1 Introduction

The widespread adoption of electric vehicles (EVs) in urban logistics has been propelled by the urgent need for sustainable transportation solutions amid environmental and regulatory pressures. This shift has transformed traditional route planning challenges, such as the Traveling Salesperson Problem (TSP), by introducing EV-specific constraints like limited battery capacity and the need for efficient charging infrastructure. A key innovation in this domain is the development of wireless charging lanes—road segments that enable EVs to recharge dynamically while in motion—offering a promising solution to alleviate range anxiety and boost operational efficiency.

This study proposes the **Electric Traveling Salesperson Problem with Wireless Charging Lanes (ETSP-WCL)**, an extension of the classic TSP tailored for EVs. The ETSP-WCL leverages both stationary charging stations and wireless charging lanes to minimize total operational costs, encompassing energy consumption from charging and time-related expenses tied to operational efficiency. By integrating wireless charging infrastructure, the model optimizes route planning to balance energy use and travel time, providing a sustainable framework for urban EV logistics. This research draws on prior advancements in network optimization and charging infrastructure deployment, contributing a practical approach to enhance the scalability and efficiency of EV-based delivery systems.

2 Literature Review

The field of EV routing has evolved significantly, driven by the need to address sustainability in transportation systems. Initial efforts, such as the **Green Vehicle Routing Problem (GVRP)** introduced by [2], tackled range limitations and sparse refueling options for alternative fuel vehicles. This was followed by the **Electric Vehicle Routing Problem (EVRP)** by [11], which incorporated full recharges, and later by [9]'s **Electric Traveling Salesperson Problem (ETSP)**, which allowed partial recharges and employed a Variable Neighborhood Search (VNS) approach. Further refinements came with [6], which developed an Adaptive Large Neighborhood Search algorithm for EVRP with partial recharges, emphasizing flexible charging strategies.

Charging infrastructure advancements have also shaped EV routing research. Studies like [3] and [7] explored diverse recharge technologies, while [8] and [4] introduced nonlinear charging functions to model real-world battery dynamics. Location-routing problems, addressed by [5] and [10], optimized the placement of charging or battery swapping stations. A notable leap forward came with [1], which integrated wireless charging lanes into transportation networks, using a user equilibrium model to optimize their deployment. Recent works, such as [12] on nonlinear battery behavior in EVRP and [13] on electric bus scheduling, have continued to broaden the scope of EV routing challenges.

Despite these advances, the integration of wireless charging lanes into EV routing problems, particularly without factoring in battery degradation, remains underexplored. The **ETSP-WCL** addresses this gap by focusing on energy and time cost optimization, building on the wireless charging insights of [1] and the computational efficiency of matheuristic approaches like [14]. The table below summarizes key studies, highlighting the position of this work.

3 Problem Settings

The **Electric Traveling Salesperson Problem with Wireless Charging Lanes (ETSP-WCL)** extends the traditional Electric Traveling Salesperson Problem (ETSP) by incorporating dynamic wireless charging lanes into the optimization framework. The objective is to determine an optimal route for a single electric vehicle (EV) that visits all required customer locations exactly once, minimizing total operational costs. These costs are defined as:

- **Energy Cost**: Expenses associated with energy consumed from stationary charging stations and wireless charging lanes.
- **Time Cost**: Costs proportional to the total tour time, reflecting the operational value of time in logistics.

In the ETSP-WCL, the EV can recharge at stationary charging stations and dynamically on wireless charging lanes, where it can adjust its travel speed to balance energy gain and travel time. The model tracks the state of charge to ensure it remains within acceptable limits but does not account for battery degradation effects. Key decisions in the solution include:

- **Route Selection**: The sequence of customer visits, including optional detours to access charging infrastructure.
- **Charging Decisions**: Where and how much to charge at stationary stations.
- **Energy Gain on Wireless Lanes**: Amount of energy recharged dynamically while traversing charging lanes.
- **Speed on Wireless Lanes**: Optimal travel speed to trade off between charging efficiency and time spent.

This formulation leverages wireless charging infrastructure to enhance energy efficiency and operational performance, offering a sustainable approach to EV routing in urban logistics.

4 Problem Description

The Electric Traveling Salesperson Problem with Wireless Charging Lanes (ETSP-WCL) involves a single electric vehicle (EV) that must:

- Start at a depot (node 0).
- Visit each customer in a set N exactly once.

- Optionally visit charging stations in a set F multiple times to manage its battery state of charge (SOC).
- Return to the depot (node $n+1$, physically the same as node 0).
- Utilize wireless charging lanes on specific arcs for dynamic recharging.
- Minimize total operational costs, including energy costs from stationary charging (at depots and charging stations), wireless charging, and time costs.

To allow multiple visits to charging stations, each station is duplicated into multiple copies, ensuring each visit is distinct and adheres to the single-option charging rule.

5 Model Formulation

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5.1 Sets and Indices

- N: Set of customer nodes.
- F: Set of physical charging stations.
- $F' = \{f_k \mid f \in F, k = 1, \ldots, m\}$: Set of duplicated charging station nodes, where m is the maximum number of visits per station (e.g., $m = |N|$).
- $V' = \{0\} \cup N \cup F' \cup \{n+1\}$: Set of all nodes, with 0 as the starting depot and $n+1$ as the ending depot.
- A': Set of arcs connecting nodes in V'.
- $A'^w \subseteq A'$: Subset of arcs with wireless charging capability.
- K: Set of charging options (0 for no charging, 1 for slow charging, 2 for fast charging).

Each $f_k \in F'$ represents a potential visit to physical station f. Arcs in A' allow feasible routes. A'^w includes wireless charging arcs.

5.2 Parameters

- c_{ik}: Cost per unit of energy for charging option k at node $i \in F'$.
- r_{ik}: Charging rate for option k at node $i \in F'$.
- τ_i: Service time at customer node $i \in N$.
- s_{ij}: Travel time on arc $(i, j) \in A'$.
- d_{ij}: Distance of arc $(i, j) \in A'$.
- h: Energy consumption rate per unit distance.
- Q: Battery capacity. available for wireless charging.
- $\beta_{ij} = \eta_w P_{ij} \alpha_{ij}$: the effective charging rate (energy per unit time) for arc (i, j), considering efficiency, power, and the available charging fraction, where
  - η_w: Wireless charging efficiency.
  - P_{ij}: Wireless charging power on arc $(i, j) \in A'^w$.
  - α_{ij}: Fraction of arc $(i, j) \in A'^w$
- c_w: Cost per unit of energy for wireless charging.
- c_t: Cost per unit of time for the total tour duration.
- M: A large constant for big-M constraints.

5.3 Decision Variables

- **Routing:**
  - $x_{ij} \in \{0, 1\}$: 1 if arc $(i, j) \in A'$ is traversed, 0 otherwise.
- **Charging:**
  - $\phi_i \geq 0$: Charging time at node $i \in F'$.
  - $w_{ik} \in \{0, 1\}$: 1 if charging option $k \in K$ is selected at node $i \in F'$, 0 otherwise.
- **Time:**
  - $t_i \geq 0$: Arrival time at node $i \in V'$.
  - $s_{ij} \geq 0$: Travel time on arc $(i, j) \in A'$.
- **State of Charge (SOC):**
  - $y_i^a \geq 0$: SOC upon arrival at node $i \in V'$.
  - $y_i^d \geq 0$: SOC upon departure from node $i \in V'$.
- **Wireless Charging:**
  - $z_{ij} \in \{0, 1\}$: 1 if the EV charges on wireless arc $(i, j) \in A'^w$, 0 otherwise.

Key Feature: Variables like ϕ_i, w_{ik}, t_i, y_i^a, and y_i^d are defined per node $i \in F'$, enabling unique values for each visit to a charging station.

5.4 Objective Function

Minimize the total operational cost:

$$\min \sum_{i \in F'} \sum_{k \in K} c_{ik} r_{ik} \phi_i w_{ik} + c_w \sum_{(i,j) \in A'^w} \beta_{ij} s_{ij} z_{ij} + c_t t_{n+1} \tag{1}$$

- First term: Cost of energy from stationary charging at depots and charging station copies.
- Second term: Cost of energy from wireless charging on arcs.
- Third term: Time cost based on arrival time at the ending depot.

5.5 Flow and Routing Constraints

- Depart from depot 0 exactly once:

$$\sum_{j \in V'} x_{0j} = 1 \tag{2}$$

- Arrive at depot $n+1$ exactly once:

$$\sum_{i \in V'} x_{i,n+1} = 1 \tag{3}$$

- Each customer $i \in N$ visited exactly once (outgoing):

$$\sum_{j \in V'} x_{ij} = 1, \quad \forall i \in N \tag{4}$$

- Each customer $i \in N$ visited exactly once (incoming):

$$\sum_{j \in V'} x_{ji} = 1, \quad \forall i \in N \tag{5}$$

- Flow conservation at charging station copies $i \in F'$:

$$\sum_{j \in V'} x_{ji} = \sum_{j \in V'} x_{ij} \leq 1, \quad \forall i \in F' \tag{6}$$

  Note: Each copy $i \in F'$ is visited at most once, but multiple copies allow multiple visits to the same station.

5.6 Time Feasibility Constraints

- Time progression for customer nodes $i \in N$:

$$t_j \geq t_i + \tau_i + s_{ij} - M(1 - x_{ij}), \quad \forall (i,j) \in A' \tag{7}$$

- Time progression for charging nodes $i \in F'$:

$$t_j \geq t_i + \phi_i + s_{ij} - M(1 - x_{ij}), \quad \forall (i,j) \in A' \tag{8}$$

- For $i = 0$ (starting depot):

$$t_j \geq t_i + s_{ij} - M(1 - x_{ij}), \quad \forall (i,j) \in A' \tag{9}$$

- Starting condition:

$$t_0 = 0 \tag{10}$$

- Travel time bounds:

$$\frac{d_{ij}}{U_{\max,ij}} x_{ij} \leq s_{ij} \leq \frac{d_{ij}}{U_{\min,ij}} x_{ij}, \quad \forall (i,j) \in A' \tag{11}$$

5.7 SOC Consistency Constraints

- Non-wireless arcs $(i,j) \in A' \setminus A'^w$:

$$y_j^a \leq y_i^d - h d_{ij} + Q(1 - x_{ij}) \tag{12}$$

- Wireless arcs $(i,j) \in A'^w$:

$$y_j^a \leq y_i^d - h d_{ij} + \beta_{ij} s_{ij} z_{ij} + Q(1 - x_{ij}) \tag{13}$$

- SOC update at charging nodes $i \in F'$:

$$y_i^d = y_i^a + \sum_{k \in K} r_{ik} \phi_i w_{ik} \tag{14}$$

- SOC unchanged at customer nodes $i \in N$:

$$y_i^d = y_i^a \tag{15}$$

- At starting depot $i = 0$:

$$y_0^d = Q, \quad y_0^a = y_0^d \tag{16}$$

  Starts fully charged, no charging occurs

- At ending depot $i = n + 1$:

$$y_{n+1}^d = y_{n+1}^a \tag{17}$$

  No charging upon arrival.

5.8 Charging Rate Constraints

- Select at most one charging option per visit:

$$\sum_{k\in K} w_{ik} = \sum_{j\in V'} x_{ji}, \quad \forall i \in F' \tag{18}$$

Note: If node i is visited ($\sum_j x_{ji} = 1$), at most one $w_{ik} = 1$.

5.9 Wireless Charging Decision

- Charge only if arc is traversed:

$$z_{ij} \le x_{ij}, \quad \forall (i,j) \in A'^{w} \tag{19}$$

5.10 Battery Capacity and SOC Limits

- SOC bounds:

$$0 \le y_i^a \le Q, \quad 0 \le y_i^d \le Q, \quad \forall i \in V' \tag{20}$$

5.11 Domain Constraints

- Variable domains:

$$\begin{gathered} x_{ij} \in \{0,1\}, \quad z_{ij} \in \{0,1\}, \quad w_{ik} \in \{0,1\}, \\ t_i \ge 0, \quad \phi_i \ge 0, \quad s_{ij} \ge 0, \quad y_i^a \ge 0, \quad y_i^d \ge 0 \end{gathered}$$

References

[1] Z. Chen, F. He, and Y. Yin. Optimal deployment of charging lanes for electric vehicles in transportation networks. *Transportation Research Part B: Methodological*, 91:344–365, 2016.

[2] S. Erdoğan and E. Miller-Hooks. A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):100–114, 2012.

[3] Á. Felipe, M. T. Ortuño, G. Righini, and G. Tirado. A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. *Transportation Research Part E: Logistics and Transportation Review*, 71:111–128, 2014.

[4] A. Froger, J. E. Mendoza, O. Jabali, and G. Laporte. Improved formulations and algorithmic components for the electric vehicle routing problem with nonlinear charging functions. *Computers & Operations Research*, 104:256–294, 2019.

[5] J. Hof, M. Schneider, and D. Goeke. Solving the battery swap station location-routing problem with capacitated electric vehicles using an avns algorithm for vehicle-routing problems with intermediate stops. *Transportation Research Part B: Methodological*, 97:102–112, 2017.

[6] M. Keskin and B. Çatay. Partial recharge strategies for the electric vehicle routing problem with time windows. *Transportation Research Part C: Emerging Technologies*, 65:111–127, 2016.

[7] M. Keskin and B. Çatay. A matheuristic method for the electric vehicle routing problem with time windows and fast chargers. *Computers & Operations Research*, 100:172–188, 2018.

[8] A. Montoya, C. Guéret, J. E. Mendoza, and J. G. Villegas. The electric vehicle routing problem with nonlinear charging function. *Transportation Research Part B: Methodological*, 103:87–110, 2017.

[9] R. Roberti and M. Wen. The electric traveling salesman problem with time windows. *Transportation Research Part E: Logistics and Transportation Review*, 89:32–52, 2016.

[10] M. Schiffer and G. Walther. The electric location routing problem with time windows and partial recharging. *European Journal of Operational Research*, 260(3):995–1013, 2017.

[11] M. Schneider, A. Stenger, and D. Goeke. The electric vehicle routing problem with time windows and recharging stations. *Transportation Science*, 48(4):500–520, 2014.

[12] Y. Zang, M. Wang, and H. Qi. A column generation tailored to electric vehicle routing problem with nonlinear battery depreciation. *Computers & Operations Research*, 137:105527, 2022.

[13] Y. Zhou, Q. Meng, and G. P. Ong. Electric bus charging scheduling for a single public transport route considering nonlinear charging profile and battery degradation effect. *Transportation Research Part B: Methodological*, 159:49–75, 2022.

[14] R. B. İslim and B. Çatay. An effective matheuristic approach for solving the electric traveling salesperson problem with time windows and battery degradation. *Engineering Applications of Artificial Intelligence*, 132:107943, 2024.