EST 130: BASICS OF ELECTRICAL AND ELECTRONICS ENGINEERING

MODULE - 3

AC circuits:

The path for the flow of alternating current is called an AC circuit. This alternating current is ~~called~~ ~~an AC~~ used for domestic and industrial purposes. In an AC circuit, the value of the magnitude and the direction of current and voltage is not constant, it changes at regular interval of time.

* Phasor representation of sinusoidal quantities.

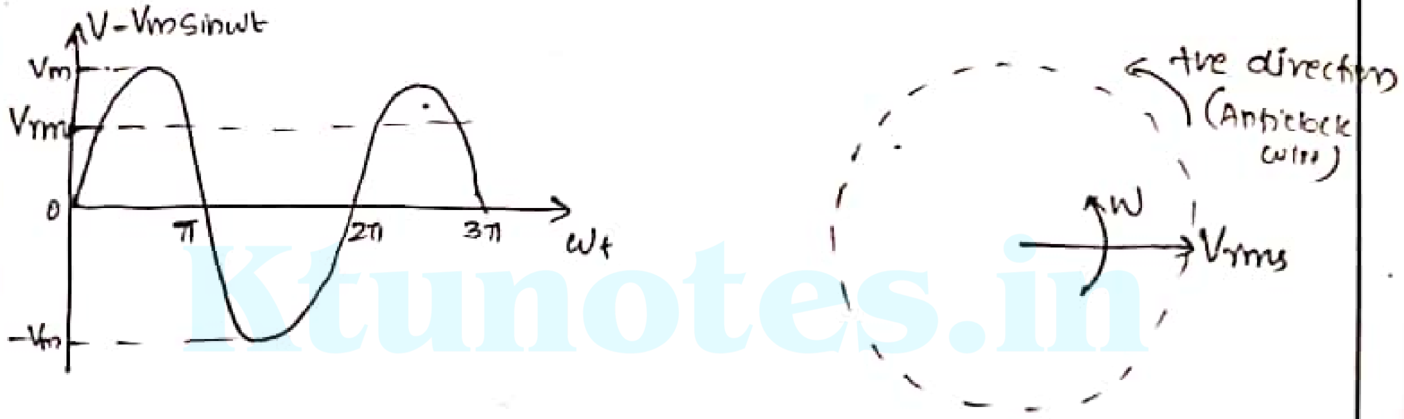
↳ A simple and more direct method of dealing with sinusoidal quantities is to use the phasor representation. It is easier compared to plotting ac waveforms or analytical method using trigonometric identities.

" A phasor looks like a vector and it continuously rotates at system angular frequency (ω rad/sec) in positive anticlock wise direction "

↳ Phasor diagrams are representing alternating current and voltage of the same frequency as vectors or phasors with the phase angle between them.

↳ The length of the phasor is usually the RMS value of the alternating quantity.

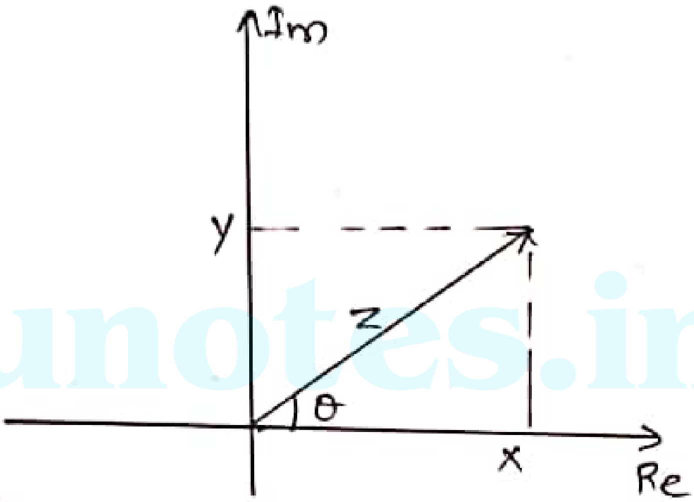
Eg: Voltage waveform and its Vector.



↳ One complete rotation of a phasor (360° rotation) is equivalent to one full cycle of the corresponding waveform.

* Different forms of phasor representation

1. Rectangular form
2. Trignometric Form
3. Polar Form
4. Exponential form



1. Retangular Form

In rectangular form a phasor is represented as,

$Z = x \pm jy$

the magnitude of the phaser is, $Z = \sqrt{x^2+y^2}$

and angle $\theta = \tan^{-1}\frac{y}{x}$

2. Trignometric Form

In trignometric form a phasor is represented by,

$Z = r(\cos\theta \pm j\sin\theta)$

where $r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1} \frac{y}{x}$

3. Polar Form

In polar form a phasor is represented by,

$Z = r\angle\theta$

where $r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

4. Exponential form

Here a phasor is represented by,

$Z = re^{j\theta}$

where $r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

* Phasor Addition and Substraction

Addition and Substraction of complex numbers can be performed conveniently only when both numbers are in the rectangular form.

Let take two complex numbers.

$$z_1 = x_1 + jy_1 \quad , \quad z_2 = x_2 + jy_2$$

i) Addition: $z = z_1 + z_2$

$$= (x_1 + jy_1) + (x_2 + jy_2)$$

$$= (x_1 + x_2) + j(y_1 + y_2)$$

The magnitude of $z = |z| = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$

$$\theta = \tan^{-1}\left(\frac{y_1 + y_2}{x_1 + x_2}\right)$$

Eg: $V_1 = 2 + 3j$ and $V_2 = 1 + 2j$

$$V_3 = V_1 + V_2 = (2+1) + (2+3)j = \underline{3 + 5j}$$

(ii) Substraction: $z = z_1 - z_2 = (x_1 + jy_1) - (x_2 + jy_2)$

$$= (x_1 - x_2) + j(y_1 - y_2)$$

$$|z| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\theta = \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

Eg: $V_1 = 4 + 3j$ and $V_2 = 2 + 2j$

$$V_3 = V_1 - V_2 = (4-2) + j(3-2) = \underline{2 + j}$$

Multiplication and Division of Phasors

Multiplication and division can be easily done if they are in polar form

Multiplication

If $z_1 = r_1 \angle \theta_1$ and $z_2 = r_1 \angle \theta_2$

then $z_1 z_2 = r_1 r_2 \angle \theta_1 + \theta_2$

Eg: $V_1 = 2\angle 30^\circ$ and $V_2 = 6\angle 20^\circ$

$V_3 = V_1 \cdot V_2 = 2 \times 6 \angle (30+20) = \underline{\underline{12\angle 50^\circ}}$.

$\cancel{\frac{z_1}{z_2}} =$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

Eg: $V_1 = 6\angle 30^\circ$ and $V_2 = 2\angle 10^\circ$

$$V_3 = \frac{V_1}{V_2} = \frac{6}{2} \angle (30^\circ - 10^\circ) = \underline{\underline{3\angle 20^\circ}}\ V$$

Numericals

1. Express the following in polar form

(a) $1+j\sqrt{3}$ (b) $4-j5$

Ans: (a) $1+j\sqrt{3}$

here $x=1$, $y=\sqrt{3}$

$r=\sqrt{x^2+y^2}=\sqrt{1+3}=2$

$\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)=60^\circ$

in polar form $= r\angle\theta = 2\angle 60^\circ$

(b) $4-j5$

here $x=4$, $y=-5$

$r=\sqrt{4^2+(-5)^2}=\sqrt{16+25}=\sqrt{41}$

$\theta=\tan^{-1}\left(\frac{-5}{4}\right)=308^\circ 40'$

polar form is $r\angle\theta=\sqrt{41}\angle 308^\circ 40'$

2. Express the following in the rectangular form

(a) $10\angle 3.5^\circ$ (b) $450\angle 94^\circ$

Ans: (a) $10\angle 3.5^\circ = 10\ \cos(3.5^\circ + j\sin 3.5^\circ)$

$= 10(0.998 + j0.06)$

$= 9.98 + j0.61$

(b) $450\angle 94^\circ = 450(\cos 94^\circ + j 94^\circ)$

$= 450(-0.698 + j0.998)$

$= 31.4 + j449.1$

3. Consider voltage phasors $\vec{V_1} = 2+j3$ and $\vec{V_2} = 1-j4$ represent both voltages in complex plane and find

(i) Magnitude of both voltages

(ii) Sum of $\vec{V_1}$ and $\vec{V_2}$

(iii) Product of $\vec{V_1}$ and $\vec{V_2}$

Ans: (i) $\vec{V_1} = 2+j3\ V$

$\vec{V_2} = 1-j4\ V$

(ii) Sum $\vec{V_1} + \vec{V_2} = (2+j3)+(1-j4) = \underline{3-j\ V}$

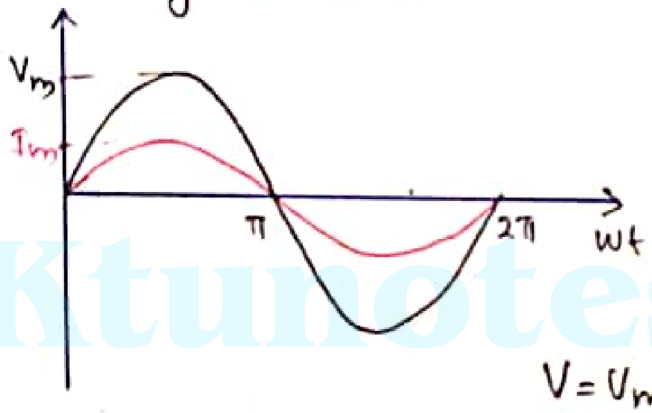
(iii) Product $\vec{V_1} \cdot \vec{V_2} = (\sqrt{13}\angle 56.3^\circ)(\sqrt{17}\angle -75.9^\circ)$

$$\vec{V_1} \cdot \vec{V_2} = \sqrt{221} \angle (56.3^\circ + -75.9^\circ)$$

$$= \sqrt{221} \angle -19.6^\circ \text{ V}$$

* Concept of phase difference between ac quantities

Phase difference is the angular displacement between two alternating quantities



$V = V_m \sin \omega t$ $\quad \vec{V} = V_m \angle 0$

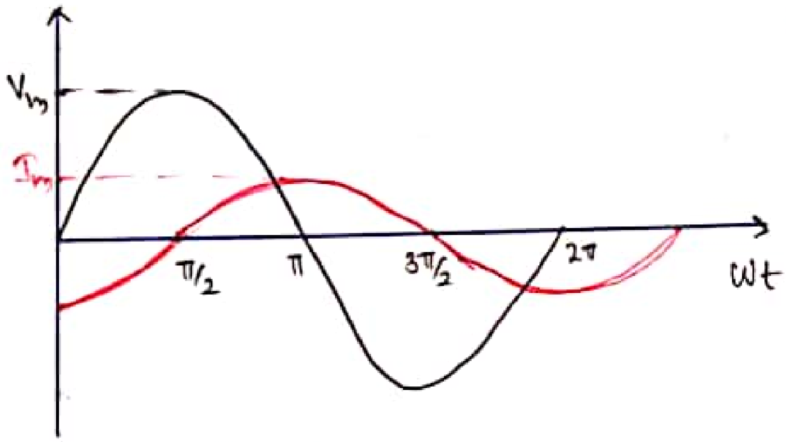
$I = I_m \sin \omega t$ $\quad \vec{I} = I_m \angle 0$

↳ If two quantities are said to be in phase with each other if they reaches maximum value or zero cross over point at the same time.

∴ Phase angle difference between $\vec{V}$ and $\vec{I} = 0^\circ$

↳ One alternating quantity leads another quantity if it reaches maximum value or zero cross over point earlier than the other quantity.

↳ A lagging quantity is one which reach its maximum value or zero cross over point later than other quantity.

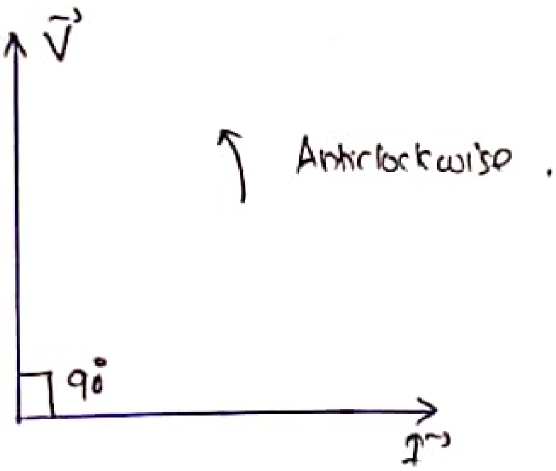


↳ Voltage touches zero cross over point at $\omega t = 0$

↳ Current reaches zero cross over point at $\omega t = \pi/2$ rad

↳ Current lags the voltage by an angle $\pi/2$ rad or (90°)

↳ Voltage leads the current by an angle $\pi/2$ rad.



$V_P = V_m \sin(\omega t + 90^\circ)$ $\quad\quad \vec{V} = V_m \angle 90^\circ$

$I = I_m \sin \omega t$ $\quad\quad \vec{I} = I_m \angle 0^\circ$

* **Power in an AC circuit**

Electric power is defined as the rate at which electrical energy is consumed in an electrical circuit

Instantaneous Power:

It is the power dissipated by the body at a given instant of time.

Positive Power :→ Power flows from source to load

$$S \xrightarrow{+} L$$

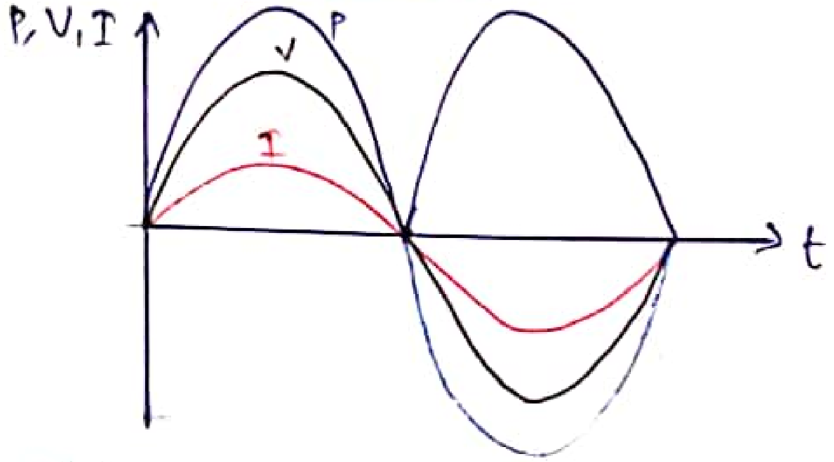
Negative Power :→ Power flows from load to source

$$S \xleftarrow{-} L$$

↳ An AC circuit contains resistors inductors and capacitors, as a result the current will either lag or lead the voltage by a certain angle called, phase angle ϕ. This phase difference should be considered while calculating power

AC power
- → Active power / True power / real power (P)
- → Reactive power / Imaginary power / wattless power / (Q)
- → Apparent power / total power / (S)

(i) Active power (P)



↳ It is the actual or real power consumed by an ac circuit

↳ Unit is watts

↳ Real power is consumed by resistor only

$\therefore P = I^2R$ watts

↳ It is the product of voltage and in phase component of current

$P = VI\cos\phi$ watts

↳ V and I are RMS values

↳ ϕ phase angle difference V and I

* Properties of active power

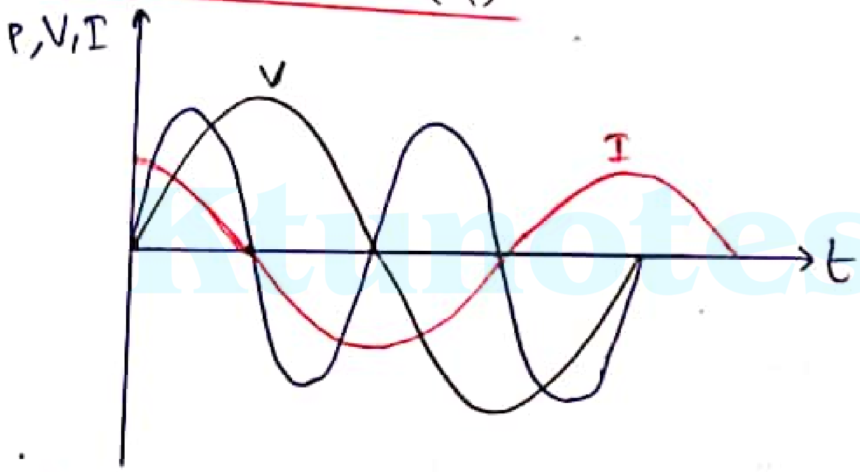
↳ It is always positive

↳ It does not change it's direction

↳ Power flow is always from source to load

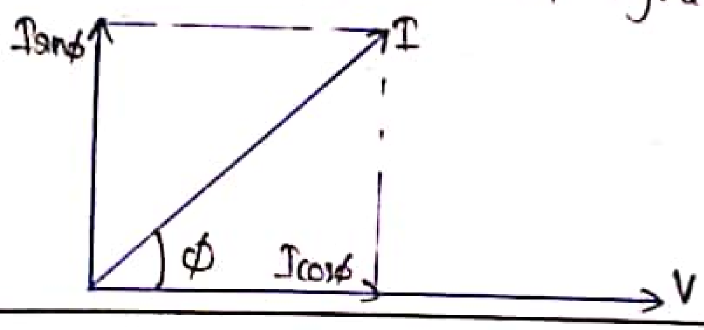
↳ Denoted by letter "P" and measured in watts

(ii) Reactive Power (Q)



↳ Reactive power is associated with reactive components, capacitors and inductors

↳ Reactive power is stored and released by reactive components in every ac half cycle.



↳ It is the product of voltage and reactive component of current

↳ Unit is Volt Ampere Reactive (VAR)

$$Q = VI \sin\phi \quad VAR.$$

* Properties of Reactive Power (Q)

↳ It is positive as well as negative

↳ It is not used for useful work. It only represent the power which goes back and forth without doing any useful work.

(iii) Apparent power

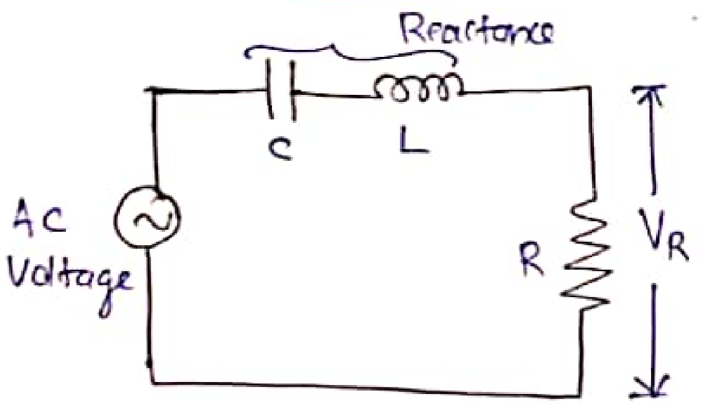
↳ Apparent power is the combination of both active power and reactive power

↳ It is the product of RMS values of current and voltage $\boxed{S = VI}$ VA

↳ Unit is Volt Ampere (VA)

$$\boxed{S^2 = P^2 + Q^2}$$

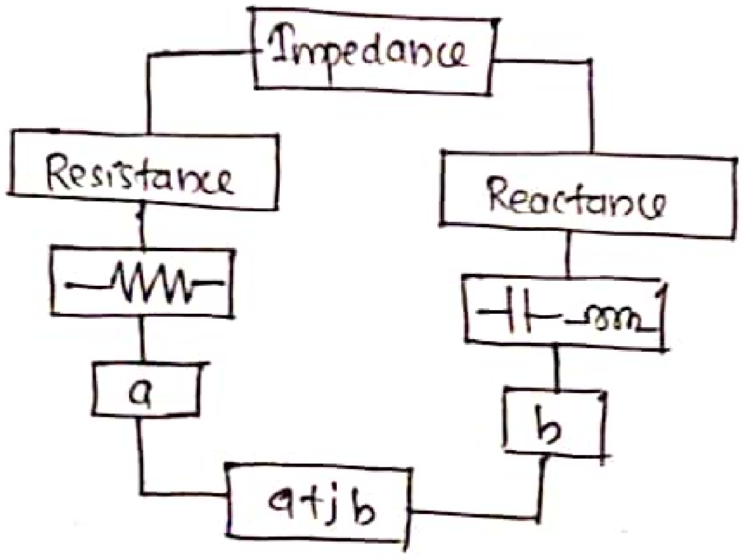
* Concept of Impedance (Z)



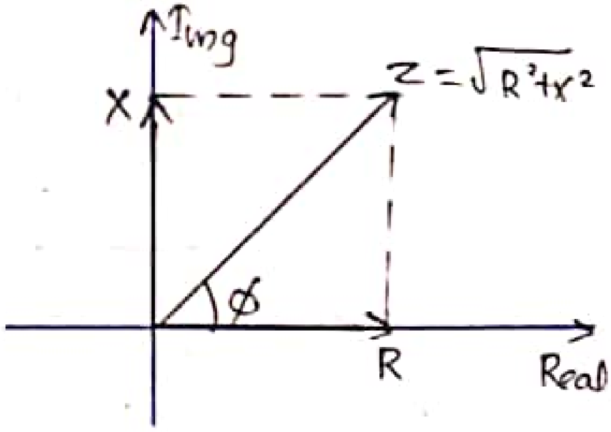
Impedance $Z = \dfrac{\text{Voltage}}{\text{Current}}$

Impedance Z = Resistance + Reactance.

↳ Impedance is the effective resistance of an electric circuit or component to alternating current arising from the combined effect of ohmic resistance and reactances (offered by C and L)



$\boxed{Z = R + jX}$ Ω (unit)

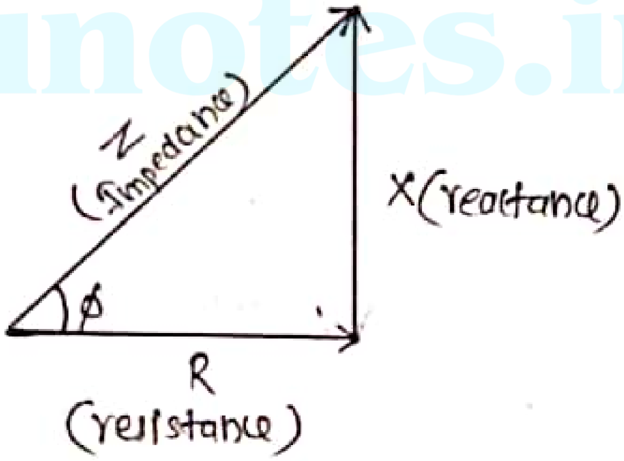


R = Resistance (Ω)

X = Reactance (Ω)

↳ In polar form, $\vec{Z} = |Z| \angle \phi$

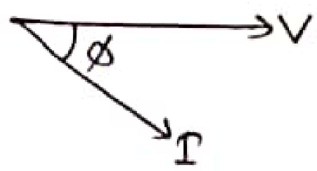
↳ Resistance (R), reactance (X), and Impedance (Z) can be represented as a right angled triangle called Impedance triangle.



* Concept of a power factor

(i) Power factor is the cosine of the angle between voltage and current in the given ac circuit

power factor ⇒ $\boxed{P.f = \cos\phi}$



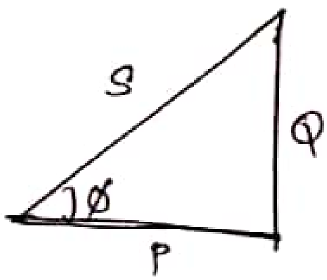
a, If current lags voltage then power factor is "lagging p.f"

b, If current leads voltage then power factor is "leading p.f"

c, If current and voltage are in phase then "Unity P.f"

(ii) Power factor is the ratio of active power to the apparent power in an ac circuit

$$\boxed{P.f = \frac{P}{S} = \frac{VI\cos\phi}{VI} = \cos\phi}$$



Power triangle

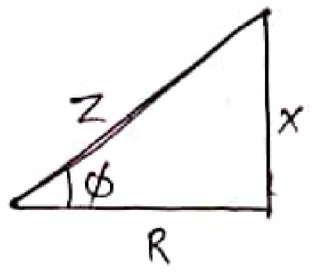
$P = VI\cos\phi$

$Q = VI\sin\phi$

$S = VI$

(iii) Power factor is the ratio of resistance to the impedance, in an ac circuit

$$\boxed{pf = \frac{R}{Z}}$$



Impedance Δle

↳ Value of p.f ranges from 0 to 1

$\cos\phi = 0$ means zero p.f

$\cos\phi = 1$ means unity p.f.

Concept

↳ The total power consumed by an ac circuit has two components

(i) Active power → responsible for doing useful work (Eg: heating an iron box Rotating a motor)

(ii) Reactive power → Stored and released continuously by inductors and capacitors

↳ The value of power factor represents the portion of total power available for useful work.

Eg: If p.f ($\cos\phi$) = 0.6 means 60% of the total power available for useful work. Rest 40% is stored and released by C and L continously (imaginery power)

↳ Practically power factor of an ac electrical system is less than 1

Eg: p.f of motor $\cos\phi = 0.6$

↳ Power factor is less than 1 since motor is an inductive load

↳ Power factor of an incandescent lamp $\simeq 1$ (unity) ($\simeq$ Pure resistance)

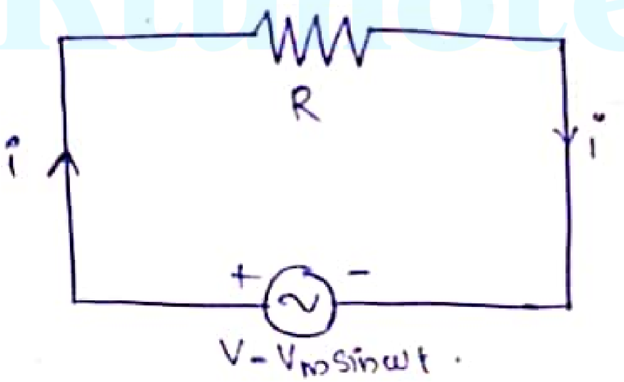
* Analysis of Simple AC Circuits

(i) AC through a pure resistive circuit

In an AC circuit the ratio of voltage to current depends upon the supply frequency, phase angle and phase difference. In an AC resistive circuit, the value of resistance of the resistor will be same irrespective of the supply frequency.

Let the alternating voltage applied across the ckt,

$$V = V_m \sin \omega t \quad \text{—(1)}$$



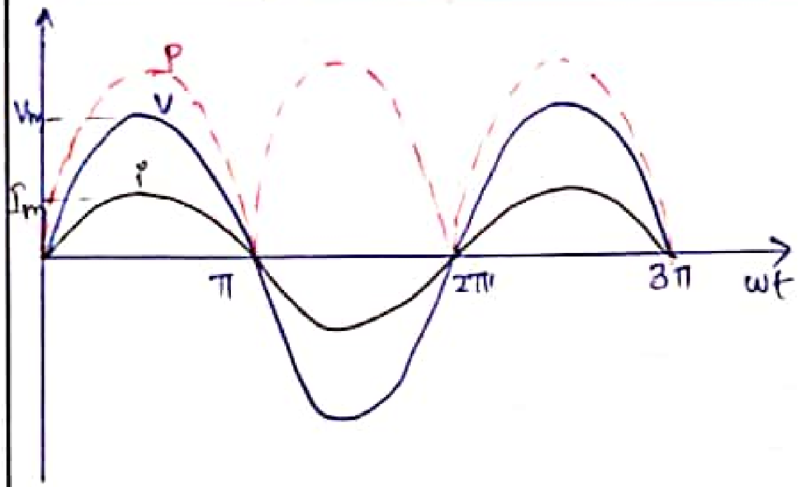
Instantaneous current,

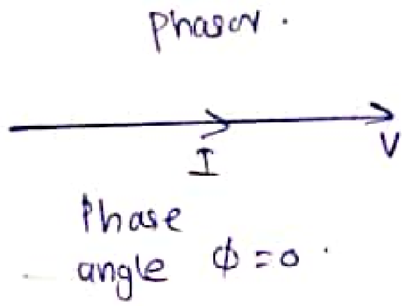
$$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$$

The value of current will be maximum when $\omega t = 90°$

ie, $i = I_m \sin \omega t$ —(2)

Phase angle and waveform of resistive circuit





From equation (1) and (3), it is clear that there is no phase difference between applied voltage and the current flowing through a purely resistive circuit, i.e. Phase angle between voltage and current is zero. Hence in an AC ckt containing pure resistance, current is in phase with the voltage as shown in the above figure.

Instantaneous power $p = (V_m \sin\omega t)(I_m \sin\omega t)$

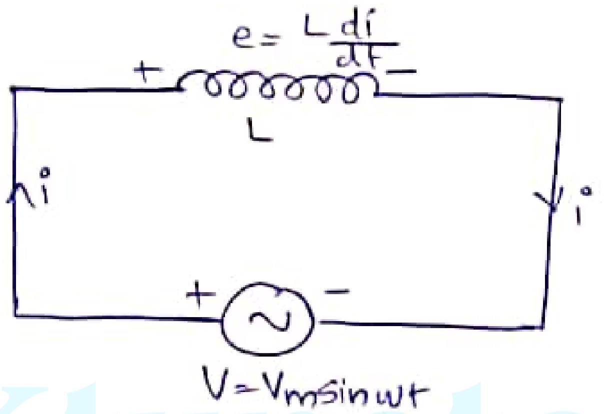
$$p = \frac{V_m I_m}{2} 2\sin^2\omega t = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} (1 - \cos 2\omega t)$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos 2\omega t$$

$$P = V_{rms} \cdot I_{rms}.$$

(ii) AC through a pure inductive circuit

Consider a pure inductive circuit (a choke coil) with inductance of 'L' henry. An ideal or pure coil has zero resistance and capacitance



↳ When an ac current flows through an inductance coil, an alternating magnetic feild is produced. Therefore according to Farraday's Laws of Electromagnetic Induction. a self induced emf 'e' is developed across inductance

$e = -L\frac{di}{dt}$. This opposes the applied voltage 'V'.

The ckt will take some time to come to steady state condition after settling up transient response at the instant the circuit is first switched on.

∴ we have $v = -e = L\frac{di}{dt}$ wher $\boxed{V = V_m \sin \omega t}$

$$i = \frac{1}{L}\int v\,dt = \frac{1}{L}\int V_m \sin\omega t \cdot dt$$

$$i = \frac{V_m}{L}\int \sin\omega t \cdot dt = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{-V_m}{\omega L}\sin(\pi/2 - \omega t) = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

$$\boxed{i = I_m \sin(\omega t - \pi/2)} \quad \text{where } I_m = \frac{V_m}{\omega L}$$

where $I_m = \frac{V_m}{\omega L} = \frac{V_m}{2\pi f L} = \frac{V_m}{X_L}$

The term $\omega L = 2\pi f L$ plays the same role as a resistance in pure resistive ckt and is known as inductive reactance, X_L

ie $\boxed{X_L = \omega L = 2\pi f L}$ ohms.

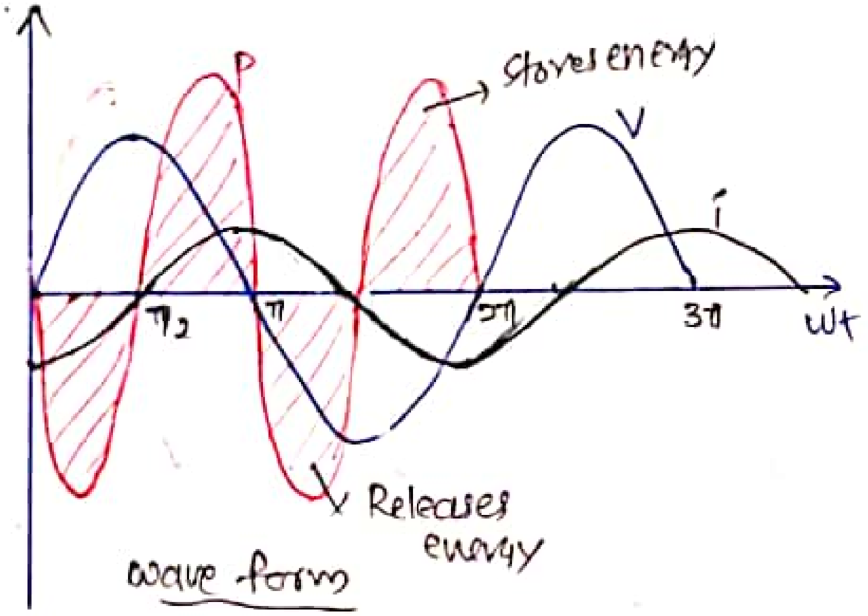
where L = Inductance of coil in Henry

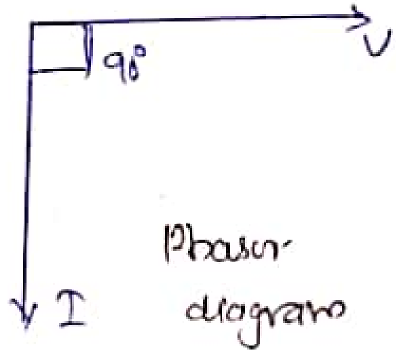
From the equations of V and I we can plot the waveforms

" In one full cycle an inductor absorbs zero average power. Inductor stores energy for one half cycle (10msec) and then releases energy for another half cycle (10msec) "

Energy is temporarily stored in inductor as magnetic energy. Inductor absorbs only reactive power (Q)

ie P (active power) = 0



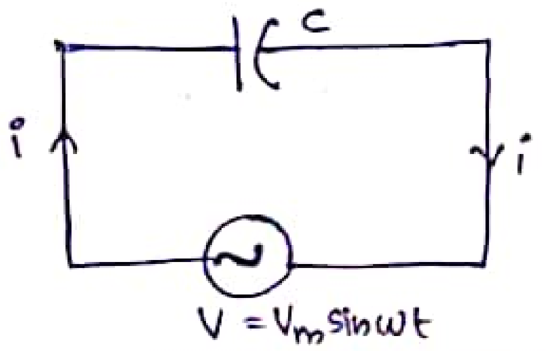


Power factor of a pure inductor $\cos\phi = \cos 90 = 0$ lag

It means an inductor deals with only reactive (Imaginary) power Q.

(iii) AC through a pure capacitor

Consider a pure capacitive ckt as shown in figure below. Let 'c' be capacitance in Henry. An ideal capacitor has zero resistance + zero Inductance



Let the applied ac voltage $V = V_m \sin\omega t$

For a capacitor V-I relation is, $i = C\frac{dV}{dt}$

$$\therefore i = C\frac{d(V_m \sin\omega t)}{dt} = V_m C \frac{d(\sin\omega t)}{dt}$$

$$i = V_m C\omega \cos(\omega t) = (V_m C\omega)\sin(\omega t + \pi/2)$$

$$i = \frac{V_m}{(1/\omega C)}\sin(\omega t + \pi/2) = I_m(\sin\omega t + \pi/2)$$

$$\boxed{i = I_m \sin(\omega t + \pi/2)}$$

where $I_m = \frac{V_m}{(1/\omega C)} = \frac{V_m}{\frac{1}{2\pi f C}} = \frac{V_m}{X_C}$

the term "$\frac{1}{\omega C} = \frac{1}{2\pi f C}$" plays the same role as a resistance in pure resistive ckt and is known as capacitive reactance 'X_C'

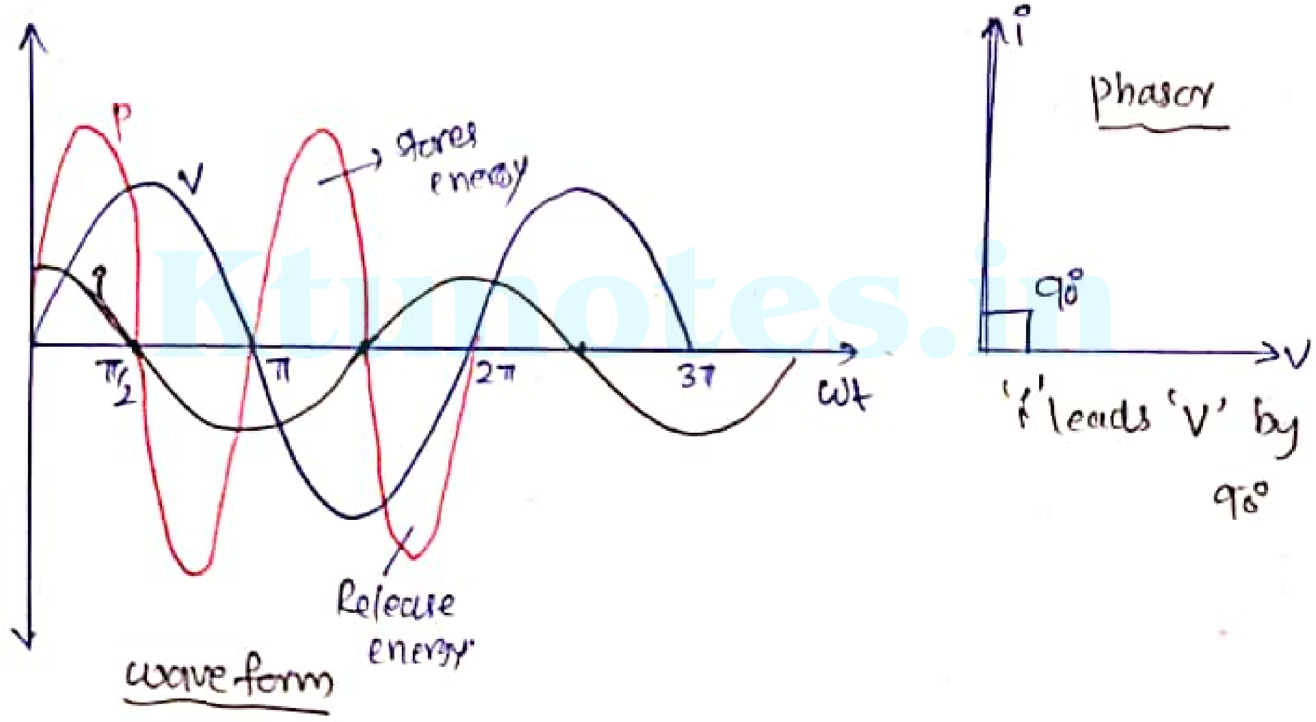
ie, $\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$ ohm

where C = capacitance in Henry

↳ From the equations of V and i we can plot the waveforms. It is clear that 'i' leads 'V' by an angle 90° or $\pi/2$ radian.

"In one full cycle a capacitor absorbs zero average power. It stores energy as electrostatic energy for one half cycle and releases this in another half cycle".

∴ A capacitor also deals with only reactive or imaginary power (Q)



* Numerical Problems

1) A 220V, 50Hz single phase sinusoidal voltage produces a current of 2.2A in a purely inductive coil. Determine (i) Inductive reactance of the coil

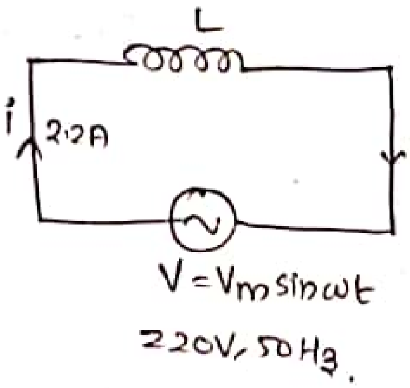
(ii) Inductance , (iii) Power absorbed

(iv) Expression for applied voltage and current

ans: Given $V = 220V$ (rms value)

$i = 2.2A$ (rms value)

$f = 50Hz$.



we have

(i) $Z = X_L = \frac{V}{i} = \frac{220}{2.2} = \underline{\underline{100\,\Omega}}$

$X_L = 100 = 2\pi f L = 2\times\pi\times 50\times L$

(ii) $\Longrightarrow L = \frac{100}{2\pi\times 50} = \underline{\underline{0.318\,H}}$

(iii) Power absorbed (average power), $P = 0$ in pure inductive ckt.

(iv) $V = V_m \sin\omega t = 220\sqrt{2}\times \sin(2\pi\times 50t)$

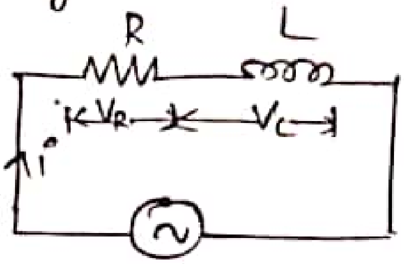
$$\underline{\underline{V = 311.13 \sin(314t) \text{ Volt}}}$$

$i = I_m \sin(\omega t - \pi/2) = 2.2\times\sqrt{2}\times\sin(314t - \pi/2)$

$$\underline{\underline{i = 3.11 \sin(314t - \pi/2)\ A}}$$

* Ac Through Series R-L circuit

Let V = RMS value of applied voltage



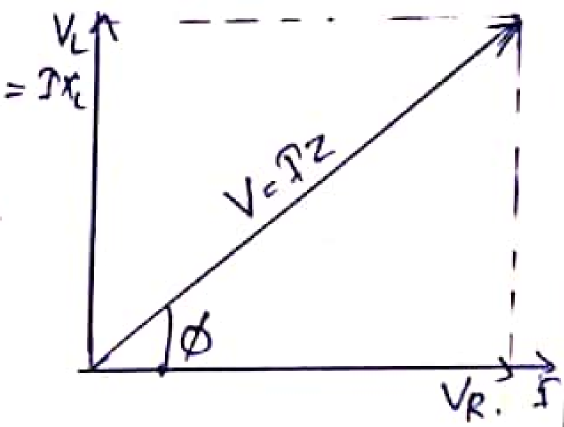
I = RMS value of current

$V_R = IR$ = voltage drop across R (in phase with I)

$V_L = IX_L$ = Voltage drop across L (leads by 90° to I)

where $X_L = 2\pi f L = \omega L$ = Inductive reactance.

↳ Figure shows the representation of two voltages in phasor diagram.



∴ The applied voltage 'V' is the phasor sum of the two voltages.

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

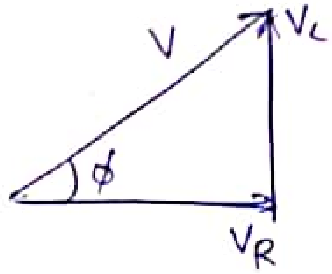
$$= I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z}$$

where, Z = Impedance $= \sqrt{R^2 + X_L^2}$ = opposition offered by series RL circuit to the flow of alternating current.

→ From the phasor diagram, it is clear that, voltage leads the current by an angle ϕ.

$$\therefore \tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}.$$



$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)} = \text{Phase angle}$$

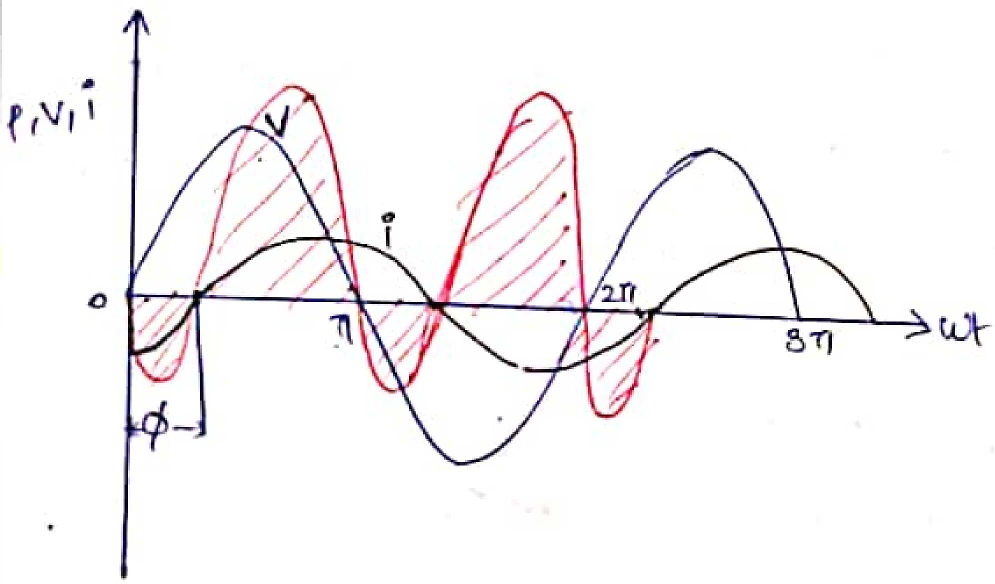
→ If applied voltage is $V = V_m \sin\omega t$, then the current expression will be, $i = I_m \sin(\omega t - \phi)$

→ Instantaneous power, $P = Vi = (V_m \sin\omega t)(I_m \sin(\omega t - \phi))$

$$\Rightarrow P = \frac{V_m I_m}{2}\, 2\sin\omega t \cdot \sin(\omega t - \phi) \qquad \left[\because 2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)\right]$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$= V_{rms} \cdot I_{rms} \cdot \cos\phi - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos(2\omega t - \phi)$$



↳ Avg. power consumed is,

$$P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi + 0 \Rightarrow P = V_{rms} \cdot I_{rms} \cdot \cos\phi$$

$$\Rightarrow \boxed{P = VI\cos\phi}$$

where $\cos\phi$ = power factor of the circuit

↳ From phasor diagram, $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$

↳ Power factor is defined as, the cosine of the angle b/w voltage and current in ~~an~~ ac circuit.

↳ In this circuit, only power is actually consumed in resistance only, Inductance does not consume any power $P = VI\cos\phi = (IZ) \cdot I\left(\frac{R}{Z}\right) = I^2R$

Numerical

A coil having a resistance of 10Ω and an inductance of 0.2H is connected across a 200V, 50Hz supply. Calculate (i) reactance and impedance of the coil. (ii) current, (iii) phase difference b/w the current and applied voltage (iv) power factor. Also draw the phasor diagram showing V and I.

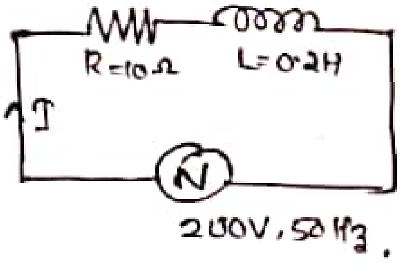
Ans: (i) Reactance $X_L = 2\pi f L = 2 \cdot 314 \times 50 \times 0.2$

$= \underline{62.8\ \Omega}$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (62.8)^2}$

$= \underline{63.59\ \Omega}$

(ii) current, $I = \frac{200}{63.59} = \underline{3.146A}$



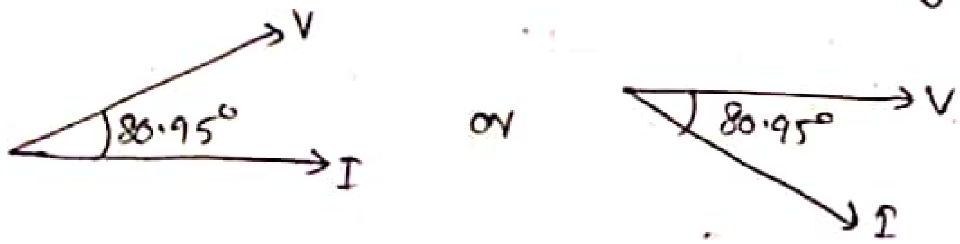
(iii) phase difference

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 80.95°$

(iv) Power factor, $\cos\phi = 0.157$ lagging

V) Phasor diagram:-

Current lags behind voltage by an angle 80·95°.



2. The current in series circuit $R = 8\Omega$ and $L = 40mH$ lags the applied voltage by 70°. Determine the source frequency and the impedance Z.

Ans: $R = 8\Omega$ and $L = 40mH$, $\phi = 70°$

$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}} = \cos 70° = \frac{8^2}{\{8^2 + 2\pi f \times (40 \times 10^{-3})^2\}}$

$\Rightarrow f^2 = 7654.748$

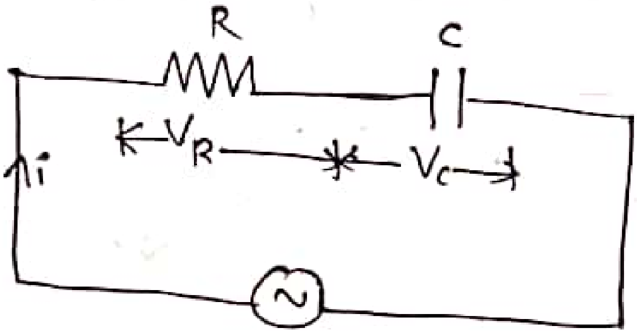
$\therefore f = \underline{87.49 Hz}$

$Z = R + jX_C = (8 + j21.98) = 23.39\angle 70°$

$|Z| = \sqrt{8^2 + (21.98)^2} = 23.39$, $\phi = \tan^{-1}\left(\frac{21.98}{8}\right) = \underline{70°}$.

* AC through Series R-C circuit:

Let an ac supply is given to a series combination of the R and C, for which a current 'i' is drawn by the circuit.

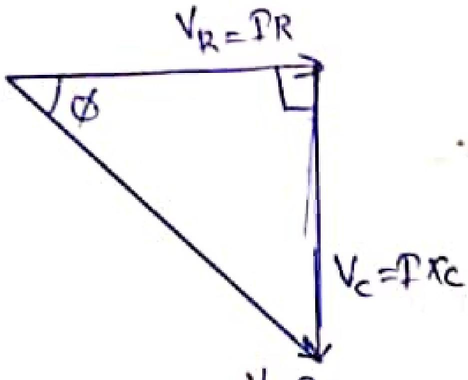


$V = V_m \sin\omega t$.

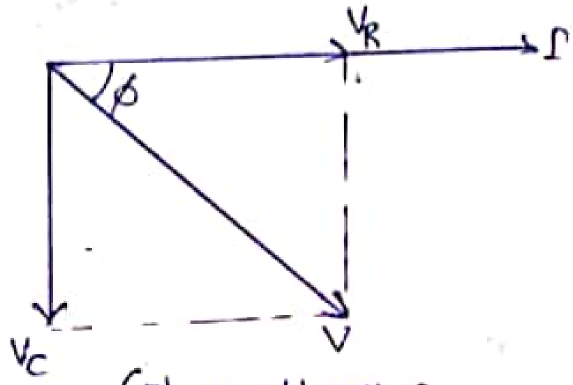
↳ V_R = Voltage drop across resistor = IR

V_C = Drop across capacitor = IX_C

where, X_C = capacitive reactance = $\frac{1}{2\pi fC}$



(Voltage triangle)



(phasor diagram)

↳ From the phasor diagram it is clear that the voltage lags current by an angle ϕ.

↳ From the voltage triangle,

$$\boxed{V = \sqrt{V_R^2 + V_C^2}} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2} = IZ$$

where $\boxed{Z = \sqrt{R^2 + X_C^2}}$ → Impedance of series R-C ckt.

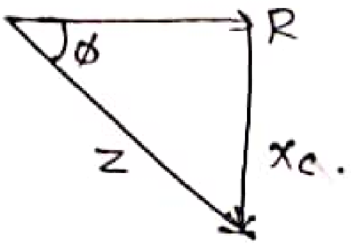
where. $Z = R + jX_C$

↳ Again from triangle, $\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}$

$$= \phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Power factor of R-C series circuit.

$$\boxed{\cos\phi = \frac{R}{Z} = \frac{V_R}{V} \text{ lead.}}$$

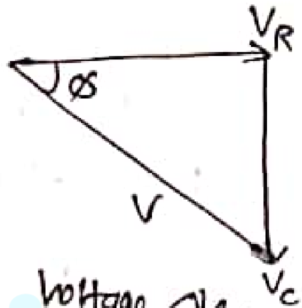


Impedance Δle

Power consumed or active power $P = VI\cos\phi = I^2R$. watt

Reactive power $Q = VI\sin\phi$ VAR

Apparent power $S = VI$ VA



Voltage Δle.

↳ In an RC circuit current 'I' leads to voltage 'V' by an angle ϕ less than $90°$.

* **Numerical**

1, The waveform of the voltage and current of a ckt are given by, $e = 120\sin(314t)$ and $i = 10\sin(314t + \pi/6)$. calculate the values of the resistance, capacitance which are connected in series to form the circuit. Also draw waveforms of current, voltage and phasor diagrams. Calculate power consumed by the circuit.

Ans: $e = 120 \sin(314t)$ comparing with $V = V_m \sin\omega t$

$\therefore V_m = 120V$ and $\omega = 314 \Rightarrow 2\pi f = 314$

$\Rightarrow f = 50Hz$.

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{120}{\sqrt{2}} = 84.55V$

$i = 10 \sin(314t + \pi/6)$ comparing with $i = I_m \sin(\omega t + \phi)$

$\therefore I_m = 10A$, $\phi = \pi/6 = 30°$

$\therefore I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07A$

Now $|Z| = \frac{V}{I} = \frac{89.55}{7.07} = 12\Omega$.

From the expressions V and i, it is clear that, current leads voltage by 30°, so the R-C series ckt is capacitive in nature.

$\therefore Z = |Z| \angle -\phi = 12 \angle -30° = (10.393 - j6)\Omega$

Comparing with $Z = R - jX_c$, we have:-

$\boxed{R = 10.393\Omega}$ and $X_c = 6\Omega$

$\Rightarrow \frac{1}{2\pi fc} = 6$

$\therefore C = \frac{6}{2\pi \times 50} = 530.45\mu F$

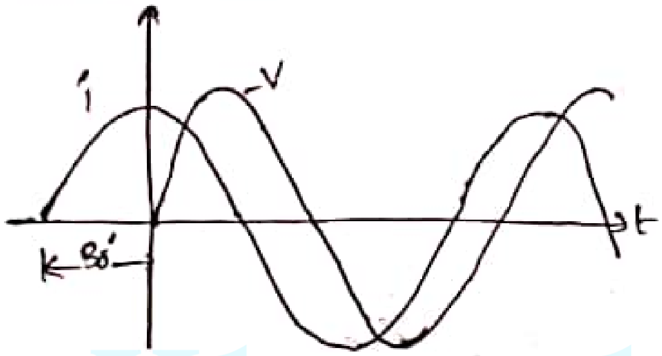
∴ Power consumed by the ckt

$$P = VI\cos\phi$$

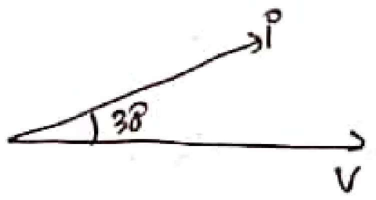
$$\Rightarrow P = 84.85 \times 7.07 \cos 30^\circ$$

$$= 519.52\,W$$

Waveforms are,

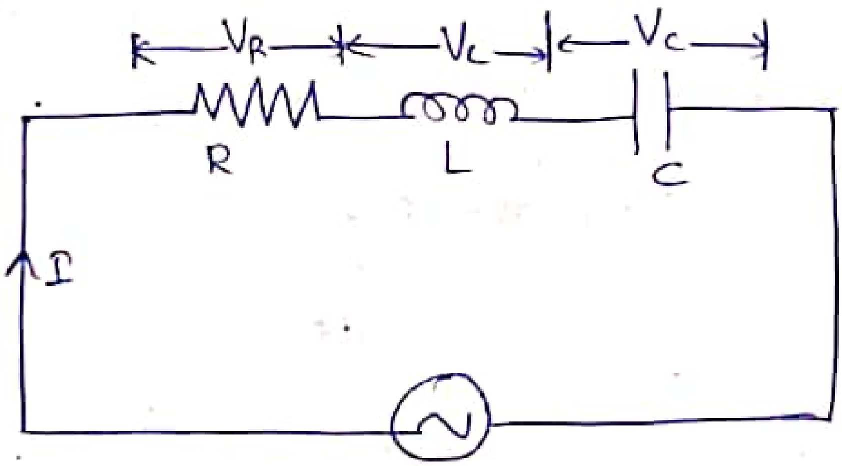


(waveform)



(Phasor diagram)

* AC through Series RLC circuit



$V = V_m \sin\omega t$

↳ Consider a circuit consisting of resistance 'R' and pure inductance 'L' and capacitance 'C', connected in series with each other across ac supply $V = V_m \sin\omega t$

↳ Let V and I are the rms values of voltage and current then,

$V_R = IR \rightarrow$ Drop across R (in phase with I)

$V_L = IX_L \rightarrow$ Drop across 'L' (leading I by 90°)

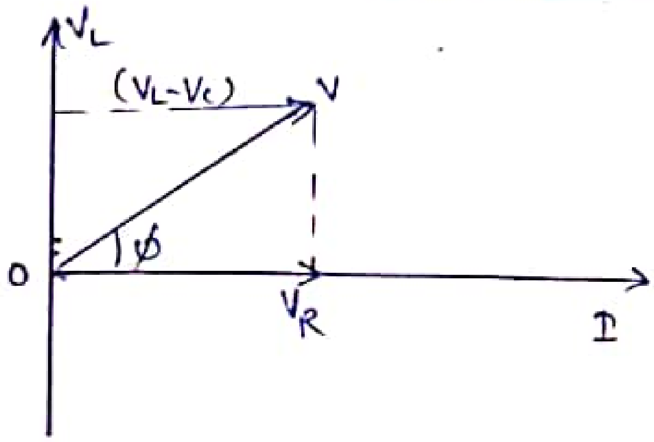
$V_C = IX_C \rightarrow$ Drop across 'C' (lagging I by 90°)

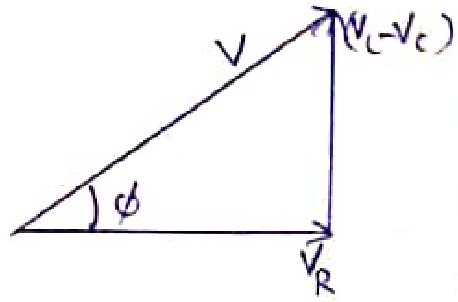
↳ Now the applied voltage can be written as,

$$\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$$

→ The phasor diagrams depends on the magnitudes of V_L and V_C which ultimately depends on X_L and X_C

Case 1: ($X_L > X_C$) (Inductive circuit)





↳ From voltage triangle $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$= \sqrt{(I R)^2 + (I X_L - I X_C)^2}$$

$$\Rightarrow V = I\sqrt{R^2 + (X_L - X_C)^2} \rightarrow V = IZ$$

where $z = \sqrt{R^2 + (X_L - X_C)^2}$

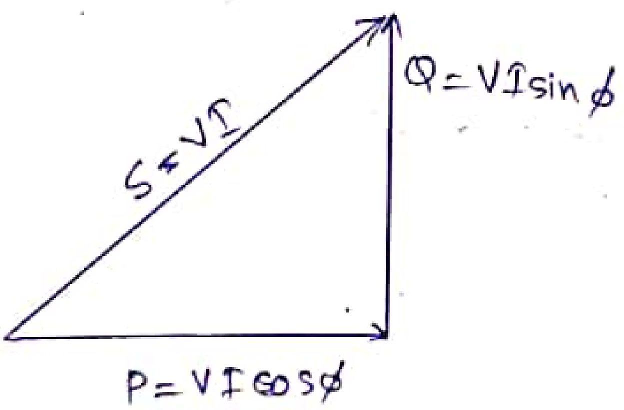
→ In general, the impedance of RLC series ckt is,

$$\boxed{Z = R + jX = R + j(X_L - X_C)} = |Z| \angle \phi$$

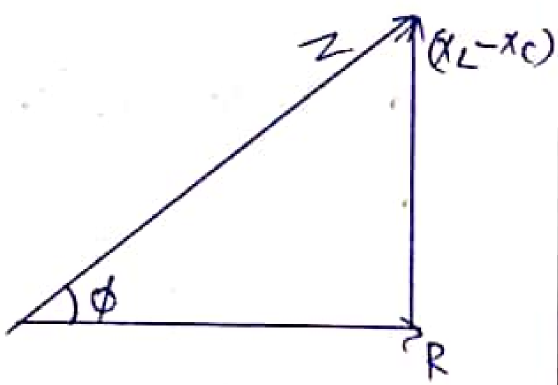
where X = Net reactance of the circuit $= X_L - X_C$

→ Here, $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$ and $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

→ The power factor is given by $\cos\phi = \frac{R}{Z}$



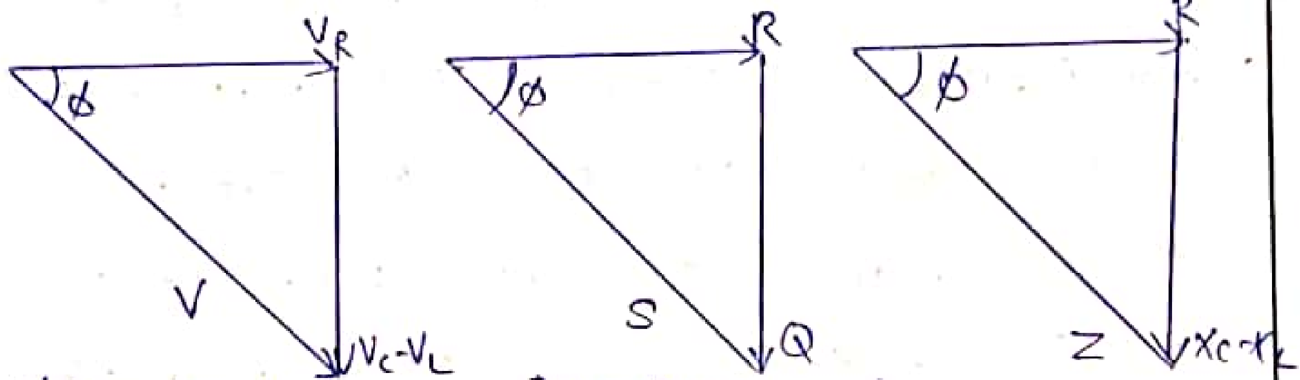
(Power triangle)



(Impedance triangle)

Case - II : ($X_L < X_C$) (capacitive circuit)

↳ The net reactance is negative and hence the angle ϕ is negative



Voltage triangle   Power triangle   Impedance triangle

Case - III ($X_L = X_C$) (Purely resistive circuit) :-

↳ Here the net reactance is zero and the circuit behaves as purely resistive circuit. The impedance will be $\boxed{Z = R}$ and the voltage and current are in phase.

↳ Now, if applied voltage is $V = V_m \sin \omega t$, then the resulting current equation will be,

$$i = I_m \sin(\omega t \pm \phi).$$

Here '+' sign indicates for capacitive circuit ($X_L < X_C$)

'—' sign indicates for inductive circuit ($X_L > X_C$)

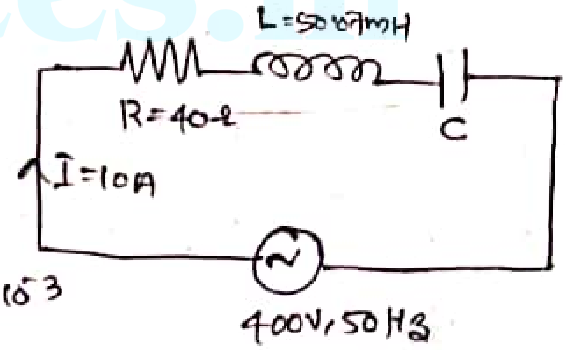
Numerical :

1, A series circuit having pure resistance of 40 Ω, pure inductance of 50·07 mH and a capacitor is connected across a 400V, 50 Hz, AC supply. This RLC combination draws a current of 10A. Calculate (i) Power factor of the circuit and (ii) capacitor value

Ans: $V = 400V$, $f = 50Hz$

$I = 10A$, $R = 40\Omega$

$L = 50{\cdot}07 mH$.



We know that

$X_L = 2\pi f L = 2\times\pi\times50\times50{\cdot}07\times10^{-3}$

$X_L = \underline{15{\cdot}73\,\Omega}$

and $X_C = \frac{1}{2\pi f C}$

Now from given datas, $|Z| = \frac{|V|}{|I|} = \frac{400}{10} = \underline{40\Omega}$.

$\therefore Z = R + j(X_L - X_C) \longrightarrow |Z| = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow 40 = \sqrt{(40)^2 + (15.73 - X_C)^2}$$

$$\Rightarrow 1600 = [1600 + (15.73 - X_C)^2]$$

$$X_C = 15.73\ \Omega \Rightarrow \frac{1}{2\pi f C} = 15.73$$

$$\Rightarrow C = 2.023 \times 10^{-4}\ F$$

$\therefore$ Impedance, $Z = 40 + j(15.73 - 15.73)$

$$= 40 + j0 = 40\angle 0^\circ$$

$\therefore$ Power factor, $\cos\phi = \cos 0^\circ = 1$

* Three phase AC systems

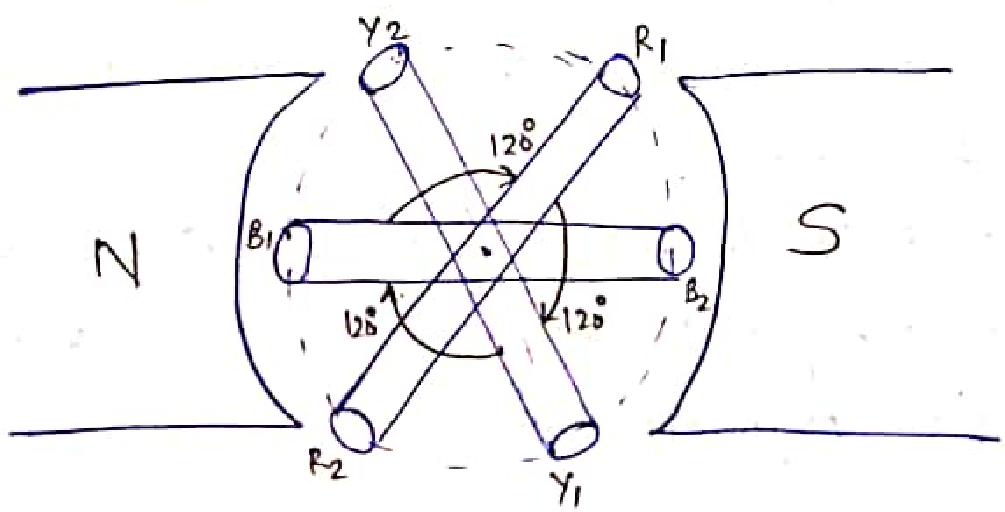
↳ A single phase ac voltage can be generated by rotating a turn made up of two conductors in a magnetic feild. Such a machine is called as single phase alternator.

↳ But practically in many applications, they need many ac voltages simultaneously such types of systems are called as. polyphase system

↳ To develop polyphase system the armature winding in an alternator is devided in to number of phase required.

↳ If armature is devided in to three coil, then three seperate a.c. voltages will be generated having same magnitude and frequency but having phase difference of $360°/3 = 120°$ with respect to each other. Now all three voltages with a phase difference of 120° supply a three phase load. Such a supply system is called three phase system.

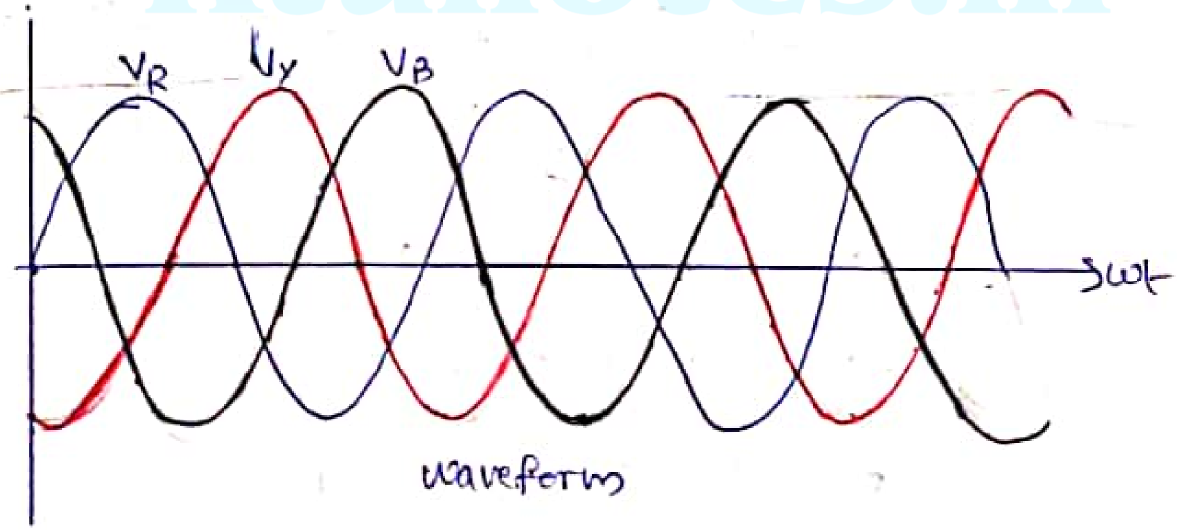
* Generation of three phase voltages.



↳ Three phase voltage is generated by three-phase alternator

↳ The armature of the alternator having coils which are devided in to three groups in such a way so that they are displaced by $120°$ electrical apart from each other.

↳ Let V_R, V_Y and V_B are the three independent voltages induced in coils R_1-R_2, Y_1-Y_2 and B_1-B_2 respectively. All of them are displaced from one another by $120°$.



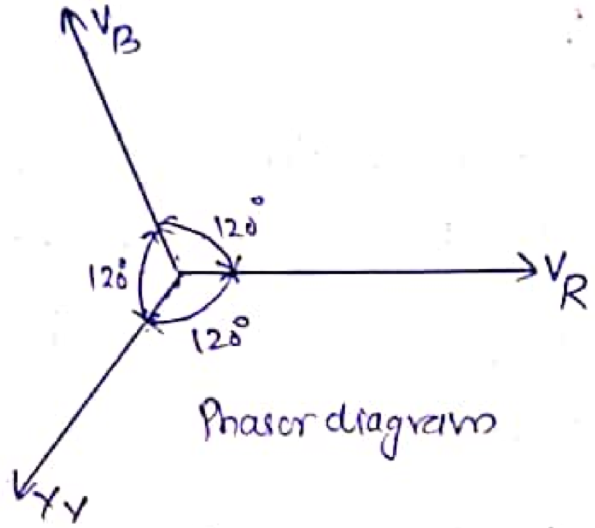
waveform

↳ The equation for the three induced voltages are,

$$V_R = V_m \sin\omega t$$

$$V_Y = V_m \sin(\omega t - 120°)$$

$V_B = V_m \sin(\omega t + 120^\circ)$



Phasor diagram

→ The sum of all the voltages at any instant is zero.

$\therefore V_R + V_Y + V_B = V_m \sin \omega t + V_m \sin(\omega t - 120^\circ) + V_m \sin(\omega t + 120^\circ)$

$= V_m [\sin \omega t + \sin \omega t \cdot \cos 120 - \cos \omega t \cdot \sin 120^\circ$

$+ \sin \omega t \cdot \cos 120 + \cos \omega t \cdot \sin 120]$

$= V_m (\sin \omega t + 2 \sin \omega t \cdot \cos(120^\circ))$

$= V_m [\sin \omega t + 2 \sin \omega t (\frac{-1}{2})]$

$= V_m \times 0$

$= 0$

$$\boxed{\therefore V_R + V_Y + V_B = 0}$$

ie, $V_R + V_Y = -V_B$

* Symmetrical System :-

↳ A three phase system in which the three voltages are of same magnitude and frequency and displaced from each other by 120 phase angle is called as symmetrical systems.

* Phase Sequence :

The sequence in which the voltages in three phases reach their maximum positive values is called phase - sequence. Generally the phase sequence is R-Y-B.

* Advantages of 3 phase System:

(i) Output of three - phase machine is always greater than single phase machine. So for a given size and voltage, a 3-ph alternator occupies less space and less cost too.

(ii) Three - phase system needs less copper or less conducting material than 1-ϕ system for a given volt-amp.

(iii) Rotating magnetic field can be produced, which makes a three phase motor self-starting, by using three phase system.

(iv) This system gives steady output.

(v) 1ϕ can be obtained from 3ϕ system but three-phase can't be obtained from 1ϕ.

(vi) Power factor of 1ϕ motors is poor than three-phase motors.

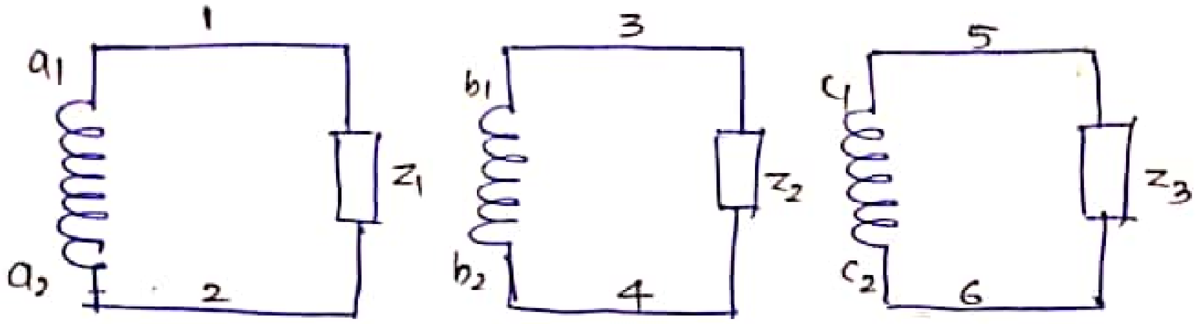
* Interconnection of three phases

↳ In a three phase AC generator, there are three windings. Each winding has two terminals.

↳ If a seperate load is connected across each phase winding as show in the figure below, then each phase supplies as independent load through a pair of wires.

↳ Thus six wires will be required to connect the load to a generator.

↳ This will make the whole system complicated and costly



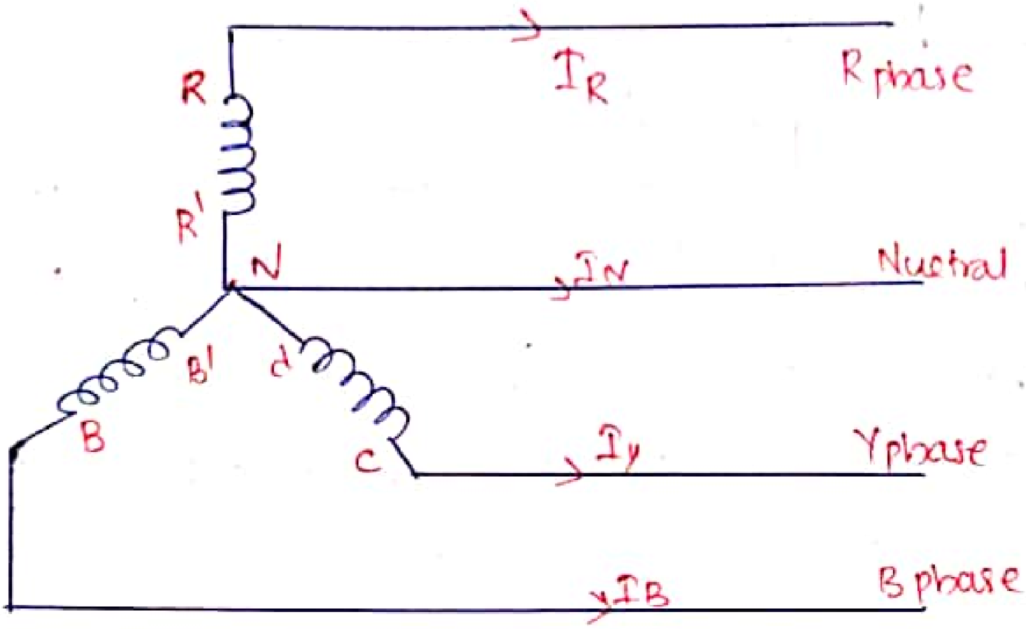
↳ Therefore, in order to reduce the number of line conductors, the three phase windings of an AC generator are interconnected.

↳ Interconnection of three phases can be done in two different ways.

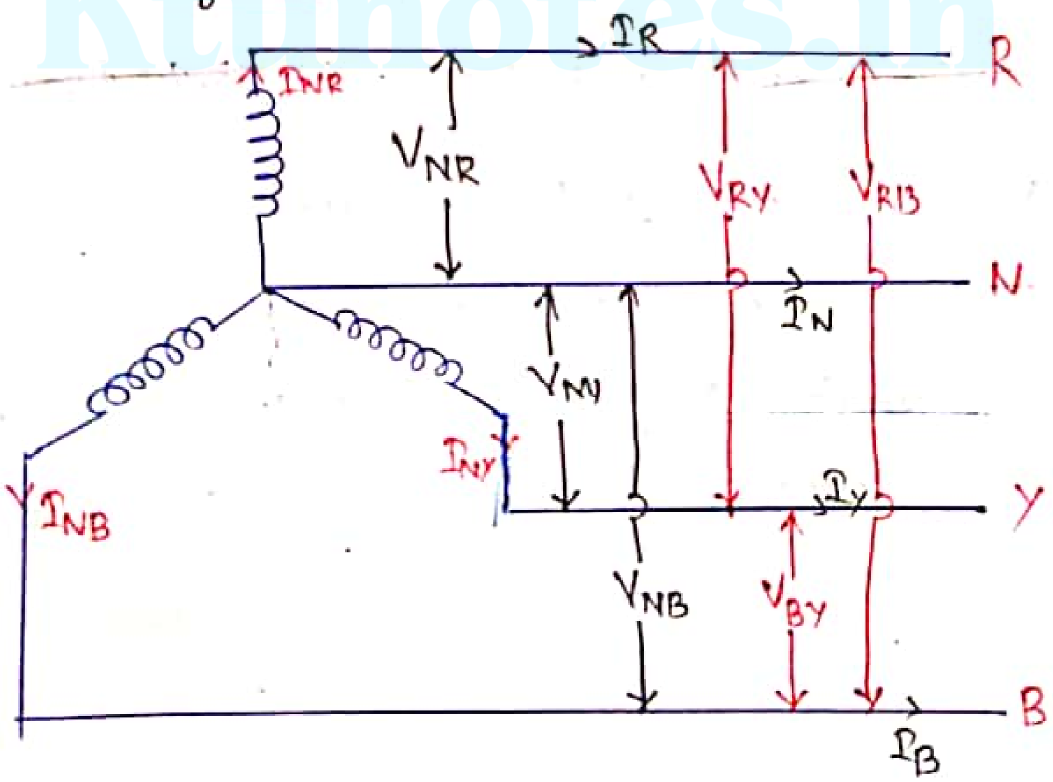
(i) Star connection or wye (Y) connection
(ii) Delta connection or mesh (Δ) connection

(i) Star connection of three phases

↳ It is a type of connection where one terminal each winding is connected to a common point and the remaining three terminals are connected to the circuit.



* Derivation of relation between line and phase voltages, line and phase currents in a star connected system.



Phase voltage: potential difference (voltage) b/w any phase line and nuetral [V_{NR}, V_{NY} and V_{NB}]

Line voltages: Potential difference between any two phase lines [V_{RY}, V_{YB}, V_{BR}]

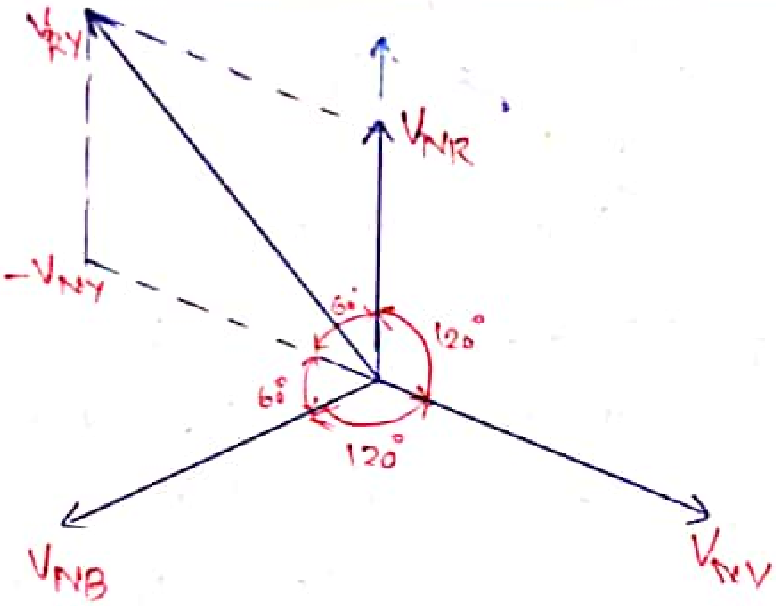
Phase currents: Current flowing through phase winding [I_{NR}, I_{NY}, I_{NB}]

Line currents: Current flowing through phase lines [I_R, I_Y, I_B]

↳ Relation b/w line and phase voltages.

↳ If three phase is connected to a balanced system then equal amount of current will flows through each phase

↳ There fore, phase voltages V_{NR}, V_{NY} and V_{NB} are equal in magnitude but displaced by an angle $120°$.



$$|V_{NR}| = |V_{NY}| = |V_{NB}| = V_{ph}$$

$$V_{RY} = V_{NR} + -V_{NY}$$

$$V_L = \sqrt{V_{NR}^2 + V_{NY}^2 + 2V_{NR} \cdot V_{NY} \cdot \cos\theta}$$

$$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph} \cdot V_{ph} \cdot \cos 60}$$

$$V_L = \sqrt{2V_{ph}^2 + 2V_{ph}^2 \cdot \cos 60}$$

$$V_L = \sqrt{3V_{ph}^2}$$

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

↳ Relation b/w line and phase currents.

$$I_{NR} = I_R$$
$$I_{NY} = I_Y$$
$$I_{NB} = I_B$$

$$I_{NR} = I_{NY} = I_{NB} = I_{ph}$$
$$I_R = I_Y = I_B = I_L$$

$$\boxed{I_L = I_{ph}}$$

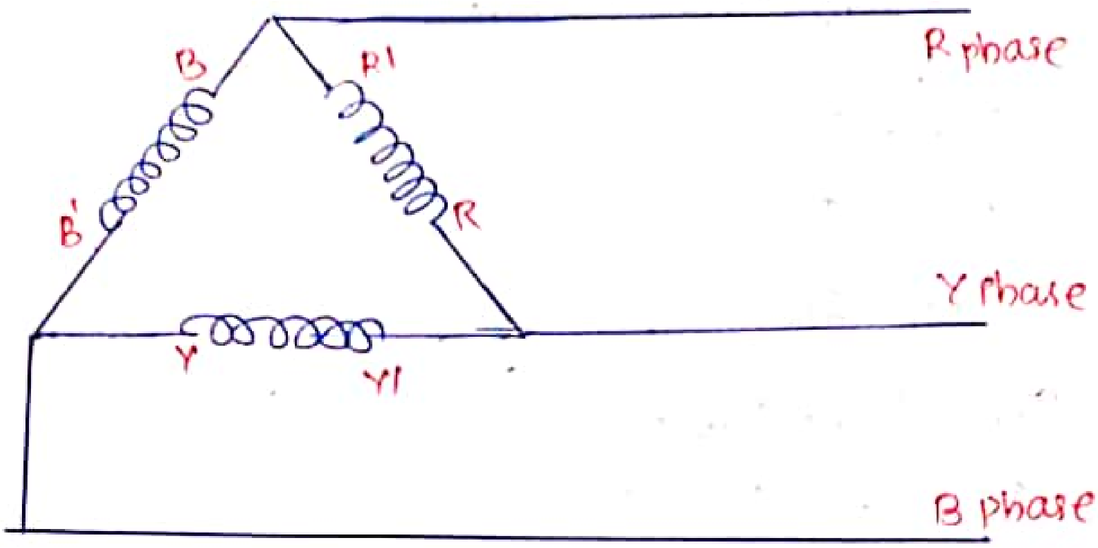
Power, $P = 3\, V_{ph} \cdot I_{ph} \cdot \cos\phi$

$\boxed{P = \sqrt{3}\, V_L\, I_L \cos\phi}$ watts.

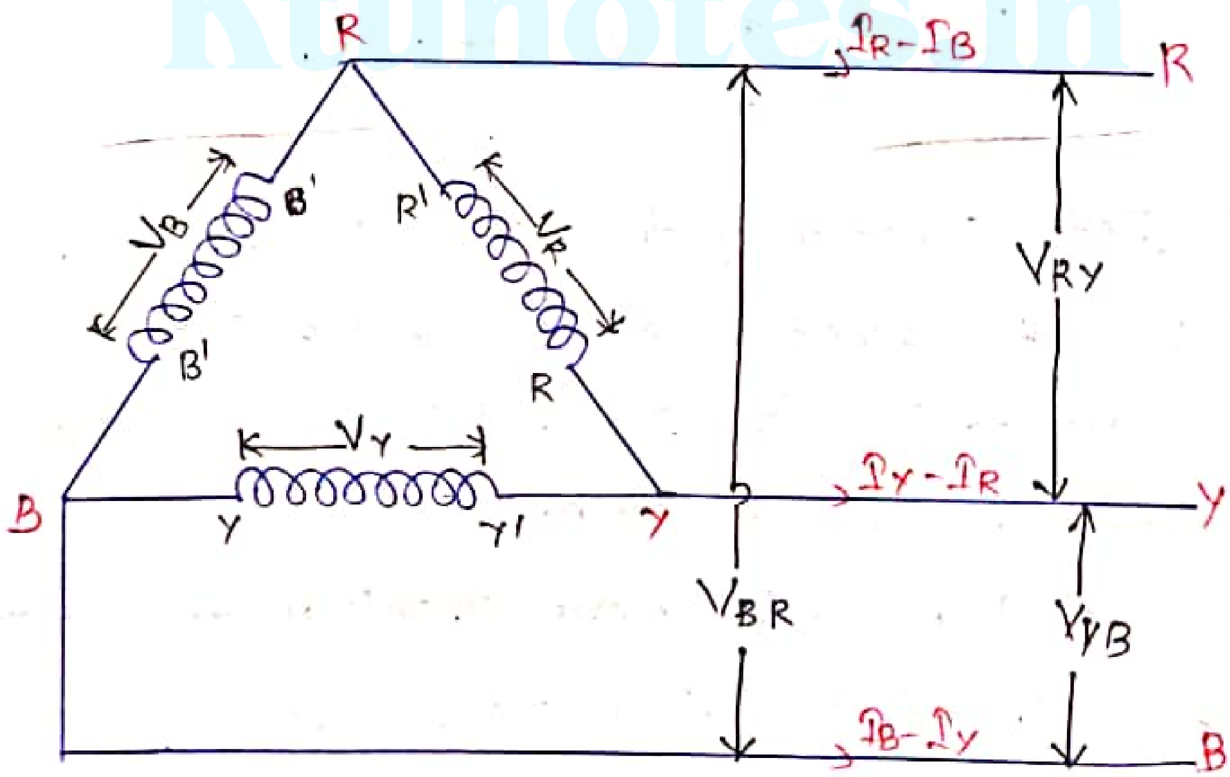
(ii) Delta Connection of three phases:

↳ In delta connection starting end of one coil is connected to the finishing end of other and so on to form a closed mesh.

↳ There is no neutral point in this ~~mesh~~ connection there fore it is a three phase three wire system.



* **Derivation of relation between line and phase voltages, line and phase currents in a Delta connected system.**



Line quantities:

a) Lin voltages :- $V_{RY} = V_{YB} = V_{BR} = V_L$

b) Line currents :- $I_R - I_B = I_Y - I_R = I_B - I_Y = I_L$

Phase quantities:

a) Phase voltages :- $V_R = V_Y = V_B = V_{ph}$

b) Phase currents :- $I_R = I_Y = I_B = I_{ph}$

In a balanced 3 phase system,

$$|I_R| = |I_Y| = |I_B| = I_{ph}$$

$$I_R - I_B = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cdot \cos\theta}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} \cdot I_{ph} \cdot \cos 60}$$

$$I_L = \sqrt{3 I_{ph}^2} = \sqrt{3}\, I_{ph}$$

$$\boxed{\therefore I_L = \sqrt{3}\, I_{ph}}$$

Power consumed in 3 ϕ system,

$$P = 3 V_{ph} I_{ph} \cdot \cos\phi$$

$$= 3 V_L \times \frac{I_L}{\sqrt{3}} \cos\phi$$

$$\boxed{P = \sqrt{3} V_L I_L \cos\phi.} \text{ watts.}$$

Numerical

1, Three identical choke coils are connected as a delta load to a three-phase supply. The line current drawn from the supply is 15 A and total power consumed is 7.5 kW. The KVA input to the load is 10 KVA. Find out.

(i) Line and phase voltages (ii) Impedance/phase (iii) Reactance/phase, (iv) resistance/phase (v) Power factor, (vi) Phase current, (vii) Inductance (if frequency is 50 Hz)/phase.

Ans: Coils are connected in Δ connection.

$I_L = 15A$. $P_{real} = 7.5kW$, $KVA = 10KVA$ = ip to the load.

(i) We know, total power $= \sqrt{3} V_L I_L$

$$\Rightarrow 10 \times 10^3 = \sqrt{3} V_L \times 15$$

$$V_L = 385 \text{ V} = V_{ph}$$

(ii) $$|Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{385}{(I_L/\sqrt{3})} = \frac{385}{15/\sqrt{3}} = 44.456\,\Omega$$

∴ We have active power, $P_{real} = \sqrt{3} V_L I_L \cos\phi$

$$\Rightarrow \cos\phi = \frac{7.5\times10^3}{\sqrt{3}(385)(15)} = 0.75 \text{ lagging}$$

$\phi = \cos^{-1}(0.75) = 41.42^\circ$ (for lagging it is +ve)

$\therefore Z_{ph} = |Z_{ph}|\angle\phi = 44.456\angle 41.42^\circ$

$= (33.33 + j29.41)\,\Omega$

(iii) X_L per phase $= X_{Lph} = 29.41\,\Omega$

(iv) $R_{ph} = 33.33\,\Omega$

(v) Power factor $= 0.75$ lagging

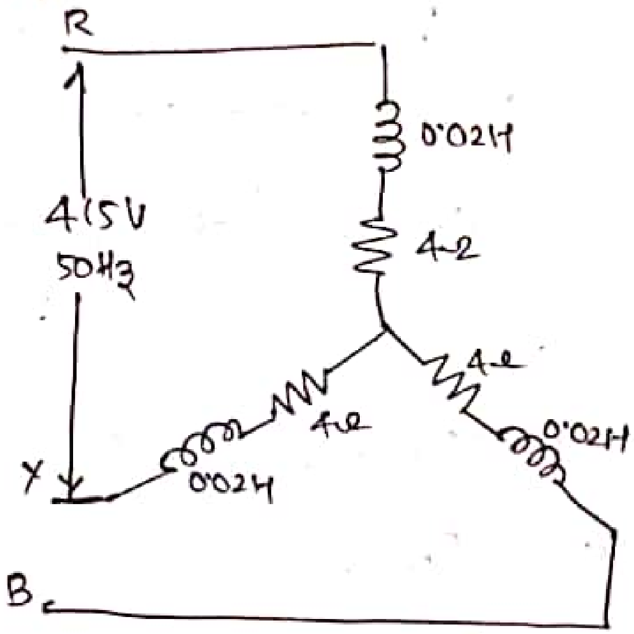
(vi) $I_{ph} = I_L/\sqrt{3} = 15/\sqrt{3} = 8.66\,A$

(vii) $L_{ph} = \frac{X_{Lph}}{2\pi f} = \frac{29.41}{2\times\pi\times50} = 93.617\,mH$

2. A balanced three phase load consists of three coils each having resistance of $4\,\Omega$ and inductance of 0.02H. It is connected to a 415V, 50Hz, 3 phase ac supply. Determine the phase voltage, Phase current, power factor and active power when the loads are connected in

(i) star (ii) delta.

Ans: (i) Star connection



$V_L = 415V$

$f = 50Hz.$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}}$

$= 239.6V$

$Z_{ph} = \sqrt{R^2+X_L^2}$

$= \sqrt{R^2+(2\pi f L)^2} = \sqrt{4^2+(2\pi\times50\times0.02)^2}$

$Z_{ph} = 7.45\Omega$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{7.45} = 32.2A$

Power factor $\cos\phi = \frac{R}{Z} = \frac{4}{7.45}$

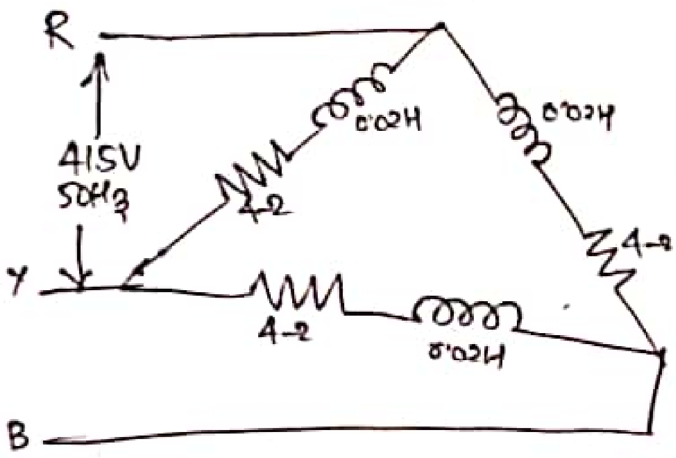
$= 0.538$ lag

Active power taken $P = 3 V_{ph} \cdot I_{ph} \cdot \cos\phi$

$= 3 \cdot 239.6 \times 32.2 \times 0.538$

$= 12.4 kW$

(ii) Delta connection



$V_L = V_{ph} = 415V$

$Z_{ph} = 7.45\Omega$

$I_{ph} = \frac{V_{ph}}{Z_{ph}}$

$= \frac{415}{7.45} = 55.7A$

Power factor $= \frac{R}{Z} = \frac{4}{7.45} = 0.538$ lag

Active power, $P = 3 V_{ph} I_{ph} \cdot \cos\phi$

$P = 3 \times 415 \times 55.7 \times 0.538$

$P = 37.3 kW$