Computational Physics Homework Assignment #4

Random Number, Random Walk, and Electromagnetism

December 17, 2024;Due December 25, 2024

Reading Assignment

1. Read lecture notes and references;
2. Study sample programs, prepare your own programs with any languages you prefer or simply copy sample programs but make them work.

Computation Assignments (Total Points 100 = 30 + 25 + 45)

1. **Random Number Generator** (Points: 30)
   Write a program to test random number generator and select 10 sets of (m, a, c) from the table given for testing, and make comparisons on them. You should use more than 3 testing methods.

m	a	c
214326	1807	45289
244944	1597	51749
233280	1861	49297
175000	2661	26979
121500	4081	25673
145800	3661	30809
139968	3877	29573
214326	3613	45289
714025	1366	150889
134456	8121	28411
243000	4561	51349
259200	7141	54773
233280	9301	49297
714025	4096	150889
1771815	2416	374441
510300	17221	107839
312500	36261	66037
217728	84589	45989
2147483647	16807	0
4294967296	1664525	1013904223
4294967296	69069	0

2. **Random walks in two dimensions** (Points: 5, 10, 10)
Enumerate all the random walks on a triangular lattice for step $N = 4$ and obtain exact results for $< x(N) >$, $< y(N) >$ and $< \Delta R^2(N) >=< x^2(N) > + < y^2(N) > - < x(N) >^2 - < y(N) >^2$, for

   (a) simple random walk.

   (b) non-reversal random walk.

   (c) self-avoiding random walk. (Total number of walks is 618).

3. **Numerical solution of the potential within a rectangular region.** (Points: 15, 15, 15)
Determine the potential $V(x, y)$ in a square region with linear dimension $L_x = L_y = 9$, as specified below:
Lattice: 9×9, 4 conners are (1,1), (9,1), (9,9), (9,1).
Boundaries: $V(x = 1, 2, ..., 9; y = 0) = V(x = 1, 2, ..., 9; y = 10) = 9$
$V(x = 0; y = 1, 2, ..., 9) = V(x = 10; y = 1, 2, ..., 9) = 5.$

   (a) Let $n_x = n_y = 9$ and run your "Jacobi Relaxation" program. How many iterations are necessary to achieve 1% accuracy with optimal relaxation parameter? Repeat it for $n_x = n_y = 72$.

   (b) Let $n_x = n_y = 9$ and run your "Jacobi Relaxation" program with two different guesses of the initial potential at the interior points, one guess is by taking simply average of the boundary values while the other one is by using the random number generator, $V(x, y) = V_{max}$ RAND[-1,1], and take $V_{max} = 20$. Which one do you consider to be a poor initial guess? What is the effect of a poor initial guess? Are the final results independent of your initial guess?

   (c) Imagine coloring the alternate points of a grid red and black, so that the grid resembles a checkerboard. Modify your "Jacobi Relaxation" program so that all the red points are updated first, and then all the black points are updated. This ordering is repeated for each iteration. Do your results converge any more quickly than before? $n_x = n_y = 45$.