Tutorial - 4

$a_0 = 1$ for $n = 1, 2, 3 \ldots$

(1) $a_n = 3a_{n-1} + 2$

$a_n x^n = 3a_{n-1} x^n + 2x^n$

$a_n x^n = 3x a_{n-1} x^{n-1} + 2x^n$

$$\sum_{n=\cancel{0}}^{\infty} a_n x^n = 3x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + 2\sum_{n=1}^{\infty} x^n$$

$$\boxed{\sum_{n=1}^{\infty} x^n = \frac{x}{1-x}}$$

$G(x) - a_0$

$= \cancel{3x}$

$= \cancel{a_0} + a_1 x + a_2 x^2 \ldots \quad \cancel{-a_0}$

$G(x) - a_0 = 3x\,(G(x)) + 2\left(\frac{x}{1-x}\right)$

$G(x) - 3x\,(G(x)) = 2\cdot\left(\frac{x}{1-x}\right) + 1$

$= G(x)(1-3x) = 2\cdot\left(\frac{x}{1-x}\right) + 1 \qquad \frac{2x}{(1-x)} + 1$

$\cancel{G(x) = \frac{2x}{(1-x)(1-3x)} + \frac{1}{1-3x}}$

$\cancel{G(x) = \frac{2x+}{(1-x)(1-3x)}}$

$G(x)(1-3x) = \frac{2x}{1-x} + \frac{1}{1}$

$G(x)(1-3x) = \frac{2x + 1(1-x)}{1-x}$

$G(x)(1-3x) = \frac{2x+1-x}{1-x}$

$G(x)(1-3x) = \frac{1+x}{1-x}$

$G(x) = \frac{1+x}{(1-x)(1-3x)}$

$\cancel{Ax}$

$\frac{A}{ax+b} + \frac{B}{cx+d}$

$= \frac{A}{1-x} + \frac{B}{1-3x}$

$= \frac{A(1-3x) + B(1-x)}{(1-x)(1-3x)}$

$= \frac{A - 3xA + B - Bx}{(1-x)(1-3x)}$

$$\frac{1+x}{(1-x)(1-3x)} = \frac{A+B+x(-3A-B)}{(1-x)(1-3x)}$$

$A+B = 1$

$1 = -3A-B$

$-3A - B = 1$

$A + B = 1$

$-4A = 2$

$-3A - \cancel{B} = 1$

$A + \cancel{B} = 1$

$-2A = 2$

$\boxed{A = -1}$

$-1+B = 1$

$B = 2$

$\boxed{B = 2}$

$$G(x) = \frac{-1}{1-x} + \frac{2}{1-3x}$$

$$-1(1-x)^{-1} + 2(1-3x)^{-1}$$

$$-1\sum_{n=0}^{\infty} 1^n x^n + 2\sum_{n=0}^{\infty} 3^n x^n$$

$$\sum_{n=0}^{\infty} x^n (-1 \cdot 1^n + 2\cdot 3^n)$$

$$= \sum_{n=0}^{\infty} (-1^n + 2\cdot 3^n)$$

$$\sum_{n=0}^{\infty} (2\cdot 3^n - 1)$$

$$\boxed{a_n = 2\cdot 3^n - 1}$$

$= a_0 = 2\cdot 3^0$

$= 2\cdot 1 = 2$

$2\cdot 3 - 1 + P$

② $a_n = 3a_{n-1} + 4^{n-1}$

$a_0 = 1$

$n = 1, 2, 3 \ldots$

$a_n x^n = 3a_{n-1} x^n + 4^{n-1} x^n$

$a_n x^n = 3x\, a_{n-1} x^{n-1} + 4^{n-1} x^n$

① $a_n = 3a_{n-1} + 2 \quad a_0 = 1$

① $a_n x^n = 3a_{n-1} x^n + 2x^n$

$a_n x^n = 3x\, a_{n-1} x^{n-1} + 2x^n$

$$\sum_{n=0}^{\infty} a_n x^n = 3x\sum_{n=0}^{\infty} a_{n-1} x^{n-1} + 2\sum_{n=0}^{\infty} x^n$$

$G(x) = a_0$

$\cancel{a_0} + a_1 x + a_2 x^2 + \ldots - \cancel{a}$

$\Rightarrow G(x) - 3x\,G(x) = 2\left(\frac{x}{1-x}\right) + 1$

$= G(x)(1-3x) = 2\left(\frac{x}{1-x}\right) + 1$

$G(x)(1-3x) = \frac{2x}{1-x} + 1$

$G(x)(1-3x) = \frac{2x + 1(1-x)}{1-x}$

$G(x)(1-3x) = \frac{1+x}{1-x}$

$$G(x) = \frac{1+x}{(1-x)(1-3x)}$$

$$\frac{1+x}{(1-x)(1-3x)} = \frac{A+B+x(-3A-B)}{(1-x)(1-3x)}$$

$A+B = 1$

$1 = -3A-B$

$-3A-\cancel{B} = 1$
$A+\cancel{B} = 1$

$-2A = 2$

$\boxed{A=-1}$

$\boxed{B=2}$

$$G(x) = \frac{-1}{1-x} + \frac{2}{1-3x}$$

$$-1(1-x)^{-1} + 2(1-3x)^{-1}$$

$$-1\sum_{n=0}^{\infty} 1^n x^n + 2\sum_{n=0}^{\infty} 3^n x^n$$

$$\sum_{n=0}^{\infty} x^n\left(-1\cdot 1^n + 2\cdot 3^n\right)$$

$$\sum_{n=0} \left(-1^n + 2\cdot 3^n\right)$$

$$\sum_{n=0}^{\infty}(2\cdot 3^n - 1)$$

$$\boxed{a_n = 2\cdot 3^n - 1}$$

$= a_0 = 2\cdot 3^0$
$= 2\cdot 1 = 2$

$2\cdot 3^n - 1\cdot 1^n$

$2\cdot 3^0 - 1\cdot 1^0$

② $a_n = 3a_{n-1} + 4^{n-1}$

$a_0 = 1$

$n = 1, 2, 3 \ldots$

$a_n x^n = 3a_{n-1}x^n + 4^{n-1}x^n$

$a_n x^n = 3x\, a_{n-1}x^{n-1} + 4^{n-1}x^n$

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①

$a_n = 3a_{n-1} + 2 \quad a_0 = 1$

$a_n x^n = 3a_{n-1}x^n + 2x^n$

$a_n x^n = 3x\, a_{n-1}x^{n-1} + 2x^n$

$$\sum_{n=0}^{\infty} a_n x^n = 3x\sum_{n=0}^{\infty} a_{n-1}x^{n-1} + 2\sum_{n=0}^{\infty} x^n$$

$G(x) = a_0$

$\cancel{a_0} + a_1x + a_2x^2 + \ldots - \cancel{a_0}$

$\Rightarrow G(x) - 3x\,G(x) = 2\left(\frac{x}{1-x}\right) + 1$

$= G(x)(1-3x) = 2\left(\frac{x}{1-x}\right) + 1$

$G(x)(1-3x) = \frac{2x}{1-x} + 1$

$G(x)(1-3x) = \frac{2x + 1(1-x)}{1-x}$

$G(x)(1-3x) = \frac{1+x}{1-x}$

$$G(x) = \frac{1+x}{(1-x)(1-3x)}$$

$\Rightarrow a_1x + a_2x^2 + \ldots \quad 3[a_0x + a_1x^2 + a_2x^3 \ldots]$

$+2[x + x^2 + x^3 + \ldots]$

$G(x) - a_0 = 3x\,G(x) + 2x\,(1 + x + x^2 + \ldots)$

$G(x) - 1 = 3x\,G(x) + 2x \frac{1}{1-x}$

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② ~~$a_n = 3a_{n-1} + 4^{n-1}$~~

② $a_n = 7a_{n-1} - 10a_{n-2} + 6 + 8n$

$$\boxed{\begin{aligned} a_n x^n &= 7a_{n-1}x^n - 10a_{n-2}x^n + 6x^n + 8n \cdot x^n \\ a_n x^n &= 7x\,a_{n-1}x^{n-1} - 10x^2 a_{n-2}x^{n-2} + 6 + 8 \end{aligned}}$$

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④

② $a_n = 3a_{n-1} + 4^{n-1}$

$a_0 = 1 \qquad n = 1, 2, 3 \ldots$

$a_n x^n = 3a_{n-1}x^n + 4^{n-1}x^n$

$a_n x^n = 3x\,a_{n-1}x^{n-1} + 4x^{n-1}\,4^{n-1}$

$\sum_{n=1}^{\infty} a_n x^n = 3x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + x \sum_{n=1}^{\infty} 4^{n-1} x^{n-1}$

$G(x) - a_0 = 3x\,G(x) + x(1 + 4x + 4x^2 + 4x^3 + \ldots)$

$G(x) - a_0 = 3x\,G(x) + x\left(\frac{1}{1-4x}\right)$

$G(x) - 3x\,G(x) = a_0 + \frac{x}{1-4x}$

$G(x)(1 - 3x) = \frac{1}{1} + \frac{x}{1-4x} \qquad \frac{1 - 4x + x}{1-4x}$

$$G(x) = \frac{1-3x}{1-4x\,\cancel{(1-3x)}}$$

$$G(x) = \frac{1}{1-4x}$$

$$G(x) = \sum_{n=0}^{\infty} 4^n x^n$$

$$\boxed{a_n = 4^n}$$

$$a_0 = 4^0 = 1$$

(3)

(4) $a_n = 3a_{n-1} + n \qquad a_0 = 1$

$$a_n x^n = 3x\, a_{n-1} x^{n-1} + n \cdot x^n$$

$$\sum_{n=1}^{\infty} a_n x^n = 3x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} n \cdot x^n$$

$$G(x) - a_0 = 3x\,(G(x)) + x + 2x^2 + 3x^3 + 4x^4 + \cdots$$

$$G(x) - 1 = 3x\,(G(x)) + x\,(1 + 2x + 3x^2 + 4x^3 + \cdots)$$

$$G(x) - 1 = 3x\,G(x) + \frac{x}{(1-2x)}$$

$$G(x) - 3x\,G(x) = \frac{x}{1-2x} + 1$$

$$G(x)\,(1-3x) = \frac{x + 1 - 2x}{1-2x}$$

$$G(x) = \frac{1-x}{1-2x\;(1-3x)}$$

$$G(x) = \frac{1-x}{(1-2x)(1-3x)}$$

$$\frac{1-x}{(1-2x)(1-3x)} = \frac{A}{1-2x} + \frac{B}{1-3x}$$

$$\frac{1-x}{(1-2x)(1-3x)} \qquad \frac{A(1-3x) + B(1-2x)}{(1-2x)(1-3x)}$$

$$1 - x = A - 3Ax + B - 2xB$$

$$1 - x = A + B + x(2B - 3A)$$

$$1 - x = A + B + x(-2B - 3A)$$

$$(A + B = 1) \times 2$$
$$-3A - 2B = -1$$

$$2A + 2B = 2$$
$$-3A - 2B = -1$$

$$-A = 1$$

$$\boxed{A = -1}$$

$$A + B = 1$$
$$-1 + B = 1$$
$$B = 1 + 1$$
$$\boxed{B = 2}$$

$$G(x) = \frac{-1}{1-2x} + \frac{2}{1-3x}$$

$$= -1 \sum_{n=0}^{\infty} 2^n x^n + 2 \sum_{n=0}^{\infty} 3^n x^n$$

$$a_n = -1 \cdot 2^n + 2 \cdot 3^n$$

$$\boxed{a_n = 2 \cdot 3^n - 1 \cdot 2^n}$$

(3) $a_n = 5a_{n-1} - 6a_{n-2}$ $\quad a_0 = 6, \; a_1 = 30$

$$a_n x^n = 5a_{n-1} x^n - 6a_{n-2} x^n$$

$$a_n x^n = 5x\, a_{n-1} x^{n-1} - 6x^2 a_{n-2} x^{n-2}$$

$$\sum_{n=2}^{\infty} a_n x^n = 5x \sum_{n=2}^{\infty} a_{n-1} x^{n-1} - 6x^2 \sum_{n=2}^{\infty} a_{n-2} x^{n-2}$$

$$G(x) - a_0 - a_1 x = 5x(G(x) - a_0) - 6x^2 (G(x))$$

$$G(x) - a_0 - a_1 x = 5x G(x) - 5x a_0 - 6x^2 G(x)$$

$$G(x) - 5x G(x) + 6x^2 G(x) = a_0 + a_1 x - 5x a_0$$

$$G(x)(6x^2 - 5x + 1) = a_0(1 - 5x) + a_1 x$$

$$G(x)(6x^2 - 3x - 2x + 1) = 6(1 - 5x) + 30x$$

$$G(x)(6x^2 - 3x - 2x + 1) = 6 \cancel{-30x} \cancel{+30x}$$

$$G(x)\; 3x(2x-1) - 1(2x-1) = 6$$

$$G(x)(3x-1)(2x-1)$$

$$G(x) = \frac{6}{(3x-1)(2x-1)}$$

$$G(x) = \frac{6}{(1-3x)(1-2x)}$$

$$\frac{6}{\cancel{(3x-1)(2x-1)}} = \frac{A}{-3x+1} + \frac{B}{-2x+1}$$

$$\frac{6}{(3x-1)(2x-1)} = \frac{A(-2x+1)+B(-3x+1)}{(-3x+1)(-2x+1)}$$

$$6 = A(-2x+1) + B(-3x+1)$$

$$6 = -2Ax + A + -3xB + B$$

$$6 = A + B + x(-2A - 3B)$$

$$\boxed{A+B=6}$$

$$-2A - 3B = 0$$
$$A + B = 6 \quad \times 3$$

$$-2A - \cancel{3B} = 0$$
$$3A + \cancel{3B} = 18$$

$$A = 18$$

$$18 + B = 6$$
$$B = 6 - 18$$
$$\boxed{B = -12}$$

$$G(x) = \frac{A}{-3x+1} + \frac{B}{-2x+1}$$

$$G(x) = \frac{18}{-3x+1} + \frac{-12}{-2x+1}$$

$$18(1-3x)^{-1} - 12(1-2x)^{-1}$$

$$18 \sum_{n=0}^{\infty} 3^n x^n - 12 \sum_{n=0}^{\infty} 2^n x^n$$

$$\sum_{n=0}^{\infty} [18 \cdot 3^n - 12 \cdot 2^n]$$

$$\sum_{n=0}^{\infty} 6[3 \cdot 3^n - 2 \cdot 2^n] x^n$$

$$\boxed{a_n = 6(3^{n+1} - 2^{n+1})}$$

# Tutorial - 7

* There are 26 choices of three positions (A to Z)
there are $n*m$ ways to do both things.

* $26*26*26 = (26)^3 = 17576$

10 choices of three positions (0 to 9)

$\therefore 10\times10\times10 = 10^3 = 1000.$

$= 17576 \times 1000$

$= 17576000$

$\therefore$ There can be 17,576,000 different license plates with

② No. of choice for cupcakes = 20
No. of choice for donuts = 10
No. of choice for muffins = 15
Total - 45 = 20+10+15

③ Formulae for Permutations with repetition

$P(n: n_1 : n_2 : \ldots n_k) = n! / (n_1! \times n_2! \times \ldots n_k!)$

$n = 6;\ n_1 = 2,\ n_2 = 2,\ n_3 = 1,\ n_4 = 1$

$P(6, 2, 2, 1, 1) = 6! / (2! \times 2! \times 1! \times 1!)$

$P(6, 2, 2, 1, 1) = \cancel{2}\ \dfrac{(6\times5\times4\times3\times2\times1)}{(2\times1)\times(2\times1)\times1\times1}$

$= \dfrac{720}{4\times2\times1} \Rightarrow \dfrac{720}{8} \Rightarrow 900$

(4) " INDEPENDENCE"

Total = letters

$= 12!$

(a) $P = \dfrac{11!}{3!\,2!\,4!}$

(b) $\dfrac{3!}{4!}$

(c) No. of

3A There are 6 digits is made of digits

9, 2, 6, 0, 0, 2

The no. of possible password is

$$\Rightarrow \frac{6!}{2!\,2!}$$

$$\Rightarrow \frac{6\times5\times\cancel{4}\times3\times\cancel{2}\times1}{\cancel{2}\times1\times\cancel{2}\times1}$$

$$\Rightarrow 180$$

Each password takes 5 seconds

$= 180\times5$

$= 900$ seconds

Minutes $= \dfrac{900}{60} = 15$ minutes.

4A:- "INDEPENDENCE"

I - 1
N - 3
D - 2
E - 4
P - 1
C - 1

Total arrangements $= \frac{12!}{3!\,4!\,2!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1}$

$= 16,63,200$

(a) Word start with P

P_ _ _ _ _ _ _

No. of arrangements $= \frac{11!}{3!\,4!\,2!} \Rightarrow 138600$

$\Rightarrow \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1}$

$\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1}$

(b) ~~Word~~ Arranging vowels

Vowels = E = 4
I - 1

No. of arrangments $= \frac{5!}{4!} = 5$

Take the vowels as one unit =