
KTU
NOTES

Module : IV

Equation of motion (Translation)

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$u \rightarrow$ initial velocity - m/s

$v \rightarrow$ final velocity - m/s

$a \rightarrow$ accelaration

$s \rightarrow$ displacement

$t \rightarrow$ time

Circular Motion

Initial velocity $= \omega_0$

final velocity $= \omega_1$

acceleration $= \alpha$

Time $= t$.

Displacement $= \theta$

- $\omega_1 = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

~~oraes~~

- $\omega_1^2 = \omega_0^2 + 2\alpha\theta$

1. A particle moves from A to B along a straight line at a constant speed of 12 m/s and then returns from B to A along BA at a constant speed of 6 m/s.

a) find the average speed during the entire motion.

b) Average velocity during the entire motion.

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

Total distance travelled = 2AB

Total displacement = 0

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

$$= \frac{AB}{12} \quad (A \rightarrow B).$$

$$\text{Time} = \frac{AB}{6} \quad (B \rightarrow A)$$

$$\text{Total time} = \frac{AB}{12} + \frac{AB}{6}$$

$$= \frac{3AB}{12} = \frac{AB}{4}$$

$$\text{Average speed} = \frac{2AB}{\frac{AB}{4}} = 8 \text{ m/s}.$$

$$\text{Average velocity} = \frac{0}{\frac{AB}{4}} = 0.$$

2. A body moves with an initial velocity 4 m/s and at a uniform acceleration of 3 m/s². Find the distance travelled in 5 s.

$u = 4$

$a = 3 \text{ m/s}^2$

$t = 5$.

ank

$s = ut + \frac{1}{2}at^2$

$s = 4 \times 5 + \frac{1}{2} \times 3 \times 5^2$

$= 20 + \frac{75}{2}$

$= \underline{\underline{57.5}}$

The initial velocity of a body moving with a deccelaration of $2 m/s^2$ is $88 m/s$. Find the interval of time in which the body will come to rest and distance moved in this interval.

$a = -2 m/s^2$

$u = 88 m/s$

$v = 0$

$t = ?$

$v = u + at$

$0 = 88 + -2 \times t$

$t = \frac{88}{2} = \underline{\underline{44 s}}$

[illegible]

$v^2 = u^2 + 2as$

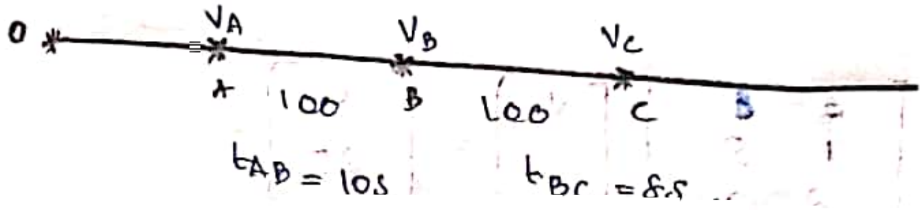
$\frac{0^2 - 88^2}{2 \times -2} = s$

$s = \frac{7744}{4} = \underline{1936}\ m$

4. Three marks A, B and C spaced a distance of 100m are made along a straight road. A car starting from rest and accelerating uniformly passes the marks A and takes 10s to reach the mark B and further 8s to reach the mark C. Calculate

i, Find the magnitude of accelaration of the car.

ii, Velocity of car at A, velocity of car at B and velocity of C.



$A \rightarrow B$

$S = ut + \frac{1}{2} at^2$

$100 = V_A \times 10 + \frac{1}{2} a \times 10^2$

$100 = V_B \times 8 + \frac{1}{2} \times a \times 8^2$

$100 = 10V_A + 50a \Rightarrow 10 = V_A + 5a$ — ①

$100 = 8V_B + 32a \Rightarrow 25 = 2V_B + 8a$ — ②

$A \rightarrow B \Rightarrow v = u + at$

$V_B = V_A + a \times 10$

$V_B = V_A + 10a$ — ③

② $\Rightarrow$ $25 = 2(V_A + 10a) + 8a$

$25 = 2V_A + 20a + 8a$

$25 = 2V_A + 28a$ — ④

$$\begin{bmatrix} 1 & 5 \\ 2 & 28 \end{bmatrix} \begin{bmatrix} V_A \\ a \end{bmatrix} = \begin{bmatrix} 10 \\ 25 \end{bmatrix}$$

$$v_A = \frac{155}{18} = \underline{\underline{8.61\ m/s}}$$

$$a = \frac{5}{18} = \underline{\underline{0.27\ m/s^2}}$$

② ⑤ $\Rightarrow$ $v_B = v_A + 10a$

$$= 8.61 + 10 \times 0.27$$

$$= \underline{\underline{11.31}}.$$

$B \rightarrow C$

$$v^2 = u^2 + 2as$$

$$v_C^2 = v_B^2 + 2a \times 100$$

$$= (11.31)^2 + 2 \times 0.27 \times 100$$

$$= (11.31)^2 + 54$$

$$v_C = \sqrt{181.91}$$

$$= \underline{\underline{13.48}}$$

Distance OA = $v^2 = u^2 + 2as$

$$s = \frac{v_{AB}^2 - 0^2}{2 \times 0.27}$$

$$S = \frac{(8.61)^2}{0.54}$$

$$= \underline{\underline{133.81}}$$

5. A truck starts from a place at an accelaration of $2 m/s^2$. A car passes the same place after 5s with a uniform speed of 20 m/s. Find the time taken in which the car overtakes the truck ?

Note:

When the car overtakes the truck. The distance travelled by car and truck is same.

$u_t = 0$

$a_t = 2 m/s$

Let the time taken by the car to overtake the truck be Ts.

· The time travelled by the ~~car~~ truck is t+5

accelaration of car = 0.

$$s_T = s_C$$

$$u_t t_t + \frac{1}{2} a_t \times (t_t)^2 = v_c t_c + \frac{1}{2} \times a_c \times (t_c)^2$$

$$0 + \frac{1}{2} \times 2 \times (t+5)^2 = 20 \times t + \frac{1}{2} \times 0 \times (t_c)^2$$

$$(t+5)^2 = 100.$$

$$t^2 + 10t + 25 = 20t.$$

$$t^2 - 10t + 25 = 0$$

$$\underline{\underline{t = 5}}$$

6. Two trains A and B leave the same station on parallel tracks. The train A starts with a uniform accelaration of $0.2\,m/s^2$ and attains a maximum speed of 45 km/hr. The train being leaves 60 s after with an uniform accelaration of $0.4\,m/s^2$ to attain a maximum speed of 72 km/hr. When and where the train will overtakes train A.

Motion under Gravity :

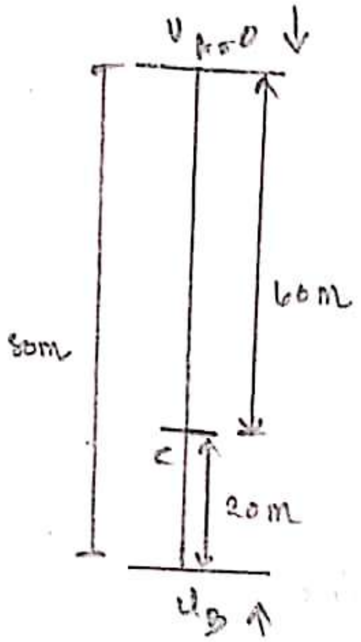
In the case of freely falling body accelaration will be g - accelaration due to gravity.

when a body moves upward $a = -g$

when a body moves downward $a = +g$.

7. A particle is drop from the top of a tower 80 m high at the same time another particle is projected upwards from the base of the tower and meets the 1st particle at a high of 20m from the base. Find the velocity with which the 2nd particle is projected upwards.

Vice versa



Time taken by A to travel 60m and B to travel 20m will be the same.

Considering the downward motion of A,

$$s = ut + \tfrac{1}{2}at^2$$

$$60 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$60 = 4.9t^2$$

$$t^2 = 12.24$$

$$t = 3.4s$$

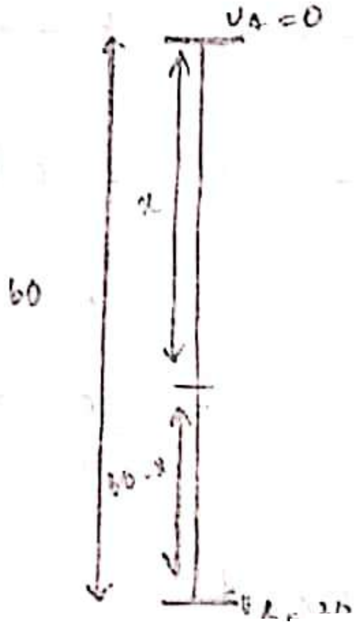
considering upward motion of B,

$$s = ut + \tfrac{1}{2}at^2.$$

$$20 = u \times 3.5 + \frac{1}{2} \times -9.8 \times (3.5)^2$$

$$u = \underline{\underline{22.86 \text{ m/s}}}$$

8. A stone is dropped from top of a tower 60 m height. At the same time another is thrown upward from the foot of tower with velocity of 30 m/s. When and where the stones cross each other.



Time taken by A to travel AC =

Time taken B to travel BC.

Considering the downward motion,

$$s = ut + \tfrac{1}{2}at^2$$

$$x = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$x = 4.9t^2$$

Considering the upward motion

$$s = ut + \tfrac{1}{2}at^2$$

$$(60-x) = 30 \times t + \frac{1}{2} \times -9.8 \times t^2$$

$$60 - x = 30t - 4.9t^2$$

$$60 - \cancel{4.9t^2} = 30t - \cancel{4.9t^2}$$

$$60 = 30t$$

$$t = \frac{60}{30} = 2$$

$$x = 4.9 \times 2^2$$

$$= 19.6$$

9) A stone is drawn from the top of the tower, reaches the ground in 8s. Find

i, Height of the tower

ii, velocity of the particle when it reaches the ground.

Given,

$u = 0$

$t = 8$

i, $S = ut + \frac{1}{2}at^2$

$= \frac{1}{2} \times 9.8 \times 8^2$

$= \underline{313.6}$

ii, $v^2 = u^2 + 2as$

$v^2 = 2 \times 9.8 \times 313.6$

$v = \underline{\underline{78.4}}$ m/s.

10) A stone is drawn vertically upward it reaches the maximum height 12 m/s. Determin

i. the velocity with which the stone was thrown.

ii, The time taken to reach the maximum height.

$\downarrow a = g$

$\uparrow a = -g$

iii, Total time taken by the stone to return to the ground surface after projected up.

At, maximum height final velocity will zero.

$$a = -g$$

i, $v^2 = u^2 + 2as$

$$0^2 = u^2 + 2 \times -9.8 \times 12$$

$$u^2 = 235.6$$

$$u = 15.33$$

ii, $V = u + at$

$$0 = 15.33 - 9.8t$$

$$15.33 = 9.8t$$

$$t = \underline{1.56}$$

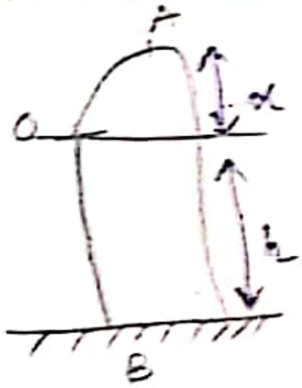
t

$\uparrow + \downarrow =$

3.12

iii, Total time taken $= \underline{3.12}$

A stone is projected upward from the roof a building with a velocity of 19.6 m/s. and other stone is drawn downward from the same point 3s later. If both the stones reaches the ground at same time. Determine the height of the building.



Given $u = 19.6 m/s$

Considering the motion OA,

time taken to travel OA

$$v = u + at$$

$$\frac{0 - 19.6}{-9.8} = t$$

$$\underline{\underline{t_1 = 2}}$$

D. OA, $= x$

$$s = ut + \frac{1}{2}at^2$$

$$s = 19.6 \times 2 + \frac{1}{2} \times -9.8 \times 2^2$$

$$= \underline{\underline{19.6}}$$

Let t be the total time taken by the stone to strike the ground.

t_1 - time taken to reach max. height

t_2 = time taken by the stone to strike the ground from max height..

$T = t_1 + t_2$

consider the motion from A to B.

Total distance $x + h = h + 19.6$

we have,

$t = t_1 + t_2$

$t_2 = t - t_1$

$u = 0$

$s = ut + 1/2 at^2$

$19.6 + h = \frac{1}{2} \times 9.8 \times (t-2)^2.$

$19.6 + h = 4.9 (t-2)^2.$

Considering the motion stone dropped after 3 seconds

$u = 0$

$t = t - 3$

$a = 9.8$

$s = H$

$s = ut + \frac{1}{2}at^2$

$H = \frac{1}{2} \times 9.8 \times (t-3)^2$

$H = 4.9\,(t-3)^2$

$19.6 + h = 4.9(t-2)^2$

$- \quad H = 4.9\,(t-3)^2$

$19.6 = 4.9[(t-2)^2 - (t-3)^2]$

$19.6 = 4.9[\cancel{t^2} - 4t + 4 - \cancel{t^2} + 6t - 9]$

$19.6 = 4.9[2t - 5]$

$19.6 = 9.8t - 24.5$

$44.1 = 9.8t$

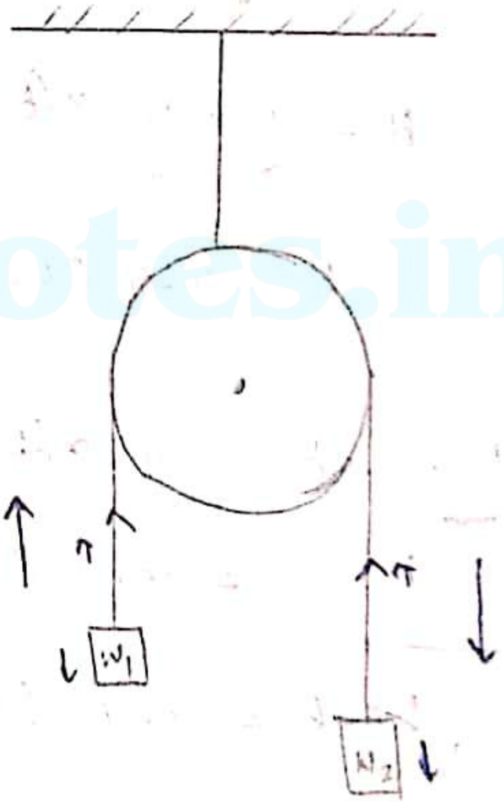
$t = 4.5\,s$

$H = 4.9\,(t-3)^2$

$H = 4.9\,(4.5-3)^2$

$H = 11.025\,m$

Motion of connected bodies:



Considering the upward motion of w_1.

$T - w_1 = ma$ (Net force)

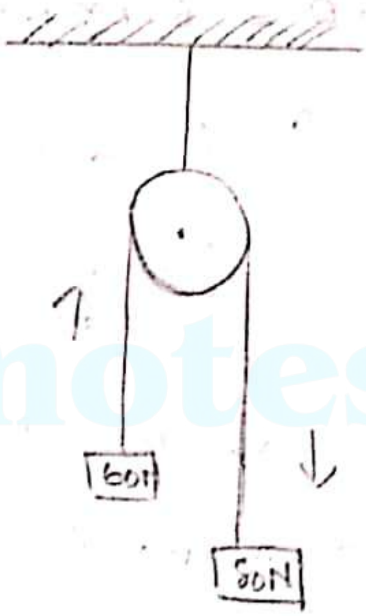
$T - w_1 = \frac{w_1}{g}\,a$

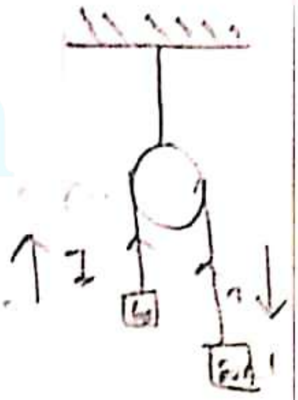
Considering the downward motion of w_2.

$$w_2 - T = ma \quad \text{(Net force)}$$

$$w_2 - T = \frac{w_2}{g} a .$$

12) Find the tension in the string and accelaration of the blocks shown in figure.





Considering upward motion of 60N.

$$T - 60 = ma$$

$$T - 60 = \frac{60}{9.8} \times a$$

$$T - 60 = 6.1a$$

2nd,

$$80 - T = \frac{80}{9.8} a$$

$$80 - T = 8.1a$$

$$T - 60 = 6.1a$$

$$+ 80 - T = 8.1a$$

$$20 = 14.2a$$

~~14.2a~~

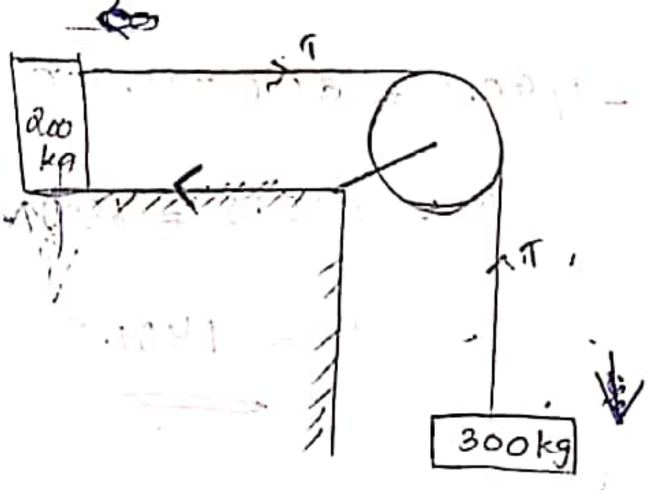
$$a = 1.4\ m/s^2$$

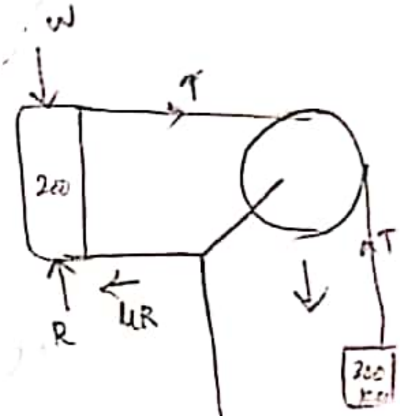
$$T - 60 = 6.1 \times 1.4$$

$$T - 60 = 8.5$$

$$T = 68.59\ N$$

13) Two blocks are joined by an inextensible string as shown in figure. If the system is released from the rest. Determine the velocity of the block after it has moved 2m. Assume coefficient of friction b/w plane and block A is 0.25.





Considering the motion of 200kg towards right.

$T - \mu R = ma$

$R = mg$

$= 200 \times 9.8$

$= \underline{1960}$

$T - 0.25 R = 200a$

$T - 490.5 = 200a$ — ①

Downward motion,

$300 \times 9.8 - T = 300a$

$2943 - T = 300a$ — ②

$T - 490.5 = 200a$

$-T + 2943 = 300a$

$2452.5 = 500a$

$a = \underline{4.905}$

$$T - 490.5 = 200 \times 4.905$$

$$T - 490.5 = \cancel{1471.5}\ 981$$

$$T = \underline{1471.5}$$

$u = 0$

$a = 4.9\ m/s^2$

$s = 2m$

$v = ?$

$$v^2 = u^2 + 2as$$

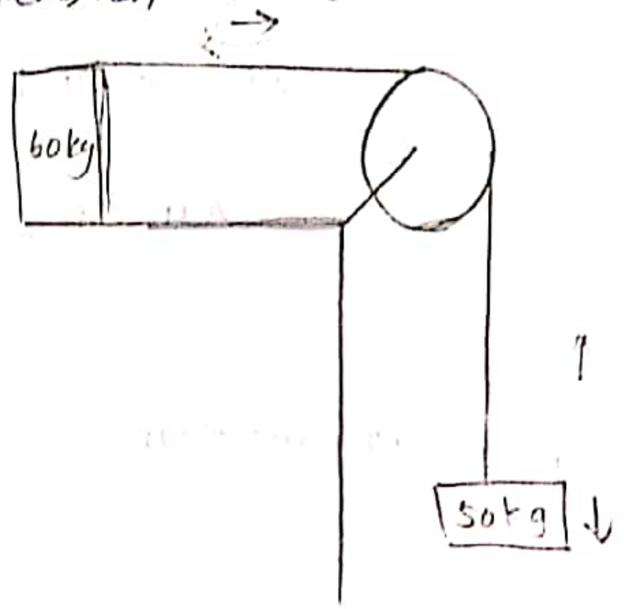
$$v^2 = 2 \times 4.9 \times 2$$

$$v^2 = 19.62$$

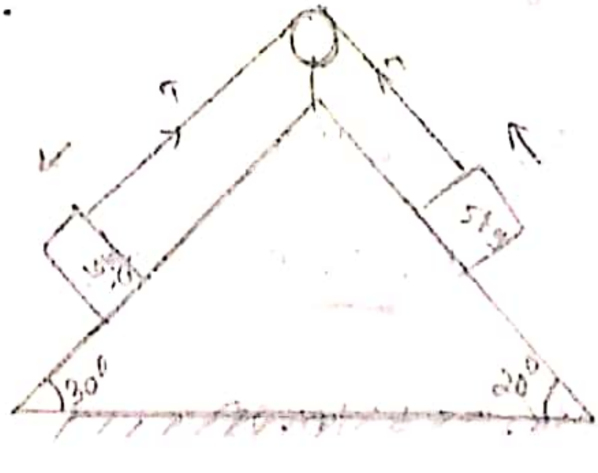
$$v = \underline{4.429}$$

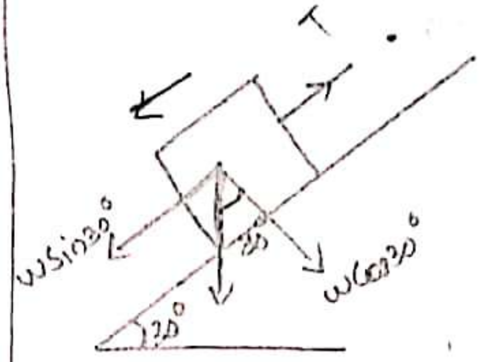
14) A mass of 60kg lies on a smooth horizontal table. It is connected by a light string passing over a smooth pulley on the edge of a table to a mass 50kg hanging freely.

the tension in the string.



15) Two smooth inclined planes whose inclination with horizontal are 30° and 20° placed back to back. Two blocks of mass 10kg and 5kg are placed on them. And are connected with a spring as shown in figure. Calculate tension in the string and accelaration of the block.

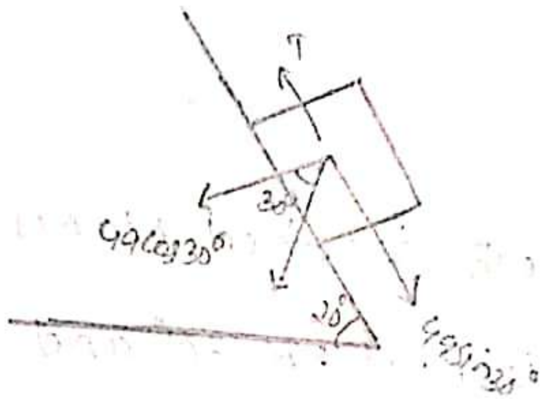




$$98\sin 30 - T = \frac{98}{9.8}a$$

$$49 - T = 10a$$

considering upward motion,



$$T - 49\sin 20 = \frac{49}{9.8}a$$

$$T - 16.75 = 5a$$

$$49 - T = 10a$$

$$T - 16.75 = 5a$$

$$32.25 = 15a$$

$$a = \underline{2.15}$$

$$T - 16.75 = 10.75$$

$$T = 27.5 //$$

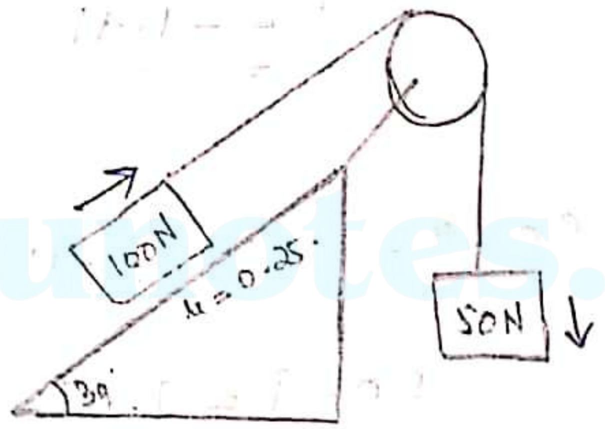
Reaction exerted on the bodies will be

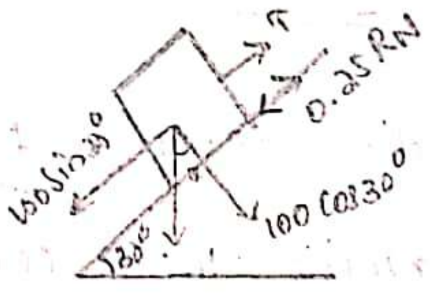
equal to w Cos α .

$$98 \cos 30 = 84.87$$

$$49 \cos 30 = 42.43$$

16) Calculate the tension and accelaration of the given figure .





$R = 100 \cos 30$

$= 86.6$

$$-100 \sin 30° + T - 0.25 RN = \frac{100}{9.8} a$$

$$-50 + T - 21.65 = 10.20 a$$

$$-71.65 + T = 10.20 a$$

Downward

$$50 - T = \frac{50}{9.8} \times a$$

$$50 - T = 5.10a$$

$$-71.65 + T = 10.20a$$

$$50 - T = 5.10a$$

$$-21.65 = 15.3a$$

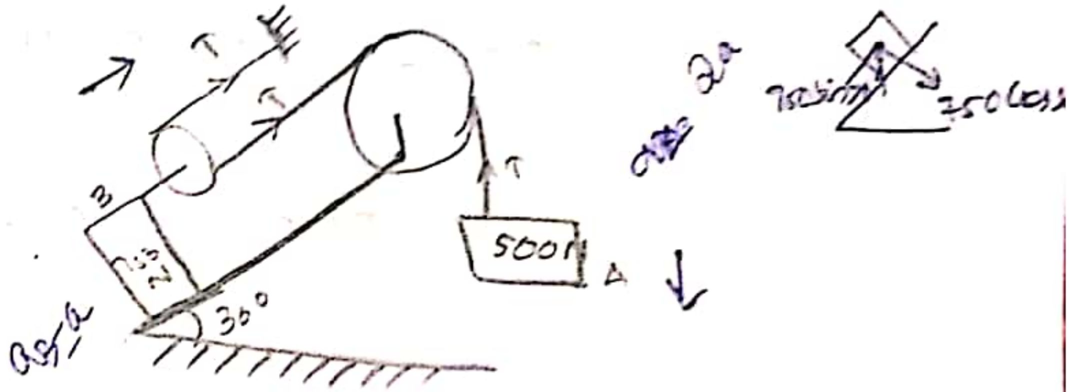
$$a = -1.41$$

$$50 - T = 5.10 \times -1.41$$

$$50 - T = 7.07$$

$$T = 57.07$$

17) The system of bodies shown in figure starts from rest. Determine the accelaration of the body B and tension in the string supporting the body A.



Note: For a given vertical displacement of body A, the displacement of B along inclined plane will be half of that of A. therefore accelaration of B will be half of that of ~~B~~ A

$$2T - 750 \sin 30 = \frac{750}{9.81} \times a_B.$$

$$2T - 375 = 76.45 a_B .$$

Downward,

$$500 - T = \frac{500}{9.81} a_A$$

$$500 - T = 50.96 a_A .$$

$a_B = a$

$a_A = 2a$ (Regarding displacement)

$$2T - 375 = 76.45 a$$

$500 - T = \cancel{\frac{200}{g}}\ 50.96 \times 2a$

$500 - T = 101.92a$

$2T - 375 = 76.45a$

$-2T + 1000 = 203.84a$

$+$

$625 = 280a$

$a = \underline{\underline{2.23}}$

$2T - 375 = 76.45 \times 2.23$

$2T - 375 = 170.47$

$2T = 545.47$

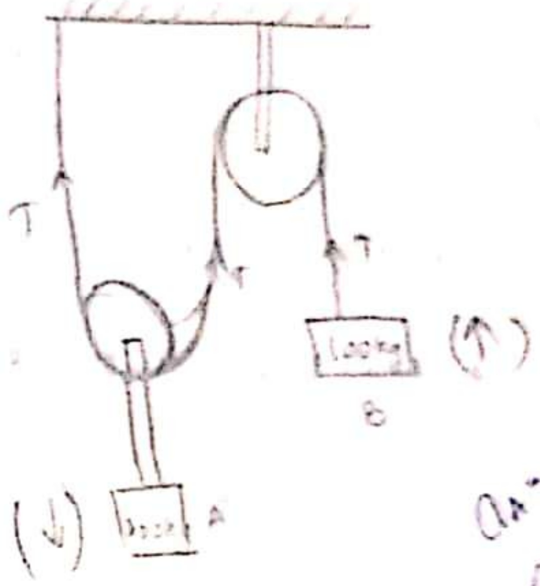
$T = \underline{\underline{272}}.73$

Accelaration of $A = 2 \times 2.3$

$= 4.6$

18) Find the tension and accelaration of each block of mass 300kg and 100kg as shown in. 2 Blocks are connected by light string and weight

pulley -



$a_A = a$

$a_B = 2a$

Downward direction,

$a_A = a$

$a_B = 2a$

$$300 \times 9.81 - 2T = 300 \times a_A$$

$$T - 100 \times 9.81 = 100 a_B$$

$$300 \times 9.81 - 2T = 300a$$

$$2943 - 2T = 300a$$

$$-1960 + 2T = 400a$$

+

$$983 = 700a$$

$$a = \underline{1.40}$$

$a_B = 2a$

$a_B = 2 \times 1.40$

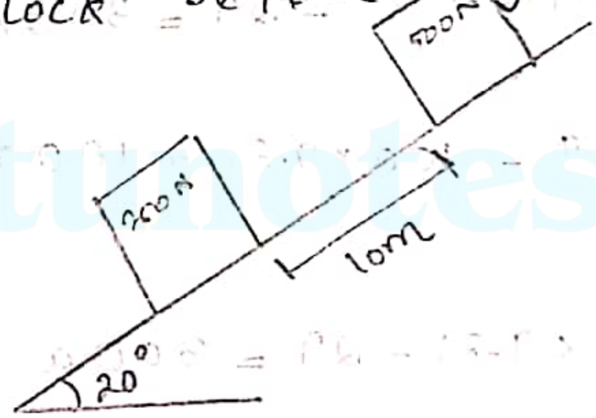
$a_B = 2.5$

$$2943 - 2T = 420$$

$$T = 1261.5$$

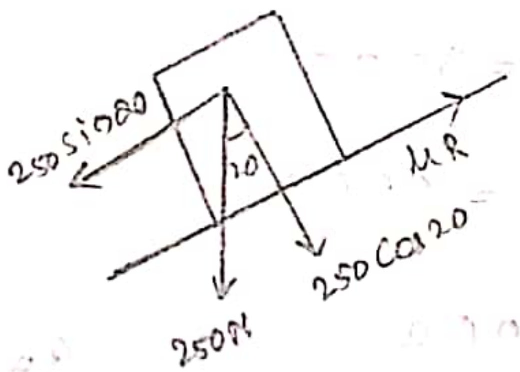
Motion of bodies on inclined plane

19) A 20° inclined plane is shown in figure. The coefficient of friction b/w the plane and block A is 0.3 and it is 0.2 b/w the plane and block B. If both the blocks are released simultaneously from rest. Calculate the time taken and distance travelled by each block before they collide.



$a_A = a$

$a_B = 2a$

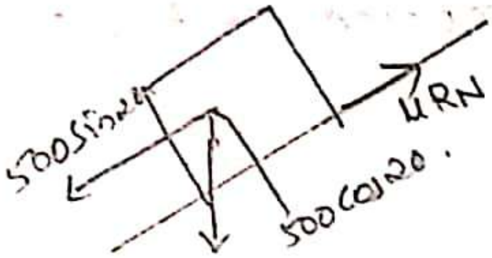


$250 \sin 20 - 0.3R = \frac{250}{9.8} a_A$

$85.5 - 70.47 = 25.51 a_A$

$15.03 = 25.51 a_A$

$a_A = 0.589$



$$500 \sin 20 - 0.2R = \frac{500}{9.8} a_B .$$

$$171.01 - 93.96 = 51.02 a_B$$

$$77.04 = 51.02 a_B$$

$$a_B = \underline{1.50}$$

Let x be the distance travelled by A at instant of collision. Then the distance travelled by B = $x + 10$.

Initial velocity u of A and B = 0.

Considering the motion of A.

$u = 0$

$S = x$

$a = 0.58$

$S = ut + \frac{1}{2}at^2$

$x = \frac{1}{2} \times 0.58 \times t^2$

$x = 0.29t^2$ —— (a)

Considering motion B,

$u = 0$

$S = x + 10$

$a = 1.5$

$S = ut + \frac{1}{2}at^2$

$x + 10 = \frac{1}{2} \times 1.5 \times t^2$

$x + 10 = 0.77t^2$ —— (b)

(a) ÷ (b)

$x = 0.29t^2$

$\div \; x + 10 = 0.77t^2$

$T - W_{\cdot\cdot}$

$W - T_{\cdot}$

$x = 0.29$

$x + 10 = 0.77$

$0.77x = 0.29x + 2.9$

$0.48x = 2.9$

$x = 6.39$

Distance A $= 6.39$

Distance by B $= 16.39$.

50 Sin30 x 9.81 −

$0.4 R_A =$

50 Cos30 9.81

$6.39 = 0.29t^2$

50A

$t = 4.69$

20) A body of mass 50 kg slides down a rough inclined plane whose inclination to the horizontal is 30°. If the coefficient of friction b/w plane and body is 0.4. Find the accelaration of the body.

1.50

D Alemberts Principle.

It is the application of newton second law of motion. A problem in dynamics can be converted into static equilibrium problem using D Alemberts principle.

According to newton 2nd law of motion $F=ma$, which can be written as $F-ma$.

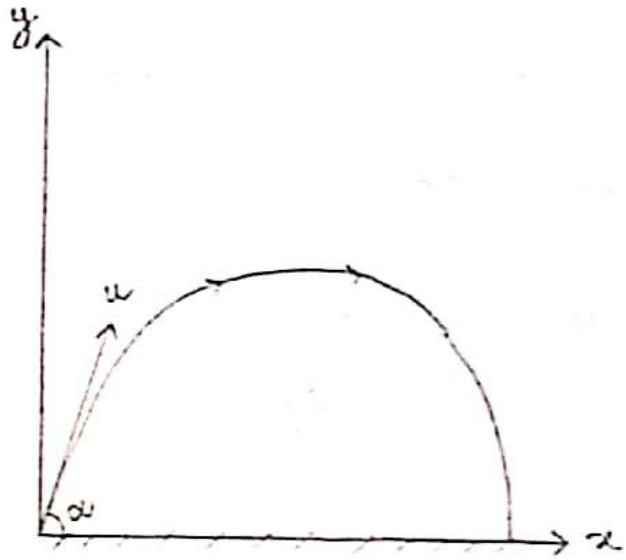
The term $-ma$ is called inertial force and it will be always acts opposite to the direction of motion.

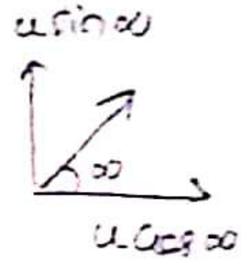
D. Alemberts principle states that the resultant of system of forces acting in a body is in dynamic equilibrium with inertial force.

$F-ma=0$: Equation of dynamic equilibrium

Curvilinear motion (projectile motion).

motion of a body along a curved path is called curvilinear motion.





$u_x = u\cos\alpha$

$u_y = u\sin\alpha$

Time of flight :

It is the total time take by projectile for instant of projection upto projectile hits the plane again.

$$T = \frac{2u\sin\alpha}{g}$$

Horizontal range :

It is distance along the plane b/w the point of projection and the point at which projectile hits the plane at the end of the

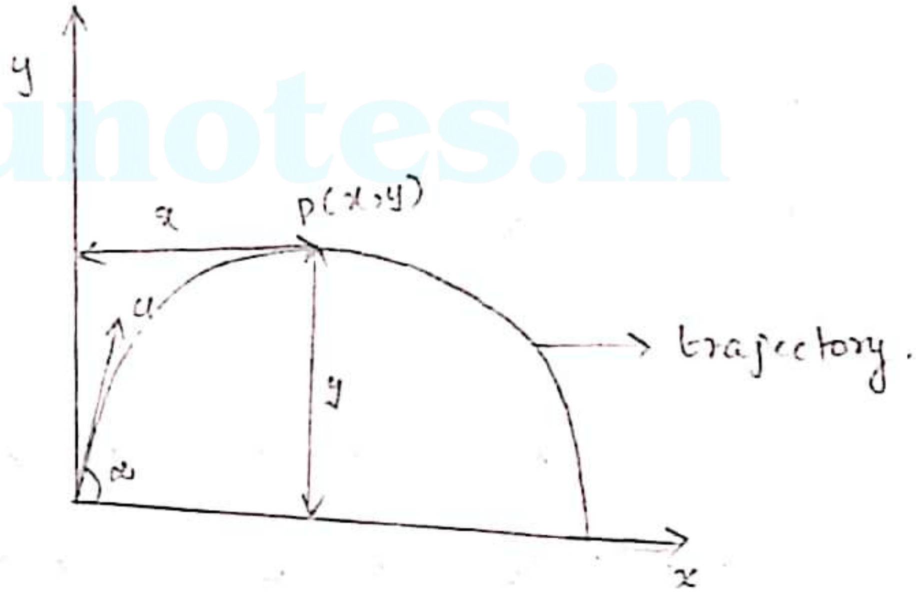
Journey

$$R = \frac{u^2 \sin 2\alpha}{g}.$$

Maximum height :

It is the maximum vertical for displacement for projectile during the motion

$$H_{max} = \frac{u^2 \sin^2\alpha}{2g}$$



$$y = x\tan\alpha - \frac{1}{2}\frac{gx^2}{u^2\cos^2\alpha}$$

21) A particle is projected with initial velocity at angle of 75° with horizontal. Determine i. maximum height attain by particle

ii, Horizontal range of particle

iii, Time taken by the particle to reach the heighest for time of flight,

Given,

$u = 60\,m/s$

$\alpha = 75^\circ$.

i, $H_{max} = \dfrac{u^2 \sin^2\alpha}{2g}$

$= \dfrac{60^2 \sin^2 75}{2 \times 9.8}$

$= \underline{\underline{171.36}}$

ii, $R = \dfrac{u^2 \sin 2\alpha}{g}$

$= \dfrac{60^2 \sin 2\times 75}{9.8}$

$= \underline{\underline{183.67}}$

At maximum height $v = 0$

$$u = 60 \sin\alpha$$

$$\underline{\underline{u = 57.95}}$$

$$a = -9.8$$

$$\cancel{\frac{u \sin\alpha}{g}}$$

$$v = u + at$$

$$0 = \cancel{60}\,57.95 + -9.8\,t$$

$$57.95 = 9.8\,t$$

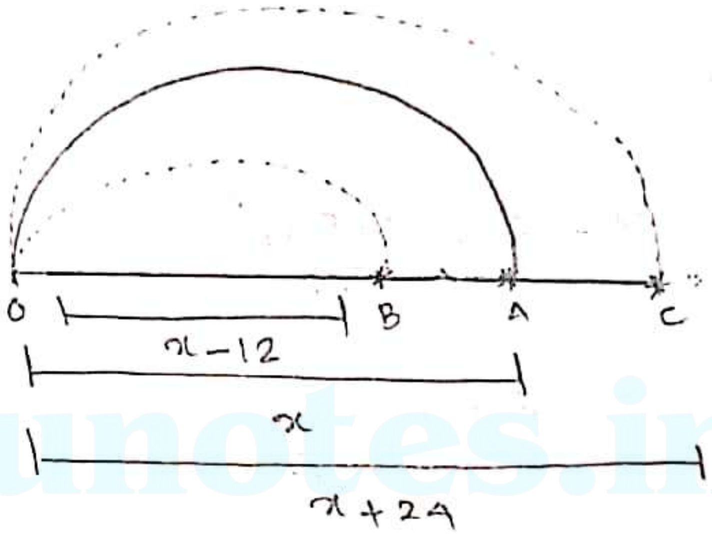
$$\underline{\underline{t = 5.91}}$$

iii $\Rightarrow$ $T = \dfrac{2u\sin\alpha}{g}$

$$= \frac{2 \times 60 \sin 75}{9.8} = \underline{\underline{11.82}}$$

22) A projectile is aimed at a mark on the horizontal plane through the point of projection. And falls 12m short when the

angle of projection 15°. When it tried again it overshoots the mark by 24m when the angle of projection is 45°. Find the correct angle of projection to hit the wall. Velocity of projection is constant in all the cases.



$$R = \frac{u^2 \sin 2\alpha}{g}$$

when $\alpha = 15°$

$$R = x - 12$$

$$x - 12 = \frac{u^2 \sin 2 \times 15}{9.8}$$

$$x - 12 = \frac{0.5u^2}{9.8} \quad -①$$

when $\alpha = 45^0$

$$R = x + 24$$

$$x + 24 = \frac{u^2 \sin(2 \times 45)}{g}$$

$$x + 24 = \frac{u^2}{9.8} \quad \text{—②}$$

① ÷ ②

$$x - 12 = \frac{0.5\, \cancel{u^2}}{\cancel{9.8}}$$

$$\div \; x + 24 = \frac{\cancel{u^2}}{\cancel{9.8}}$$

$$\frac{x - 12}{x + 24} = \frac{0.5}{1}$$

$$x - 12 = 0.5x + 12$$

$$x - 0.5x = 24$$

$$0.5x = 24$$

$$\underline{\underline{x = 48m}}$$

$$x + 24 = \frac{u^2}{9.8}$$

$$48 + 24 \times 9.8 = u^2$$

$$u = \cancel{16.52}\ 26.56$$

$$R = \frac{u^2 \sin 2\alpha}{g}$$

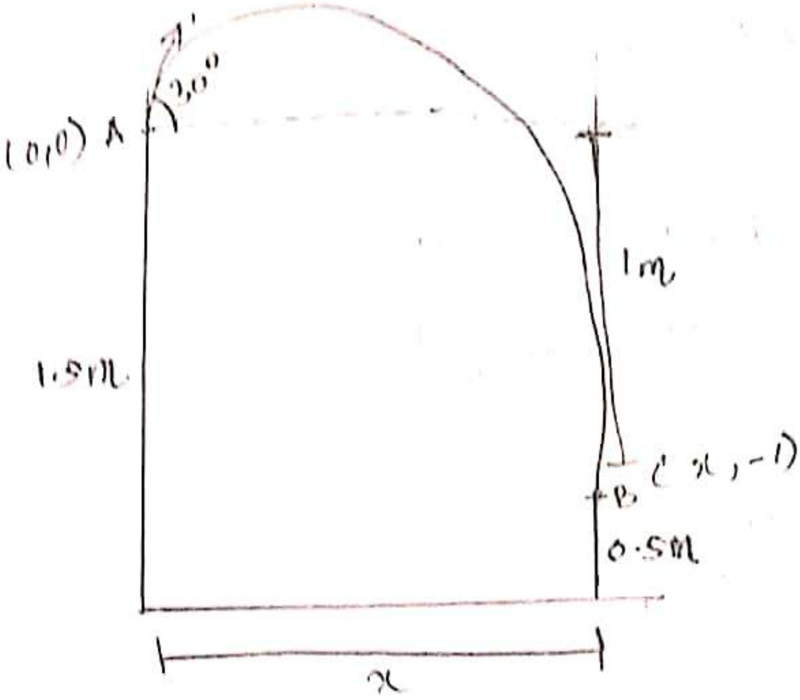
$$48 = \frac{26.56^2 \sin 2\alpha}{9.8}$$

$$470.4 = 705.43 \sin 2\alpha$$

$$\sin 2\alpha = 0.6$$

$$\alpha = 20.91$$

imp 23) A cricket ball hits at a height of 1.5 m from the ground by a batsman with a velocity of 20 m/s, at angle of 30° to the horizontal was caught by a fieldman at a height of 150cm from the ground. Find out the distance b/w the 2 players.



coordinates of B will be $(x, -1)$

$\alpha = 30$

$u = 20$

$$Y = \tan\alpha\, x - \frac{1}{2}\frac{g x^2}{u^2 \cos^2\alpha}$$

$$-1 = \tan 30\, x - \frac{1}{2} \times \frac{9.8 \times x^2}{20^2 \cos^2 30}$$

$$-1 = 0.57x - \frac{9.8x^2}{600}$$

$$-600 = 342x - 9.8x^2$$

$$x = \underline{\underline{37.00}}$$

<u>Work energy Equation:</u>

work energy principle states that work done by a system of forces acting on a body during the displacement is equal to change in kinetic energy of the body during the same displacement.

Consider a body of mass m moving with a velocity u. Let s be the displacement and v is the final velocity. Let F be the resultant force.

$$F = ma$$

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{ds} \times \frac{ds}{dt}$$

$$a = v \cdot \frac{dv}{ds}$$

$$F = m \cdot v \frac{dv}{ds}$$

$$F \cdot ds = mv\,dv$$

Integrating on both sides,

$$\int_0^s F \cdot ds = \int_u^v mv\,dv.$$

$$\left[F \cdot S\right]_0^S = \left[m \frac{v^2}{2}\right]_u^v$$

$$F \cdot (S-0) = m \left[\frac{v^2 - u^2}{2}\right]$$

$$F \cdot S = \frac{mv^2}{2} - \frac{mu^2}{2}$$

workdone = change in K.E.

Impulse:

Large ~~distance~~ force acting on a ~~small~~ short period of time, the force is called an impulsive force

momentum:

It is the product of mass and velocity

Impulse:

The impulse of force F acting on a short time interval t_1 to t_2 is defined by

$$\int_{t_1}^{t_2} F \cdot dt$$

$$F = ma$$

$$F = m\frac{dv}{dt}$$

$$F = \frac{d}{dt}(mv)$$

where mv is called momentum.

$$F\,dt = d(mv)$$

Integrating on both sides

$$\int F.dt = \int d(mv)$$

$$\int F.dt = \int m\,dv$$

$$\int_{t_1}^{t_2} F.dt = \int_{v_1}^{v_2} m\,dv$$

$$F\,[t]_{t_1}^{t_2} = m[v]_{v_1}^{v_2}$$

$$f(t_2-t_1) = m(v_2-v_1).$$

$$Ft = m(v_2-v_1)$$

Impulse = change in momentum

Module 4

KRISHNENDU SIVADAS ASSISTANT PROFESSOR

TRANSLATIONAL KINEMATICS

SYLLABUS

- **Dynamics – rectilinear translation - equations of kinematics(review) kinetics – equation of motion – D'Alembert's principle – motion on horizontal and inclined surfaces, motion of connected bodies. Impulse momentum equation and work energy equation (concepts only).**
- **Curvilinear translation - equations of kinematics –projectile motion(review), kinetics – equation of motion. Moment of momentum and work energy equation (concepts only).**

DYNAMICS

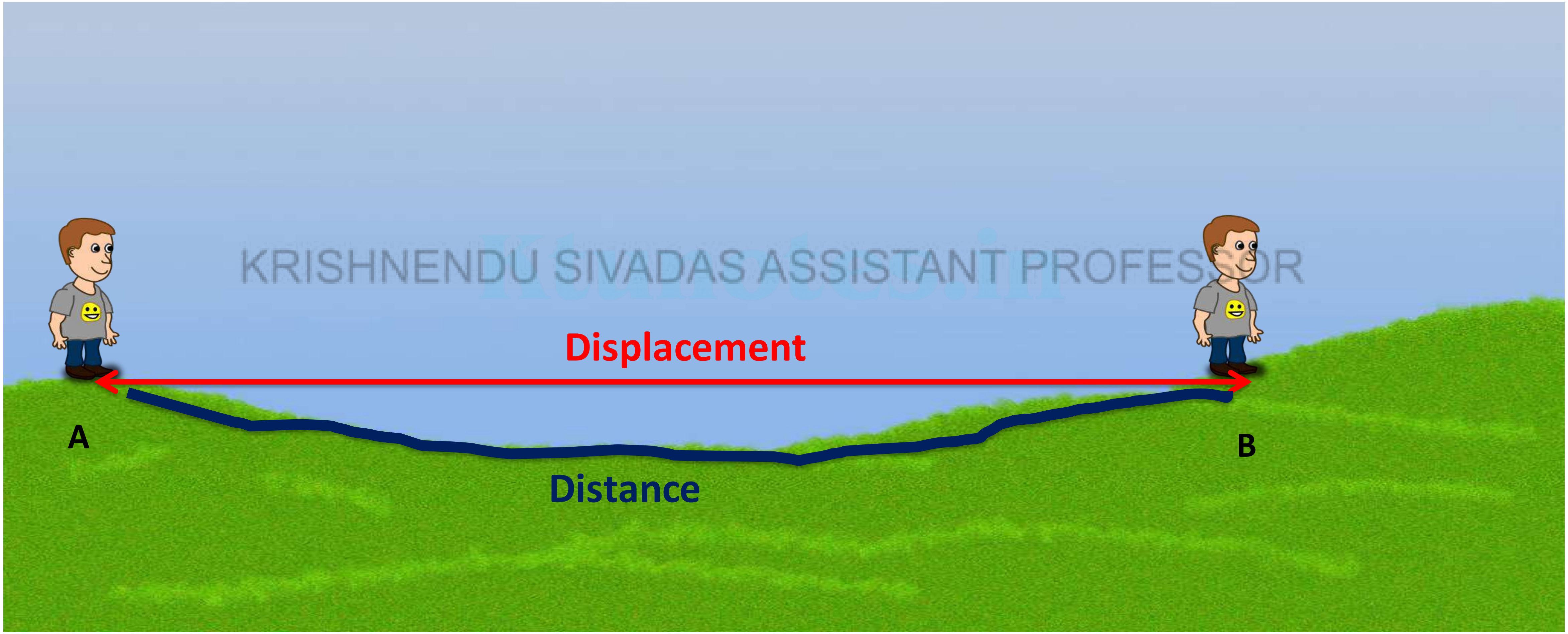
- **Dynamics is the branch of mechanics that deals with the motion of bodies under the action of forces.**

- **It can be sub-divided into two distinct parts:**
- **Kinetics – branch of dynamics that deals with the study of motion of bodies with reference to forces causing it.**
- **Kinematics - branch of dynamics that deals with the study of motion of bodies without the reference to the forces causing it.**

Rectilinear translation

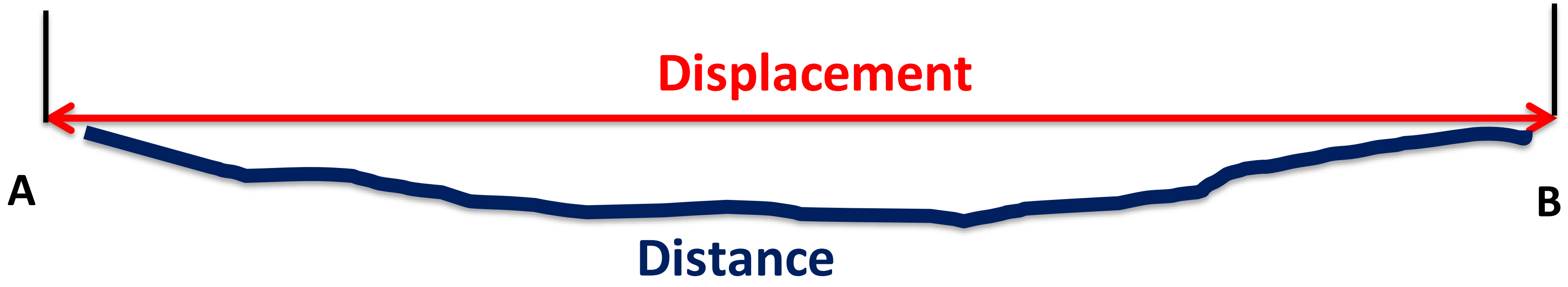
- **When a particle moves along a straight line, the motion is called rectilinear translation**
- **Kinematics of rectilinear translation of a particle is characterized by specifying its displacement, velocity and acceleration.**

Displacement



Displacement

- The change of position of a particle with respect to a certain fixed reference point is known as displacement.
- Consider a particle that moves from A to B along a curved path in tine t seconds.
- The length of path travelled by the particle between A and B is called distance covered by particle in t seconds.
- The shortest distance between A and B is called displacement of particle in t seconds.



Distance and Displacement

Distance	Displacement
Total distance travelled by a particle in the given interval of time is called distance	The shortest distance covered by a particle in the given time is called displacement.
Scalar quantity	Vector quantity
Always a positive value	It can be a positive or negative value

Velocity

- The rate of change of position of a particle with respect to time is called velocity
- Consider the motion of a particle along a straight line. At time ‘t’, the particle is at A which is at a distance x from the reference point O.
- At time $(\mathrm{t}+\Delta t)$,the particle is at B, which is at a distance of $(\mathrm{x}+\Delta x)$ from reference point O.
- Then, average velocity is given by:

$$V_{AV} = \frac{\Delta x}{\Delta t}$$

Instantaneous velocity

- **Instantaneous velocity: The velocity of a particle at a particular point on the line is called instantaneous velocity of the particle.**

- **It is the velocity in the limit Δt tends to zero.**

$$V = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}$$

$$V = \frac{dx}{dt}$$

Acceleration

- The rate of change of velocity of a particle with respect to time is called acceleration.
- When the velocity of a particle decreases, the acceleration will be negative.
- Negative acceleration is called retardation or deceleration
- Acceleration is a vector quantity
- If V and (V+dV) are the velocities of a particle at time t and $(t + \Delta t)$seconds respectively, then average acceleration of the particle over time interval Δt is given by:

$$a_{av} = \frac{\Delta V}{\Delta t}$$

Instantaneous acceleration

- **Instantaneous acceleration: The acceleration of a particle at a particular point on the line is called instantaneous acceleration of the particle.**

- **It is the acceleration in the limit Δt tends to zero.**

$$a = \lim_{\Delta t \to 0} \left(\frac{\Delta V}{\Delta t}\right) = \frac{dV}{dt}$$

$$a = \frac{dV}{dt}$$

Acceleration and displacement

- **Acceleration** , $a = \frac{dV}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

$$a = \frac{d^2x}{dt^2}$$

- **Acceleration, a=** $\frac{dV}{dt} \times \frac{dx}{dx} = \frac{dV}{dx} \times \frac{dx}{dt} = \frac{dV}{dx} \times V$

$$a = V.\frac{dV}{dx}$$

Equations of kinematics

Equations of motion	Free falling body	Body moves upawards
V = u + at	V = u + gt	V = u – gt
$V^2 = u^2 + 2as$	$V^2 = u^2 + 2gh$	$V^2 = u^2 - 2gh$
S = ut + $\frac{1}{2}at^2$	h = ut + $\frac{1}{2}gt^2$	h = ut - $\frac{1}{2}gt^2$

Velocity-Time curve

- **In the velocity-time curve, the abscissa represents time of motion and the ordinate represents the velocity.**
- **Velocity V = $\frac{dS}{dt}$**

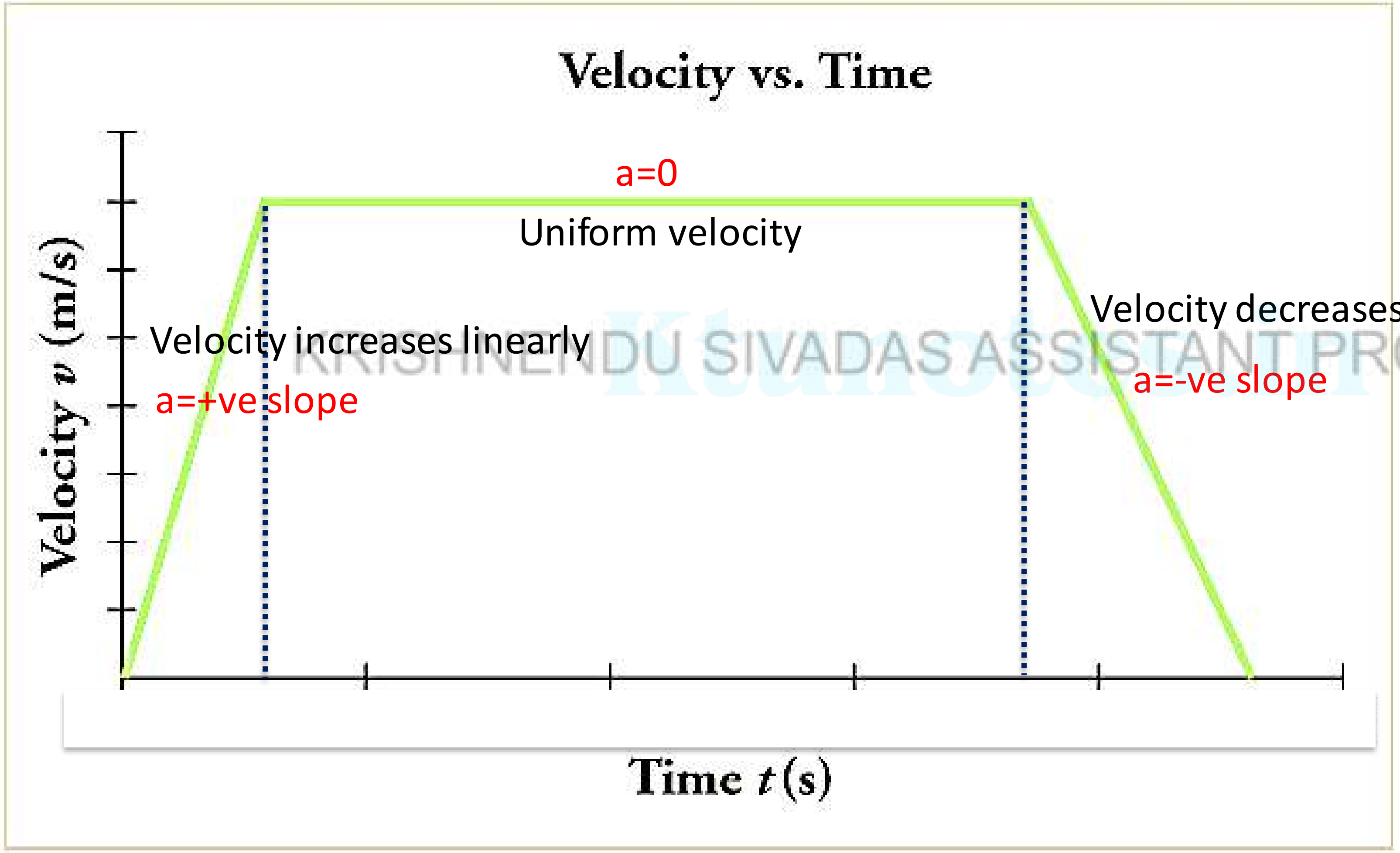
$$dS = V\,dt$$

$$\int dS = \int V\,dt$$

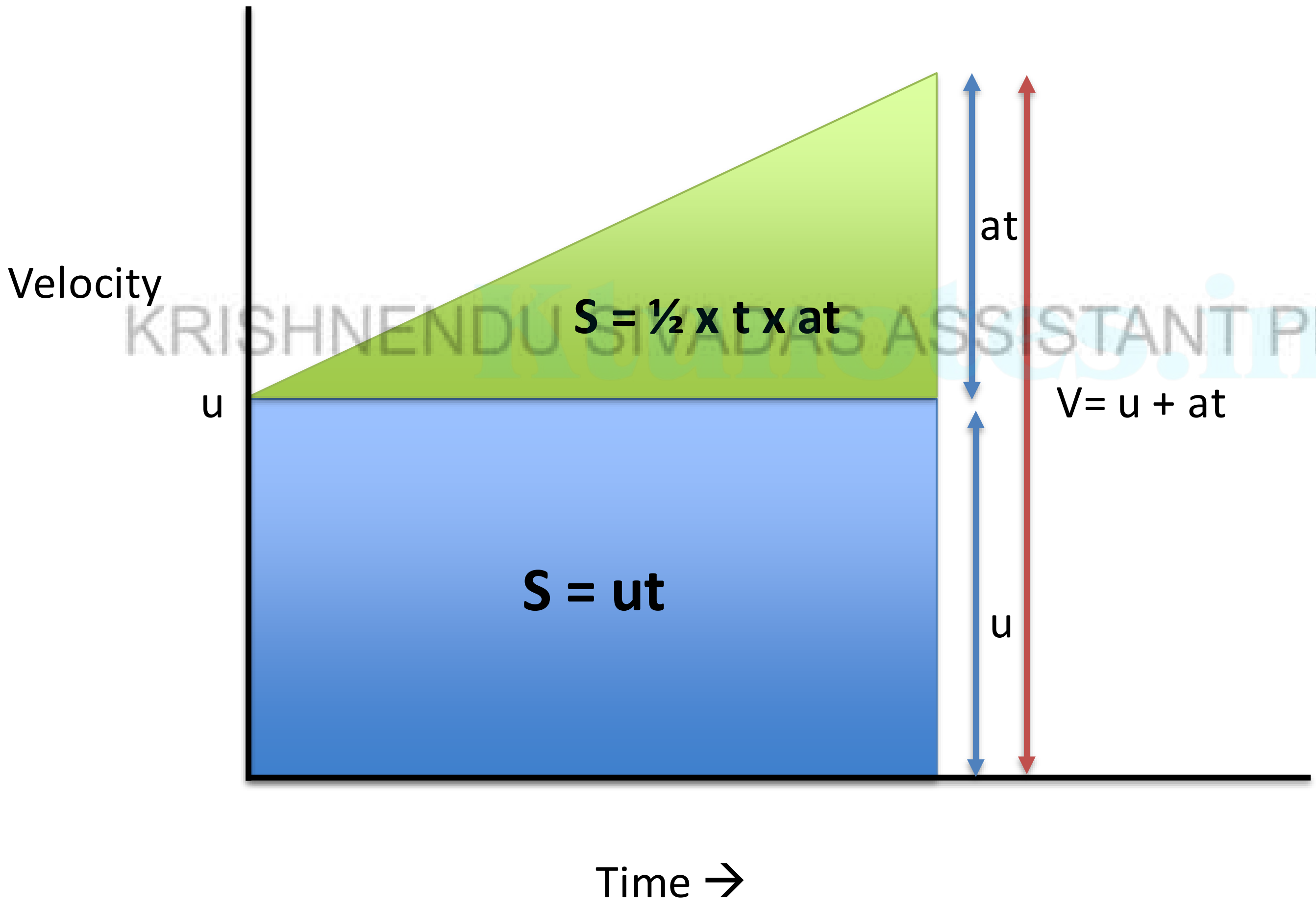
$$S = \int V\,dt$$

- **The area under velocity-time graph represents the displacement.**
- **The slope of velocity time curve is $\frac{dV}{dt}$ = a i.e. acceleration**

Velocity-Time curve



When velocity increases from initial velocity u



Motion of a particle with variable acceleration

- **The equations of motion are only applicable when the particle moves with uniform acceleration.**
- **When a particle is acted upon by a force which varies with time, the acceleration also varies with time.**
- **The displacements, velocity and acceleration are functions of time.**
- **When S= f(t)**

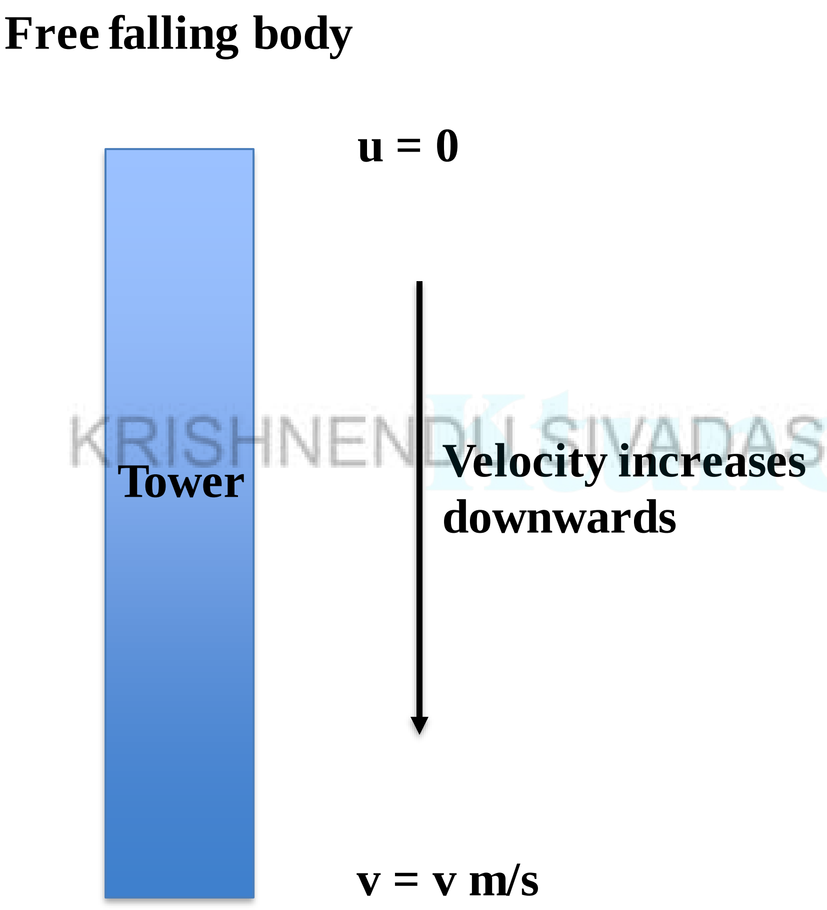
$$V = \frac{dS}{dt} = \frac{d}{dt}\,f(t)$$

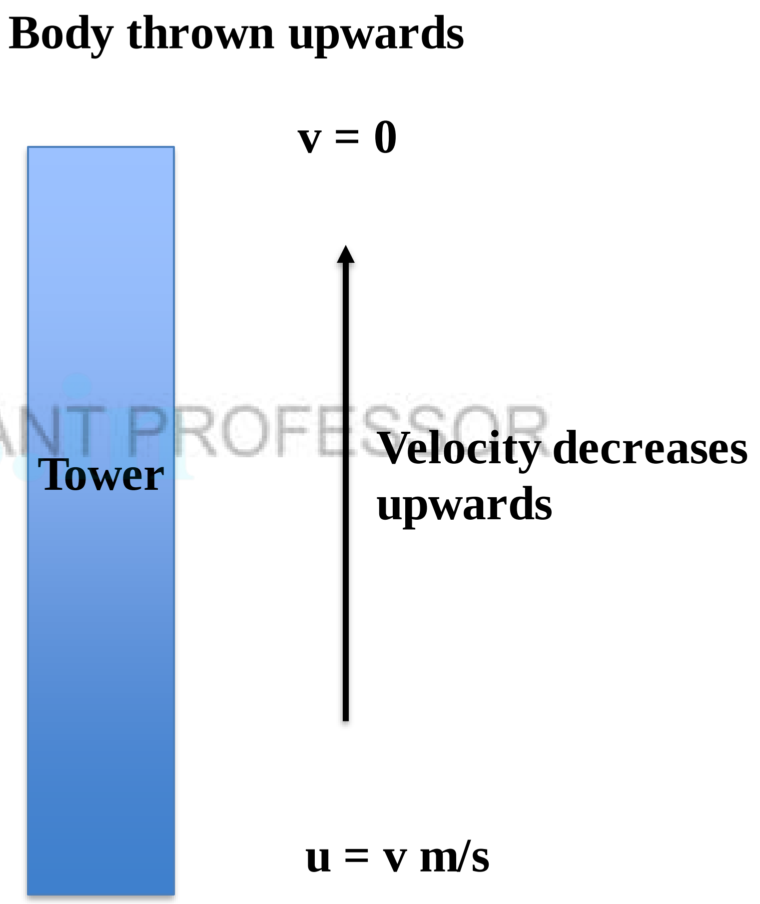
$$a = \frac{dV}{dt} = \frac{d}{dt}\frac{d}{dt}\,f(t) = \frac{d^2}{dt^2}\,f(t)$$

NUMERICALS

KRISHNENDU SIVADAS ASSISTANT PROFESSOR

CONCEPT #1


Free falling body
u = 0
Tower
Velocity increases downwards
v = v m/s


Body thrown upwards
v = 0
Tower
Velocity decreases upwards
u = v m/s

1) A stone is dropped from the top of the tower, 60 m high. At the same time another stone is thrown upwards from the foot of the tower with a velocity 30 m/s. When and where the two stones cross each other?

Given : height of tower, h = 60 m.

$$u_1 = 0, u_2 = 30 \text{ m/s}$$

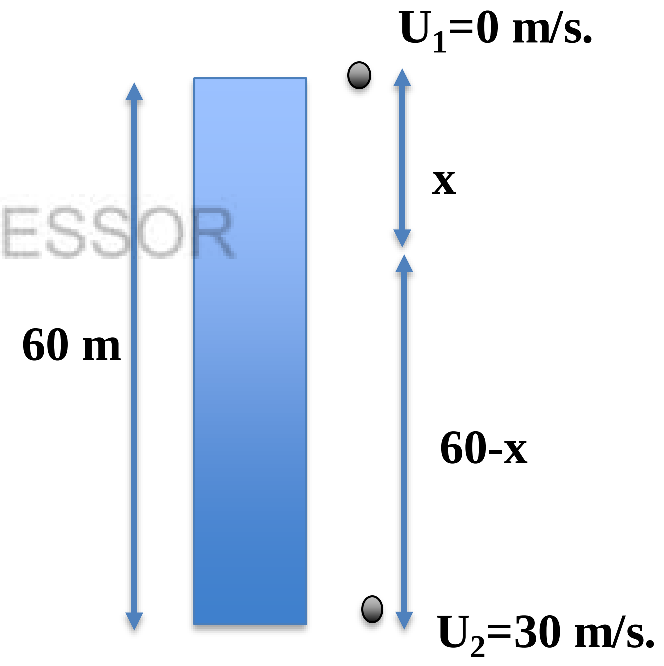
$$t_1 = t_2 = t$$

Let x be the distance from the top of the tower where the two stones cross each other.

$$\boxed{x = u_1 t + \frac{1}{2} gt^2}$$

$$= 0 + \frac{1}{2} gt^2 \text{---------- (i)}$$

$$\boxed{60 - x = u_2 t - \frac{1}{2} gt^2} \text{-------- (ii)}$$



adding equations (i) and (ii)

$$60 = u_2 \times t$$

$$t = \frac{60}{30} = 2 \text{ s}$$

$$\boxed{x = u_1 t + \frac{1}{2} g t^2}$$

$$= 0 + \frac{1}{2} \times 9.81 \times 2^2$$

$$= 19.62 \text{ m}$$

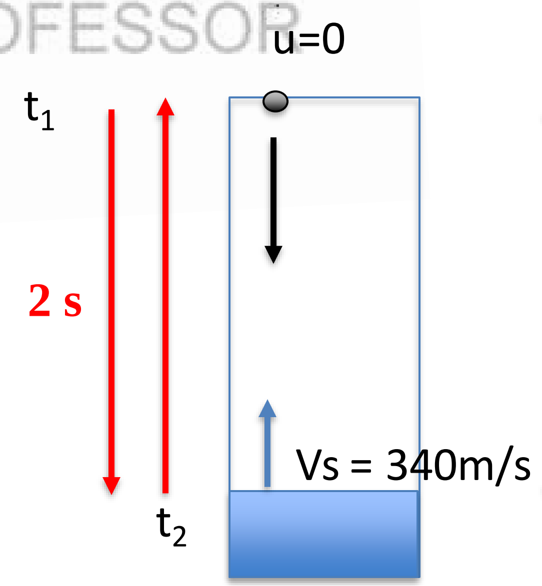
The two stones will cross each other at a distance of 19.62 m from the top of the tower, after 2 seconds.

2) A stone dropped into a well is heard to strike the water after 2 seconds. Find depth of the well, if velocity of sound is 340 m/s.

Solution:

Velocity of sound, $V_s = 340$ m/s.

Let h be the depth of well and t_1 is the time taken by the stone to reach the bottom of well and t_2 is the time taken by the sound to reach the top of well.



$$t_1 + t_2 = 2s$$

$$\boxed{h = \text{velocity of sound} \times \text{time.}}$$

$$= 340 \times t_2 = 340\,(2 - t_1)\text{.......(i)}$$

$$\boxed{h = u\,t + \frac{1}{2}\,g\,t^2}$$

$$h = 0 + \frac{1}{2} \times 9.81 \times t_1^{\,2}\text{.............(ii)}$$

From eqns. (i) and (ii)

$$\boxed{340\,(2-t_1) = \frac{1}{2} \times 9.81 \times t_1^2}$$

$$69.32\,(2-t_1) = t_1^2$$

$$t_1^2 + 69.32\,t_1 - 138.64 = 0$$

$$t_1 = 1.95 \text{ s}$$

$$t_2 = 2 - t_1 = 2 - 1.95 = 0.05 \text{ s}$$

Therefore $\boxed{h = 340 \times t_2}$

$$= 340 \times 0.05$$

$$= 17 \text{ m}$$

3) A train travels between two stopping stations, 7 km apart in 14 minutes. Assuming motion is of uniform acceleration for one part of journey and uniform retardation for the rest. Find the maximum speed.

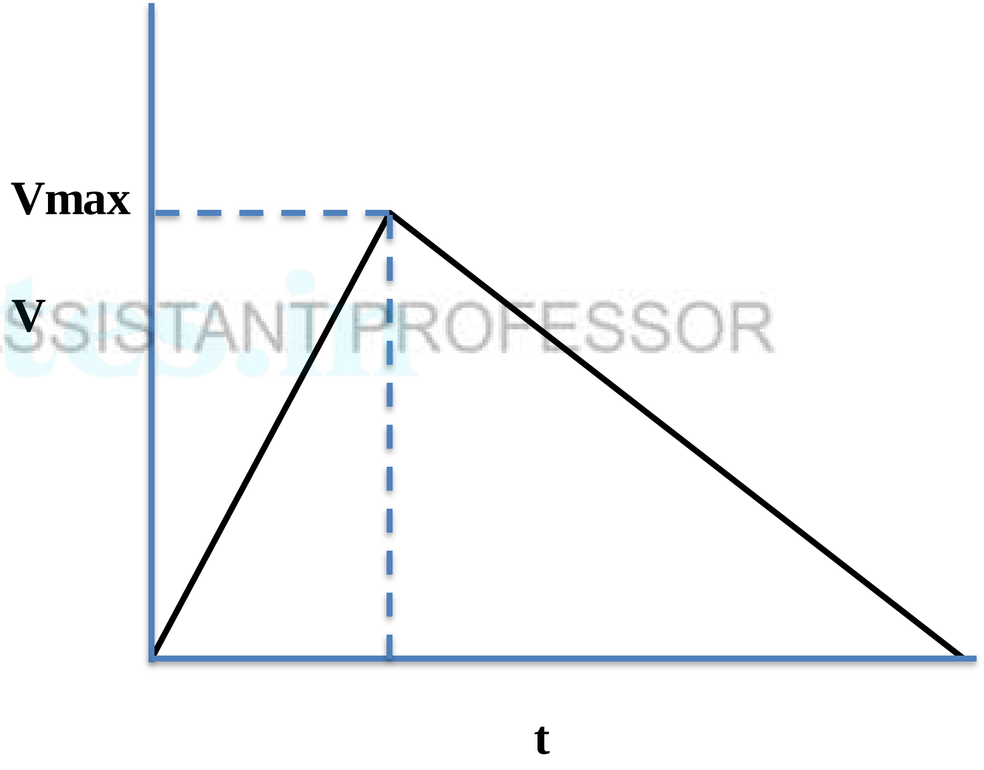
$$S = 7 \text{ km.},$$

$$t = 14 \text{ min.} = \left(\frac{14}{60}\right) h$$

$$S = \frac{1}{2} \times t \times V_{max}.$$

$$7 = \frac{1}{2} \times \left(\frac{14}{60}\right) V_{max}.$$

$$V_{max} = 60 \text{ kmph}$$



4) A car travelling at 40kmph sights a distant signal at 150 m and comes uniformly to rest at signal. It remains at rest for 20 s. As allowed by the signal, it uniformly accelerates and attain 40 kmph in 250 m. Calculate the time lost due to signal.

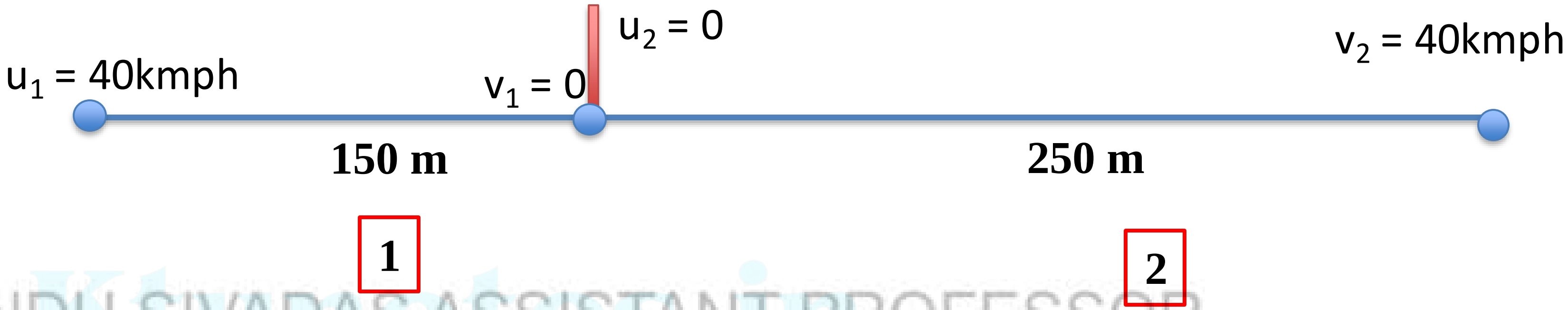
<u>**Given data:**</u>

$u_1 = 40\text{kmph}$

$= 40\text{x}\,\frac{5}{18} = 11.11\ m/s$

$v_1 = 0$

$S_1 = 150\text{ m}$

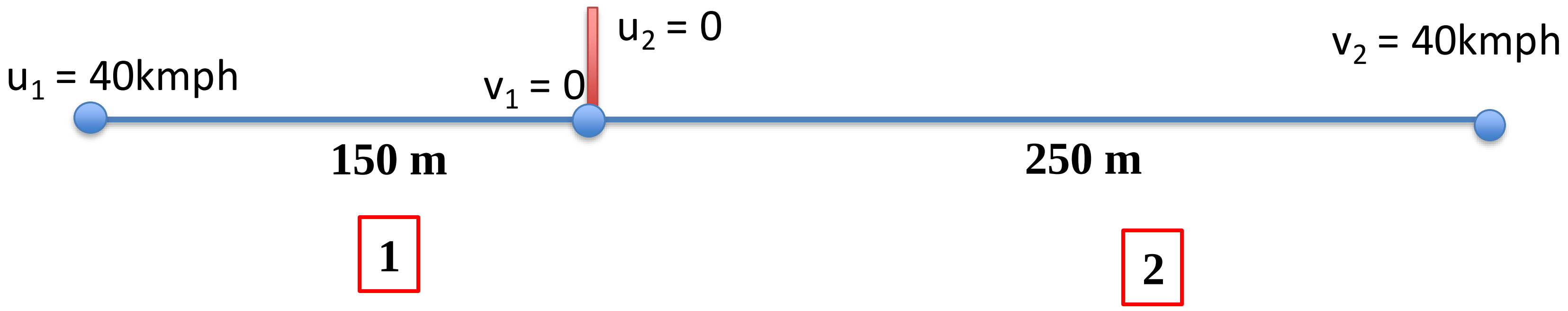


$u_2 = 0$

$v_2 = 40\text{kmph}$

$= 40\text{x}\,\frac{5}{18} = 11.11\ m/s$

$S_2 = 250\text{ m}$



Let t_1 be the time taken by the car to reach signal.

t_2 = time at signal = 20 s

t_3 = time taken to attain speed 40kmph

Total time = $t_1+t_2+t_3$

Time required to cover distance T = distance /velocity = (150+250)/11.11 = 36 s

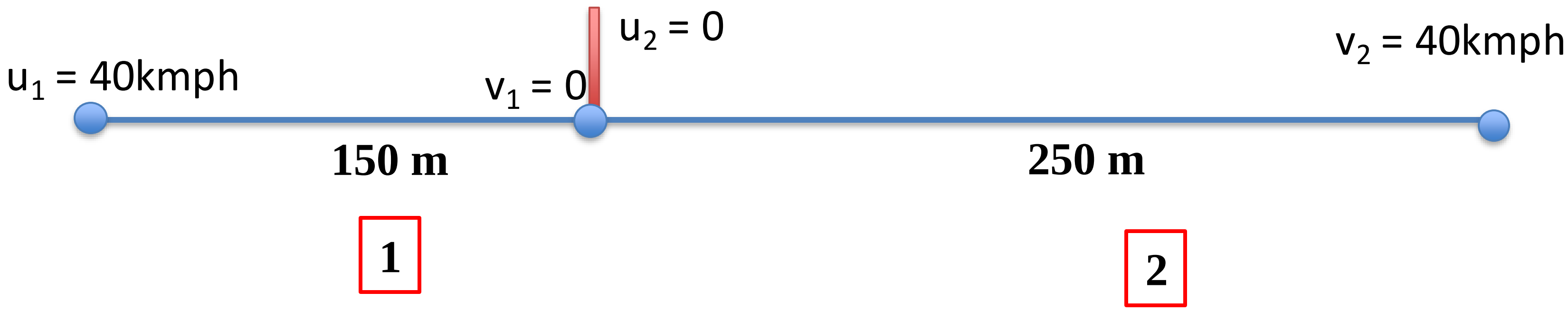
Therefore time lost at signal = Total time – 36

To find t_1

$$\mathbf{V_1^2 = u_1^2 + 2a_1s_1}$$

$$0 = 11.11^2 + 2a_1x150$$

a_1 = -0.41 m/s^2



$V_1 = u_1 + a_1t_1$

$0 = 11.11 - 0.41t_1$

$t_1 = 27$ s

To find t_2

$$V_2^2 = u_2^2 + 2a_2s_2$$

$$11.11^2 = 0 + 2a_1x250$$

$a_1 = 0.247\ m/s^2$

$V_2 = u_2 + a_2t_2$

$11.11 = 0 + 0.247\ t_2$

$t_2 = 45$s

Therefore time lost at signal = Total time – 36 = (27+20+45) – 36 = 56 s

5) The motion of a particle along a straight line is defined as S=25t + $5t^2 - 2t^3$, where S is in metres and t is in seconds. Find (i) velocity and acceleration at the start; (ii)the time the particle reaches maximum velocity and iii) maximum velocity of the particle

Hint:

S=25t + $5t^2 - 2t^3$

(i) At start means t=0

V= ds/dt = 25 m/s

a = dV/dt = 10 m/s

(ii) Maximum velocity means dV/dt =0

t = 0.83 s

(iii)Maximum velocity is at t=0.83s

Vmax = 29.17 m/s

6) The displacement of a particle is given by S= t^3 - $3t^2$ +2t + 5. find the time at which the acceleration is zero and time at which velocity is 2m/s.

Hint

- S= t^3 - $3t^2$ +2t + 5
- a =dV/dt
- V= dS/dt

Ans:

- Time at which acceleration is zero= 1 s
- Time at which velocity is 2 m/s = 2 s

7) A point is moving in a straight line with acceleration given by a = 15t-20. It passes through a reference point at t=0 and another point 30 m away after an interval of 5 seconds. Calculate displacement, velocity and acceleration of the point after a further interval of 5 seconds.

Given data:

a= 15t-20

To find : S,V and a at t=5 s

Solution:

a= 15t-20

a = dV/dt

dV= a.dt

V= ∫ a.dt

= ∫ (15t–20)dt

$= \frac{15t^2}{2}$ - 20t + C1 = $\mathbf{7.5t^2}$ **- 20t +C1**

V= dS/dt

dS= V.dt

S= $\int$ V.dt

$= \int (7.5t^2 - 20t + C1).dt$

$= \frac{7.5t^3}{3} - \frac{20t^2}{2} + C1\ .\ T + C2$ = **$2.5\ t^3 - 10\ t^2 + C1.t + C2$**

<u>At t=0, S=0</u>

$S = 2.5\ t^3 - 10\ t^2 + C1.t + C2$

0 = 0- 0 + 0 + C2

C2 = 0

<u>At t=5 s, S=30 m</u>

$S = 2.5\ t^3 - 10\ t^2 + C1.t + C2$

$30 = (2.5 \times 5^3) - (10\ x\ 5^2) + 5\ C1$

C1 = -6.5

Displacement , velocity and acceleration at the end of 10 s

Put t = 10 s

Displacement S= 2.5 t^3 - 10 t^2+ C1.t +C2

S = 2.5 $x10^{\ 3}$ - 10 $x10^{\ 2}$- 6.5 x 10 + 0

= **1435 m**

Velocity V = 7.5t^2 - 20t +C1

V= 7.5x 10^2 - 20x10 - 6.5

= **543.5 m/s**

Acceleration a = 15t-20

a= (15x10) -20

= **130 m/s^2**

KINETICS

Kinetics is the study forces acting on a body in motion with reference to the forces acting on the body.

Solution of problems in Kinetics

Three approaches

- Newton's second law
- Work-energy principles
- Impulse and momentum

Newton's second law

- According to Newton's Second law the rate of change of momentum is directly proportional to the applied force and the motion takes place in the direction in which the force acts.

$$F = ma$$

- When a system of forces act on a body, the above statement can be stated as:

$$R = m\,a$$

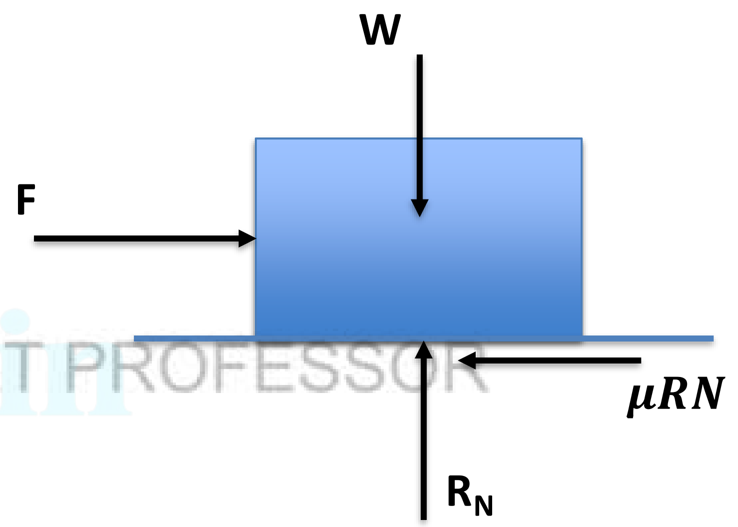
1. A block weighing 1000 N rest on a horizontal plane. Find the magnitude of the force required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction between the block and plane is 0.25.

Given data:

W = 1000 N

A= 2.5 m/s^2

μ = 0.25



To find: Magnitude of force (F)

Solution:

Net force in horizontal direction = m a

$F - \mu R_N = m a$

$F = ma + \mu R_N$

$= \frac{W}{g} a + \mu R_N$

$= \frac{1000}{9.81} \times 2.5 + 0.25 \times 1000 =$ **504.84 N**

Motion is along the horizontal plane, therefore,

Net force in horizontal direction= m a

Net force in vertical direction = 0

$W = \mathbf{R_N}$

$\mathbf{R_N} = 1000$ N

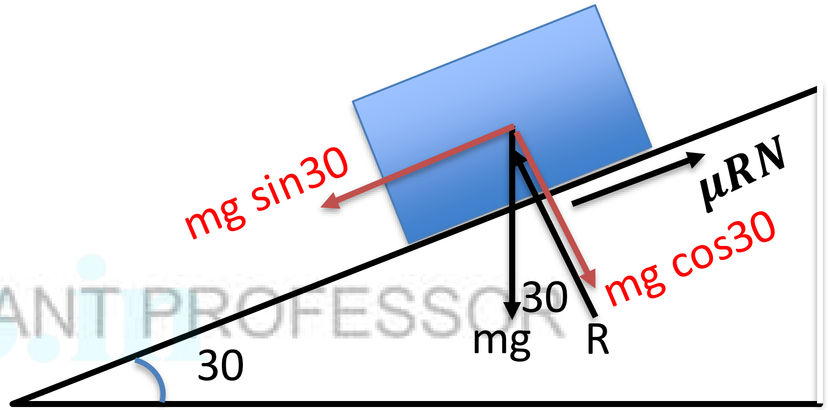
2. A body of mass 50 kg slides down a rough inclined plane whose inclination to the horizontal is 30. If the coefficient of friction between the plane and the body is 0.4, determine acceleration of the body.

Given data:

m = 50 kg

Angle of inclination = 30

μ = 0.4



To find: acceleration of the body (a)

Solution:

Motion is along the plane, therefore,

Net force along the plane= m a

Net force perpendicular to the plane = 0

Net force perpendicular to the inclined plane = 0

$\boldsymbol{R_N}$= mg cos 30 = **50x 9.81 cos 30**

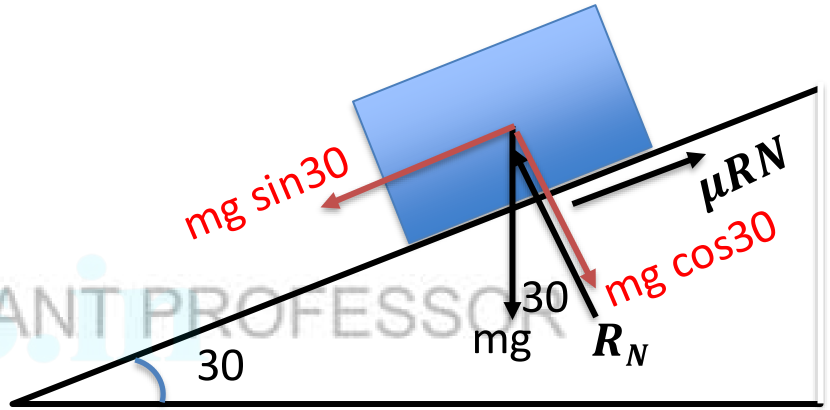
Net force along the inclined plane = ma

mg sin30 - μRN = ma

(50x9.81xsin30) - (0.4 x 50x 9.81 cos 30) = ma

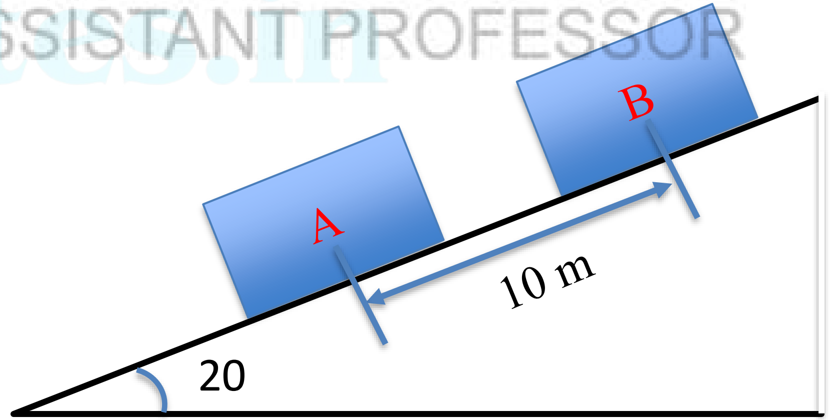
ma = 75.336

a = 75.336/50 = **1.51 m/s^2**



Assignment Question: 1

Two blocks A and B of weight 250 N and 500 N respectively are held stationary 10 m apart on a 20° inclined plane as shown in figure. The co-efficient of friction between the plane and block A is 0.3 while it is 0.2 between the plane and block B. If the blocks are released simultaneously, calculate the accelerations of the two blocks.



Answers:

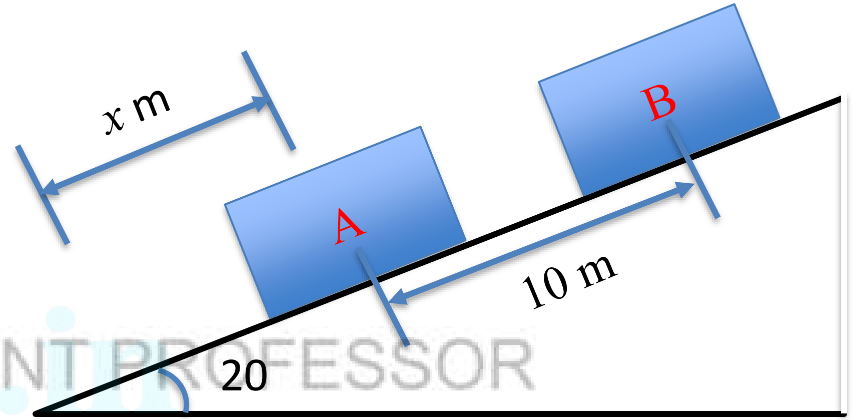
Acceleration of block A = **0.59 m/s^2**

Acceleration of block B = **1.51 m/s^2**

3. In the previous problem, calculate time taken and distance travelled by each block before they are at verge of collision. [Take acceleration of block A and B as 0.59 m/s^2 and 1.51 m/s^2 respectively.

Distance travelled by block A in t seconds = x m

Distance travelled by block B in t seconds = (10 + x) m



$$S = ut + \frac{1}{2}at^2$$

Block A:

$x = 0 + \frac{1}{2} 0.59\, t^2$ ……..(i)

Block B:

$10 + x = 0 + \frac{1}{2} 1.51\, t^2$……..(ii)

Subtracting (ii)- (i)

$10 = (\frac{1}{2} 1.51\, t^2) – (\frac{1}{2} 0.59\, t^2)$

t= 4.66 sec

To find distance travelled by block A:

Substituting the value of t in eqn.(i);

x = 0 + $\frac{1}{2}0.59\, t^2$

x = 0 + $\frac{1}{2}0.59\, x\, (4.66)^2$

x = 6.41 m

Distance travelled by block B:

10+ x = **16.41 m**

KINETICS

MOTION OF CONNECTED BODIES

MOTION OF CONNECTED BODIES

- **Consider two bodies connected by a light inextensible string passing over a smooth pulley.**
- **Since the pulley is smooth, the tension in the string on both sides of pulley will be the same.**
- **The body of greater mass moves downwards and other mass moves upwards.**
- **Considering the motion of each body separately and applying Newton's law of motion, F= ma, acceleration can be determined.**
- **NOTE: Force in the direction of motion should be taken as positive and force opposite to the direction of motion should be taken as negative.**

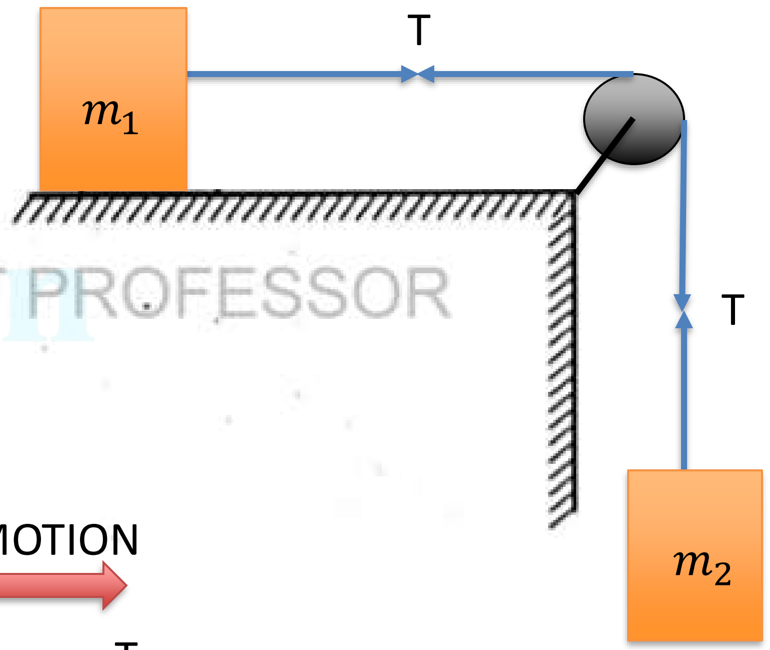
1. A mass of 60 kg lies on a smooth horizontal table. It is connected to a fine string passing over a smooth guide pulley on the edge of the table to a mass 50 kg hanging freely. Find the tension in the string and acceleration of the system.

Given data:

m_1=60 kg

m_2= 50 kg

Smooth horizontal table, so no friction



To find : Tension (T) and acceleration (a)

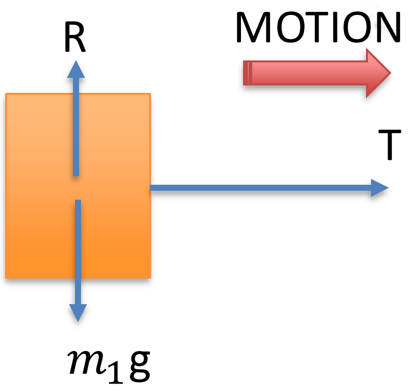
Solution:

Consider mass m_1

Net force in horizontal direction = m_1a

T = m_1a = 60 a ……..(i)

Mass m_1



Consider mass m_2

Net force in vertical direction = m_2a

m_2g - T = m_2a

(50x9.81)- 60 a = 50 a

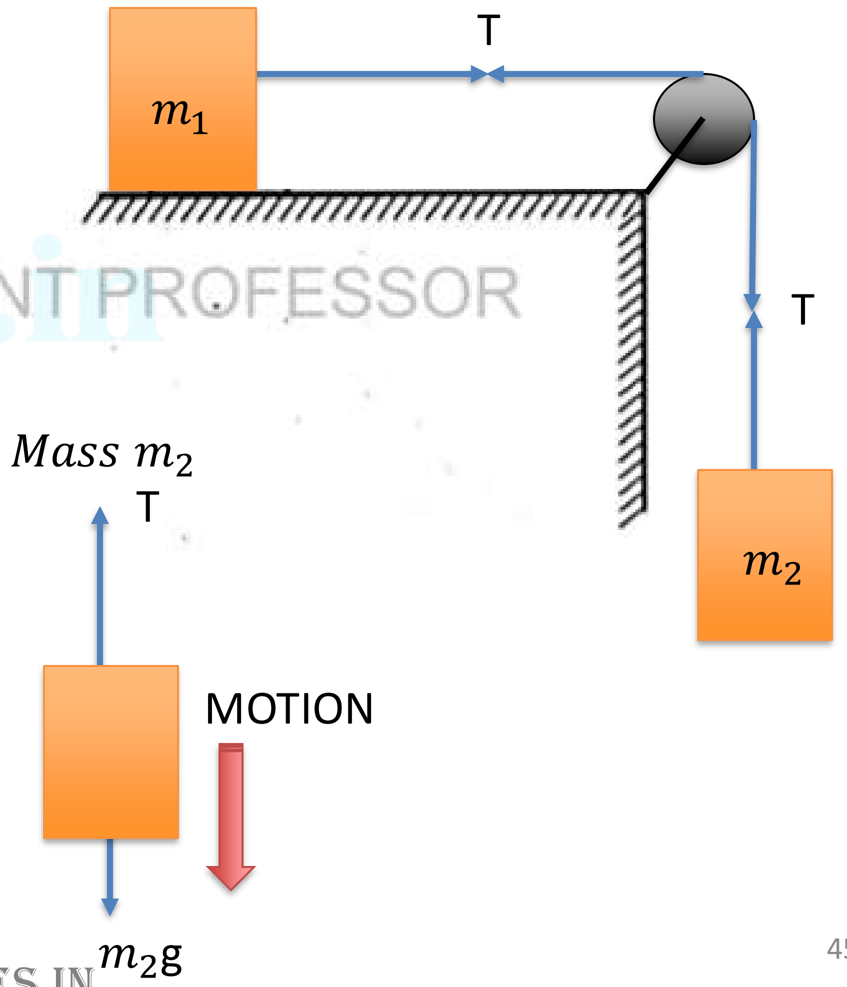
50 x9.81 = 110 a

a= 4.46 m/s^2

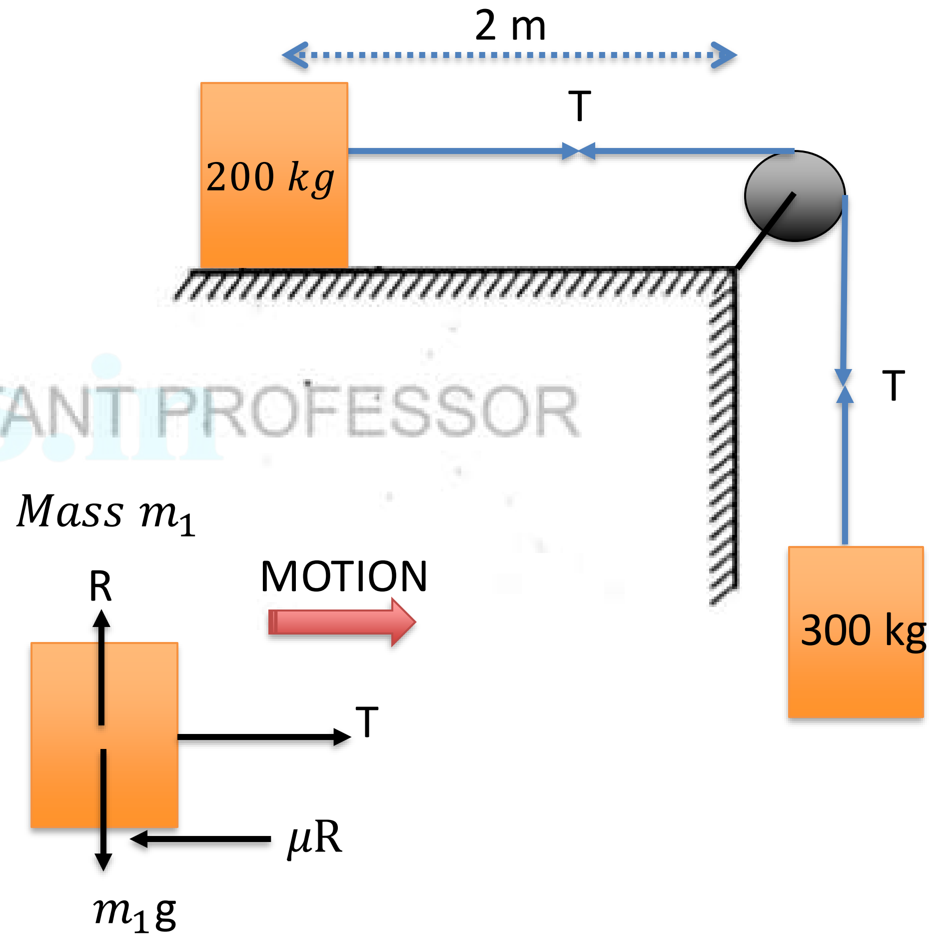
From equation (i)

Tension T = 60 a

T = 60 x 4.46 = **267.7 N**



2. Two blocks are joined by an inextensible string as shown in figure. If the system is released from rest, determine the velocity of block after it has moved 2 m. Assume the co-efficient of friction between the block and the plane is 0.25. The pulley is weightless and frictionless.



Given data:

m_1=200 kg

m_2= 300 kg

μ = 0.25

To find : Velocity of the block

Solution:

Consider mass m_1

R= m_1g = 200 x9.81

Net force in horizontal direction = m_1a

T -μR = m_1a

T– (0.25x200x9.81)= 200 a……..(i)

Consider mass m_2

Net force in vertical direction = m_2a

m_2g -T = m_2a

(300 x9.81) -T = 300a……..(ii)

Adding (i) and (ii)

(300 x9.81)– (0.25x200x9.81)= 200a+300a

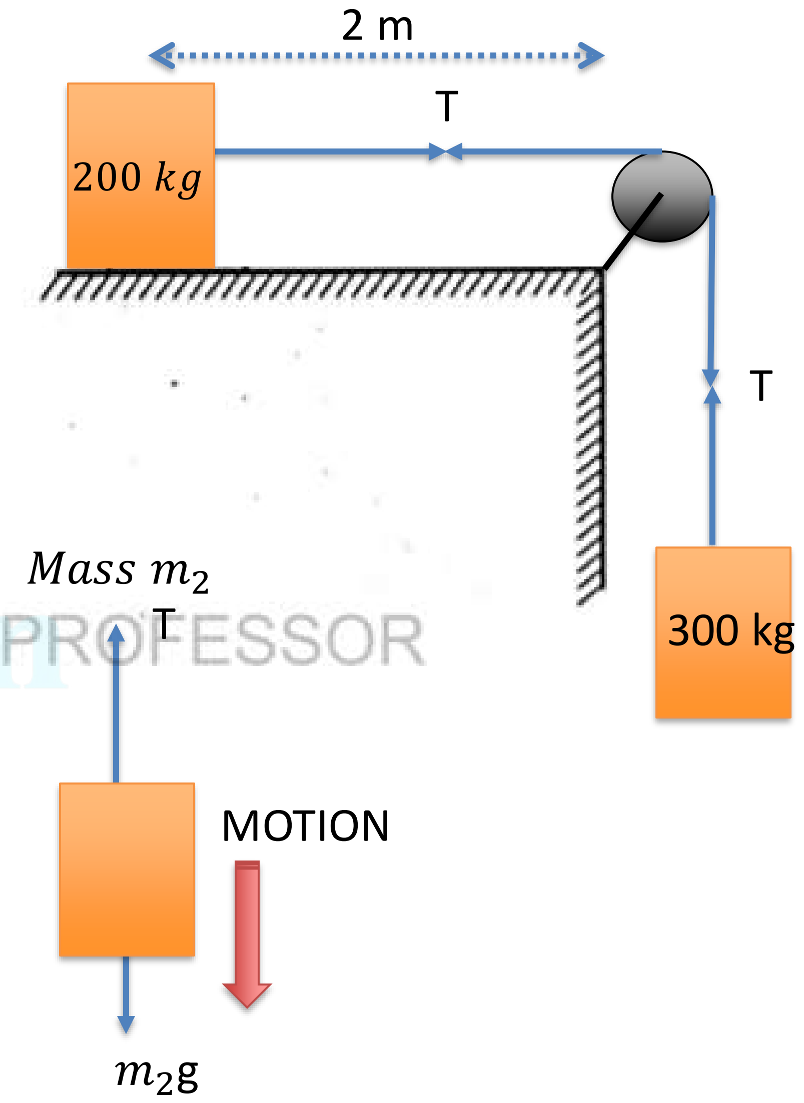
a= 4.905 m/s²

To find Velocity (S and a known)

$$\mathbf{V^2 = u^2 + 2as}$$

$$\mathbf{V^2 = 0 + (2\ x\ 4.905\ x\ 2)}$$

V= 4.43 m/s



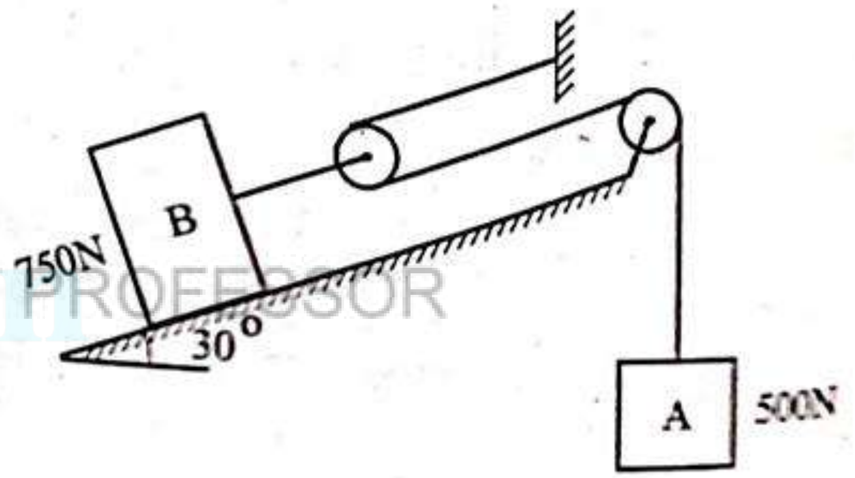
3. The system of bodies shown in figure starts from rest. Determine the acceleration of body B and tension in string supporting body A

Given data:

W_a= 500 N

W_b= 750 N

For the given vertical displacement of body A, the displacement of body B along inclined plane will be half that of A



$x_B = \frac{1}{2} x_A$

$a_B = \frac{1}{2} a_A$

To find: acceleration of body B (a_B) and Tension in body A (T_A)

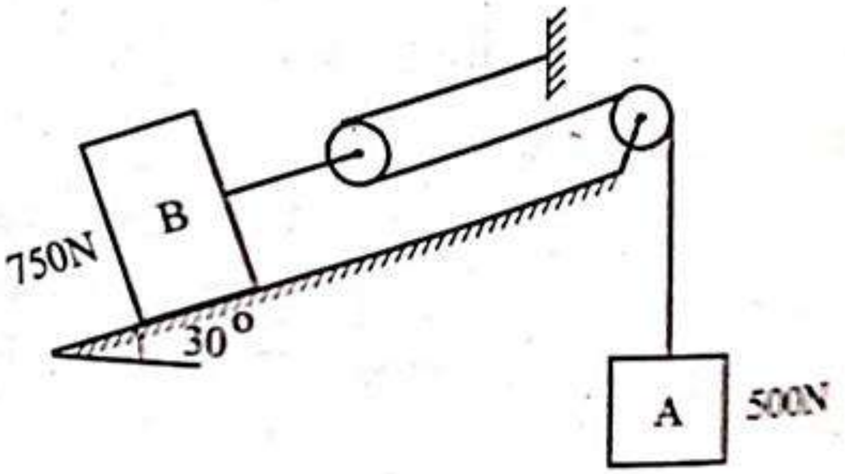
Consider the downward motion of body A,

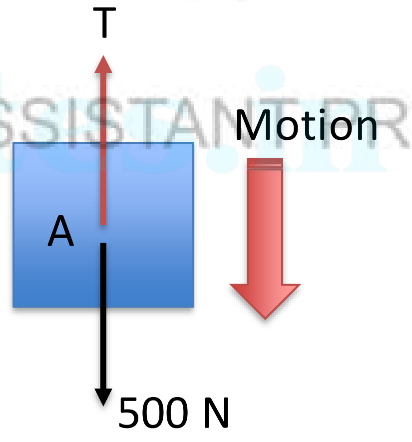
Net force = mass × acceleration.

$$m_A g - T = m_A \times a_A$$

$$500 - T = \frac{500}{9.81} \times a_A = \frac{500}{9.81} \times 2\, a_B$$

$$500 - T = \frac{1000}{9.81} a_B \text{ --------(i)}$$





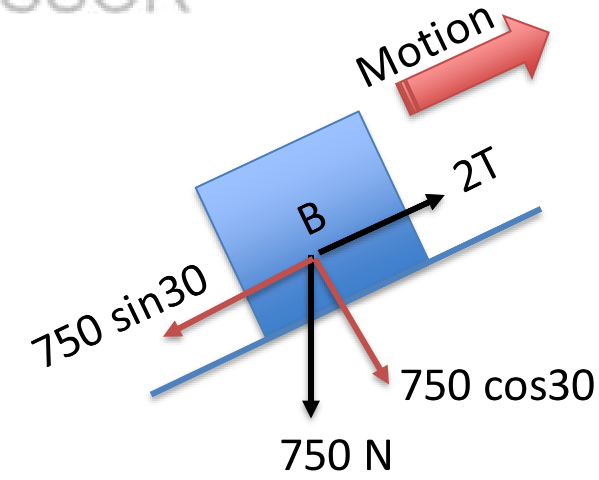
Consider the motion of body B, up the inclined plane.

Net force = mass × acceleration.

$$2T - m_B g \sin\theta = m_B \times a_B$$

$$2\,T - 750 \times \sin 30 = \frac{750}{9.81} \times a_B$$

$$T - 187.5 = \frac{375}{9.81} \times a_B \text{ ----- (ii)}$$



Adding equations (i) and (ii)

$$312.5 = \frac{1375}{9.81} \times a_B$$

$$a_B = 2.23 \text{ m/s}^2$$

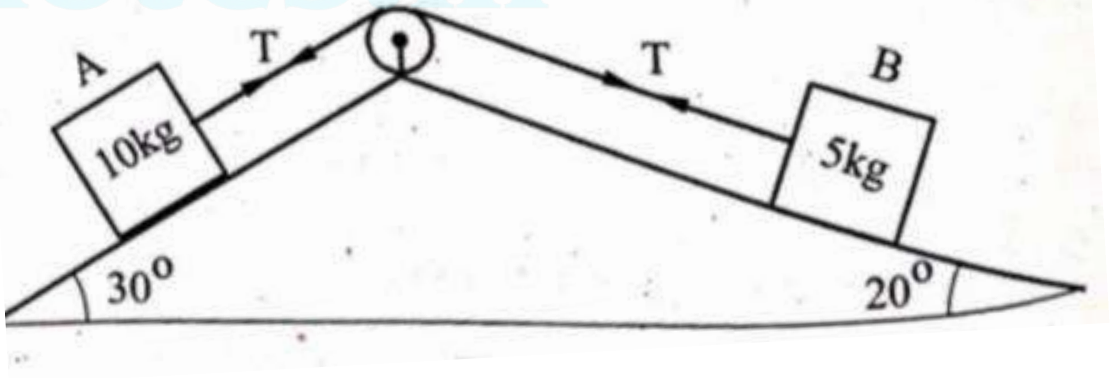
From eqn (ii)

$$T - 187.5 = \frac{375}{9.81} \times 2.23.$$

$$T = 272.74 \text{ N.}$$

Assignment Question: 2

Two smooth inclined planes whose inclinations with horizontal are 30° and 20° are placed back to back. Two bodies of mass 10kg and 5 kg are placed on them and are connected by a string as shown in figure. Calculate the tension in the string and acceleration of the bodies.



Answers:

Tension in string = **27.477 N**

Acceleration a = **2.16 m/s^2**

4. Determine the tension in the string and acceleration of the two bodies of mass 300kg and 100kg connected by a string and frictionless and weightless pulley as shown in figure.

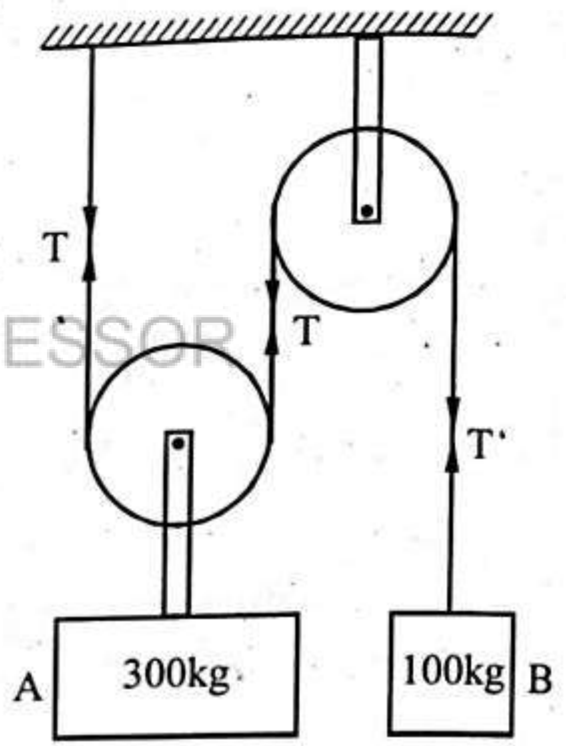
Given data:

m_a= 300kg

m_b= 100kg

The downward displacement of 300 kg will be only half of the upward displacement of 100 kg mass

$a_A = \frac{1}{2}\, a_B$



To find: Tension in string, acceleration of two bodies

Consider the downward motion of body A

$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$m_A g - 2T = m_A \times a_A$$

$$300 \times 9.81 - 2T = 300 \times a_A$$

$$150 \times 9.81 - T = 150\ a_A \text{ ------------(i)}$$

Consider the upward motion of body B

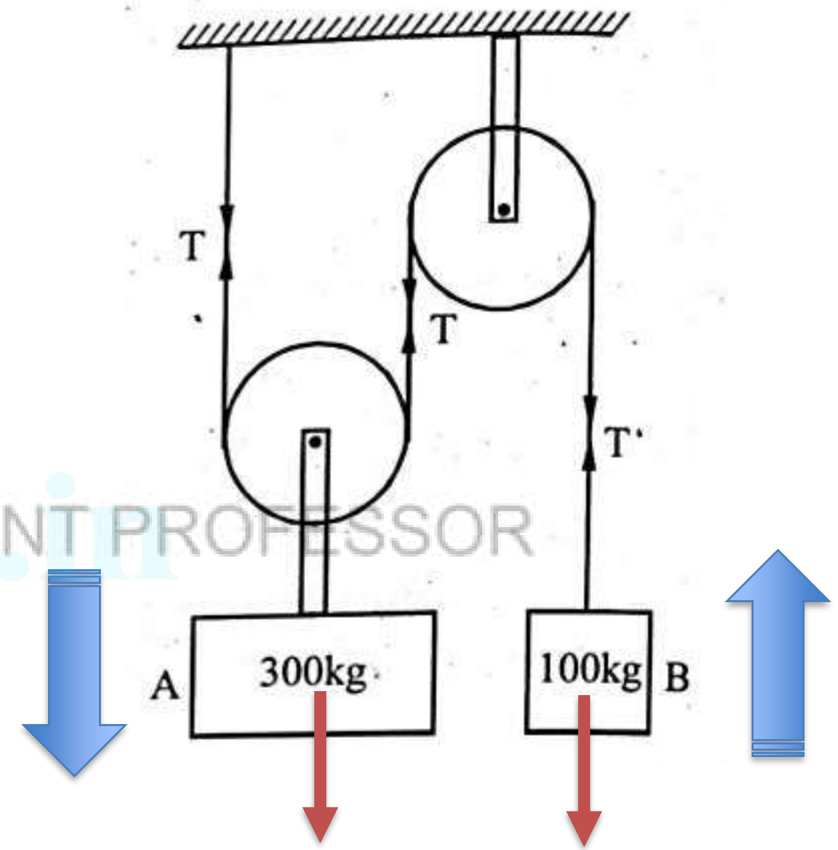
$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$T - m_B g = m_B\ a_B$$

$$= m_B \times 2\ a_A$$

$$T - 100 \times 9.81 = 100 \times 2 \times a_A$$

$$T - 100 \times 9.81 = 200\ a_A \text{ ----------(ii)}$$



From equations (i) and (ii)

$$50 \times 9.81 = 350\, a_A$$

$$a_A = 1.4 \text{ m/s}^2$$

$$a_B = 2.8 \text{ m/s}^2$$

Tension in string = 200 a_A + (100 x9.81)

= 1261 N

5. Two equal weights W are connected by a string passing over a frictionless pulley. A small weight w is attached to one side, as shown in figure causing that the weight to fall. Determine the acceleration of the system assuming that the weights starts from rest

Consider the upward motion of weight W,

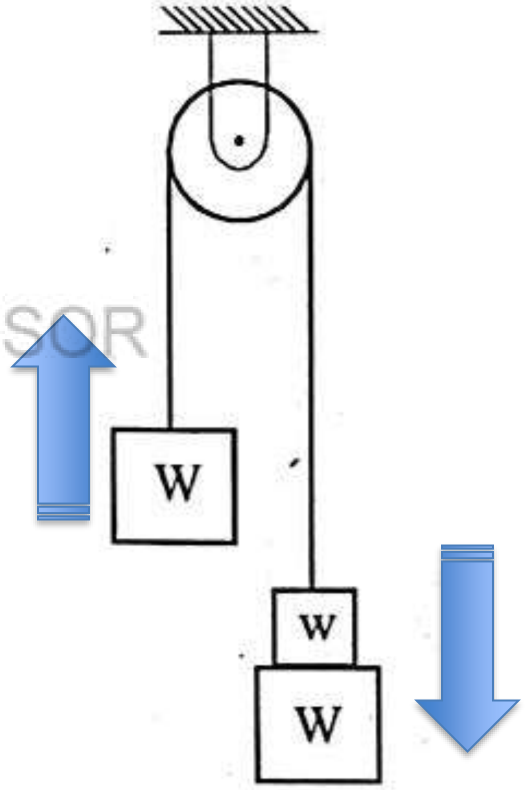
$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$T - W = \frac{W}{g} \times a \quad \text{.......} \quad \text{(i)}$$

Consider the downward motion of weight (W+w)

$$\text{Net force} = \text{mass} \times \text{acceleration}$$

$$W + w - T = \frac{W + w}{g} \times a \quad \text{.........} \quad \text{(ii)}$$



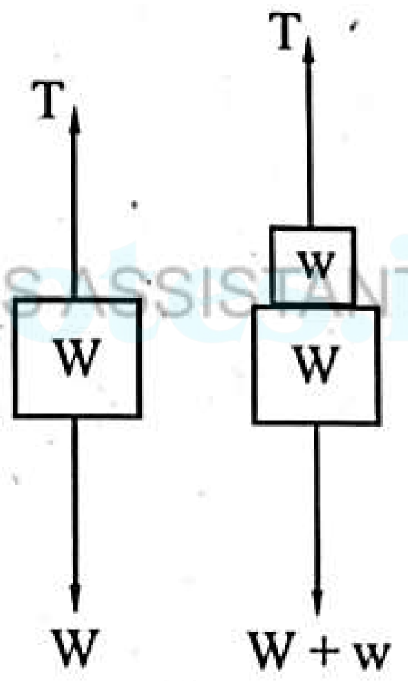
Adding equations (i) and (ii)

$$T - W + W + w - T = \frac{W}{g} \times a + \left(\frac{W + w}{g}\right) \times a$$

$$w = (W + W + w) \times \frac{a}{g}$$

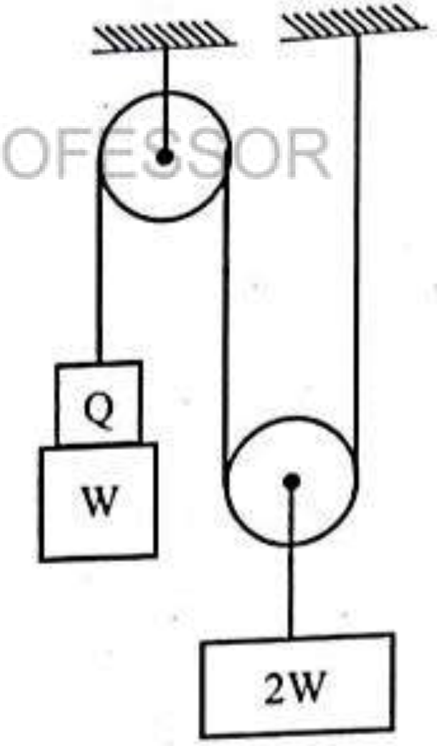
$$= (2W + w) \times \frac{a}{g}$$

$$a = \frac{wg}{2W + w}$$



Assignment question : 3

Weights W and 2W are supported in a vertical plane by a string and pulleys arranged as shown in figure. Find the magnitude of an additional weight Q applied to the weight W which will have a downward acceleration 0.981 m/s^2



Answer:

Q =0.167 W

D’Alembert’s Principle

D’Alembert’s principle

- D’Alembert’s principle is an application of Newton’s second law to a moving body.
- A problem in dynamics can be converted into an equivalent problem in static using D’Alembert’s principle.
- F= ma can be written as F-ma =0
- The term (-ma) is called the inertial force. Negative because it is opposite to direction of acceleration.
- The magnitude of inertial force is equal to product of mass and acceleration and it acts in the direction opposite to the direction of acceleration. F_i= -ma
- F = ma can be written as:
- F-ma =0 or F+ (-ma)= 0 or F + F_i=0
- D’Alembert’s principle states that the resultant of a system of force acting on a body in motion is in dynamic equilibrium with the inertia force.

1. A force of 300 N acts on a body of mass 150kg. Calculate the acceleration of the body using D'Alembert's principle.

Given data:

F= 300 N

m = 150 kg

Acc. To D'Alembert's principle,

F+(-ma) = 0

300 + (-150xa)= 0

300 = 150 a

A= 2 m/s^2

2. A system of weights connected by strings passing over pulleys A and B is shown in figure. Find the acceleration of the three weights P,Q and R using D'Alembert's principle.

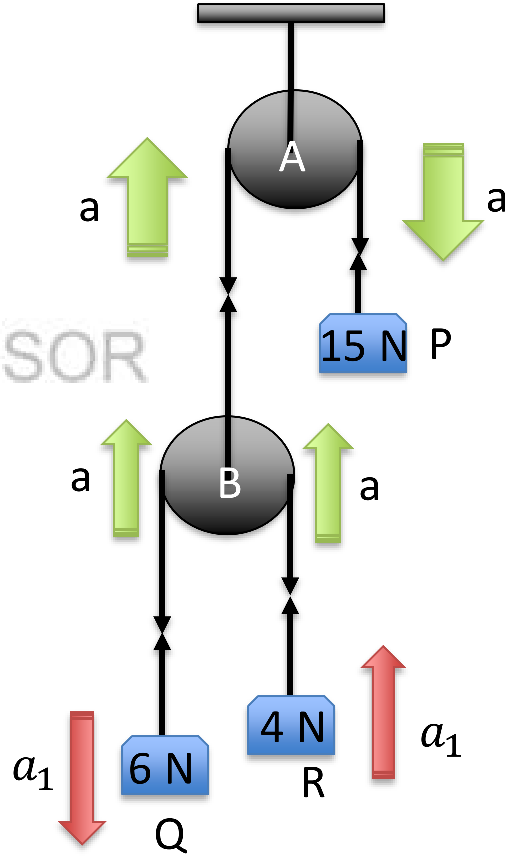
Given data:
Pulley A and B
Weights P,Q,R

To find: acceleration of P,Q and R

Solution:
Downward acceleration of body P = a
Upward acceleration of pulley B = a

Downward acceleration of body Q with respect to pulley B = a_1
Upward acceleration of body R with respect to pulley B = a_1

Absolute acceleration of body Q = a_1-a
Absolute acceleration of body R = a_1 +a



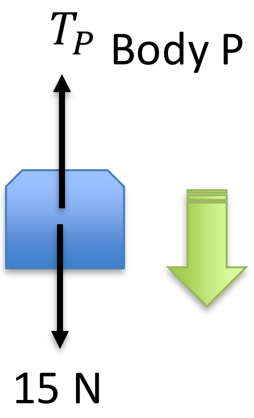
Consider body P :

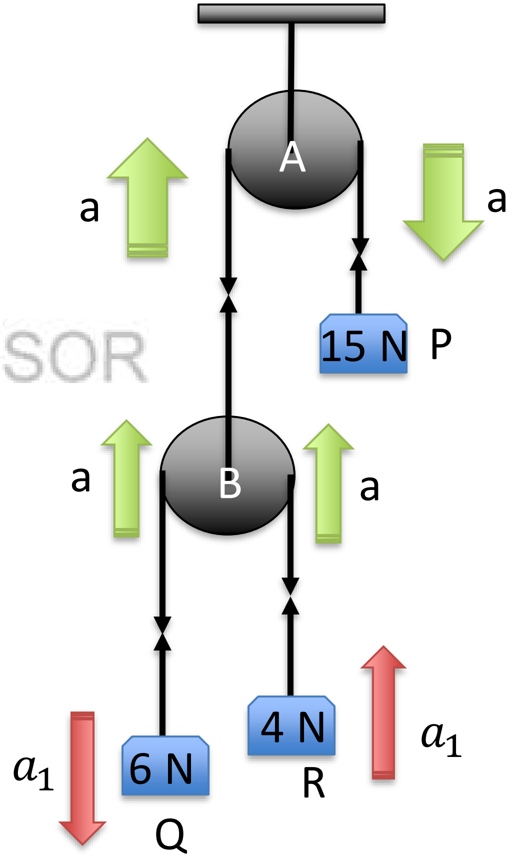
Acc. to D'Alembert's principle:

F+ (-ma)= 0

$(15 - T_P) + (-\frac{15}{9.81} \text{ x a}) = 0$

$T_P = 15 - 1.53\,a$ …….(i)





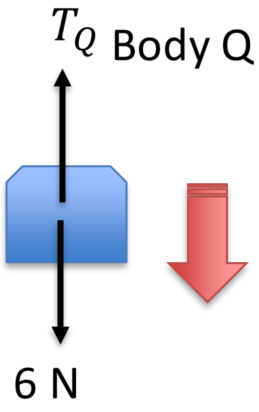
Consider body Q :

Acc. to D'Alembert's principle:

F+ (-ma)= 0

$(6 - T_Q) + (-\frac{6}{9.81} \text{ x } (a_1\text{-a}) = 0$

$T_Q = 6 - 0.612\,(a_1\text{-a})$ …….(ii)



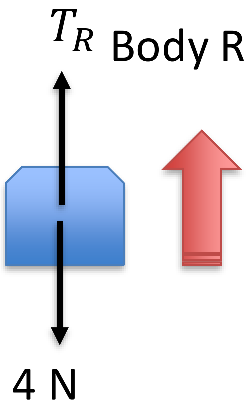
Consider body R :

Acc. to D'Alembert's principle:

F+ (-ma)= 0

$(T_R - 4) + (-\frac{4}{9.81} \text{ x } (a_1 + \text{a})) = 0$

$T_R = 4 + 0.407\,(a_1 + \text{a})$ …….(iii)



We know tension along pulley B is same,

Therefore: $T_Q = T_R$

Equating (ii) and (iii)

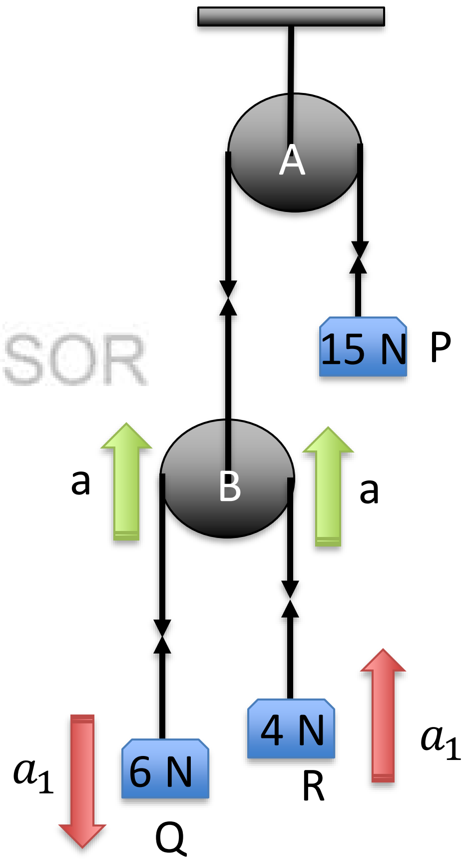
$6 – 0.612\,(a_1\text{-a}) = 4 + 0.407\,(a_1 + \text{a})$

$2 = 0.407\,(a_1 + \text{a}) + 0.612\,(a_1\text{-a})$

$2 = 1.02\,a_1 - 0.2\text{ a}$

$1.02\,a_1 = 2+0.2\text{ a}$

$a_1 = 1.96+0.196\text{ a}$……..(iv)



Consider pulley B: (weightless, so mass=0)

Acc. to D'Alembert's principle:

F+ (-ma)= 0

F=0

-2 T_Q + T_P = 0

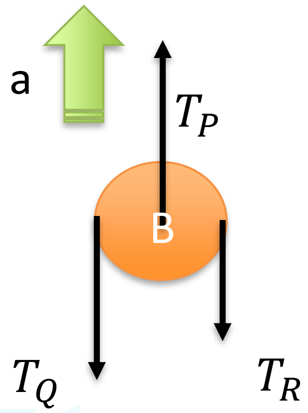
T_P= 2 T_Q …….(v)

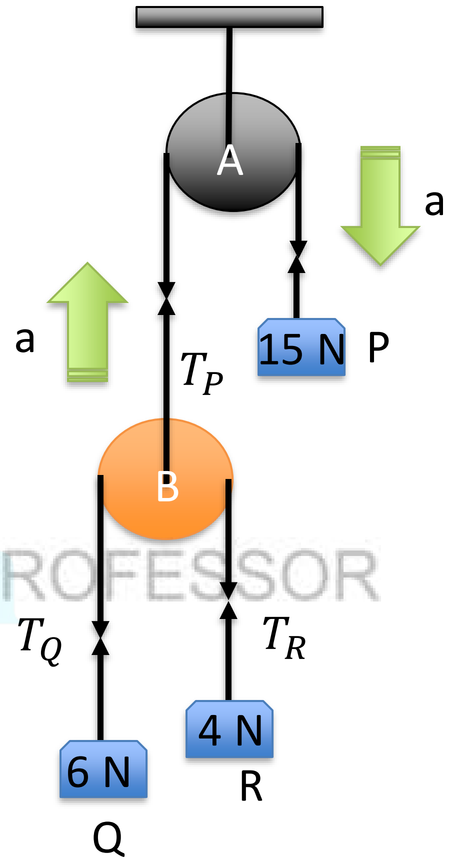
15 - 1.53 a =2[6 – 0.612 (a_1-a)]

15 - 1.53 a =12 – 1.224a_1 +1.224 a

3- 2.75 a =-1.224 a_1

a_1 =2.25a – 2.45……(vi)





From eqn. (iv and vi)

1.96+0.196 a = 2.25a – 2.45

a= 2.15 m/s²

a_1 = 2.25a – 2.45= **2.38 m/s²**

Downward acceleration of body P = a = **2.15 m/s^2**

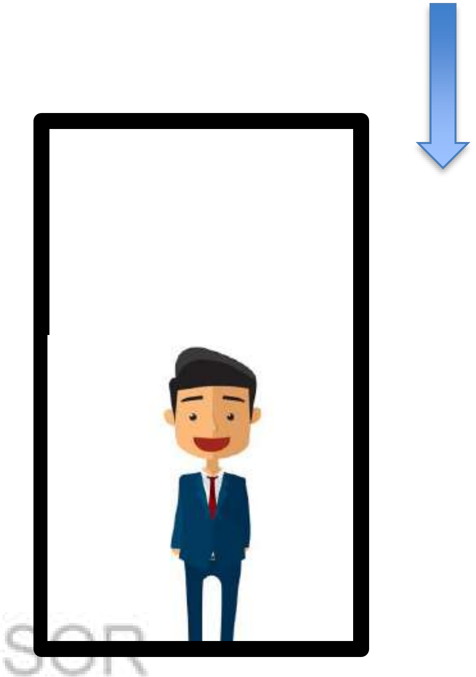
Absolute acceleration of body Q = a_1-a = 2.38-2.15 = **0.23 m/s^2**

Absolute acceleration of body R = a_1 +a = 2.38+2.15 = **4.53m/s^2**

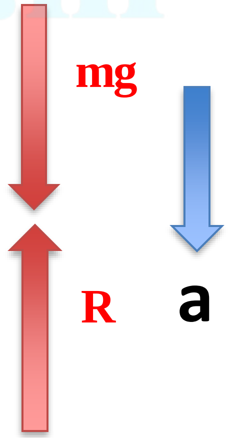
MOTION OF A LIFT

KRISHNENDU SIVADAS ASSISTANT PROFESSOR

Motion of a lift in downward direction

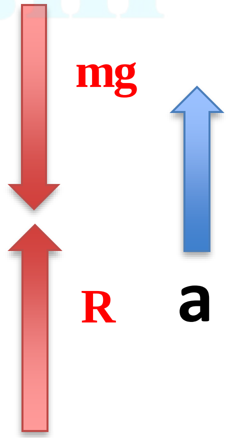


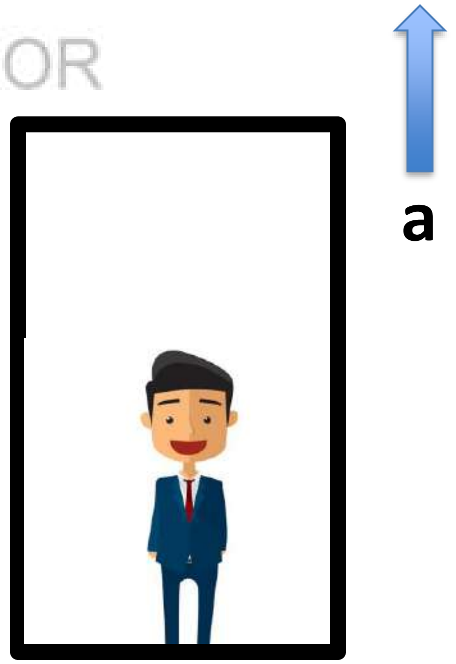
- Consider the motion of a lift with acceleration in the downward direction.
- Let W be the weight of the man and R be the reaction of force applied by the man on the floor of the lift.
- Acceleration is downwards and hence inertia force acts in the upward direction.
- For dynamics equilibrium:
- F- ma = 0
- $W - R - \frac{W}{g} a = 0$
- $R = W - \frac{W}{g} a = W[1 - \frac{a}{g}]$



Motion of a lift in Upward direction

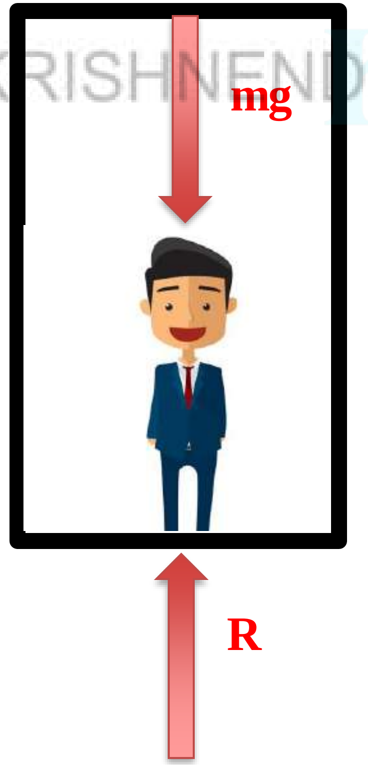
- **When acceleration is upwards, the direction of inertia force is downwards.**
- **For dynamic equilibrium;**
- **F-ma= 0**
- **$R - W - \frac{W}{g}a = 0$**
- **$R = W[\ 1+ \frac{a}{g}a]$**



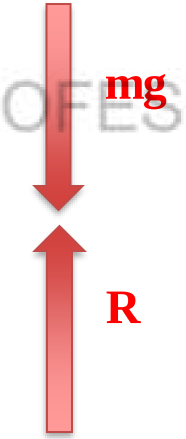


Motion of a lift with uniform velocity

- When a lift moves with uniform velocity, the acceleration of lift is zero.
- A man standing on the floor of the lift exerts force equal to its own weight on the lift.



R=mg



a=0

NOTE:

When lift accelerates in downward direction	R = W[1- $\frac{a}{g}$]	Man exerts less force
When lift accelerates in upward direction	R = W[1+ $\frac{a}{g}$]	Man exerts more force
Lift moving with uniform velocity	R = mg	
When lift accelerates	Direction of inertia force is opposite to that of acceleration	
When lift decelerates	Direction of inertia force is same direction to that of deceleration	

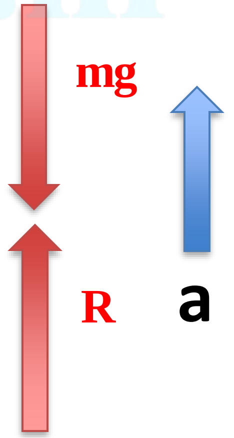
1. A lift has an upward acceleration of 1.2 m/s^2. What force will a man weighing 750 N exerts on the floor of the lift? What force would he exert if the lift had an acceleration of 1.2 m/s^2 downwards. What upward acceleration would cause his weight to exert a force of 900 N on the floor?

Case 1: When lift moves upwards

Given data:

Upward acceleration, a = 1.2 m/s^2

Weight of man W= 750 N



To find: Force exerts on the floor of lift (R)

Solution:

$$R = W[1+ \frac{a}{g}]$$

$$= 750\ [1+ \frac{1.2}{9.81}] = 841.74\ N$$

Case 2: When lift moves downwards

Given data:

Downward acceleration, a = 1.2 m/s^2

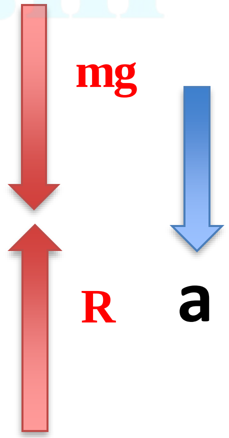
Weight of man W= 750 N

To find: Force exerts on the floor of lift (R)

Solution:

$$R = W[1- \frac{a}{g}]$$

$$= 750\ [1- \frac{1.2}{9.81}] = 658.26\ N$$



Case 3: When lift moves upwards

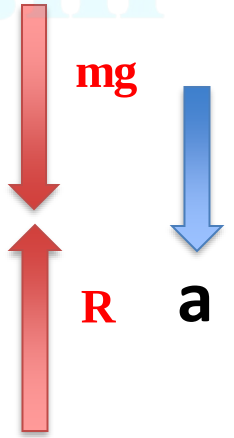
Given data:

Weight of man W= 750 N

Reaction force exerted on floor R= 900 N

To find: Upward acceleration a

Solution:



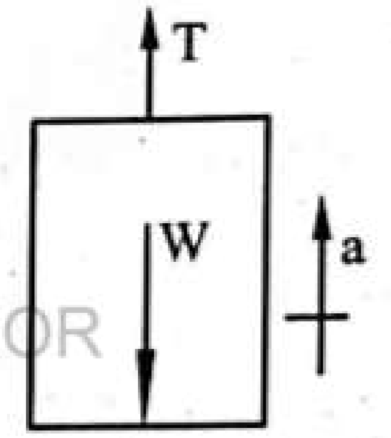
$$R = W[1+ \frac{a}{g}]$$

$$900 = 750\ [1+ \frac{a}{9.81}]$$

$$1.2 = 1+ \frac{a}{9.81}$$

a= 1.962 m/s² → $a = 1.962\ m/s^2$

2. An elevator of total weight 5000 N starts to move upwards with a constant acceleration of 1 m/s^2 Find the force in the cable during the acceleration motion. Also find the force at the floor of the elevator under the feet of a man weighing 600 N when elevator moves up with auniform retardation of 1 m/s^2



Case 1: Elevator moves upward with acceleration

Given data:

Weight of elevator = 5000 N

a= 1m/s^2

To find: Force in the cable during acceleration motion (T)

Solution:

For dynamic equilibrium; F-ma=0

T-W- $\frac{Wa}{g}$ =0

T = W[1+ $\frac{a}{g}$]

= 5000[1+ $\frac{1}{9.81}$] = **5509.68 N**

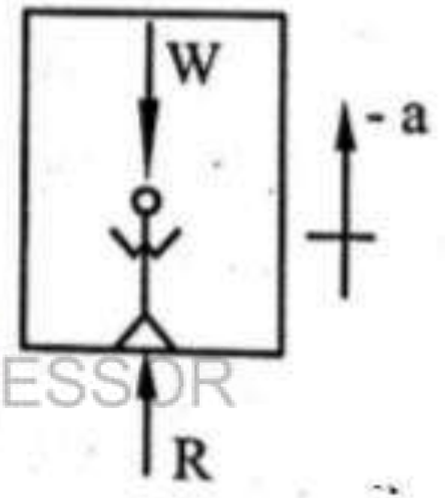
Case 2: Elevator moves upward with deceleration

Inertial force is upwards.

Given data:

Weight of man = 600 N

a= 1m/s^2



To find: Force at the floor of elevator R

Solution:

$$R = W[1- \frac{a}{g}]$$

$$= 600[1- \frac{1}{9.81}] = 538.84 \text{ N}$$

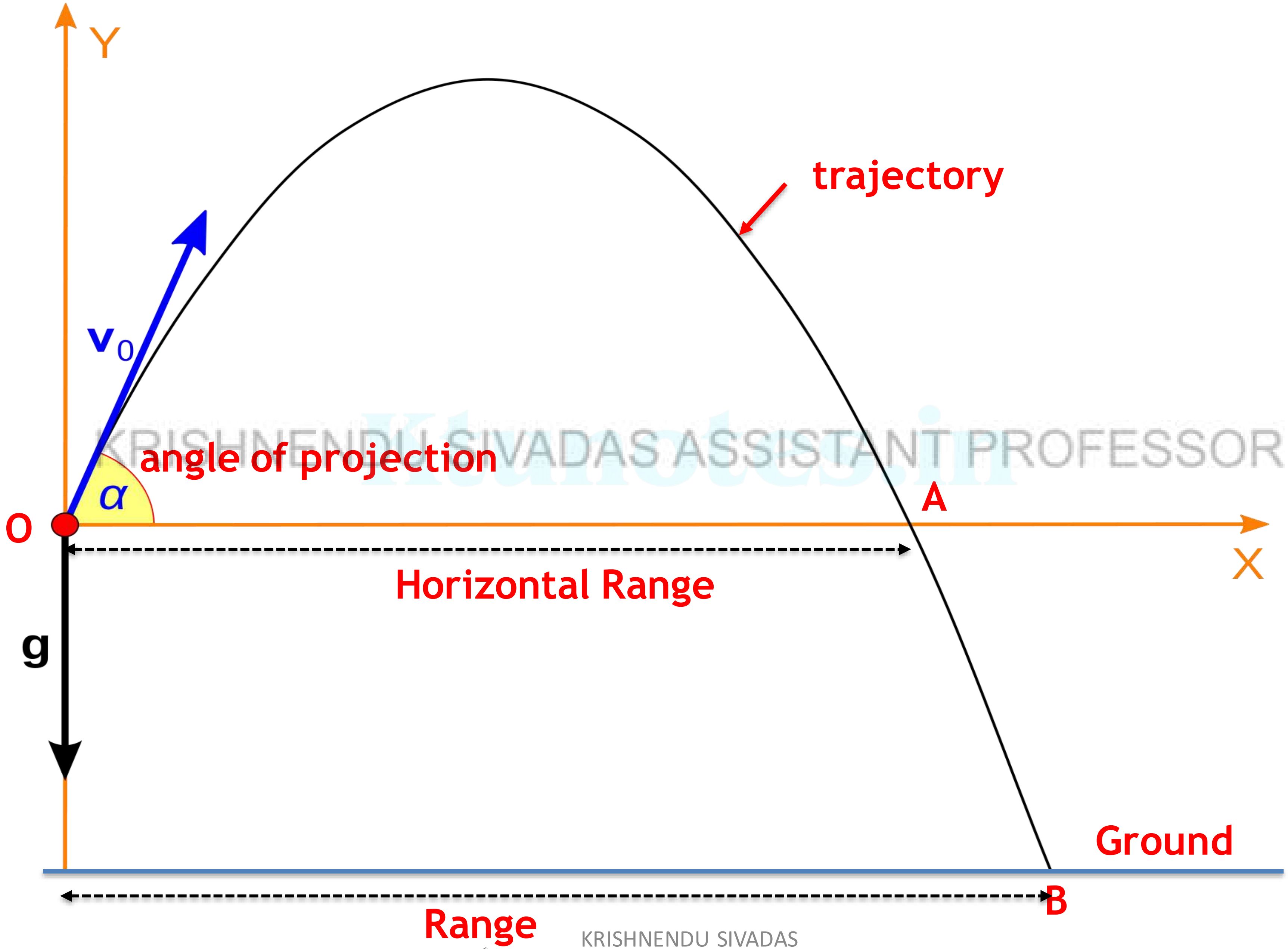
PROJECTILE MOTION

KRISHNENDU SIVADAS ASSISTANT PROFESSOR

Motion of a Projectile

- **A particle which is projected into space at an angle to the horizontal is called a projectile.**
- **The path traced by the projectile is called trajectory.**
- **The velocity with which the projectile is projected into space is called velocity of projection.**
- **The angle at which the projectile is projected is called angle of projection.**
- **The time during which the projectile is in motion is called its time of flight.**
- **It is the interval of time at which projectile is projectile is projected and it hits the ground.**

- **Range** is the horizontal distance between the point of projection and the point where the projectile strikes the ground.
- **Horizontal range** is the horizontal distance between the point of projection and the point where **horizontal line through the point of projection meets the trajectory.**


Y
trajectory
$\mathbf{v}_0$
angle of projection
α
O
A
X
Horizontal Range
g
Ground
B
Range

Case 1: Motion of a projectile vertically into space

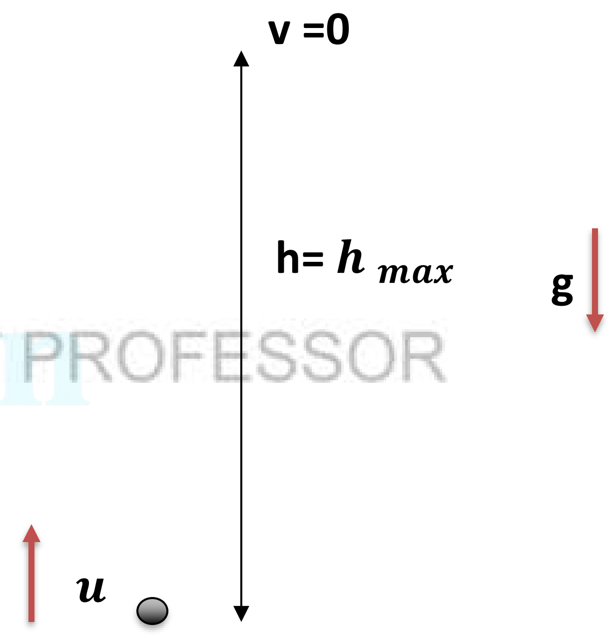
- **Let u be the velocity of projection**
- **Angle of projection = α = 90°**
- **The velocity of a particle at a certain height can be obtained using the relation:**

$$v^2 = u^2 - 2gh$$

- **When h= h_{max} , v =0**

$$0 = u^2 - 2gh_{max}$$

$$\boxed{h_{max} = \frac{u^2}{2g}}$$



Motion of a projectile vertically into space

- The time to attain maximum height can be obtained using the relation;

$$v = u - gt$$

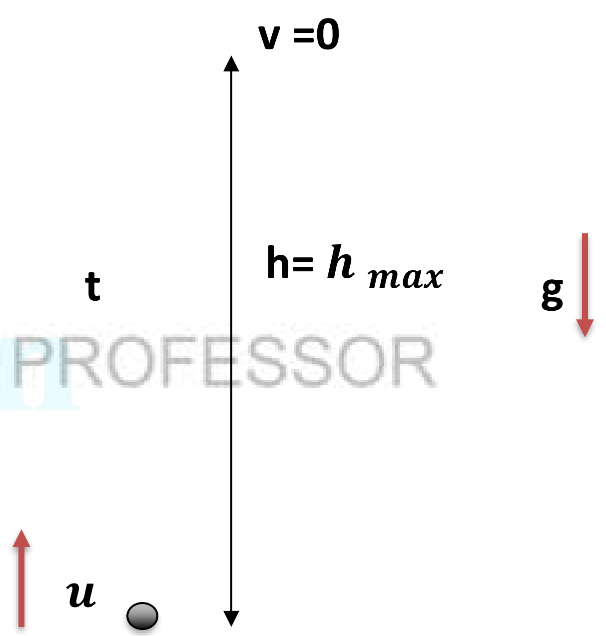
- When h= h_{max} , v =0

$$0 = u - gt$$

$$t = \frac{u}{g}$$

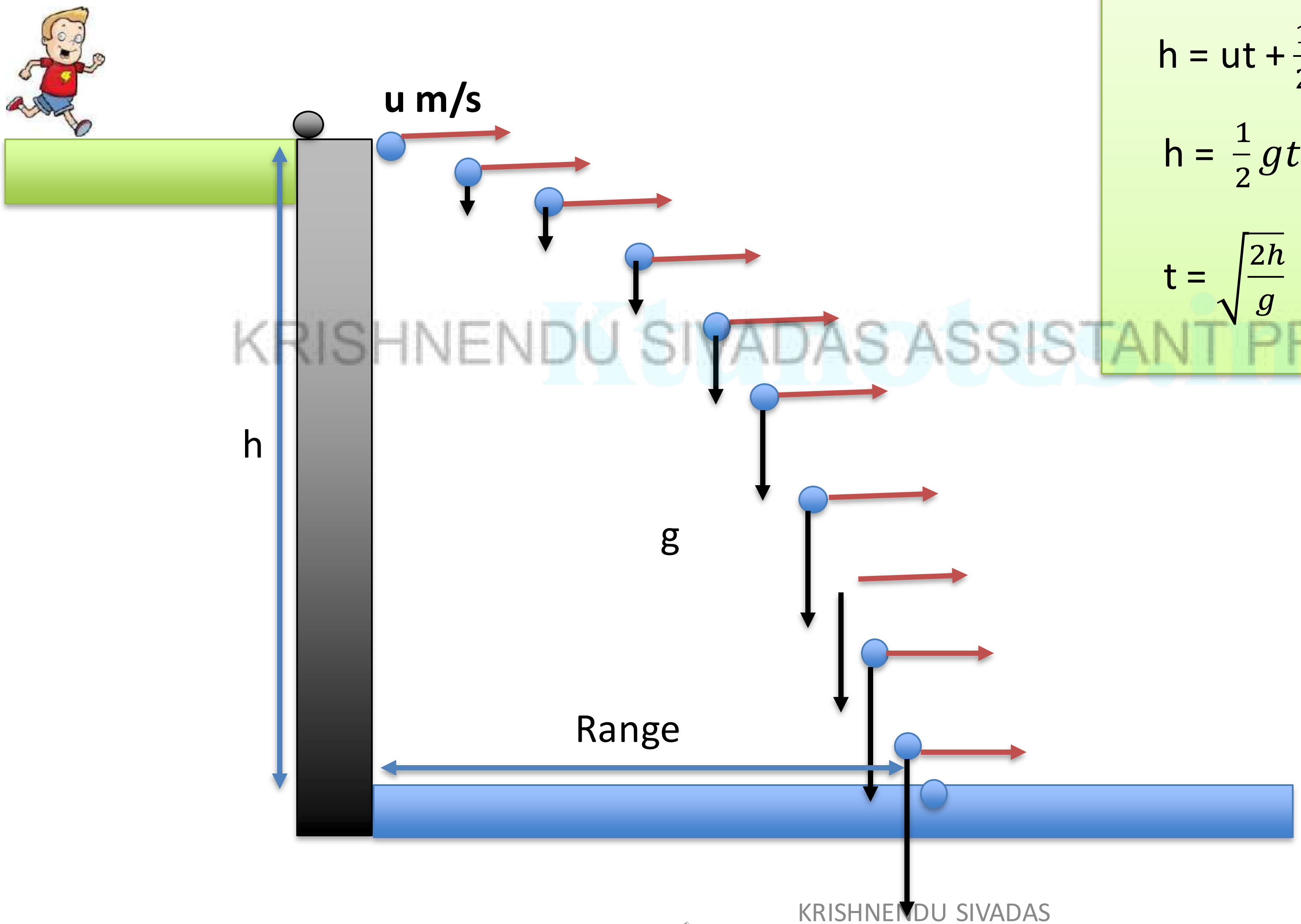
- Time of flight T = 2t = $\frac{2u}{g}$

$$\boxed{T = 2t = \frac{2u}{g}}$$



Range R =0

Case 2:
Motion of a particle horizontally into space



vertical motion

h = ut + $\frac{1}{2}gt^2$

h = $\frac{1}{2}gt^2$

t = $\sqrt{\frac{2h}{g}}$

horizontal motion

R = ut + $\frac{1}{2}at^2$

R = ut

R = u $\sqrt{\frac{2h}{g}}$

Motion of a particle horizontally into space

- **Consider a particle thrown horizontally from a point A, at a height h above the ground.**
- **At any instant, the particle is subjected to:**
- **i) Horizontal motion with constant velocity u**
- **Ii) Vertical downward motion with initial velocity zero and acceleration due to gravity g**

Motion of a particle horizontally into space

Consider the vertical motion

- **Using general expression:**

  h = ut + $\frac{1}{2}gt^2$

  h = 0 + $\frac{1}{2}gt^2$

- **Time of flight T =** t = $\sqrt{\frac{2h}{g}}$

Consider horizontal motion

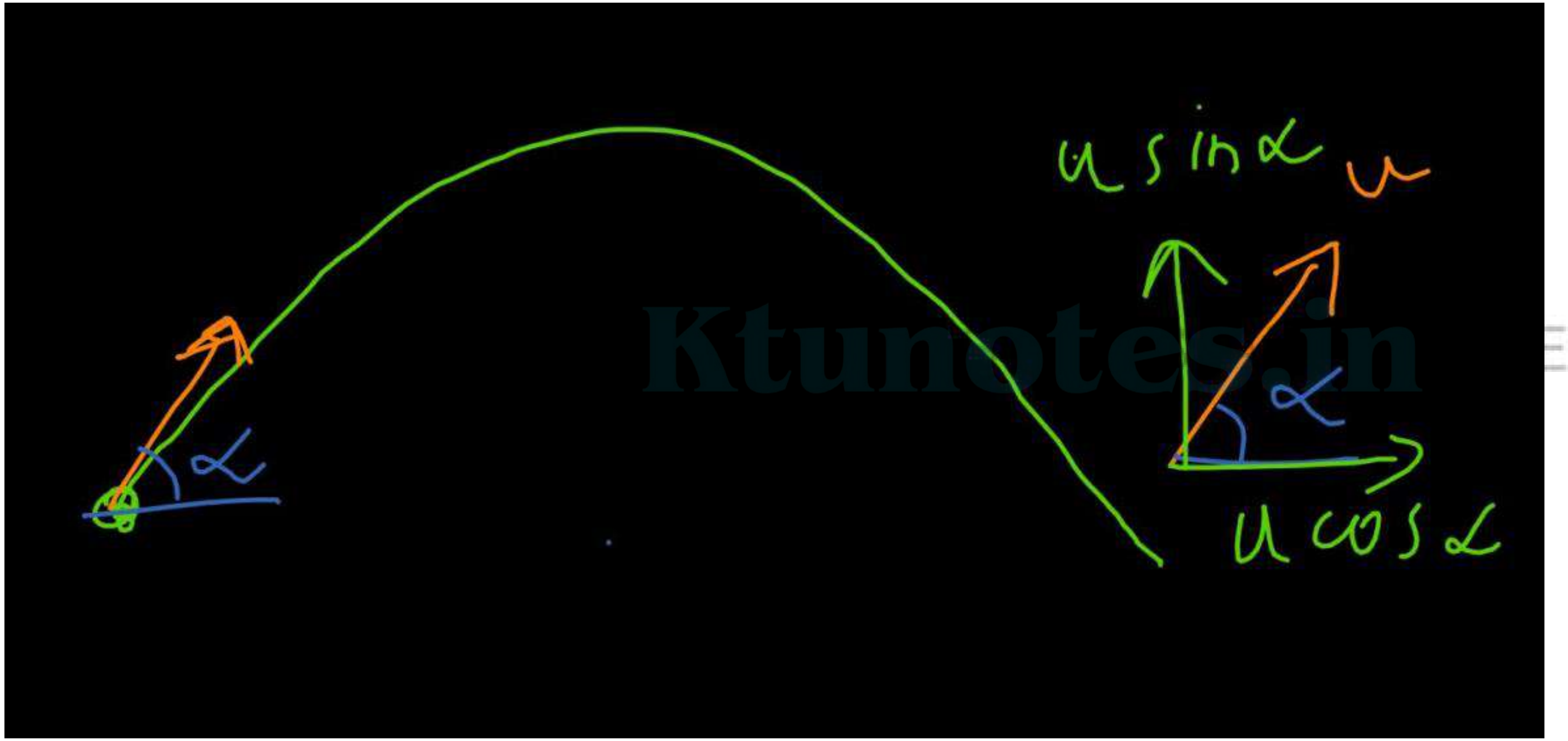
- **During the time of flight the particle moves horizontally with uniform velocity u m/s**
- **Range = u x T**

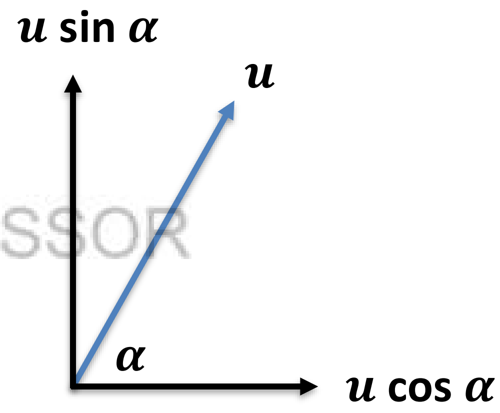
  R = u $\sqrt{\frac{2h}{g}}$

Case 3: Inclined projection on a levelled ground

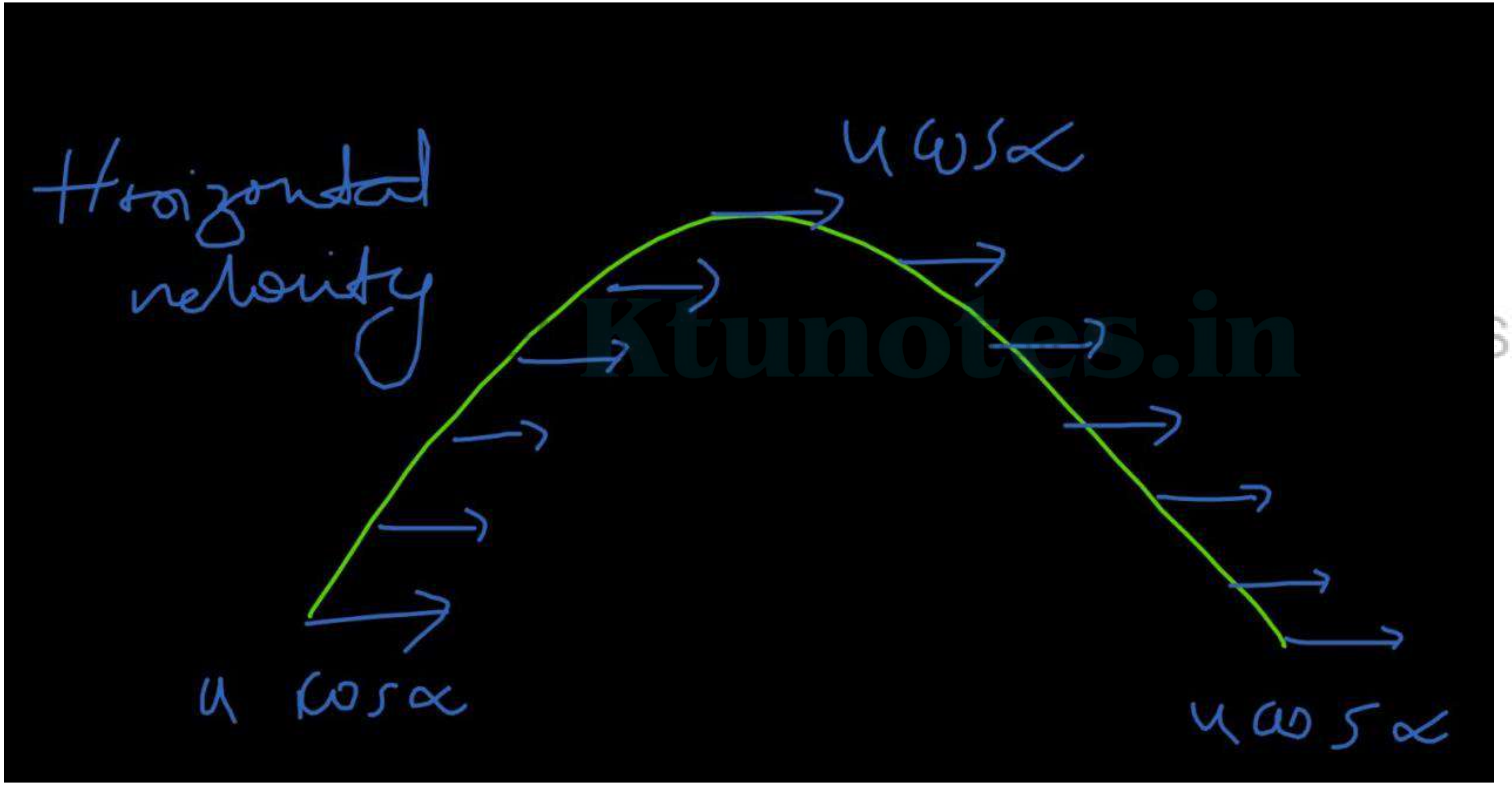
- Consider the motion of a projectile projected from O with a velocity of projection u and angle of projection α.
- The projectile has motion in vertical as well as horizontal directions.
- Since there is no force in the horizontal direction, horizontal component of velocity remains constant throughout the flight.
- Horizontal velocity component u cos α = constant
- The vertical component of velocity decreases due to gravity force.
- The height of projectile from the ground h at any instant of time t seconds is given by:
- $h = (u \sin \alpha)t - \frac{1}{2}gt^2$
- Where u sin α is the initial velocity in the vertical direction.

Inclined projection on a levelled ground with initial velocity u m/s. It can be resolved into vertical and horizontal components.

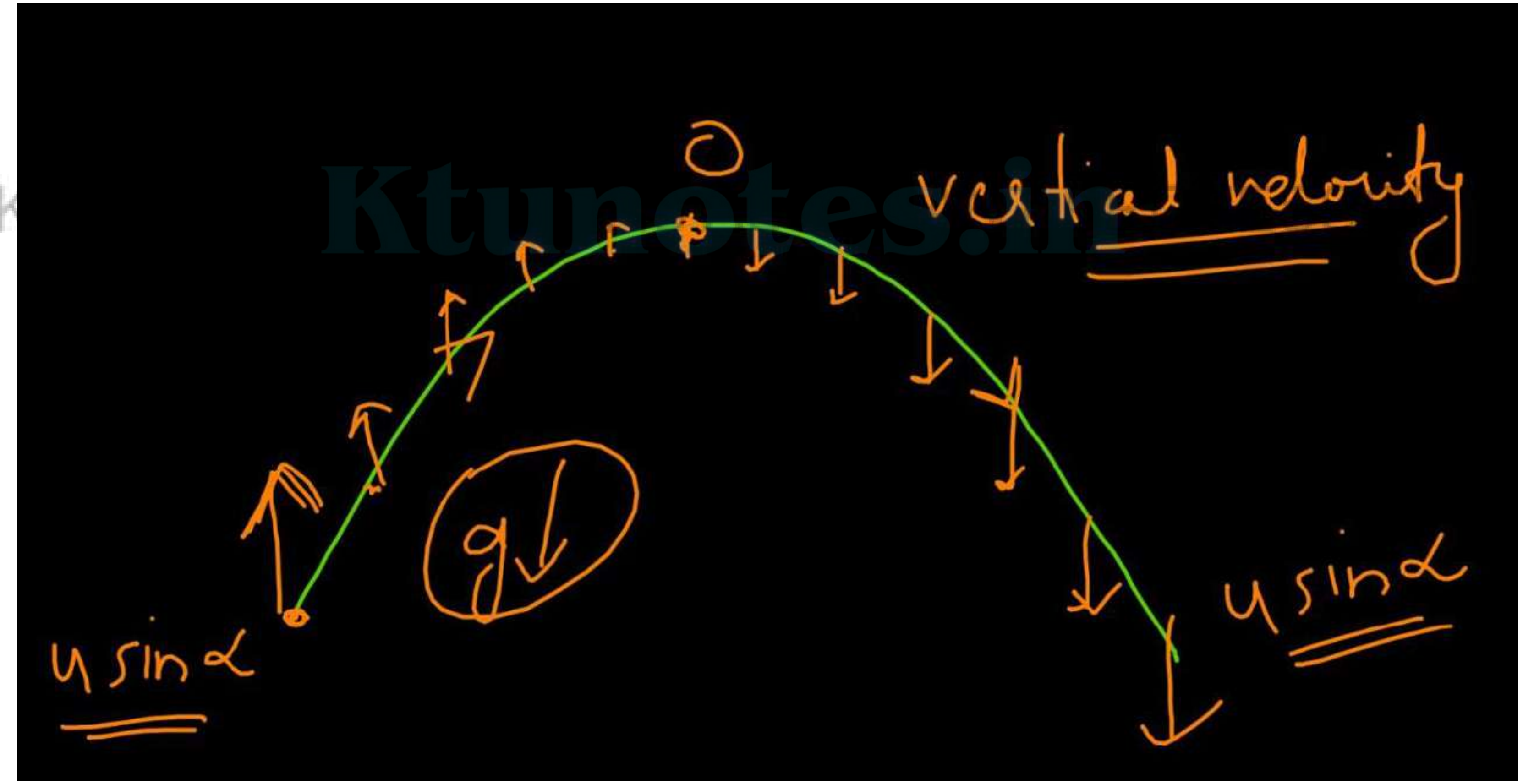


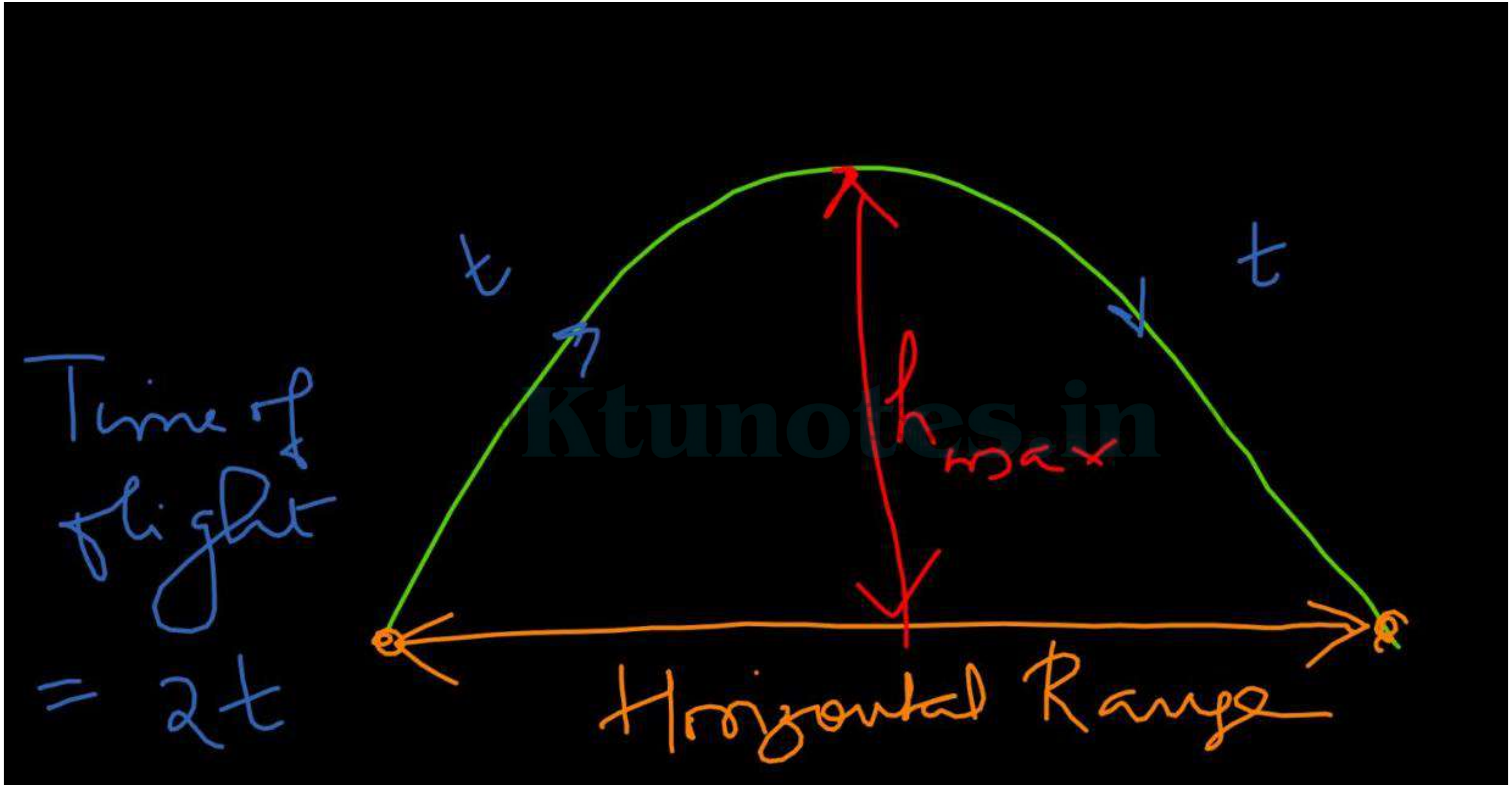


Constant horizontal velocity



- **Velocity in the vertical direction changes during ascend and descend due to the action of gravity.**
  During ascend, initial vertical velocity is $u \sin \alpha$.
  Since it is moving against gravity, vertical velocity decreases to zero at the highest point during ascend.
  And the during descend, the vertical velocity increases as it is moving in the direction of gravity.





t
t
Time of flight = 2t
h_{max}
Horizontal Range

Inclined projection on a levelled ground

- **Let x be the horizontal distance travelled in t seconds and y is the vertical distance travelled in t seconds.**
- **Horizontal distance x = ut + $\frac{1}{2}at^2$**
- **x = ut = (u cos α)t**
- **t = $\frac{x}{\text{u cos } \alpha}$**

- **Vertical distance h= ut - $\frac{1}{2}gt^2$**

$$y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{1}{2} g \left(\frac{x}{u \cos \alpha}\right)^2$$

$$y = x \tan \alpha - \frac{1}{2} \frac{g x^2}{u^2 \cos^2 \alpha}$$

Maximum height

Expression for maximum height.

Initial velocity in the vertical direction is u sin α. At maximum height the vertical velocity is zero. ie, at $h = h_{max}$, $V = 0$

Using the relation for vertical motion,

$$V^2 = u^2 - 2gh$$

$$0 = (u \sin \alpha)^2 - 2gh_{max}$$

$$\therefore h_{max} = \frac{u^2 \sin^2 \alpha}{2g}$$

Time to attain maximum height (t)

Expression for time to attain maximum height.

Using the relation,

$$V = u - g\,t$$

$$0 = (u \sin \alpha) - g\,t$$

$$t = \frac{u \sin \alpha}{g}$$

Time of flight, $T = 2\,t = 2 \times \dfrac{u \sin \alpha}{g}$

$$T = \frac{2\,u \sin \alpha}{g}$$

Horizontal Range

Expression for horizontal range, R.

Horizontal range R is the horizontal distance travelled in 2 t seconds where t is the time taken to attain the maximum height. Since the horizontal component of velocity is constant and equal to u cos α,

$$R = (u \cos \alpha) \times 2t$$

$$= 2u \cos \alpha \times \frac{u \sin \alpha}{g}$$

$$= \frac{u^2 \, 2 \sin \alpha \cos \alpha}{g}$$

$$R = \frac{u^2}{g} \sin 2\alpha$$

- **Trajectory equation:**

$$y = x \tan\alpha - \frac{1}{2}\frac{g x^2}{u^2 \cos^2\alpha}$$

- **Maximum height**

$$h_{max} = \frac{u^2 \sin^2\alpha}{2g}$$

- **Time to attain maximum height**

$$t = \frac{u \sin \alpha}{g}$$

- **Time of flight**

$$T = \frac{2 u \sin \alpha}{g}$$

- **Horizontal range**

$$R = \frac{u^2}{g} \sin 2\alpha$$

Question 1

A pilot flying his bomber at a height of 1000 m with uniform horizontal velocity of 30 m/s wants to strike a target on the ground. At what distance from the target, he should release the bomb?

Solution: Consider the vertical motion of the bomb. Initial vertical velocity = 0.

$$h = u\,t + \frac{1}{2}\,g\,t^2$$

$$1000 = 0 + \frac{1}{2}\,g \times t^2$$

$$t = \sqrt{\frac{2000}{g}} = 14.29 \text{ s}$$

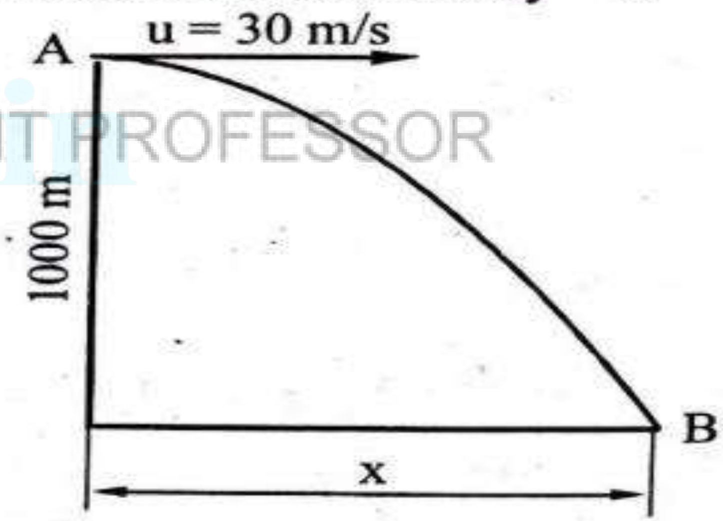


Fig. 4.53

Since the horizontal velocity remains constant, the horizontal distance moved in 14.29 seconds is velocity × time.

$$x = 30 \times 14.29 = 428.7 \text{ m.}$$

3) An aeroplane is flying at a height of 200 m with a horizontal velocity of 70 m/s. A shot is fired from a gun from the ground when the aeroplane is exactly above the gun. What should be the minimum initial velocity of the shot and the angle of elevation in order to hit the aeroplane.

Solution:

Let u_A be the velocity of plane and u_S be the velocity of projection of shot at an angle of projection α.

When the shot hits the aeroplane, the horizontal distance travelled by the shot and plane will be the same.

$$\therefore u_A \times t = u_S \cos\alpha \times t$$

$$u_A = u_S \cos\alpha \text{ ---------(i)}$$

The vertical component of u_S should be such that, it should go upto a height of 200 m.

Using the relation, $$V^2 = u^2 - 2\,g\,h;$$

$$0 = (u_S \sin\alpha)^2 - 2 \times g \times 200$$

$$u_S \sin\alpha = 62.64 \text{----------} (ii)$$

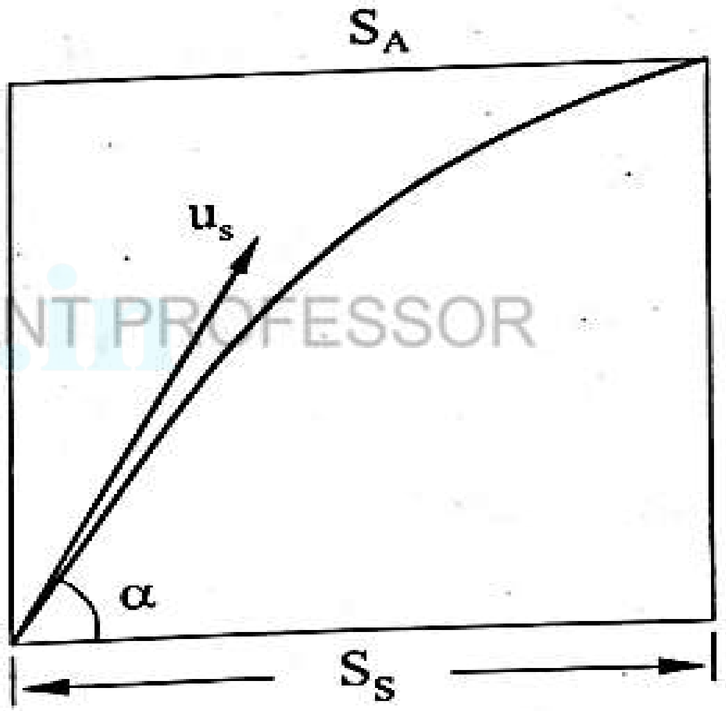


Fig. 4.54

$$\frac{u_S \sin\alpha}{u_S \cos\alpha} = \frac{62.64}{70} = 0.895$$

$$\tan\alpha = 0.895$$

$$\alpha = 41.82^0$$

From equation (i)

$$u_A = u_S \cos\alpha$$

$$70 = u_S \cos 41.82$$

$$\therefore\ u_S = 93.93 \text{ m/s}$$

3) A particle is projected with agiven velocity u at an angle of elevation α from the origin. It passes through two points (15,8) and (40,9) on its path. Find the greatest height reached by the particle and its range

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha} \qquad x_1 = 15, \quad y_1 = 8$$

$$8 = 15\tan\alpha - \frac{1}{2}\times\frac{9.81\times 15^2}{u^2\cos^2\alpha}$$

$$15\tan\alpha - 8 = \frac{1}{2}\times\frac{9.81\times 15^2}{u^2\cos^2\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$x_2 = 40, \quad y_2 = 9$

$$9 = 40\tan\alpha - \frac{1}{2}\times\frac{9.81\times 40^2}{u^2\cos^2\alpha}$$

$$40\tan\alpha - 9 = \frac{1}{2}\times\frac{9.81\times 40^2}{u^2\cos^2\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

$$\frac{40\tan\alpha-9}{15\tan\alpha-8}=\frac{40^2}{15^2}$$

$$40\tan\alpha-9=7.11\left(15\tan\alpha-8\right)$$

$$=106.65\tan\alpha-56.88$$

$$66.65\tan\alpha=47.88$$

$$\tan\alpha=0.72$$

$$\alpha=35.75^\circ$$

From equation (i),

$$15\tan 35.75-8=\frac{1}{2}\times\frac{9.81\times 15^2}{u^2\cdot\cos^2 35.75}$$

$$u=24.47\text{ m/s}$$

Greatest height, $h_{max} = \dfrac{u^2 \sin^2 \alpha}{2g}$

$$= \frac{24.47^2 \sin^2 35.75}{2 \times 9.81}$$

$$= 10.42 \text{ m}$$

The range $R = \dfrac{u^2 \sin 2\alpha}{g}$

$$= \frac{24.47^2 \times \sin(2 \times 35.75)}{9.81}$$

$$= 57.88 \text{ m}$$