11
        lazy_op(int i, int j, ll x)
```

```
26
            return x;
27
        }
28
    };
29
30
    template <class t>
31
    class SegTreeLazy
32
33
34
        vector<ll> arr, st, lazy;
        int n;
35
36
        void build(int u, int i, int j)
37
38
            if (i == j)
39
            {
40
                st[u] = arr[i];
41
42
                return;
43
            int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
44
            build(l, i, m);
45
            build(r, m + 1, j);
46
            st[u] = t::op(st[1], st[r]);
47
        }
48
49
        void propagate(int u, int i, int j, ll x)
50
51
            // nota, las operaciones pueden ser un and, or, ..., etc.
52
            st[u] += t::lazy_op(i, j, x); // incrementar el valor (+)
53
            // st[u] = t::lazy_op(i, j, x); // setear el valor
54
            if (i != j)
55
            {
56
                // incrementar el valor
57
                lazy[u * 2 + 1] += x;
58
                lazy[u * 2 + 2] += x;
59
                // setear el valor
60
                //lazy[u * 2 + 1] = x;
61
                //lazy[u * 2 + 2] = x;
62
63
            lazy[u] = 0;
64
        }
65
66
        ll query(int a, int b, int u, int i, int j)
67
68
            if (j < a or b < i)
69
                return t::neutro;
70
            int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
71
            if (lazy[u])
72
                propagate(u, i, j, lazy[u]);
73
            if (a \le i \text{ and } j \le b)
74
                return st[u];
75
            11 x = query(a, b, 1, i, m);
76
            11 y = query(a, b, r, m + 1, j);
77
            return t::op(x, y);
78
        }
79
80
```

```
81
         void update(int a, int b, ll value,
                     int u, int i, int j)
 82
 83
             int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
 84
             if (lazv[u])
 85
                 propagate(u, i, j, lazy[u]);
86
             if (a \le i \text{ and } j \le b)
87
                 propagate(u, i, j, value);
 88
             else if (i < a \text{ or } b < i)
 89
                 return;
90
91
             else
92
93
                 update(a, b, value, l, i, m);
94
                 update(a, b, value, r, m + 1, j);
                 st[u] = t::op(st[1], st[r]);
 95
            }
96
        }
97
 98
      public:
99
         SegTreeLazy(vector<11> &v)
100
101
102
             arr = v;
             n = v.size();
103
             st.resize(n * 4 + 5);
104
             lazy.assign(n * 4 + 5, 0);
105
             build(0, 0, n - 1);
106
        }
107
108
109
         11 query(int a, int b)
         {
110
             return query(a, b, 0, 0, n - 1);
111
        }
112
113
114
         void update(int a, int b, ll value)
         {
115
116
             update(a, b, value, 0, 0, n - 1);
117
118 };
 4.1.2 Iterative
 1 | #include "../../headers/headers.h"
 2
 3 // It requires a struct for a node (e.g. prodsgn)
    // A node must have three constructors
 5 //
             Arity 0: Constructs the identity of the operation (e.g. 1 for
          prodsgn)
    1//
 6
             Arity 1: Constructs a leaf node from the input
   1//
             Arity 2: Constructs a node from its children
 7
    1//
 8
   // Building the Segment Tree:
             Create a vector of nodes (use constructor of arity 1).
 10
    1//
             ST<miStructNode> mySegmentTree(vectorOfNodes);
11
12 // Update:
13 //
            mySegmentTree.set_points(index, myStructNode(input));
```

```
15
   //
            mySegmentTree.query(1, r); (It searches on the range [1,r), and
         returns a node.)
16
    // Logic And Query
17
    struct ANDQ
18
19
        ll value:
20
        ANDQ() { value = -111; }
21
        ANDQ(11 x) \{ value = x; \}
22
        ANDQ(const ANDQ &a,
23
             const ANDQ &b)
24
25
            value = a.value & b.value;
26
        }
27
    };
28
29
    // Interval Product (LiveArchive)
    struct prodsgn
31
32
        int sgn;
33
        prodsgn() { sgn = 1; }
34
        prodsgn(int x)
35
        {
36
            sgn = (x > 0) - (x < 0);
37
38
        prodsgn(const prodsgn &a,
39
                const prodsgn &b)
40
41
42
            sgn = a.sgn * b.sgn;
43
44
45
    // Maximum Sum (SPOJ)
    struct maxsum
47
48
        int first, second;
49
        maxsum() { first = second = -1; }
50
        maxsum(int x)
51
52
            first = x;
53
            second = -1;
54
        }
55
        maxsum(const maxsum &a,
56
                const maxsum &b)
57
58
            if (a.first > b.first)
59
60
                first = a.first;
61
                 second = max(a.second,
62
                              b.first);
63
64
65
            else
66
                first = b.first:
67
```

```
second = max(a.first,
 68
                               b.second):
 69
 70
        }
 71
 72
         int answer()
73
             return first + second;
74
 75
    };
 76
77
78
     // Range Minimum Query
     struct rming
 79
80
81
         int value;
         rming() { value = INT_MAX; }
 82
         rminq(int x) { value = x; }
83
84
         rminq(const rminq &a,
               const rminq &b)
 85
         {
 86
             value = min(a.value,
87
                         b.value);
 88
 89
     };
 90
91
     template <class node>
92
     class ST
93
94
         vector<node> t;
95
         int n;
96
97
     public:
 98
         ST(vector<node> &arr)
99
100
101
             n = arr.size();
             t.resize(n * 2);
102
             copy(arr.begin(), arr.end(), t.begin() + n);
103
             for (int i = n - 1; i > 0; --i)
104
105
                 t[i] = node(t[i << 1], t[i << 1 | 1]);
        }
106
107
         // 0-indexed
108
109
         void set_point(int p, const node &value)
110
111
             for (t[p += n] = value; p > 1; p >>= 1)
                 t[p >> 1] = node(t[p], t[p ^ 1]);
112
        }
113
114
         // inclusive exclusive, 0-indexed
115
         node query(int 1, int r)
116
117
118
             node ansl, ansr;
             for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1)
119
120
                 if (1 & 1)
121
                     ansl = node(ansl, t[l++]);
122
```

5 Dynamic Programming

6 Graphs

7 Mathematics

7.1 Modular Arithmetic

7.1.1 Chinese Remainder Theorem

```
1 | #include "../../headers/headers.h"
   11 inline mod(l1 x, l1 m) { return ((x %= m) < 0) ? x + m : x; }</pre>
   | 11 inline mul(11 x, 11 y, 11 m) { return (x * y) % m; }
   ll inline add(ll x, ll y, ll m) { return (x + y) % m; }
    // extended euclidean algorithm
    // finds g, x, y such that
    // a * x + b * y = g = GCD(a,b)
   11 gcdext(11 a, 11 b, 11 &x, 11 &y)
11
       11 r2, x2, y2, r1, x1, y1, r0, x0, y0, q;
12
       r2 = a, x2 = 1, y2 = 0;
13
       r1 = b, x1 = 0, y1 = 1;
14
       while (r1)
15
16
            q = r2 / r1;
17
            r0 = r2 \% r1;
18
            x0 = x2 - q * x1;
19
            y0 = y2 - q * y1;
20
            r2 = r1, x2 = x1, y2 = y1;
21
            r1 = r0, x1 = x0, y1 = y0;
22
23
24
       11 g = r2;
25
       x = x2, y = y2;
26
       if (g < 0)
            g = -g, x = -x, y = -y; // make sure g > 0
27
       // for debugging (in case you think you might have bugs)
       // assert (g == a * x + b * y);
29
       // assert (g == __gcd(abs(a),abs(b)));
30
31
32
33
    // CRT for a system of 2 modular linear equations
```

```
// We want to find X such that:
38 // 1) x = r1 \pmod{m1}
   // 2) x = r2 (mod m2)
  // The solution is given by:
41 // sol = r1 + m1 * (r2-r1)/g * x' (mod LCM(m1,m2))
  // where x' comes from
43 // m1 * x' + m2 * y' = g = GCD(m1,m2)
44 // where x' and y' are the values found by extended euclidean
        algorithm (gcdext)
   // Useful references:
   // https://codeforces.com/blog/entry/61290
47 // https://forthright48.com/chinese-remainder-theorem-part-1-coprime-
48 // https://forthright48.com/chinese-remainder-theorem-part-2-non-
         coprime-moduli
   // ** Note: this solution works if lcm(m1,m2) fits in a long long (64
   pair<11, 11> CRT(11 r1, 11 m1, 11 r2, 11 m2)
   | {
51
52
       11 g, x, y;
        g = gcdext(m1, m2, x, y);
53
       if ((r1 - r2) % g != 0)
           return {-1, -1}; // no solution
55
       11 z = m2 / g;
56
       11 \ lcm = m1 * z:
57
       ll sol = add(mod(r1, lcm), m1 * mul(mod(x, z), mod((r2 - r1) / g, z)
58
       // for debugging (in case you think you might have bugs)
       // assert (0 <= sol and sol < lcm);</pre>
       // assert (sol % m1 == r1 % m1);
       // assert (sol % m2 == r2 % m2);
62
        return {sol, lcm}; // solution + lcm(m1,m2)
63
64
   | }
65
    // CRT for a system of N modular linear equations
    // Args:
69
           r = array of remainders
           m = array of modules
71
           n = length of both arrays
   // Output:
74 //
           a pair {X, lcm} where X is the solution of the sytemm
           X = r[i] \pmod{m[i]} for i = 0 \dots n-1
75
           and lcm = LCM(m[0], m[1], ..., m[n-1])
           if there is no solution, the output is \{-1, -1\}
    // ** Note: this solution works if LCM(m[0],...,m[n-1]) fits in a long
         long (64 bits)
   pair<11, 11> CRT(11 *r, 11 *m, int n)
       11 r1 = r[0], m1 = m[0];
81
        repx(i, 1, n)
82
83
           11 r2 = r[i], m2 = m[i];
84
```

```
11 g, x, y;
            g = gcdext(m1, m2, x, y);
86
            if ((r1 - r2) % g != 0)
87
                return {-1, -1}; // no solution
88
           11 z = m2 / g:
89
90
            ll sol = add(mod(r1, lcm), m1 * mul(mod(x, z), mod((r2 - r1) / g
                 , z), z), lcm);
            r1 = sol;
92
            m1 = 1cm;
93
94
        // for debugging (in case you think you might have bugs)
        // assert (0 <= r1 and r1 < m1);</pre>
96
        // rep(i, n) assert (r1 % m[i] == r[i]);
97
        return {r1, m1};
99 }
```

7.2 Primality Checks

7.2.1 Miller Rabin

```
1 #include "../../headers/headers.h"
   ll mulmod(ull a, ull b, ull c)
4
        ull x = 0, y = a % c;
        while (b)
6
            if (b & 1)
               x = (x + y) \% c;
10
            y = (y << 1) \% c;
            b >>= 1;
11
12
        return x % c;
13
14
15
    11 fastPow(11 x, 11 n, 11 MOD)
16
17
       ll ret = 1;
18
        while (n)
19
20
            if (n & 1)
21
                ret = mulmod(ret, x, MOD);
22
            x = mulmod(x, x, MOD);
23
24
            n >>= 1;
       }
^{25}
        return ret;
26
27
28
    bool isPrime(ll n)
29
30
        vi a = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
31
32
        if (binary_search(a.begin(), a.end(), n))
33
            return true:
34
```

```
35
       if ((n \& 1) == 0)
36
37
           return false:
38
39
        for (11 m = n - 1; !(m \& 1); ++s, m >>= 1)
40
41
42
       int d = (n - 1) / (1 << s);
43
44
       for (int i = 0; i < 7; i++)
45
46
           11 fp = fastPow(a[i], d, n);
47
           bool comp = (fp != 1);
48
           if (comp)
49
                for (int j = 0; j < s; j++)
50
51
                    if (fp == n - 1)
52
53
                        comp = false;
54
                       break;
55
56
57
                    fp = mulmod(fp, fp, n);
58
59
            if (comp)
60
61
               return false;
62
       return true;
63
64 }
7.2.2 Sieve of Eratosthenes
1 | #include "../../headers/headers.h"
2
   // O(n log log n)
   vi sieve(int n)
4
5
        vi primes;
6
7
        vector<bool> is_prime(n + 1, true);
8
        int limit = (int)floor(sqrt(n));
9
       repx(i, 2, limit + 1) if (is_prime[i]) for (int j = i * i; j <= n; j
10
             += i)
           is_prime[j] = false;
11
12
       repx(i, 2, n + 1) if (is_prime[i]) primes.eb(i);
13
14
15
        return primes;
16 }
7.2.3 trialDivision
```

1 #include "../../headers/headers.h"

2

```
3 // O(sqrt(n)/log(sqrt(n))+log(n))
   vi trialDivision(int n, vi &primes)
5
        vi factors;
        for (auto p : primes)
            if (p * p > n)
                break:
10
            while (n \% p == 0)
11
12
13
                primes.pb(p);
                if ((n /= p) == 1)
14
                    return factors;
15
16
       }
17
        if (n > 1)
18
19
            factors.pb(n);
20
        return factors;
^{21}
22 }
```

7.3 Others

7.3.1 Polynomials

```
#include "../../headers/headers.h"
    template <class T>
    class Pol
5
6
   private:
        vector<T> cofs;
        int n;
    public:
10
        Pol(vector<T> cofs) : cofs(cofs)
11
        {
12
            this->n = cofs.size() - 1;
13
        }
14
15
        Pol<T> operator+(const Pol<T> &o)
16
17
            vector<T> n_cofs;
18
            if (n > o.n)
19
20
^{21}
                n_cofs = cofs;
                rep(i, o.n + 1)
22
23
                    n_cofs[i] += o.cofs[i];
24
25
            }
26
            else
27
28
                n_cofs = o.cofs;
                rep(i, n + 1)
30
```

```
31
                    n_cofs[i] += cofs[i];
32
33
34
            return Pol(n_cofs);
35
36
37
        Pol<T> operator-(const Pol<T> &o)
38
39
            vector<T> n_cofs;
40
            if (n > o.n)
41
42
                n_cofs = cofs;
43
44
                rep(i, o.n + 1)
45
                    n_cofs[i] -= o.cofs[i];
46
47
            }
48
            else
49
50
                n_cofs = o.cofs;
51
                rep(i, n + 1)
52
53
                    n_cofs[i] *= -1;
54
                    n_cofs[i] += cofs[i];
55
56
57
            return Pol(n_cofs);
58
       }
59
60
        Pol<T> operator*(const Pol<T> &o) //Use Fast Fourier Transform when
61
             we implement it
62
63
            vector<T> n_cofs(n + o.n + 1);
            rep(i, n + 1)
64
65
            {
                rep(j, o.n + 1)
66
67
                    n_{cofs}[i + j] += cofs[i] * o.cofs[j];
68
69
70
            return Pol(n_cofs);
71
       }
72
73
74
        Pol<T> operator*(const T &o)
75
            vector<T> n_cofs = cofs;
76
            for (auto &cof : n_cofs)
77
            {
78
                cof *= o;
79
80
            return Pol(n_cofs);
81
82
83
        double operator()(double x)
84
```

```
85
              double ans = 0;
 86
              double temp = 1;
 87
              for (auto cof : cofs)
 88
 89
                  ans += (double)cof * temp;
 90
 91
                  temp *= x;
 92
 93
              return ans;
         }
 94
 95
         Pol<T> integrate()
 96
 97
              vector\langle T \rangle n_cofs(n + 2);
 98
 99
              repx(i, 1, n_cofs.size())
100
                  n_{cofs[i]} = cofs[i - 1] / T(i);
101
102
              return Pol<T>(n_cofs);
103
         }
104
105
         double integrate(T a, T b)
106
107
              Pol<T> temp = integrate();
108
              return temp(b) - temp(a);
109
110
111
         friend ostream &operator<<(ostream &str, const Pol &a);</pre>
112
113
114
     ostream &operator<<(ostream &strm, const Pol<double> &a)
115
116
         bool flag = false;
117
118
         rep(i, a.n + 1)
119
              if (a.cofs[i] == 0)
120
                  continue;
121
122
              if (flag)
123
                  if (a.cofs[i] > 0)
124
                       strm << " + ";
125
                  else
126
                       strm << " - ";
127
              else
128
                  flag = true;
129
              if (i > 1)
130
131
                  if (abs(a.cofs[i]) != 1)
132
                       strm << abs(a.cofs[i]);</pre>
133
                  strm << "x^" << i;
134
135
              else if (i == 1)
136
137
                  if (abs(a.cofs[i]) != 1)
138
                       strm << abs(a.cofs[i]);</pre>
139
```

```
140 strm << "x";
141 }
142 else
143 {
144 strm << a.cofs[i];
145 }
146 }
147 return strm;
148 }
```

7.3.2 Factorial Factorization

```
#include "../../headers/headers.h"
2
   // O(n)
    umap<int, int> factorialFactorization(int n, vi &primes)
4
5
        umap<int, int> p2e;
6
        for (auto p : primes)
7
8
            if (p > n)
9
                break:
10
            int e = 0;
            int tmp = n;
12
            while ((tmp /= p) > 0)
13
                e += tmp;
14
            if (e > 0)
15
16
                p2e[p] = e;
17
        return p2e;
18
19 }
```

8 Geometry

8.1 Vectors/Points

```
#include "../../headers/headers.h"
2
    const double PI = acos(-1);
3
4
    struct vector2D
5
   {
6
        double x, y;
7
8
        vector2D &operator+=(const vector2D &o)
9
10
11
            this->x += o.x;
            this->y += o.y;
12
13
            return *this;
       }
14
15
        vector2D &operator==(const vector2D &o)
16
17
18
            this->x -= o.x:
```

```
19
            this->y -= o.y;
           return *this:
20
       }
21
22
        vector2D operator+(const vector2D &o)
23
24
           return \{x + o.x, y + o.y\};
25
       }
26
27
       vector2D operator-(const vector2D &o)
28
29
       {
           return \{x - o.x, y - o.y\};
30
       }
31
32
       vector2D operator*(const double &o)
33
       {
34
           return \{x * o, y * o\};
35
       }
36
37
        bool operator==(const vector2D &o)
38
39
           return x == o.x and y == o.y;
40
       }
41
42
        double norm2() { return x * x + y * y; }
43
        double norm() { return sqrt(norm2()); }
44
        double dot(const vector2D &o) { return x * o.x + y * o.y; }
45
       double cross(const vector2D &o) { return x * o.y - y * o.x; }
46
       double angle()
47
       {
48
            double angle = atan2(y, x);
49
            if (angle < 0)
50
               angle += 2 * PI;
51
           return angle;
52
       }
53
54
       vector2D Unit()
55
56
           return {x / norm(), y / norm()};
57
       }
58
    };
59
60
    /* Cross Product -> orientation of vector2D with respect to ray */
    /* ============ */
63
    // cross product (b - a) x (c - a)
    11 cross(vector2D &a, vector2D &b, vector2D &c)
65
66
       11 dx0 = b.x - a.x, dy0 = b.y - a.y;
67
       11 dx1 = c.x - a.x, dy1 = c.y - a.y;
68
       return dx0 * dy1 - dx1 * dy0;
       // return (b - a).cross(c - a); // alternatively, using struct
70
71 }
72
```

```
73 // calculates the cross product (b - a) x (c - a)
74 // and returns orientation:
75 // LEFT (1): c is to the left of ray (a -> b)
76 // RIGHT (-1): c is to the right of ray (a -> b)
   // COLLINEAR (0): c is collinear to ray (a -> b)
   // inspired by: https://www.geeksforgeeks.org/orientation-3-ordered-
    int orientation(vector2D &a, vector2D &b, vector2D &c)
79
80
        11 tmp = cross(a, b, c);
81
        return tmp < 0 ? -1 : tmp == 0 ? 0 : 1; // sign
82
83
84
85
    /* Check if a segment is below another segment (wrt a ray) */
    /* ========== */
    // i.e: check if a segment is intersected by the ray first
    // Assumptions:
   // 1) for each segment:
    // p1 should be LEFT (or COLLINEAR) and p2 should be RIGHT (or
         COLLINEAR) wrt rav
92 // 2) segments do not intersect each other
   // 3) segments are not collinear to the ray
94 // 4) the ray intersects all segments
    struct Segment
    {
96
        vector2D p1, p2;
97
98
    #define MAXN (int)1e6 //Example
    Segment segments[MAXN]; // array of line segments
    bool is_si_below_sj(int i, int j)
    { // custom comparator based on cross product
102
        Segment &si = segments[i];
103
        Segment &sj = segments[j];
104
        return (si.p1.x \ge sj.p1.x) ? cross(si.p1, sj.p2, sj.p1) > 0 : cross
105
             (sj.p1, si.p1, si.p2) > 0;
107
   // this can be used to keep a set of segments ordered by order of
         intersection
    // by the ray, for example, active segments during a SWEEP LINE
    set<int, bool (*)(int, int)> active_segments(is_si_below_sj); // ordered
110
    /* =========== */
111
    /* Rectangle Intersection */
    /* ======= */
113
    bool do_rectangles_intersect(vector2D &dl1, vector2D &ur1, vector2D &dl2
114
         , vector2D &ur2)
115 {
        return max(dl1.x, dl2.x) <= min(ur1.x, ur2.x) && max(dl1.y, dl2.y)
116
             <= min(ur1.y, ur2.y);
117
    /* ======= */
119
120 /* Line Segment Intersection */
```

```
/* ======= */
     // returns whether segments p1q1 and p2q2 intersect, inspired by:
    // https://www.geeksforgeeks.org/check-if-two-given-line-segments-
123
     bool do_segments_intersect(vector2D &p1, vector2D &q1, vector2D &p2,
124
          vector2D &q2)
125
         int o11 = orientation(p1, q1, p2);
126
         int o12 = orientation(p1, q1, q2);
127
        int o21 = orientation(p2, q2, p1);
128
129
         int o22 = orientation(p2, q2, q1);
         if (o11 != o12 and o21 != o22) // general case -> non-collinear
130
              intersection
             return true;
131
132
        if (o11 == o12 \text{ and } o11 == 0)
        { // particular case -> segments are collinear
133
             vector2D dl1 = \{\min(p1.x, q1.x), \min(p1.y, q1.y)\};
134
             vector2D ur1 = {max(p1.x, q1.x), max(p1.y, q1.y)};
135
             vector2D d12 = \{\min(p2.x, q2.x), \min(p2.y, q2.y)\};
136
             vector2D ur2 = \{\max(p2.x, q2.x), \max(p2.y, q2.y)\};
137
             return do_rectangles_intersect(dl1, ur1, dl2, ur2);
138
        }
139
        return false;
140
141
142
     /* ======= */
143
     /* Circle Intersection */
144
     /* ======= */
145
     struct Circle
146
147
148
         double x, y, r;
149
     bool is_fully_outside(double r1, double r2, double d_sqr)
150
151
         double tmp = r1 + r2;
152
153
        return d_sqr > tmp * tmp;
154
     bool is_fully_inside(double r1, double r2, double d_sqr)
155
156
        if (r1 > r2)
157
             return false;
158
         double tmp = r2 - r1;
159
         return d_sqr < tmp * tmp;</pre>
160
161
     bool do_circles_intersect(Circle &c1, Circle &c2)
162
163
        double dx = c1.x - c2.x;
164
         double dy = c1.y - c2.y;
165
         double d_sqr = dx * dx + dy * dy;
166
         if (is_fully_inside(c1.r, c2.r, d_sqr))
167
168
             return false;
        if (is_fully_inside(c2.r, c1.r, d_sqr))
169
170
             return false;
         if (is_fully_outside(c1.r, c2.r, d_sqr))
171
             return false;
172
```

```
173
        return true;
174
    ۱,
175
     /* ====== */
176
     /* vector2D - Line distance */
177
     /* ====== */
     // get distance between p and projection of p on line <- a - b ->
    double point_line_dist(vector2D &p, vector2D &a, vector2D &b)
180
181
        vector2D d = b - a;
182
183
        double t = d.dot(p - a) / d.norm2();
        return (a + d * t - p).norm();
185
186
     /* ======= */
187
    /* vector2D - Segment distance */
188
     /* ======= */
     // get distance between p and truncated projection of p on segment a ->
190
    double point_segment_dist(vector2D &p, vector2D &a, vector2D &b)
191
192
        if (a == b)
193
            return (p - a).norm(); // segment is a single vector2D
194
        vector2D d = b - a:
                                 // direction
195
        double t = d.dot(p - a) / d.norm2();
196
        if (t <= 0)
197
            return (p - a).norm(); // truncate left
198
199
            return (p - b).norm(); // truncate right
200
        return (a + d * t - p).norm();
201
202
203
204
     /* Straight Line Hashing (integer coords) */
205
    /* ======== */
206
    // task: given 2 points p1, p2 with integer coordinates, output a unique
     // representation \{a,b,c\} such that a*x + b*y + c = 0 is the equation
     // of the straight line defined by p1, p2. This representation must be
     // unique for each straight line, no matter which p1 and p2 are sampled.
211
    struct Line
    {
212
213
        int a, b, c;
214
    |};
    int gcd(int a, int b)
    { // greatest common divisor
217
        a = abs(a);
        b = abs(b):
218
219
        while (b)
220
221
            int c = a:
222
            a = b;
223
            b = c \% b:
        }
224
225
        return a;
226 }
```

```
227
    Line getLine(vector2D p1, vector2D p2)
228
         int a = p1.y - p2.y;
229
         int b = p2.x - p1.x;
230
         int c = p1.x * (p2.y - p1.y) - p1.y * (p2.x - p1.x);
231
         int sgn = (a < 0 | | (a == 0 \&\& b < 0)) ? -1 : 1;
232
         int f = gcd(a, gcd(b, c)) * sgn;
233
         a /= f:
234
         b /= f;
235
         c /= f;
236
237
         return {a, b, c};
238 }
```

8.2 Calculate Areas

8.2.1 Integration via Simpson's Method

```
1 #include "../../headers/headers.h"
    //0(Evaluate f)=g(f)
    //Numerical Integration of f in interval [a,b]
    double simpsons_rule(function<double(double)> f, double a, double b)
6
        double c = (a + b) / 2;
        double h3 = abs(b - a) / 6;
        return h3 * (f(a) + 4 * f(c) + f(b));
10
11
    //0(n g(f))
12
    //Integrate f between a and b, using intervals of length (b-a)/n
    double simpsons_rule(function<double(double)> f, double a, double b, int
          n)
15
        //n sets the precision for the result
16
        double ans = 0;
17
        double step = 0, h = (b - a) / n;
18
        rep(i, n)
19
20
            ans += simpsons_rule(f, step, step + h);
21
            step += h;
22
        }
23
24
        return ans;
25 }
```

8.2.2 Green's Theorem

```
#include "../../headers/headers.h"

// O(1)
// Circle Arc
double arc(double theta, double phi)
{
    }
}
```

```
10 // Line
11 double line(double x1, double y1, double x2, double y2)
12 {
13 }
```

8.3 Pick's Theorem

Given a simple polygon (no self intersections) in a lattice such that all vertices are grid points. Pick's theorem relates the Area A, points inside of the polygon i and the points of the border of the polygon b, in the following way:

$$A = i + \frac{b}{2} - 1$$

9 Strings

9.1 Trie

```
1 | #include "../../headers/headers.h"
2
    /* Implementation from: https://pastebin.com/fyqsH65k */
3
   struct TrieNode
4
   {
5
        int leaf; // number of words that end on a TrieNode (allows for
6
             duplicate words)
7
        int height; // height of a TrieNode, root starts at height = 1, can
             be changed with the default value of constructor
        // number of words that pass through this node,
 8
        // ask root node for this count to find the number of entries on the
9
        // all nodes have 1 as they count the words than end on themselves (
10
             ie leaf nodes count themselves)
11
        TrieNode *parent; // pointer to parent TrieNode, used on erasing
12
        map<char, TrieNode *> child;
13
        TrieNode(TrieNode *parent = NULL, int height = 1):
14
            parent(parent),
15
            leaf(0),
16
            height(height),
17
            count(0), // change to -1 if leaf nodes are to have count 0
18
                 insead of 1
19
            child()
        {}
20
21
    };
^{22}
23
     * Complexity: O(|key| * log(k))
25
   TrieNode *trie_find(TrieNode *root, const string &str)
26
27
        TrieNode *pNode = root;
28
```

```
for (string::const_iterator key = str.begin(); key != str.end(); key
29
        {
30
            if (pNode->child.find(*key) == pNode->child.end())
31
                return NULL:
32
            pNode = pNode->child[*key];
33
        }
34
        return (pNode->leaf) ? pNode : NULL; // returns only whole word
35
        // return pNode; // allows to search for a suffix
36
37
38
39
     * Complexity: O(|key| * log(k))
40
41
42
    void trie_insert(TrieNode *root, const string &str)
43
44
        TrieNode *pNode = root;
        root -> count += 1;
45
        for (string::const_iterator key = str.begin(); key != str.end(); key
46
        {
47
            if (pNode->child.find(*key) == pNode->child.end())
48
                pNode->child[*key] = new TrieNode(pNode, pNode->height + 1);
49
            pNode = pNode->child[*key];
50
            pNode -> count += 1;
51
       }
52
        pNode->leaf += 1;
53
54
55
56
     * Complexity: O(|key| * log(k))
57
58
    void trie_erase(TrieNode *root, const string &str)
59
60
        TrieNode *pNode = root;
61
        string::const_iterator key = str.begin();
62
        for (; key != str.end(); key++)
63
64
            if (pNode->child.find(*key) == pNode->child.end())
65
66
            pNode = pNode->child[*key];
67
68
        pNode->leaf -= 1;
69
        pNode->count -= 1;
70
        while (pNode->parent != NULL)
71
72
            if (pNode->child.size() > 0 || pNode->leaf)
73
74
            pNode = pNode->parent, key--;
75
            pNode->child.erase(*key);
76
            pNode->count -= 1;
77
78
79
```

```
#include "../../headers/headers.h"
2
3
   vi prefix(string &S)
   {
4
        vector<int> p(S.size());
5
6
        for (int i = 1; i < S.size(); ++i)</pre>
7
8
            p[i] = p[i - 1];
9
            while (p[i] > 0 \&\& S[p[i]] != S[i])
10
                p[i] = p[p[i] - 1];
11
            if (S[p[i]] == S[i])
12
13
                p[i]++;
14
15
        return p;
   }
16
17
    vi KMP(string &P, string &S)
18
19
        vector<int> pi = prefix(P);
20
        vi matches;
21
        int n = S.length(), m = P.length();
22
        int j = 0, ans = 0;
23
        for (int i = 0; i < n; ++i)
24
25
            while (j > 0 && S[i] != P[j])
26
                j = pi[j - 1];
27
            if (S[i] == P[i])
28
29
                ++j;
30
            if (j == P.length())
31
32
                /* This is where KMP found a match
33
                 * we can calculate its position on S by using i - m + 1
                 * or we can simply count it
35
36
                ans += 1; // count the number of matches
37
                matches.eb(i - m + 1); // store the position of those
38
                // return; we can return on the first match if needed
39
                // this must stay the same
40
                j = pi[j - 1];
41
            }
42
43
        return matches; // can be modified to return number of matches or
44
             location
45 }
```