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Lab Section: 1A **Id No:** 2000080061

Pre-lab Session work (15M)	In-Lab Session work (15M)	Post Lab session work (10M)	Viva (10M)	Total Marks 50M

Date: **Signature of the Instructor** **Marks awarded**

Signal Processing-Lab (20AD3107)

Lab 6- Sampling Theorem

Lab Report

Objective: The objective of this Lab is to understand concepts and observe the effects of periodically sampling a continuous signal at different sampling rates, changing the sampling rate of a sampled signal, aliasing, and anti-aliasing filters.

Introduction: The signals we use in the real world, such as our voices, are called "analog" signals. To process these signals in digital computers or digital systems, we need to convert the signals to "digital" form. While an analog signal is continuous in both time and amplitude, a digital signal is discrete in both time and amplitude. To convert a signal from continuous time to discrete time, a process called ***sampling*** is used. The value of the signal is measured at certain intervals in time. Each measurement is referred to as a sample. The analog signal is also quantized in amplitude, but that process is ignored in this demonstration.

Sampling Theorem: The sampling theorem can be defined in two ways as below.

Time domain Statement: A band limited signal having no frequency components higher than f_m Hz may be completely recovered from the knowledge of its samples taken at the rate of at least 2 f_m samples per second.

Frequency domain Statement: A band limited signal having no frequency components higher than f_m Hz is completely described by its sample values at uniform intervals less than or equal to 1 / 2 f_m seconds apart.

Basic theory: A simple process of sampling and reconstruction is illustrated in Fig1.

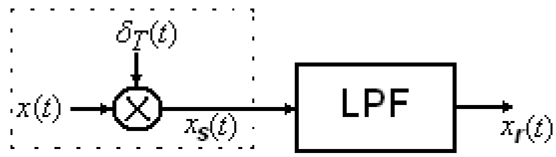
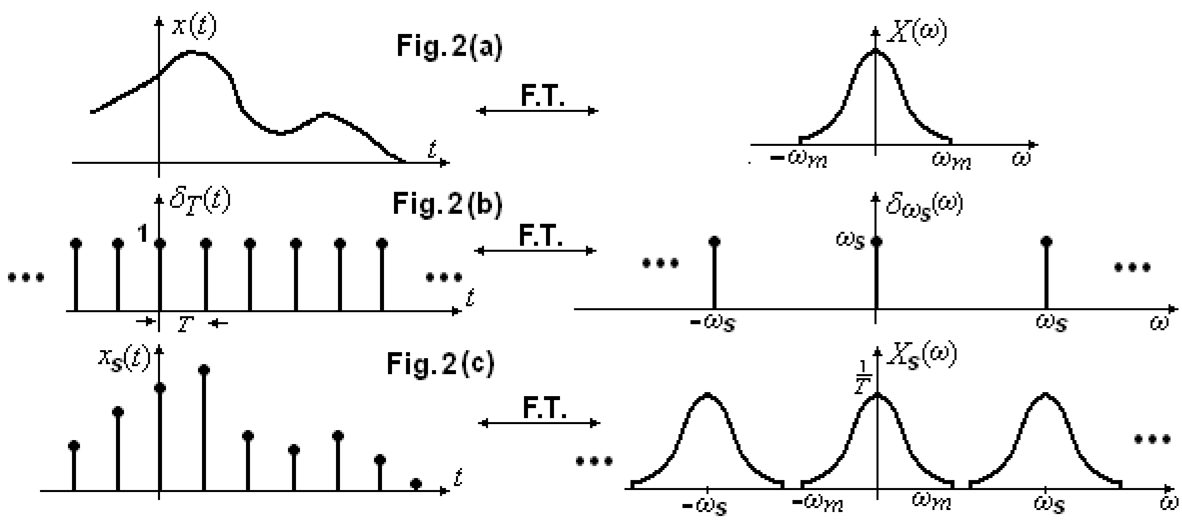


Fig.1. Sampling Process Reconstruction Process

The uniform sampling and reconstruction process is illustrated in Fig.1. Let us consider a band limited signal $x(t)$ having no frequency components beyond f_m Hz, i.e., $X(\omega)$ is zero for $|\omega| > \omega_m$, where $\omega_m = 2\pi f_m$. When this signal is multiplied by a periodic impulse function $\delta_T(t)$ (with period 'T'), the product yields a sequence of impulses located at uniform intervals of T seconds. The strength of resulting impulses is equal to the value of $x(t)$ at the corresponding instants.



The Fourier transforms of the band limited signal $x(t)$ and the impulse train $\delta_T(t)$ are $X(\omega)$ and $\delta_{\omega_s}(\omega)$ shown in Fig 2(a) and (b) respectively. The product of $x(t)$ and $\delta_T(t)$ yields a discrete time signal $x_s(t)$ as shown in Fig 2(c). The corresponding spectrum $X_s(\omega)$ can be determined by frequency convolution theorem as below.

We know that the Fourier Transform of periodic impulse train is also periodic function $\delta_{\omega_s}(\omega)$ and can be written as the sum of impulses located at $\omega = 0, \pm\omega_s, \pm\omega_s, ...$

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) \xleftrightarrow{\text{F.T.}} \omega_s \delta_{\omega_s}(\omega) = \omega_s \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)$$

Then the sampled signal is given by

$$\boxed{\begin{aligned} x_s(t) &= x(t)\delta_T(t) \\ &= \sum_{n=-\infty}^{\infty} x_n \delta(t-nT) \end{aligned}} \xleftrightarrow{\text{F.T.}} \boxed{\begin{aligned} X_s(\omega) &= \frac{1}{2\pi}\left[X(\omega) * \omega_s \delta_{\omega_s}(\omega)\right] \\ &= \frac{\omega_s}{2\pi}\left[X(\omega) * \delta_{\omega_s}(\omega)\right] = \frac{1}{T}\left[X(\omega) * \delta_{\omega_s}(\omega)\right] \end{aligned}}$$

Thus the spectrum $X_s(\omega)$ is obtained by convolving $X(\omega)$ and $\delta_{\omega_s}(\omega)$ represents $X(\omega)$ repeating every ω_s rad/sec. It is obvious from $X_s(\omega)$ shown in Fig 2(c), that $X(\omega)$ will repeat periodically without overlapping provided $\omega_s \geq 2\omega_m$ or $\frac{2\pi}{T} \geq 2(2\pi f_m)$.

That is the sampling rate $f_s = \frac{1}{T}$ is given as $f_s \geq 2 f_m$.

Hence the sampling rate should at least be equal to the twice of max frequency component present in the signal $x(t)$. This means that at least two samples per second are needed for a complete recovery of the signal from $x_s(t)$. Therefore the minimum sampling rate $f_s = 2 f_m$. This minimum rate of sampling is known as Nyquist sampling rate.

Signal Recovery from its samples: It can be shown that the original signal $x(t)$ can be recovered by passing its sampled version through a LPF with a cut off frequency $f_c = f_m$ or $\omega_c = \omega_m$).

Let the function $x(t)$ is sampled at Nyquist rate ω_s, then we have $\omega_s = 2\omega_m$.

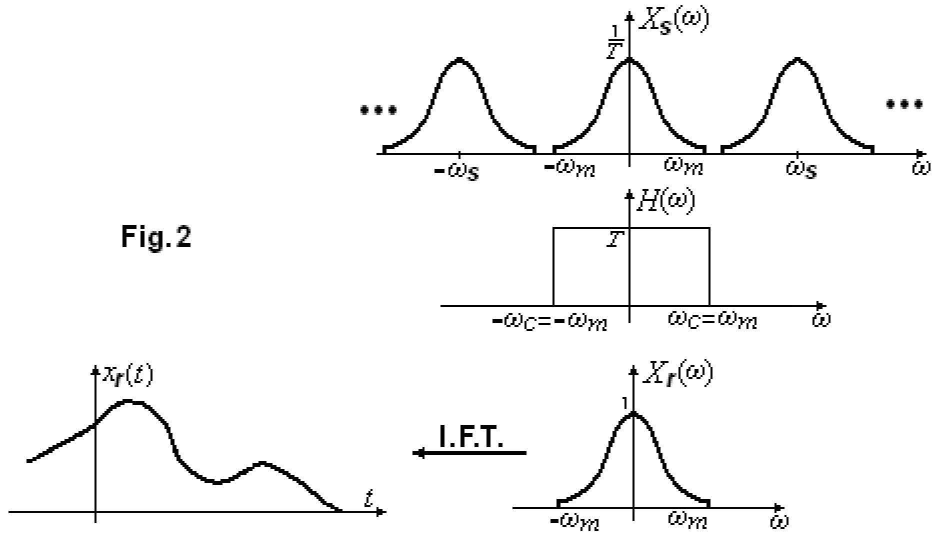
Therefore $X_s(\omega) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(\omega - m\omega_s) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(\omega - 2m\omega_m)$.

The LPF transfer function is given by $H(\omega) = \begin{cases} T, |\omega| < \omega_c \\ 0, \ |\omega| > \omega_c \end{cases}$

Then $X_r(\omega) = X_s(\omega)H(\omega)$ or

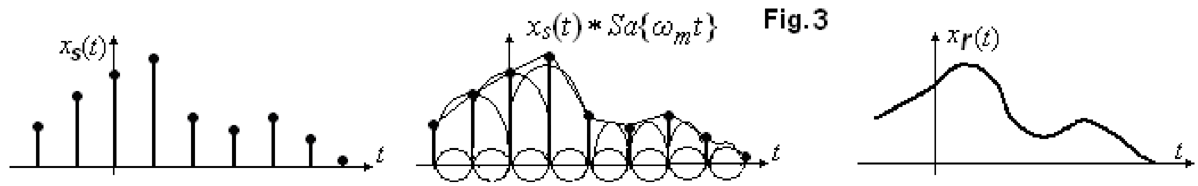
$$\begin{aligned} x_r(t) &= x_s(t) * \text{I.F.T}\{H(\omega)\} \\ &= x_s(t) * T\frac{\omega_s}{2\pi} Sa\{\omega_m t\} = x_s(t) * Sa\{\omega_m t\} \\ &= \sum_{n=-\infty}^{\infty} x_n \delta(t-nT) * Sa\{\omega_m t\} = \sum_{n=-\infty}^{\infty} Sa\{\omega_m (t-nT)\} = \sum_{n=-\infty}^{\infty} Sa(\omega_m t - n\pi) \end{aligned}$$

Fig.2



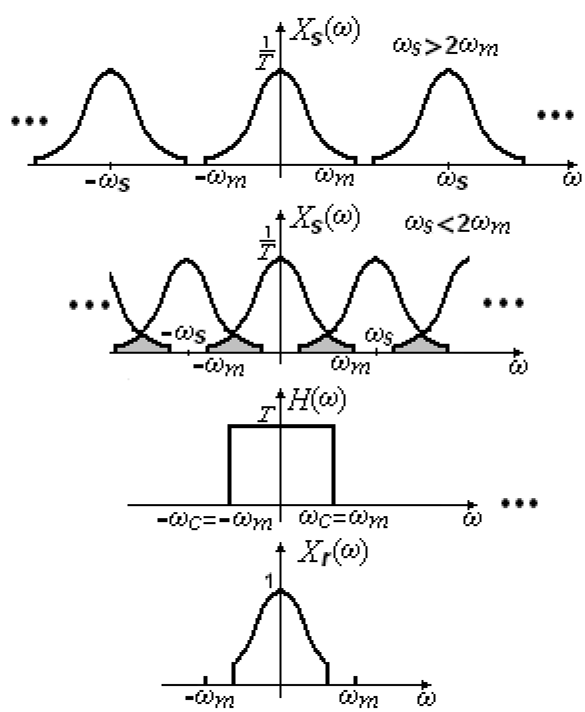
This equation represents the function $x_r(t)$ (or equivalent to $x(t)$) can be constructed by multiplying its samples x_n with a sampling function $Sa(\omega_m t - n\pi)$ and adding the multiplied values. This procedure is referred to as ideal band limited interpolation using the sync function. The construction of $x_r(t)$ (or equivalent to $x(t)$) is shown in Fig 2. and Fig 3.

Fig.3



The effect of Under Sampling: Aliasing:

As explained in the previous section, with $\omega_s \geq 2\omega_m$ (or $f_s \geq 2f_m$), the spectrum of sampled signal consists of scaled replications of the spectrum of $x(t)$ and thus forms the basis for the sampling theorem.



When $\omega_s < 2\omega_m$, $X(\omega)$ the spectrum of $x(t)$ is no longer replicated in $X_s(\omega)$ and thus no longer recoverable by low pass filtering. This effect in which the individual terms overlap is referred to as aliasing. It is also called ***frequency folding effect***. This is illustrated in the figure. The effect of under sampling, whereby higher frequencies are reflected into lower frequencies. Hence to avoid aliasing, it should be ensured that (i) $x(t)$ is strictly band limited and (ii) ω_s must be greater than $2\omega_m$.

Examples:

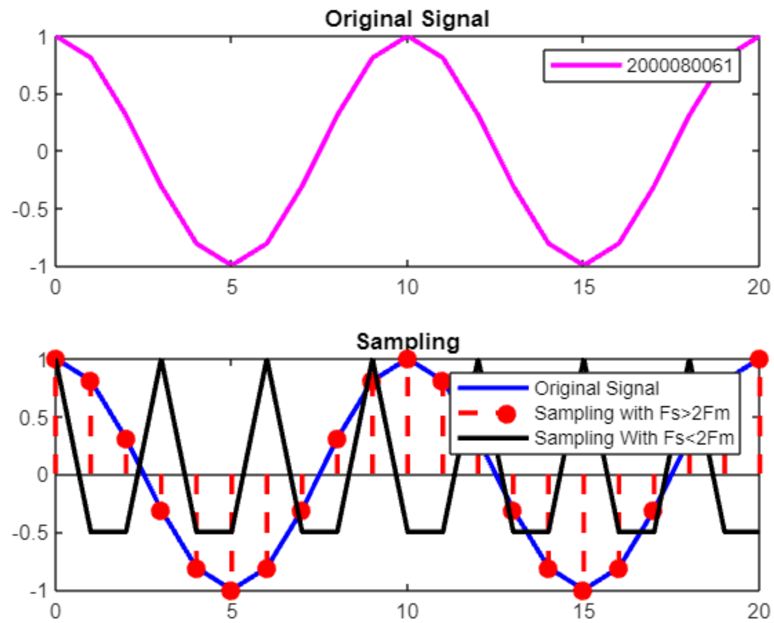
Ex9.1: Time domain sampling demonstration

CODE

```
clear all; close all; clc;
f0=1000;
fs1=10000;
fs2=1500;
n=0:1:20;
x=cos(2*pi*f0*n/fs1);
x1=cos(2*pi*f0*n/fs2);
figure ();
subplot(2,1,1);
plot(n,x, 'm','LineWidth',2); title('Original Signal');
legend("2000080061")

subplot(2,1,2);plot(n,x, 'b','LineWidth',2);hold on
stem(n,x,'--r','fill','LineWidth',2);
plot(n,x1,'k','LineWidth',2);
title('Sampling');
legend('Original Signal','Sampling with Fs>2Fm','Sampling With Fs<2Fm');
```

OUTPUT



Ex9.2: Frequency domain sampling demonstration

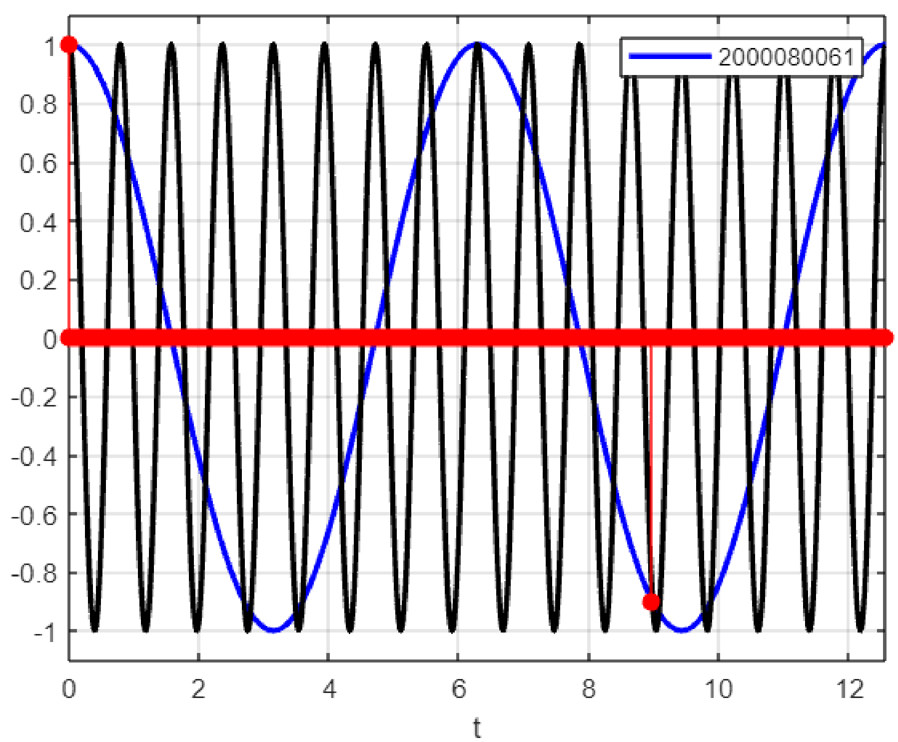
Solution:

CODE

```
clear all; close all; clc;

omega_0 = 1;omega_s = 7;
T = 2* pi/omega_0;
t = 0:0.01:2*T;
x1 = cos(omega_0* t);
x2 = cos((omega_0 + omega_s)*t);
N = length(t);
Ts = 2*pi/omega_s;
M = fix(Ts/0.001);
imp = zeros(1,N);
for k = 1:M:N-1.
    imp(k) = 1;
end
xs = imp.*x1;
plot(t,x1,'b',t,x2,'k','LineWidth',2); hold on
stem(t,imp.*x1,'r','filled');
axis([0 max(t) -1.1 1.1]);
xlabel('t');
legend("2000080061");
grid
```

OUTPUT



Lab Exercise

Exercise1: Consider a multitone signal

$$s(t) = 1.2\sin(380\pi t + \frac{\pi}{3}) + 2\cos(840\pi t - \frac{\pi}{5}).$$

Suppose this signal is sampled with the following sampling frequencies.

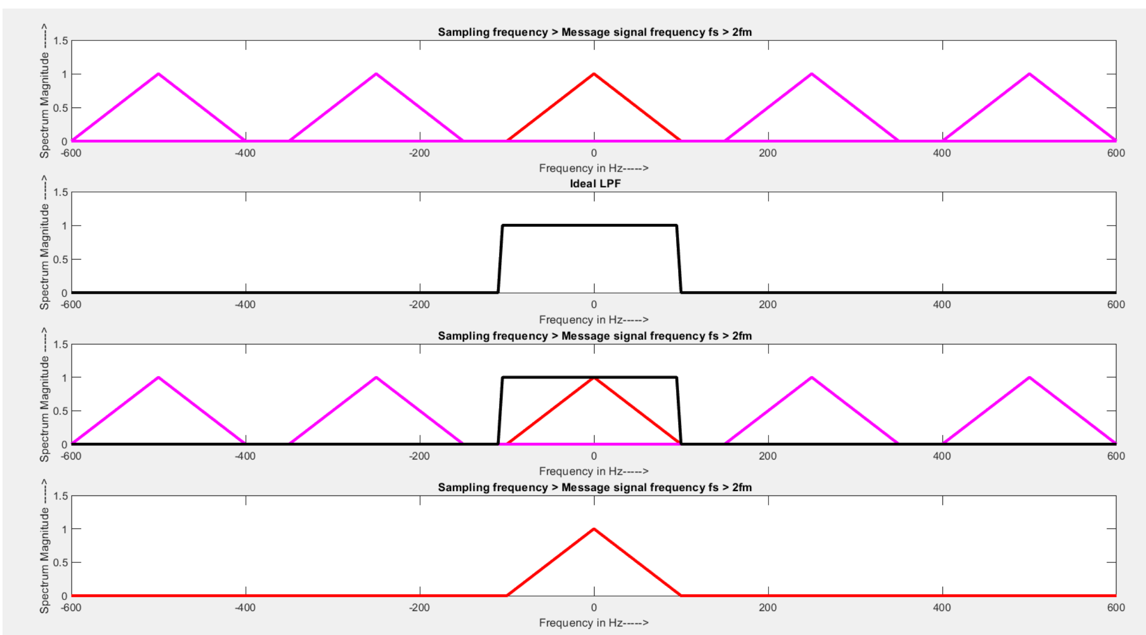
(a) 600 sam/sec (b) 1200 sam /sec

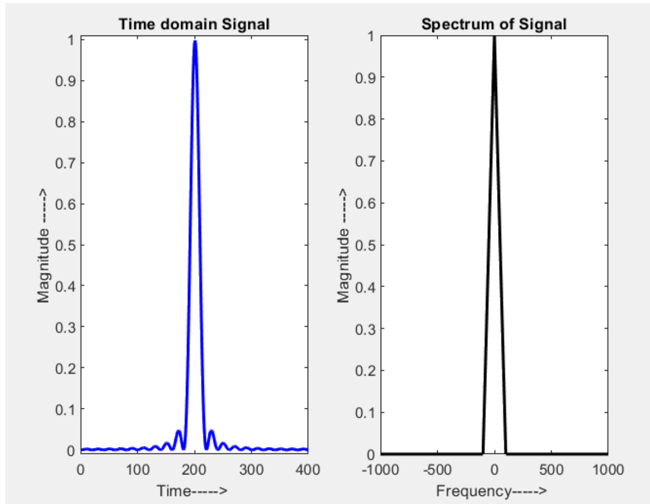
Is there any aliasing effect? Develop Matlab codes, sketch and label them.

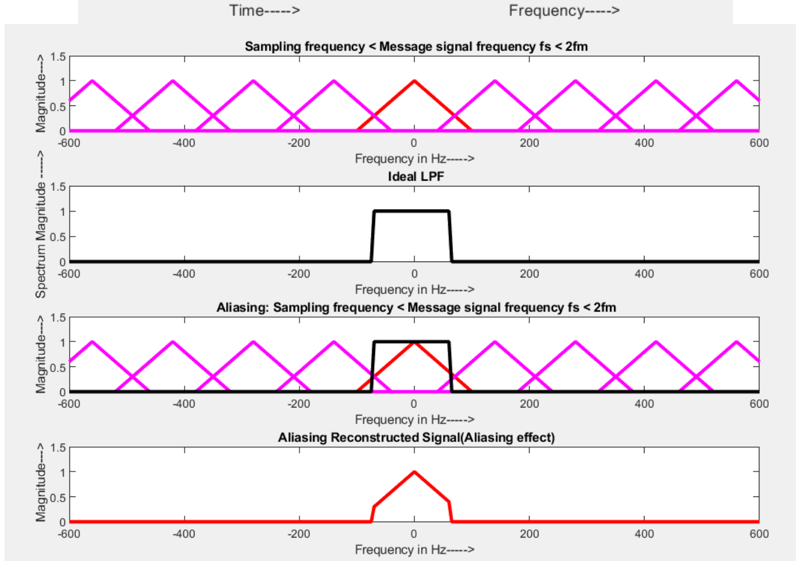
CODE

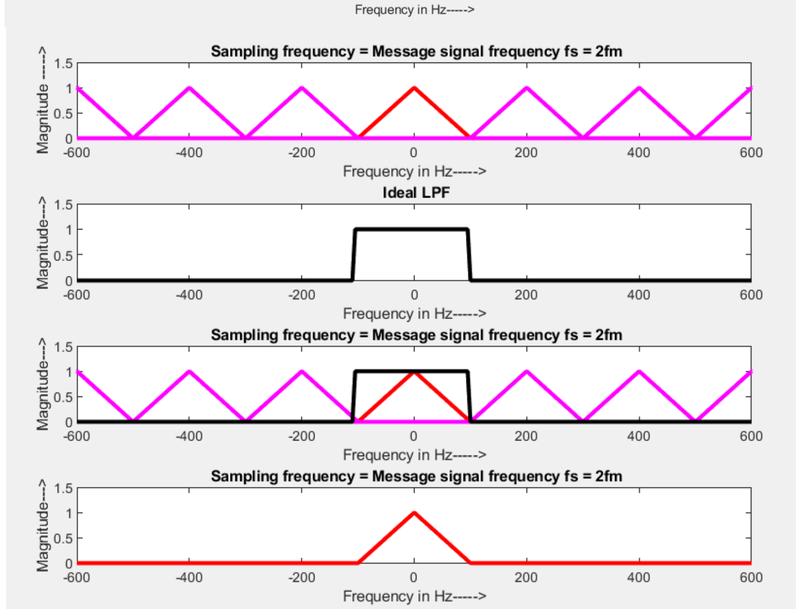
```
% 2000080061
clear all; close all; clc;
fm =  100;
fmax = 1000;
N =  4;
fs_high = 250; fs_equal=200; fs_low = 140;
Sampl_demo(fm, fs_high, fs_equal, fs_low, fmax, N);
```

OUTPUT




Time domain Signal
Magnitude ----->
Time----->
Spectrum of Signal
Magnitude ----->
Frequency----->


Sampling frequency < Message signal frequency fs < 2fm
Magnitude--->
Frequency in Hz----->
Ideal LPF
Spectrum Magnitude ----->
Frequency in Hz----->
Aliasing: Sampling frequency < Message signal frequency fs < 2fm
Magnitude--->
Frequency in Hz----->
Aliasing Reconstructed Signal(Aliasing effect)
Magnitude--->
Frequency in Hz----->


Sampling frequency = Message signal frequency fs = 2fm
Magnitude ----->
Frequency in Hz----->
Ideal LPF
Magnitude--->
Frequency in Hz----->
Sampling frequency = Message signal frequency fs = 2fm
Magnitude--->
Frequency in Hz----->
Sampling frequency = Message signal frequency fs = 2fm
Magnitude--->
Frequency in Hz----->

CODE

```
clc; clear all; close all;
f0=600;
f1=1200;
n=0:1:20;
x=1.2*sin((380*pi*n)/f0+pi/3)+2*cos((840*pi*n)/f0-pi/5);
x1=1.2*sin((380*pi*n)/f1+pi/3)+2*cos((840*pi*n)/f1-pi/5);
figure()
subplot(2,1,1);
plot(n,x,'m','linewidth',2);
title('original signal');
legend('2000080061');

subplot(2,1,2);
plot(n,x,'b','linewidth',2);hold on
stem(n,x,'--r','fill','linewidth',2);
plot(n,x1,'k','linewidth',2);
title('sampling');
legend('original signal','sampling with 600','sampling with 1200')
```

OUTPUT