Neural Circuits for Fast Poisson Compressed Sensing the in the Olfactory Bulb

As reviewed by Chaebeom Sheen

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cauchybs@snu.ac.kr

서 울 대 학 교
SEOUL NATIONAL UNIVERSITY

1. Introduction

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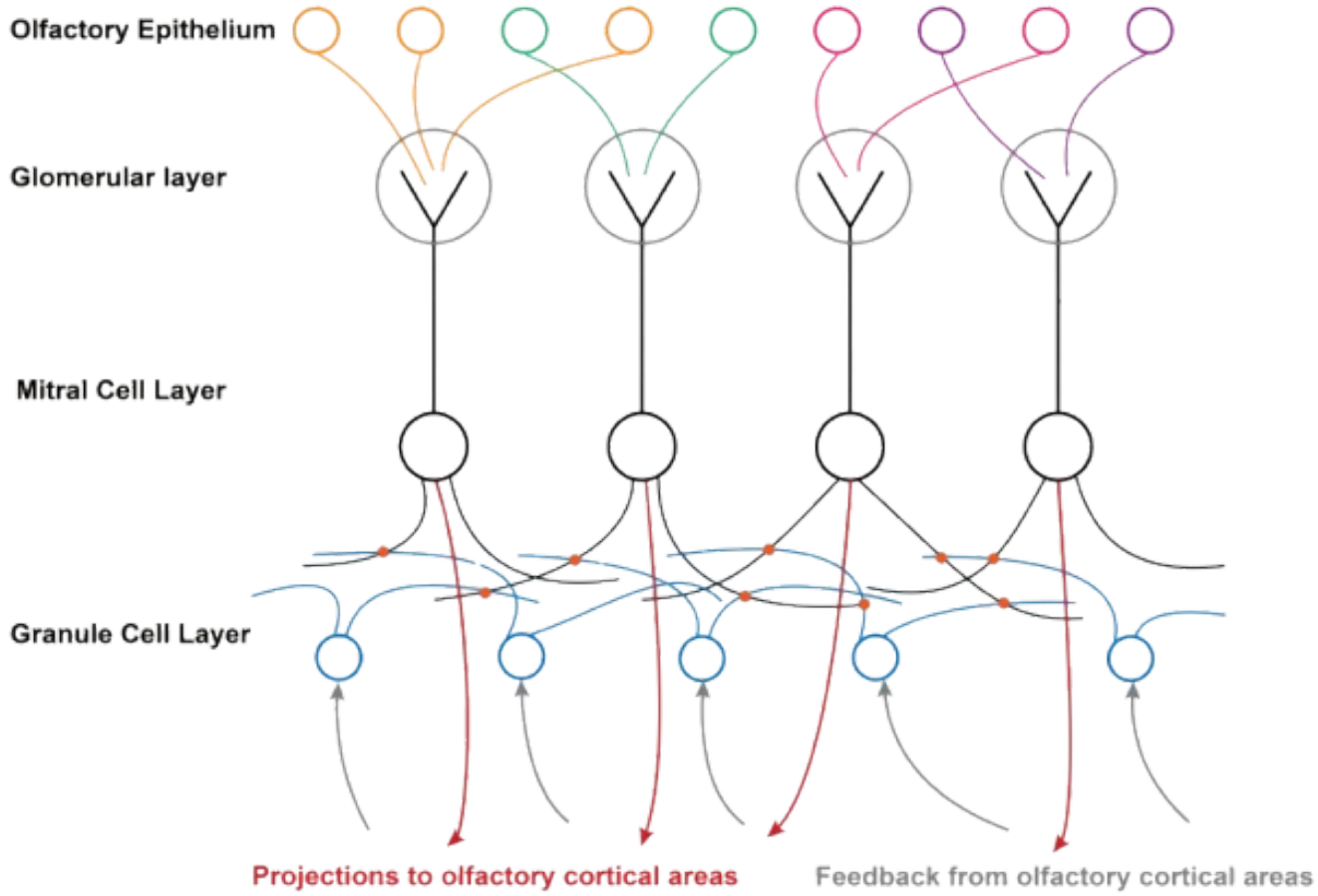
- Vision and audition are the most studied sensory modalities in neuroscience.Encoding for these modalities can be derived from *orientation* or *frequency*, which map directly onto neural representations.
- The neural encoding of olfaction is less understood.
- The authors propose a Poisson Compressed Sensing Model to understand the neural encoding of olfaction in the mammalian olfactory bulb (primarily mouse and human).
  - Why Compressed Sensing?: With only a few hundred different types of olfactory receptors, humans can distinguish between millions of different odors.
  - **Strengths**:
    a. The PCS model *maps directly* onto the circuits of the bulb, without requiring an axis-aligned coding which is biologically implausible.
    b. The PCS model allows fast, accurate inference by considering the geometry of the olfactory bulb.

1. Anatomy of the Olfactory Bulb



2. Methods

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- An important distinction between Poisson compressed sensing and earlier models is that earlier, Gaussian models assumed axis-aligned coding (where one granular neuron is responsible for one odorant), which is biologically implausible.
- The goal of the PCS model is Bayesian inference of the odorant concentration $\mathbf{c} \in \mathbb{R}^{n_{\text{odor}}}$ given a single spike train in the oral epithelium $\mathbf{s} \in \mathbb{R}^{n_{\text{OSN}}}$.
- It is assumed that the mean activity of the OSN is given by a linear-affine function,

$$\mathbf{s}|\mathbf{c} \sim \text{Poisson}(\mathbf{r} + \mathbf{A}\mathbf{c})$$

$$\mathbf{c} \sim \text{Gamma}(\boldsymbol{\alpha}, \boldsymbol{\lambda})$$

- Given this likelihood and prior, the MAP (maximum a posteriori) estimate of the concentration is given by gradient ascent as

$$\dot{\mathbf{c}}(t) = \nabla_c \left[\log p(\mathbf{s}|\mathbf{c})p(\mathbf{c})\right] + \boldsymbol{\eta}(t)$$

- Where $\boldsymbol{\eta}(t)$ is n_{odor}-dimensional white noise with zero mean and $\mathbb{E}[\boldsymbol{\eta}_j(t)\boldsymbol{\eta}_{j'}(t')] = 2\delta_{jj'}\delta(t-t')$.

2. Methods

- Based on our prior and likelihood, computing the gradient of the log posterior is straightforward

$$\nabla_{\mathbf{c}} \log p(\mathbf{s}|\mathbf{c}) = \mathbf{A}^T \left(\frac{\mathbf{s}}{\mathbf{r} + \mathbf{A}\mathbf{c}} - 1 \right)$$

$$\nabla_{\mathbf{c}} \log p(\mathbf{c}) = \frac{\boldsymbol{\alpha} - 1}{\mathbf{c}} - \boldsymbol{\lambda}$$

- However, in this setup there is one-to-one mapping between nurons and odorants. Assume a population n_g of neurons which map to $\mathbf{c}$ through a matrix $\mathbf{\Gamma} \in \mathbb{R}^{n_{\text{odor}} \times n_g}$ so that $\mathbf{c} = \mathbf{\Gamma}\mathbf{g}$.
- A classic result in stochastic gradient MCMC tells us that given $\mathbf{\Gamma}\mathbf{\Gamma}^T$ is positive definite and τ_g is the time constant of the neurons, the following dynamics will converge to a MAP.

$$\tau_g \dot{\mathbf{g}}(t) = \Gamma^T \nabla_{\mathbf{g}} \log p(\mathbf{g}|\mathbf{s}) + \boldsymbol{\xi}(t)$$

- Inputting the above we have

$$\tau_g \dot{\mathbf{g}}(t) = (A\Gamma)^T \left(\frac{\mathbf{s}}{\mathbf{r} + \mathbf{A}\mathbf{\Gamma}\mathbf{g}} - 1 \right) + \Gamma^T \left(\frac{\boldsymbol{\alpha} - 1}{\mathbf{\Gamma}\mathbf{g}} - \boldsymbol{\lambda} \right) + \boldsymbol{\xi}(t)$$

2. Methods

- This dynamic system contains divisive non-linearities, which are difficult to simulate.
- To linearize the system, we introduce two new populations of neurons $\mathbf{p}$ of size n_{OSN} and $\mathbf{b}$ of size n_{odor} which have as their fixed points $\dfrac{\mathbf{s}}{\mathbf{r}+\mathbf{A\Gamma g}}$ and $\dfrac{\boldsymbol{\alpha}-1}{\mathbf{\Gamma g}}$ respectively.
- The dynamics of $\mathbf{p}$ and $\mathbf{b}$ are given by

$$\tau_p \dot{\mathbf{p}}(t) = \mathbf{s} - (\mathbf{r} + \mathbf{A\Gamma g}) \odot \mathbf{p}$$

$$\tau_b \dot{\mathbf{b}}(t) = \boldsymbol{\alpha} - 1 - \mathbf{\Gamma g} \odot \mathbf{b}$$

- where $\odot$ is element-wise multiplication. In the steady state regime where $\tau_p, \tau_b \downarrow 0$, the dynamics of $\mathbf{g}$ are given by the initial equation.

2. Methods

- Putting all of this together, the dynamics of the system are given by

$$\mathbf{c} = \Gamma \mathbf{g}$$

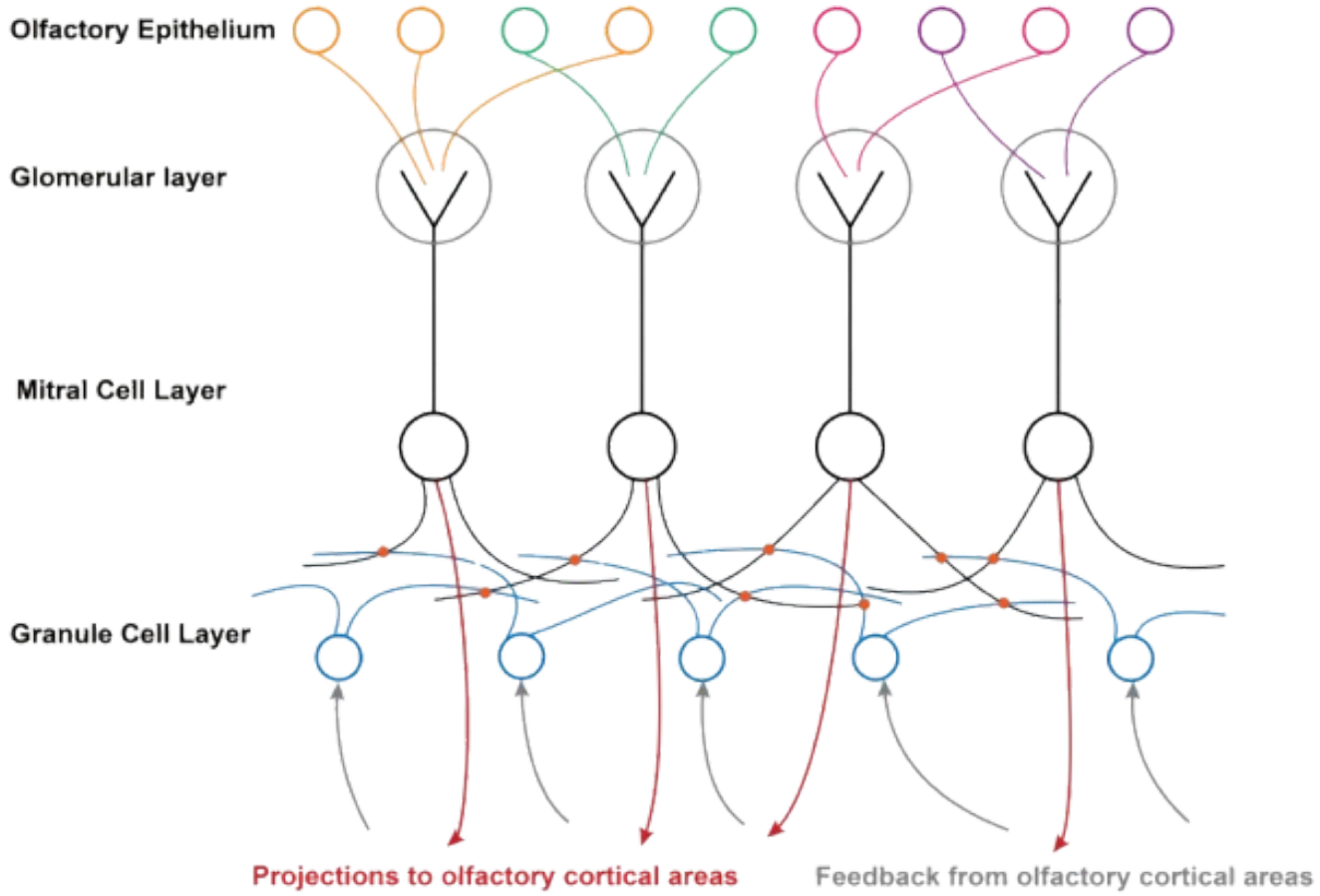
$$\tau_p \dot{\mathbf{p}}(t) = \mathbf{s} - (\mathbf{r} + \mathbf{A}\Gamma\mathbf{g}) \odot \mathbf{p}$$

$$\tau_b \dot{\mathbf{b}}(t) = \boldsymbol{\alpha} - 1 - \boldsymbol{\Gamma}\mathbf{g} \odot \mathbf{b}$$

$$\tau_g \dot{\mathbf{g}}(t) = (A\Gamma)^T (p - 1) + \Gamma^T (\mathbf{b} - \boldsymbol{\lambda}) + \boldsymbol{\xi}(t)$$

- where $\boldsymbol{\xi}(t)$ is n_g-dimensional white noise with zero mean and $\mathbb{E}[\boldsymbol{\xi}_j(t)\boldsymbol{\xi}_{j'}(t')] = 2\tau_g \delta_{jj'} \delta(t - t')$.
- The converge to that MAP is relegated to the appendix, and will not be discussed here.
- The authors show that $\mathbf{p}$, which encodes the divisive difference between prediction and observation, are **projection neurons** (mitral cells), $\mathbf{g}$, which encodes the MAP, are **granular neurons**, and $\mathbf{b}$, which encodes the prior, is **cortical feedback** from the brain.
- Meanwhile $\mathbf{A}$ encodes the sensitivity of the OSN to the odorant, while $\mathbf{A\Gamma}$ encodes the synaptic weights of the mitral and granule-cells, coupled by dendrodendritic synapses.

2. Methods

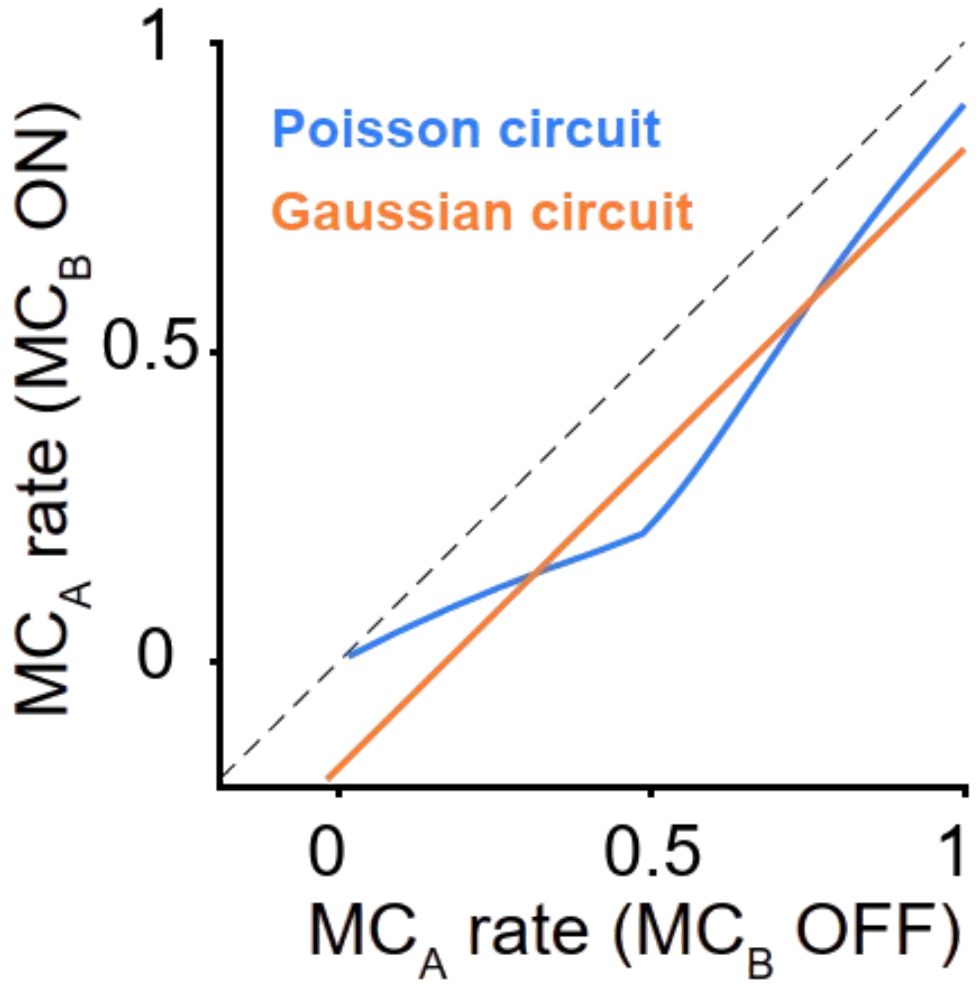

Olfactory Epithelium
Glomerular layer
Mitral Cell Layer
Granule Cell Layer
Projections to olfactory cortical areas
Feedback from olfactory cortical areas

2. Methods

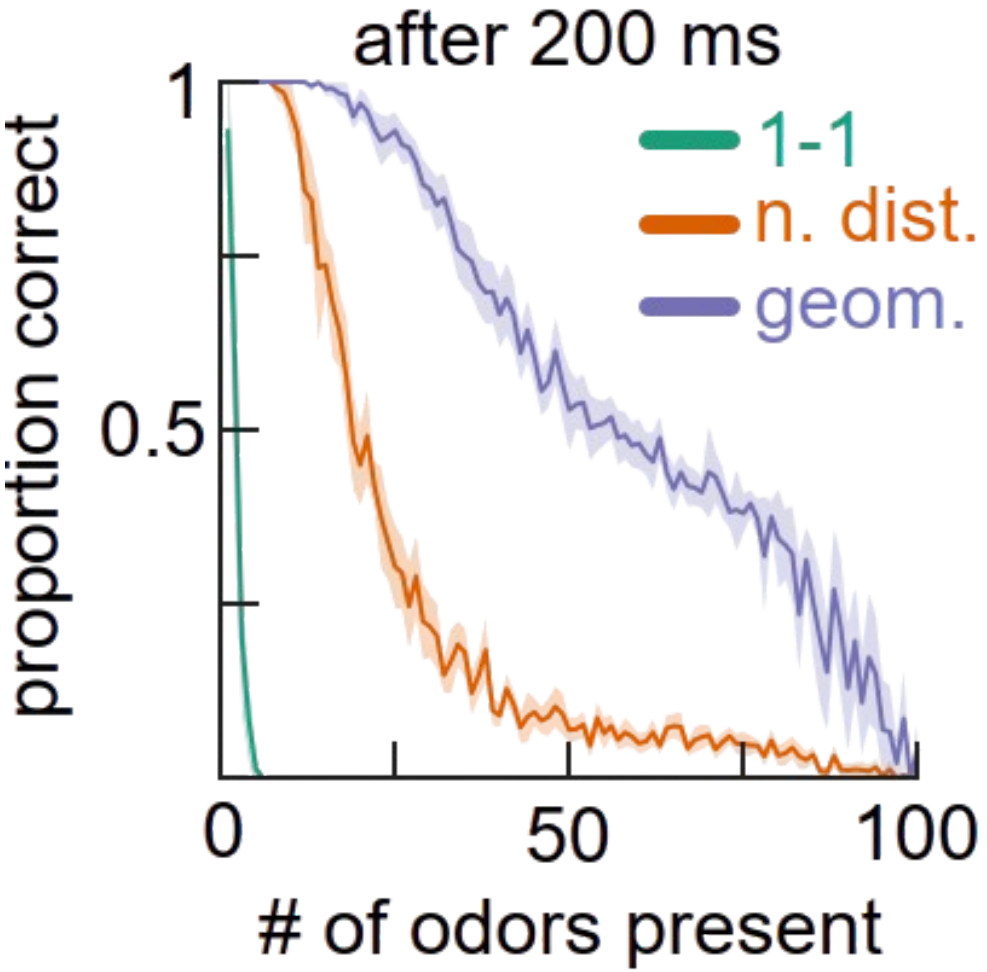
- $\mathbf{A}$ can be selected using Calcium imaging.
- The choice of Γ is more difficult, as our model contains simplifications.
- For fast inference, the authors realize that the firing rates of each mitral cell should be independent of the firing of other mitral cells, thus we choose Γ so that $\Gamma\Gamma^T = aA^TA$ with $a \in \mathbb{R}_+$ some constant. This is called **geometry-aware code**, as it respects the geometry of mitral firing.
- Code which does not satisfy this condition is called **naively-distributed**. As a control, the authors select Γ so that $\Gamma\Gamma^T = k\mathbf{I}$.
- This is further compared with **one-to-one** coding where $\Gamma = \mathbf{I}$ with $n_g = n_{\text{odor}}$.

3. Results

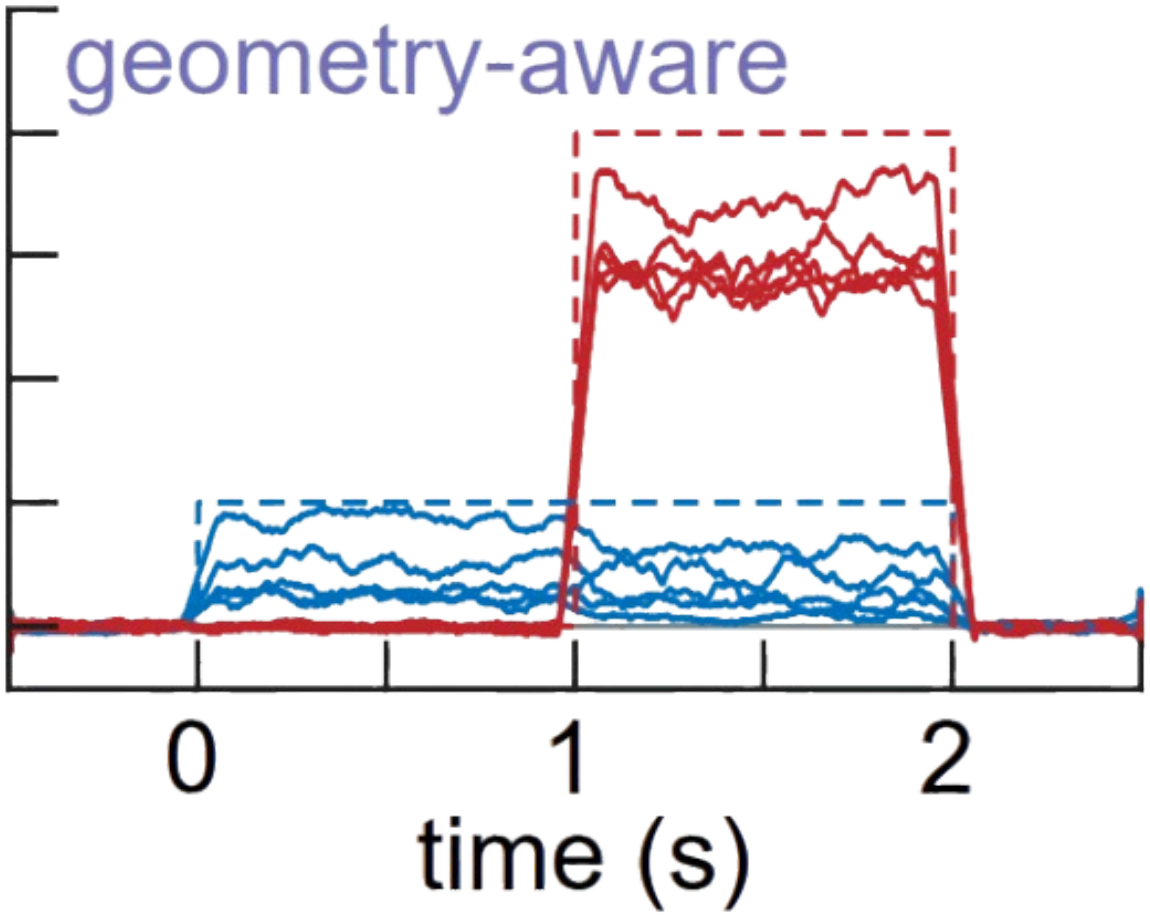
3. Results: PMC shows state-dependent inhibition



3. Results: Quick and accurate inference is possible



3. Results: Dynamic Inference in a noisy world is possible



4. Conclusion

4. Conclusion

- The authors demonstrate a Poisson Compressed Sensing model for the olfactory bulb which is both biologically plausible and computationally efficient.
  - The model is based on a Bayesian inference of the MAP of the odorant concentration given a spike train in the olfactory epithelium.
  - The MAP is found by Langevin dynamics approximating gradient ascent in the granular neurons.
  - The model respects the anatomy of olfactory bulb, in terms of the dendrodendritic coupling of mitral and granule neurons as well as the synapses in the glomerulus.
  - Geometry-aware coding is shown to be more efficient than one-to-one coding, and shows Bayesian inference in biological time scales (200ms).
- Significance
  - The new model naturally respects the microanatomy of the olfactory bulb.
  - The authors demonstrate distributed coding can lead to faster inference than axis-aligned coding.
- Limitations
  - Synaptic plasticity is not considered, and the way in which $\mathbf{A}$ and $\mathbf{A}\Gamma$ are learned remains unclear.