



# KTU **NOTES**

The learning companion.

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NOTIFICATIONS | SOLVED QUESTION PAPERS**

12/01/2020  
Tuesday

## MODULE - I

### ELEMENTARY CONCEPTS OF ELECTRIC CIRCUITS

Basic concepts in Electrical Engineering

#### > Electric current

Electric current is the rate of flow of electric charges.

$i = \frac{dq}{dt}$ , where  $q$  is elementary electric charge. The unit of charge is coulomb.  
The unit of current is Ampere.

$$1 \text{ Ampere} = 1 \text{ coulomb/sec.}$$

#### > Electromotive force

In order to move electrons along a conductor, some amount of work is required. The work required is supplied by an electromotive force provided by a battery or a similar device.

#### > Potential difference

Potential difference is the difference b/w the voltages at two ends of a conductor.

The voltage across the element  $V_{ab}$  is given by,

$$V_{ab} = \frac{W}{Q}$$

W-work done

Q-charge.

#### > Resistance

Electric resistance is the property of a material by which it opposes the flow of current. Energy is dissipated across a resistor.

in the form of heat.

The resistance of a conductor :

> Is directly proportional to its length  $l$ :

$$R \propto l$$

> Is inversely proportional to area of cross

section  $A$ .  $R \propto \frac{1}{A}$ .

> depends on the nature of conductor.

> depends on the temperature.

$$R \propto \frac{l}{A}$$

$$R = \frac{\rho l}{A}$$

$\rho$  is called resistivity or specific resistance

$$\rho = \frac{RA}{l}$$

> Capacitor

capacitor is an electronic component used to store electric charge.

$$C = \frac{\epsilon A}{d} \quad C = \frac{q}{V}$$

Unit = Farad (F)

Energy stored in capacitor,  $E = \frac{1}{2} CV^2$

> Inductor

Inductance is the property of materials by which it opposes any change in the current flowing through it.

$$V = L \cdot \frac{di}{dt}$$

Energy stored in inductor,  $E = \frac{1}{2} Li^2$

## → Ohm's Law.

Ohm's Law states that at constant temperature the current through any conductor is directly proportional to the potential difference b/w its ends.

$$V \propto I$$

$$V = IR$$

## > Conductance

It is the reciprocal of resistance (R) and is the measure of the ease to the ~~resistance~~ strength flow of current through a substance. It is denoted by  $\alpha$ .

$$\alpha = \frac{1}{R}$$

unit is Siemens or mhos

$$\alpha = \frac{I}{\rho A} = \frac{\sigma A}{l}$$

$\sigma = \frac{1}{\rho}$  is conductivity or specific conductance

## > Electrical Power

It is defined as the rate of doing work. It is expressed in watts (W) and is given by,

$$P = V \times I = (IR) \times I = I^2 R$$

$$P = V \times I = V \times \frac{V}{R} = \frac{V^2}{R}$$

## > Electrical Energy

It is defined as the work done by the electrical power. It is expressed in Joules and is given by,  $E = \text{Power} \times \text{Time}$

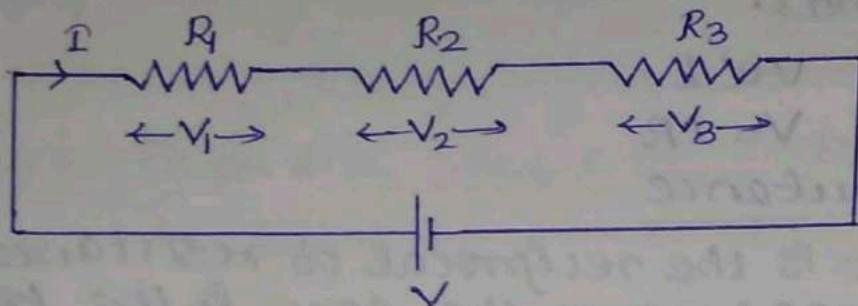
$$E = V \times I \times t \text{ Joules}$$

$$E = I^2 R t = \frac{V^2}{R} t \text{ Joules}$$

$$1 \text{ Joule} = 1 \text{ watt-second}$$

## \* Resistance in series

The circuit in which resistances are connected end to end so that there is only one path for current flow, is called a series circuit.



$$R_{\text{eq}} = R_1 + R_2 + R_3$$

$$\text{From fig, } V_s = V_1 + V_2 + V_3$$

$$V = I R_{\text{eq}}$$

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$$

$$V_s = V_1 + V_2 + V_3$$

$$I R_{\text{eq}} = IR_1 + IR_2 + IR_3$$

$$\therefore R_{\text{eq}} = \underline{\underline{R_1 + R_2 + R_3}}$$

$$I = \frac{V}{R_{\text{eq}}}$$

$$V_1 = IR_1 = \frac{V}{R_{\text{eq}}} \cdot R_1 = V \cdot \frac{R_1}{R_1 + R_2 + R_3}$$

$$V_2 = V \cdot \frac{R_2}{R_1 + R_2 + R_3}$$

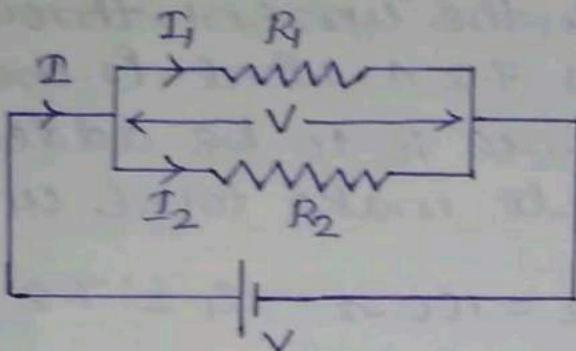
$$V_3 = V \cdot \frac{R_3}{R_1 + R_2 + R_3}$$

Voltage drop in each resistance =

main voltage × corresponding resistance  
Total resistance

"Voltage division rule states that voltage drop across any resistance in a series circuit is proportional to the ratio of its resistance to the total resistance."

### \* Resistances in Parallel



$$I = I_1 + I_2$$

$$V = I_1 R_1 = I_2 R_2 = I R_{\text{Req}}$$

$$\frac{1}{R_{\text{Req}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{\text{Req}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 R_1 = I R_{\text{Req}}$$

$$= I \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

$$I_1 = I \cdot \frac{R_1 \cdot R_2}{R_1 (R_1 + R_2)} = I \cdot \frac{R_2}{R_1 + R_2}$$

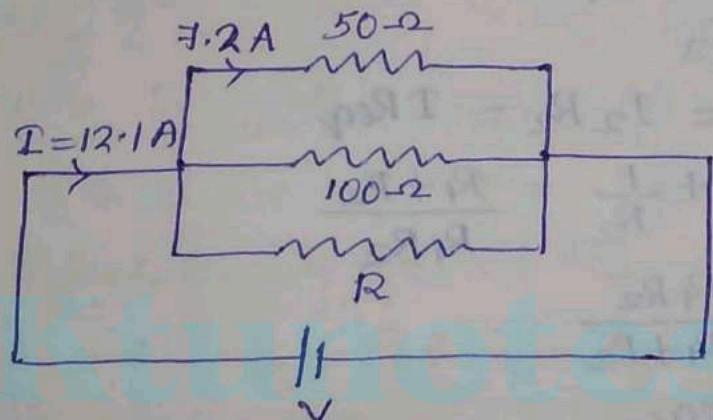
$$I_2 = I \cdot \underline{\underline{\frac{R_1}{R_1 + R_2}}}$$

current through each branch  $I_{\text{Req}, \text{each}} = \frac{\text{main current} \times \text{Resistance of the other branch}}{\text{total resistance}}$

"current division rule states that current through one branch is equal to the ratio of resistance of the other branch to the total resistance multiplied by line current (total main current)"

Q:1: A  $50\text{-}\Omega$  resistor is in parallel with  $100\text{-}\Omega$  resistor and the current through  $50\text{-}\Omega$  resistance is  $7.2\text{ A}$ . What is the value of a third resistance  $R$  to be added in parallel to the circuit to make total current as  $12.1\text{ A}$ .

$$\rightarrow R_1 = 50\text{-}\Omega, R_2 = 100\text{-}\Omega, I_1 = 7.2\text{ A}$$



using current division rule,  
voltage drop across  $50\text{-}\Omega$ ,

$$V = IR = 7.2 \times 50 = \underline{\underline{360\text{ V}}}$$

Total current,  $I = 12.1\text{ A}$

$$R = 100\text{-}\Omega \quad V = 360\text{ V}$$

$$I_2 = \frac{360}{100} = \underline{\underline{3.6\text{ A}}}$$

$$I = I_1 + I_2 + I_3$$

$$I_3 = I - I_1 + I_2$$

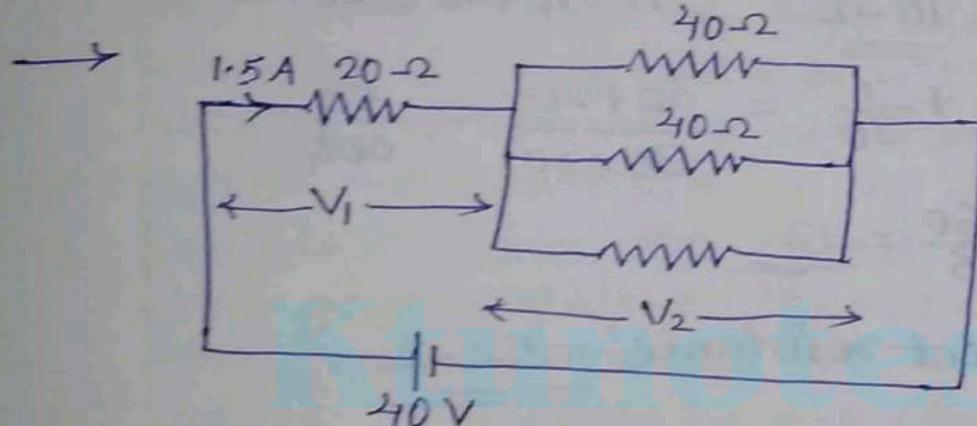
$$= 12.1 - 7.2 + 3.6$$

$$= \underline{\underline{1.3\text{ A}}}$$

$$I_3 = 1.3 \text{ A}, V = 360 \text{ V}$$

$$R_3 = \frac{V}{I_3} = \frac{360}{1.3} = \underline{\underline{277\Omega}}$$

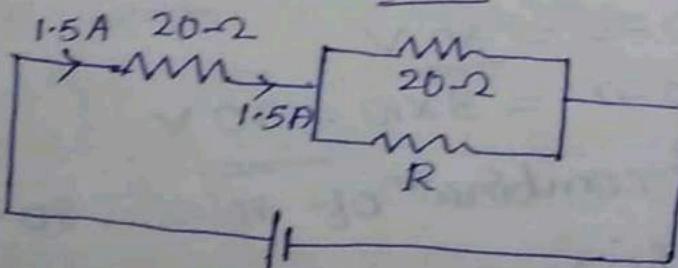
Q:2: A resistance of  $20\Omega$  is connected in series with a combination of two resistances arranged in parallel each having a value of  $40\Omega$ . Determine the resistance  $R$  which is connected in parallel across the parallel combination so that total current drawn by the circuit is  $1.5 \text{ A}$  with applied voltage of  $40 \text{ V}$ .



voltage across  $20\Omega$  resistor,

$$V_1 = 1.5 \times 20 = 30 \text{ V}$$

$$V_2 = 40 - V_1 = \underline{\underline{10 \text{ V}}}$$



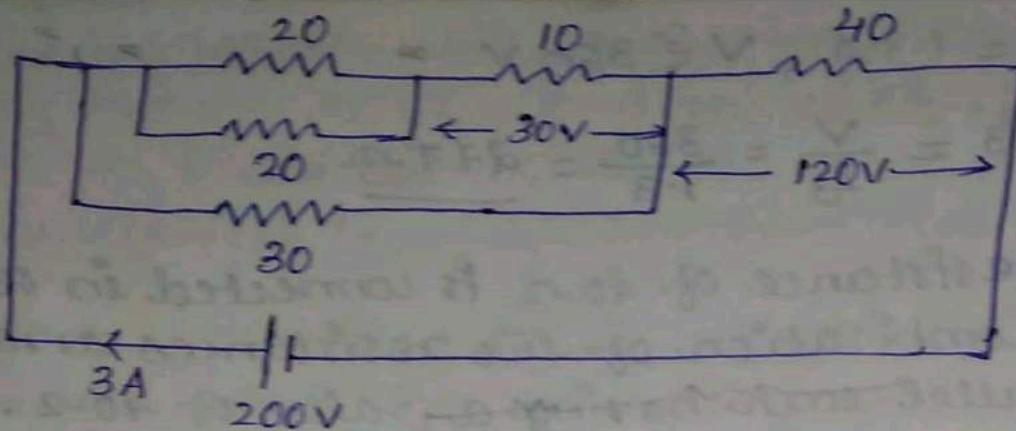
$$V_2 = 10 \text{ V}$$

$$I_1 = \frac{V_2}{R} = \frac{10}{20} = \underline{\underline{0.5}}$$

$$I_2 = 1.5 - 0.5 = \underline{\underline{1 \text{ A}}}$$

$$R = \frac{V}{I} = \frac{10}{1} = \underline{\underline{10\Omega}}$$

Q: 8:



→ Find the equivalent resistance of the circuit given and also find voltage drop across each resistance.

$$R_{eq} = \frac{20}{2} = \underline{\underline{10\Omega}} \quad R = 10 + 10 = 20\Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{30} = \frac{30+20}{30 \times 20} = \frac{50}{600}$$

$$\therefore R_{eq} = \frac{60}{5} = \underline{\underline{12\Omega}}$$

$$\therefore \text{Equivalent resistance} = \underline{\underline{52\Omega}}$$

$$V \text{ across } 40\Omega = 3 \times 40 = \underline{\underline{120V}}$$

$$I \text{ across } 11\Omega \text{ combination} = 3A$$

$$V \text{ across } 11\Omega \text{ combination} = \underline{\underline{80V}}$$

$$V \text{ across } 30\Omega = \underline{\underline{80V}}$$

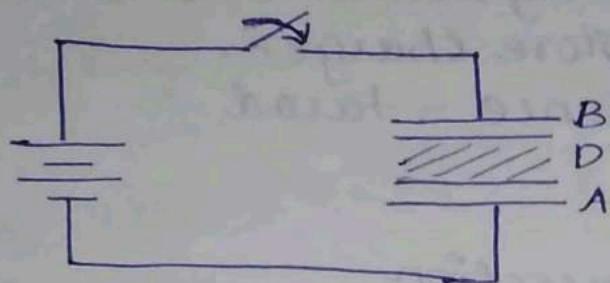
$$V \text{ across } 10\Omega = \underline{\underline{30V}}$$

$$V \text{ across } 11\Omega \text{ combination} = \underline{\underline{80V}}$$

$$\therefore V \text{ across each } 20\Omega = \underline{\underline{50V}}$$

## \* Capacitors

capacitor consists of two thin parallel plates of conducting material separated by a dielectric material.



A and B are the capacitor plates and D is the dielectric material. capacitor is capable of storing charges when a voltage source connected across the capacitor as shown in fig. Electrons from -ve terminals of the battery accumulated on plate A and the excess ens produces -ve charge on plate A. then plate B losses ens as they move towards +ve terminal of the battery hence it maintains +ve charge.

### Charging

Built-up of voltage across the capacitor plates due to the accumulation of ens when a voltage source is provided is said to be charging of capacitor. voltage across the capacitor become equal to source voltage when the capacitor is fully charged. capacitor remains charged even after the disconnection of source voltage.

### Discharging

The capacitor discharges when a conducting path is provided across the capacitor plates without connecting any voltage source.

amount of charge,  $Q$ , stored in a capacitor is proportional to the charging voltage. i.e,

$$Q \propto V$$

$$Q = CV$$

where  $C$  is the capacitance. It is the ability of a capacitor to store charge.

unit of capacitance = Farad

$$C = \frac{AE}{d}$$

$A \rightarrow$  area of cross section

$d \rightarrow$  distance b/w capacitor plates

$\epsilon \rightarrow$  absolute permittivity.

$$Q = CV$$

$$dq = C dV$$

$$i dt = C dV$$

$$i = C \cdot \underline{\frac{dV}{dt}}$$

$$i = \frac{dq}{dt}$$

$$dq = i \cdot dt$$

→ Energy stored in a capacitor

When a voltage applied across the parallel plates, charge stored by the capacitor is given by,

$$Q = CV$$

and the workdone by transferring  $dq$  coulombs of charge can be expressed as,

$$dw = V dq$$

$$\text{also, } dq = C dV$$

∴ total workdone in raising the potential of capacitor to applied voltage,  $V$  is given by,

$$W = \int_0^V dw$$

$$W = \int_0^V vcdv$$

$$= C \int_0^V vdv = C \left[ \frac{v^2}{2} \right]_0^V$$

$$\therefore W = \underline{\underline{\frac{1}{2} CV^2}}$$

### \* Inductors.

Inductors are made of coils having 'N' no: of turns. The core of the coil may be air or any magnetic material which is placed inside the coil. When the coil is wound on an iron core it is called iron core inductor.

Inductance of a coil is directly proportional to square of no: of turns.

$$L \propto N^2$$

Inductance is the ability of a coil to induce emf in it when the current flowing through the coil changes.

1 Henry is defined as ability to induce 1 volt emf when the current through it changes in the rate 1 ampere/sec. according to Faraday's Law of Electromagnetic Induction,

$$e \propto \frac{di}{dt}$$

$$e = L \cdot \frac{di}{dt}$$

When a steady dc current flows through the inductor, no emf is induced in that inductor since there is no change in current.

→ Energy stored in an inductor

$$V = e = L \cdot \frac{di}{dt}$$

Power stored in an inductor is,

$$P = Vi$$

$$= L \cdot \frac{di}{dt} i = L \cdot i \cdot \frac{di}{dt}$$

Energy at any instant,

$$E = Pxt$$

$$E = \int_0^t P dt = \int_0^t L i \frac{di}{dt} dt$$

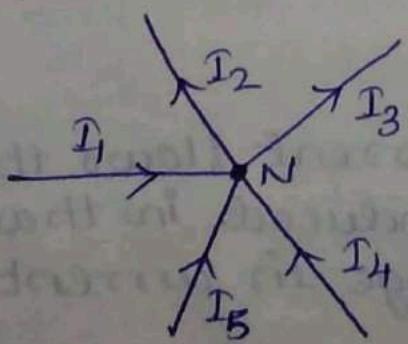
$$= L \int_0^t i di$$

$$\underline{\underline{E = \frac{1}{2} L i^2}}$$

\* Kirchhoff's Laws

(i) Kirchhoff's Current Law

In any electrical network algebraic sum of currents meeting at any junction is equal to zero. i.e., sum of currents entering the node is equal to sum of currents leaving the node.



$$\sum i = 0$$

$$I_1 + I_4 + I_5 - I_2 - I_3 = 0$$

$$\underline{\underline{I_1 + I_4 + I_5 = I_2 + I_3}}$$

leaving current -ve  
entering current +ve

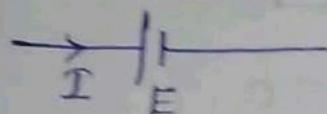
## ② Kirchhoff's Voltage Law

In a closed mesh, algebraic sum of voltage drop across a resistor and algebraic sum of emf acting across the circuit is equal to zero.

$$\text{ie, } \sum \text{emf} + \sum IR = 0$$

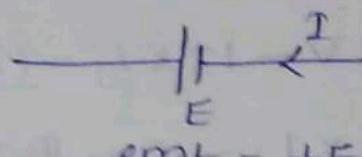
Rules for applying KVL

(I) Fall of potential



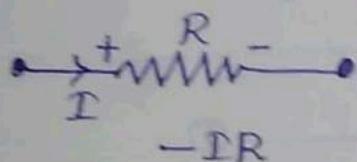
$$\text{emf} = -E$$

Rise of potential

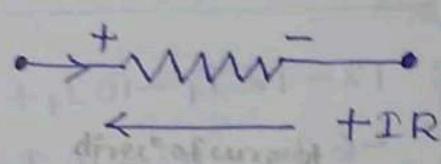


$$\text{emf} = +E$$

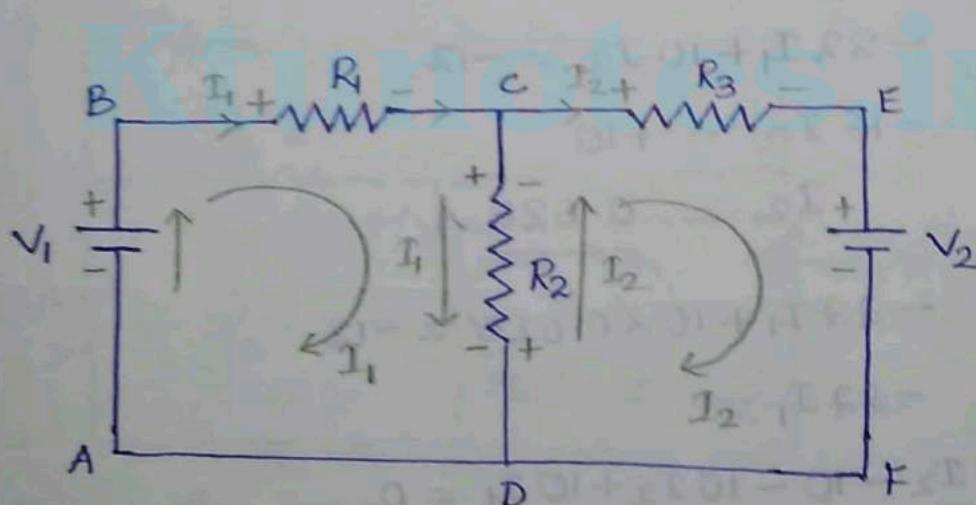
(II)



$$-IR$$



$$+IR$$



ABCDA

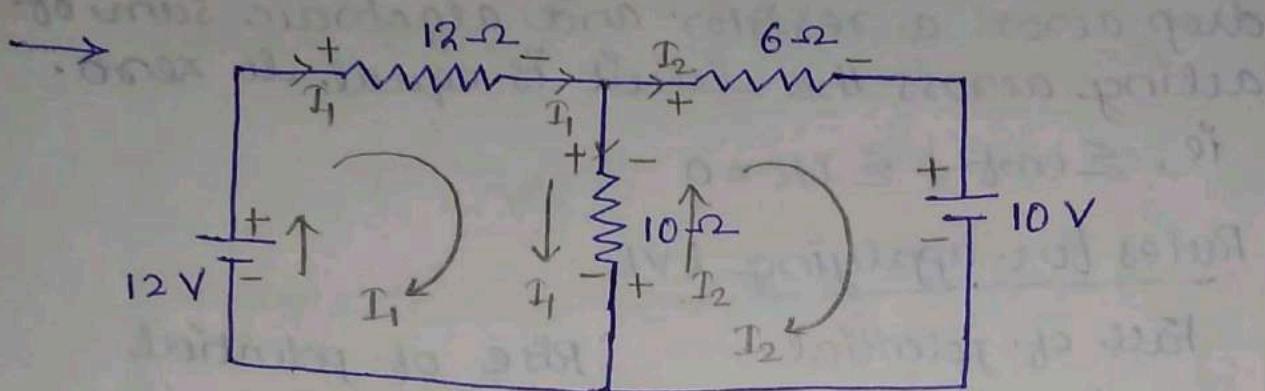
$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0$$

C E F D C

$$- I_2 R_3 - V_2 - (I_2 - I_1) R_2 = 0$$

.....

Q:- In the circuit given find out current through each resistor and voltage drop across each resistor.



$$+12V - 12 \times I_1 - (I_1 - I_2) \times 10 = 0$$

$$-6 \times I_2 - 10 - (I_2 - I_1) \times 10 = 0$$


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$$12 - 12I_1 - 10I_1 + 10I_2 = 0 \quad \text{--- (1)}$$

$$-6I_2 - 10 - 10I_2 = 0 \quad \text{--- (2)}$$

$$\textcircled{1} \Rightarrow -22I_1 + 10I_2 = -12$$

$$\textcircled{2} \Rightarrow -16I_2 = +10$$

$$I_2 = \underline{\underline{0.625 \text{ A}}}$$

$$-22I_1 + 10 \times 0.625 = -12$$

$$-22I_1 =$$

$$\textcircled{3} \Rightarrow -6I_2 - 10 - 10I_2 + 10I_1 = 0$$

$$\textcircled{4} \Rightarrow (10I_1 - 16I_2 = 10) \times 10$$

$$\textcircled{5} \Rightarrow (-22I_1 + 10I_2 = -12) \times 16$$

$$\textcircled{6} \Rightarrow 100I_1 - 160I_2 = 100 + \\ -352I_1 + 160I_2 = -192$$


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$$-252I_1 = -92$$

$$I_1 = \frac{-92}{-252} = \underline{\underline{0.365 \text{ A}}}$$

$$100I_1 - 160I_2 = 100$$

$$100 \times 0.365 - 160I_2 = 100$$

$$\therefore I_2 = -0.396 \text{ A}$$

-ve sign indicates that direction of current is opposite to the assumed direction.

$$\text{current across } 10\Omega = I_1 + I_2$$

$$I_1 + I_2 = 0.365 + 0.396 = 0.761 \text{ A}$$

voltage drop across  $12\Omega$  resistor,

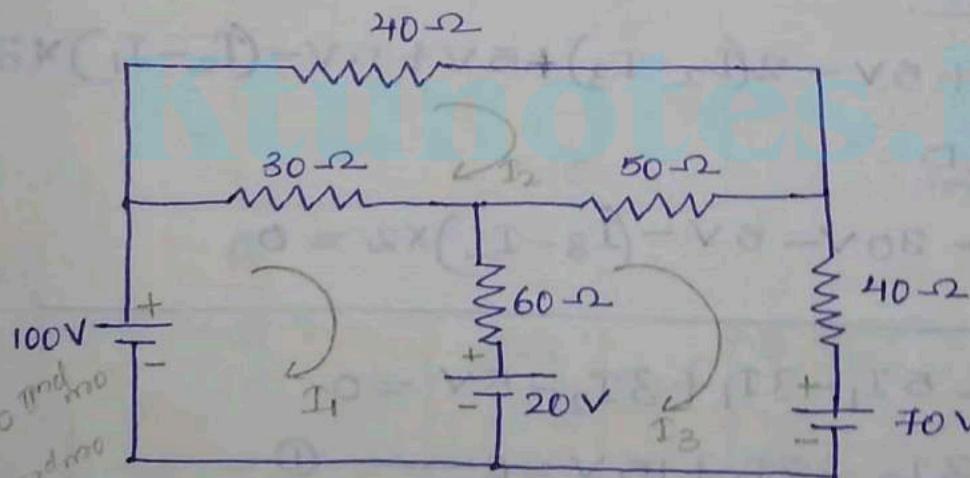
$$V = IR = 0.365 \times 12 \\ = 4.38 \text{ V}$$

voltage drop across  $10\Omega$  resistor,

$$V = 0.761 \times 10 = 7.61 \text{ V}$$

voltage drop across  $6\Omega$  resistor,

$$V = -0.396 \times 6 = -2.376 \text{ V}$$



Inspection method,

$$R = \begin{bmatrix} 90 & -30 & -50 \\ -30 & 120 & -50 \\ -60 & -50 & 150 \end{bmatrix} \quad \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \\ -50 \end{bmatrix}$$

$$\text{Ist mesh } 90I_1 - 30I_2 - 60I_3 = 80$$

$$\text{IInd mesh } -30I_1 + 120I_2 - 50I_3 = 0$$

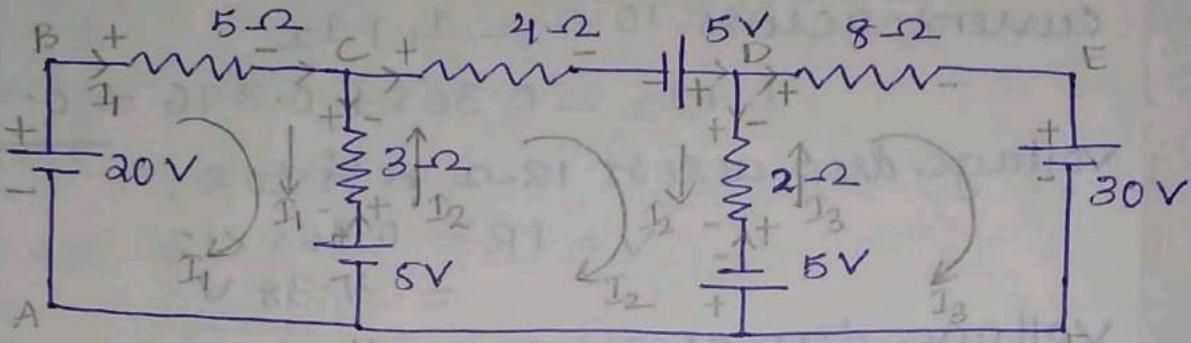
$$\text{IIIrd mesh } -60I_1 - 50I_2 + 150I_3 = 0$$

$$\therefore I_1 = \underline{1.91 A}$$

$$\therefore I_2 = \underline{0.92 A}$$

$$\therefore I_3 = \underline{1.074 A}$$

Q:- Determine the mesh currents in the circuit given



ABC HA

$$20V - 5I_1 - (I_1 - I_2) \times 3 - 5V = 0$$

CDGHC

$$-4I_2 + 5V - 2(I_2 - I_3) + 5V + 5V - (I_2 - I_1) \times 3 = 0$$

DEFGD

$$-8I_3 - 30V - 5V - (I_3 - I_2) \times 2 = 0$$

$$20V - 5I_1 - 3I_1 + 3I_2 - 5V = 0$$

$$3I_2 - 8I_1 + 15V = 0 \quad \text{--- } ①$$

$$-4I_2 + 5V - 2I_2 + 2I_3 - 3I_2 + 3I_1 = 0$$

$$15V - 9I_2 + 2I_3 + 3I_1 = 0 \quad \text{--- } ②$$

$$-8I_3 - 30V - 5V - 2I_3 + 2I_2 = 0$$

$$2I_2 - 10I_3 - 35V = 0 \quad \text{--- } ③$$

$$-8I_1 + 3I_2 + 15V = 0 \quad \text{--- (1)}$$

$$3I_1 - 9I_2 + 2I_3 + 15V = 0 \quad \text{--- (2)}$$

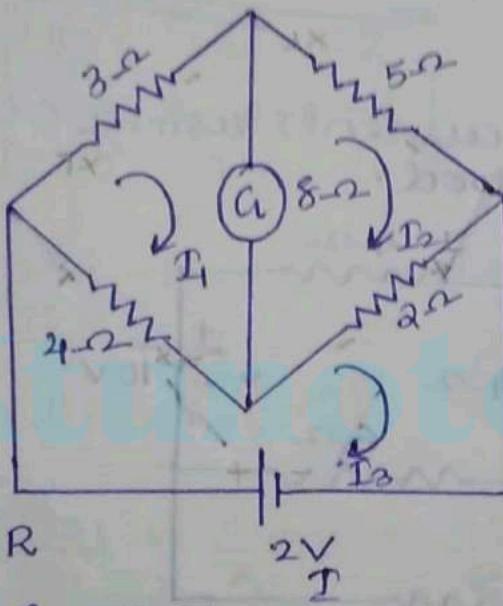
$$2I_2 + 10I_3 - 35V = 0 \quad \text{--- (3)}$$

$$\therefore I_1 = \underline{0.558 A}$$

$$\therefore I_2 = \underline{1.822 A}$$

$$\therefore I_3 = \underline{3.135 A}$$

Q: Determine current flowing through the galvanometer.



$$\begin{bmatrix} 15 & -8 & -4 \\ -8 & 15 & -2 \\ -4 & -2 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}$$

$$15I_1 - 8I_2 - 4I_3 = 0$$

$$-8I_1 + 15I_2 - 2I_3 = 0$$

$$-4I_1 - 2I_2 + 6I_3 = 2$$

$$I_1 = \underline{0.282 A}$$

$$I_2 = \underline{0.230 A}$$

$$I_3 = \underline{0.598 A}$$

$$I_A = I_1 - I_2 = \underline{0.053 A}$$

Cramer's rule

$$A \Rightarrow \begin{bmatrix} 15 & -8 & -4 \\ -8 & 15 & -2 \\ -4 & -2 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & -8 & -4 \\ 0 & 15 & -2 \\ 2 & -2 & 6 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 15 & 0 & -4 \\ -8 & 0 & -2 \\ -4 & 2 & 6 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 15 & -8 & 0 \\ -8 & 15 & 0 \\ -4 & -2 & 2 \end{bmatrix}$$

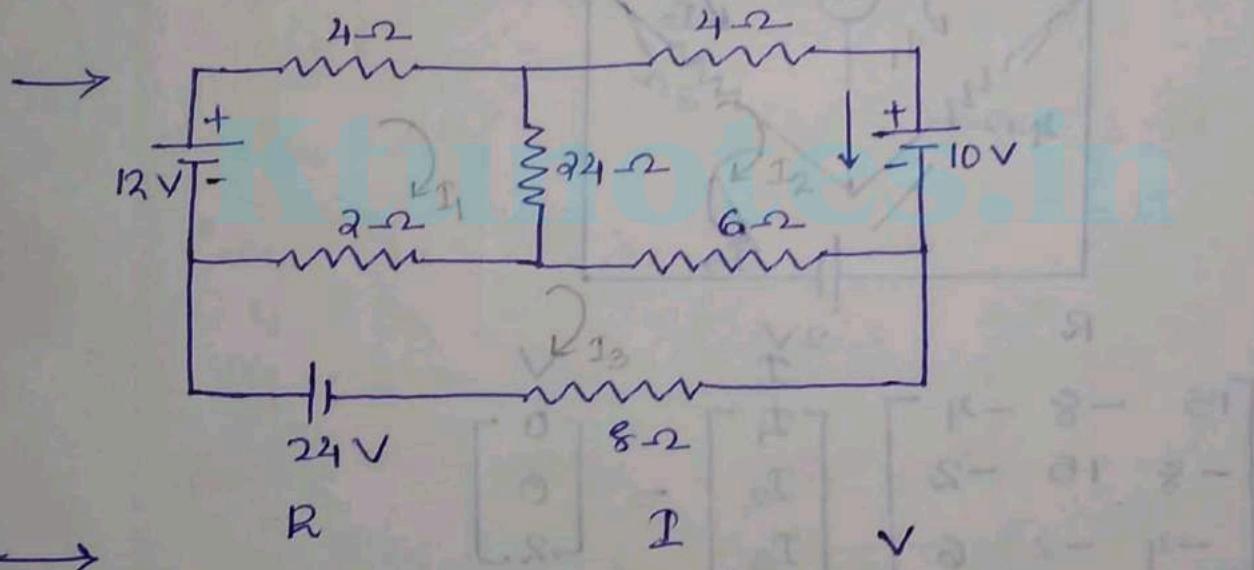
$$|A| = ?, \quad |A_1| = ?$$

$$I_2 = \frac{|A_2|}{|A|}$$

$$I_3 = \frac{|A_3|}{|A|}$$

Hess analysis  
or Maxwell's loop  
current method.

Q:- Solve for loop currents using Cramer's rule.  
or matrix method.



$$A = \begin{bmatrix} 30 & -24 & -2 \\ -24 & 34 & -6 \\ -2 & -6 & 16 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -10 \\ 24 \end{bmatrix}$$

$$|A| = 30 \times [16 \times 34 - (-6 \times -6)] - 24 \times [-24 \times 16 - (-6 \times -2)]$$

$$+ -2(-24 \times 6 - -2 \times 34)$$

$$= 5312$$

$$A_1 = \begin{bmatrix} 12 & -24 & -2 \\ -10 & 34 & -6 \\ 24 & -6 & 16 \end{bmatrix}$$

$$\begin{aligned} |A_1| &= 12 \times (34 \times 16 - -6 \times -6) - 24 (-10 \times 16 - -6 \times 24) + \\ &\quad - 2 (-10 \times -6 - 24 \times 34) \\ &= \underline{\underline{5312}} \end{aligned}$$

$$I_1 = \frac{|A_1|}{|A|} = \frac{\cancel{5312}}{\cancel{5312}} = \frac{\underline{\underline{1.359}}}{\underline{\underline{A}}} = 1.359 A$$

$$A_2 = \begin{bmatrix} 30 & 12 & -2 \\ -24 & -10 & -6 \\ -2 & 24 & 16 \end{bmatrix}$$

$$\begin{aligned} |A_2| &= 30 \times (-10 \times 16 - -6 \times 24) - 12 (-24 \times 16 - -2 \times -6) + \\ &\quad - 2 (-24 \times 24 - -2 \times -10) \\ &= \underline{\underline{5464}} \end{aligned}$$

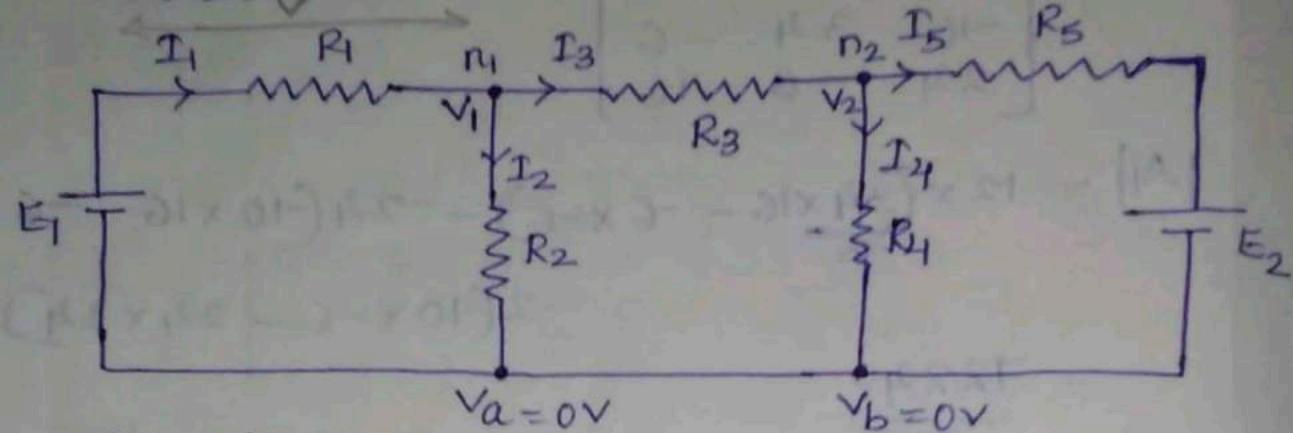
$$I_2 = \frac{|A_2|}{|A|} = \frac{\underline{\underline{5464}}}{\underline{\underline{5312}}} = \underline{\underline{1.028}} A = 1.028 A$$

$$A_3 = \begin{bmatrix} 30 & -24 & 12 \\ -24 & 34 & -10 \\ -2 & -6 & 24 \end{bmatrix}$$

$$|A_3| = \underline{\underline{10920}}$$

$$I_3 = \frac{|A_3|}{|A|} = \frac{\underline{\underline{10920}}}{\underline{\underline{5312}}} = \underline{\underline{2.055}} A = 2.055 A$$

\* Node Voltage Method



According to KCL,

$$I_1 = I_2 + I_3$$

$$I_1 - I_2 - I_3 = 0$$

$$\left( \frac{E_1 - V_1}{R_1} \right) - \left( \frac{V_1 - 0}{R_2} \right) - \left( \frac{V_1 - V_2}{R_3} \right) = 0$$

$$\frac{E_1 - V_1}{R_1} - \frac{V_1}{R_2} - \frac{V_1}{R_3} + \frac{V_2}{R_3} = 0$$

$$\frac{E_1}{R_1} - V_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{V_2}{R_3} = 0 \quad \textcircled{1}$$

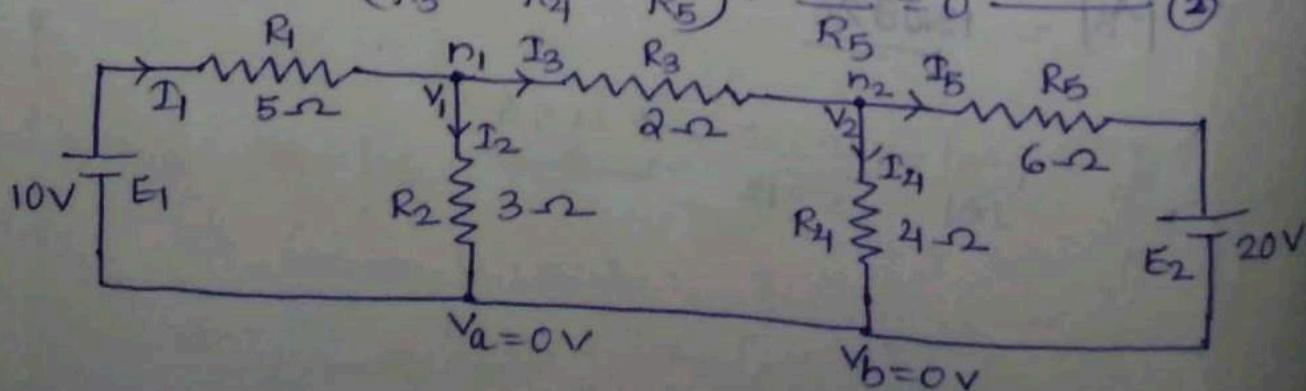
consider node 'n2' ;

$$I_3 - I_4 - I_5 = 0$$

$$\left( \frac{V_1 - V_2}{R_3} \right) - \left( \frac{V_2 - 0}{R_4} \right) - \left( \frac{V_2 - E_2}{R_5} \right) = 0$$

$$\frac{V_1}{R_3} - \frac{V_2}{R_3} - \frac{V_2}{R_4} - \frac{V_2}{R_5} + \frac{E_2}{R_5} = 0$$

$$\frac{V_1}{R_3} - V_2 \left( \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right) + \frac{E_2}{R_5} = 0 \quad \textcircled{2}$$



$$\textcircled{1} \Rightarrow \frac{10}{5} - V_1 \left( \frac{1}{5} + \frac{1}{3} + \frac{1}{2} \right) + \frac{V_2}{2} = 0$$

$$\textcircled{2} \Rightarrow \frac{V_1}{2} - V_2 \left( \frac{1}{2} + \frac{1}{4} + \frac{1}{6} \right) + \frac{30}{6} = 0$$

$$\textcircled{1} \Rightarrow 2 - V_1 \left( \frac{31}{30} \right) + \frac{V_2}{2} = 0$$

$$\textcircled{2} \Rightarrow \frac{V_1}{2} - V_2 \left( \frac{11}{12} \right) + 3 \cdot 33 = 0$$

$$-\frac{31}{30} (V_1) + \frac{V_2}{2} = -2$$

$$\frac{V_2}{2} - \frac{31 V_1}{30} = -2$$

$$\frac{30 V_2 - 62 V_1}{60} = -2 \Rightarrow 30 V_2 - 62 V_1 = -120 \quad \textcircled{3}$$

$$\frac{V_1}{2} - \frac{11 V_2}{12} = -3 \cdot 33$$

$$\frac{12 V_1 - 22 V_2}{24} = -3 \cdot 33$$

$$\Rightarrow 12 V_1 - 22 V_2 = -79.92 \quad \textcircled{4}$$

$$\therefore V_2 = \underline{\underline{6.369}} \text{ V}$$

$$\therefore V_1 = \underline{\underline{5.017}} \text{ V}$$

$$I_1 = \frac{E_1 - V_1}{R_1} = \frac{10 - \cancel{6.369}}{5} = \underline{\underline{0.996}} \text{ A}$$

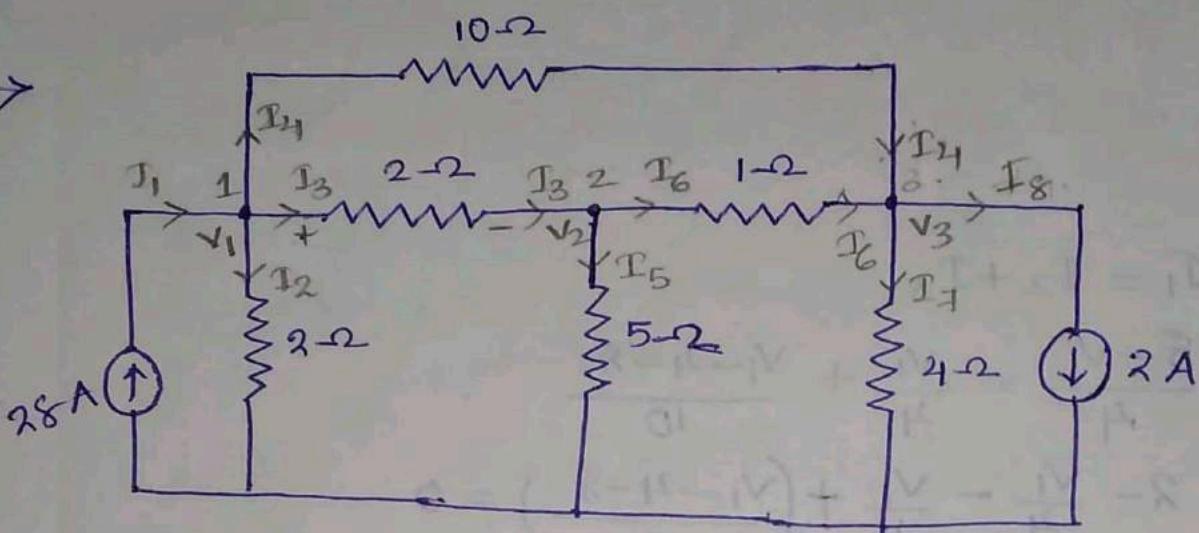
$$I_2 = \frac{V_1}{R_2} = \frac{5.017}{3} = \underline{\underline{1.672}} \text{ A}$$

$$I_3 = \frac{V_1 - V_2}{R_3} = \frac{5.017 - 6.369}{2} = \underline{\underline{0.676}} \text{ A}$$

$$I_4 = \frac{V_2}{R_4} = \frac{6.369}{4} = \underline{\underline{1.59}} \text{ A}$$

$$I_5 = \frac{V_2 - E_2}{R_5} = \frac{6.369 - 30}{6} = \underline{\underline{3.27}} \text{ A}$$

\* Use nodal analysis to form network equations and solve nodal voltages using matrix method. Also calculate current in different branches.



Node 1,

$$I_1 - I_2 - I_3 - I_4 = 0$$

$$28A - \frac{V_1}{2} - \left( \frac{V_1 - V_2}{2} \right) - \left( \frac{V_1 - V_3}{10} \right) = 0$$

$$28 - \frac{V_1}{2} - \frac{V_1}{2} + \frac{V_2}{2} - \frac{V_1}{10} + \frac{V_3}{10} = 0$$

$$28 - V_1 \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{10} \right) + \frac{V_2}{2} + \frac{V_3}{10} = 0$$

$$28 - \frac{11V_1}{10} + \frac{V_2}{2} + \frac{V_3}{10} = 0$$

$$28 = \frac{11V_1}{10} - \frac{V_2}{2} - \frac{V_3}{10} \quad (1)$$

Node 2,

$$I_3 - I_5 - I_6 = 0$$

$$\left( \frac{V_1 - V_2}{2} \right) - \frac{V_2}{5} - \left( \frac{V_2 - V_3}{1} \right) = 0$$

$$\frac{V_1}{2} - \frac{V_2}{2} - \frac{V_2}{5} - \frac{V_2}{1} + \frac{V_3}{1} = 0$$

$$\frac{V_1}{2} - V_2 \left( \frac{1}{2} + \frac{1}{5} + 1 \right) + \frac{V_3}{1} = 0$$

$$\frac{V_1}{2} - \frac{17V_2}{10} + V_3 = 0 \quad (2)$$

Node 3,

$$T_6 + T_{24} - T_3 - T_8 = 0$$

$$\left( \frac{V_2 - V_3}{1} \right) + \left( \frac{V_1 - V_3}{10} \right) - \frac{V_3}{2} - 2 = 0$$

$$\frac{V_2}{1} - \frac{V_3}{1} + \frac{V_1}{10} - \frac{V_3}{10} - \frac{V_3}{2} = 2$$

$$V_2 + \frac{V_1}{10} - V_3 \left( \frac{1}{1} + \frac{1}{10} + \frac{1}{2} \right) = 2$$

$$\frac{V_1}{10} + V_2 - \frac{27V_3}{20} = 2 \quad (3)$$

~~$$V_1 = 28.415 V$$~~

~~$$V_2 = 5.565 V$$~~

~~$$V_3 = 4.746 V$$~~

$$\underline{\underline{V_1 = 36 V}} \quad \underline{\underline{V_2 = 20 V}} \quad \underline{\underline{V_3 = 16 V}}$$

$$I_2 = \frac{V_1}{2} = \frac{36}{2} = \underline{\underline{18 \Omega}}$$

$$I_3 = \frac{V_1 - V_2}{2} = \frac{36 - 20}{2} = \underline{\underline{8 \Omega}}$$

$$I_{24} = \frac{V_1 - V_3}{10} = \frac{36 - 16}{10} = \underline{\underline{2 \Omega}}$$

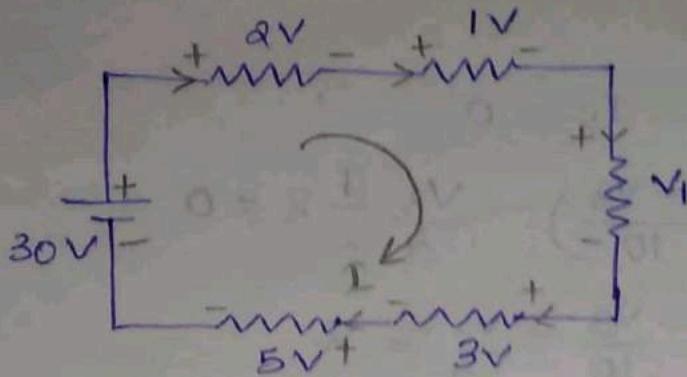
$$I_5 = \frac{V_2}{5} = \frac{20}{5} = \underline{\underline{4 \Omega}}$$

$$I_6 = \frac{V_2 - V_3}{1} = \frac{20 - 16}{1} = \underline{\underline{4 \Omega}}$$

$$I_7 = \frac{V_3}{4} = \frac{16}{4} = \underline{\underline{4 \Omega}}$$

- \* Determine unknown voltage drop  $V_1$  in the given circuit.



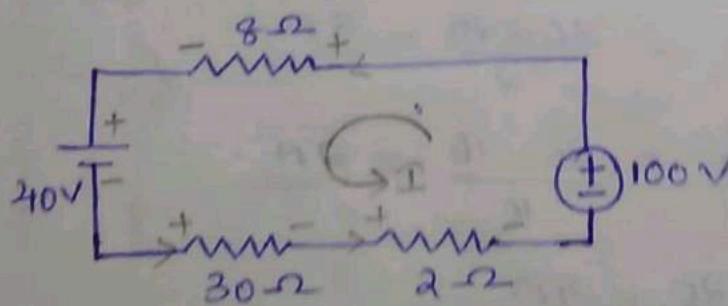
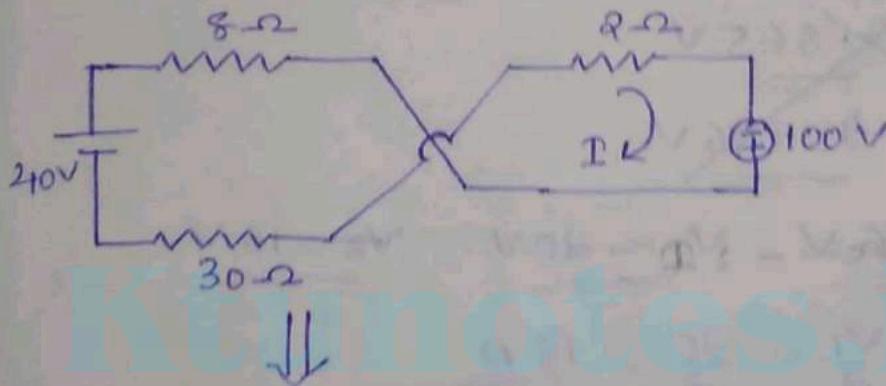


$$30 - 2I - 1 - V_1 - 3 - 5 = 0$$

$$19 - V_1 = 0$$

$$\therefore \underline{V_1 = +19 \text{ V}}$$

\* Find out current  $I$  and obtain drop across  $3\Omega$ .



$$-8I - 40 - 30I - 2I = 0 + 100 = 0$$

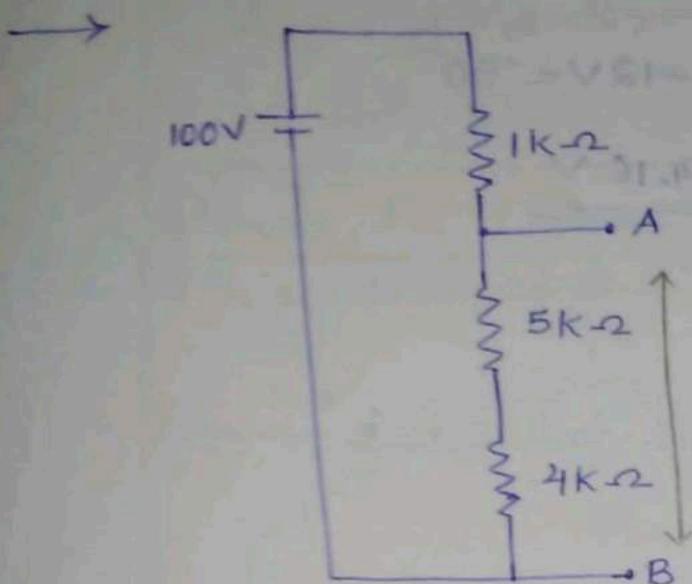
$$-40I - 40 = 0$$

$$-40I + 60 = 0$$

$$\therefore \underline{I = 1.5 \text{ A}}$$

$$\underline{V = 30I = 45 \text{ V}} \Rightarrow \text{drop across } 3\Omega.$$

\* Find voltage across A and B in the circuit given.



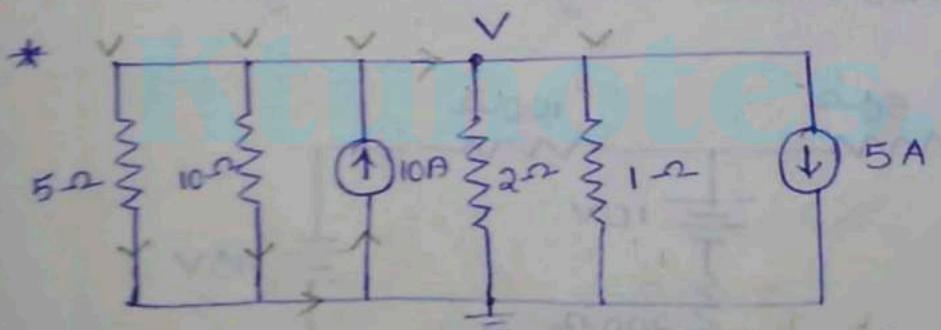
This is a series circuit  
Total  $R = 10k\Omega$

$$V_{AB} \sim ?$$

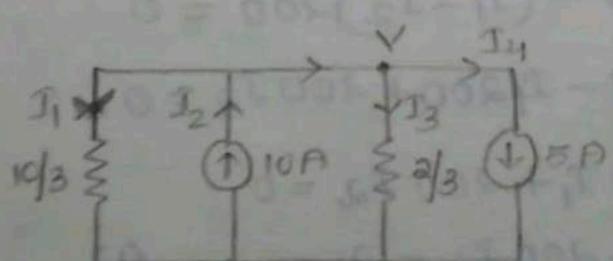
$$R = 15 + 4 = 9k\Omega$$

Voltage drop in each resistance =  
main voltage  $\times \frac{\text{corresponding resistance}}{\text{Total resistance}}$

$$V = 100 \times \frac{9}{10} = 90 \text{ V}$$



Find out current through each branch in the given circuit.



~~$I_1 + I_2 - I_3 - I_4 = 0$~~

$$\frac{V}{10/3} + 2A - \frac{V}{2/3} - 5 = 0$$

$$\frac{3V}{10} - \frac{3V}{2} + 5 = 0$$

$$\frac{3V}{10} - \frac{15V}{10} = -5$$

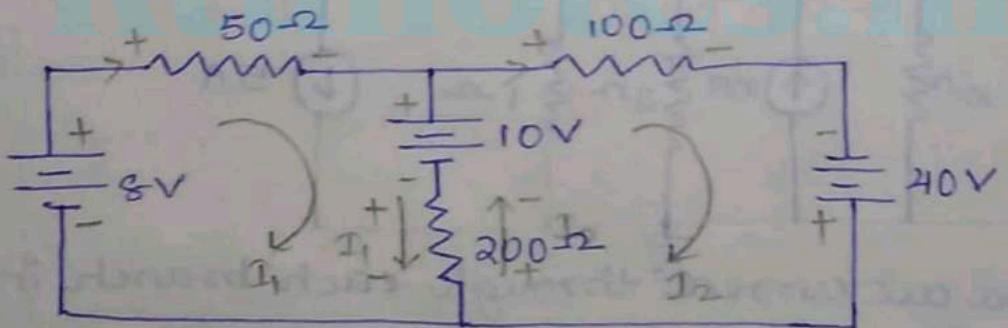
$$-\frac{12V}{10} = 5 \Rightarrow -12V = -50$$

$$V = \underline{\underline{4.16\text{ V}}}$$

$$I_1 = \frac{4.16}{10/3} =$$

$$I_2 =$$

- \* Determine power dissipated in all the resistors in the given circuit using mesh analysis.



$$8V - 50I_1 - 10V - (I_1 - I_2)200 = 0$$

$$8V - 50I_1 - 10V - I_1 200 + 200I_2 = 0$$

$$-2V - 250I_1 + 200I_2 = 0$$

$$-250I_1 + 200I_2 = 2 \quad (0)$$

$$-100I_2 + 40 - (I_2 - I_1)200 + 10 = 0$$

$$-100I_2 + 40 - 200I_2 + 200I_1 + 10 = 0$$

$$-300I_2 + 200I_1 + 50 = 0$$

$$200I_1 - 300I_2 = -50 \quad (2)$$

$$I_1 = 0.268 \text{ A}$$

$$I_2 = 0.345 \text{ A}$$

$$\boxed{\text{Power, } P = I^2 R = V I = \frac{V^2}{R}}$$

$$\begin{aligned} \text{Power dissipated in } 50\Omega &= (0.268)^2 \times 50 \\ &= 3.59 \text{ W} \end{aligned}$$

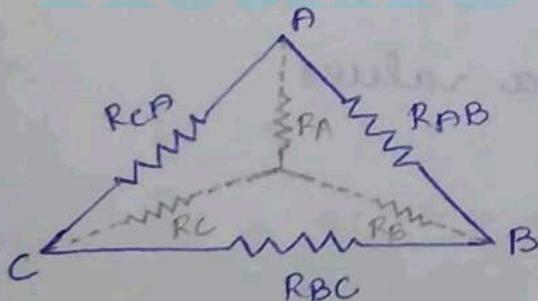
$$\begin{aligned} \text{Power dissipated in } 200\Omega &= (I_2 - I_1)^2 \times 200 \\ &= (0.077)^2 \times 200 \\ &= 1.185 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Power dissipated in } 100\Omega &= (0.345)^2 \times 100 \\ &= 11.902 \text{ W} \end{aligned}$$

~~W.H.P.~~

### Star-Delta Transformation

④ Delta  $\rightarrow$  Star

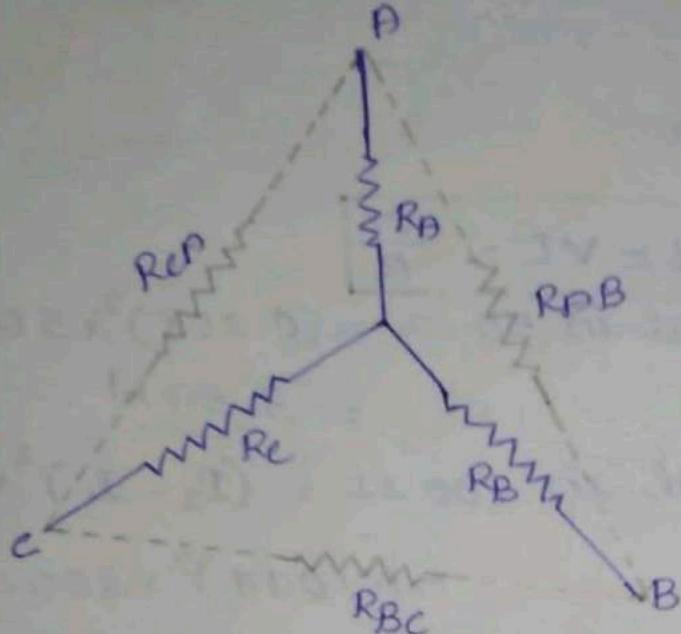


$$R_A = \frac{R_{AB} \cdot R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_B = \frac{R_{AB} \cdot R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_C = \frac{R_{CA} \cdot R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

(2) Star  $\rightarrow$  delta

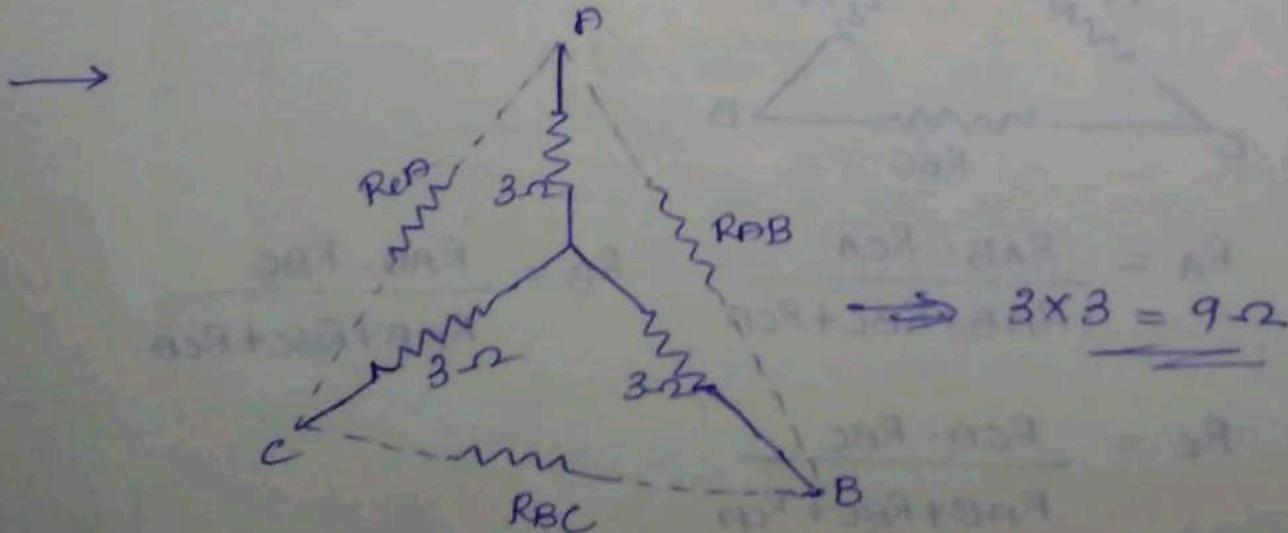


$$R_{AB} = R_A + R_B + \frac{R_A \cdot R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B \cdot R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C \cdot R_A}{R_B}$$

\* Find equivalent delta values.

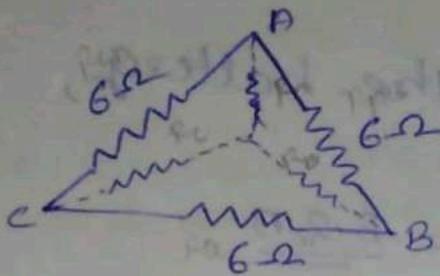


$$R_{AB} = 3 + 3 + \frac{3 \times 3}{3} = 9 \Omega$$

$$R_{BC} = 3 + 3 + \frac{3 \times 3}{3} = 9 \Omega$$

$$R_{CA} = 3 + 3 + \frac{3 \times 3}{3} = 9 \Omega$$

\* Find out equivalent star values.

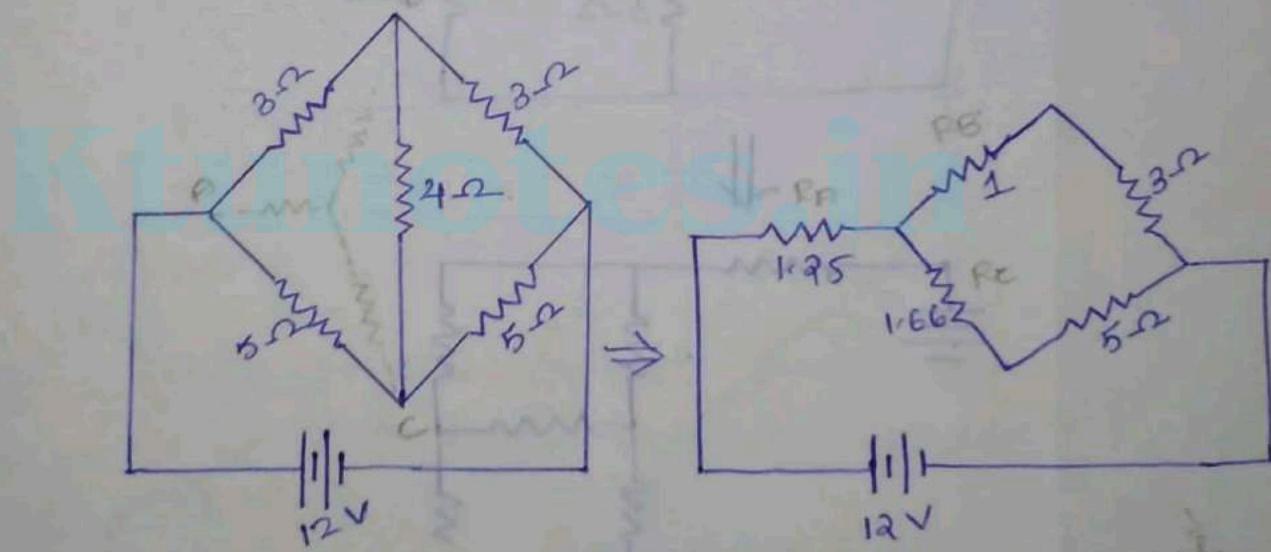
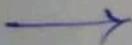


$$\Rightarrow \frac{6}{3} = \underline{\underline{2\Omega}}$$

$$R_{PAB} = \frac{6 \times 6}{6+6+6} = \underline{\underline{2\Omega}}$$

$$R_{BC} = \frac{6 \times 6}{6+6+6} = \underline{\underline{2\Omega}} \quad R_{CA} = \frac{6 \times 6}{6+6+6} = \underline{\underline{2\Omega}}$$

\* Calculate current supplied by the battery in the given circuit.

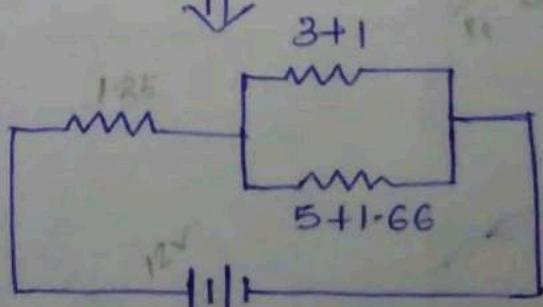


$$R_P = \frac{3 \times 5}{3+4+5} = \underline{\underline{1.25\Omega}}$$

$$R_B = \frac{3 \times 4}{3+4+5} = \underline{\underline{1\Omega}}$$

$$R_C = \frac{5 \times 4}{3+4+5} = \underline{\underline{1.66\Omega}}$$

$$\frac{1}{R_P} = \frac{1}{4} + \frac{1}{6.66} \Rightarrow \frac{1}{R_P} = \underline{\underline{0.144\Omega^{-1}}}$$



$$R_{eq} = \frac{3 \cdot 75}{2}$$

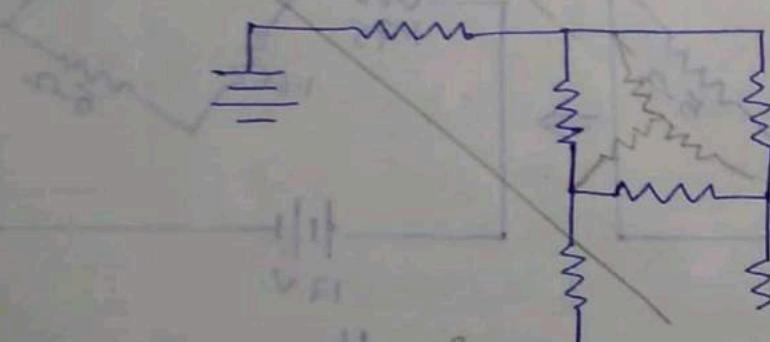
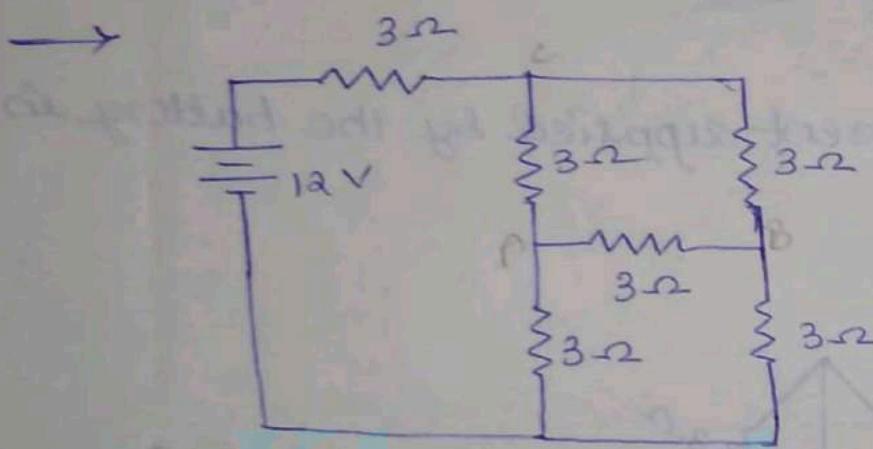
$$V = 12 \text{ V}$$

current supplied in the battery,

$$I = \frac{V}{R}$$

$$= \frac{12}{3.75} = \underline{\underline{3.2 D}}$$

- \* calculate current supplied by the battery to the network.



$$2+1+\bar{R}_S = 1+3 = 2_{f=2}$$

$$R_S = T_3 \approx 10$$

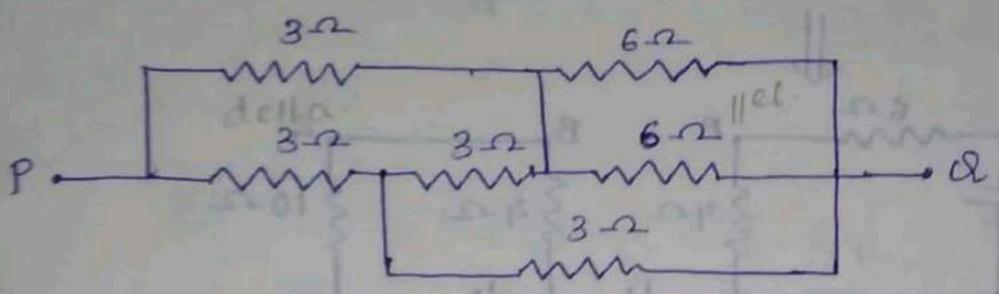
$$2 + \frac{1}{\beta_D} = \frac{1}{2} + \frac{1}{2}$$

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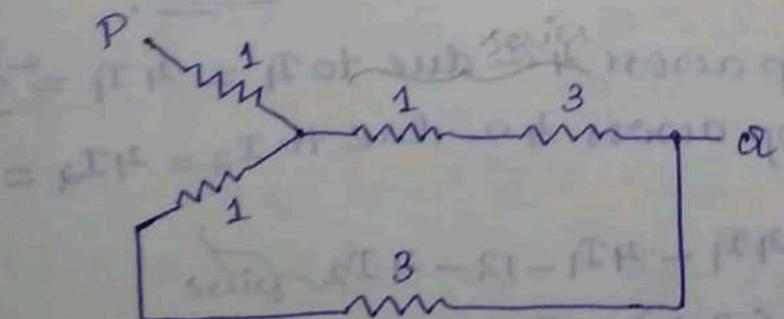
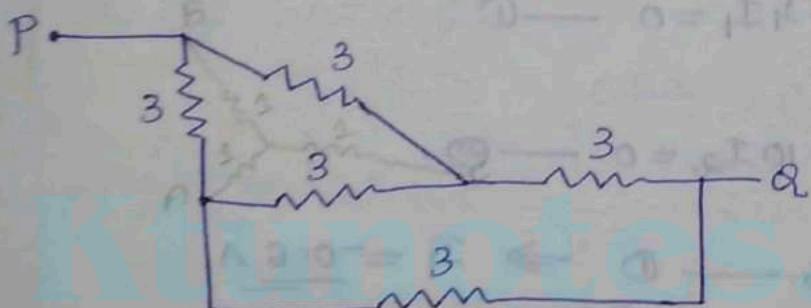
$$R_{eq} = 3 + 1 + 2 = \underline{\underline{6\Omega}}$$

$$I = \frac{V}{R} = \frac{12}{6} = 2A$$

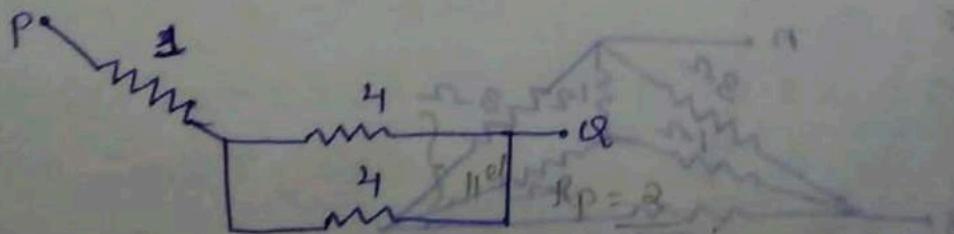
\* calculate equivalent resistance across the terminals P and Q using star-delta transformation.



$$R_p = \frac{R}{3} = \frac{6}{3} = \underline{\underline{2\Omega}}$$

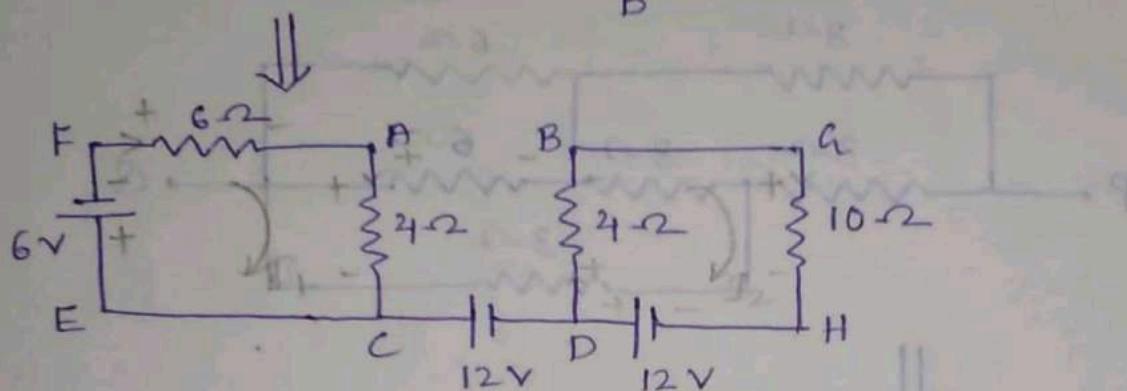
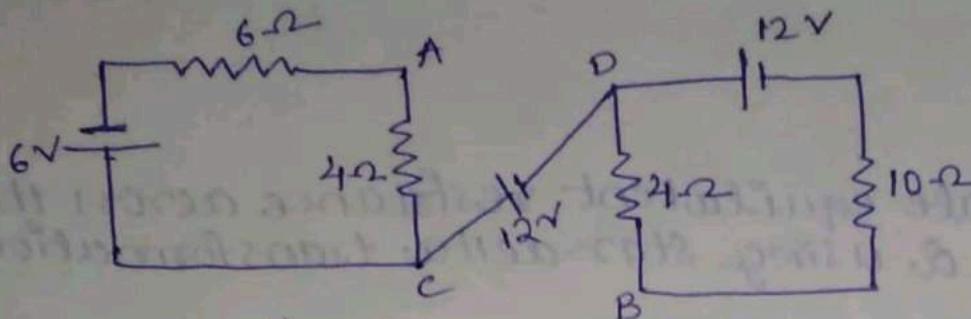


$$R_s = 1 + 1 = \underline{\underline{2\Omega}}$$



$$R_{eq} = 1 + 3 = \underline{\underline{3\Omega}}$$

\* What is the voltage b/w the terminals A and B.



ACEFA.

$$-6 - 6I_1 - 4I_1 = 0 \quad \text{--- (1)}$$

BDHGB.

$$12 - 4I_2 - 10I_2 = 0 \quad \text{--- (2)}$$

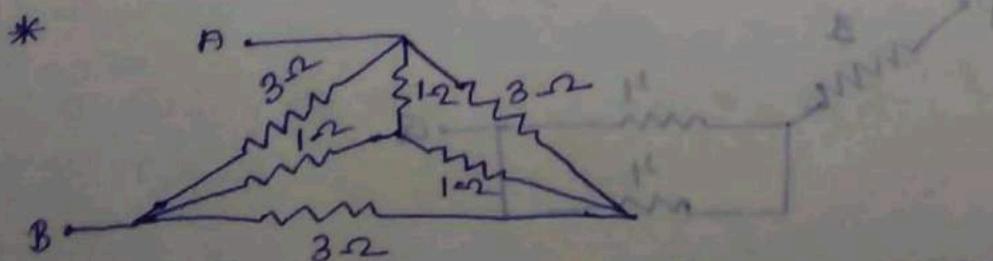
$$-10I_1 = 6 \quad \text{--- (1)} \Rightarrow I_1 = \underline{-0.6 \text{ A}}$$

$$-14I_2 = -12 \quad \text{--- (2)} \Rightarrow I_2 = \underline{0.857 \text{ A}}$$

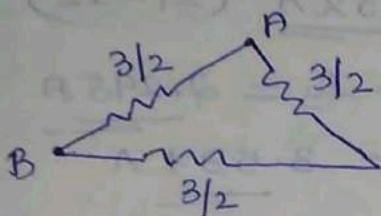
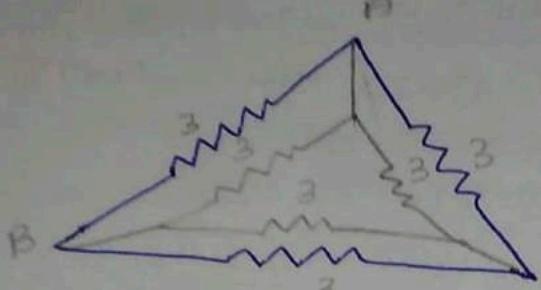
voltage drop across  $4\Omega$  due to  $I_1 = 4I_1 = \underline{-2.4 \text{ V}}$

voltage drop across  $4\Omega$  due to  $I_2 = 4I_2 = \underline{3.428 \text{ V}}$

$$\begin{aligned} V_{AB} &= 12 - 4I_1 - 12 - 4I_2 \\ &= -(2.4) - 12 - 3.428 = \underline{-13.028 \text{ V}} \end{aligned}$$



obtain equivalent resistance across the terminals A and B.

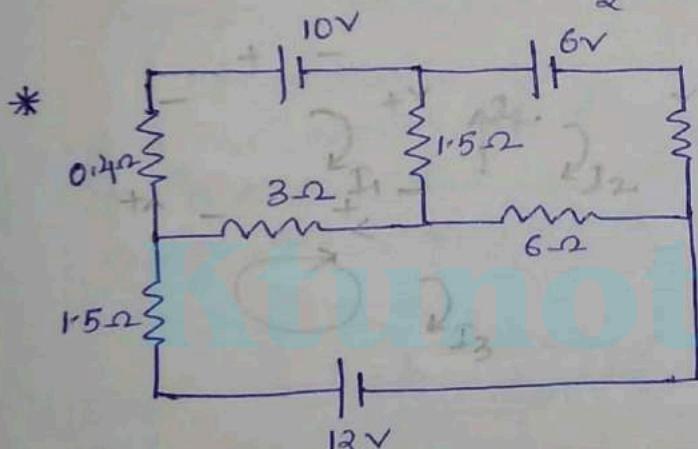


$$R_p = \frac{R}{n} = \frac{3}{3} = 1 \Omega$$

$$R_s = \frac{3}{2} + \frac{3}{2} = \frac{6}{2} = 3 \Omega$$

$$\text{Req b/w A \& B} = \frac{\frac{3}{2} \times 3}{\frac{3}{2} + 3} = 1 \Omega$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$



obtain all the branch currents in the circuit.

$$-\frac{(I_1 - I_3)3}{\pi^2} - 0.4I_1 - 10 - (I_1 - I_2)1.5 = 0 \quad \textcircled{1}$$

$$-6 - 0.3I_2 - (I_2 - I_3)6 - (I_2 - I_1)1.5 = 0 \quad \textcircled{2}$$

$$12 - 1.5I_3 - (I_3 - I_1)3 - (I_3 - I_2)6 = 0 \quad \textcircled{3}$$

$$\textcircled{1} \Rightarrow -3I_1 + 3I_3 - 0.4I_1 - 10 - 1.5I_1 + 1.5I_2 = 0$$

$$-4.9I_1 + 1.5I_2 + 3I_3 = 10$$

$$\textcircled{2} \Rightarrow -6 - 0.3I_2 - 6I_2 + 6I_3 - 1.5I_2 + 1.5I_1 = 0$$

$$1.5I_1 - 7.8I_2 + 6I_3 = 6$$

$$\textcircled{3} \Rightarrow 12 - 1.5I_3 - 3I_3 + 3I_1 - 6I_3 + 6I_2 = 0$$

$$3I_1 + 6I_2 - 10.5I_3 = -12$$

$$I_1 = -3.537 \text{ A}$$

$$I_2 = -2.405 \text{ A}$$

$$I_3 = -1.342 \text{ A}$$

Mesh 1

$$\text{Current through } 1.5\Omega = 1.132 \text{ A} \quad (I_1 - I_2)$$

$$" \quad 3\Omega = I_1 - I_3 = 2.295 \text{ A}$$

$$" \quad 0.4\Omega = I_1 = 3.537 \text{ A}$$

Mesh 2

$$\text{Current through } 1.5\Omega = I_2 - I_1 =$$

