

Date: \_\_\_\_\_

Name: \_\_\_\_\_

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Answer all questions in the area provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum Mark: 10]

**In this question you will explore the convergence of infinite sequences and a mathematical test for determining if a specific sequence converges.**

Consider the following sequence:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$$

(a) (1 point) Find the common ratio between consecutive terms of the sequence.

**Solution:** The common ratio is  $\frac{1}{2}$ , or 0.5

A1

(b) (1 point) Hence, find the 4<sup>th</sup> and 5<sup>th</sup> terms of the sequence.

**Solution:** The fourth and fifth terms of the sequence are  $\frac{1}{16}$  and  $\frac{1}{32}$ , respectively.

A1

(c) (1 point) Hence, state whether the sequence converges (i.e. approaches a finite value) or diverges (i.e. tends to infinity).

**Solution:** The sequence converges.

A1

(d) (1 point) Hence, state a general formula for the  $k^{\text{th}}$  element of the sequence.

**Solution:** One possible formula is

$$T_k = \left(\frac{1}{2}\right)^k$$

A1

(e) (1 point) Find a general formula for the following sequence:

$$1, 2, 4, 8 \dots$$

**Solution:** One possible formula is

$$T_k = 2^{k-1}$$

A1

- (f) (1 point) Hence, state whether the sequence converges or diverges.

**Solution:** The sequence diverges.

A1

- (g) (1 point) Calculate the ratio between consecutive terms for terms 1 to 5 in the following sequence in decimal:

$$1, 4, 9, 16 \dots$$

where  $T_k = k^2$

**Solution:** The ratios are as follows:

$$4, 2.25, 1.777 \dots, 1.5625$$

A1

- (h) (2 points) Show that the equation  $y = \frac{(x+1)^2}{x^2}$  has a horizontal asymptote at  $y = 1$

**Solution:**

$$y = \frac{(x+1)^2}{x^2} \tag{1}$$

$$= \frac{x^2 + 2x + 1}{x^2} \tag{2}$$

$$= 1 + \frac{2}{x} + \frac{1}{x^2} \tag{3}$$

M1

Since  $\frac{2}{x}$  and  $\frac{1}{x^2}$  both have asymptotes at  $y = 0$ , then  $1 + \frac{2}{x} + \frac{1}{x^2}$  has an asymptote at  $y = 1$ .

R1

- (i) (1 point) Hence, state what the ratio between terms approaches in the sequence in (g)

**Solution:** The ratio between terms approaches 1.

A1