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Module II

Moment

Rotating In addition to other tendency to move a body, force may also tend to rotate a body about an axis.

The turning tendency of a force about a point is called the moment of the force about the point.

It is measured by the pdt of the force & the distance of its line of action from the point.

$$M = Fd$$

⇒ Moment is a vector quantity.

- Magnitude.
- Direction.
- Axis of rotation.

⇒ Unit: Nm

⇒ Depending upon the relative position of the force's moment centre, the moment of a force will be either clockwise or counter clockwise.

⇒ If a clockwise moment is taken as +ve, then the anticlockwise moment taken as -ve.

Resultant of co-planar // force system

⇒ Line of action // to each other

⇒ Magnitude of resultant can be found out by adding up all the forces using a sign convention for the sense of force.

$$R = \sum F.$$

⇒ Location of resultant can be determined by applying principle of moments (Varignon's theorem)

Principle of moments (Varignon's theorem)

The moment of the resultant of a system of non-collinear coplanar forces acting on a body about any point is equal to the algebraic sum of the moments of all the forces forming the system about the same point.

$$AE = AG + GE = AG + BH$$

$$AD \cos \theta = AB \cos \theta_1 + BD \cos \theta_2$$

$$R \cos \theta = P \cos \theta_1 + Q \cos \theta_2$$

Multip. by OA

$$R \cdot OA \cdot \cos \theta = P \cdot OA \cdot \cos \theta_1 +$$

$$Q \cdot OA \cdot \cos \theta_2$$

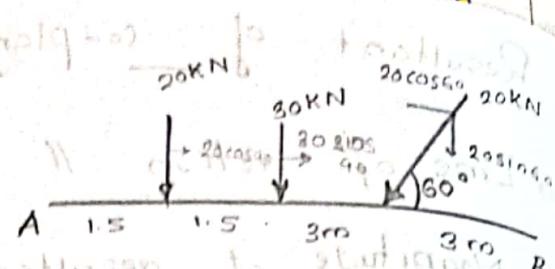
$$Rh = Ph_1 + Qh_2$$

i.e. Moment of R about O =

sum of moment of P, E, Q about O.

Hence proved.

9. Determine the resultant force, position of the resultant force, direction of resultant.



$$\sum F_x = -20 \cos 60^\circ = -10 \text{ N}$$

$$\begin{aligned} \sum F_y &= -20 \sin 60^\circ + -30 \sin 90^\circ - 20 \sin 90^\circ \\ &= -20 \frac{\sqrt{3}}{2} - 30 - 20 \end{aligned}$$

$$= -10\sqrt{3} - 50 \text{ N}$$

$$\begin{aligned} \text{Resultant force} &= \sqrt{(-10\sqrt{3})^2 + (-50)^2} \\ &= -67.32 \text{ N} \end{aligned}$$

(resultant force) acting to the left

$$R = \sqrt{\sum F_x^2 + \sum F_y^2} = \sqrt{(-10)^2 + (-67.32)^2}$$

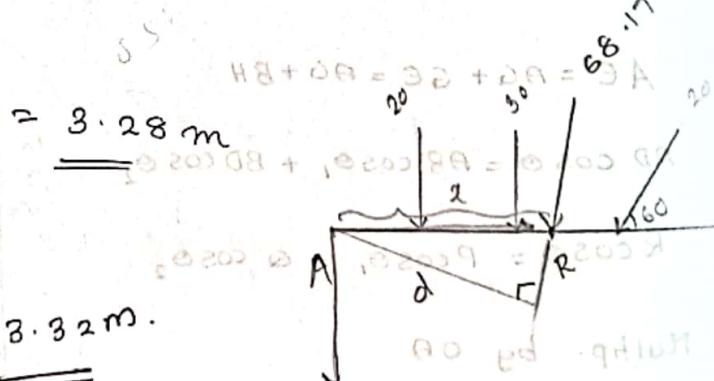
Resultant force = 68.17 N

Resultant force acts to the left of the beam

$$R \times d = 20 \times 1.5 + 30 \times 3 + 20 \times 1.5 \sin 60^\circ \times \frac{6}{6}$$

$$= 223.92$$

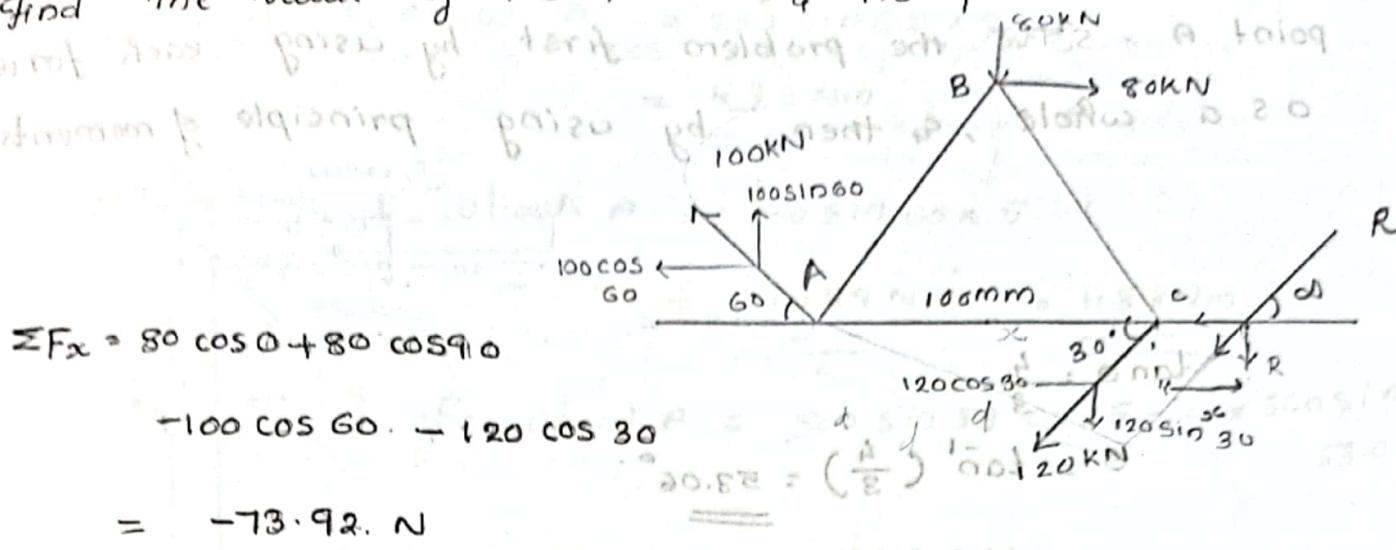
$$d = \frac{223.92}{68.17}$$



$$d = \frac{\sum M}{\sum F_y} = 3.28 \text{ m}$$

$$y = \frac{\sum M}{\sum F_x} = 22.39 \text{ m}$$

Find the value of resultant & its position.



$$\sum F_x = 80 \cos 0 + 80 \cos 90$$

$$-100 \cos 60 - 120 \cos 30$$

$$= -73.92 \text{ N}$$

$$\sum F_y = 80 \sin 0 - 80 \sin 90 + 100 \sin 60 - 120 \sin 30$$

$$= -53.39 \text{ N}$$

$$R = \sqrt{(73.92)^2 + (-53.39)^2} = 91.18 \text{ N}$$

$$\alpha = \tan^{-1} \left(\frac{53.39}{73.92} \right) = 35.84^\circ$$

~~$$R_d = \frac{50 \times 80 \sin 90}{\cos 60} = 100 \text{ kN}$$~~

~~$$R_d = 100 \text{ kN}$$~~

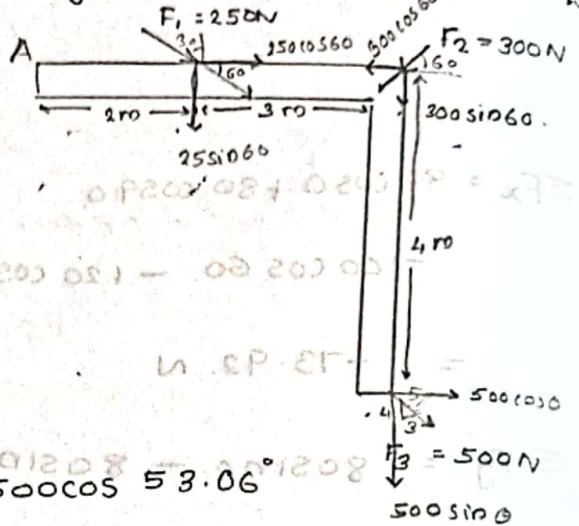
~~$$P_{APR} = P_{PPA} + P_{PAI}$$~~

~~$$P_{APR} = P_{PPA} + P_{PAI} + P_{PAI}$$~~

~~$$P_{APR} = 2.8282 \text{ kN}$$~~

~~$$P_{APR} = 2.8282 \text{ kN}$$~~

Determine the moment of each of the 3 forces about point A. Solve the problem first by using each force as a whole, & then by using principle of moments.



$$\tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1} \left(\frac{4}{3} \right) = 53.06^\circ$$

$$\sum F_x = 250 \cos 60 - 300 \cos 60 + 500 \cos 53.06 \\ = 215.48 \text{ N}$$

$$\sum F_y = -250 \sin 60 - 300 \sin 60 - 500 \sin 53.06 \\ = -815.94 \text{ N}$$

$$R = \sqrt{(215.48)^2 + (-815.94)^2} = 918.24 \text{ N}$$

$$Rd = 250 \sin 60 \times 2 + 300 \sin 60 \times 5 + 500 \sin 53.06 \times 4 \\ 4 \times 500 \cos 53.06$$

$$= 433.042 + 1299.03 + 10807.67 - 196.2$$

$$= 433 + 1299 + 10807.67 - 196.2$$

$$= 2528.2 \text{ NM}$$

$$d = \frac{2528.2}{918.24} = 2.75 \text{ m}$$

$$\text{Moment of } F_1 \text{ about } A = 250 \sin 60 \times 2 \\ = 433 \text{ Nm}$$

$$\text{Moment of } F_2 \text{ about } A = 300 \sin 60 \times 5$$

$$= 1299 \text{ Nm} = 1.3 \text{ kNm}$$

$$\text{Moment of } F_3 \text{ about } A = 500 \sin 53.06 \times 5 - 4 \times 500 \sin \frac{53.06}{53.06}$$

$$= 796.2 \text{ Nm}$$

$$= 796.2 \text{ Nm}$$

a. Determine the moment of the couple acting on the machine member shown below.

$$\tan \theta = \frac{3}{4}$$

$$\theta = \tan^{-1} \left(\frac{3}{4} \right) = 36.86^\circ$$

$$\sum F_x = 150 \cos 36.86 - 150 \cos 36.86 = 0$$

$$\sum F_y = 150 \sin 36.86 - 150 \sin 36.86 = 0$$

$$\sum M_A = 150 \sin 36.86 \times 2 + 5 \times 150 \sin 36.86 = 88.232P$$

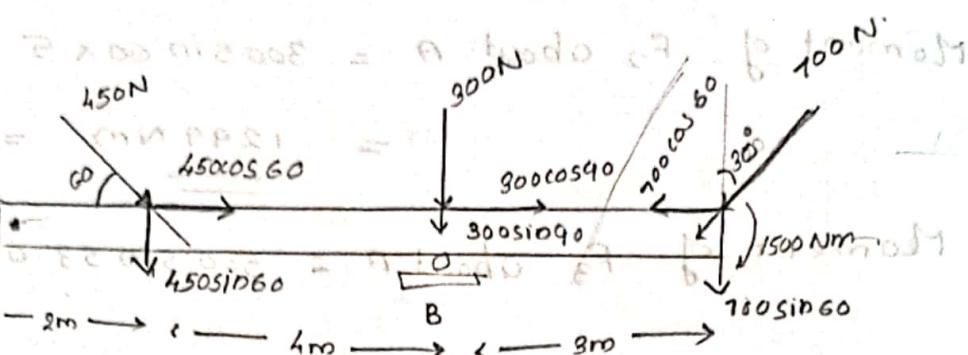
$$R = 0$$

$$Rd = 150 \sin 36.86 \times 2 + 5 \times 150 \sin 36.86 + 88.232P = 179.96 + 449.89 + 0.8 = 630.65$$

$$= \frac{630.65}{88.232P} = b$$

$$2.48 = \left(\frac{630.65}{88.232P} \right) 100 = 0$$

Q. Replace 8 forces acting on the shaft beam by a single resultant force. Specify where the force acts, measured from end A.



$$\sum F_x = 450\cos 60 + 300\cos 90 - 700\cos 60$$

$$= -125 \text{ N}$$

$$\begin{aligned} \sum F_y &= -450\sin 60 - 300\sin 90 - 700\sin 60 \\ &= -1295.93 \text{ N} \end{aligned}$$

$$R = \sqrt{(125)^2 + (1295.93)^2} = 1301.94 \text{ N}$$

$$\begin{aligned} R_d &= 2 \times 450\sin 60 + 6 \times 300\sin 90 + 9 \times 700\sin 60 \\ &= 719.42 + 1800 + 5455.96 + 1500 \\ &= 9535.38 \end{aligned}$$

$$d = \frac{9535.38}{1301.94} = 7.33 \text{ m}$$

$$\alpha = \frac{\sum M}{\sum F_y}$$

$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = 84.5^\circ$$

Q) 3 forces 200N, 300N & 400N act along 3 sides of an equilateral triangle taken in order. Find the magnitude & line of action of the resultant force.

$$\sum F_x = 200 \cos 60^\circ + 400 \cos 60^\circ - 300 \cos 60^\circ \\ = -150 \text{ N}$$

$$\sum F_y = -200 \sin 60^\circ + 300 \sin 60^\circ + 400 \sin 60^\circ \\ = -86.6 \text{ N}$$

$$R = \sqrt{(-150)^2 + (-86.6)^2} = 173.2 \text{ N}$$

~~$\theta_{\text{eq}} = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right) = 30^\circ$~~

~~$\sum M_A = -300 \sin 60^\circ \times AB \\ = -259.81 \times AB$~~

$$\sum M_A = -300 \sin 60^\circ \times AB \\ = -259.81 \times AB$$

$$\sum M_B = Rx \\ x = \frac{\sum M_B}{R} = \frac{259.81 AB}{173.2}$$

~~$\sum M_B = Rx - 200 \times 6 - 300 \times 6 + 200 \times 10 = 272$~~

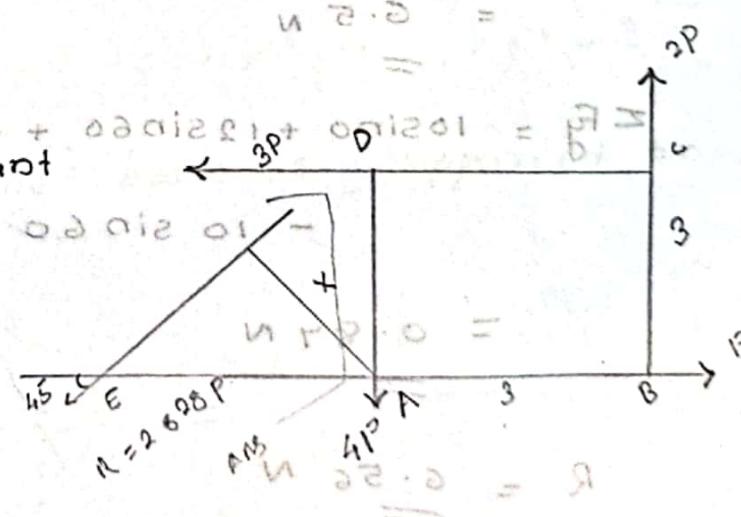
~~$x = \frac{\sum M_B}{R} = \frac{259.81 AB}{173.2}$~~

Q) Find out the resultant

& its position

$$\sum F_x = P \cos 60^\circ - 3P \cos 60^\circ \\ = -2P$$

$$\sum F_y = -4P \sin 60^\circ + 2P \sin 60^\circ \\ = -2P$$



$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

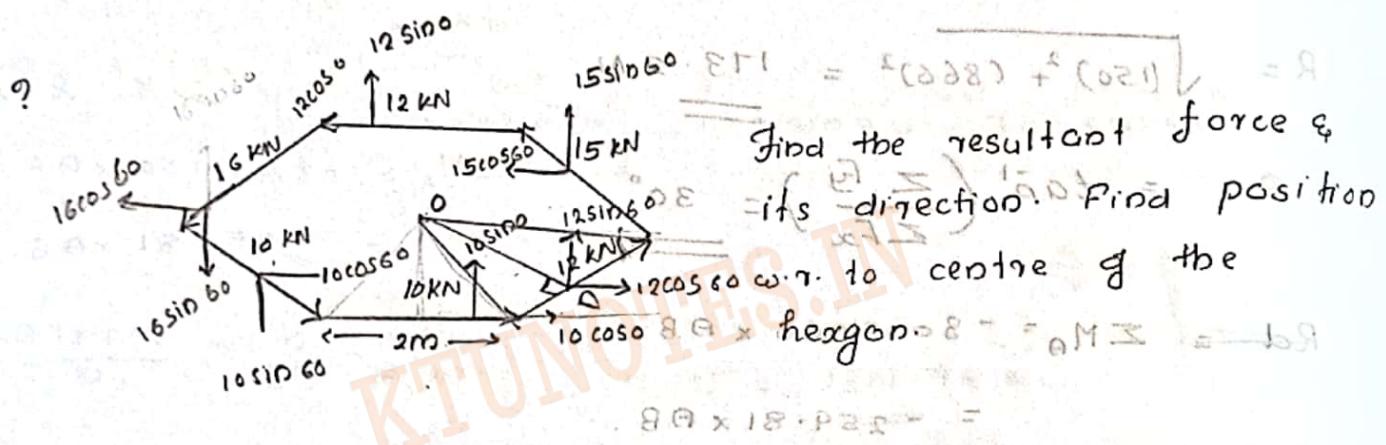
$$= \sqrt{(2P)^2 + (-2P)^2} = 2.828P$$

$$\alpha = \tan^{-1} \left(\frac{\Sigma F_y}{\Sigma F_x} \right)$$

$$= \tan^{-1} \left(\frac{-2P}{2P} \right) = 45^\circ$$

$$Rx = 5Px \alpha$$

$$x = \frac{5Px \alpha}{2.828 P} = 1.768 \text{ m}$$



$$\Sigma F_x = 10 \cos 60 + 12 \cos 60 - 15 \cos 60 - 12 \cos 60 - 16 \cos 60$$

$$= \frac{89.12 \text{ kN}}{5.828} = 15.3 \text{ kN}$$

$$= 6.5 \text{ N}$$

$$\Sigma F_y = 10 \sin 60 + 12 \sin 60 + 15 \sin 60 + 12 \sin 60 - 16 \sin 60$$

$$= 10 \sin 60$$

$$= 0.84 \text{ N}$$

$$R = 6.56 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{0.84}{6.5} \right)$$

$$= 82.5^\circ \text{ or } 7.62^\circ$$

$$\Sigma M = R d = - (10 + 12 + 15 + 12 + 16 + 10) \times 2 \sin 60$$

$$= - 129.75 \text{ NM}$$

$$d = \frac{-129.75}{6.56} = 19.8 \text{ m}$$

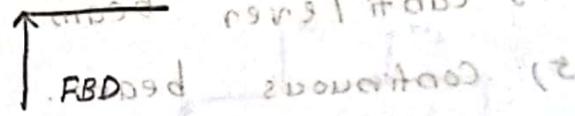
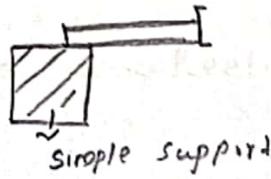
~~$$x = \frac{\Sigma M}{\Sigma F_y} = \frac{129.75}{87} = 1.498 \text{ m}$$~~

~~$$y = \frac{\Sigma M_0}{\Sigma F_x} = \frac{129.75}{-6.5} = 19.96 \text{ m}$$~~

Types of Supports

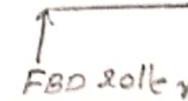
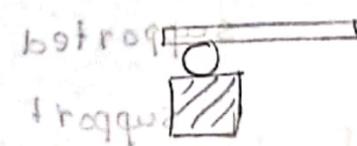
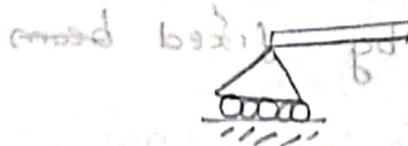
1) Simple support.

If one end of the beam rests on a fixed support, the support is known as simple supported beam.



2) Roller support.

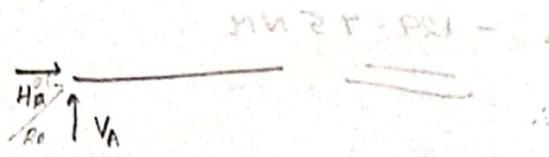
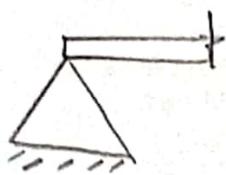
Here one end of the beam is supported on a roller.



3) Hinged support.

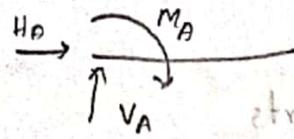
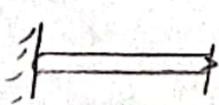
The beam does not move either along or normal

to the axis but can rotate.



4) Fixed Support.

The beam is not free to rotate or slide along the length of ~~the beam~~ ~~not~~ in the direction normal to the beam. Therefore 3 reaction components can be observed. Also known as built-in support.

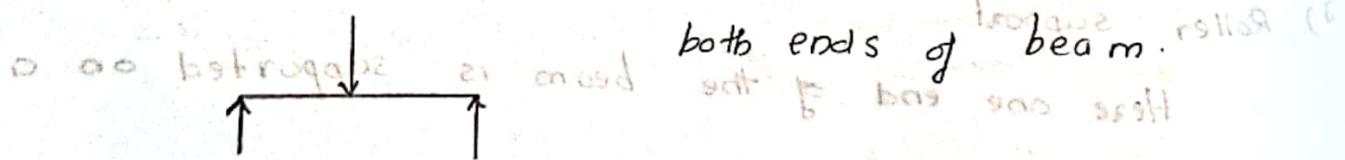


torque for effect

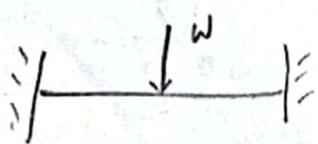
Types of beam

- 1) Simply supported beam
- 2) fixed beam
- 3) overhanging beam
- 4) cantilever beam
- 5) continuous beam.

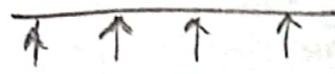
1) Simply supported beams: Supports are provided at



a) Fixed beam : when both end of forces are supported by fixed beam support

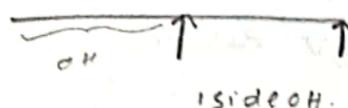
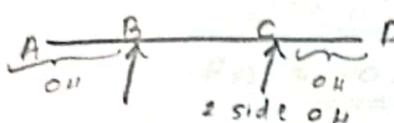


3) overhanging : when beam is supported by more than 2 points



4) Cantilever beam : one end of the beam is based & other end is kept free.

5) Overhanging beam : when the beam is projecting beyond the surface it is known as overhanging beam.



Types of loads

⇒ concentrated load or point load.

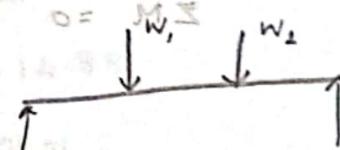
⇒ uniformly distributed load (UDL)

⇒ uniformly varying load (UVL)

⇒ external moment.

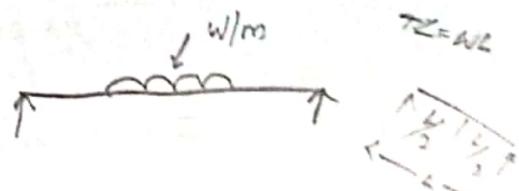
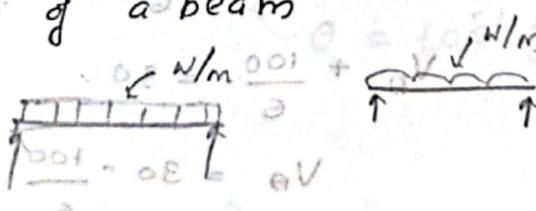
⇒ general loading

Concentrated load : when the loads are applied certain points in a wheel.

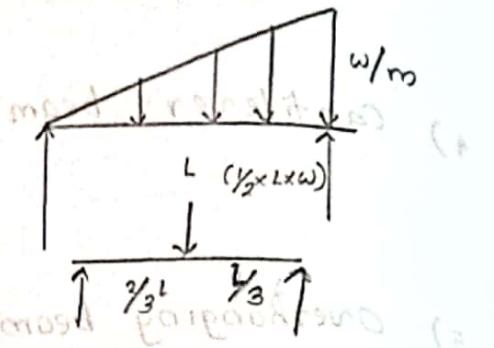


Uniformly distributed : The load which have same intensity of load over a certain length of beam.

length of a beam

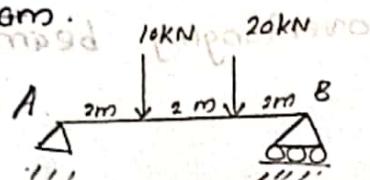


Uniformly varying load : In this type of load, load intensity vary from one point to other.

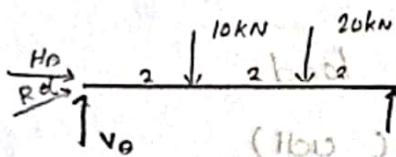


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erstet fest set b bis

6/2/20
part of a fixed end set with
the support of the beam set dropped
? Draw the diagram.



calculate support reaction at A & B.



$$\sum F_x = 0 \Rightarrow H_A = 0$$

$$\sum F_y = 0 \Rightarrow V_A - 10 - 20 + V_B = 0$$

$$V_A + V_B - 30 = 0$$

zurrol f 299

$$\sum M = 0 \Rightarrow 10 \times 2 + 20 \times 4 - V_B \times 6 = 0$$

$$20 + 80 - 6V_B = 0$$

$$100 - 6V_B = 0$$

$$6V_B = 100$$

$$V_B = \frac{100}{6} = 16.67 \text{ N}$$

$$V_A + \frac{100}{6} = 30$$

$$V_A = 30 - \frac{100}{6}$$

$$= \frac{80}{6} = 13.3 \text{ N}$$

beam f 299 ziel al : hochreiter (299)

$$\text{reaction of total set } R_B = \sqrt{10^2 + (13.3)^2} = 13.3 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{V_B}{H_B} \right) = \underline{\underline{90^\circ}} \text{ not defined.}$$

Calculate

$$\sum F_x = 0 \Rightarrow H_B + 20 \cos 60^\circ = 0$$

$$H_B = \underline{\underline{10}}$$

$$\sum F_y = 0 \Rightarrow V_B - 10 - 20 \sin 60^\circ = 0$$

$$V_B = 10 + 20 \sin 60^\circ = 10 + 20 \cdot \frac{\sqrt{3}}{2} = 10 + 10\sqrt{3}$$

$$V_B = 10 + 10\sqrt{3} = 10 + 17.3 = 27.3$$

$$gV \times V_B + V_{B3} = 27.3 \quad \underline{\underline{gV = M}}$$

$$\sum M = 0 \Rightarrow 2 \times 10 + 4 \times 20 \sin 60^\circ = 6 V_B$$

$$20 + 40\sqrt{3} = 6 V_B$$

$$6 V_B = 20 + 40\sqrt{3}$$

$$V_B = 14.88 \text{ KN}$$

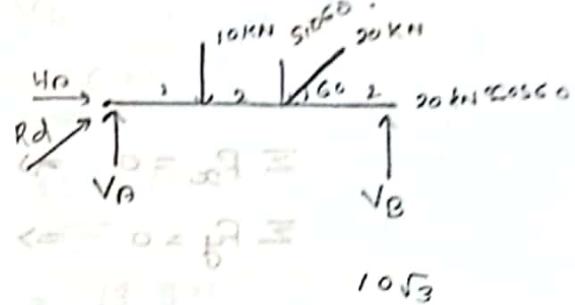
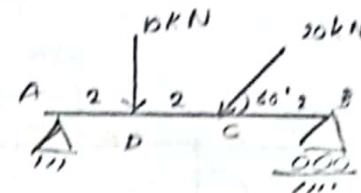
$$V_B = 27.3 - 14.88$$

$$V_B = \sqrt{R^2 - (12.42)^2} = \underline{\underline{12.42 \text{ KN}}}$$

$$R = \sqrt{H_B^2 + V_B^2} = \underline{\underline{15.95 \text{ KN}}}$$

$$\theta = \tan^{-1} \left(\frac{12.42}{10} \right) = \underline{\underline{51.11^\circ}}$$

$$V_B = R_B = 14.88 \text{ KN}$$



$$gV \times V_B + V_{B3} = 27.3 \quad \underline{\underline{gV = M}}$$

$$\sum M = 0 \Rightarrow 2 \times 10 + 4 \times 20 \sin 60^\circ = 6 V_B$$

$$20 + 40\sqrt{3} = 6 V_B$$

$$6 V_B = 20 + 40\sqrt{3}$$

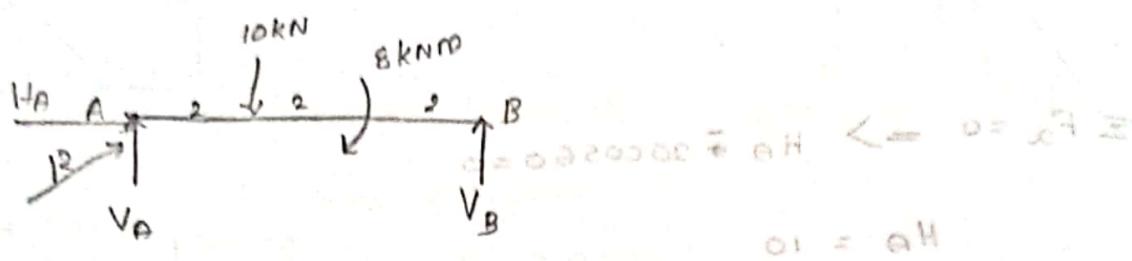
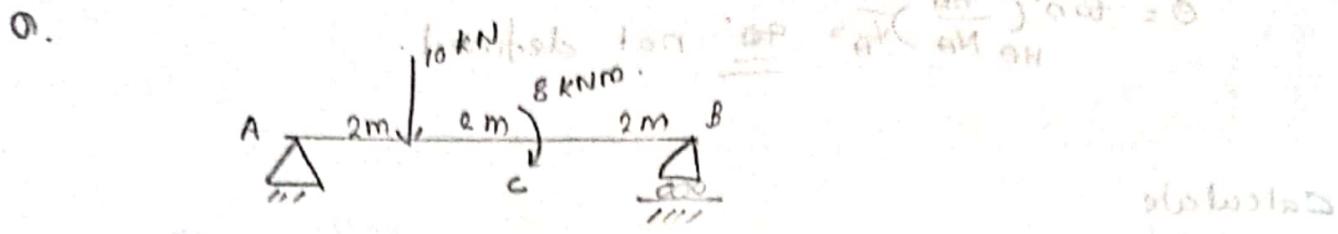
$$V_B = 14.88 \text{ KN}$$

$$V_B = 27.3 - 14.88$$

$$V_B = \sqrt{R^2 - (12.42)^2} = \underline{\underline{12.42 \text{ KN}}}$$

$$R = \sqrt{H_B^2 + V_B^2} = \underline{\underline{15.95 \text{ KN}}}$$

$$\theta = \tan^{-1} \left(\frac{12.42}{10} \right) = \underline{\underline{51.11^\circ}}$$



$$\sum F_x = 0 \Rightarrow H_A = 0 \quad \leftarrow R_B = V_B$$

$$\sum F_y = 0 \Rightarrow V_A + 10 + V_B = 0$$

$$V_A + V_B = 10 \quad \leftarrow V + aV$$

$$\sum M = 0 \Rightarrow 2 \times 10 + 8 - 2 \times 6 \times V_B$$

$$20 + 8 - 6 V_B \leftarrow a = M$$

$$6 V_B = 28$$

$$R_B = V_B = \frac{28}{6} = 4.67 \text{ kN}$$

$$V_B = 10 - 4.67 \\ = 5.34 \text{ kN.}$$

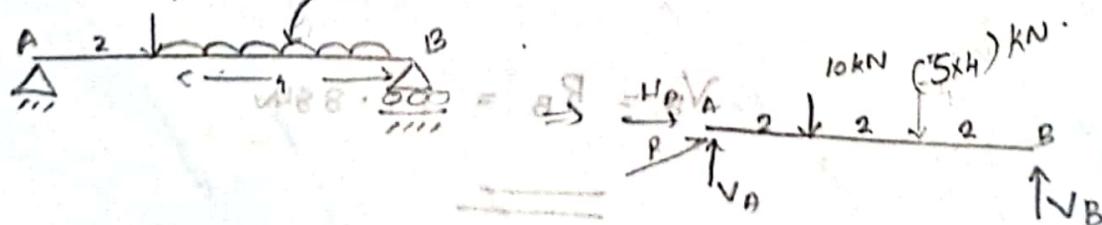
$$28.41 - 8.56 = 19.85$$

$$R = \sqrt{(5.34)^2} = 5.34 \text{ kN}$$

$$H_A = 0 \quad \leftarrow \theta = \tan^{-1}\left(\frac{H_A}{V_A}\right) = \text{not defined}$$

$$= \left(\frac{H_A + V_A}{V_A}\right)^{\frac{1}{2}} = 0$$

Q. 11.12 - $\frac{10 \text{ kN}}{2} = 5 \text{ kN/m}$



$$\sum F_x = 0 \Rightarrow H_A = 0$$

$$\sum F_y = 0 \Rightarrow V_A - 10 - 20 + V_B = 0$$

$$V_A + V_B = 30$$

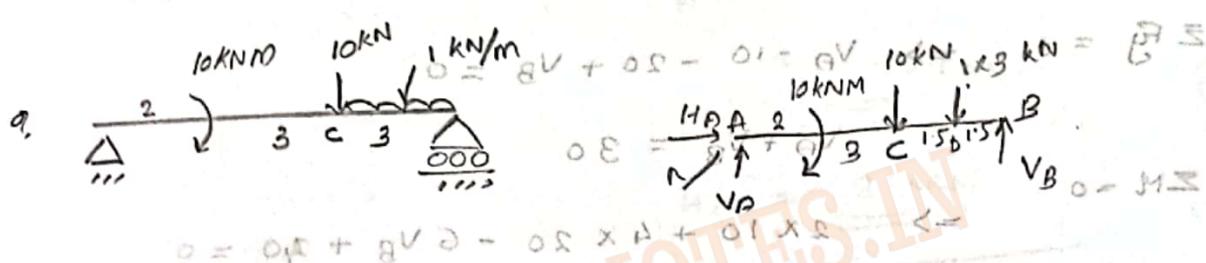
$$\sum M_o = 0 \Rightarrow 2 \times 10 + 2 \times 20 - G V_B$$

$$100 = G V_B$$

$$V_B = \underline{16.66 N}$$

$$V_A = \underline{13.33 N}$$

$$R_B = \underline{18.33 N}$$



$$\sum F_x = 0 \Rightarrow R_A + H_A = 0$$

$$\sum F_y = 0 \Rightarrow V_A + 10 + 3 + V_B$$

$$\sum F_y = 0 \Rightarrow V_A + V_B = 13$$

$$\sum M_o = 0 \Rightarrow 10 + 5 \times 10 + 6.5 \times 3 - 8 V_B$$

$$10 + 50 + 19.5 - 8 V_B$$

$$8 V_B = 79.5$$

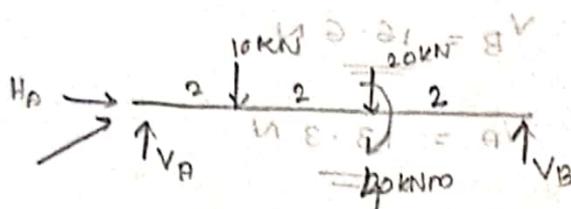
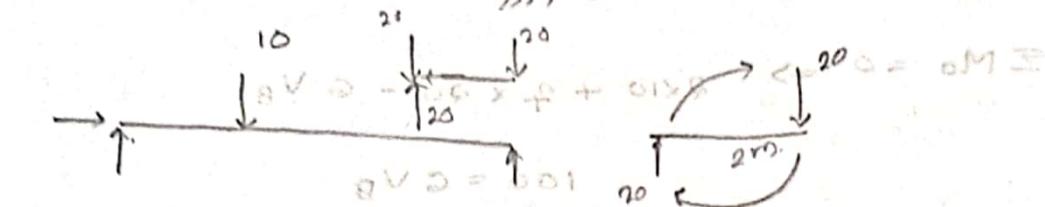
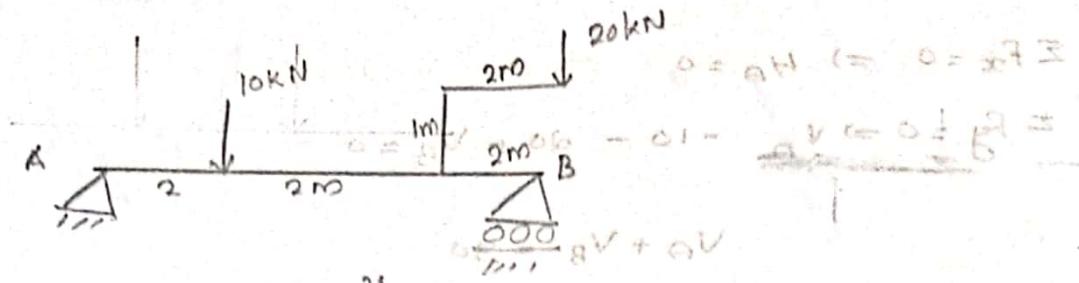
$$V_B = \underline{9.94 N}$$

$$V_A = \frac{13}{21.3} - 9.94$$

$$= \underline{8.06 N}$$

$$R = \sqrt{(13.06)^2} = \underline{\underline{13.06}}$$

Q.



$$\sum F_x = 0 \rightarrow H_{A_1} = 0$$

$$\sum F_y = 0 \Rightarrow V_A - 10 - 20 + V_B = 0$$

$$V_A + V_B = 30$$

$$\sum M = 0$$

$$\Rightarrow 2 \times 10 + 4 \times 20 - 6 V_B + 20 \cdot 2 = 0$$

$$20 + 80 + 40 - 6 V_B = 0$$

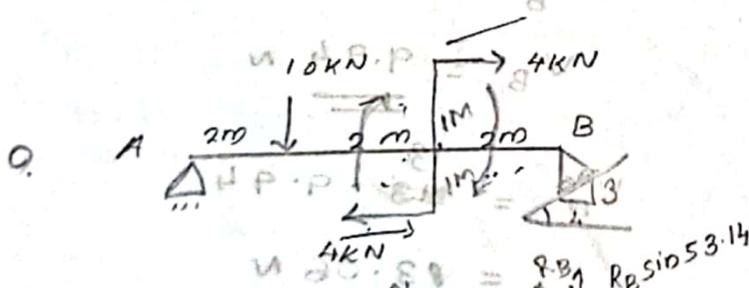
$$6 V_B = 140$$

$$V_B = 23.33 \text{ KN}$$

$$R_B = V_B = 23.3 \text{ KN}$$

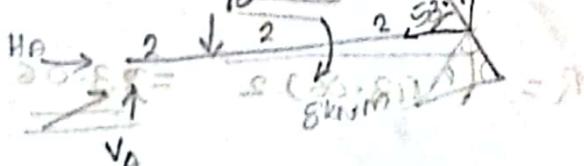
$$R_B = 23.3 \text{ KN}$$

$$R_B = 23.3 \text{ KN}$$



$$\tan \theta = \frac{3}{4}$$

$$\theta = 36.86$$



$$\sum F_x = 0 \Rightarrow H_A - R_B \cos 53.14 = 0 \therefore H_A = 0.59 R_B = 0.$$

$$\sum F_y = 0 \rightarrow V_A - 10 + R_B \sin 53.14 = 0$$

$$V_A + R_B \sin 53.14 = 10$$

$$\text{in } \text{eq. 1} \text{ case } = V_A + 0.8 R_B = 10.$$

$$\sum M_O = 0 \Rightarrow 20 \times 8 - 6 \times R_B \sin 53.14.$$

$$\text{in eq. 0} = 28 - 4.8 R_B = 0.$$

$$R_B = \frac{28}{4.8} = 5.8 \text{ kN}$$

$$(0.88 - 0P) = V_A \quad V_A = 10 - 0.8 \times 5.8$$

$$= 5.34 \text{ kN.}$$

$$H_A = \underline{\underline{3.4 \text{ kN}}}$$

$$R_A = \sqrt{(H_A)^2 + (V_A)^2}$$

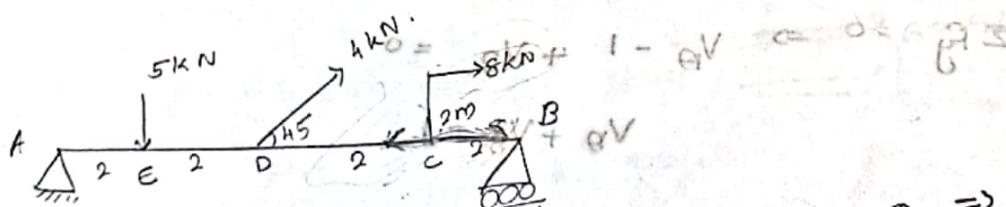
$$= \underline{\underline{6.3 \text{ kN}}}$$

$$R_B = 5.8 \text{ kN.}$$

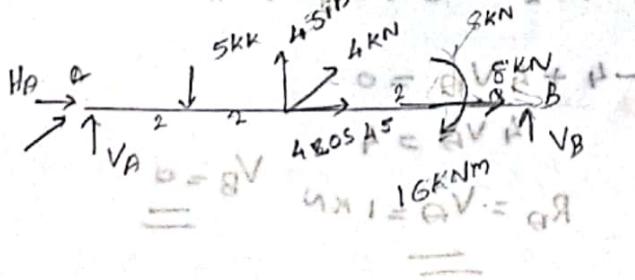
$$\theta_H = \tan^{-1} \left(\frac{H_A}{V_A} \right) = 32.48^\circ$$



Q.



$$\sum F_x = 0 \Rightarrow H_A + 4 \cos 45^\circ = 0$$



$$H_A = \underline{\underline{-10.83}} \quad 8 + H_A + \frac{4}{\sqrt{2}} = 0$$

$$\sum F_y = 0 \Rightarrow V_A - 5 + 4 \sin 45^\circ$$

$$+ V_B = 0$$

$$V_A + V_B = 5 + 8 - \frac{4}{\sqrt{2}} = 2.17$$

$$\sum M_B = 0 \Rightarrow 10 + 4 \times 4 \sin 45 - 8V_B + 16 = 0.$$

$$10 + 11.3 + 16 - 8V_B = 0$$

$$8V_B = 37.3 \quad 14.7$$

$$R_B = V_B = 4m \times 6 \frac{1.83}{=}$$

$$V_A + V_B = 2.17$$

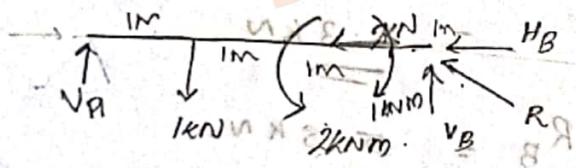
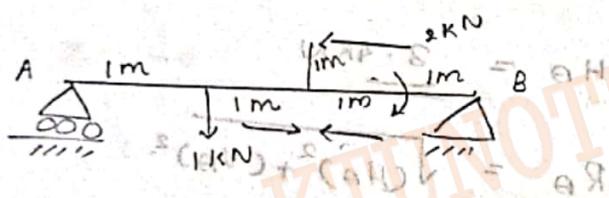
$$V_B = 0.34 \text{ kN}$$

$$R_A = \frac{8.2}{8.4} R_B = 0.84 R_B$$

$$R_A = 10.84 \text{ kN}$$

$$8.2 \times 8.0 - 01 = 90 \quad \theta_A = (90 - 88.2)$$

$$= 1.77^\circ$$



$$\sum F_x = 0 \Rightarrow H_B - 2 = 0$$

$$H_B = 2.$$

$$\sum F_y = 0 \Rightarrow V_B - 1 + V_A = 0$$

$$V_A + V_B = 1$$

$$0 = 2.1 \cdot 203A + aH \leftarrow 0$$

$$\sum M_B = 0 \Rightarrow -4 + 2 \cdot 1 - 3 \times 1 + 4V_A$$

$$0 = \frac{4}{2} + aH + 8 \quad \underline{\underline{EB.01}} = aH$$

$$-4 + 4V_A = 0$$

$$0 = 2.1 \cdot 203A + 2 - aV \leftarrow 0$$

$$4V_A = 4$$

$$0 = .8V + 2$$

$$R_B = V_A = 1 \text{ kN}$$

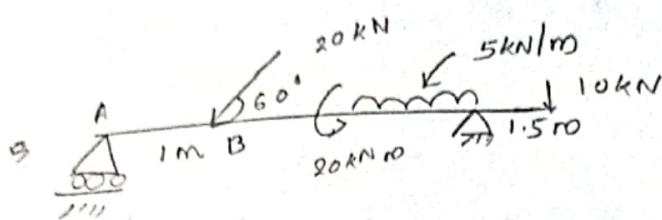
$$\underline{\underline{V_B = 0}}$$

$$F1.2 = \frac{1}{2} - 8V + 2 = 8V + aV$$

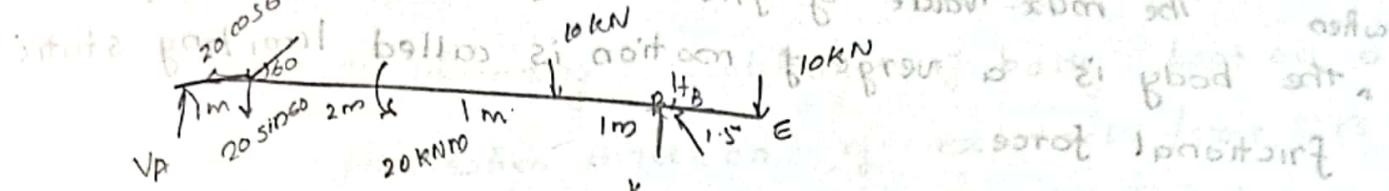
$$\underline{\underline{=}}$$

$$R_B = 2 \text{ kN}$$

θ = not defined.



rotate to correct position



$$\sum F_x = 0 \Rightarrow -H_B - 20 \cos 60 = 0$$

$$H_B = -10 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow V_B - 20 \sin 60 - 10 + V_B = 0$$

$$V_B + V_B = 37.3$$

$$\sum M_O = 0 \Rightarrow -10 \cdot 20 + 4 \times 20 \sin 60 + 5 V_B + 15 \times 10$$

$$R_A = 5 V_B = 9.77 - 16.86 \text{ kN}$$

$$V_B = 20.44 \text{ kN}$$

$$R_B = 22.77 \text{ kN}$$

$$(90 - 26.06)$$

$$= 16.07^{\circ} 63.94^{\circ}$$

reaction at roller support is shown along the line of action of reaction force

reaction at roller support is shown along the line of action of reaction force

to prevent sliding towards right, reaction at roller support is

to prevent sliding towards left, reaction at roller support is

reaction at roller support is shown along the line of action of reaction force

11/21/20

Friction

- 1) Dry friction
- 2) fluid friction.
- 3) Internal friction.

Limiting force of friction

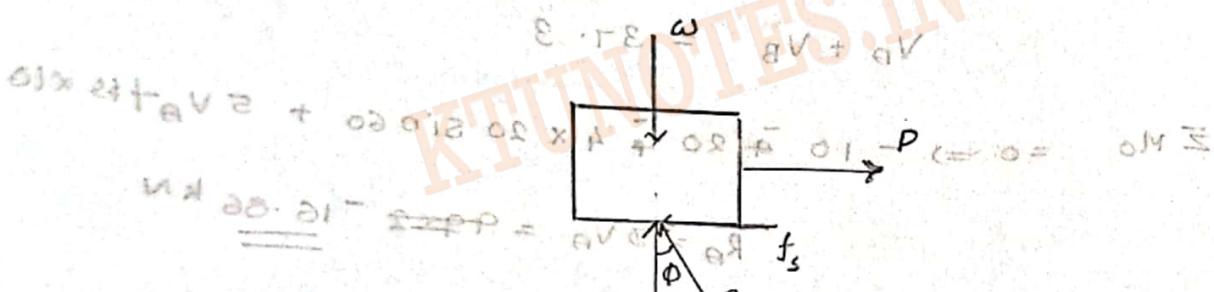
when The max. value of frictional force at surface of contact
the body is a verge of motion is called limiting static
frictional force

$$F_s = \mu_s N$$

F_s : limiting static frictional force

μ_s : coefficient of static friction.

N : Normal reaction.

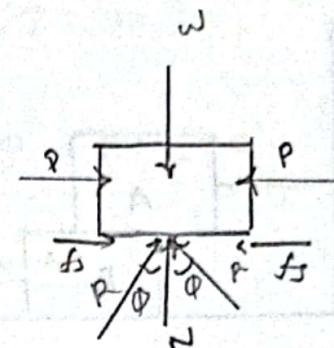
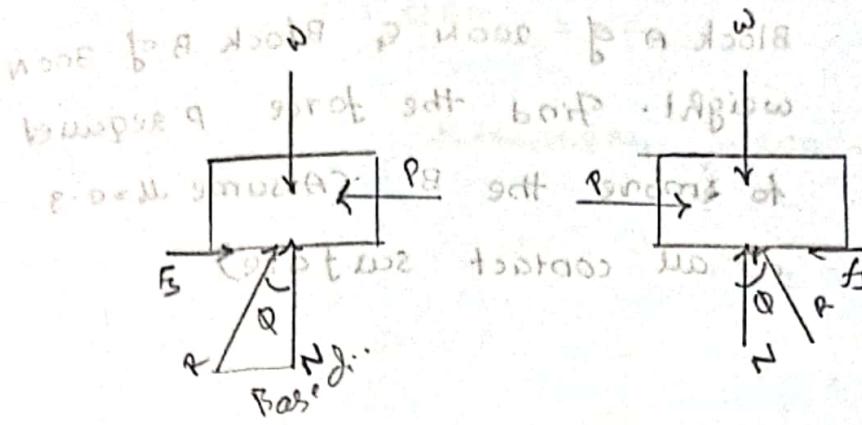


μ_s = limiting friction force

(Normal react to 2 bodies)

Angle of friction: Angle made by the resultant of normal reaction & the friction force to the normal react.

Cone of friction: The right circular cone with vertex at the point of contact of 2 surfaces, axis is the direction of the normal react (\vec{N}) & semi-vertical angle is equal angle of friction ϕ .



Consider the limiting equilibrium of body kept on a horizontal plane when direction of external force is change, the direction of resultant b. change. But the angle b/w normal reaction & resultant should be same. when the directⁿ of external force is gradually change 360° the resultant are generate a circular cone with a semi cone angle ϕ . This cone is called cone of friction. The axis of this cone will be the normal reaction & generators are the resultant of force & base radius is equal to limiting frictional force.

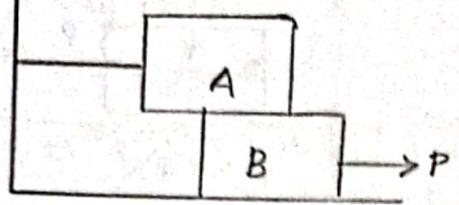
Angle of free force $\theta = \phi$

Max. inclination of the plane at which the body remain in equilibrium over the incline plane by assistance of frictional force alone.

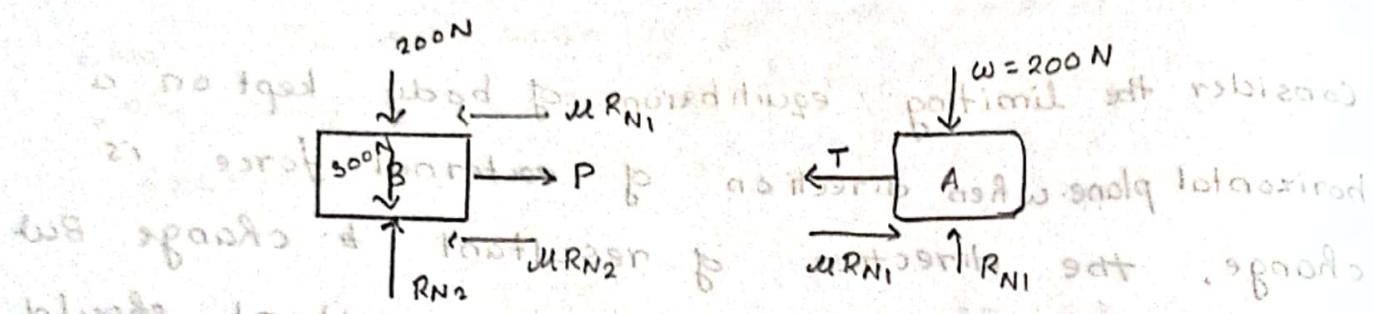
$$\tan \theta = \mu$$

$$\theta = \tan^{-1} \mu$$

$$\theta = \tan^{-1} \mu$$



Block A of 200N & Block B of 300N weight. Find the force P required to move the B. (Assume $\mu = 0.3$ at all contact surfaces)



Consider body A,

$$\text{At top of block A, } \sum F_x = 0 \Rightarrow 200 \cos 90^\circ - T \cos 0^\circ + \mu R_{N1} \cos 0^\circ - R_{N1} \cos 90^\circ = 0$$

$$\text{At bottom of block A, } \sum F_y = 0 \Rightarrow -200 \sin 90^\circ - T \sin 0^\circ + \mu R_{N1} \sin 0^\circ + \mu R_{N2} \sin 90^\circ = 0$$

$$-200 - T R_{N1} = 0 \quad \text{for static equilibrium}$$

$$R_{N1} = \frac{200}{T}$$

$$T = 60 \text{ N}$$

Consider body B,

$$\begin{aligned} \sum F_x = 0 &\Rightarrow P \cos 0^\circ - \mu R_{N1} \cos 0^\circ + 200 \cos 90^\circ \\ &+ 300 \cos 90^\circ + R_{N2} \cos 90^\circ \\ &- \mu R_{N2} \cos 0^\circ = 0. \end{aligned}$$

$$P - \mu R_{N1} - \mu R_{N2} = 0$$

$$P - \mu R_N_2 = 60$$

$$\sum F_y = 0 \Rightarrow R_N_1 + R_N_2 - 200 - 300 = 0 \Rightarrow R_N_1 + R_N_2 = 500$$

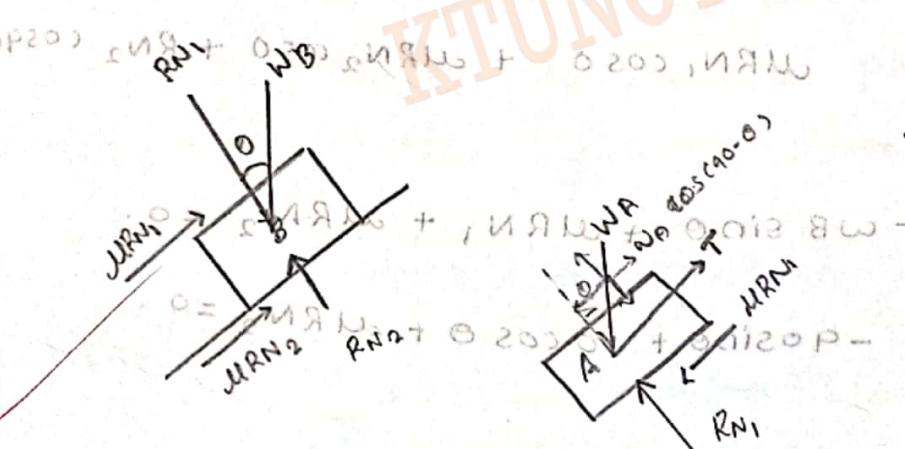
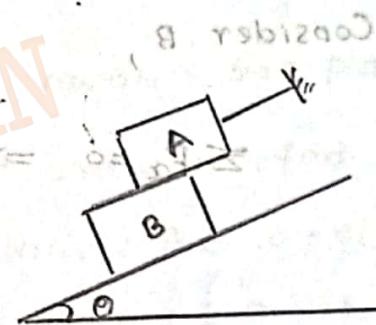
$$R_N_2 = 500 \text{ N}$$

$$R_N_1 = 300 \text{ N}$$

$$P = 210 \text{ N}$$

$$0 = 300 \cos \theta - 210 - T$$

Q. What should be the value of angle θ for the motion of the block B weighing 90N to impend down the plane. The coefficient of friction for all surfaces of contact is $\frac{1}{3}$. Block A weights 30N. The block A is held in position as show in fig.



$$\mu = \frac{1}{3}$$

29.3

$$0 = P \cos \theta - R_N_1 - R_N_2 - (0 - P) \sin \theta - \mu B \sin \theta - \mu A \sin \theta$$

Consider A,

$$\sum F_x = 0 \Rightarrow T \cos \theta - \mu B \cos \theta - \mu A \cos \theta + R_N_1 \cos \theta = 0$$

$$- \mu R_N_1 \cos \theta = 0$$

$$0 = R_N_1 + R_N_2 - \mu B \cos \theta - \mu A \cos \theta$$

$$T - \mu B \sin \theta - \mu A \sin \theta = 0$$

$$0 = R_N_1 + R_N_2 - \mu B \cos \theta - \mu A \cos \theta$$

$$T - 30 \sin \theta - \mu R_N_1 = 0$$

$$\sum F_y = 0 \Rightarrow T \sin \theta - \omega_B \sin(90^\circ - \theta) + R_{N_1} \sin 90^\circ = 0$$

$$T \sin \theta - \omega_B \cos \theta + R_{N_1} = 0$$

$$-\omega_B \cos \theta + R_{N_1} = 0$$

$$R_{N_1} = 30 \cos \theta$$

$$\sum F_x = 0 \Rightarrow T - 30 \cos \theta - \frac{1}{3} 30 \cos \theta = 0$$

$$T = 40 \cos \theta$$

$$A \text{ does not rotate} \Rightarrow T = 40 \cos \theta$$

$$B \text{ rotates} \Rightarrow T = 40 \cos \theta$$

Consider B,

$$\sum F_x = 0 \Rightarrow -\omega_B \cos(90^\circ - \theta) - R_{N_1} \cos 90^\circ +$$

$$+ \mu R_{N_1} \cos \theta + \mu R_{N_2} \cos \theta + R_{N_2} \cos 90^\circ = 0$$

$$-\omega_B \sin \theta + \mu R_{N_1} + \mu R_{N_2} = 0$$

$$-90 \sin \theta + 10 \cos \theta + \mu R_{N_2} = 0$$

$$\sum F_y = 0 \Rightarrow -\omega_B \sin(90^\circ - \theta) - R_{N_1} \sin 90^\circ +$$

$$+ \mu R_{N_1} \sin \theta + \mu R_{N_2} \sin \theta + R_{N_2} \sin 90^\circ = 0$$

$$-\omega_B \cos \theta - R_{N_1} + R_{N_2} = 0$$

$$0 = 10 \cos \theta - 90 \cos \theta + 120 \cos \theta = 0$$

$$R_{N_2} = 120 \cos \theta$$

$$R_{N_2} = 120 \cos \theta$$

$$-90 \sin \theta + 10 \cos \theta + 40 \cos \theta = 0 \quad \text{--- (1)}$$

$$-90 \sin \theta + 50 \cos \theta = 0$$

$$\frac{1}{3} \times 120$$

$$-90 \sin \theta = 50 \cos \theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{50}{90}$$

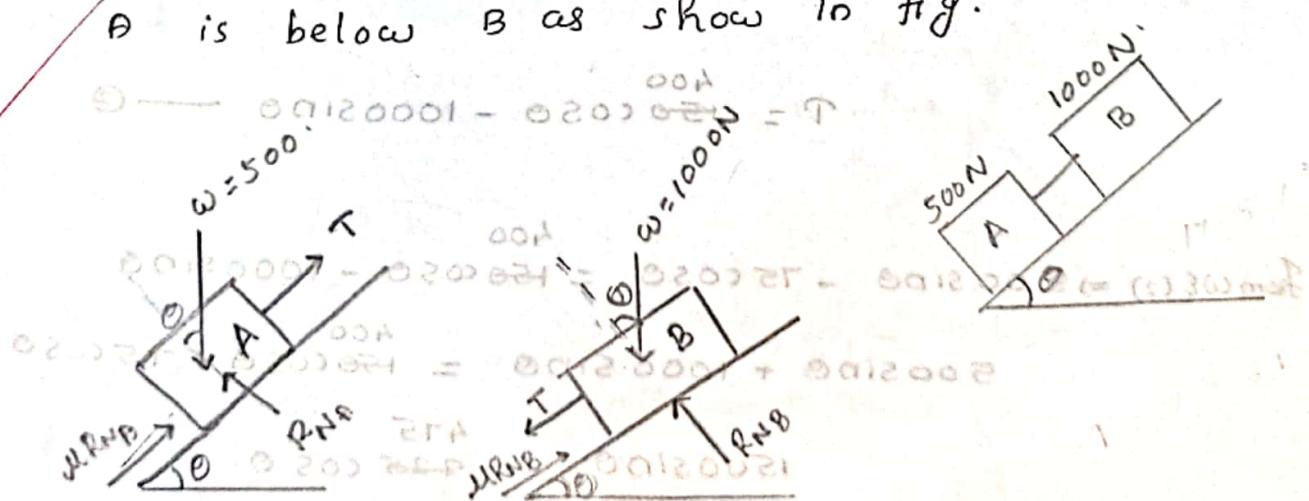
$$\tan \theta = -\frac{5}{9}$$

$$\theta = \tan^{-1} \left(\frac{5}{9} \right)$$

$$\theta = 29.05^\circ$$

B1048

Q. 2 bodies A & B of weights 500N & 1000N are placed on an inclined plane. The blocks are connected by a string parallel to the inclined plane. The coefficient of friction b/w the inclined plane block A is 0.15 & that for the block B 0.4. Find the inclination of plane when the motion is about to take place. Also calculate the tension in string. The block A is below B as shown in fig.



Block A

$$\sum F_x = 0 \Rightarrow T \cos \theta - 500 \cos(90-\theta) + \mu R_{NB} \cos \theta + R_{NB} \cos \theta = 0$$

$$T - 500 \sin \theta + \mu R_{NB} = 0$$

$$T + \mu R_{NB} = 500 \sin \theta$$

$$\sum F_y = 0 \Rightarrow T \sin \theta - 500 \sin(90-\theta) + \mu R_{NB} \sin \theta + R_{NB} \sin \theta = 0$$

$$T - 500 \cos \theta + R_{NB} = 0$$

$$\frac{T}{P} = \frac{R_{NB}}{P} = 500 \cos \theta$$

$$\left(\frac{P}{P}\right) \text{Ans} = 0 \quad T = 500 \sin \theta - 0.15 \times 500 \cos \theta$$

$$T = 500 \sin \theta - 75 \cos \theta \quad \text{--- (1)}$$

Block B

$$\sum F_x = 0 \Rightarrow -1000 \cos(90-\theta) - T \cos \theta + \mu R_{NB} \cos \theta + R_{NB} \cos \theta = 0$$

$$-1000 \sin \theta - T + \mu R_{NB} = 0$$

$$-1000 \sin \theta - T + \mu R_{NB} = 1000 \sin \theta$$

$$-1000 \sin \theta - T \sin \theta + \mu R_{NB} \sin \theta + R_{NB} \sin \theta = 0$$

$$-1000 \cos \theta + R_{NB} = 0$$

$$R_{NB} = 1000 \cos \theta$$

$$T = \frac{400}{150} \cos \theta - 1000 \sin \theta \quad \text{--- (2)}$$

$$\text{From (2)} \Rightarrow 500 \sin \theta - 75 \cos \theta = \frac{400}{150} \cos \theta - 1000 \sin \theta$$

$$500 \sin \theta + 1000 \sin \theta = \frac{400}{150} \cos \theta + 75 \cos \theta$$

$$1500 \sin \theta = \frac{475}{225} \cos \theta$$

$$\tan \theta = \frac{475}{1500}$$

8 books form a

$$\theta = \tan^{-1} \left(\frac{3}{40} \right) = 4 \times 0.25 = 3.9 \times 1^\circ$$

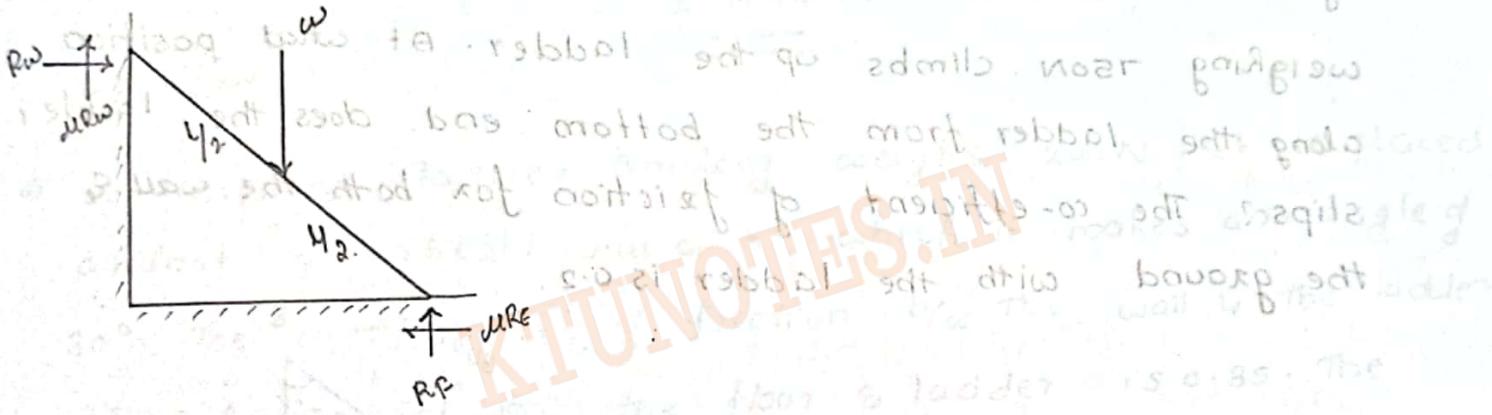
$$\theta = \tan^{-1} \left(\frac{14}{60} \right) = 0.25 = 8 \times 0.25$$

$$\theta = \underline{\underline{17.57^\circ}}$$

$$T = \frac{500 \sin(17.57)}{500 \cos(17.57) - 75 \cos(17.57)}$$

$$= \underline{\underline{19.43 \text{ N}}}$$

- Q. Ladder friction



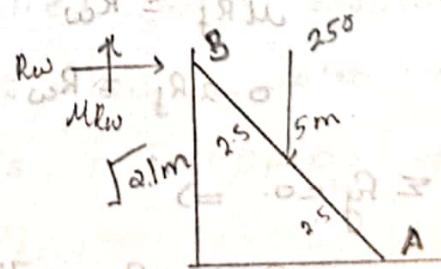
- A uniform ladder of 250 kg long weighting 250N is placed against a smooth vertical wall with a lower end 2m from the wall. The co-efficient of friction b/w ladder & wall is 0.25. ST the ladder remains in equilibrium in this position.

$$l = 5 \text{ m.}$$

$$w = 250 \text{ N.}$$

$$\Sigma F_x = 0 \Rightarrow R_w = \mu R_f = 62.5 \text{ N}$$

$$\Sigma F_y = 0 \Rightarrow R_f = 250 \text{ N}$$



Moment about B,

$$R_f \times AC - 250 \times 1 - F \times BC = 0$$

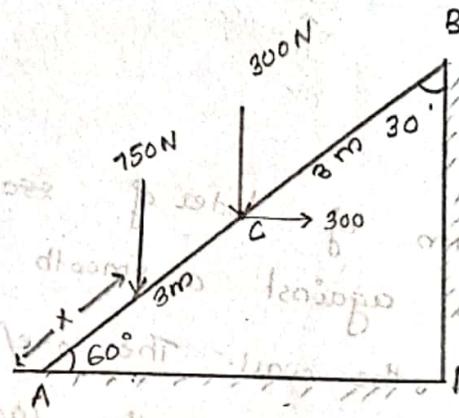
$$250 \times 2 - 250 = F \times 4.58$$

$$F = 54.59 N$$

$$=$$

Since the frictional force at A is less than the limiting frictional force of $62.5 N$, the ladder will remain in equilibrium.

- ? A uniform ladder 8m long weighing $300N$, is resting against a wall with which it makes 30° . A man weighing $750N$ climbs up the ladder. At what position along the ladder from the bottom end does the ladder slip? The co-efficient of friction for both the wall & the ground with the ladder is 0.2 .



$$\mu = 0.2$$

$$\sum F_x = 0$$

$$\mu R_f = R_w$$

$$\mu R_f = R_w$$

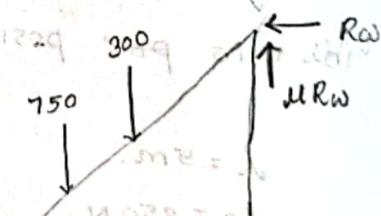
$$0.2 R_f = R_w$$

$$\sum F_y = 0 \Rightarrow$$

$$R_f + \mu R_w - 750 - 300 = 0 \quad \mu R_f$$

$$R_f + \mu R_w = 1050$$

$$\frac{R_w}{0.2} + 0.2 R_w = 1050$$



$$5 \cdot 2 R_w = 1050$$

$$R_w = 201.92 \text{ N}$$

$$R_f = 1009.6 \text{ N}$$

$$\sum M = 0 \Rightarrow$$

$$750 x \cos 60 + 300 \times 3 \cos 60 - 2 R_w \times 6 \cos 60$$

$$= R_w \cdot 6 \sin 60.$$

$$0 = 6R_w - 9 + \frac{750x}{2} + 450 - 0.6R_w - 5.19R_w$$

$$0 = 0.4R_w + 450 - 5.19R_w$$

$$0 = 0.25R_w + 450$$

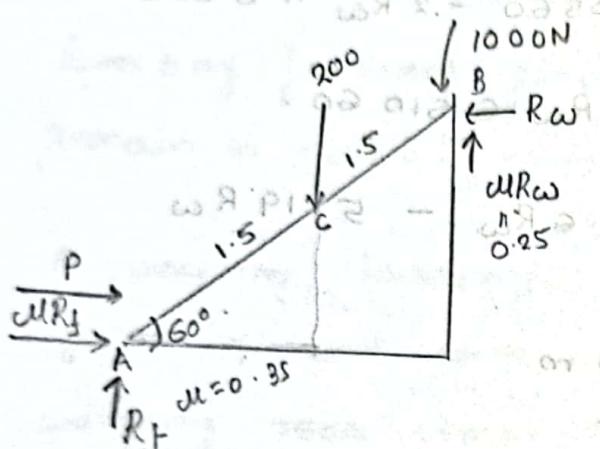
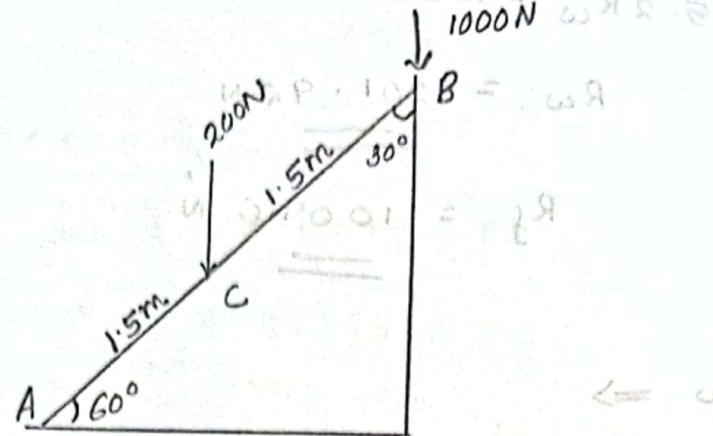
$$x = 1.92 \text{ m.}$$

Q. A uniform ladder 8m long weighs 200N. It is placed against a vertical wall with which it makes an angle of 30° . The coefficient of friction b/w the wall & the ladder is 0.25 & that b/w the floor & ladder is 0.35. The ladder in addition to its own weight has to support a load of 100N at its top end. Find

(i) The horizontal force P to be applied to the ladder at the floor level to prevent slipping.

✓ (ii) If the force P is not applied what should be the minimum inclination of the ladder with the horizontal so that there is no slipping of it with the load at its top end.

$$N P \cdot \mu = R_w$$



$$\sum F_x = 0 \Rightarrow \mu R_f + P - R_w = 0$$

$$0.35 R_f + P - R_w = 0.$$

~~$$\sum F_y = 0 \Rightarrow R_f + \mu R_w - 200 = 1000$$~~

$$R_f + 0.25 R_w = 1200$$

$$0.35 R_f - R_w = -P \quad \text{--- (1)}$$

$$R_f + 0.25 R_w = 1200 \quad \text{--- (2)}$$

$$\sum M = 0, \quad -R_w \times 3 \sin 60 + \mu R_w \times 3 \cos 60 + 1000 \times 3 \cos 60$$

$$200 \times 0.25 \cos 60$$

$$= -2.59 R_w + 0.375 R_w + 1500$$

$$= -2.965 R_w + 1650 \quad \text{--- (3)}$$

$$2.965 R_w = 1650$$

$$R_w = 554.49 \text{ N}$$

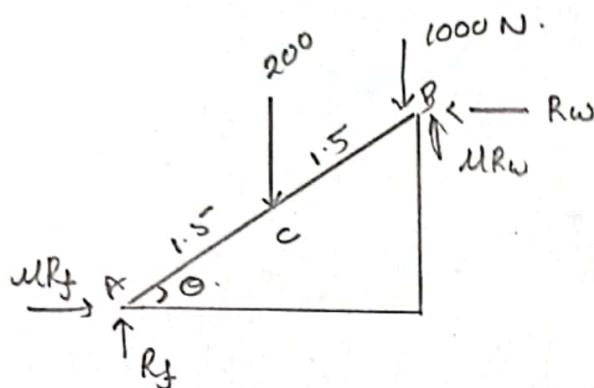
$$R_f = 1200 - 139.12$$

$$\therefore R_f = \underline{\underline{1060.88 \text{ N}}}$$

$$0.35 \times 1060.88 - 554.49 = -P$$

$$\therefore P = \underline{\underline{18.182}}$$

$$P = \underline{\underline{18.18 \text{ N}}}$$



$$\sum F_x = 0 \Rightarrow \mu R_f - R_\omega = 0$$

$$0.35 R_f = R_\omega$$

$$\sum F_y = 0 \Rightarrow R_f + \mu R_\omega - 1200 = 0$$

$$(0.35 R_f) + \mu R_\omega = 1200$$

$$2.85 R_\omega + 0.25 R_\omega = 1200$$

$$3.1 R_\omega = 1200$$

$$R_\omega = 387.09 \text{ N}$$

$$R_f = \underline{\underline{1105.97 \text{ N}}}$$

$$\text{at } \sum M = 0 \Rightarrow \text{reaction force } R_f \text{ at } 3\cos\theta$$

$$= R_\omega \times 3\sin\theta + \mu R_\omega 3\cos\theta + 1000 \times 3\cos\theta$$

$$\text{at } 1105.97 \sin\theta + 290.3175 + 3000 \cos\theta + 3000 \cos\theta = 0$$

$$-1161.27 \sin\theta - 290.3175 + 3000 \cos\theta + 3000 \cos\theta = 0$$

$$-1167 \cdot 27 \sin\theta = -3009 \cdot 6825 \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{3009 \cdot 6825}{1167 \cdot 27}$$

$$= 2.578$$

$$\theta = \tan^{-1}(2.578)$$

$$= 68.8^\circ$$

Module 3

$$\text{Square} = a^2$$

$$\text{Cube} = a^3$$

$$\text{Cuboid} = lwh$$

$$\text{Sphere} = \frac{4}{3}(\pi r^3)$$

$$\text{Cylinder} = \frac{1}{2}\pi^2 h$$

$$\text{Hollow cylinder} = \frac{\pi}{4} \times h(D^2 - d^2)$$

Diff. b/w centre of gravity & centroid

⇒ The term centre of gravity applies to bodies with mass & weight. centroid applies to plane area.

⇒ centre of gravity of a body is the point through which the resultant gravitational force acts for any orientation of body whereas centroid is the point in the plane area such that the moment of area about any axis through that point is zero.

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