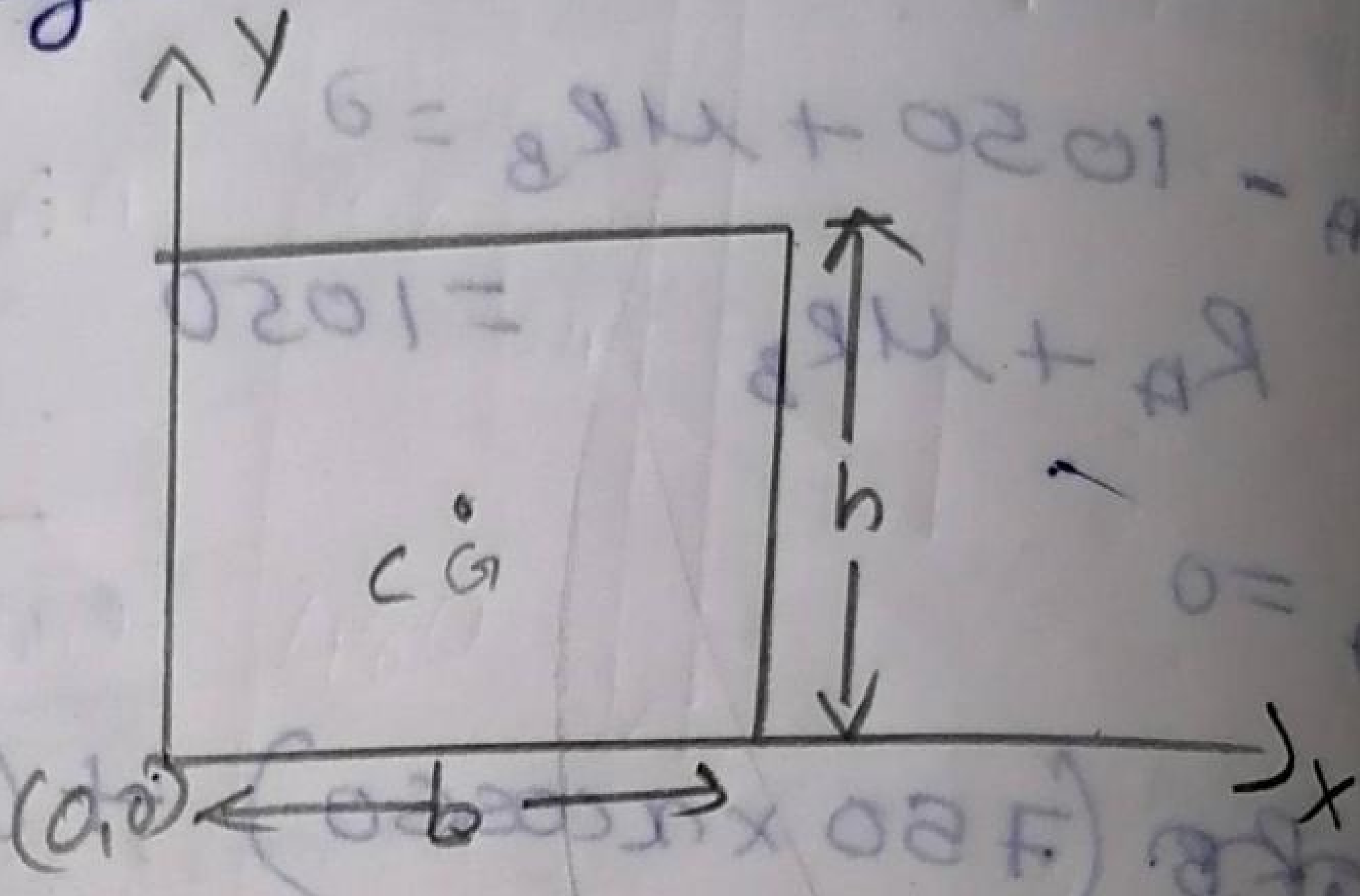
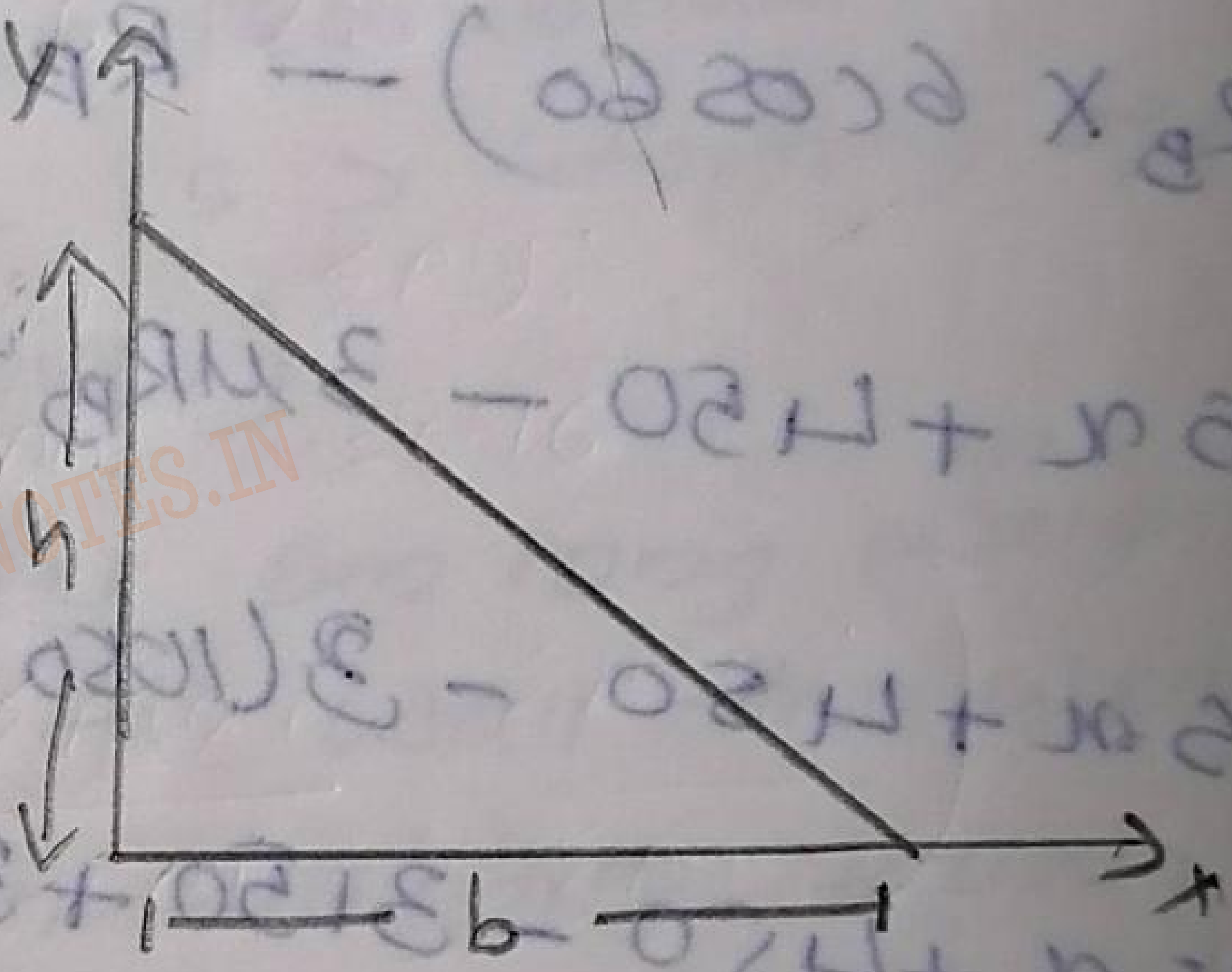
MODULE - 3

Centroid of rectangle

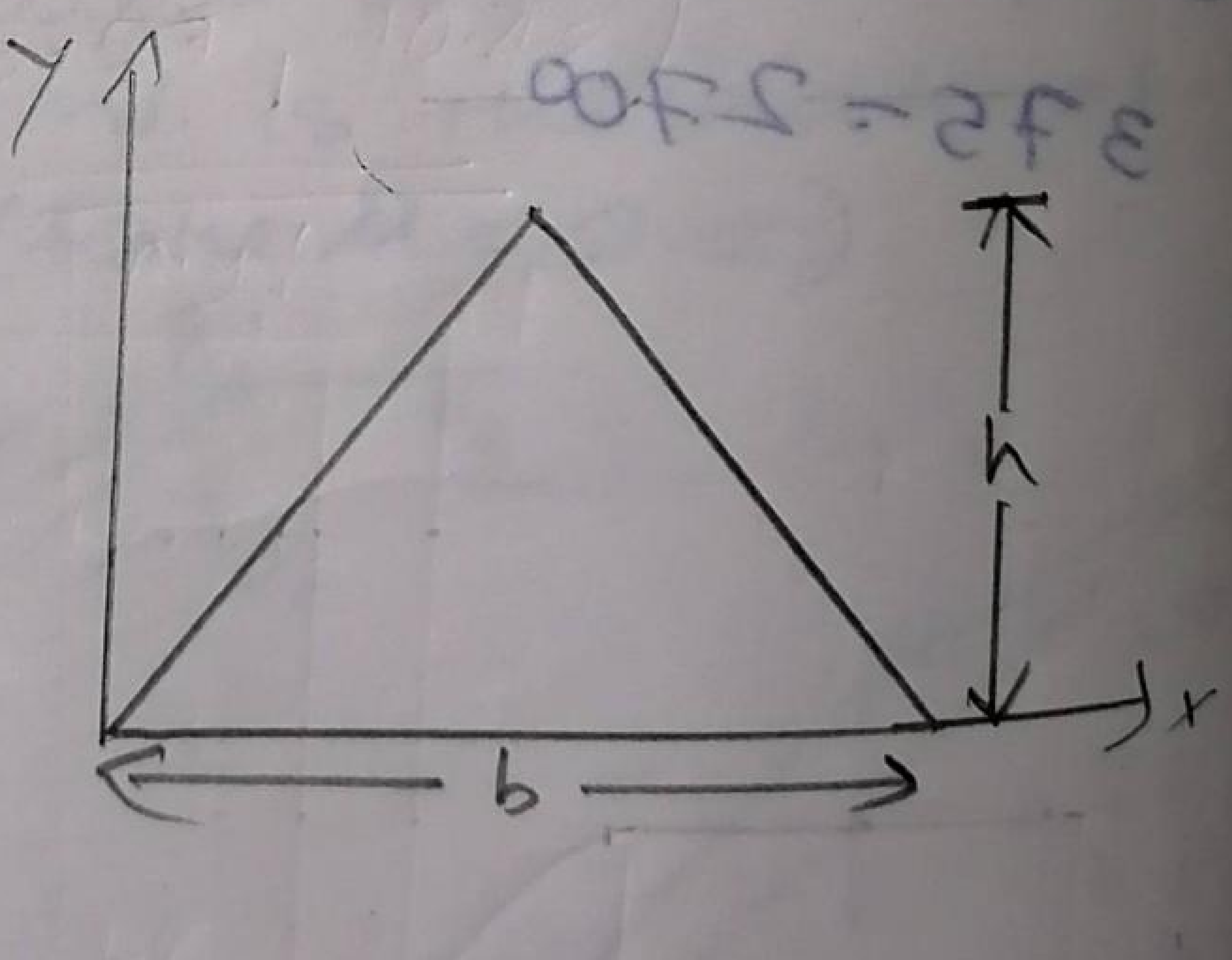
$(x, y) = (b/2, h/2)$



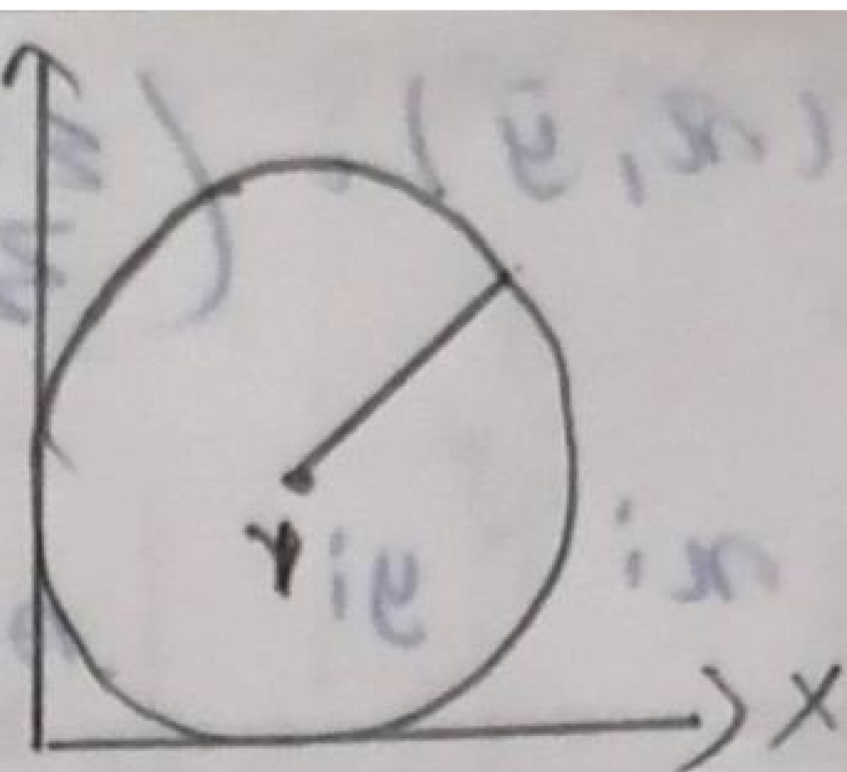
$(x, y) = (b/3, h/3)$



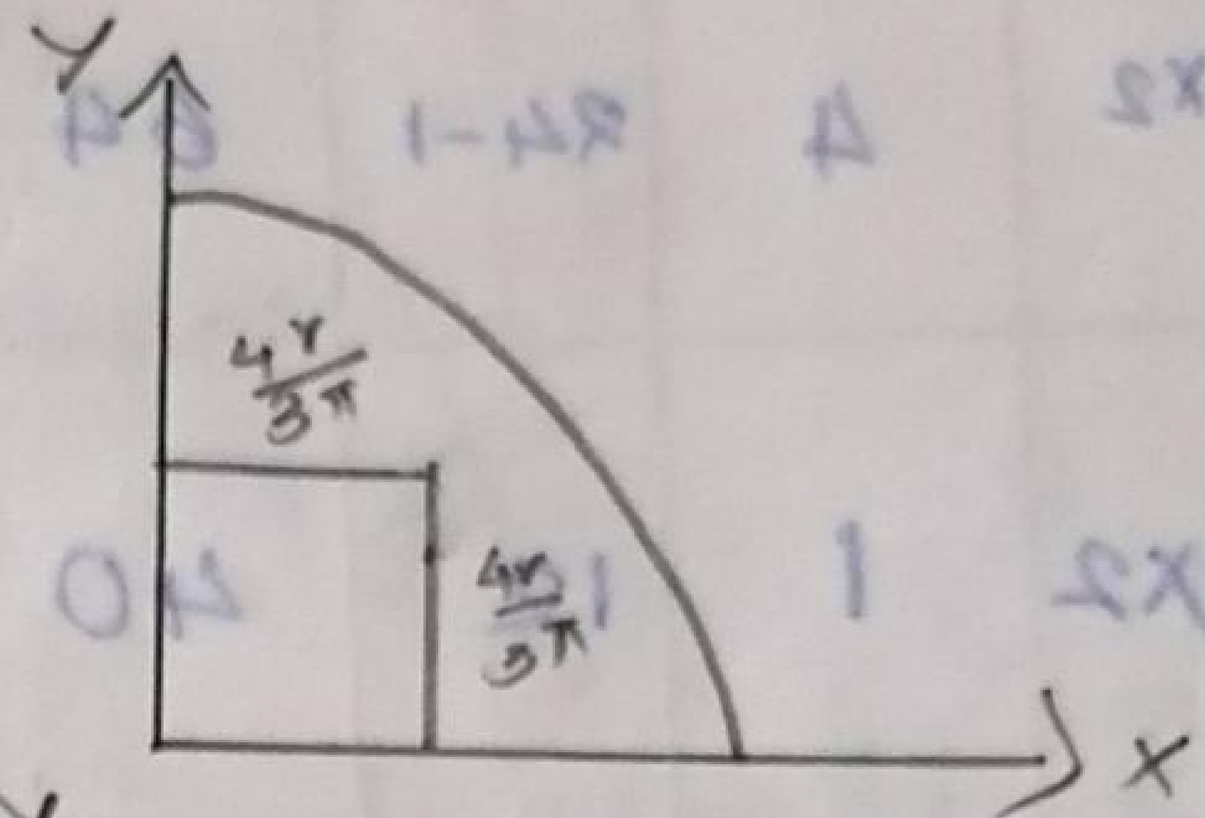
$(x, y) = \left(\frac{b}{2}, \frac{h}{3}\right)$



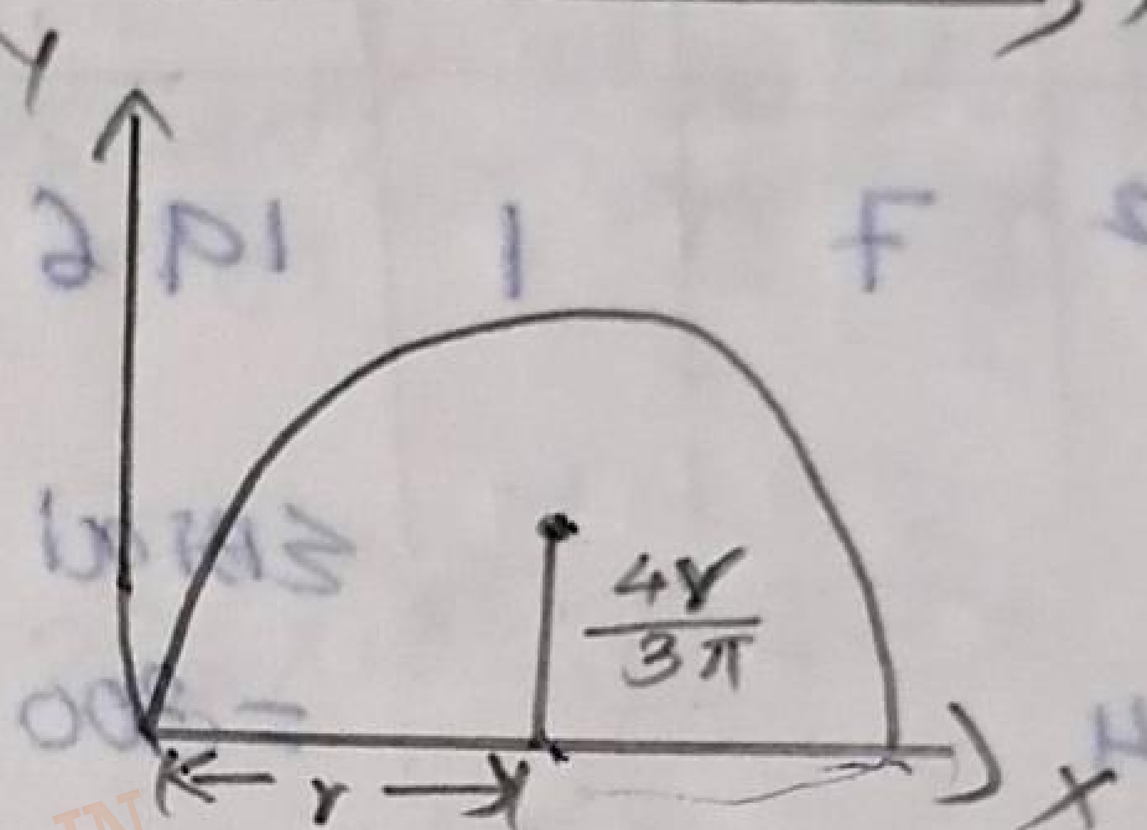
$(x,y) = (r,r)$



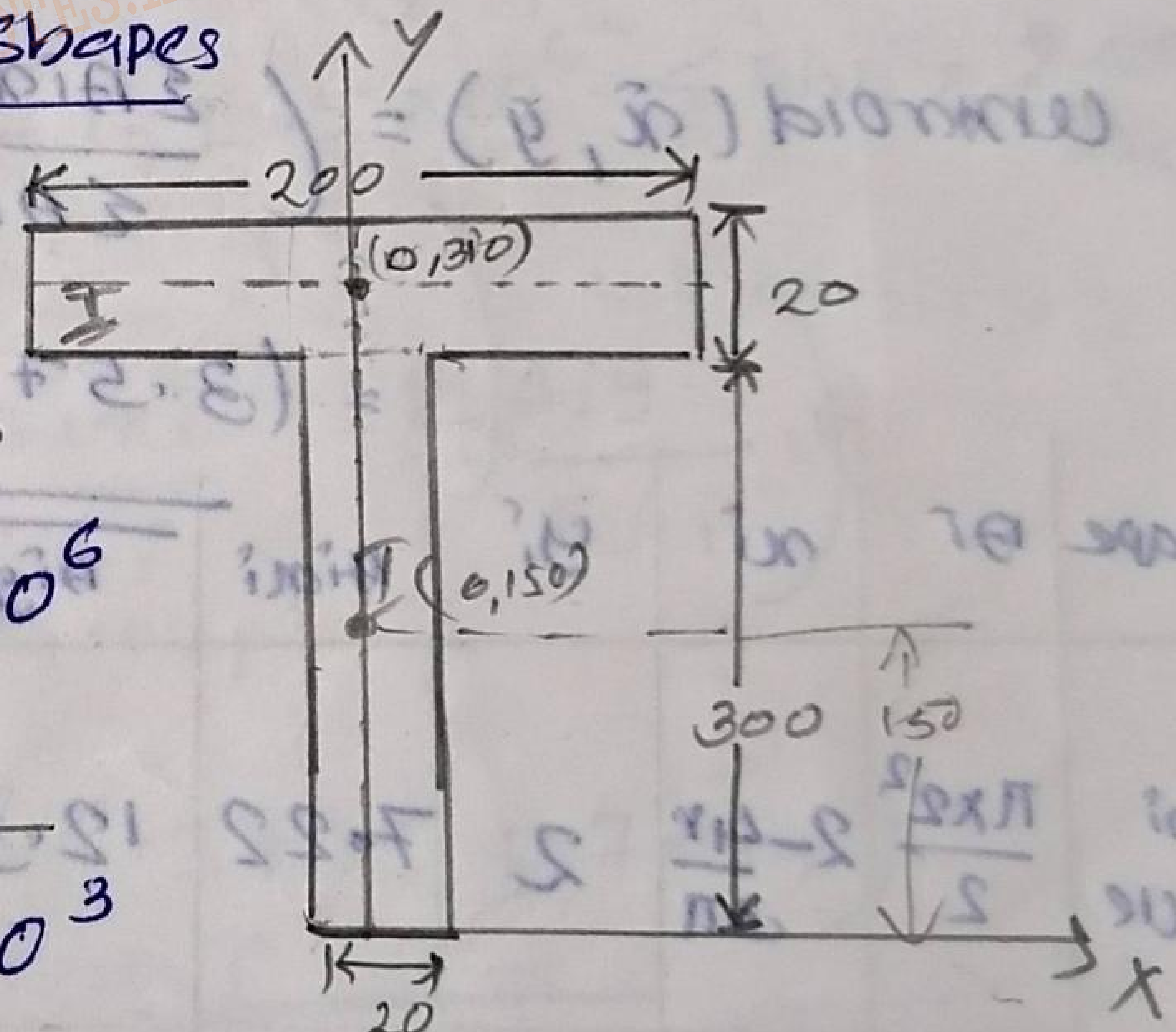
$(x,y) = \left(\frac{4r}{3\pi}, \frac{4r}{3\pi}\right)$



$(x,y) = \left(r, \frac{4r}{3\pi}\right)$

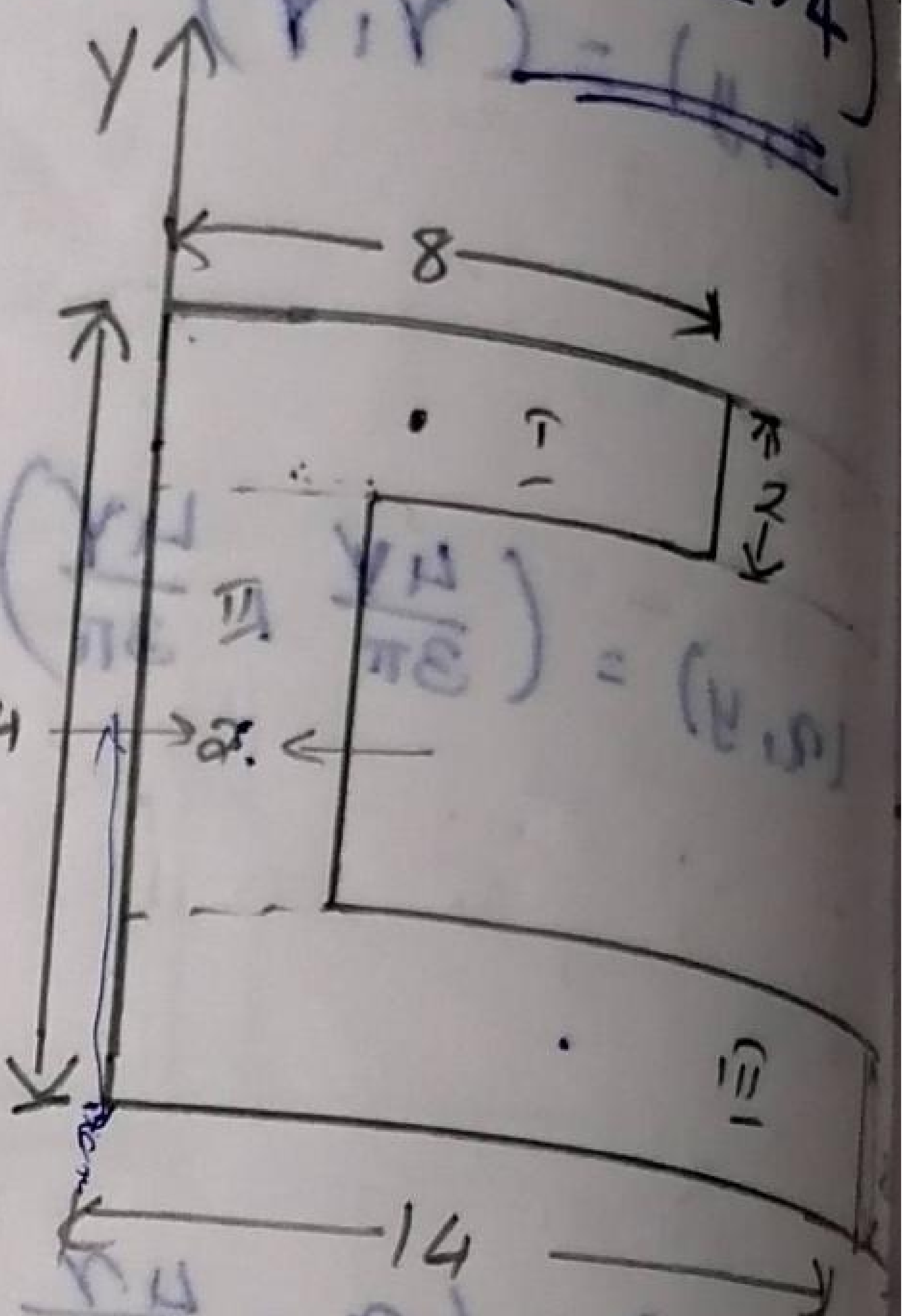


Centroid of composite shapes



Shape	A_i	x_i	y_i	A_ix_i	A_iy_i
I	200×20	0	310	0	1.24×10^6
II	300×20	0	150	0	900×10^3
	$\Sigma A_i = 10\times10^3$			$\Sigma A_ix_i = 0$	$\Sigma A_iy_i = 2.14\times10^6$

Centroid $(\bar{x}, \bar{y}) = \left(\frac{\sum A_i x_i}{\sum A_i}, \frac{\sum A_i y_i}{\sum A_i}\right) = (0, 2)$



Shape	A_i	x_i	y_i	A_ix_i	A_iy_i
I	8x2	4	24-1	64	368
II	20x2	1	12	40	480
III	14x2	7	1	196	28

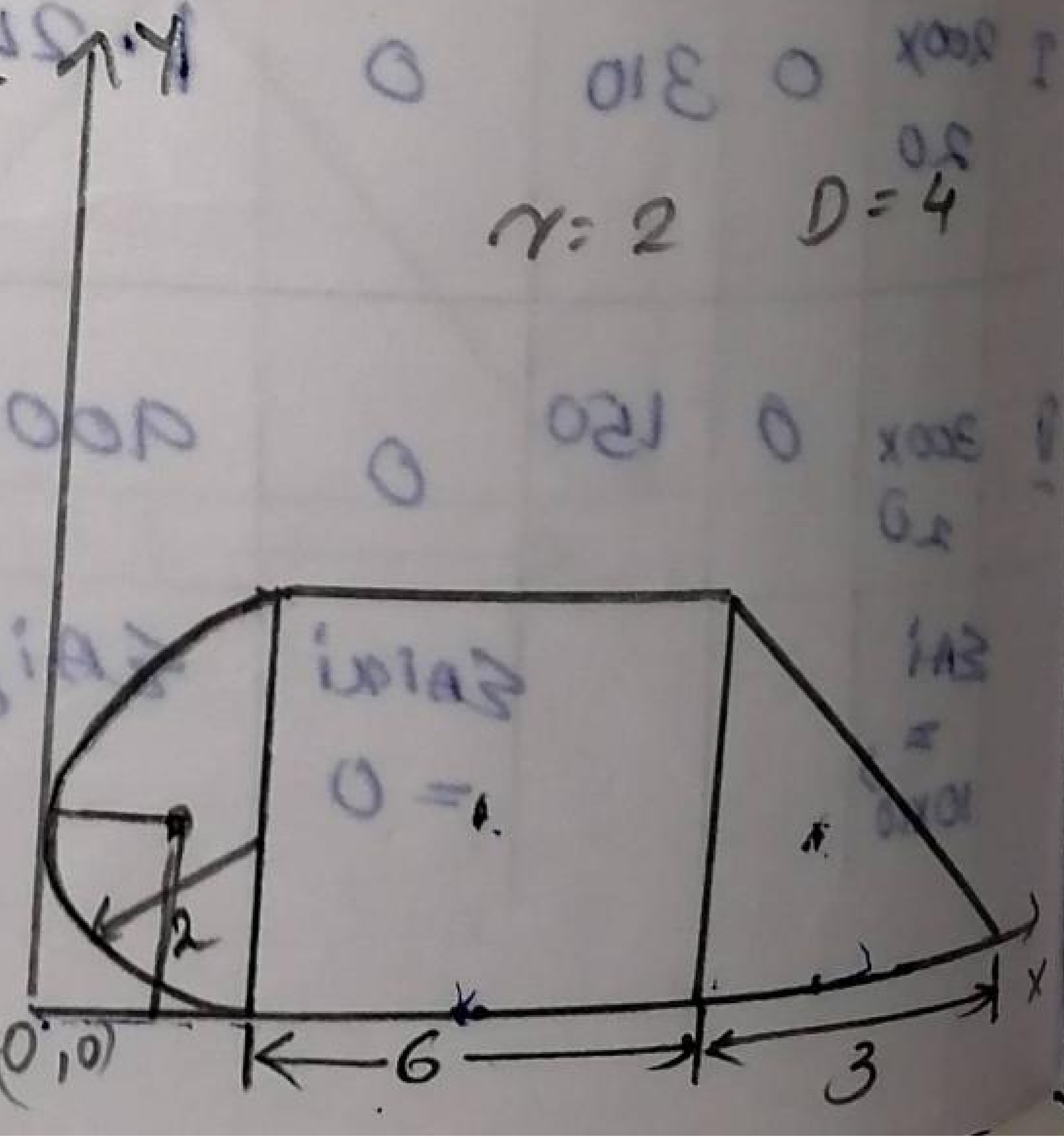
$\sum A_i = 84$ $\quad \sum A_i x_i = 300$ $\quad \sum A_i y_i = 876$

Centroid $(\bar{x}, \bar{y}) = \left(\frac{\sum A_i x_i}{\sum A_i}, \frac{\sum A_i y_i}{\sum A_i}\right)$

$= (3.57, 10.42)$

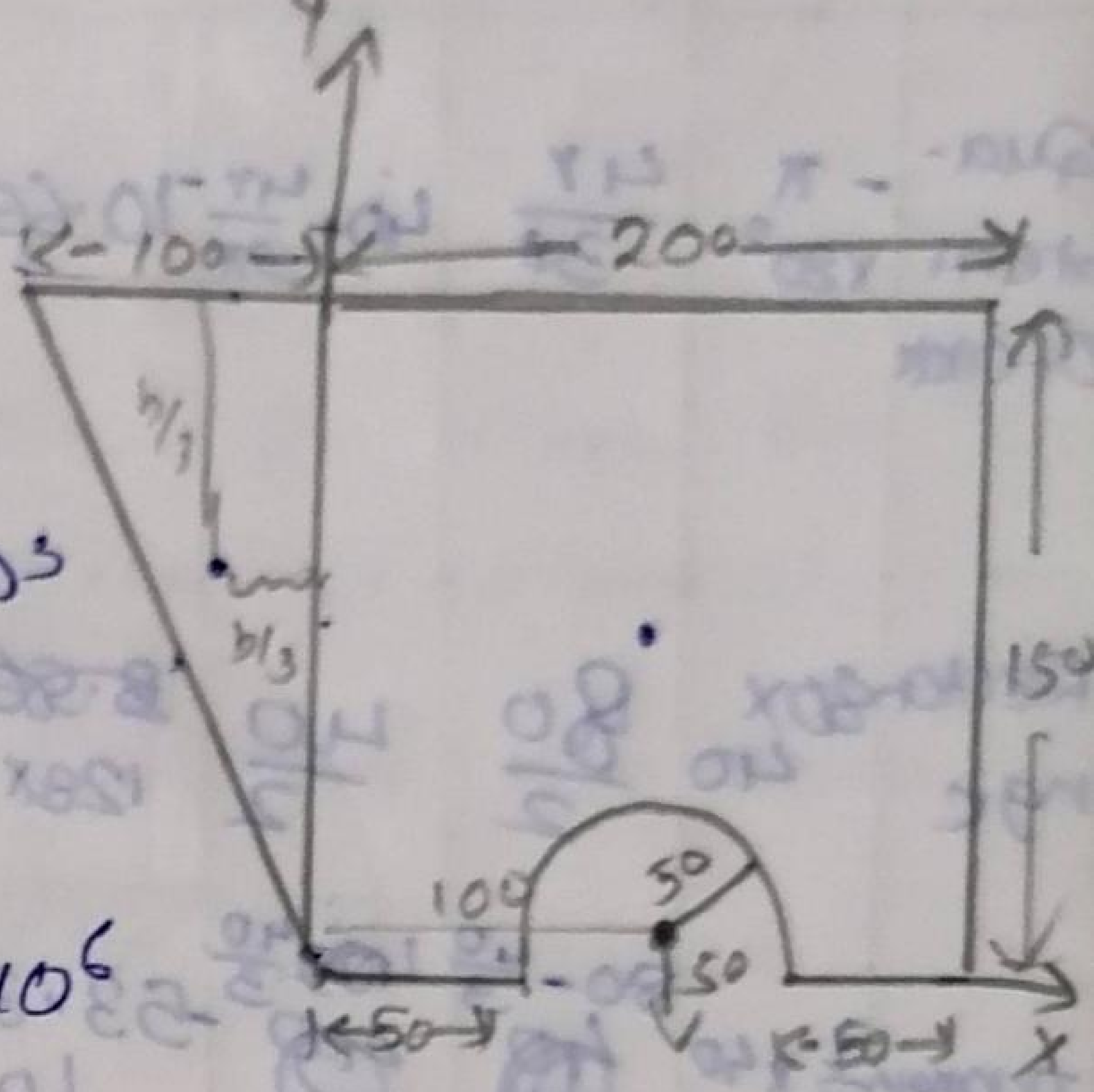
Shape	A_i	x_i	y_i	A_ix_i	A_iy_i
Semi circle	$\frac{\pi \times 2^2}{2}$	$2-\frac{4r}{3\pi}$	2	7.22	12.56
Rectangle	6x4	$2+\frac{6}{2}$	2	120	48
Triangle	$\frac{1}{2}\times 3 \times 4$	$2+6+\frac{3}{3}$	$\frac{4}{3}$	54	8

$r = 2 \quad D = 4$



$\Sigma A_i =$ $\Sigma A_i x_i$ $\Sigma A_i y_i =$

Centroid $(\bar{x}, \bar{y}) =$



Shape	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
Triangle	$\frac{1}{2} \times 100 \times 150$	$\frac{-100}{3}$	$150 - \frac{h}{3}$ (150)	-250×10^3	7050×10^3
rectangle	200×150	$\frac{200}{2}$	$\frac{150}{2}$	3×10^6	2.25×10^6
Circle	$-\pi \times 50^2$	100	50	-785×10^3	-39.25×10^3

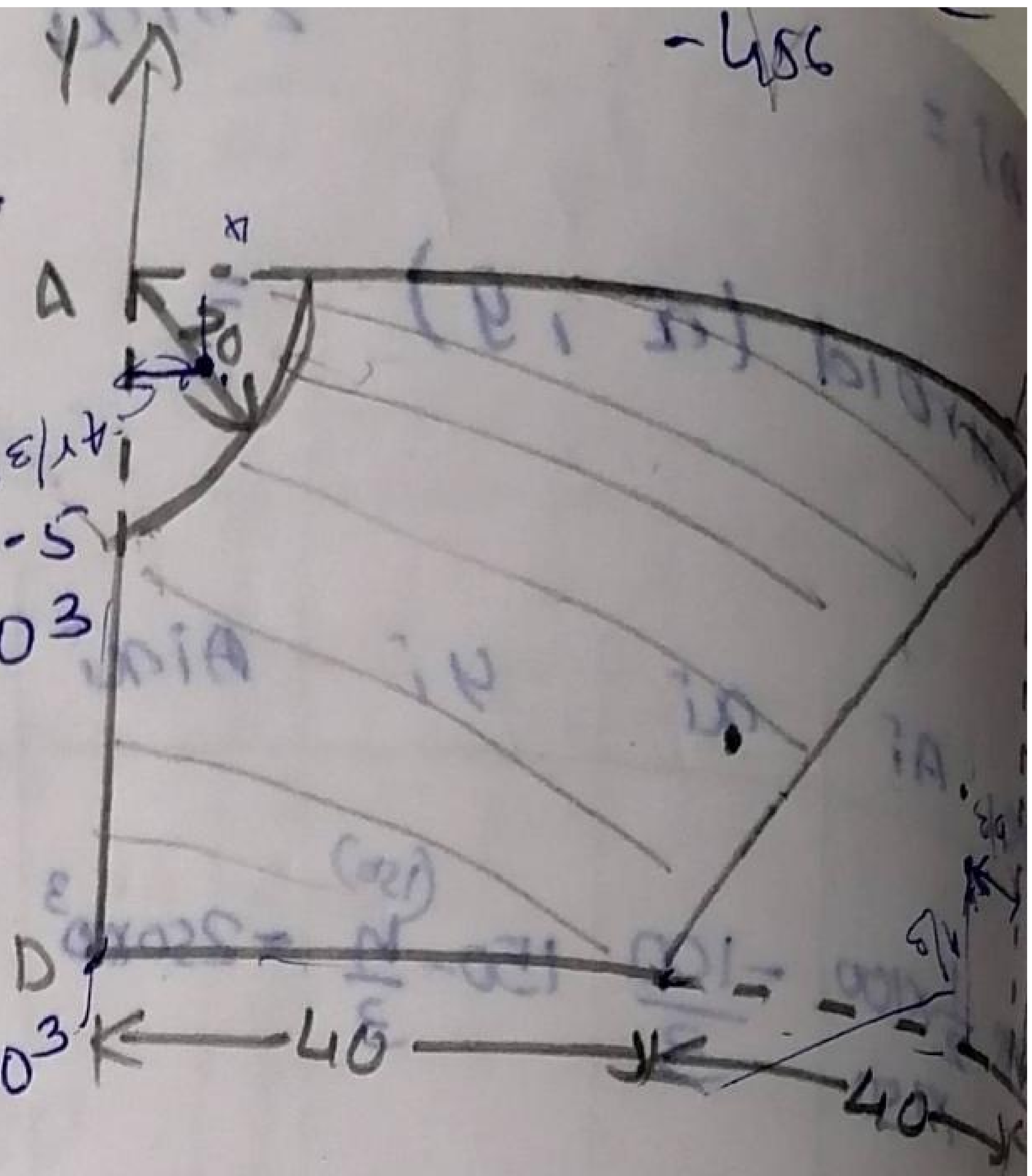
$\Sigma A_i = 29650$ $\quad \Sigma A_i x_i = 1.965 \times 10^6$ $\quad \Sigma A_i y_i = 2.6075 \times 10^6$

$$\text{Centroid}(\bar{x}, \bar{y}) = \left(\frac{\Sigma A_i x_i}{\Sigma A_i}, \frac{\Sigma A_i y_i}{\Sigma A_i} \right)$$

$$= (66.27, 87.94)$$

-456

Shape	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
Quadrant of circle	$-\pi \times 20^2$	$\frac{4r}{3\pi}$	$40-\frac{4r}{3\pi}$	-10.66×10^3	-48.5×10^3
Rectangle	80×40	$\frac{80}{2}$	$\frac{40}{2}$	~~8.56~~ 128×10^3	64×10^3
Triangle	$\frac{1}{2}\times40\times40$	$80-\frac{40}{3}$	$\frac{40}{3}$	53.3×10^3	10.66×10^3



$\Sigma A_i = \cancel{3215.36\times10^6}$ 2744 $\quad \Sigma A_i x_i = 2.144\times10^6$

$\Sigma A_i y_i = 26160$

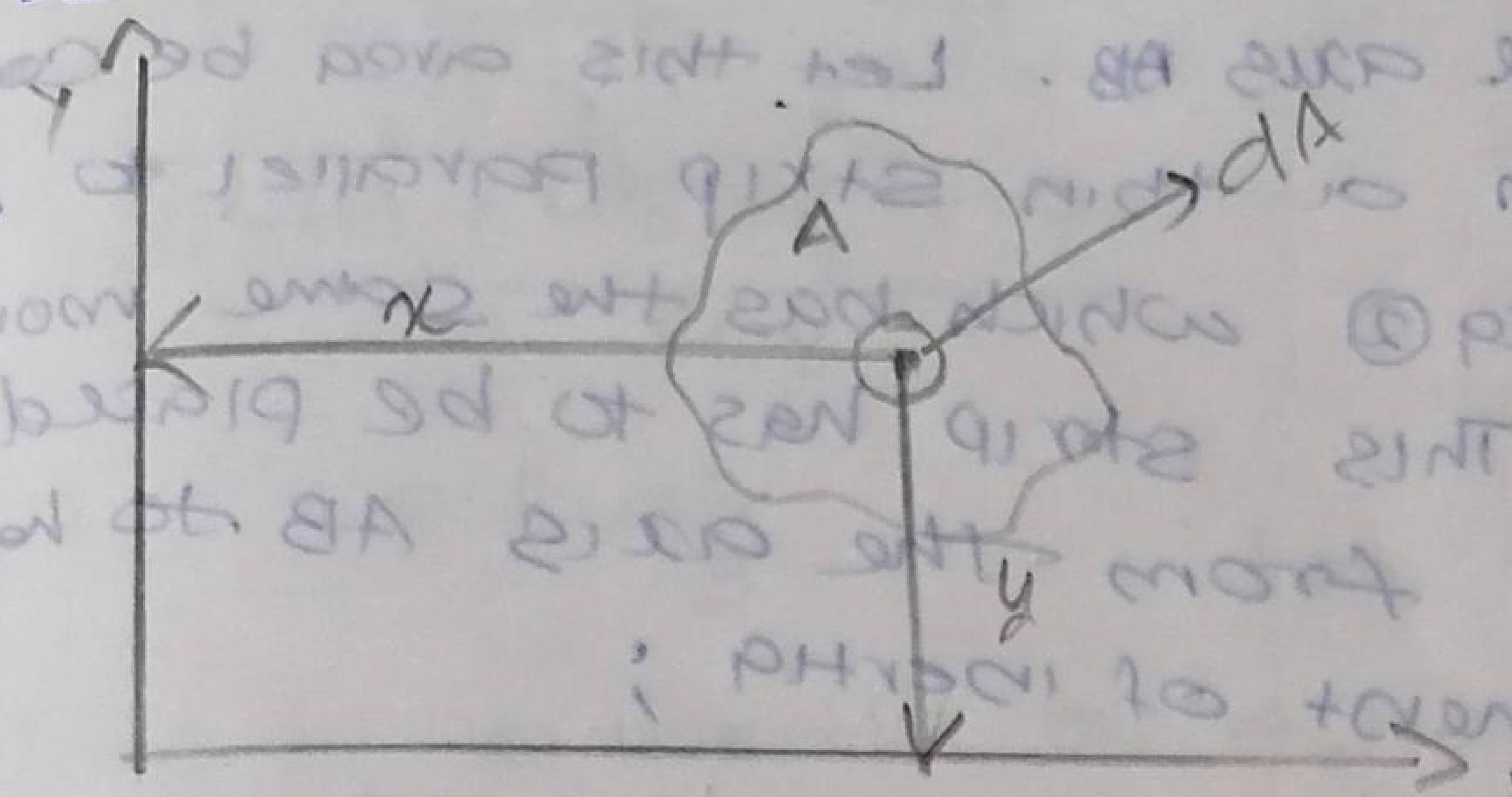
$$\text{Centroid} = \left(\frac{\Sigma A_i x_i}{\Sigma A_i}, \frac{\Sigma A_i y_i}{\Sigma A_i}\right)$$

(781.34)

Moments of Inertia

Moment of inertia is the measure of resistance to bending i.e it is applying to while dealing with deflection or deformation.

Let A be the total area of a rigid body



Let dA be the elemental area. Then moment of inertia w.r.t x axis is given by

$$I_{xx} = \Sigma y (dA)$$

$$= Ay^2$$

$$I_{yy} = Ax^2$$

Radius of Gyration

The radius of Gyration of an area about an axis is the distance of a long narrow strip whose area is equal to the area of the lamina ie whose moment of inertia remains the same as that of original area.

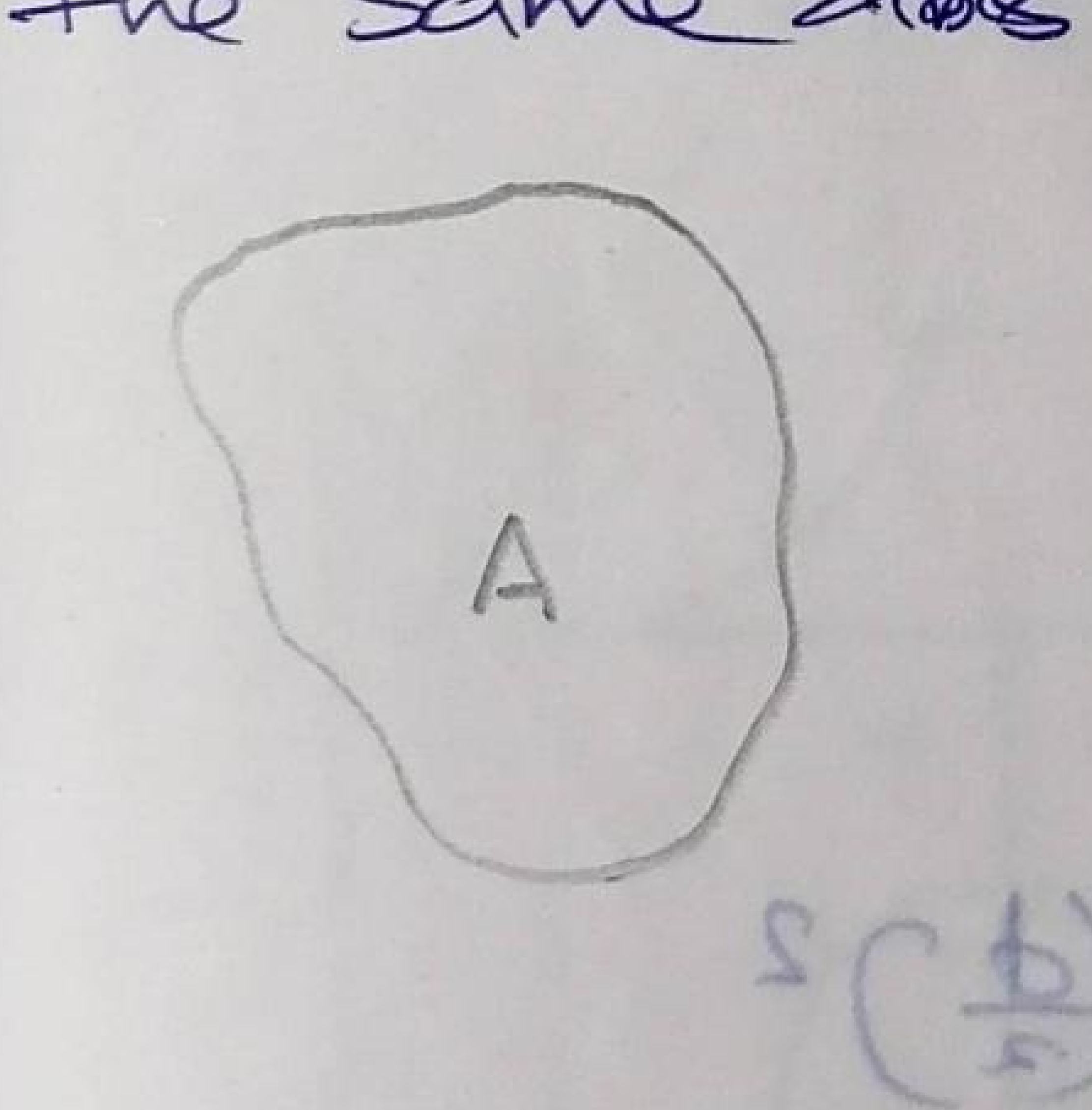


Fig 1

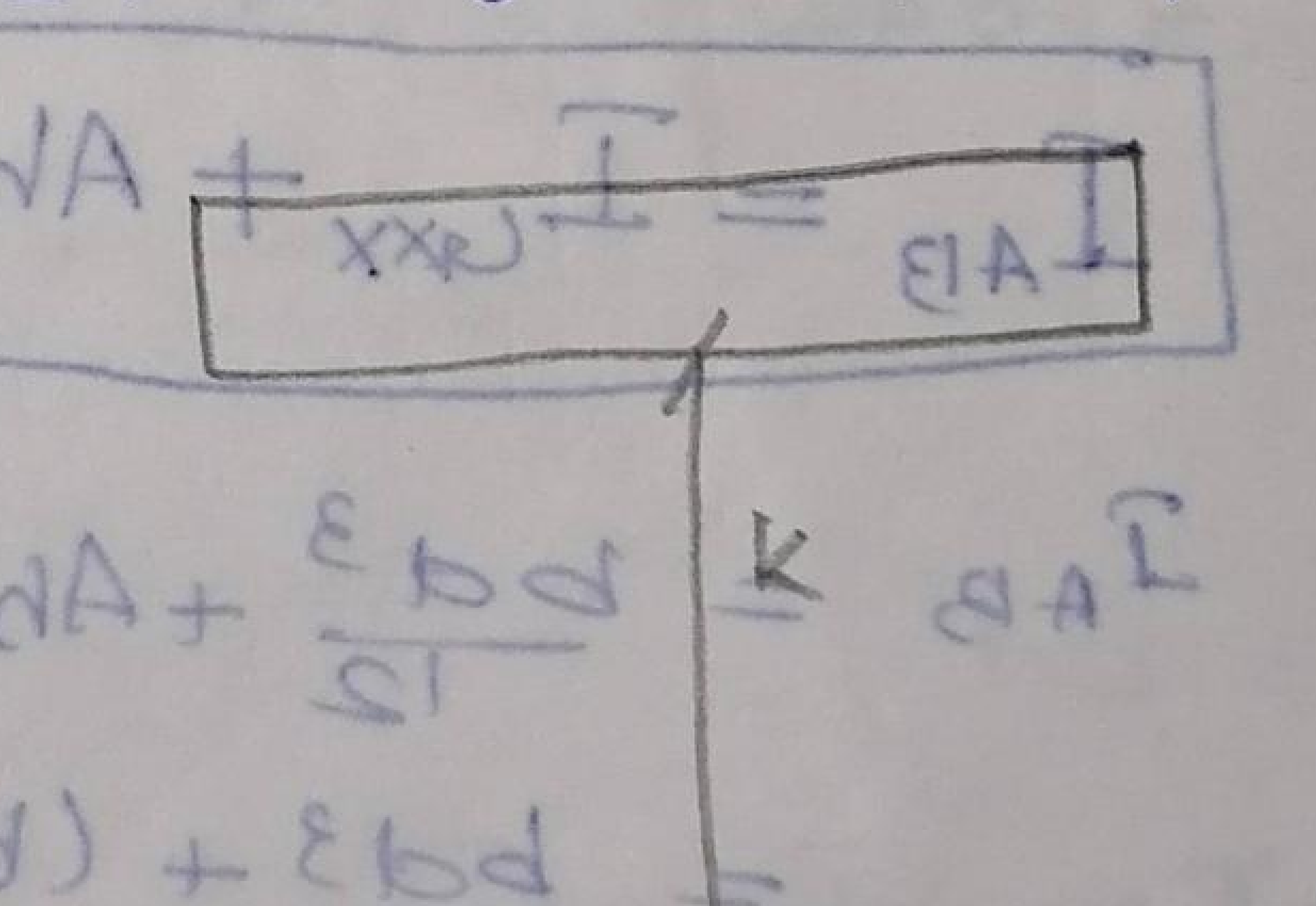


Fig 2

consider an area A which has moment of inertia I w.r.t a reference axis AB. Let this area be compressed through a thin strip parallel to AB a shown in Fig ② which has the same moment of inertia I. This strip has to be placed @ a distance k from the axis AB to have the same moment of inertia;

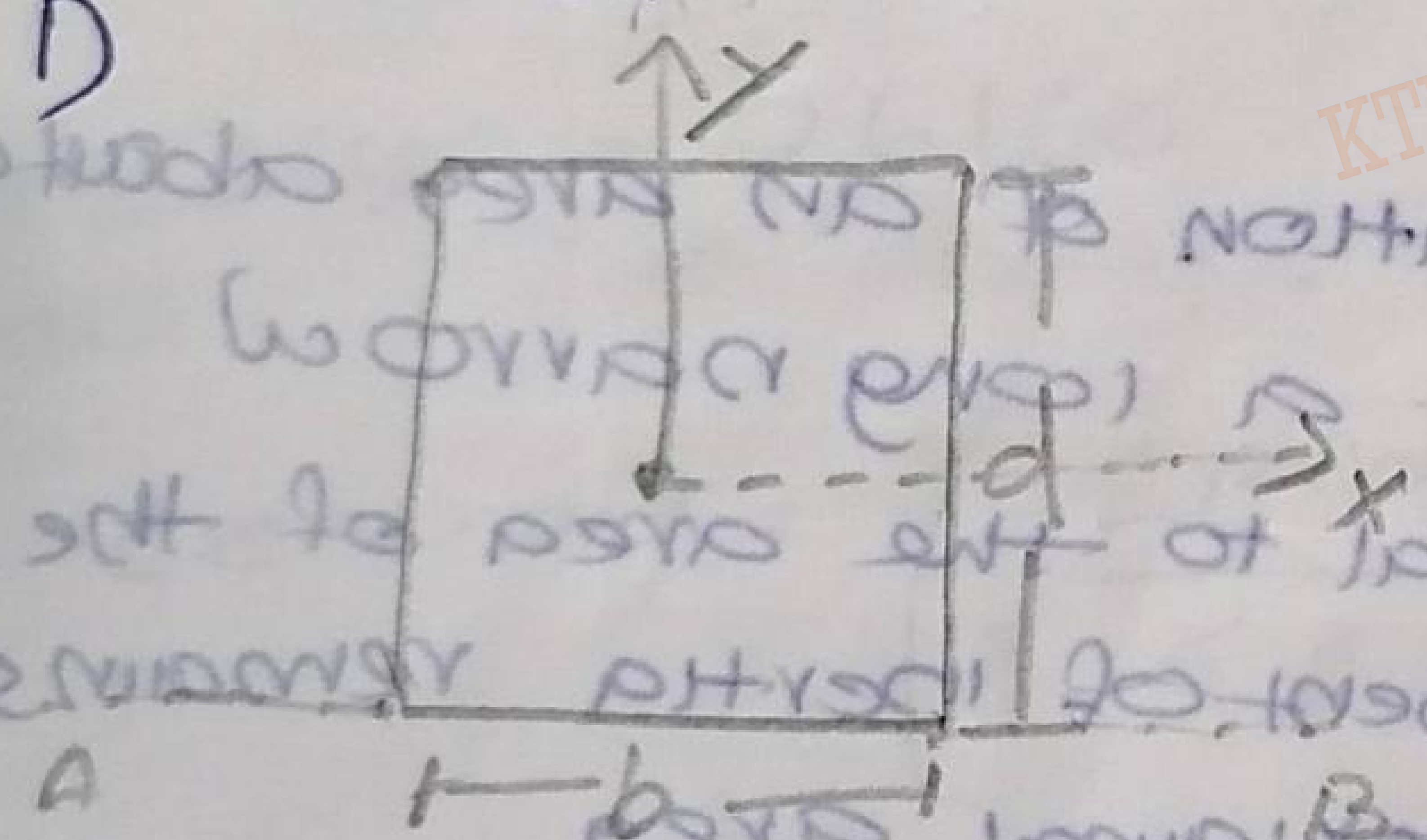
$$I = Ak^2$$

$$k = \sqrt{\frac{I}{A}}$$ where k is radius of gyration

Moment inertia of common standards

Moment inertia of rectangle

1)



$$\boxed{I_{Gxx} = \frac{bd^3}{12} \qquad I_{Gyy} = \frac{db^3}{12}}$$

$$\boxed{I_{AB} = I_{Gxx} + Ah^2}$$

$$I_{AB} = \frac{bd^3}{12} + Ah^2$$

$$= \frac{bd^3}{12} + (b \times d) \times \left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + bd \times \frac{d^2}{4}$$

$$\frac{bd + 12bd}{1\ 12} + \frac{d^2}{4}$$

$$\frac{4bd^3 + 48bd + 12d^2}{48}$$

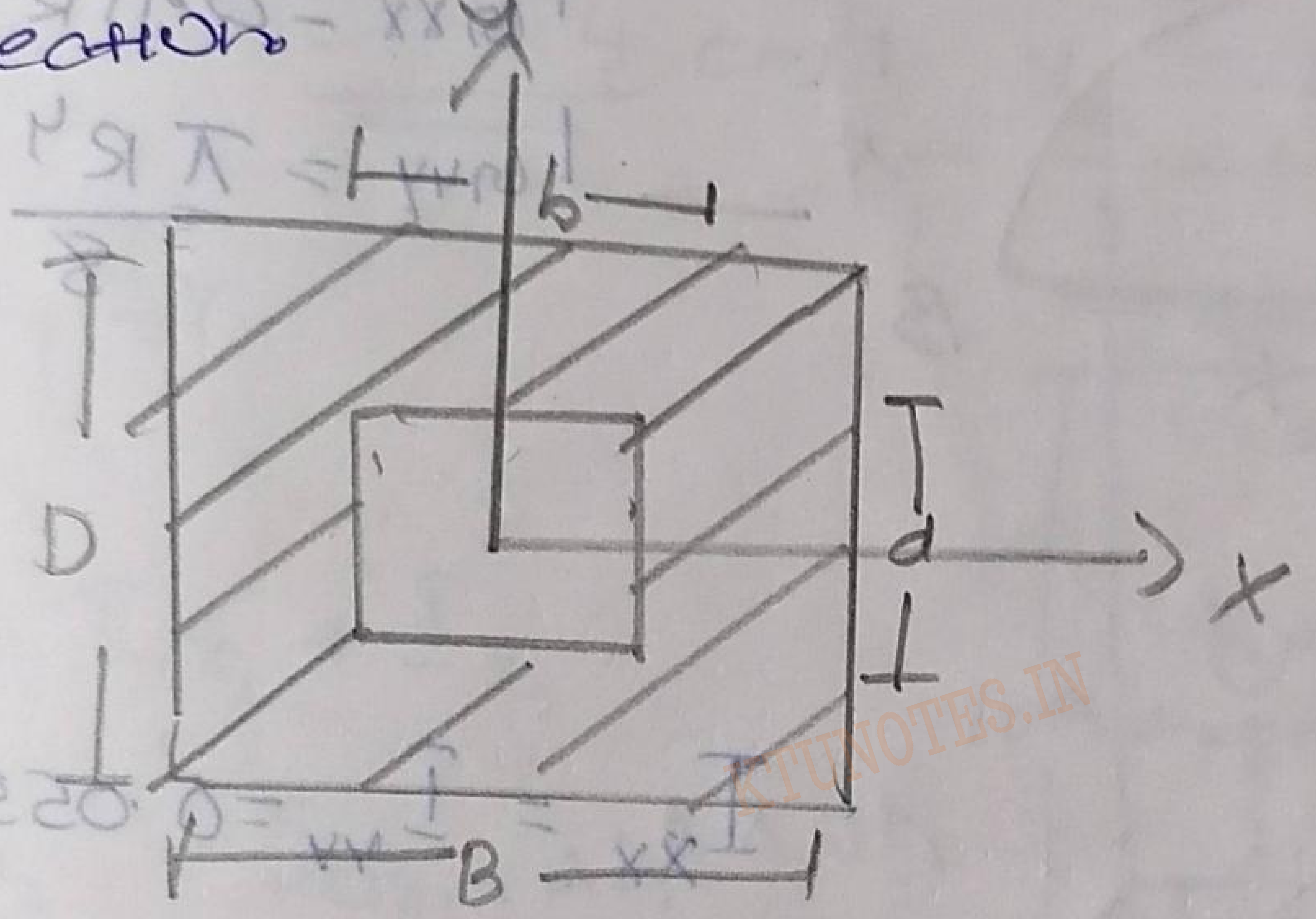
$$\frac{4bd^3 + 12bd^3}{48}$$

$$\frac{16bd^3}{48}$$

$I_{AB} = bd^3/3$

$I_{AB} = bd^3/3$

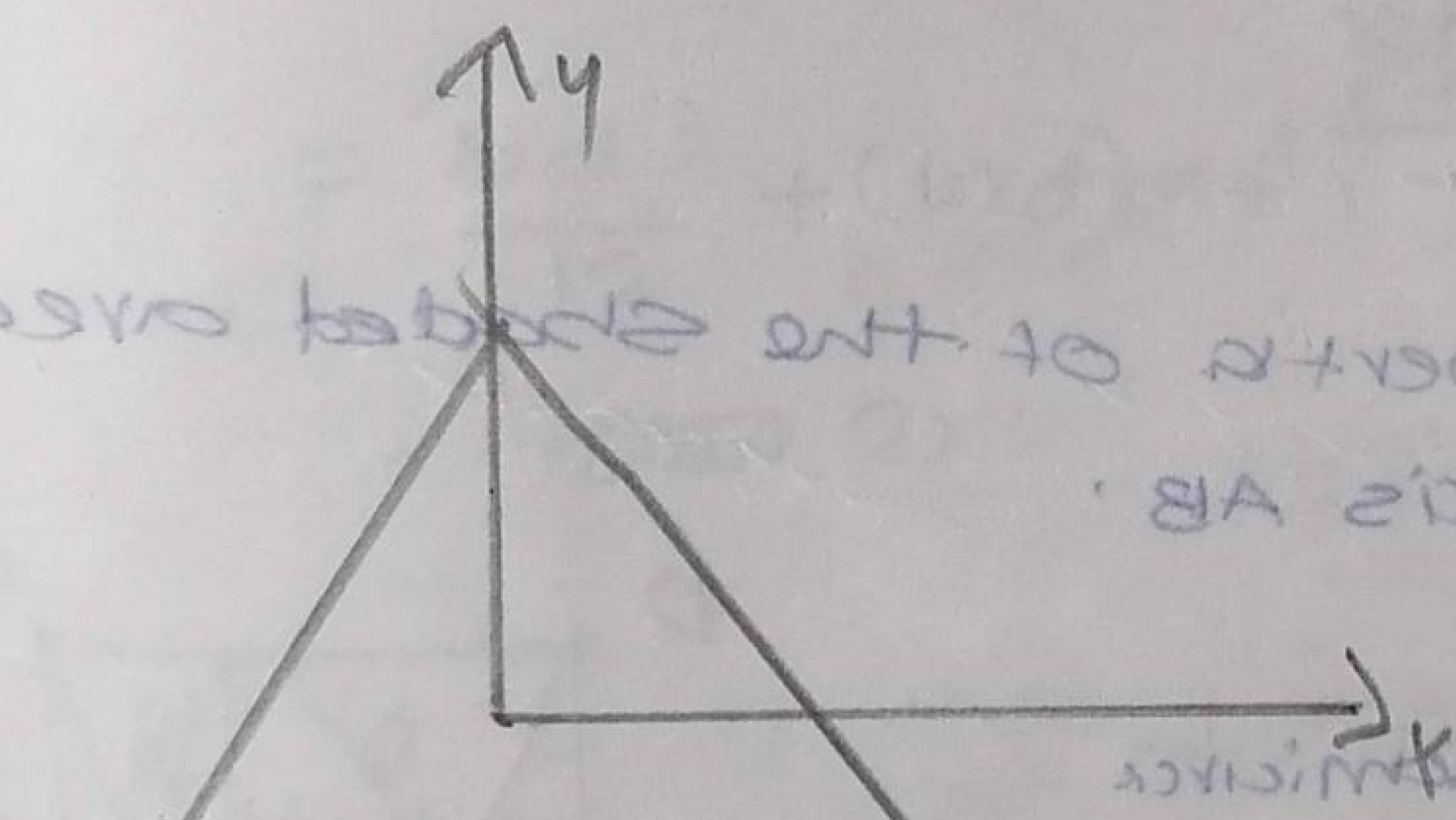
2) moment of inertia of a hollow rectangular section



$$I_{Gxx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

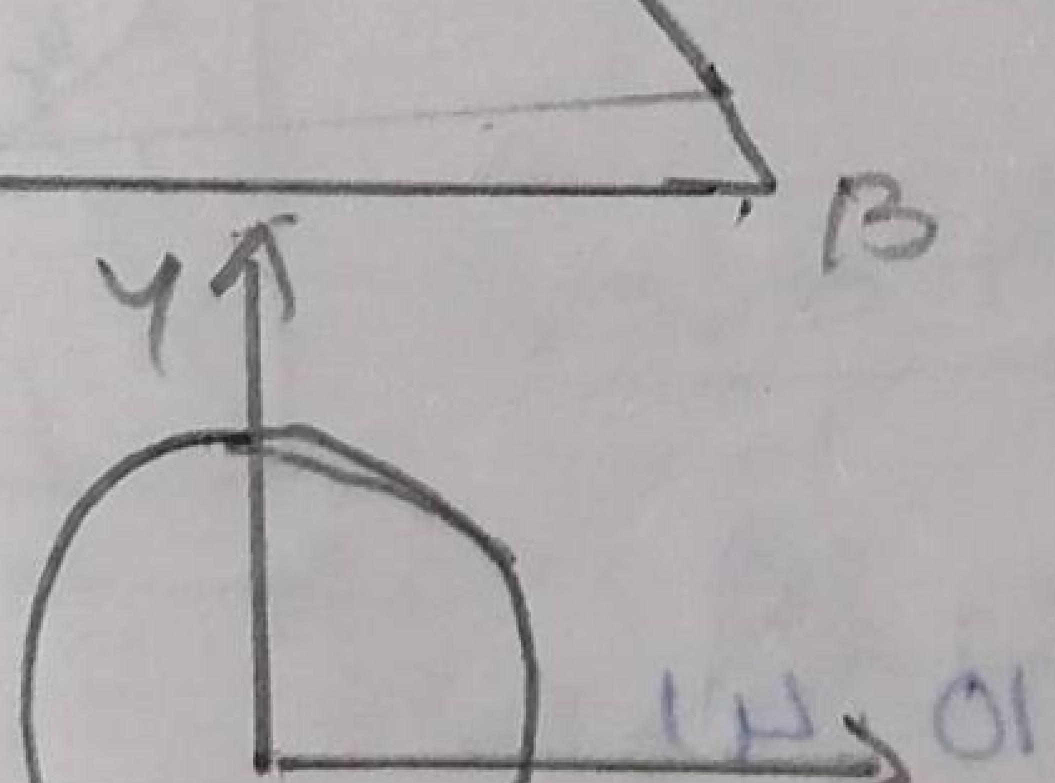
$$I_{Gyy} = \frac{DB^3}{12} - \frac{db^3}{12}$$

3



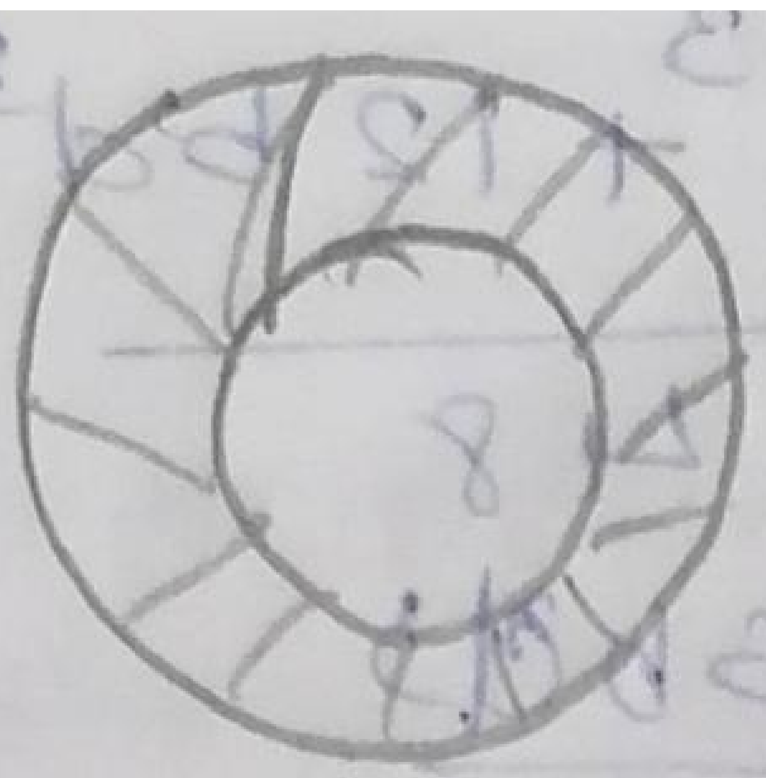
$$I_{Gxx} = \frac{bh^3}{36}$$

4



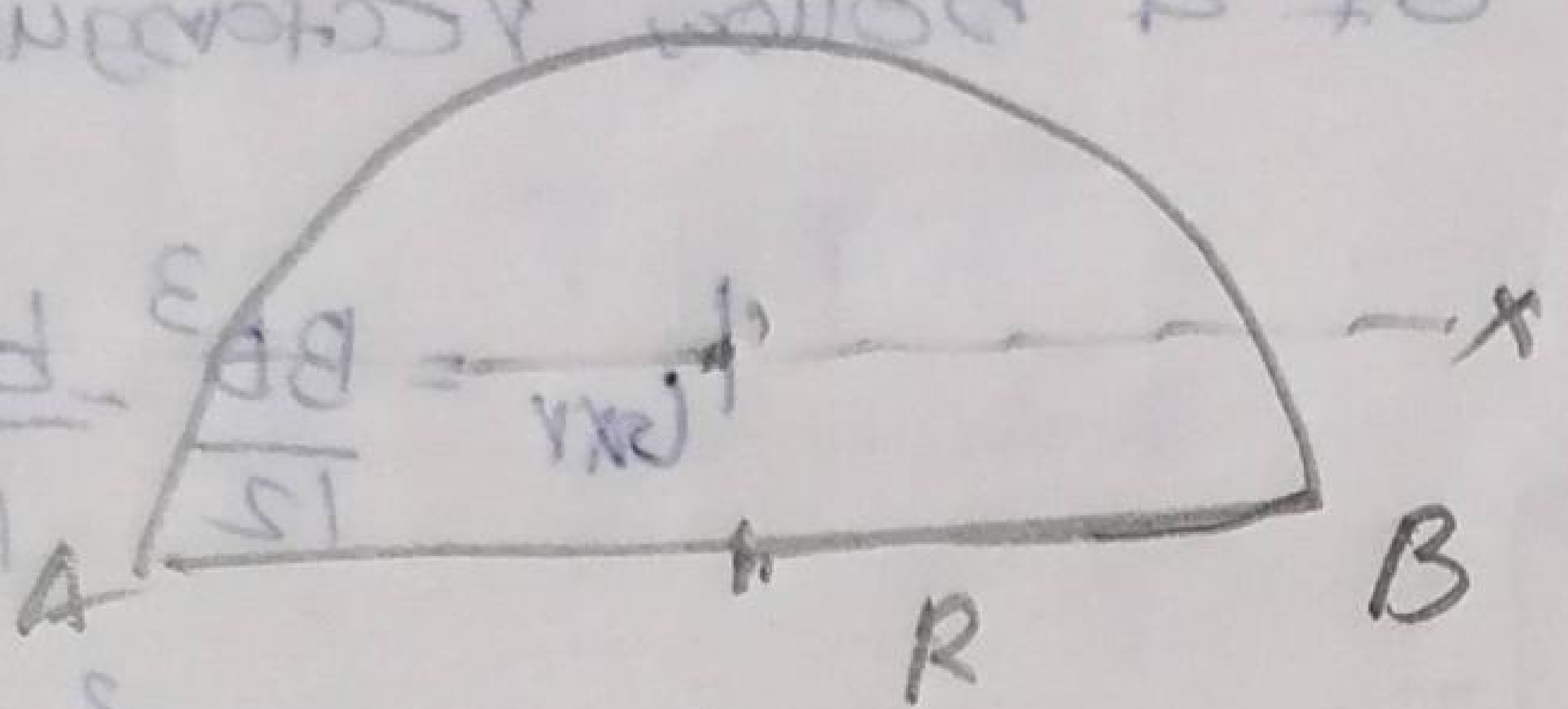
$$I_{Gxx} = I_{Gyy} = \frac{\pi d^4}{64}$$

5



$$I_{Gxx} = I_{Gyy} = \frac{\pi\left(d_0^4 - d_1^4\right)}{64}$$

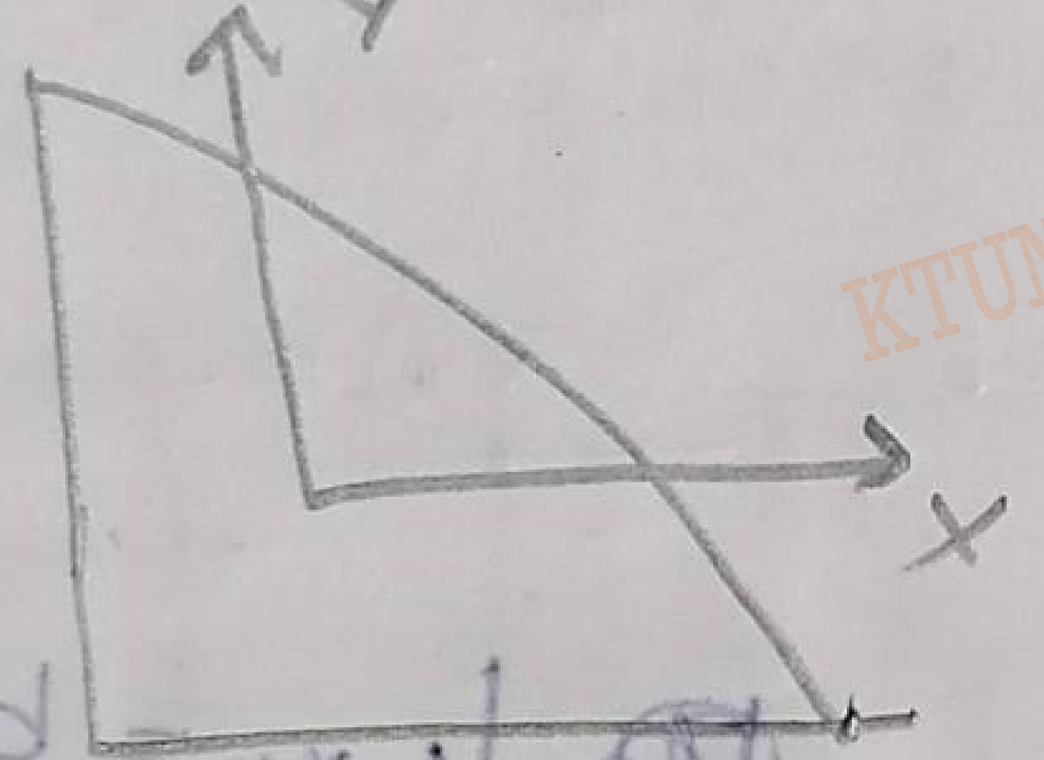
6)



$$I_{Gxx} = 0.11R^4$$

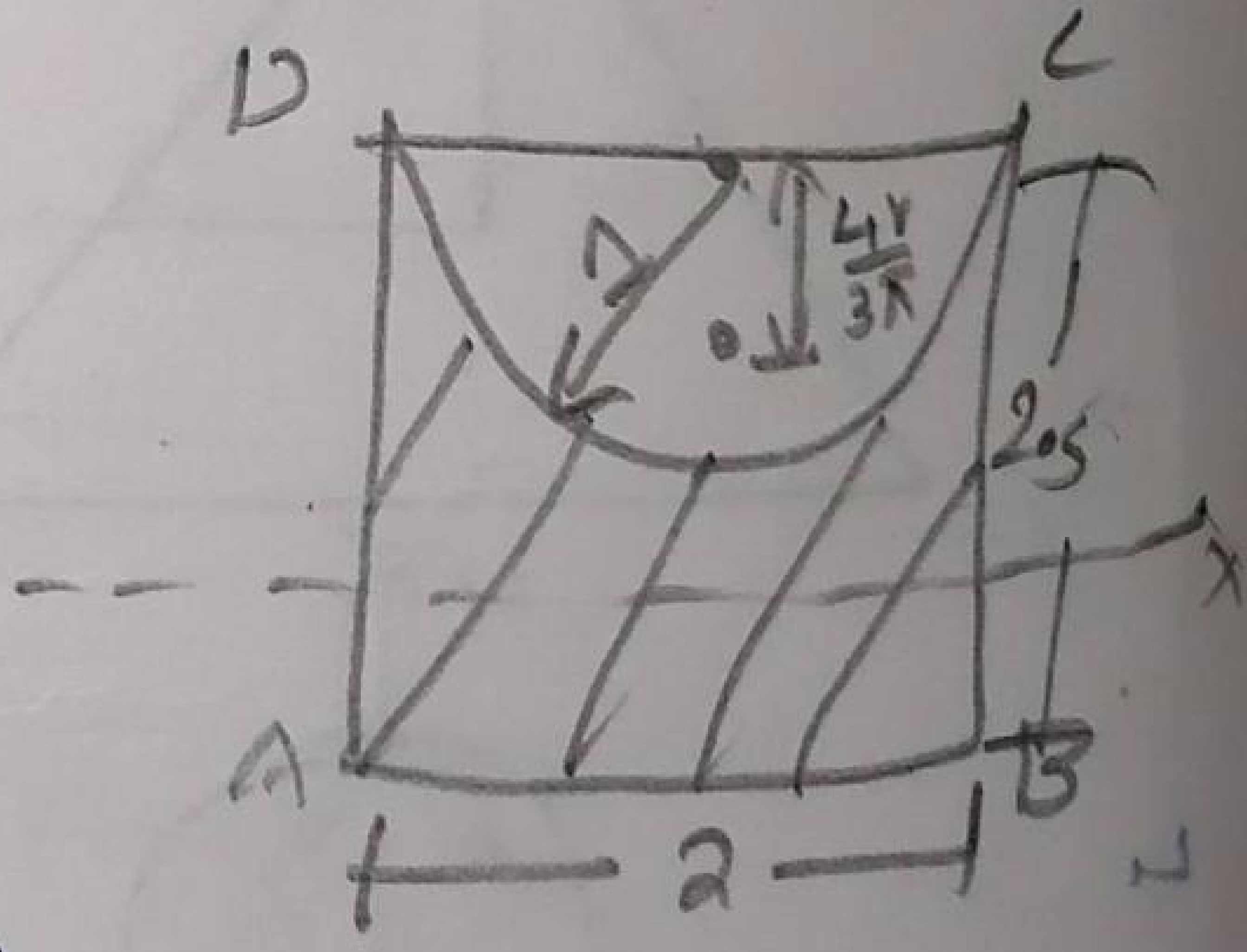
$$I_{Gyy} = \frac{\pi R^4}{8}$$

7



$$I_{xx} = I_{yy} = 0.055r^4$$

1) determine moment of inertia of the shaded area above the axis AB.



$$I_{AB} = I_{rect.} - I_{semicircle}$$

$$I_{recta} = bd^3/3$$

$$= \frac{2 \times (2.5)^3}{3} = \underline{10.41}$$

$I_{arc} = I_{Gxx} + Ah^2$ h = distance b/w xx & AB

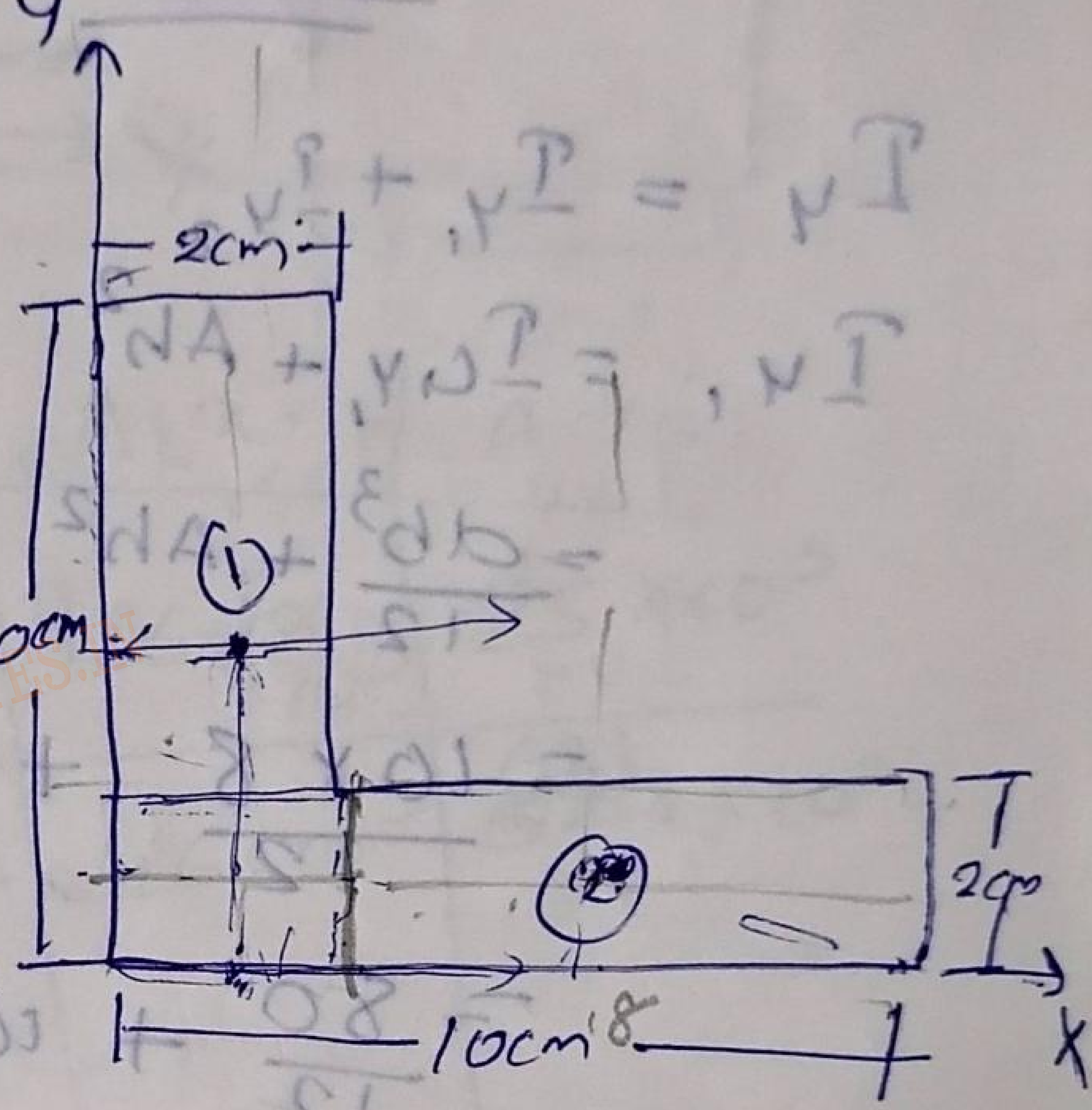
$= 0.11R^4 + \left(\frac{\pi r^2}{2}\right)\left(2.5 - \frac{4r}{3\pi}\right)^2$

$= \cancel{10.85}\,6.87$

$I_{AB} = 10.41 - \cancel{10.85}\,6.87$

$= \underline{\underline{3.54}}$ cm⁴

? I_x, I_y



$I_y = I_{x_1} + I_{x_2}$

$I_{x_1} = I_{Gx_1} + Ah^2$

$= \frac{bd^3}{12} + (b\times d)\times 5^2$

$= \cancel{10000}\ \frac{2\times 10^3}{12} + (2\times 10)\times 25$

$= \frac{2\times 1000}{12} + 500$

$= \frac{2000 + 6000}{12} = \frac{8000}{12} = 666.66$

$I_{x_2} = I_{Gx_2} + Ah^2$

$\frac{bd^3}{12} + (b\times d)\times 1$

$\frac{\cancel{8\times10}^3}{12} + 2\times\cancel{10}\,8$

$\frac{\cancel{8000}\;64}{12} + 20$

$\frac{2000+240}{12}$

$\underline{\underline{21.33}}$

$I_y = I_{y_1} + I_{y_2}$

$I_{y_1} = I_{Gy_1} + Ah^2$

$= \frac{db^3}{12} + Ah^2$

$= \frac{10\times8}{12} + db\times1$

$= \frac{80}{12} + 10\times2$

$= \underline{\underline{26.66}}$

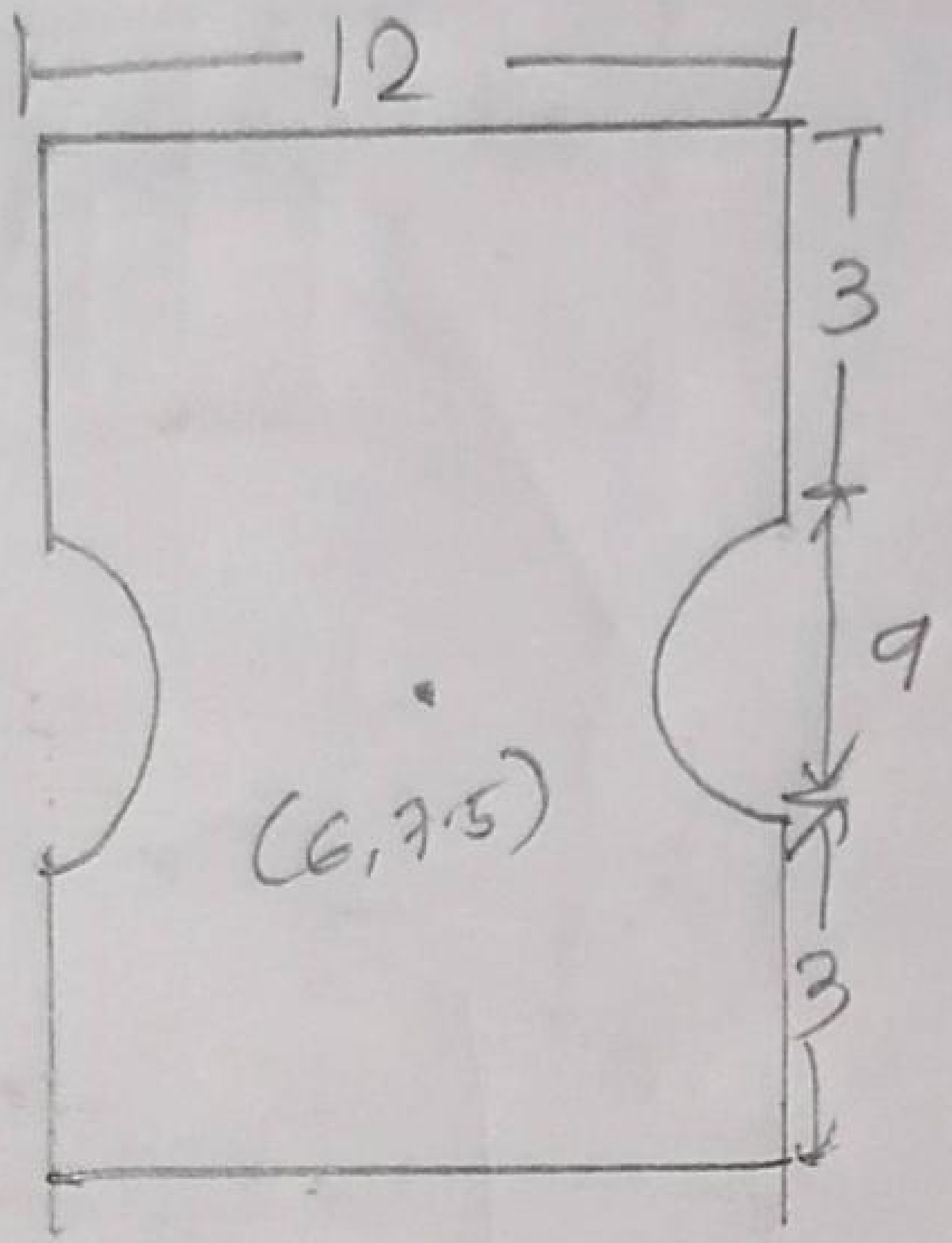
$I_{y_2} = I_{Gy_2} + Ab^2$

$= \frac{db^3}{12} + A\,\cancel{5}\times6^2$

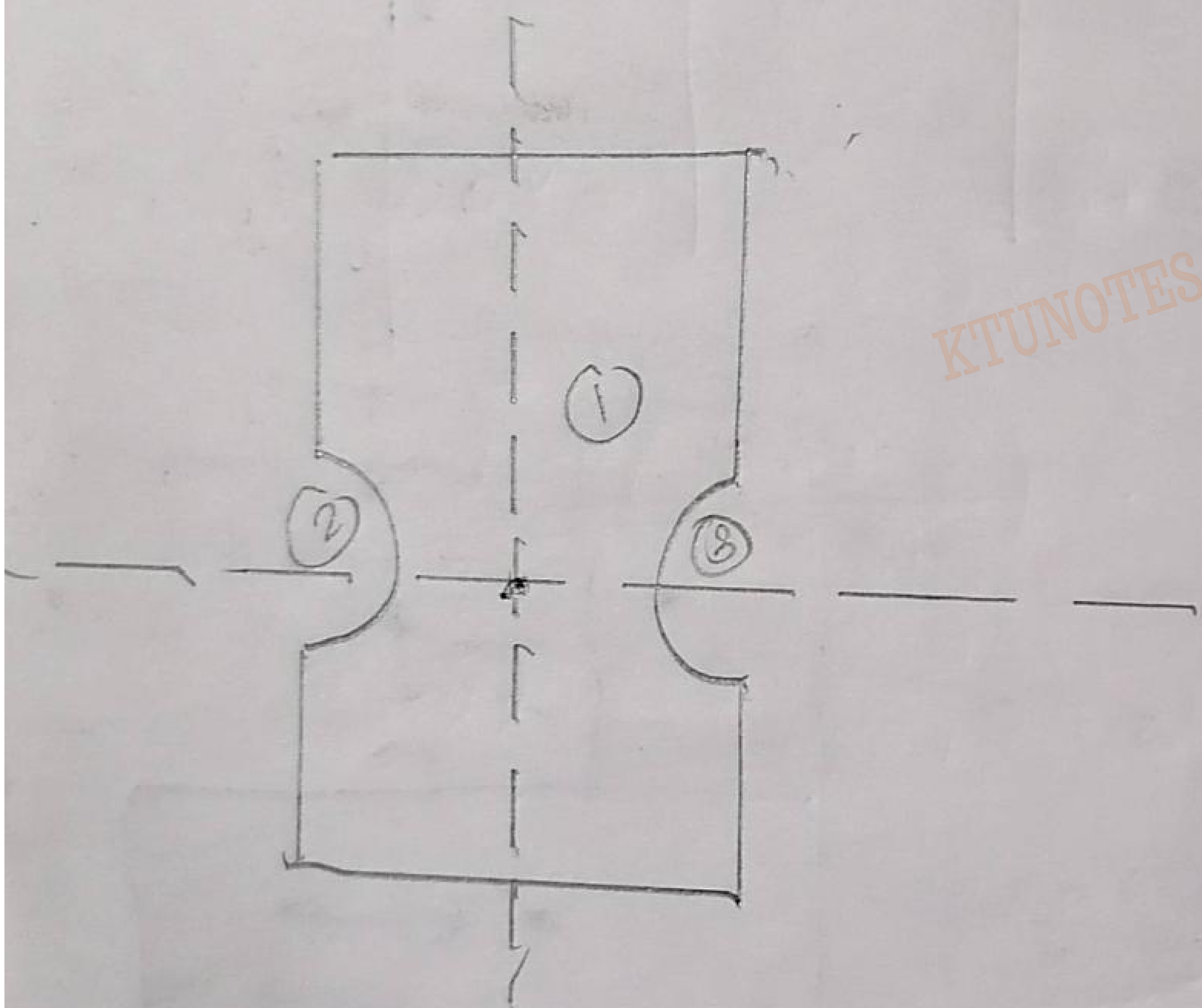
$= \frac{2\times8^3}{12} + \overset{18}{\cancel{10}}\times36$

$=$

Find moment of inertia about its centroidal axis



$(\bar{x}, \bar{y}) = (6, 7.5)$



$$I_{xx} = I_{xx_1} - I_{xx_2} - I_{xx_3}$$

$$I_{xx_1} = I_{Gxx_1} + Ah^2$$

$$= \frac{bd^3}{12} + (b \times d) \times 0$$

$$= \frac{\cancel{12} \times 15^3}{\cancel{12}} = \underline{\underline{3375 \text{ cm}^4}}$$

$$I_{xx_2} = I_{Gxx_2} + Ah^2$$

$$= 0.11R^4 + \left(\frac{\pi r^2}{2}\right) \times 0$$

$$= 0.11 \times 4.5^4$$

$$= \underline{\underline{45.106}} \text{ cm}^4$$

$$I_{xx_3} = I_{Gxx_3} + Ah^2$$

$$= 0.11R^4 + A \times 0$$

$$= 45.106 \text{ cm}^4$$

$$I_{xx} = 3284.788 \text{ cm}^4$$

$$I_{yy} = I_{yy_1} - I_{yy_2} - I_{yy_3}$$

centroid of hemisphe

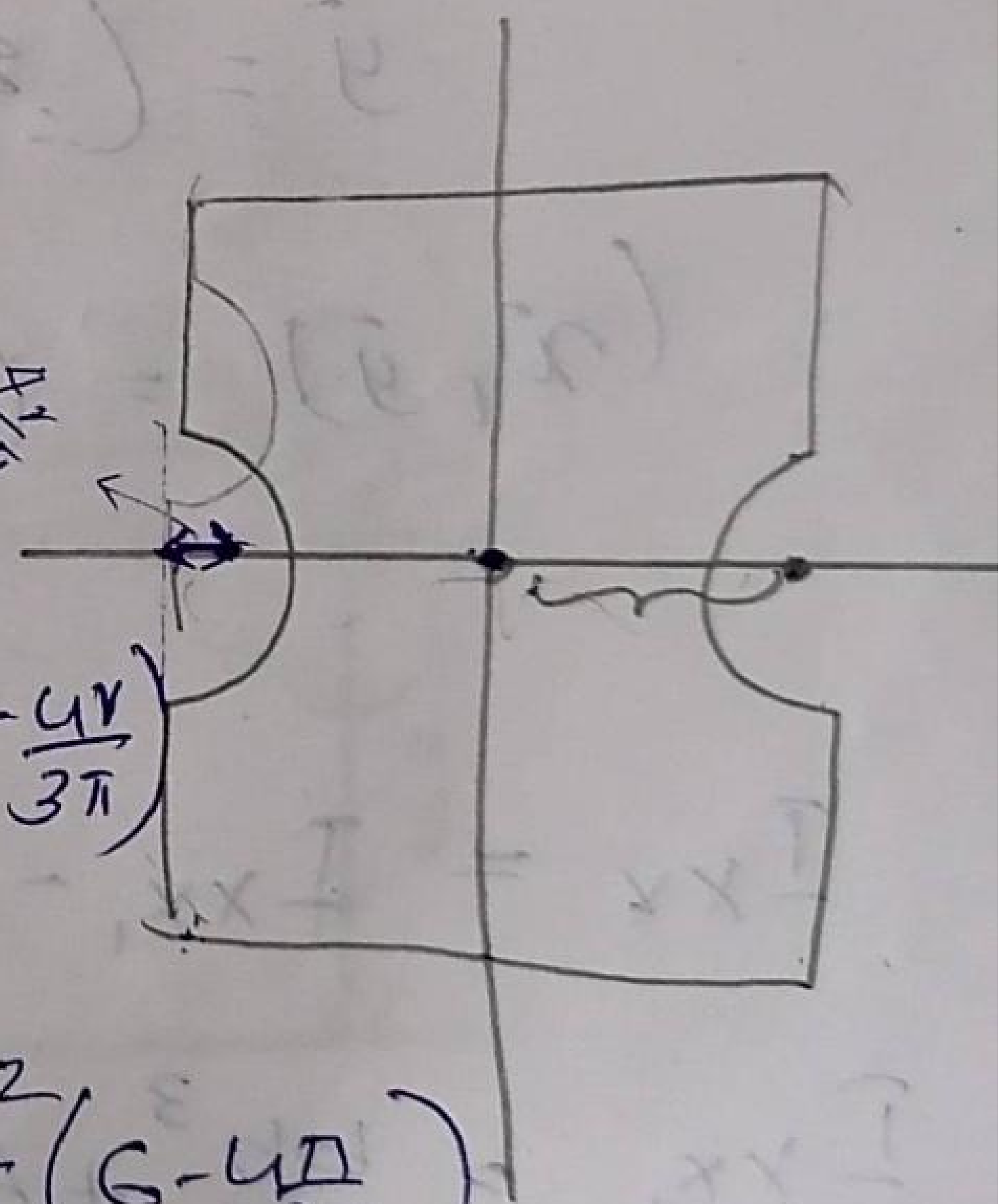
$\left(r, \frac{4r}{3\pi}\right)$

$\left(4.5, \frac{4 \times 4.5}{3 \times 3.14}\right)$

$$I_{yy_1} = I_{Gyy_1} + Ah^2$$

$$= \frac{db^3}{12} + (b \times d) \times 0$$

$$= \frac{15 \times 12^3}{12} = 2160$$



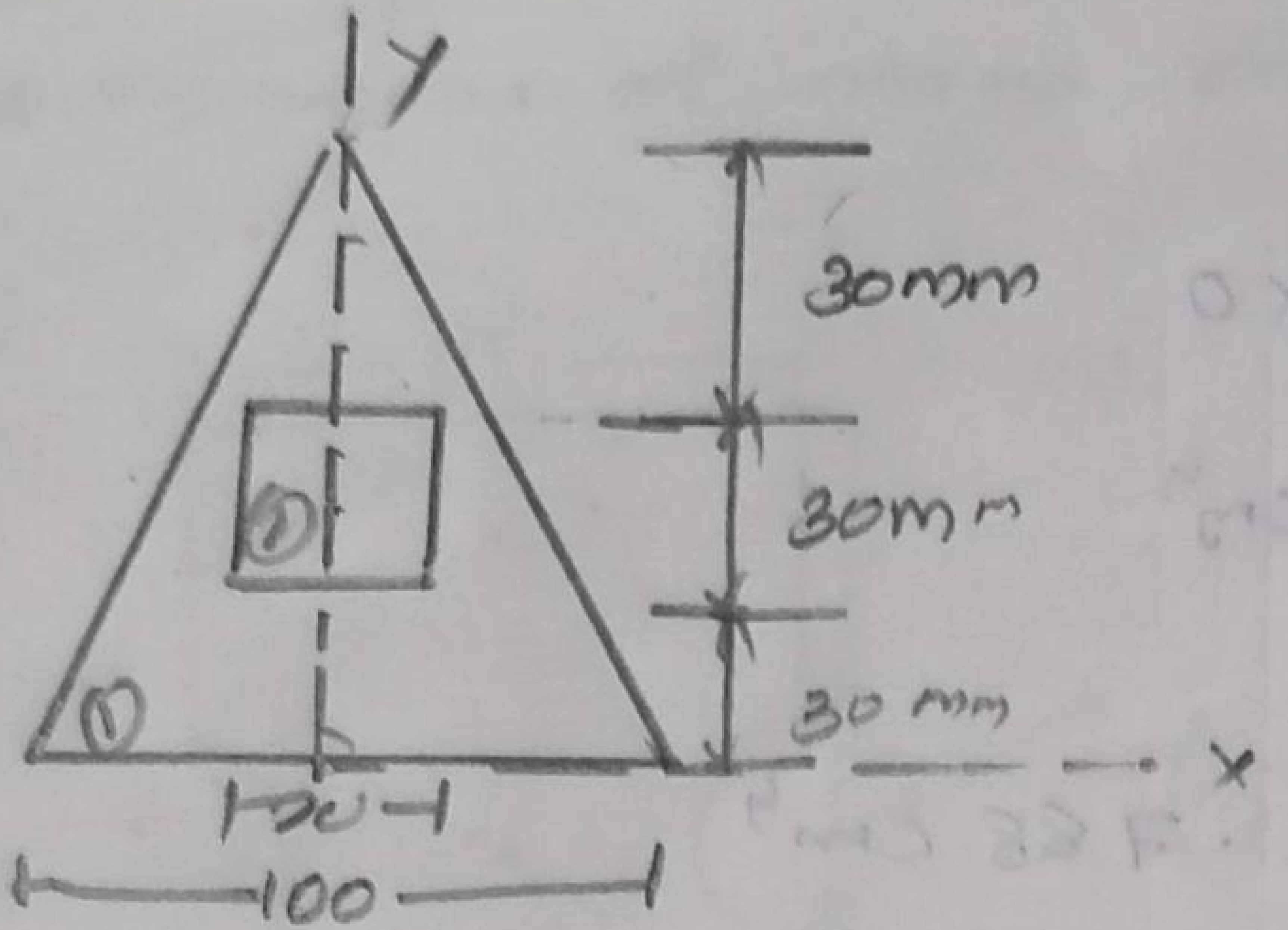
$$I_{yy_2} = I_{Gyy_2} + Ah^2$$

$$= \frac{\pi R^4}{8} + \frac{\pi R^2}{2} \times \left(6 - \frac{4r}{3\pi}\right)$$

$$= \frac{3.14 \times (4.5)^4}{8} + \frac{\pi R^2}{2}\left(6 - \frac{4r}{3\pi}\right)$$

$$= 268.71$$

$$I_{yy_3} = 268.71$$



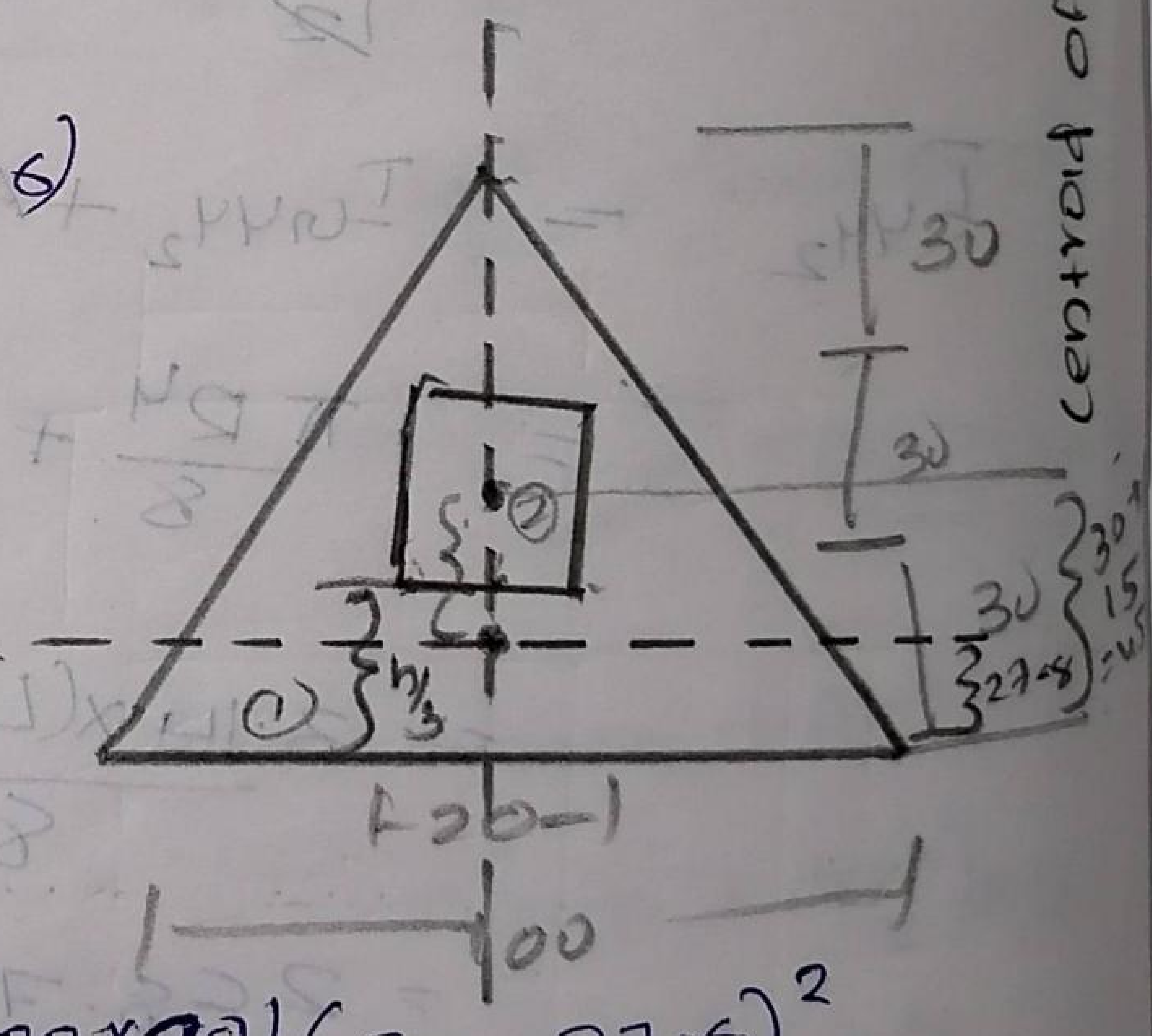
$\bar{x} = 50 \quad \left(\frac{100}{2}\right)$

Shape	A_i		y_i		$A_i y_i$
triangle	$\frac{1}{2} \times 100 \times 90$		$\frac{90}{3}$		135×10^3
rectangle	(20×30)		$30 + \frac{30}{2}$		-27×10^3

centroid of triangle is in $\frac{h}{3} = \frac{90}{3}$

$$\bar{y} = \left(\frac{\Sigma A_i y_i}{\Sigma A_i}\right) = \underline{27.6}$$

$$(\bar{x}, \bar{y}) = (50, 27.6)$$



$$I_{xx} = I_{xx_1} - I_{xx_2}$$

$$I_{xx_1} = \frac{bh^3}{36} + A_1 h_1^2$$

$$= \frac{100 \times 90^3}{36} + \left(\frac{1}{2} \times 100 \times 90\right)(30 - 27.6)^2$$

$$= \underline{2.05 \times 10^6 \text{ mm}^4}$$

$$I_{xx_2} = \frac{bd^3}{12} + A_2 b^2$$

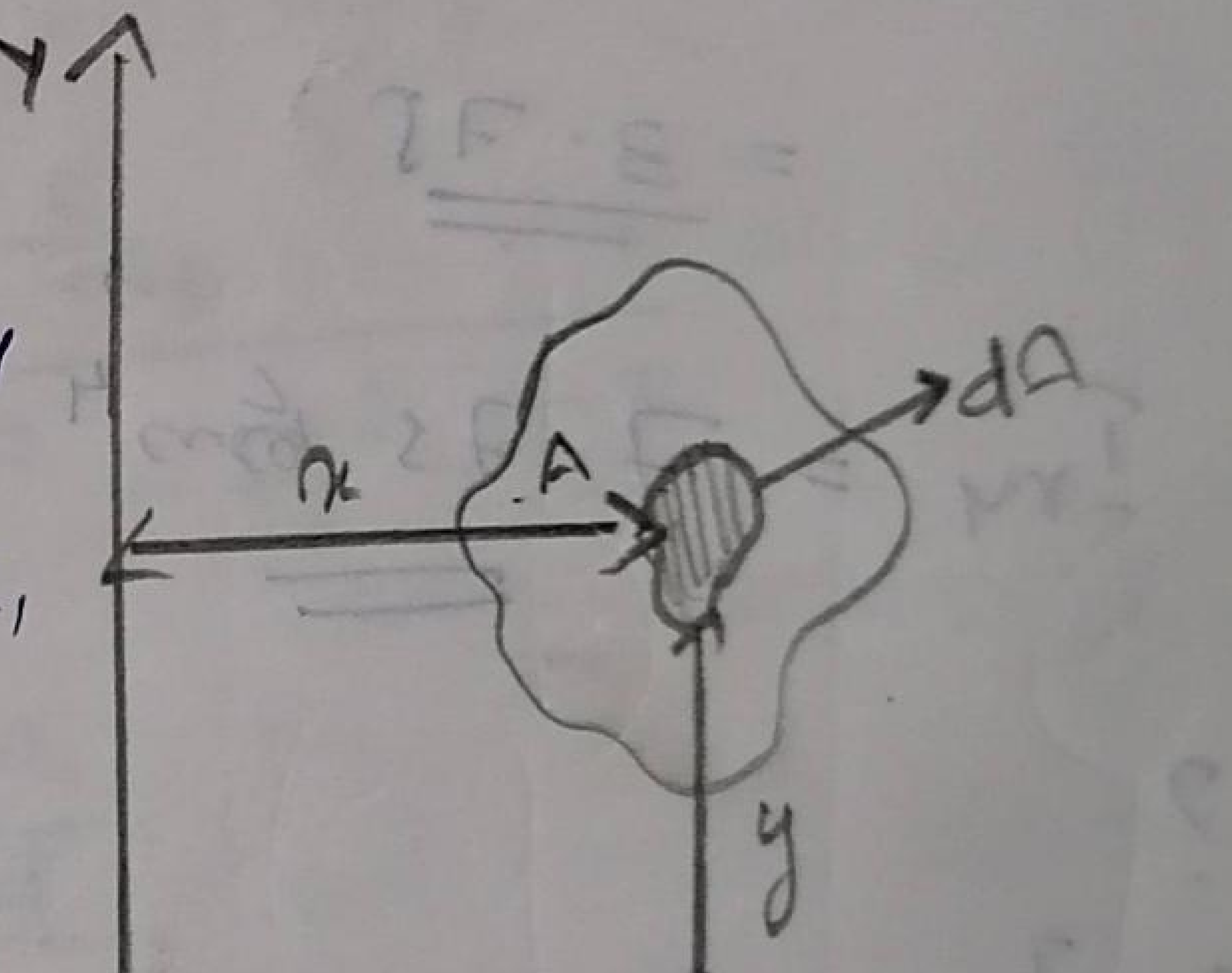
$$= \frac{20 \times 30^3}{12} + (20 \times 30) \times (45 - 27.8)^2$$

$$= 267.504 \times 10^3$$

$$I_{xx} = 1.7824 \times 10^6 \text{ mm}^4$$

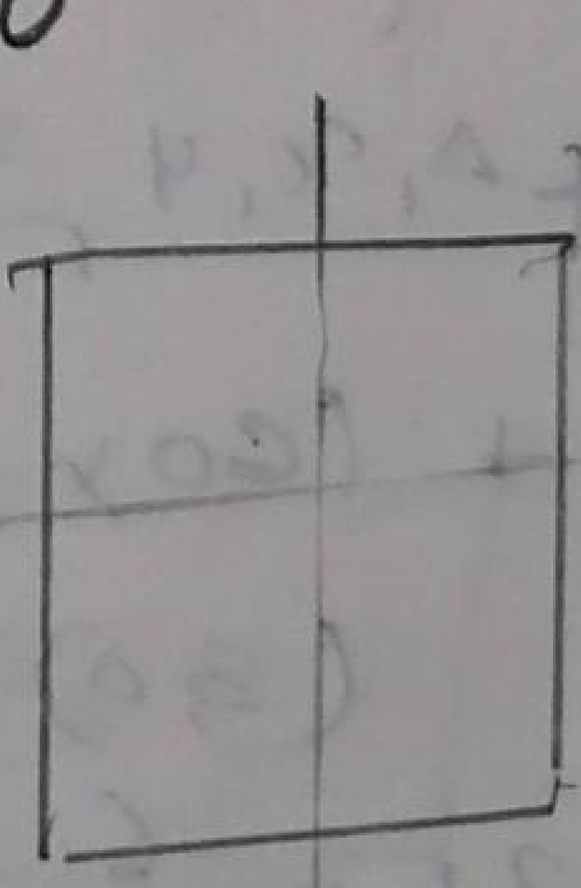
Product of Inertia

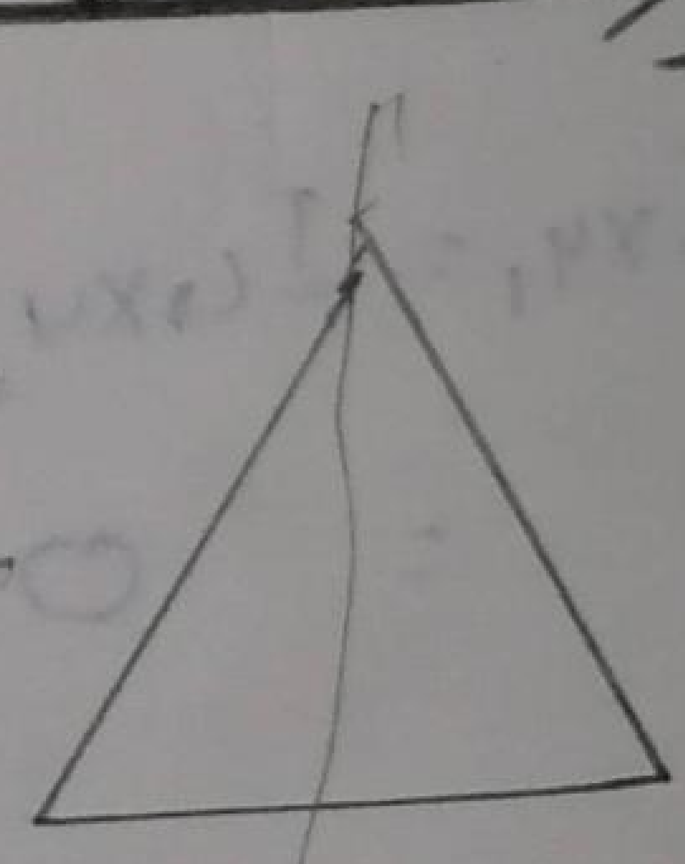
Consider ~~[illegible]~~ plane area A & ~~let~~ dA be a small elemental area whose coordinates are x and y w.r.t ~~ox~~ ox and oy axis.



$\int xy \, dA$ denoted by I_{xy}

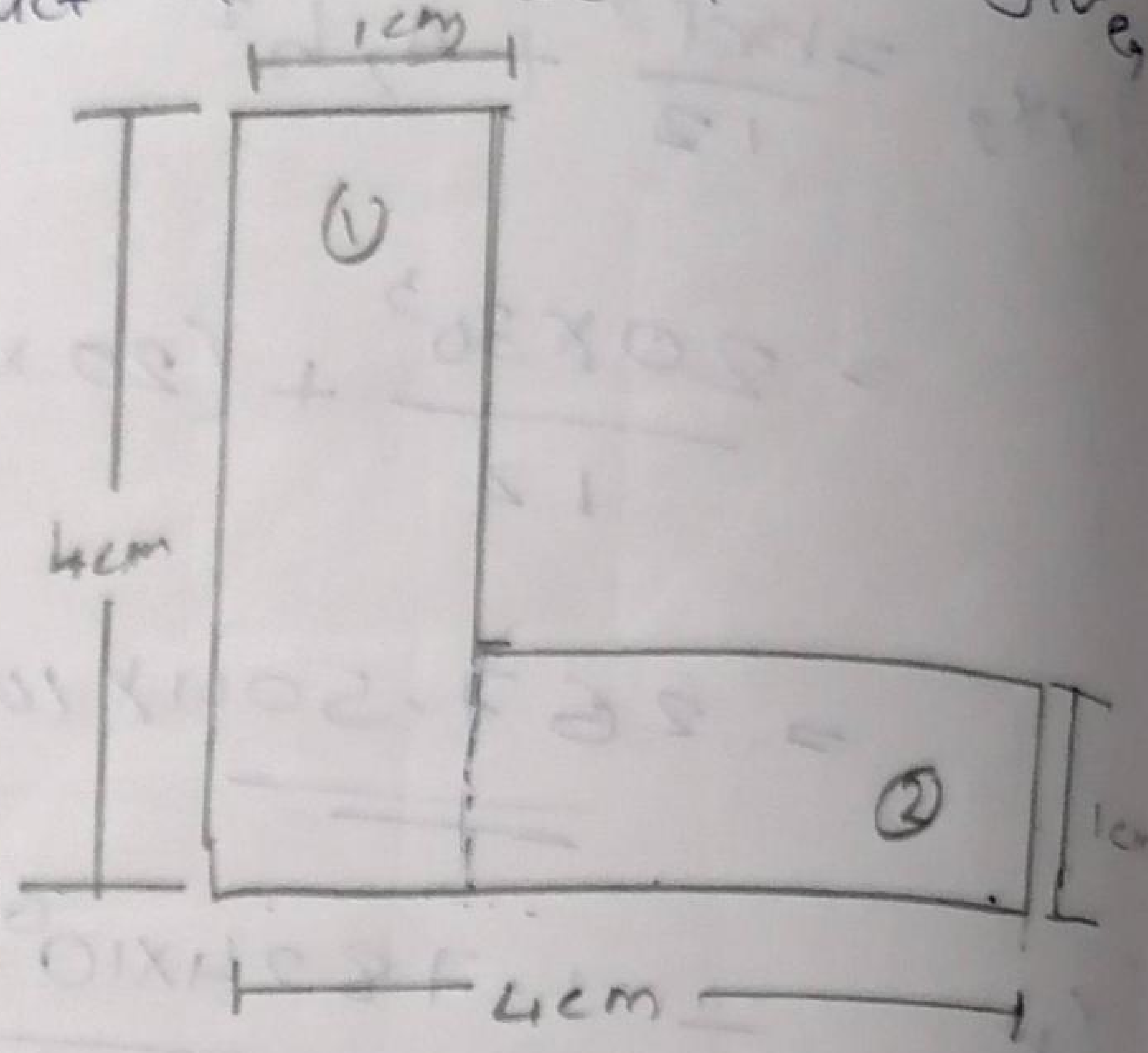
~~[illegible]~~ I_{xy} may ~~[illegible]~~ be +ve, -ve, or zero when one or both ~~the~~ axis x and y are symmetric the product of inertia of that area will be zero.





$$\boxed{I_{xy} = I_{Gxy} + \bar{x}\bar{y}A}$$

1 determine the product of inertia of the given figure.



$$I_{xy} = I_{xy_1} + I_{xy_2}$$

$$I_{xy_1} = I_{Gxy_1} + A_1 x_1 y_1$$

$$= 0 + (4\times1)\times(0.5)(2)$$

$$= \underline{\underline{4}}$$

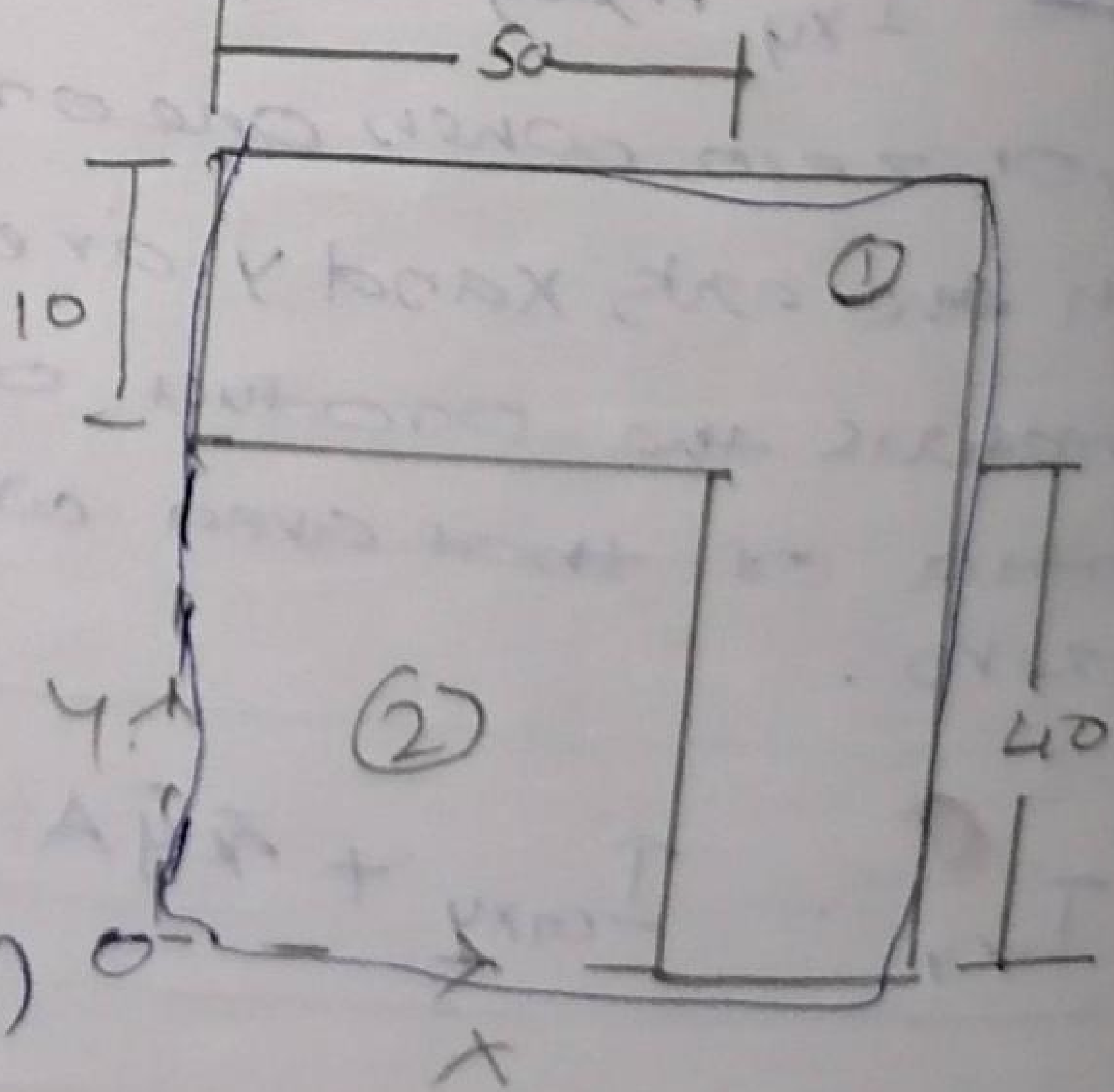
$$I_{xy_2} = I_{Gxy_2} + A_2 x_2 y_2$$

$$= 0 + (3\times1)\times\left(\frac{3}{2}+1\right)(0.5)$$

$$= \underline{\underline{3.75}}$$

$$I_{xy} = 7.75\ cm^4$$

?



$$I_{xy} = I_{xy_1} - I_{xy_2}$$

$$I_{xy_1} = I_{Gxy_1} + A_1 x_1 y_1$$

$$= 0 + (60\times50)(30)(25)$$

$$= 2.25\times10^6$$

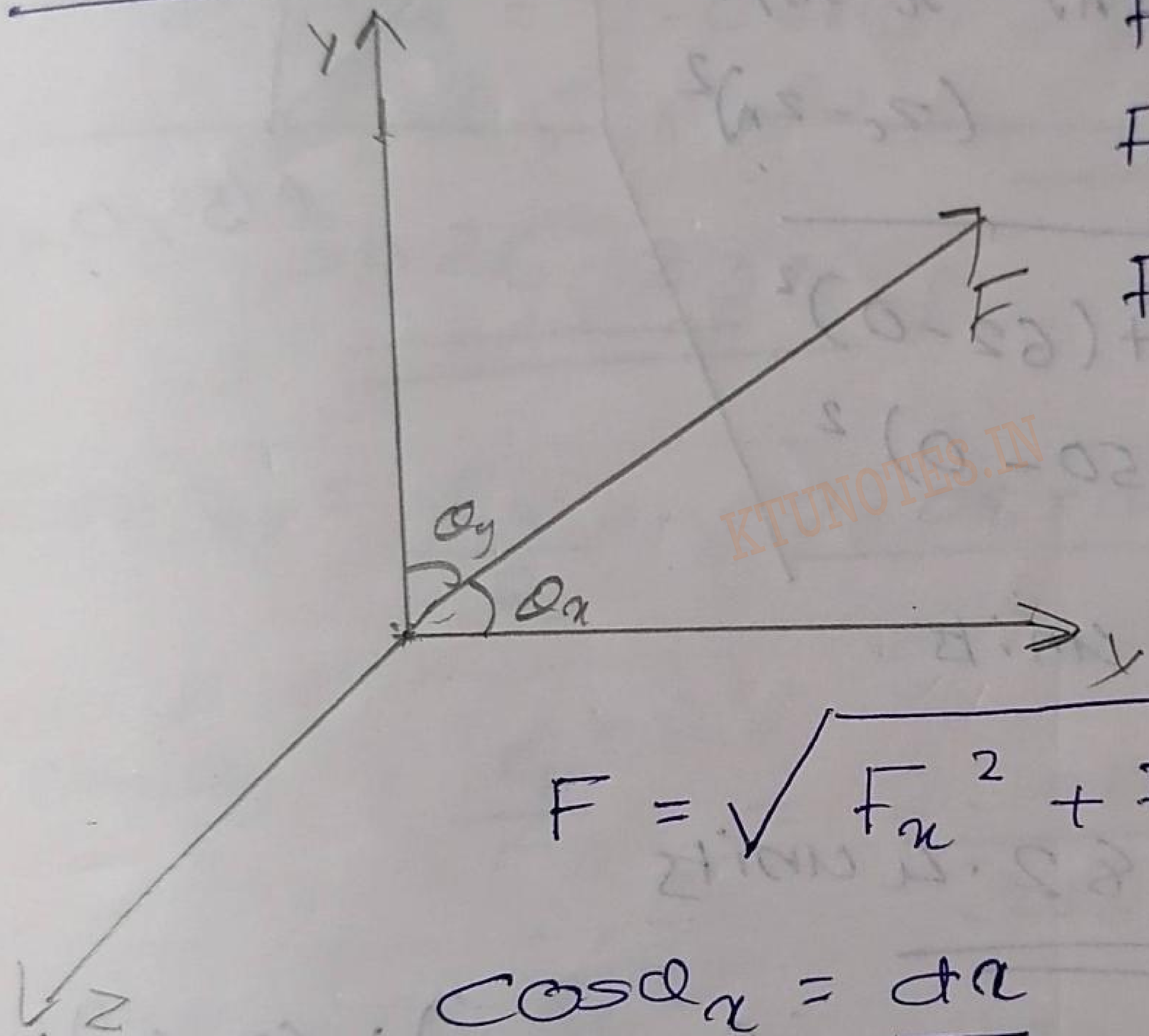
$I_{xy_2} = I_{Gxy_2} + A_2 x_2 y_1$

$= 0 + (40\times30)(25)(20)$

$= 1\times10^6$

$I_{xy} = 3.25\times10^6$

Forces in space.



$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

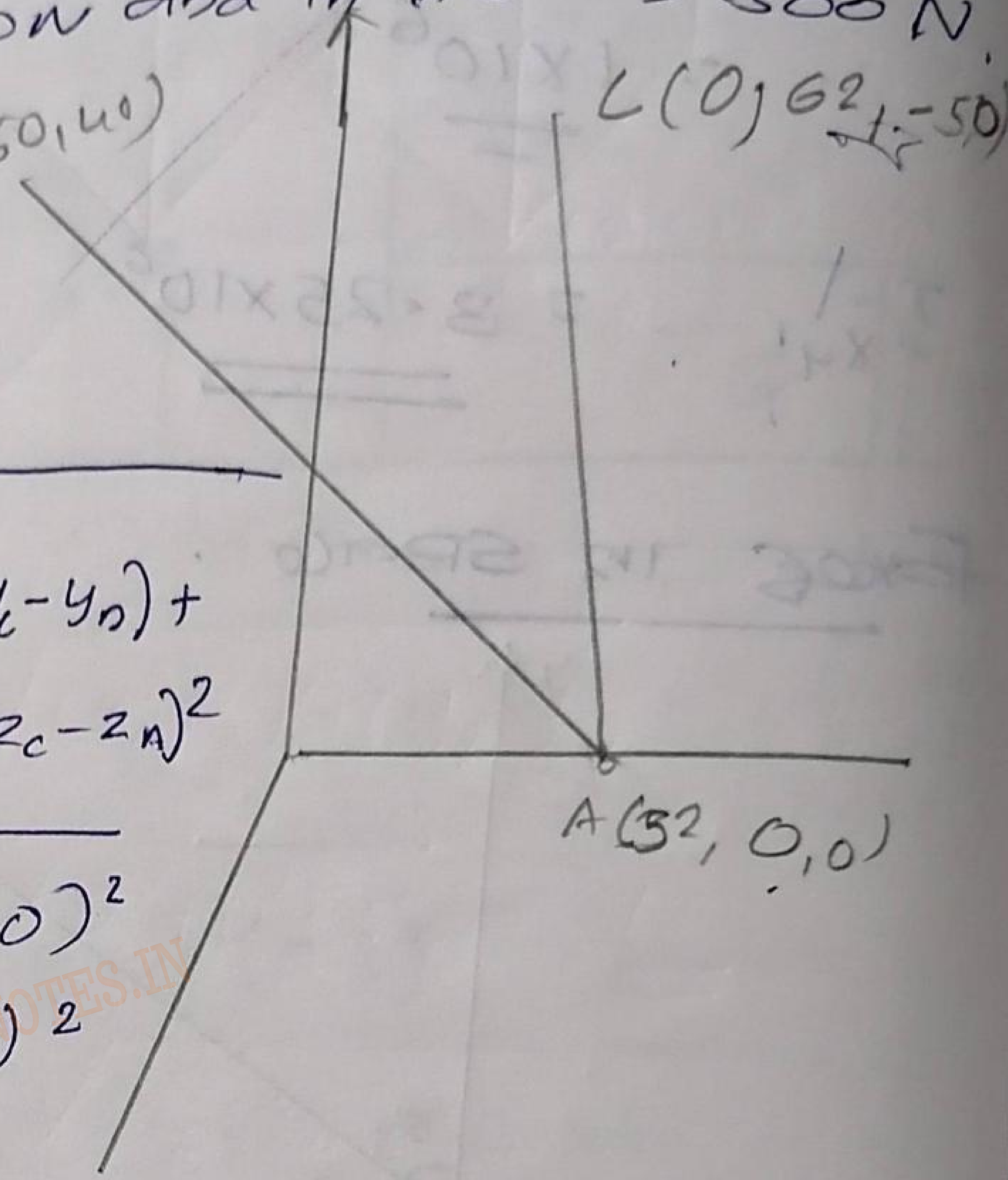
$\cos\theta_x = \frac{dx}{d}$

$\cos\theta_y = \frac{dy}{d}$

$\cos\theta_z = \frac{dz}{d}$

$$\boxed{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$$

1 Two cables AB and AC are attached ~~at A~~ as shown in figure. determine the resultant of force exerted by A : the two cables. tension in AB is 2000N and in AC is 1500N.



length of AC = $\sqrt{(x_c-x_A)^2+(y_c-y_A)+(z_c-z_A)^2}$

$=\sqrt{(0-52)^2+(62-0)^2+(-50-0)^2}$

$=95.12$ units

length of AB = 82.4 units

unit vector along AC $= \dfrac{(x_c-x_A)i+(y_c-y_A)j+(z_c-z_A)k}{95.12}$

$= \dfrac{-52i+62j-50k}{95.12}$

unit vector along AB $= \dfrac{-52i+50j+40k}{82.4 \text{ units}}$

Force vector of AC = $\dfrac{-52i + 62j - 50k}{95.12} \times 1500$

" AB = $\dfrac{-52i + 50j + 40k}{82.4} \times 2000$

$F_{AC} = (-820.01i + 977.71j - 788.4k)$

$F_{AB} = (-1260.75i + 1213.59j + 970.87k)$

$R = F_{AC} + F_{AB} = -2.08 \times 10^3 i + 2.1913 \times 10^3 j + 182.47k$

$|R| = 3026.23N$

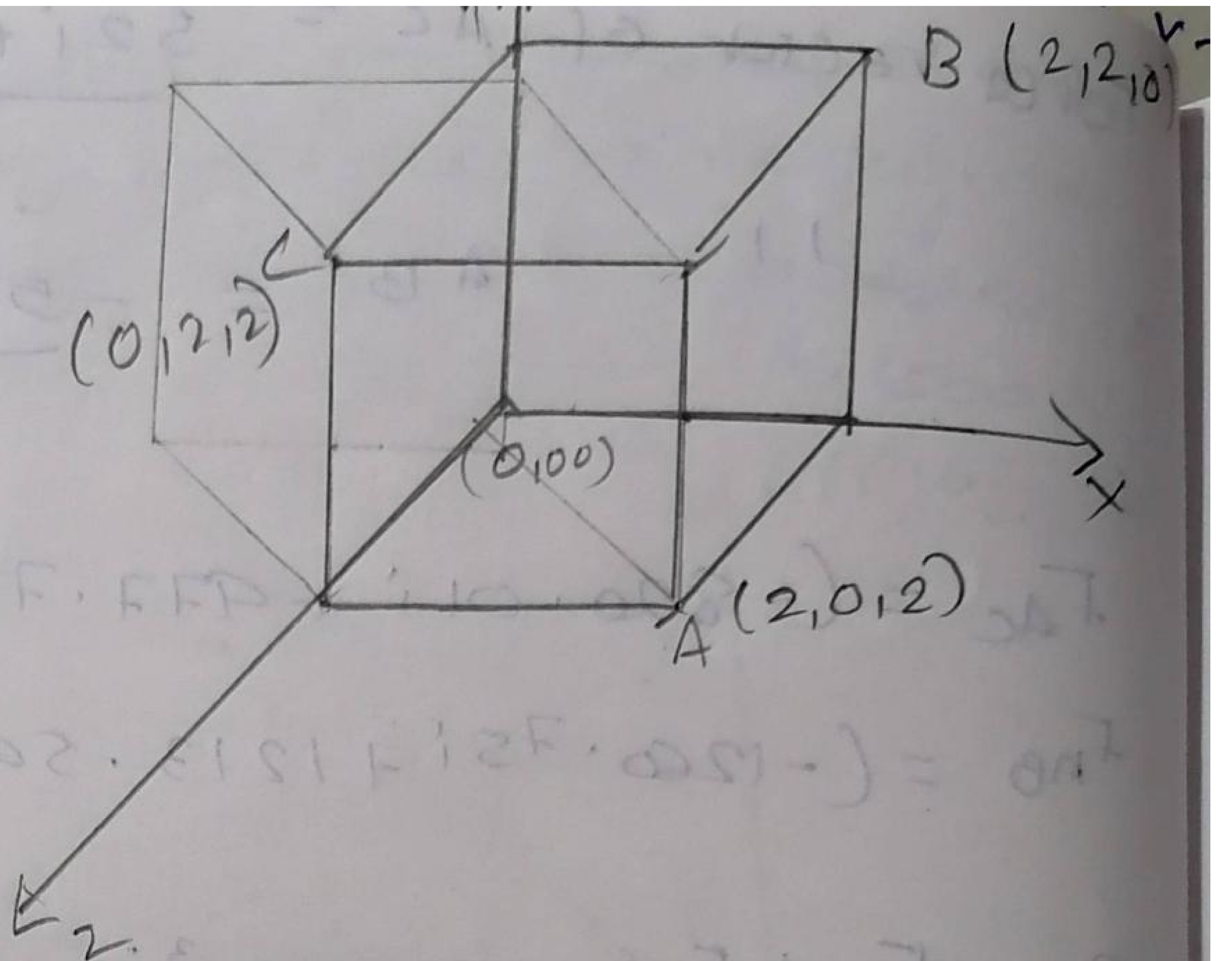
$\cos\theta_x = \dfrac{R_x}{R} = \dfrac{-2.08 \times 10^3 i}{3026.23}$

$\cos\theta_y = \dfrac{R_y}{R} = \dfrac{2.1913 \times 10^3 j}{3026.23}$

$\cos\theta_z = \dfrac{R_z}{R} = \dfrac{182.47k}{3026.23}$

2) 3 forces, 500N, 700N & 800N are acting along 3 diagonals of adjacent faces of cube of the side 2m as shown in figure determine the resultant forces

O(0,0,0)
A(2,0,2)
B(2,2,0)
C(0,2,2)



length of OA $= \sqrt{(x_A - x_o)^2 + (y_A - y_o)^2 + (z_A - y_o)^2}$

$= \sqrt{2^2 + 0^2 + 2^2} = \sqrt{8}$

length of OB $= \sqrt{(x_B - x_o)^2 + (y_B - y_o)^2 + (z_B - z_o)^2}$

$= \sqrt{4+4+0} = \sqrt{8}$

III^{rly} length OC $= \sqrt{8}$

unit vector along OA $= (x_A - x_o) i + (y_A - y_o) j + (z_A - z_o)$

$= \dfrac{2i + 2k}{\sqrt{8}}$

unit vector along OB $= \dfrac{2i + 2j}{\sqrt{8}}$

unit vector along OC $= \dfrac{2j + 2k}{\sqrt{8}}$

Force vector of $OA = \frac{2i+2k}{\sqrt{8}} \times 500$

" $OB = \frac{2i+2j}{\sqrt{8}} \times 700$

" $OC = \frac{2j+2k}{\sqrt{8}} \times 800$

$F_{OA} = 353.55i + 353.55k$

$F_{OB} = 494.97i + 494.97j$

$F_{OC} = 565.68j + 565.68k$

$R = F_{OA} + F_{OB} + F_{OC} = 848.52i + 1.06\times10^3 j + 919.23k$

$|R| = 1639.68\ N$

$\cos\theta_x = \frac{R_x}{|R|} = \frac{848.52}{1639.68}$; $\theta_x = 58.8^\circ$

$\cos\theta_y = \frac{R_y}{|R|} = \frac{1.06\times10^3}{1639.38}$; $\theta_y = 49.71^\circ$

$\cos\theta_z = \frac{R_z}{|R|} = \frac{919.23}{1639.38}$; $\theta_z = 55.89$