HW0 due 5p Fri Oct 8 2021

This assignment will be graded on participation; so long as you make an effort on each problem and upload a legible pdf to Canvas, you will receive full credit (1 point for each sub-problem). However, you are responsible for reviewing the prerequisite material needed to understand and solve the self-assessment problems, since this material will be leveraged extensively in this class.

Future assignments will be graded on both participation and correctness (1 point each, for a total of 2 points for each sub-problem).

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0. [Nakseung Choi]; [He/Him/His]

a. Approximately how many hours did you spend on this assignment?

I spent about 20 hours to go over all the prerequisite material you recommanded studying. The more I studied, the more curious I became about the materials, so I ended up spending much more time, I should have. But it was a good chance for me to go back and recall the materials, such as LaPlace transform, Fourier series & transform, signal and system, and matrix.

b. Were there specific problems that took much longer than others?

I spent most of them studying for signal and system because it's been very long time since I took the course back in undergraduate. After I finally understood the questions and solved, I realized they were just simple questions asking definitions.

c. Were you working on campus or remotely this week?

I worked both on campus and remotely this week. I tried to come to school early on Monday, Tuesday, and Wednesday, so I could read the textbook and understand the material before class. I also have a class with 100% online lectures because the professor is out of town for the family problem. So, I basically work here and there depending on where I would like to work.

d. What class meeting(s) did you participate in this week?

I participated in all the meetings for my classes this week.

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1. Welcome to Control Systems!

Purpose: your answers to these questions will help us get to know you, and may inform the examples and applications we cover in lecture and homework.

a. What degree are you pursuing, in what department, and what concentration? (e.g. BS in ECE with Systems / Controls / Robotics concentration)

MS in ECE with Power Electronics.

b. Why did you enroll in this course?

I enrolled in this course to expand my knowledge in control system and broaden my technical horizon to have a better understanding of electrical circuits, systems and products in the world. I would like to learn as much as possible and leverage those skills to become a successful electrical engineer in the future.

c. What do you hope to learn in this course?

I would like to learn everything that this course covers about control systems and would like to have a better understanding of how control systems work. So far, I am not regretting taking this course. The problems and examples that we are dealing in class are super interesting.

d. Have you taken MATH 307 or an equivalent course that covers differential equations?

Yes, I have, but I have taken it more than 5 years ago back in South Korea. So, I am spending quite a bit of time to review all recommended prerequisite courses.

e. Log in to Canvas and edit your Profile; add a headshot photo, and specify your preferred the name and pronouns we should use to refer to you. (see my example at https://canvas.uw.edu/about/3510568)

Yes, I just did.

f. What do you hope to do after you graduate?

I'd like to finish my master's degree first and then I'll see how I feel about pursuing a Ph.D.

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2. Self-assessment

ECE 447 is a senior-level course with multiple prerequisites including EE 235 (signals and systems) and MATH 308 (linear algebra), both of which have their own prerequisites -- notably, CSE 142 (programming) and MATH 126 (multivariable calculus). This course will draw heavily on the knowledge you acquired in these prerequisite courses; we will not cover background material in lecture.
The following questions will help you refresh your background knowledge from prerequisite courses. You may need to consult textbooks, websites, or other resources to answer the questions; suggested resources are indicated for each block of questions.

2.1 Calculus

Textbooks: Calculus (MIT OpenCourseWare) by Strang; Real mathematical analysis by Pugh.

a. Consider the polynomial expression ax^2+bx+c.

Note: this expression can be regarded as a quadratic (i.e. degree 2) polynomial in the variable x or an affine (i.e. degree 1) polynomial in the variable a.
a.1 Regarding the expression as a polynomial in x, what are the roots?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a.2 Regarding the expression as a polynomial in a, what are the roots?

$$a = \frac{-bx - c}{x^2}$$

a.3 Are any extra assumptions required for your answers to (a.1) or (a.2) to make sense?

a and x^2 must not be equal to zero.

b. Consider the complex number $z=re^{\theta j}\in C$, where $r,\theta\in R$.

b.1 What angle does z make with the real axis?

$$\theta$$

b.2 When $r=1,\theta=\pi/2$, what does z equal?

$$\begin{aligned} z &= r(cos\theta + \mathrm{j}\sin\theta) \\ &= 1(\cos\left(\tfrac{\pi}{2}\right) + jsin\left(\tfrac{\pi}{2}\right)) \\ &= 1j \end{aligned}$$

b.3 If we regard z as a vector in the two-dimensional Cartesian plane $R2$, what are the x- and y-coordinates of z?

$$x = cos\theta, \mathrm{y} = \sin\theta$$

c. Consider the functions $f:X\rightarrow Y$, $g:Y\rightarrow Z$.
Note: if you are unfamiliar with this notation, it is a compact way to say "f is a rule that assigns a unique value $f(x)$ in the set Y to each element x in the set X". You may be more familiar with the notation $y=f(x)$ where $x\in X$ (that is, x is an element of the set X) and $y\in Y$ (that is, y is an element of the set Y). The set X is called the ***domain of*** f.

c.1 With $h:X\rightarrow Z$ defined by $h(x)=g(f(x))$, and supposing f and g are continuously differentiable, provide an expression for $ddxh(x)$ that is valid for all $x\in X$.

$$h(x) = \frac{dg(n)}{dn} * \frac{dn}{dx}$$

c.2 Assuming f is two times continuously differentiable at x_0, give the general expression for the second-order Taylor series of f at x_0; your expression should have three terms in it corresponding to the function value, the value of its derivative, and the value of its second derivative.

$$f(x) = \sum_{n=2}^{\infty} \frac{f^2(x_0)}{2!}(x - x_0)^2$$
$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0)^1 + \frac{f''(x_0)}{2!}(x - x_0)^2$$

c.3 With $f(x)=\sin x$, compute the first-order Taylor series of f about $x_0=\pi$, i.e. only include the first two terms from your expression in (c.2).

$$f(x) = sinx$$
$$f'(x) = cosx$$
$$f''(x) = sinx$$

Putting the equations into the first-order Taylor series, I get

$$0 - (x - \pi)$$
$$= \pi + x$$

2.2 signals and systems

Textbooks: Systems, Signals, and Transforms by Phillips, Parr, and Riskin; Signals and Systems by Oppenheim, Willsky, and Hamid.

In the following problems, let S be a single-input/single-output (SISO) linear time-invariant (LTI) system with time-domain ***impulse response*** $h:\mathbb{R}\to\mathbb{R}$, let $u:\mathbb{R}\to\mathbb{R}$ be a time-domain ***input*** to the system, and let $y:\mathbb{R}\to\mathbb{R}$ be the time-domain ***output*** corresponding to input u.

a. Give a time-domain expression for y, that is, write $y(t)$ in terms of h and u; be explicit about any notation you introduce.

$$y(t) = u(t) * h(t)$$

Let $H=Fh$, $Y=Fy$, $U=Fu$ denote the Fourier transforms of the time-domain impulse response, input, and output, respectively.

b. Give a frequency-domain expression for $Y(\omega)$, that is, write $Y(\omega)$ in terms of $H(\omega)$ and $U(\omega)$.

$$Y(w) = U(w) * H(w)$$

Notice: your expression for $y(t)$ in (a.) depends on the value of h and u at ***all times*** (not just the time t), whereas your expression for $Y(\omega)$ in (b.) depends only on the value of H and U at frequency ω. This illustrates the key strength of frequency-domain analysis -- convolution in the time domain turns into multiplication in the frequency domain.

c. Give the general expression for the Fourier transformation F, that is, write $H(\omega)$ in terms of h.

$$F\{h(t)\} = H(w) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt$$

d. Give the general expression for the ***inverse*** Fourier transformation F_{-1}, that is, write $h(t)$ in terms of H.

$$F^{-1}\{H(w)\} = h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(w)e^{-jwt}dw$$

Notice: since an integral is an infinite sum, you can infer from the expressions in (c.) and (d.) that the time-domain signal h can be represented as an infinite linear combination of frequency-domain exponentials, and similarly that the frequency-domain signal H can be represented as an infinite linear combination of time-domain exponentials.

2.3 linear algebra

Textbooks: [Introduction to Applied Linear Algebra]() by Boyd and Vandenberghe; [Linear Algebra]() by Hefferon.

In the following problems, let $A=[A_{11} 0 A_{12} A_{22}] \in R_{2\times 2}$ be a matrix with ***shape*** 2×2, $B=[B_{11} B_{21}] \in R_{2\times 1}$ be a matrix with shape 2×1, and $x=[x_1 x_2] \in R_2$ be a vector with ***length*** 2.

a. Determine the eigenvalues of A.

eigenvalue = $\det(\lambda I - A) = 0$

$$A = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$
$$= \begin{bmatrix} \lambda - A_{11} & -A_{12} \\ 0 & \lambda - A_{22} \end{bmatrix}$$
$$(\lambda - A_{11})(\lambda - A_{22}) - 0 = 0$$
$$\lambda = A_{11}, A_{22}$$

b. Compute the determinant of A.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$
$$= A_{11} A_{22}$$

c. Compute Ax and $x^T A^T$ using matrix multiplication, where M^T denotes the ***transpose*** of matrix M.

$$X^T = [x_1\ x_2], A^T = \begin{bmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{bmatrix}$$
$$X^T A^T = [x_1 A_{11} + x_2 A_{12}\ x_2 A_{22}]$$

d. Why can't you compute xA or $A^T x^T$ using matrix multiplication?

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$
$$1X1 \qquad\qquad 2X2$$

Two numbers have to be a same number to do matrix multification.

e. Compute $B+x$; is the result a matrix or vector?
I assume the result is neither matrix or vector because matrix has only I but vector has real and imaginary number too.

2.4 scientific computing

Resources: [NumPy for Matlab users](); [Dive into Python](); [NumPy Manual](); [Colaboratory Notebook]()

a. Familiarize yourself with the Colaboratory Notebook and Programming in Python using the resources on the ***Computational Tools*** section of the Syllabus (Canvas home page).

In the following problems, let $A=[0-101-1]$, $B=[01]$.

b. Determine the eigenvalues of A using a numerical linear algebra library (`numpy.linalg.eigvals` in NumPy).

c. Compute the determinant of A using a numerical linear algebra library (`numpy.linalg.det` in NumPy).

d. Compute AB and B^TA^T using numerical matrix multiplication (`numpy.dot` or `@` in NumPy; ***not*** "`*`"); what are the resulting ***shapes***?

e. What happens (i.e. what error do you receive) when you try the following numerical matrix multiplications: BA or A^TB^T?

Eigenvalue.m

```
a = [ 0 1; -10 -1]
b = [ 0; 1]
c = eig(a)
D = det(a)
e = a*b
f = transpose(a)
g = transpose(b)
h = g*f
i = b*a
```

Command Window

```
>> clear
>> Eigenvalue

a =

     0     1
   -10    -1


b =

     0
     1
```

Eigenvalue.m

```
a = [ 0 1; -10 -1]
b = [ 0; 1]
c = eig(a)
D = det(a)
e = a*b
f = transpose(a)
g = transpose(b)
h = g*f
i = b*a
```

Command Window

```
c =

  -0.5000 + 3.1225i
  -0.5000 - 3.1225i


D =

    10


e =

     1
    -1
```

Eigenvalue.m

```
1 -   a = [ 0 1; -10 -1]
2 -   b = [ 0; 1]
3 -   c = eig(a)
4 -   D = det(a)
5 -   e = a*b
6 -   f = transpose(a)
7 -   g = transpose(b)
8 -   h = g*f
9 -   i = b*a
```

Command Window

```
f =

     0   -10
     1    -1


g =

     0     1


h =

     1    -1

Error using *
```

Eigenvalue.m

```
1 -   a = [ 0 1; -10 -1]
2 -   b = [ 0; 1]
3 -   c = eig(a)
4 -   D = det(a)
5 -   e = a*b
6 -   f = transpose(a)
7 -   g = transpose(b)
8 -   h = g*f
9 -   i = b*a
```

Command Window

```
h =

     1    -1

Error using  *
Incorrect dimensions for matrix multiplication. Check that the number of columns in the first
matrix matches the number of rows in the second matrix. To perform elementwise multiplication, us
'.*'.

Error in Eigenvalue (line 9)
i = b*a

Related documentation

>>
```