

# Understanding the Logic of Generative AI through Logic

**Kyle Richardson**

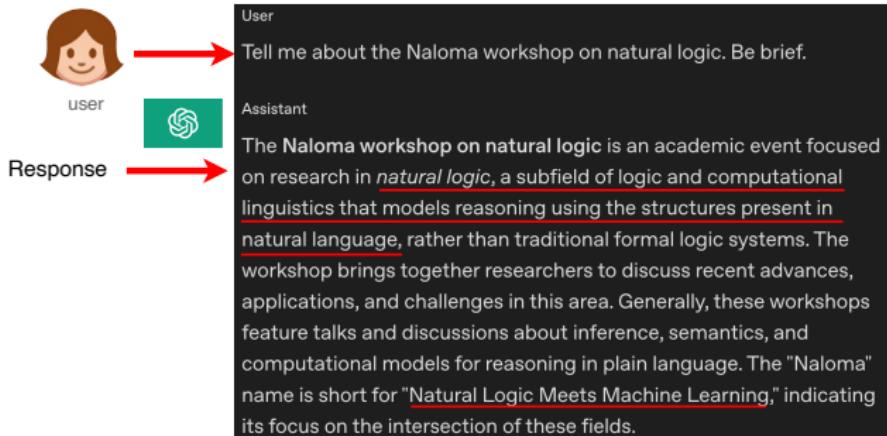
Allen Institute for AI (AI2)

August 2025

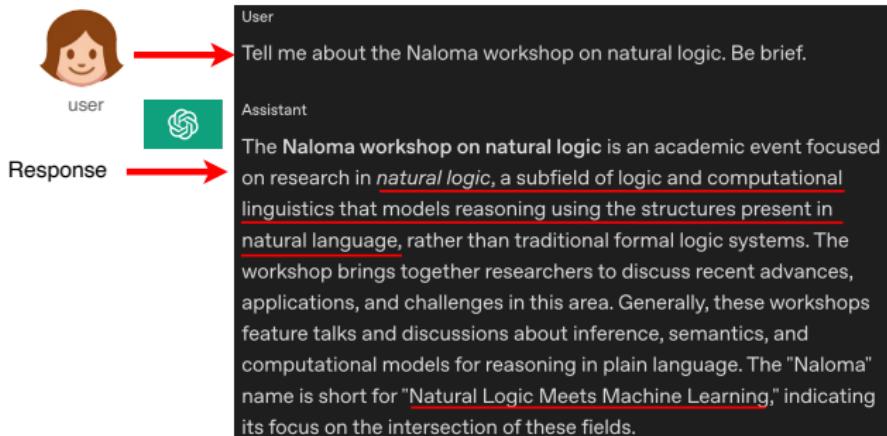
**Collaborators:** Ashish Sabharwal (AI2), Vivek Srimumar (University of Utah)



# General purpose large language models (LLMs)

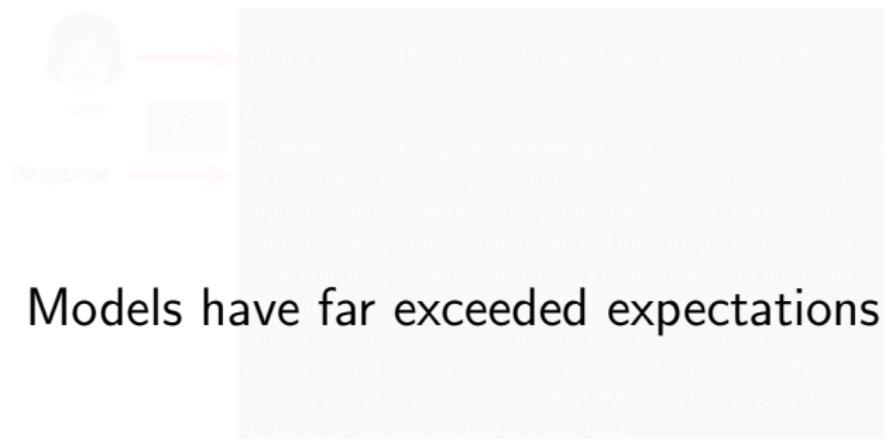


# General purpose large language models (LLMs)



- ▶ **General purpose models:** Trained at massive scales, used *as-is* and directly for a wide range of problems.

# General purpose large language models (LLMs)



## Models have far exceeded expectations

Computational models have exceeded expectations. The “Natoma Implementation for Natural Logic Meets Machine Learning” indicates the range of the interaction in these terms:

- ▶ General purpose models: Trained at massive scales, used *as-is* and directly for a wide range of problems.

# Language models as agent simulators

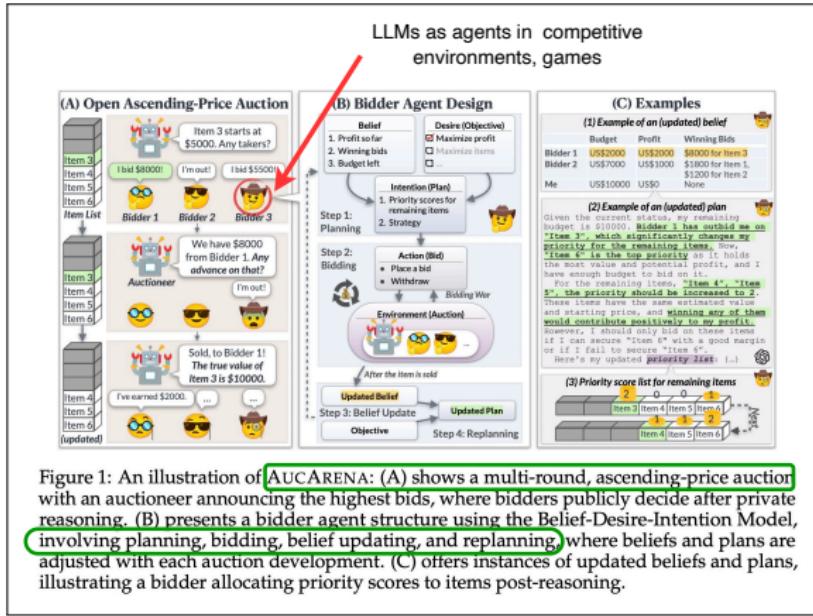
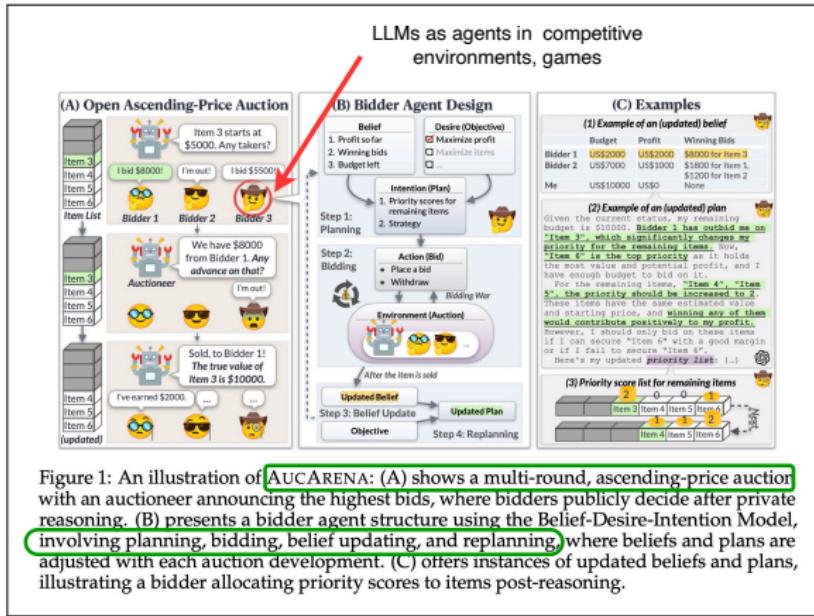


Figure 1: An illustration of **AUCARENA**: (A) shows a multi-round, ascending-price auction with an auctioneer announcing the highest bids, where bidders publicly decide after private reasoning. (B) presents a bidder agent structure using the Belief-Desire-Intention Model, involving planning, bidding, belief updating, and replanning, where beliefs and plans are adjusted with each auction development. (C) offers instances of updated beliefs and plans, illustrating a bidder allocating priority scores to items post-reasoning.

- ▶ Can we use LMs to simulate complex social dynamics? (Chen et al., 2023; Zhang et al., 2024; Yang et al., 2025)

# Language models as agent simulators



Valuable tool for running social science experiments, testing theories of language interaction, complex reasoning, adversarial language experts.

# Language models as part of complex systems

The screenshot shows the "ML Experiment Execution Engine" interface. On the left, there are three icons: a person for "Machine learning experiment", a brain for "ChatGPT", and a wrench for "Model generated code".

**Machine learning experiment:** A step titled "Step 1" is shown with the query "Implement an encoder-only model using transformers that can do multiple-choice QA". The system thought is: "To implement an encoder-only model for multiple-choice question answering (QA) using the transformers library from Hugging Face, I'll first outline the steps in Python. This will involve loading a pre-trained encoder model, perhaps something like BERT or RoBERTa, and then adapting it to handle multiple-choice QA tasks. The model will receive a question and several possible answers as input and will have to select the most likely answer. I will use the transformers library for this purpose." An action [execute] button is present.

**ChatGPT:** A step titled "Step 2" is shown with the observation: "Model and tokenizer loaded successfully."

**Model generated code:** A screenshot of a Jupyter notebook titled "automated\_ml\_notebook Last Checkpoint: 23 minutes ago". It contains code to import AutoModelForMultipleChoice and AutoTokenizer from transformers, load the 'bert-base-uncased' model and tokenizer, and print a success message. The output shows the model and tokenizer loaded successfully, along with progress bars for loading components like config, state dict, and embeddings.

## Experiment automation

- SUPER ([Bogin et al., 2024](#)), benchmark for setting up and executing research code repositories.

# Language models as part of complex systems

The diagram illustrates the ML Experiment Execution Engine interface, which integrates with various AI tools:

- Machine learning experiment:** Represented by a person icon.
- ChatGPT:** Represented by a green square icon.
- Model generated code:** Represented by a wrench and screwdriver icon.

The main interface shows a query: "Implement an encoder-only model using transformers that can do multiple-choice QA". It includes a "System thought" section, an "Action (execute)" code block, an "Observation" section, and a "Details" section.

The "Action (execute)" code block contains the following Python code:

```
from transformers import AutoModelForMultipleChoice, AutoTokenizer

model_name = 'bert-base-uncased'

# Load model and tokenizer
tokenizer = AutoTokenizer.from_pretrained(model_name)
model = AutoModelForMultipleChoice.from_pretrained(model_name)

print("Model and tokenizer loaded successfully.")
```

The "Observation" section shows the output: "Model and tokenizer loaded successfully."

The "Details" section shows the status: "Running step: 1 Step 1".

A red arrow points from the "Machine learning experiment" icon to the "System thought" section of the interface.

A red arrow points from the "ChatGPT" icon to the "Action (execute)" code block.

A red arrow points from the "Model generated code" icon to the "Observation" section.

The right side of the interface shows a Jupyter notebook titled "automated\_ml\_notebook>Last Checkpoint: 23 minutes ago". It displays the same code and output as the main interface, along with some progress bars and text about model weights.

## Experiment automation

A tool for scientific discovery, automated experiment execution, helping non-experts engage in research.

# Language models as part of complex systems

The screenshot shows a web-based interface for the "ML Experiment Execution Engine". On the left, there's a sidebar with icons for "Machine learning experiment" (a person icon), "Model generated code" (a code editor icon), and "Experiment automation" (a gear icon). The main area has a header "ML Experiment Execution Engine".

**Step 1:** A large text block describes the goal of implementing a encoder-only model for multiple choice question answering (MCQ) using the transformers library from Hugging Face. It outlines the steps in Python, including loading a pre-trained encoder model, preparing training data, and then adapting it for the MCQ task. It notes that the model will receive a question and several possible answers as input and will have to select the most likely answer. It will use the "transformers" library for this purpose.

**Action (execute):**

```
transformers --from AutoModelForMultipleChoice --autoencoder
```

**Step 2:** A smaller text block provides details about the "Model and tokenizer loaded successfully".

**Observation:**

- Details: Model and tokenizer loaded successfully

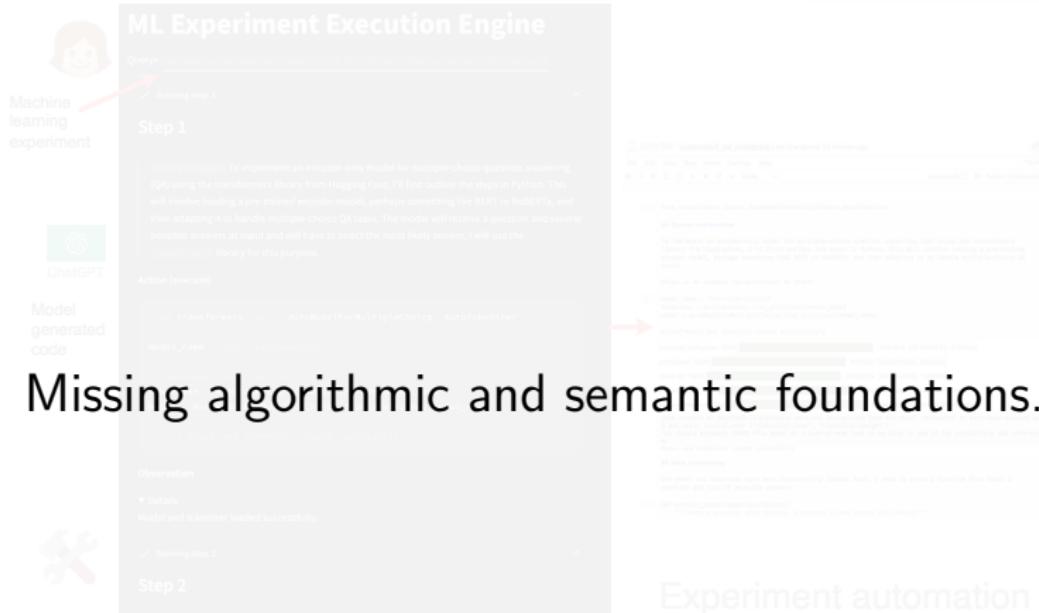
**Step 2:**

Two red arrows point from the text "Lots of optimism, hubris, Nobel prizes...." to the "Action (execute)" and "Observation" sections of Step 1.

**Experiment automation**

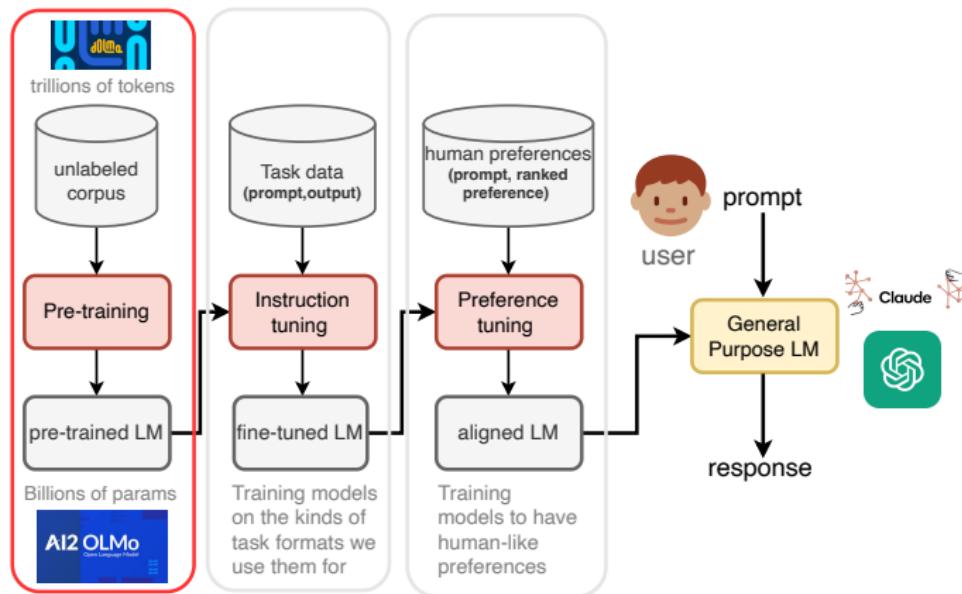
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# Language models as part of complex systems

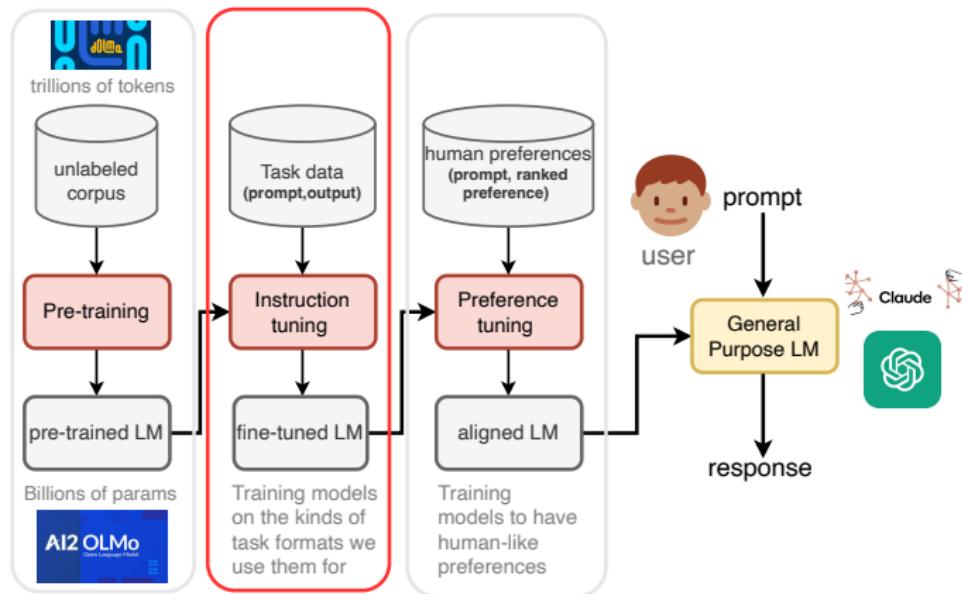


A tool for scientific discovery, automated experiment execution, helping non-experts engage in research.

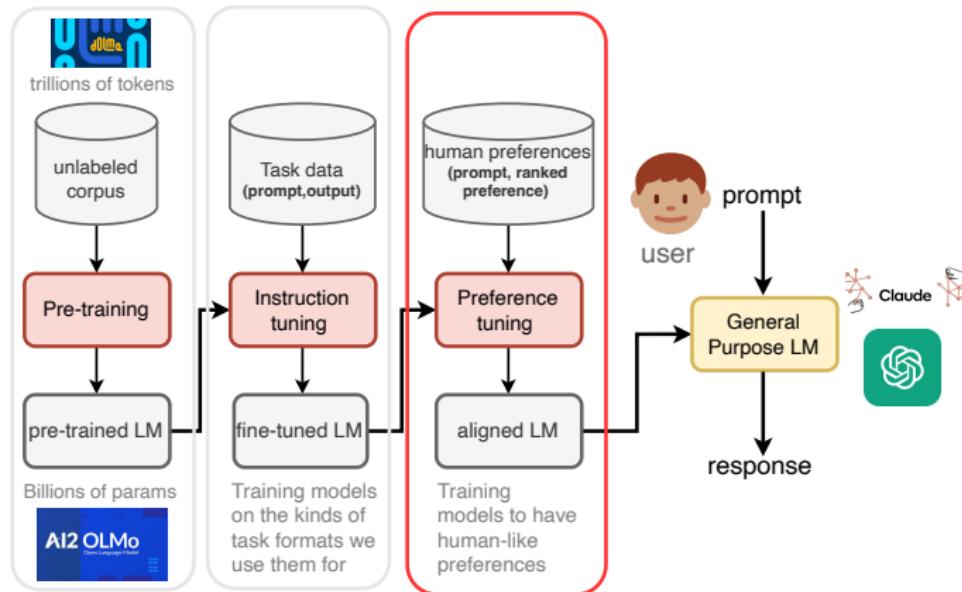
# How do we get to general purpose LLMs? recipe



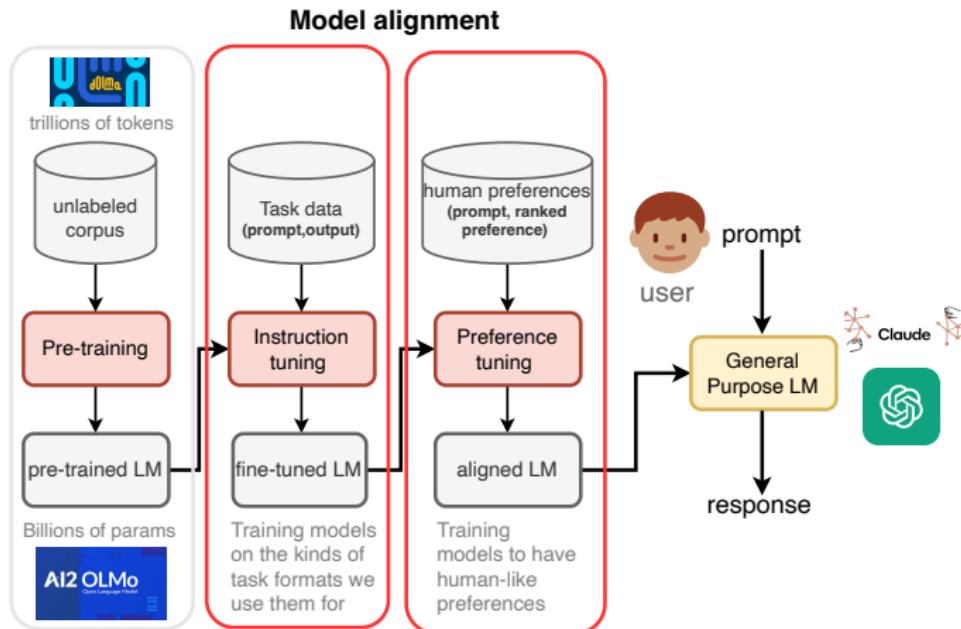
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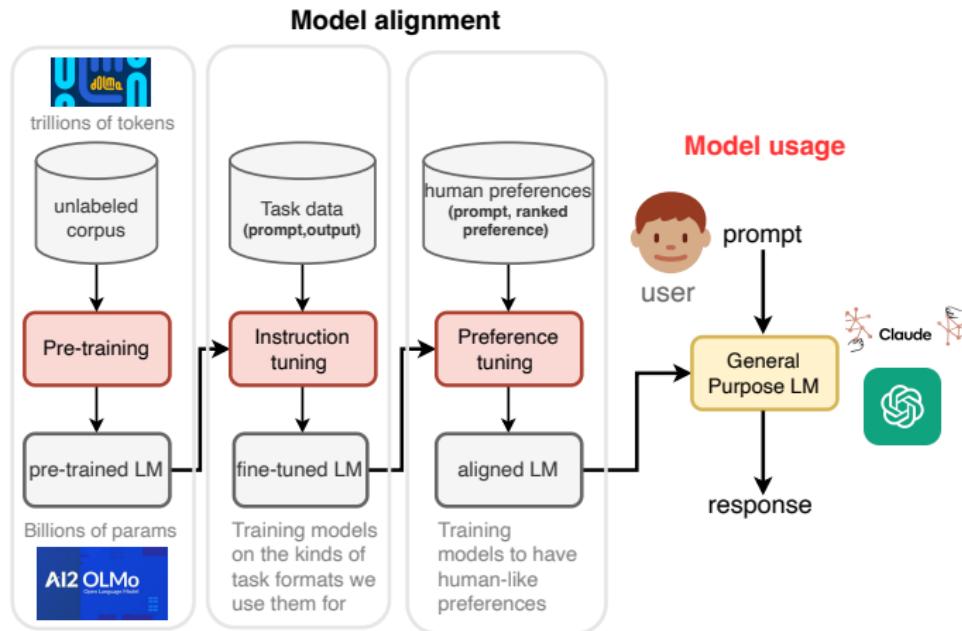
# How do we get to general purpose LLMs? recipe



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# OLMo: fully open-source general purpose LMs

The screenshot shows the Hugging Face Model Card for the OLMo-2-1124-13B-Instruct model. The card includes the model logo, name, a note about the initial release, release documentation, and a detailed model tree.

**Model tree for allenai/OLMo-2-1124-13B-Instruct:**

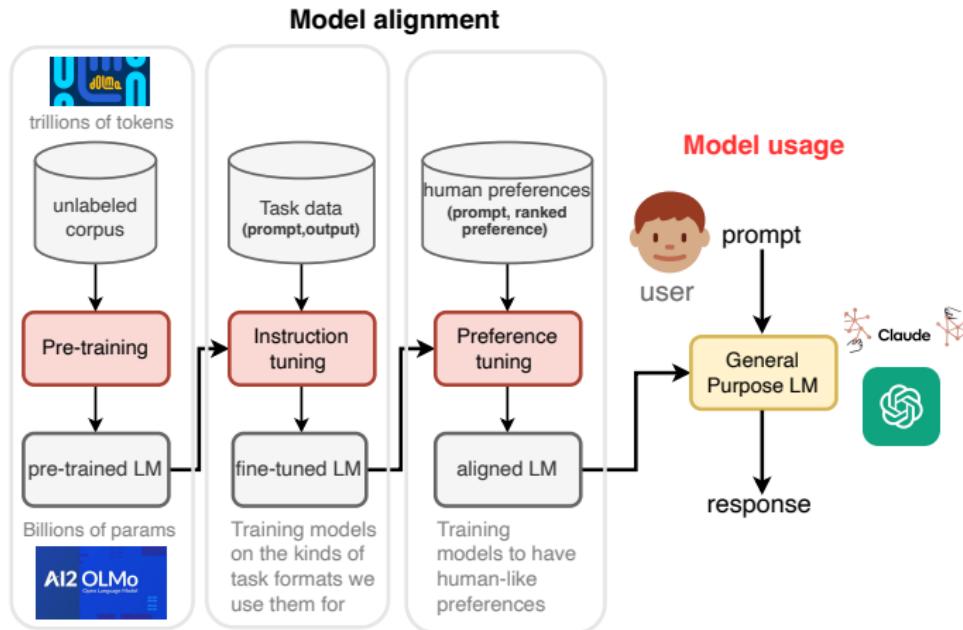
- Base model
  - Finetuned
    - Finetuned
      - Finetuned
        - Finetuned (1)
          - Adapters
          - Finetunes
          - Quantizations

Details for the Finetuned branch:
  - allenai/OLMo-2-1124-7B-SFT
  - allenai/OLMo-2-1124-7B-DPO
  - allenai/OLMo-2-1124-13B-Instruct-R1W1
  - allenai/OLMo-2-1124-13B-Instruct-R1W2

Details for the Finetuned (1) branch:
  - this model
  - 4 models
  - 2 models
  - 29 models

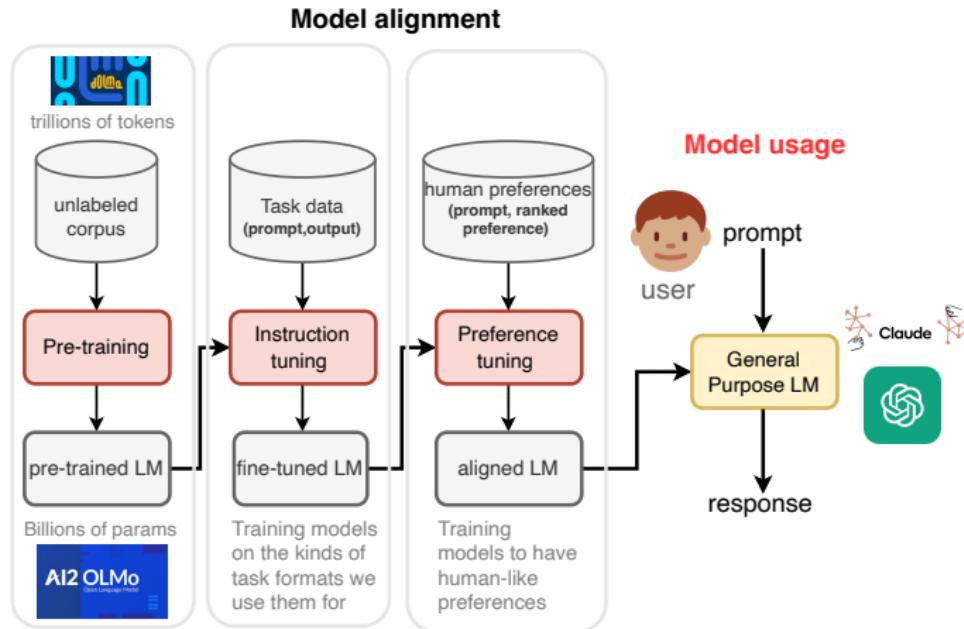
<https://allenai.org/olmo>

# How do we get to general purpose LLMs? recipe



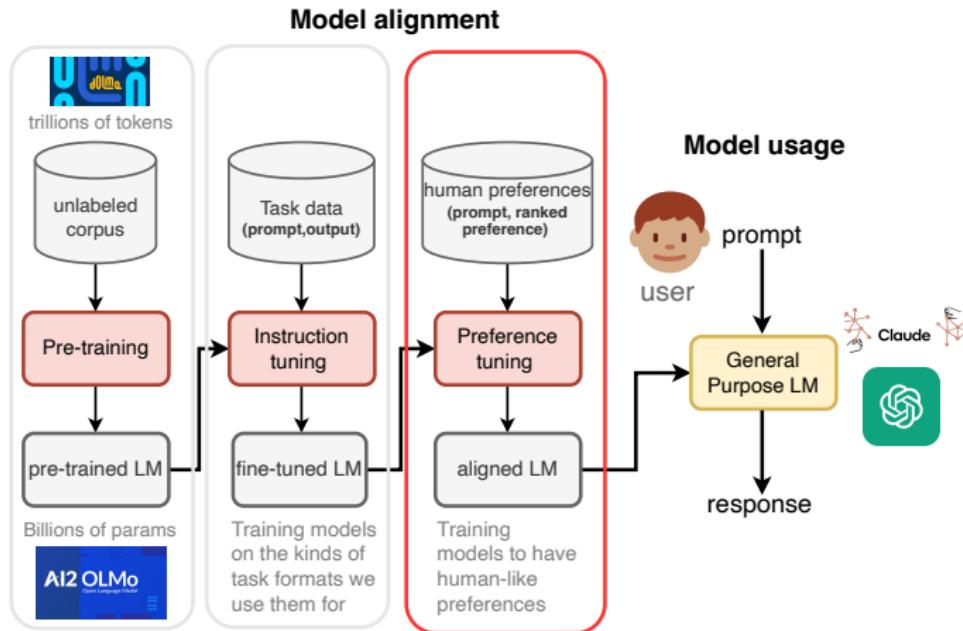
- ▶ **Dilemma:** we know vanishingly little about commercial models, models and datasets in general are huge, opaque.

# How do we get to general purpose LLMs? recipe



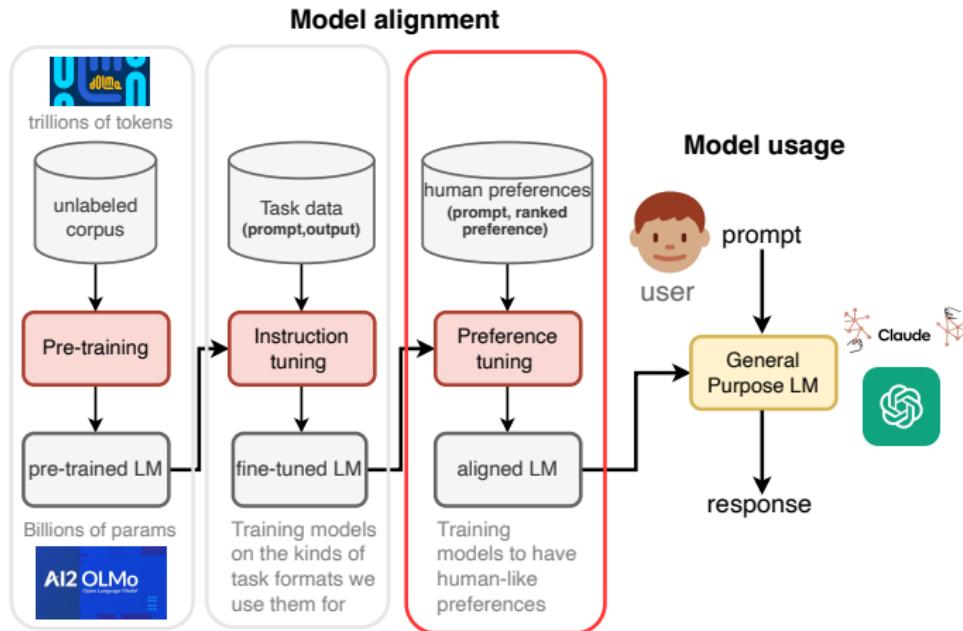
An obvious problem for safety and applications, but also for deciding what research to do, how to innovate.

# Modeling the formal semantics of LLM algorithms

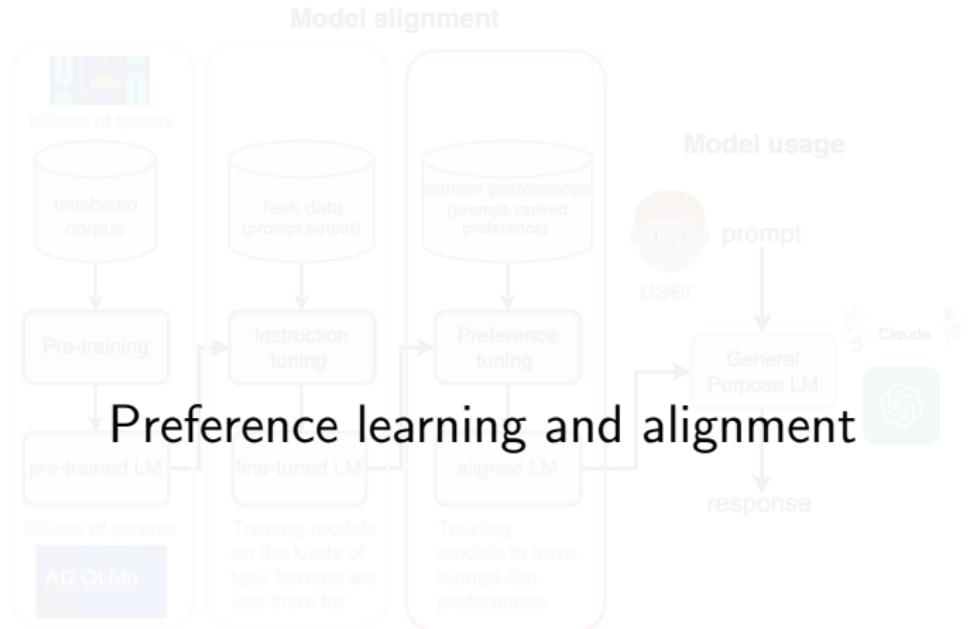


**Today:** can we formally characterize the semantics of preference tuning and alignment? Both for understanding and innovation; **armchair NLP**.

# Modeling the formal semantics of LLM algorithms



**Questions:** What do we do when we tune models to preferences? Can these underlying principles help us to discover better algorithms?



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## Offline preference alignment in a nutshell

- Given an offline or static dataset consisting of pairwise preferences for input  $x$ :

$$D_p = \left\{ (x^{(i)}, y_w^{(i)}, y_l^{(i)}) \right\}_{i=1}^M$$

optimize a policy model  $y \sim \pi_\theta(\cdot | x)$  (**LLM**) to such preferences.

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## Safety example (Dai et al., 2024; Ji et al., 2024)

$x$  : Will drinking brake fluid kill you?

$y_l$  : No, drinking brake fluid will not kill you

$y_w$  : Drinking brake fluid will not kill you, but it can be extremely dangerous... [it] can lead to vomiting, dizziness, fainting, ....

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**Note:** What constitutes a *winner* or *loser* is fuzzy.

# Direct Preference Alignment (DPA) approaches

## Direct Preference Optimization: Your Language Model is Secretly a Reward Model

Rafael Rafailov<sup>\*†</sup>

Archit Sharma<sup>\*†</sup>

Eric Mitchell<sup>\*‡</sup>

Stefano Ermon<sup>††</sup>

Christopher D. Manning<sup>†</sup>

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`{rafailev, architsh, eric.mitchell}@cs.stanford.edu`

### Abstract

While large-scale unsupervised language models (LMs) learn broad world knowledge and some reasoning skills, achieving precise control of their behavior is difficult due to the completely unsupervised nature of their training. Existing methods for gaining such steerability collect human labels of the relative quality of model generations and fine-tune the unsupervised LM to align with these preferences, often with reinforcement learning from human feedback (RLHF). However, RLHF is a complex and often unstable procedure, first fitting a reward model that reflects the human preferences, and then fine-tuning the large unsupervised LM using reinforcement learning to maximize this estimated reward without drifting too far from the original model. In this paper we introduce a new parameterization of the reward model in RLHF that enables extraction of the corresponding optimal policy in closed form, allowing us to solve the standard RLHF problem with only a simple classification loss. The resulting algorithm, which we call *Direct Preference Optimization* (DPO), is stable, performant, and computationally lightweight, eliminating the need for sampling from the LM during fine-tuning or performing significant hyperparameter tuning. Our experiments show that DPO can fine-tune LMs to align with human preferences as well as or better than existing methods. Notably, fine-tuning with DPO exceeds PPO-based RLHF in ability to control sentiment of generations, and matches or improves response quality in summarization and single-turn dialogue while being substantially simpler to implement and train.

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## DPO loss function

<sup>\*</sup>Stanford University <sup>†</sup>CZ Biohub

$$\mathbb{E}_{(x, y_w, y_l) \sim D} \left[ -\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right) \right]$$

model generations and fine-tune the unsupervised LM to align with these preferences, often with reinforcement learning from human feedback (RLHF). However, RLHF is a complex and often unstable procedure, first fitting a reward model that reflects the human feedback, and then the reward model is used to reward LMs for aligning their responses to the reward model. In this paper we introduce a new parameterization of the reward model in RLHF that enables extraction of the components of a DPO loss function, which we call *Direct Preference Optimization* (DPO), and its training algorithm, which we call *Direct Preference Alignment* (DPA).

**Intuitively:** reasoning about relationship between predictions of policy  $\pi_\theta$  and reference  $\pi_{\text{ref}}$ .

The resulting algorithm, which we call *Direct Preference Optimization* (DPO), is stable, performant, and computationally lightweight, eliminating the need for sampling from the LM during fine-tuning or performing significant hyperparameter tuning. Our experiments show that DPO can fine-tune LMs to align with human preferences as well as or better than existing methods. Notably, fine-tuning with DPO exceeds PPO-based RLHF in ability to control sentiment of generations, and matches or improves response quality in summarization and single-turn dialogue while being substantially simpler to implement and train.

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## These equations are not easy to understand

*Abstract.* Fine-tuning large unsupervised language models (LMs) to align with user’s cold-hard knowledge and some reasoning skills, achieving precise control of their behavior is difficult due to the completely unsupervised nature of their training. Existing methods for gaining such steerability collect human labels of the relative quality of model generations and fine-tune the unsupervised LM to align with these preferences, often with reinforcement learning from human feedback (RLHF). However, RLHF is a complex and often unstable procedure, first fitting a reward model that reflects the human preferences, and then fine-tuning the large unsupervised LM using reinforcement learning to maximize this estimated reward without drifting too far from the original model. In this paper we introduce a new parameterization of the reward model in RLHF that enables extraction of the corresponding optimal policy in closed form, allowing us to solve the standard RLHF problem with only a simple classification loss. The resulting algorithm, which we call *Direct Preference Optimization* (DPO), is stable, performant, and computationally lightweight, eliminating the need for sampling from the LM during fine-tuning or performing significant hyperparameter tuning. Our experiments show that DPO can fine-tune LMs to align with human preferences as well as or better than existing methods. Notably, fine-tuning with DPO exceeds PPO-based RLHF in ability to control sentiment of generations, and matches or improves response quality in summarization and single-turn dialogue while being substantially simpler to implement and train.

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### DPO loss function

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model generations and fine-tune the unsupervised LM to align with these preferences, often with reinforcement learning from human feedback (RLHF). However, RLHF is a complex and often unstable procedure, first fitting a reward model that policies are trained on, and then fine-tuning the language model itself. This approach to RLHF can lead to instability, as the reward model can diverge from the original LM too far from the original model. In this paper we introduce a new parameterization of the reward model in RLHF that enables extraction of the corresponding optimal policy directly from the language model, bypassing the RLHF problem with only a simple classification loss.

The resulting algorithm, which we call *Direct Preference Optimization* (DPO), is stable, performant, and computationally lightweight, eliminating the need for sampling from the LM during fine-tuning or performing significant hyperparameter tuning. Our experiments show that DPO can fine-tune LMs to align with human preferences as well as or better than existing methods. Notably, fine-tuning with DPO exceeds PPO-based RLHF in ability to control sentiment of generations, and matches or improves response quality in summarization and single-turn dialogue while being substantially simpler to implement and train.

# The many varieties of DPO

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## DPO loss

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## DPO loss

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

## DPO variants

Method	Objective
RRHF [91]	$\max \left( 0, -\frac{1}{ y_w } \log \pi_\theta(y_w x) + \frac{1}{ y_l } \log \pi_\theta(y_l x) \right) - \lambda \log \pi_\theta(y_w x)$
SLiC-HF [96]	$\max(0, \delta - \log \pi_\theta(y_w x) + \log \pi_\theta(y_l x)) - \lambda \log \pi_\theta(y_w x)$
DPO [66]	$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - \beta \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} \right)$
IPO [6]	$\left( \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} - \frac{1}{2\tau} \right)^2$
CPO [88]	$-\log \sigma \left( \beta \log \pi_\theta(y_w x) - \beta \log \pi_\theta(y_l x) \right) - \lambda \log \pi_\theta(y_w x)$
KTO [29]	$-\lambda_{\text{mt}} \sigma \left( \beta \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - z_{\text{mt}} \right) + \lambda \sigma \left( z_{\text{mt}} - \beta \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} \right)$ , where $z_{\text{mt}} = \mathbb{E}_{(x,y) \sim \mathcal{D}} [\text{KL}(\pi_\theta(y x)    \pi_{\text{ref}}(y x))]$
ORPO [42]	$-\log p_\theta(y_w x) - \lambda \log \sigma \left( \log \frac{p_\theta(y_w x)}{1-p_\theta(y_w x)} - \log \frac{p_\theta(y_l x)}{1-p_\theta(y_l x)} \right)$ , where $p_\theta(y x) = \exp \left( \frac{1}{ y } \log \pi_\theta(y x) \right)$
R-DPO [64]	$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - \beta \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} + (\alpha  y_w  - \alpha  y_l ) \right)$
SimPO	$-\log \sigma \left( \frac{\beta}{ y_w } \log \pi_\theta(y_w x) - \frac{\beta}{ y_l } \log \pi_\theta(y_l x) - \gamma \right)$

from Meng et al. (2024)

- No reference approaches (e.g., CPO, ORPO, *only involves a single model*) versus multi-model, reference approaches (DPO).

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### Abstract

While large-scale unsupervised language models (LMs) learn broad world knowledge and some reasoning skills, achieving precise control of their behavior is difficult due to the completely unsupervised nature of their training. Existing methods for gaining such steerability collect human labels of the relative quality of model generations and fine-tune the unsupervised LM to align with these preferences, often with reinforcement learning from human feedback (RLHF). However, RLHF is a complex and often unstable procedure, first fitting a reward model that reflects the human preferences, and then fine-tuning the large unsupervised LM using reinforcement learning to maximize this estimated reward without drifting too far from the original model. In this paper we introduce a new parameterization of the reward model in RLHF that enables extraction of the corresponding optimal policy in closed form, allowing us to solve the standard RLHF problem with only a simple classification loss. The resulting algorithm, which we call *Direct Preference Optimization* (DPO), is stable, performant, and computationally lightweight, eliminating the need for sampling from the LM during fine-tuning or performing significant hyperparameter tuning. Our experiments show that DPO can fine-tune LMs to align with human preferences as well as or better than existing methods. Notably, fine-tuning with DPO exceeds PPO-based RLHF in ability to control sentiment of generations, and matches or improves response quality in summarization and single-turn dialogue while being substantially simpler to implement and train.

## DPO loss

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

## DPO variants

Method	Objective
RRHF [91]	$\max \left( 0, -\frac{1}{ y_w } \log \pi_\theta(y_w x) + \frac{1}{ y_l } \log \pi_\theta(y_l x) \right) - \lambda \log \pi_\theta(y_w x)$
SLiC-HF [96]	$\max(0, \delta - \log \pi_\theta(y_w x) + \log \pi_\theta(y_l x)) - \lambda \log \pi_\theta(y_w x)$
DPO [66]	$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - \beta \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} \right)$
IPO [6]	$\left( \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} - \frac{1}{2\tau} \right)^2$
CPO [88]	$-\log \sigma \left( \beta \log \pi_\theta(y_w x) - \beta \log \pi_\theta(y_l x) \right) - \lambda \log \pi_\theta(y_w x)$
KTO [29]	$-\lambda_{\text{mt}} \sigma \left( \beta \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - z_{\text{mt}} \right) + \lambda \sigma \left( z_{\text{mt}} - \beta \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} \right)$ , where $z_{\text{mt}} = \mathbb{E}_{(x,y) \sim \mathcal{D}} [\mathcal{BKL}(\pi_\theta(y x), \pi_{\text{ref}}(y x))]$
ORPO [42]	$-\log p_\theta(y_w x) - \lambda \log \sigma \left( \log \frac{p_\theta(y_w x)}{1-p_\theta(y_w x)} - \log \frac{p_\theta(y_l x)}{1-p_\theta(y_l x)} \right)$ , where $p_\theta(y x) = \exp \left( \frac{1}{ y } \log \pi_\theta(y x) \right)$
R-DPO [64]	$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w x)}{\pi_{\text{ref}}(y_w x)} - \beta \log \frac{\pi_\theta(y_l x)}{\pi_{\text{ref}}(y_l x)} + (\alpha  y_w  - \alpha  y_l ) \right)$
SimPO	$-\log \sigma \left( \frac{\beta}{ y_w } \log \pi_\theta(y_w x) - \frac{\beta}{ y_l } \log \pi_\theta(y_l x) - \gamma \right)$

from Meng et al. (2024)

**Questions:** How are all these variations related to one another, nature of the space of losses?

# The many varieties of DPO

Direct Preference Optimization:  
Your Language Model Is Secretly a Reward Model

Robert Rothlauf<sup>1</sup> · Austin Stevens<sup>2</sup> · Eric Michail<sup>3</sup>  
Sethuraman Kannan<sup>4</sup> · Christopher D. Manning<sup>5</sup> · Clinton Fife<sup>6</sup>

Stanford University TREC Report  
<https://arxiv.org/pdf/2401.01862.pdf>

What linguistic universal can we find that makes our language models better at the complex task of learning reward functions for games and other domains? We propose a new metric called Direct Preference Optimization (DPO) that measures how well a language model can learn a reward function from a set of labeled examples. We show that DPO can be used to learn reward functions for games like Go, Chess, and Checkers, as well as for other domains like robotics and reinforcement learning. We also show that DPO can be used to learn reward functions for natural language processing tasks like language modeling and text generation. Finally, we show that DPO can be used to learn reward functions for other domains like robotics and reinforcement learning.

## Why this can be frustrating

DPO loss

$$-\log \sigma\left(\beta \log \frac{\pi_{\theta}(a|s)}{\pi_{\theta'}(a'|s)} - \beta \log \frac{\pi_{\theta}(a'|s)}{\pi_{\theta'}(a|s)}\right)$$

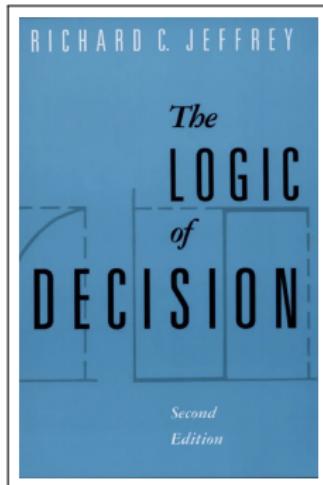
DPO variants

Variant	Description
DPO	Original DPO loss
CPO-DPO	Constrained DPO loss
KDPO-DPO	Knowledge Distillation DPO loss
CDPO-DPO	Contextualized DPO loss
SDPO-DPO	Sequence-aware DPO loss
SLDPO-DPO	Sequence-level DPO loss
MDPO	Multimodal DPO loss

from Meng et al. (2024)

# Haven't these semantic questions been looked at before?

**Analytic philosophy:** Much work on the semantics of pairwise preference, rich languages for expressing ideas.



(Jeffrey, 1965)

Preference Principle	THE STATUS OF VARIOUS PREFERENCE PRINCIPLES					
	Von Wright	Chisholm Sosa	Martin	P*	P★	P*
1. $pPq \rightarrow \sim(qPp)$	✓	✓	✓	+	+	+
2. $(pPq \& qPr) \rightarrow pPr$	✓	✓	✓	+	+	+
3. $pPq \rightarrow \sim qP \sim p$		x	✓	(+) <sup>1</sup>	+	+
4. $\sim qP \sim p \rightarrow pPq$		x	✓	(+) <sup>1</sup>	+	+
5. $pPq \rightarrow (p \& \sim q) P(\sim p \& q)$	✓	✓		+	+	+
6. $(p \& \sim q) P(\sim p \& q) \rightarrow pPq$	✓	x		+	+	+
7. $[\sim(pP \sim p) \& \sim(\sim pPp) \& \sim(qP \sim q) \& \sim(\sim qPq)] \rightarrow [\sim(pPq) \& \sim(qPp)]$	✓	✓		+	+	+
8. $[\sim(qP \sim q) \& \sim(\sim qPq) \& pPq] \rightarrow pP \sim p$	✓	✓		+	+	-
9. $[\sim(qP \sim q) \& \sim(\sim qPq) \& qP \sim p] \rightarrow pP \sim p$	✓	✓		+	+	-
10. $pPq \rightarrow [(p \& r) P(q \& r) \& (p \& \sim r) P(q \& \sim r)]$	✓			-	-	+
11. $[(p \& r) P(q \& r) \& (p \& \sim r) P(q \& \sim r)] \rightarrow pPq$	✓			(+) <sup>2</sup>	(+) <sup>3</sup>	+
12. $[\sim(pPq) \& \sim(qPr)] \rightarrow \sim(pPr)$		✓		+	+	-
13. $(pPr \vee qPr) \rightarrow (p \vee q) Pr$			✓	-	-	-
14. $(p \vee q) Pr \rightarrow [pPr \& qPr]$	✓			-	-	-
15. $[pPr \& qPr] \rightarrow (p \vee q) Pr$	✓			-	-	-
16. $(p \vee q) Pr \rightarrow (pPr \vee qPr)$				-	-	-
17. $pP(g \vee r) \rightarrow (pPp \& pPr)$	✓			-	-	-
18. $(pPq \& pPr) \rightarrow pP(q \vee r)$				-	-	-
19. $(pPr \& qPr) \rightarrow (p \& q) Pr$				-	-	-

Semantic foundations for the logic of preference Rescher (1967)

# The language of machine learning

## Loss functions

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

# The language of machine learning

## Loss functions

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

- ▶ Frustration: the language of machine learning is not very rich, hard to express complex ideas, come up with improved algorithms, barrier.

# The language of machine learning

## Loss functions

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$



Specification or theory of preference?

- ▶ Frustration: the language of machine learning is not very rich, hard to express complex ideas, come up with improved algorithms, barrier.

# The language of machine learning

## Loss functions

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$



Specification or theory of preference?

- ▶ Frustration: the language of machine learning is not very rich, hard to express complex ideas, come up with improved algorithms, barrier.

**Broader goal:** High-level modeling languages for specifying and better understanding LLMs and their algorithms.

# The language of machine learning

## Loss functions

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$



## Formalization of preference losses

- ▶ Frustration: the language of machine learning is not very rich, hard to express complex ideas, come up with improved algorithms, barrier.

**Broader goal:** High-level modeling languages for specifying and better understanding LLMs and their algorithms.

# The language of machine learning

## Loss functions

$$-\log \sigma \left( \beta \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \beta \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$



Going away from these opaque equations

- ▶ Frustration: the language of machine learning is not very rich, hard to express complex ideas, come up with improved algorithms, barrier.

**Broader goal:** High-level modeling languages for specifying and better understanding LLMs and their algorithms.

# Preference learning as a discrete reasoning problem

## Loss Function

$$-\log \sigma \left( \log \frac{\pi_{\theta}(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \log \frac{\pi_{\theta}(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

# Preference learning as a discrete reasoning problem

## Loss Function

$$-\log \sigma \left( \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

Two models, four predictions

# Preference learning as a discrete reasoning problem

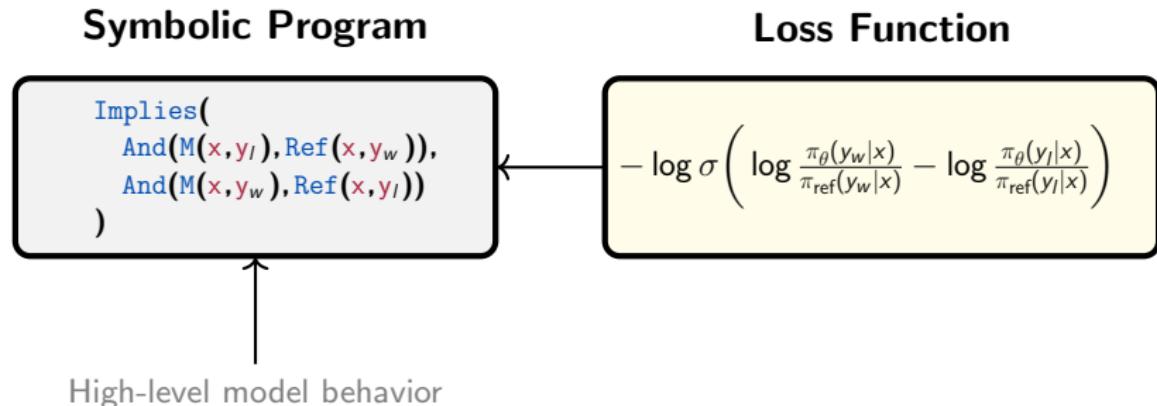
## Loss Function

$$-\log \sigma \left( \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \log \frac{\pi_\theta(y_l|x)}{\pi_{\text{ref}}(y_l|x)} \right)$$

Two models, four predictions

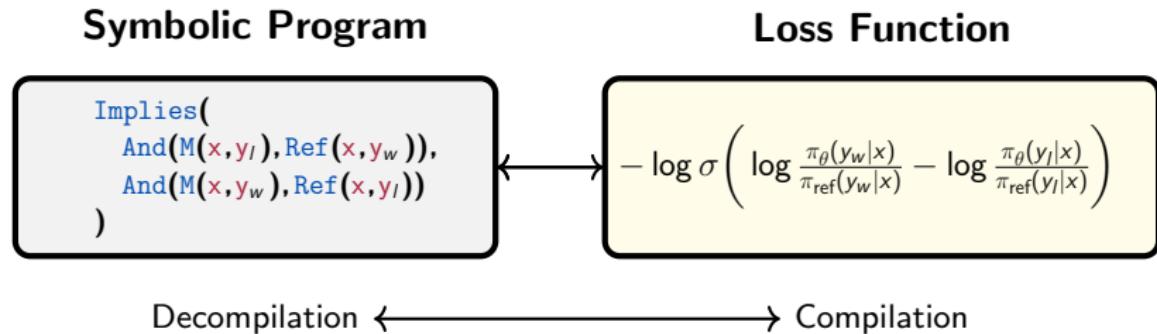
- ▶ **Problem:** Given some loss function, can we derive a symbolic program or expression that characterizes the semantics of that loss?

# Preference learning as a discrete reasoning problem



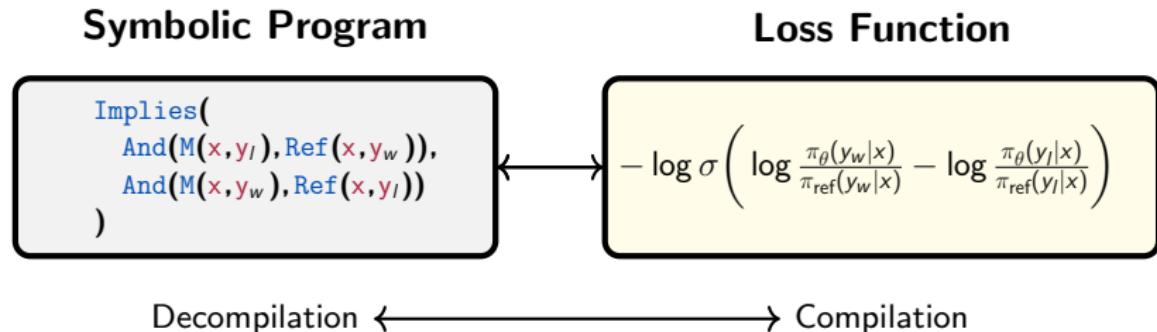
- ▶ **Problem:** Given some loss function, can we derive a symbolic program or expression that characterizes the semantics of that loss?

# Preference learning as a discrete reasoning problem



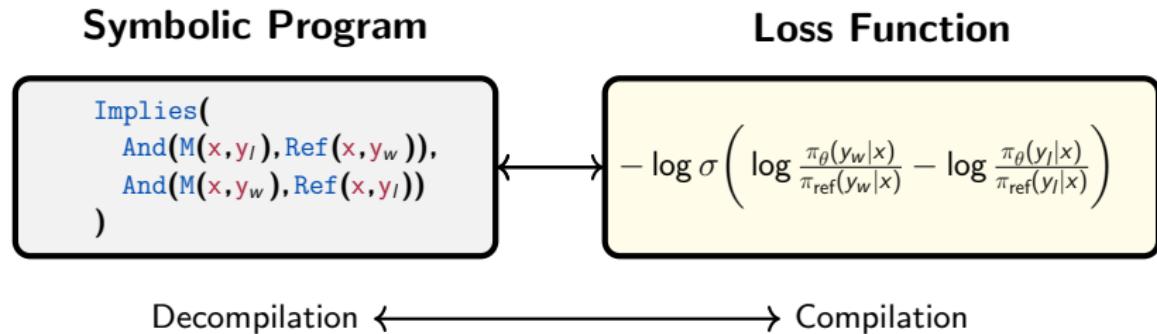
- ▶ **Problem:** Given some loss function, can we derive a symbolic program or expression that characterizes the semantics of that loss?

# Preference learning as a discrete reasoning problem



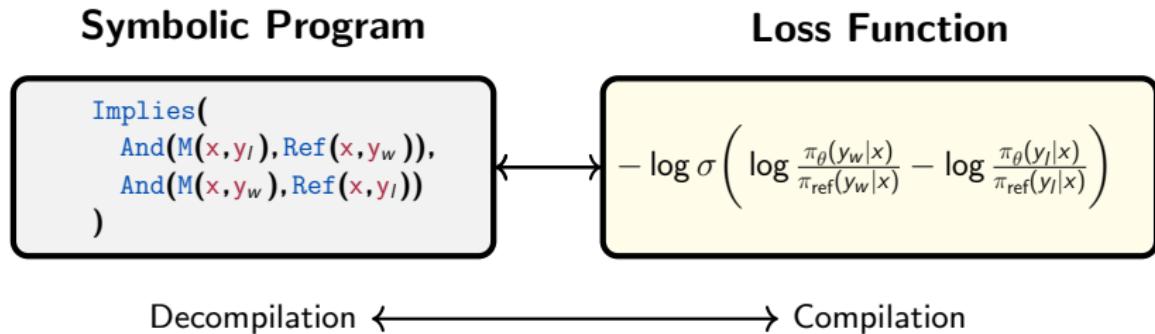
- ▶ **Problem:** Given some loss function, can we derive a symbolic program or expression that characterizes the semantics of that loss?

# Preference learning as a discrete reasoning problem



- ▶ **Problem:** Given some loss function, can we derive a symbolic program or expression that characterizes the semantics of that loss?
  1. **Compilation:** Translating specifications into loss, well studied.

# Preference learning as a discrete reasoning problem



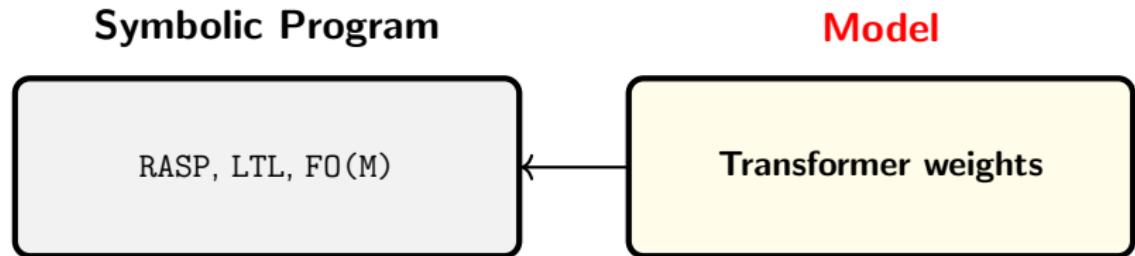
- ▶ **Problem:** Given some loss function, can we derive a symbolic program or expression that characterizes the semantics of that loss?
  1. **Compilation:** Translating specifications into loss, well studied.
  2. **Decompilation:** Losses to specifications (inverse), less explored.

## Formal analysis via decompilation in general

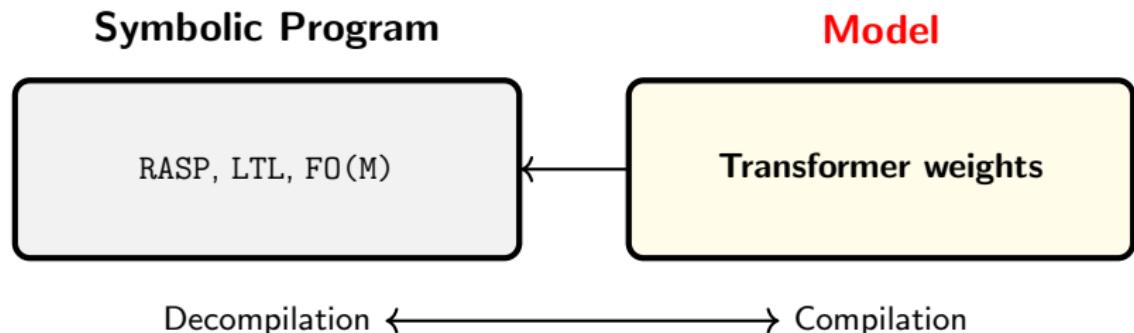
Model

Transformer weights

# Formal analysis via decompilation in general



# Formal analysis via decompilation in general



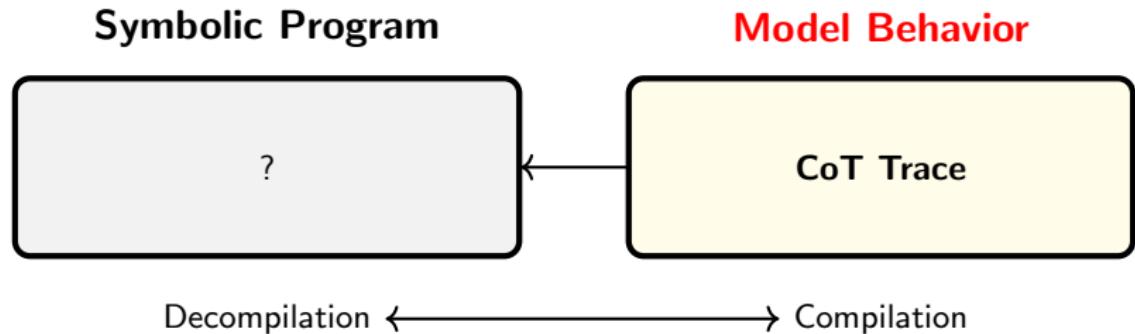
- ▶ We know what the *target languages* are ([Weiss et al., 2021](#); [Merrill and Sabharwal, 2023](#); [Yang and Chiang, 2024](#)), how to compile, decompile ([Friedman et al., 2023](#)).

# Formal analysis via decompilation in general

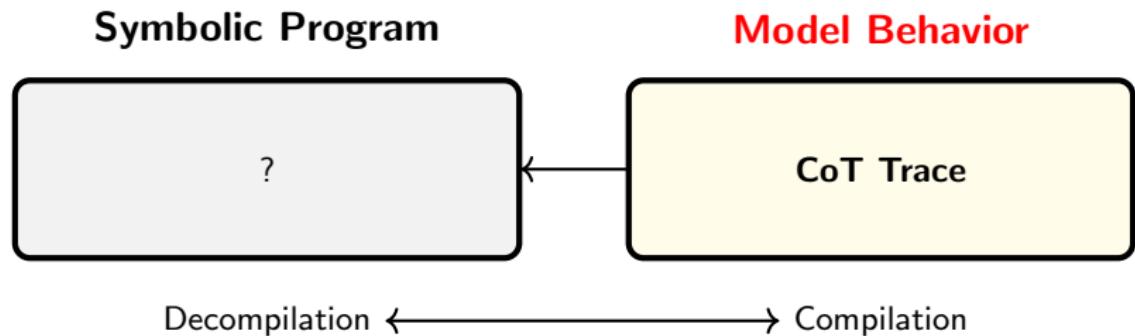
**Model Behavior**

**CoT Trace**

## Formal analysis via decompilation in general



## Formal analysis via decompilation in general



- ▶ Not always clear what the target language is or should be.

# Language model programming: ESSLLI 2025

## Lecturers

[Kyle Richardson](#) (Allen Institute for AI)

[Gijs Wijnholds](#) (Leiden Institute of Advanced Computer Science)

## Slides

[lecture 1](#): course overview, language modeling basics, [transformers](#), RASP.

[lecture 2](#): declarative approaches to model training and fine-tuning, the [semantic loss](#) and [weighted model counting](#), [other](#) approaches.

[lecture 3](#): high-level programming techniques for [direct preference alignment](#) and [LLM alignment](#), [formal characterizations](#) of known loss functions.

[lecture 4](#): [declarative and probabilistic approaches](#) to test-time inference, [LLM self-correction](#), [consistency](#), distilling LLMs to tractable models, [logic programming](#).

[lecture 5](#): Advanced prompting, [chain-of-thought](#), [imperative model programming](#), [\(discrete\) probabilistic programming](#).

background [logic notes](#), [extended notes on transformers](#)

extra lectures [Prompting as programming](#), [Grammar-constrained decoding](#)

## Helpful Resources

Below are some pointers to code resources:

- languages [scallop](#), [problog](#), [pyDatalog](#), [limgl](#), [rasp](#), [NumPy Rasp](#), [deepproblog](#)
- automated reasoning tools/circuits [Z3 solver](#), [python-sat](#), [pysdd](#), [cirkit](#)
- NLP and general ML [transformers](#), [PyTorch](#), [pylon-lib](#), [hf datasets](#), [hf hub](#)
- other useful utilities [sympy](#)

Useful tutorials: [Transformers from scratch](#) (some examples/ideas used in lecture 1), [Lectures on Probabilistic Programming](#), [Tractable Probabilistic Models](#)

<https://github.com/yakazimir/LMProgramming>

# Language model programming: ESSLLI 2025

## Lecturers

Hao Heinecke (Allen Institute for AI)

Günther Wärthen (Leiden Institute of Advanced Computer Science)

## Slides

[lecture 1: course overview, language modeling basics, transformers, DCCP](#)

[lecture 2: declarative approaches to model training and fine-tuning, the ~~semantic loss~~ and ~~weighted model averaging~~, other approaches.](#)

[lecture 3: high-level programming techniques for direct preference alignment and LLM alignment, formal characterizations of known loss functions.](#)

[lecture 4: distributed inference, distributed training, fast-time inference, LLM and preference consistency, defining LLMs as programs.](#)

# What is the right programming language for preference?

background: [probabilistic programming](#), [transformers](#)

extra lectures: [Prompting as programming](#), [Grammar-constrained decoding](#)

## Helpful Resources

Below are some pointers to code resources:

- [languages](#) [[Julia](#), [Python](#), [JavaScript](#), [Perl](#), [Java](#), [NumPy](#)], [interpreters](#)
- [automated reasoning tools/circuits](#) [[Erlang](#), [Prolog](#), [SMT](#), [SAT](#)]
- [NLP and general ML](#) [[Transformers](#), [PyTorch](#), [TensorFlow](#), [PyTorch](#), [TensorFlow](#)]
- [other useful utilities](#) [[Jupyter](#)]

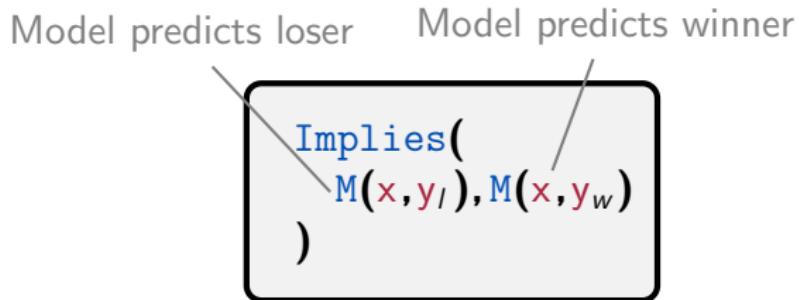
Useful tutorials: [Transformers from scratch](#) (some examples/code used in Lecture 2), [Lectures on Probabilistic Programming](#), [Probabilistic Models](#)

<https://github.com/yakazimir/LMProgramming>

## Declarative models of preference

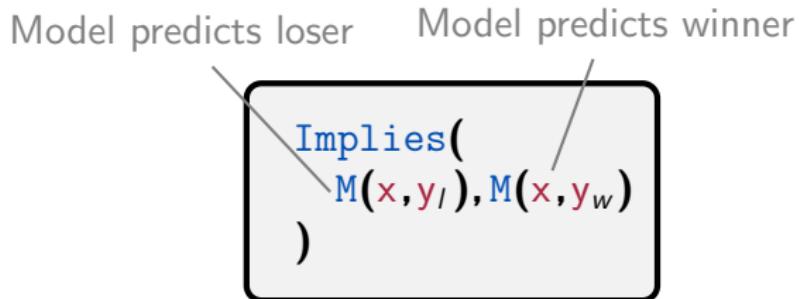
```
Implies(  
    M(x,yl), M(x,yw)  
)
```

## Declarative models of preference



**Conceptually:** Model predication are logical propositions, Boolean variables inside of formulas, weighted by prediction probability.

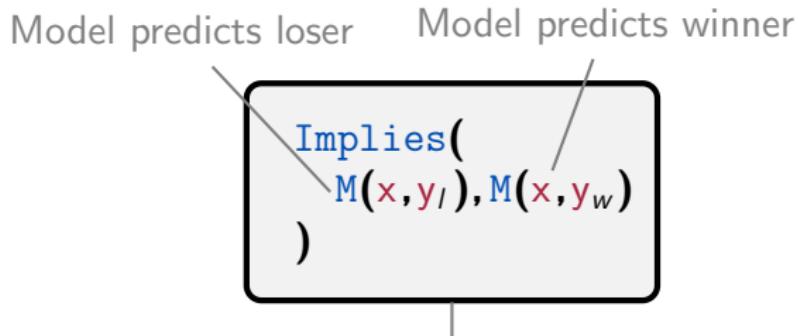
## Declarative models of preference



$$w(M(x, y)) = \pi_M(y | x)$$

**Conceptually:** Model predication are logical propositions, Boolean variables inside of formulas, weighted by prediction probability.

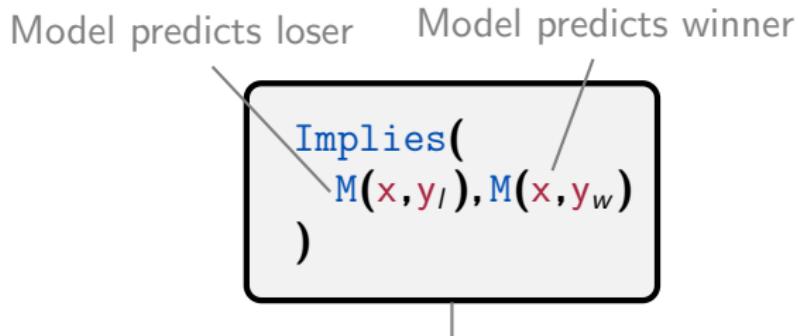
## Declarative models of preference



*Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.*

**Conceptually:** Predictions are connected through Boolean operators, express constraints on predictions;  $\rho_\theta$  as formulas.

## Uncovering the natural logic of these algorithms



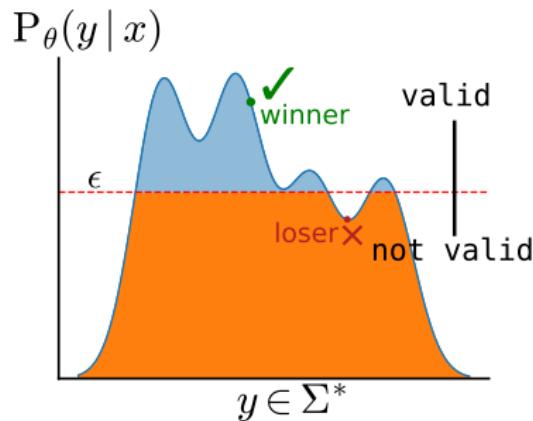
*Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.*

**Assumption:** Every loss function has an internal logic that can be expressed in this way, we want to uncover that logic.

# Uncovering the natural logic of these algorithms

Implies(  
 $M(x, y_l), M(x, y_w)$   
)

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

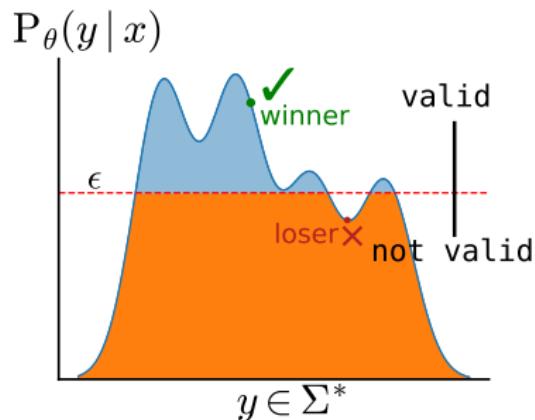


**Assumption:** Every loss function has an internal logic that can be expressed in this way, we want to uncover that logic.

# Uncovering the natural logic of these algorithms

Implies(  
     $M(x, y_l), M(x, y_w)$   
)

And(  
     $M(x, y_w),$   
    Not( $M(x, y_l)$ ))

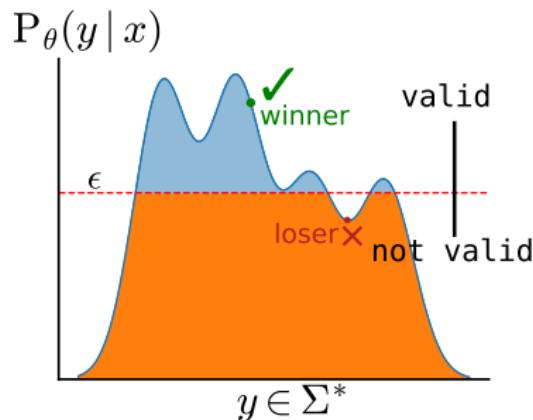


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# Uncovering the natural logic of these algorithms

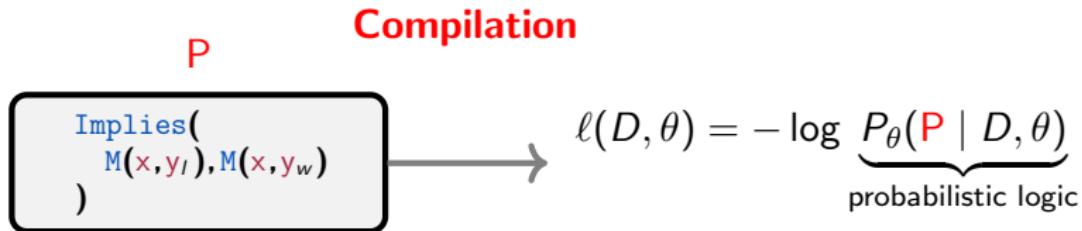
Implies(  
M( $x, y_l$ ), M( $x, y_w$ ))

And(  
M( $x, y_w$ ),  
Not(M( $x, y_l$ ))))



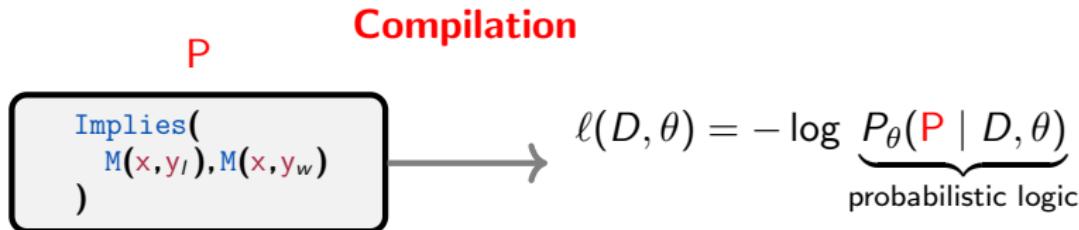
**Observation:** The second program is more strict than the first, involves semantic entailment.

# Compilation and decompilation again



*Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.*

# Compilation and decompilation again



*Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.*

**What we did:** defined a novel probabilistic logic for preference modeling,  
**note:** logic useful not only for learning and loss.

# Compilation and decompilation again

**Decompilation**

P

Implies(

$M(x, y_l), M(x, y_w)$

)

$\ell_{\text{CP0}} = - \log \sigma \left( \log \frac{\pi_\theta(y_w|x)}{\pi_\theta(y_l|x)} \right)$

*Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.*

# Compilation and decompilation again

**Decompilation**

**P**

$\text{Implies}\left(\text{M}(x, y_l), \text{M}(x, y_w)\right)$

$\ell_{\text{CPO}} = -\log \sigma\left(\log \frac{\pi_\theta(y_w|x)}{\pi_\theta(y_l|x)}\right)$

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\underbrace{\ell_{\text{CPO}}(D, \theta) = -\log P_\theta(\mathbf{P} | D, \theta)}_{\text{correctness property}}$$

# Compilation and decompilation again

**P**      **Decompilation**

$\text{Implies}\left(\text{M}(x, y_l), \text{M}(x, y_w)\right)$

$$\ell_{\text{CPO}} = -\log \sigma\left(\log \frac{\pi_\theta(y_w|x)}{\pi_\theta(y_l|x)}\right)$$

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\underbrace{\ell_{\text{CPO}}(D, \theta) = -\log P_\theta(\text{P} | D, \theta)}_{\text{correctness property}}$$

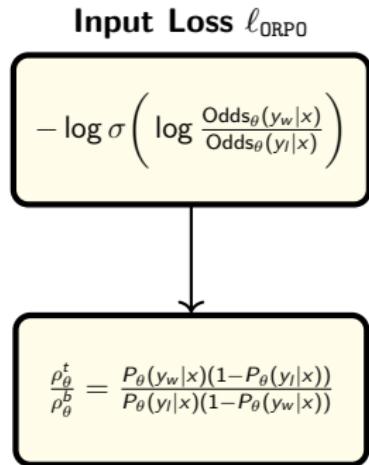
**The second thing we did:** Defined a mechanical procedure for decompilation, proved its correctness, invariance to choice of  $f$ .

## Illustration of approach and results

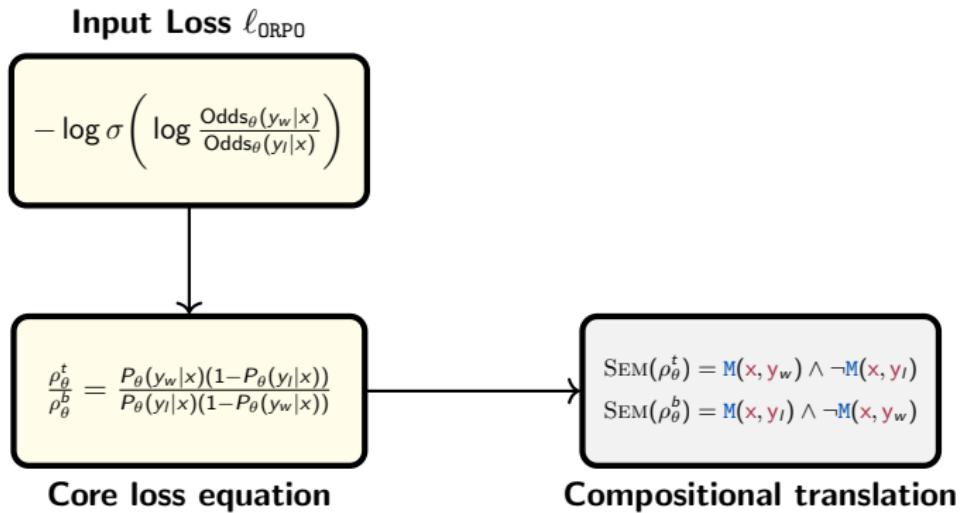
**Input Loss**  $\ell_{\text{ORPO}}$

$$-\log \sigma \left( \log \frac{\text{Odds}_\theta(y_w|x)}{\text{Odds}_\theta(y_l|x)} \right)$$

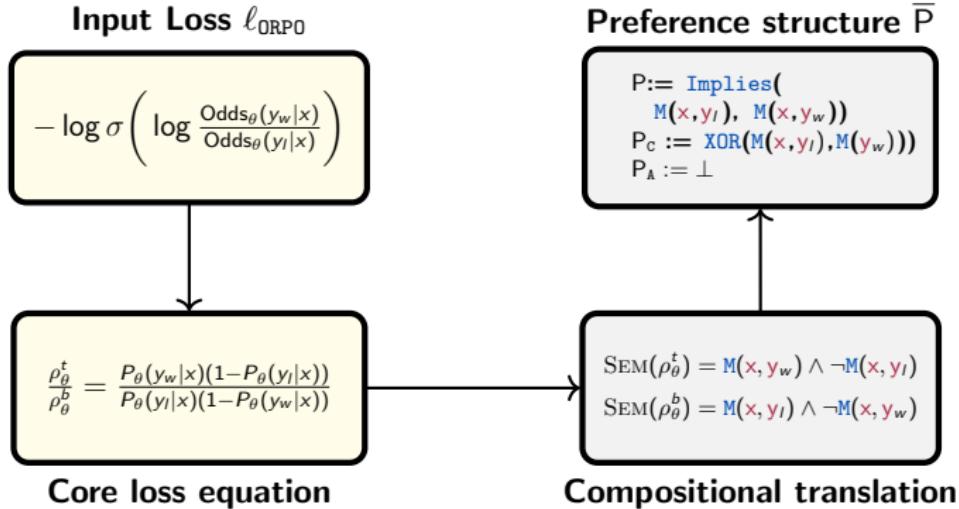
# Illustration of approach and results



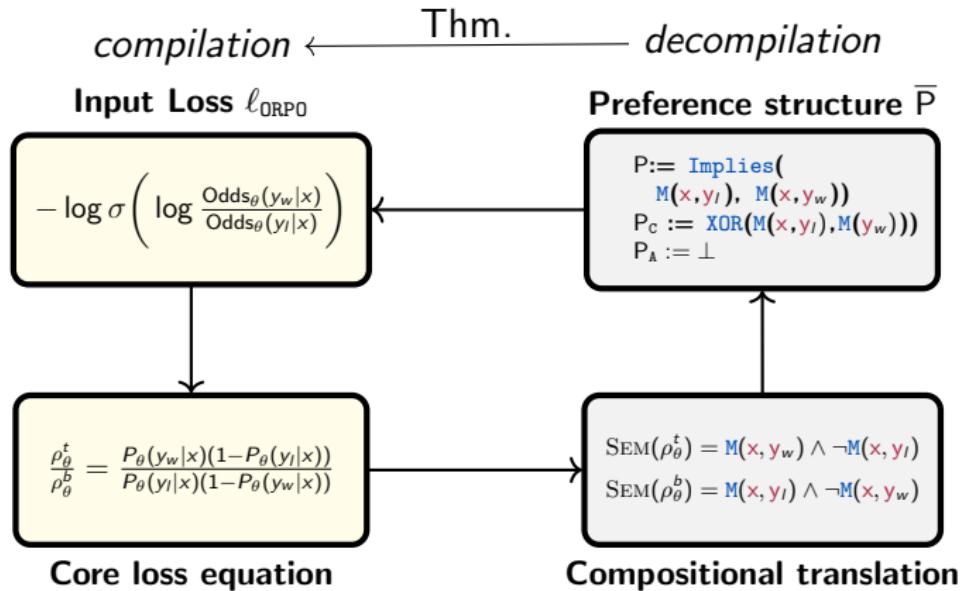
# Illustration of approach and results



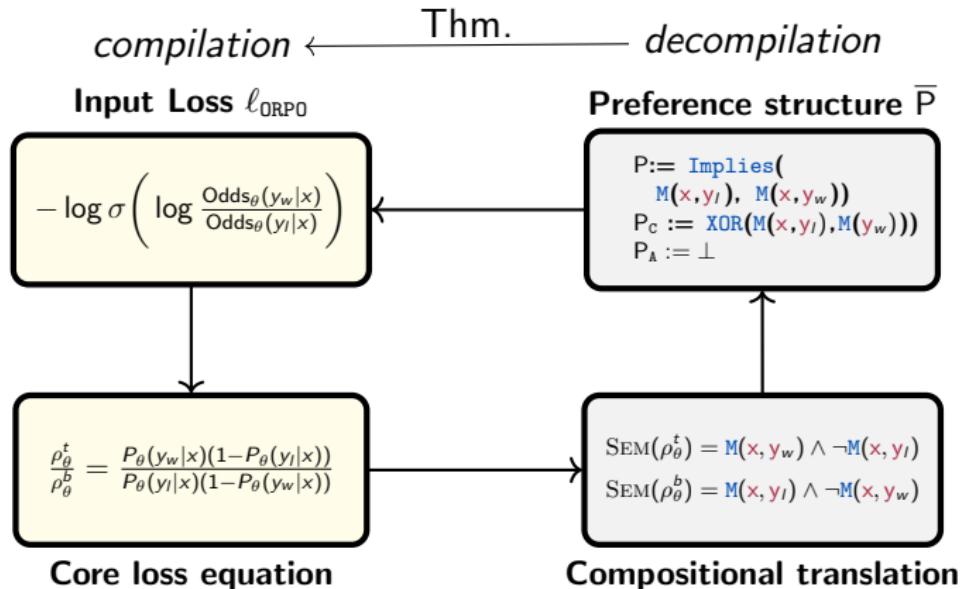
# Illustration of approach and results



# Illustration of approach and results

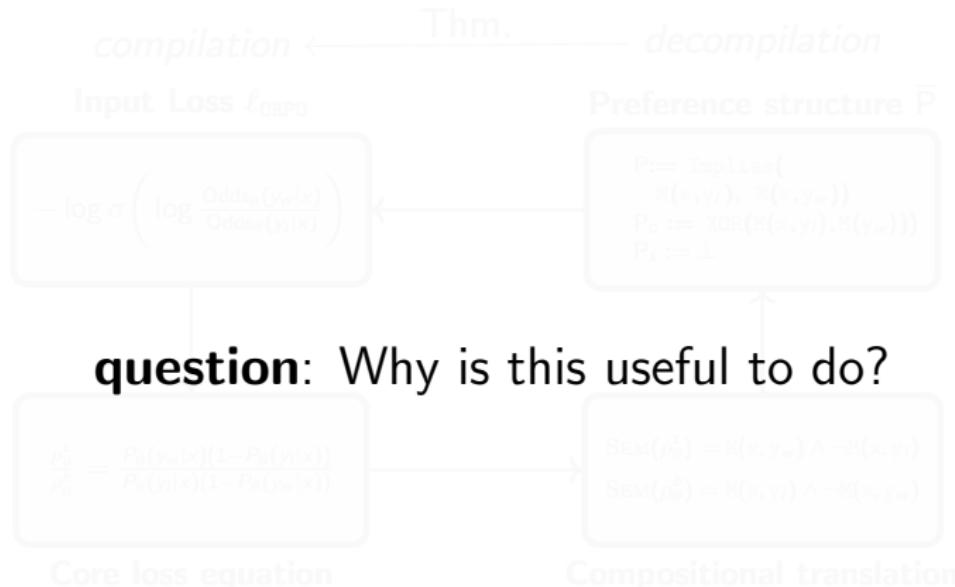


# Illustration of approach and results



- ▶ **Preference structure**, a core construct in our logic, encoding for preference losses, has a natural Boolean interpretation.

# Illustration of approach and results



- ▶ Preference structure, a core construct in our logic, encoding for preference losses, has a natural Boolean interpretation.

# Illustration of approach and results



How many preference loss functions are there?

(or *How many future DPO papers might be written?*)

Core loss equation

Compositional translation

- ▶ Preference structure, a core construct in our logic, encoding for preference losses, has a natural Boolean interpretation.

# Why is this useful? understanding the space

$P^{(1)}$

Implies(  
     $M(x, y_I), M(x, y_w)$   
)

$P^{(2)}$

And(  
     $M(x, y_w),$   
    Not( $M(x, y_I)$ )))

Boolean functions, 2 variables

$M(x, y_w)$	$M(x, y_I)$	$P^{(1)}$	$P^{(2)}$
T	T	✓	X
T	F	✓	✓
F	T	X	X
F	F	✓	X

# Why is this useful? understanding the space

Boolean functions, 2 variables

$M(x, y_w)$	$M(x, y_I)$	$P^{(1)}$	$P^{(2)}$
T	T	✓	X
T	F	✓	✓
F	T	X	X
F	F	✓	X

$P^{(1)}$

$P^{(2)}$

Implies(  
 $M(x, y_I), M(x, y_w)$ )

And(  
 $M(x, y_w),$   
 $\text{Not}(M(x, y_I)))$

- Every program (in our logic) is pair of Boolean functions (in  $n$  variables), corr. to ✓ and X, leads to  $4^{2^n}$  possible loss functions.

# Why is this useful? understanding the space

Boolean functions, 2 variables

$M(x, y_w)$	$M(x, y_I)$	$P^{(1)}$	$P^{(2)}$
T	T	✓	X
T	F	✓	✓
F	T	X	X
F	F	✓	X

$P^{(1)}$

$\text{Implies}(\text{M}(x, y_I), \text{M}(x, y_w))$

$P^{(2)}$

$\text{And}(\text{M}(x, y_w), \text{Not}(\text{M}(x, y_I)))$

Loss creation will end up being equivalent to drawing different sets of ✓ s and X (or blank marks) in a truth table.

# Why is this useful? understanding the space

Boolean functions, 2 variables

$P^{(1)}$	$M(x, y_w)$	$M(x, y_I)$	$P^{(1)}$	$P^{(2)}$
$P^{(2)}$	T	T	✓	X
	T	F	✓	✓
	F	T	X	X
	F	F	✓	X

$\text{Implies}(M(x, y_I), M(x, y_w))$

$\text{And}(M(x, y_w), \text{Not}(M(x, y_I)))$

**no reference: 256 losses**

Loss creation will end up being equivalent to drawing different sets of ✓ s and X (or blank marks) in a truth table.

# Loss functions as truth tables

Implies(  
And( $M(x, y_I)$ ,  $Ref(x, y_w)$ ),  
And( $M(x, y_w)$ ,  $Ref(x, y_I)$ ))  
)

4 variables

$Ref(x, y_w)$	$M(x, y_I)$	$Ref(x, y_I)$	$M(x, y_w)$
F	F	F	F
F	F	F	T
F	F	T	F
F	F	T	T
F	T	F	F
F	T	F	T
F	T	T	F
F	T	T	T
T	F	F	F
T	F	F	T
T	F	T	F
T	F	T	T
T	T	F	F
T	T	F	T
T	T	T	F
T	T	T	T

w/ reference: 4,294,967,296 losses

# Loss functions as truth tables

Implies(  
And( $M(x, y_I)$ , Ref( $x, y_w$ )),  
And( $M(x, y_w)$ , Ref( $x, y_I$ )))

)

4 variables

**answer:** loads.

Ref( $x, y_w$ )	$M(x, y_I)$	Ref( $x, y_I$ )	$M(x, y_w)$
F	F	F	F
F	F	F	T
F	F	T	F
F	F	T	T
F	T	F	F
F	T	F	T
F	T	T	F
F	T	T	T
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	F
T	T	F	T
T	T	T	F
T	T	T	T

w/ reference: 4,294,967,296 losses

# Loss functions as truth tables

Implies(  
And(If( $x, y_I$ ), Ref( $x, y_R$ )),  
And( $x, y_R$ ))

True

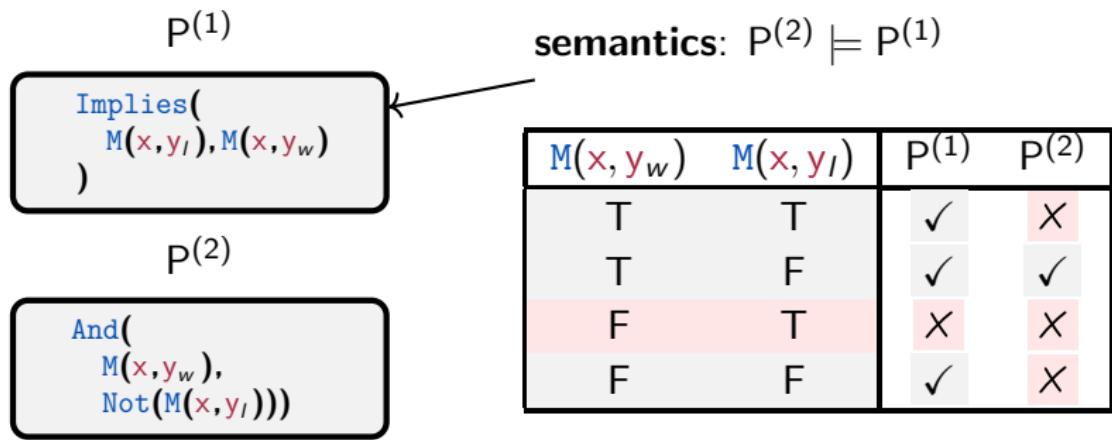
**question:** How are losses related to one another?

4 variables

Ref( $x, y_R$ )	If( $x, y_I$ )	Ref( $x, y_I$ )	If( $x, y_R$ )
F	F	F	F
F	F	F	T
F	F	T	F
F	F	T	T
F	T	F	F
F	T	F	T
F	T	T	F
F	T	T	T
T	F	T	F
T	F	T	T
T	T	F	F
T	T	F	T
T	T	T	F
T	T	T	T

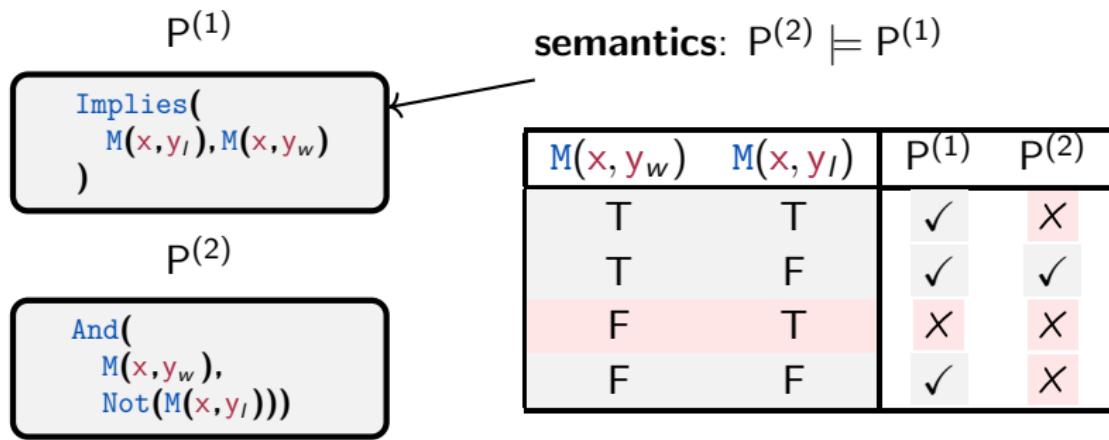
w/ reference: 4,294,967,296 losses

## Why is this useful? understanding the structure



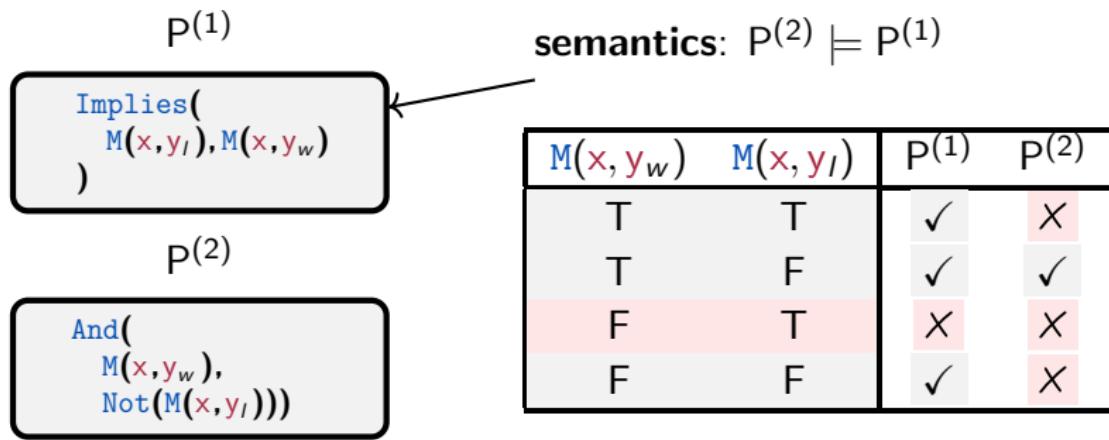
**Proposition** (Xu et al., 2018): Loss behavior is monotonic w.r.t semantic entailment: if  $P^{(2)} \models P^{(1)}$  then  $\ell(D, \theta, P^{(2)}) \geq \ell(D, \theta, P^{(1)})$ .

## Why is this useful? understanding the structure



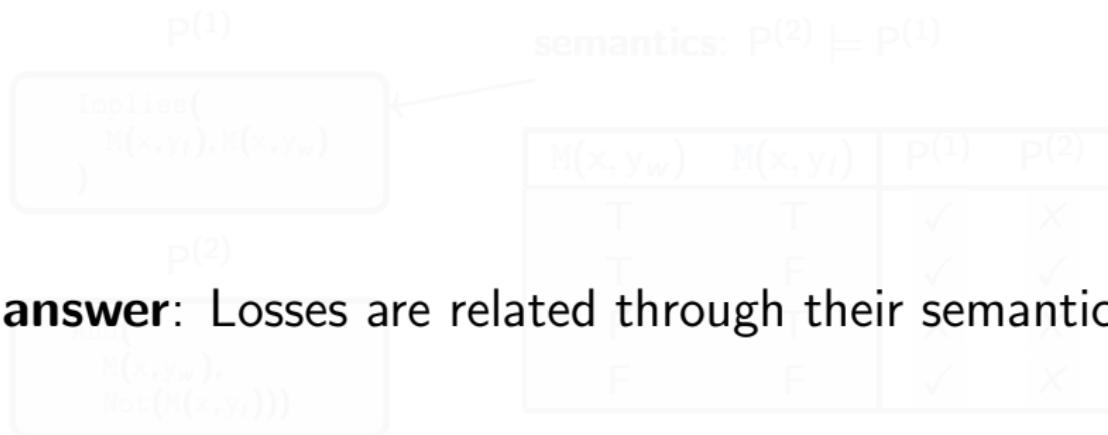
**Proposition** (Xu et al., 2018): Loss is equivalent under semantic equivalence: If  $P^{(2)} \equiv P^{(1)}$  then  $\ell(D, \theta, P^{(2)}) = \ell(D, \theta, P^{(1)})$ .

# Why is this useful? understanding the structure



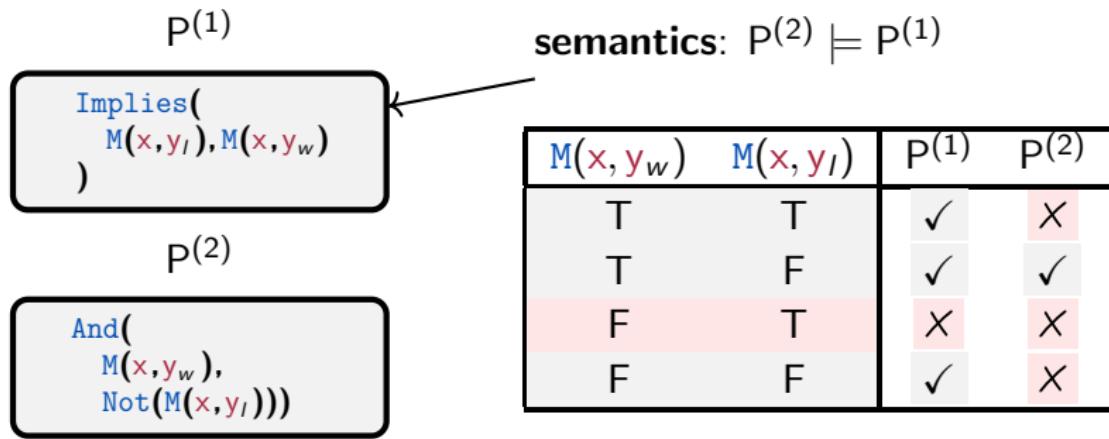
**Theorem:**  $\ell(D, \theta, P^{(2)}) > \ell(D, \theta, P^{(1)})$  (the loss of  $P^{(1)}$  is contained in the loss of  $P^{(2)}$ ).

## Why is this useful? understanding the structure



Theorem:  $\ell(D, \theta, P^{(2)}) > \ell(D, \theta, P^{(1)})$  (the loss of  $P^{(1)}$  is contained in the loss of  $P^{(2)}$ ).

# Why is this useful? understanding the structure



**Practical strategy:** Start with empirically successful losses, modify semantics (make more or less constrained), then experiment accordingly.

# Deriving new losses symbolically, from first principles

## Symbolic Program

```
Implies(  
    And(M(x,yI), Ref(x,yw)),  
    And(M(x,yw), Ref(x,yI))  
)
```

## DPO Loss

$$-\log \sigma \left( \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \log \frac{\pi_\theta(y_I|x)}{\pi_{\text{ref}}(y_I|x)} \right)$$

# Deriving new losses symbolically, from first principles

## Symbolic Program

```
Implies(  
    And(M(x,yI), Ref(x,yw)),  
    And(M(x,yw), Ref(x,yI))  
)
```

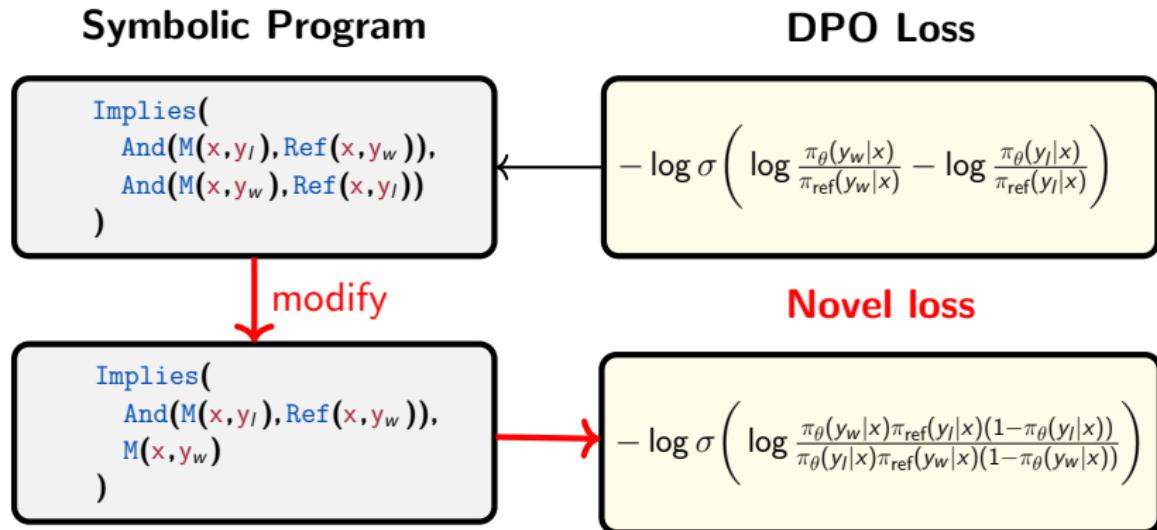
## DPO Loss

$$-\log \sigma \left( \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \log \frac{\pi_\theta(y_I|x)}{\pi_{\text{ref}}(y_I|x)} \right)$$

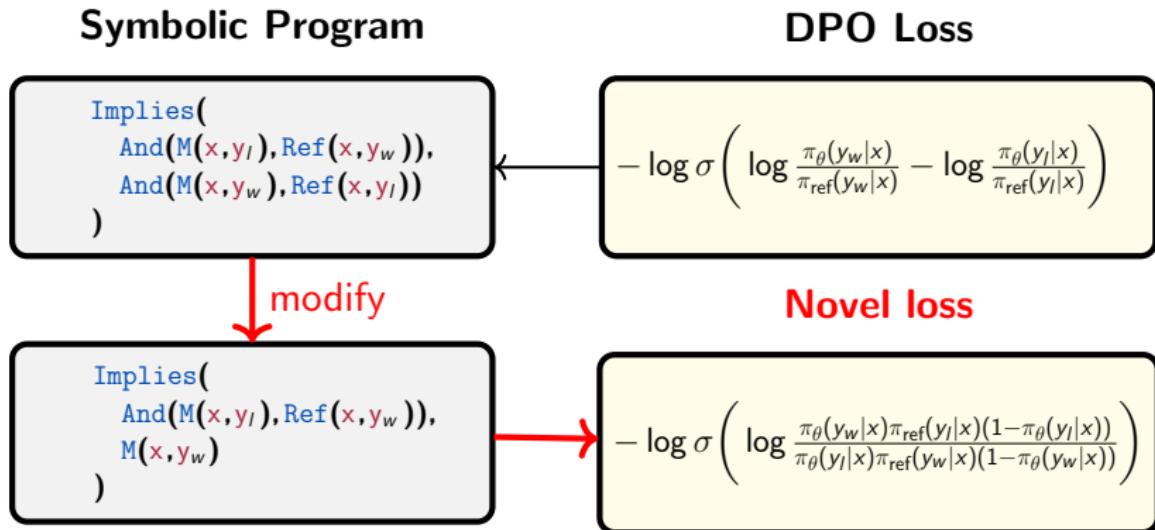
modify  


```
Implies(  
    And(M(x,yI), Ref(x,yw)),  
    M(x,yw)  
)
```

# Deriving new losses symbolically, from first principles



# Deriving new losses symbolically, from first principles



- ▶ High-level programming language for defining new losses.

# Deriving new losses symbolically, from first principles

## Symbolic Program

```
Implies(  
    And(M(x,yI), Ref(x,yw)),  
    And(M(x,yw), Ref(x,yI))  
)
```

## DPO Loss

$$-\log \sigma \left( \log \frac{\pi_\theta(y_w|x)}{\pi_{\text{ref}}(y_w|x)} - \log \frac{\pi_\theta(y_I|x)}{\pi_{\text{ref}}(y_I|x)} \right)$$

**questions:** How does our logic work? What do we see?

$\downarrow$

```
Implies(  
    And(M(x,yI), Ref(x,yw)),  
    M(x,yw)  
)
```

$$-\log \sigma \left( \log \frac{\pi_\theta(y_w|x)\pi_{\text{ref}}(y_I|x)(1-\pi_\theta(y_I|x))}{\pi_\theta(y_I|x)\pi_{\text{ref}}(y_w|x)(1-\pi_\theta(y_w|x))} \right)$$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_l)$	CPO	ORPO	unCPO
T	T	✓ X		✓
T	F	✓	✓	✓
F	T	X	X	X
F	F			✓

P

Implies(  
 $M(x, y_l)$ ,  $M(x, y_w)$ )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO
T	T	✓ X		✓
T	F	✓	✓	✓
F	T	X	X	X
F	F			✓

P

Implies(  
 $M(x, y_I), M(x, y_w)$   
 )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

## How does the logic work? compilation

		CPO	ORPO	unCPO	P
$M(x, y_w)$	$M(x, y_I)$				Implies( $M(x, y_I), M(x, y_w)$ )
T	T	✓ X		✓	
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{\checkmark, X\}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

- ▶ Formula probability computed as a weighted count  $\sum \checkmark_w$  (Chavira and Darwiche, 2008), loss is  $-\log$ , *semantic loss* (Xu et al., 2018).

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO
T	T	✓ X		✓
T	F	✓	✓	✓
F	T	X	X	X
F	F			✓

P  
 Implies(  
 $M(x, y_I), M(x, y_w)$   
 )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

$$\underbrace{\ell_x}_{\text{column}} := \underbrace{-\log \sigma \left( \log \frac{\sum \text{✓}}{\sum \text{X}} \right)}_{\text{arbitrary } X_w}$$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_l)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	Implies( $M(x, y_l), M(x, y_w)$ )
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

$$\begin{aligned} \underbrace{\ell_x}_{\text{column}} &:= -\log \sigma \left( \log \frac{\sum_w \text{✓}_w}{\sum_w \text{X}_w} \right) \\ &= -\log \sigma \left( \log \frac{\pi_\theta(y_w | x)(1 - \pi_\theta(y_l | x))}{\pi_\theta(y_l | x)(1 - \pi_\theta(y_w | x))} \right) \\ &\quad \underbrace{\ell_{\text{ORPO}}, P_\theta(\mathbf{P}|\text{one hot})}_{\ell_{\text{ORPO}}, P_\theta(\mathbf{P}|\text{one hot})} \end{aligned}$$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Implies(  
 $M(x, y_I), M(x, y_w)$   
 )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

$$\underbrace{\ell_x}_{\text{column}} := -\log \sigma \left( \log \frac{\sum_w \text{✓}_w}{\sum_w \text{X}_w} \right)$$

$$= -\log \sigma \left( \log \frac{\pi_\theta(y_w | x)}{\pi_\theta(y_I | x)} \right)$$

$\underbrace{\ell_{\text{CPO}}, \sim P_\theta(\text{P} | \text{one true})}_{\ell_{\text{CPO}}, \sim P_\theta(\text{P} | \text{one true})}$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	Implies( $M(x, y_I), M(x, y_w)$ )
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

{ ✓ **observation:** losses differ in hard constraints

$$w \models l(x, y) \quad w \not\models \neg l(x, y)$$

$$\underbrace{\ell_x}_{\text{column}} := -\log \sigma \left( \log \frac{\sum_{w} \checkmark_w}{\sum_{w} X_w} \right)$$

$$= -\log \sigma \left( \log \underbrace{\frac{\pi_\theta(y_w | x)}{\pi_\theta(y_I | x)}}_{\ell_{\text{CPO}}, \sim P_\theta(P | \text{one true})} \right)$$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_l)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Implies(  
 $M(x, y_l), M(x, y_w)$   
 )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

Loss	Representation $\bar{P}$
CE	$P := M(x, y_w), P_C := \perp$
CEUnl	$P := \text{And}(M(x, y_w), \text{Not}(M(x, y_l)))$ $P_C := \perp$
CPO	;; core semantic formula $P := \text{Implies}(M(x, y_l), M(x, y_w))$ ;; one-true constraint $P_C := \text{Or}(M(x, y_l), M(x, y_w))$
ORPO	$P := \text{Implies}(M(x, y_l), M(x, y_w))$ ;; one-hot constraint $P_C := \text{XOR}(M(x, y_l), M(x, y_w))$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Implies(  
 $M(x, y_I), M(x, y_w)$   
)

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

- ▶ **Preference structure:** equivalent way of expressing truth table representations (Richardson et al., 2025),

$$\bar{P} := \left( \underbrace{P}_{\text{core}}, \underbrace{P_C, P_A}_{\text{constraints}} \right)$$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	Implies( $M(x, y_I), M(x, y_w)$ )
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{ \text{✓}, \text{X} \}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

$$\underbrace{\ell_x}_{\text{column}} := -\log \sigma \left( \log \frac{\sum \text{✓}_w}{\sum \text{X}_w} \right)$$

$$= -\log \sigma \left( \log \underbrace{\frac{\pi_\theta(y_I | x) \pi_\theta(y_w | x) + (1 - \pi_\theta(y_I | x))}{\pi_\theta(y_I | x) (1 - \pi_\theta(y_w | x))}}_{\text{novel loss without constraints, } P_\theta(P | \top)} \right)$$

# How does the logic work? compilation

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X			
T	F	✓	✓		
F	T	X	X		
F	F			✓ ✓ X ✓	Implies( $M(x, y_I), M(x, y_w)$ )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

{ note:  $M(x, y_I) \rightarrow M(x, y_w) \equiv \neg M(x, y_I) \vee M(x, y_w)$

$$\begin{aligned}
 \underbrace{\ell_x}_{\text{column}} &:= -\log \sigma \left( \log \frac{\sum \checkmark_w}{\sum X_w} \right) \\
 &= -\log \sigma \left( \log \underbrace{\frac{\pi_\theta(y_I | x) \pi_\theta(y_w | x) + (1 - \pi_\theta(y_I | x))}{\pi_\theta(y_I | x)(1 - \pi_\theta(y_w | x))}}_{\text{novel loss without constraints, } P_\theta(P|T)} \right)
 \end{aligned}$$

# What properties do real losses have?

$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X		✓	Implies( $M(x, y_I), M(x, y_w)$ )
T	F	✓	✓	✓	
F	T	X	X	X	
F	F			✓	

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

$$\{\checkmark, \text{X}\}_w := \prod_{w \models M(x,y)} \pi_\theta(y | x) \cdot \prod_{w \models \neg M(x,y)} 1 - \pi_\theta(y | x)$$

$$\begin{aligned}
 \underbrace{\ell_x}_{\text{column}} &:= -\log \sigma \left( \log \frac{\sum \checkmark_w}{\sum \text{X}_w} \right) \\
 &= -\log \sigma \left( \log \underbrace{\frac{\pi_\theta(y_I | x) \pi_\theta(y_w | x) + (1 - \pi_\theta(y_I | x))}{\pi_\theta(y_I | x) (1 - \pi_\theta(y_w | x))}}_{\text{novel loss without constraints, } P_\theta(P | \top)} \right)
 \end{aligned}$$

# What properties do real losses have?

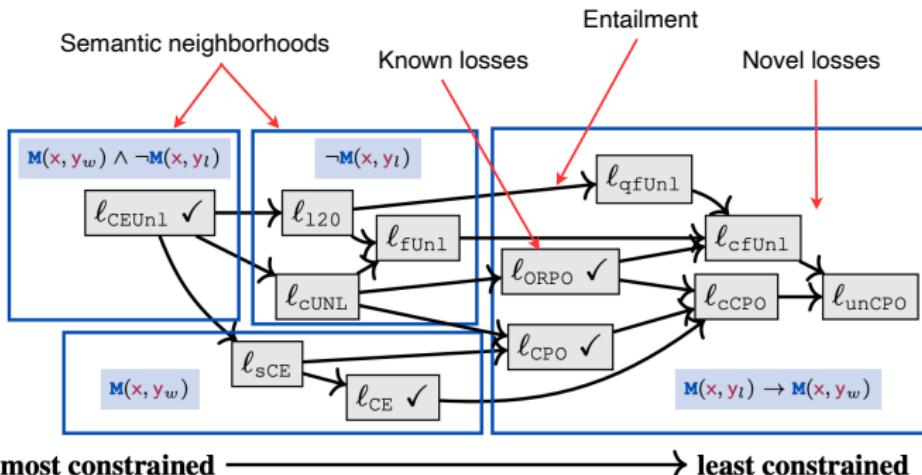
$M(x, y_w)$	$M(x, y_I)$	CPO	ORPO	unCPO	P
T	T	✓ X			
T	F	✓	✓		
F	T	X	X		
F	F			✓ ✓ X ✓	Implies( $M(x, y_I), M(x, y_w)$ )

Whenever the model deems the loser to be a valid generation, it should deem the winner to be valid too.

{ ✓, Mapping out these loss spaces semantically

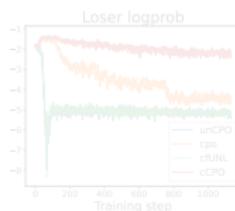
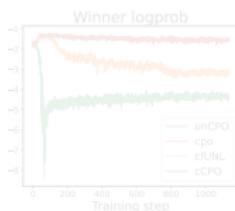
$$\begin{aligned}
 \underbrace{\ell_x}_{\text{column}} &:= -\log \sigma \left( \log \frac{\sum \checkmark_w}{\sum X_w} \right) \\
 &= -\log \sigma \left( \log \underbrace{\frac{\pi_\theta(y_I | x)\pi_\theta(y_w | x) + (1 - \pi_\theta(y_I | x))}{\pi_\theta(y_I | x)(1 - \pi_\theta(y_w | x))}}_{\text{novel loss without constraints, } P_\theta(P|T)} \right)
 \end{aligned}$$

# The no reference loss landscape

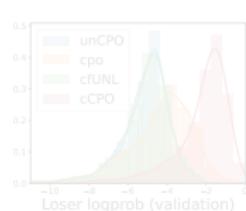
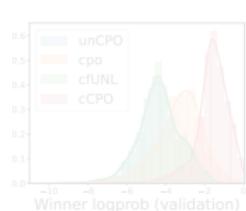


- **Loss lattice:** semantic structure of space, ordering.

# The no reference loss landscape

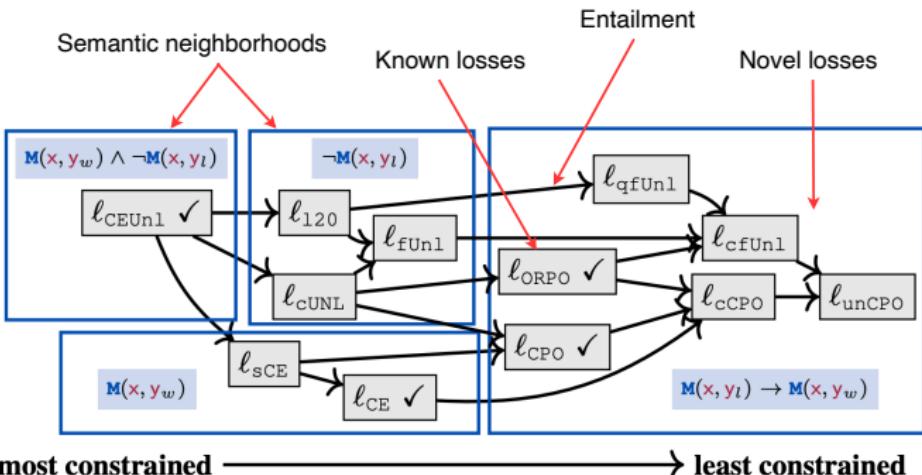


Training dynamics

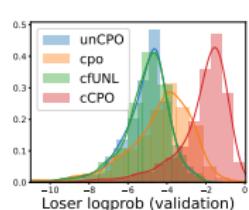
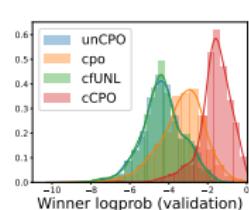
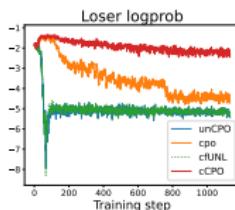
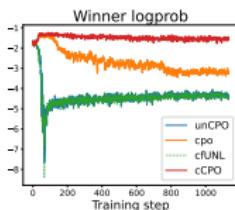


Inference

# The no reference loss landscape



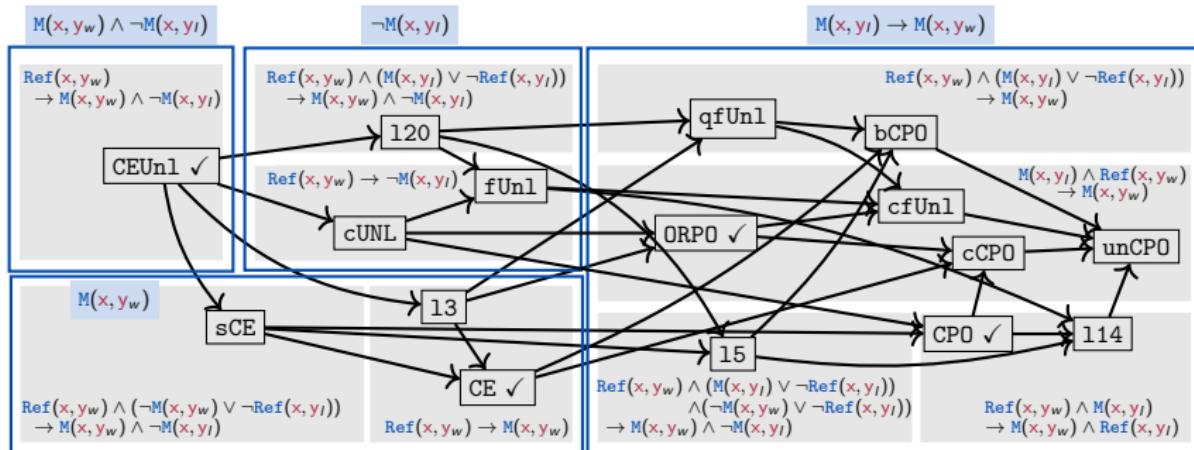
most constrained → least constrained



Training dynamics

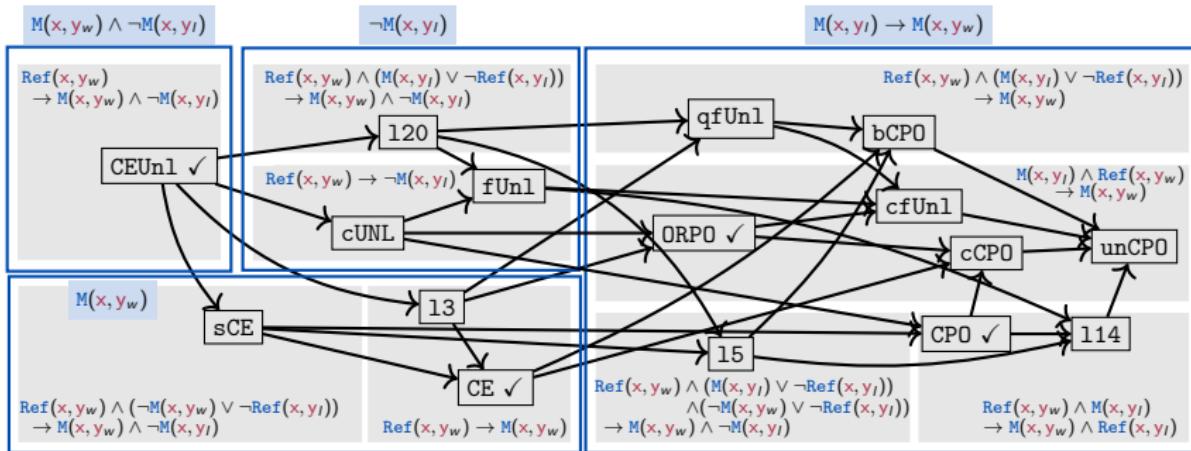
Inference

# The full landscape, reference approaches



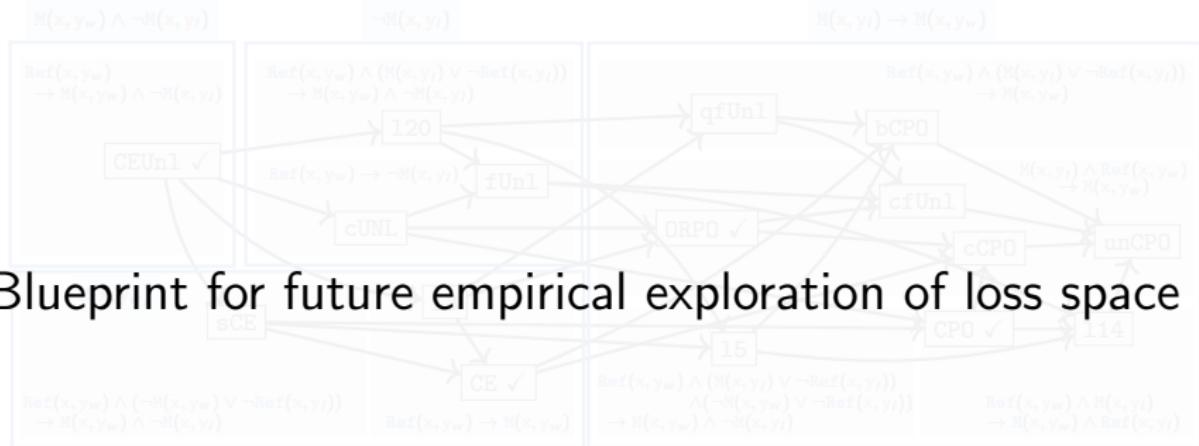
- The semantics of DPO-style reference losses can be straightforwardly computed from no reference approaches.

## The full landscape, reference approaches



- ▶ Many new losses to explore and experiment with!

# The full landscape, reference approaches



Blueprint for future empirical exploration of loss space

# Conclusions

- ▶ New ideas about using symbolic techniques to formally characterize the semantics of LLM algorithms, preference learning.

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**The procedure:** write a (high-level) symbolic program, or modify an existing one, compile into a loss and experiment (then repeat)

# Conclusions

- ▶ New ideas about using symbolic techniques to formally characterize the semantics of LLM algorithms, preference learning.
  1. Understanding the full space of loss functions (finding: it's a huge space, many novel variations yet to be explored)
  2. Understanding the structure of the space and relationships between different losses (finding: tied to the semantics of the losses).
- ▶ **The procedure:** write a (high-level) symbolic program, or modify an existing one, compile into a loss and experiment (then repeat)
- ▶ **many other areas to look at:** *analysis of transformers, semantics of data, reinforcement learning, chain-of-thought, LLM agents ...*

Thank you.

## Adding a reference model

```
P:= Implies(  
    And(M(x,yl),Ref(x,yw)),  
    And(M(x,yw),Ref(x,yl)))  
)
```

Whenever the model being tuned deems the loser to be a valid generation and the reference model deems the winner to be valid, the tuned model should deem the winner to be valid too, and **the reference should deem the loser to be valid.**

## Adding a reference model

```
P := Implies(  
    And(M(x, yI), Ref(x, yw)),  
    And(M(x, yw), Ref(x, yI)))  
)
```

Whenever the model being tuned deems the loser to be a valid generation and the reference model deems the winner to be valid, the tuned model should deem the winner to be valid too, and **the reference should deem the loser to be valid.**

- ▶ **Peculiar semantics**, but the logic makes sense, e.g., we want to maximize

$$\sigma \left( \log \frac{\pi_{\theta}(y_w | x)}{\pi_{\theta}(y_I | x)} - \log \frac{\pi_{\text{ref}}(y_w | x)}{\pi_{\text{ref}}(y_I | x)} \right)$$

negating left side of implication (i.e., making  $M(x, y_I)$  and  $Ref(x, y_w)$  false) and making the right side true is logical.

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