Module 1

Engineering Mechanics is a branch of physical science that deals with the state of rest or motion of bodies under the action of force.

Mechanics have 2 basic classification.

i) Statics ii) Dynamics.

Statics → Equilibrium of bodies under the action of forces [No accelaration ~~forces~~ motion]

~~In~~ Dynamics, deals with the motion of body under the action of force.

Dynamics has 2 subdivisions

i) kinetics ii) kinematics.

kinetics → It is the study of motion of bodyies with considering the force.

kinematics → It is the study of motion of bodies without considering force.

engineering mechanics ~~has~~ 2 is 2 types.

i) Mechanics of solids.

ii) Mechanics of fluids.

- Eng. Mechanics
  - Mechanics of solids
    - Mechanics of rigid bodies
      - Statics
      - Dynamics
        - Kinetics
        - Kinematics
    - Mechanics of deformable bodies
      - Mechanics of theory of elasticity
      - Theory of plasticity
  - Mechanics of fluids
    - Ideal fluids
    - Viscous fluids
    - Incompressible fluids

Fundamental concepts

Rigid body : The body which will not deform or the body in which deformation can be neglected in the analysis.

Particle : A body with mass but with dimensions can be neglected.

Scalars : Associated with magnitude alone
- mass, density, volume, time, energy.

Vectors : Associated with magnitude and direction
- force, displacement, velocity, accelaration.

In Statics, bodies are considered as rigid.

Principles of Mechanics

* a) Newton's laws of motion
  i) Newton's 1st law
  ii) " IInd law
  iii) " 3rd law.
* b) Newtons law of gravitation.
  c) Parellelogram law.
  d) Triangle law
  e) equillibrium law.
  f) law of transmissibility of forces.
  g) law of superposition.
  h) Lami's Theoram.
  i) law of superposition

1) Newtons laws of motion

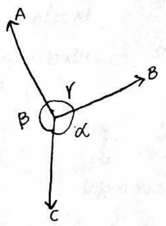
Newtons 1st law: states that every body continuous in its state of rest or uniform motion in a straight line unless it is compelled by an external force.

Newtons 2nd law: The rate of change of momentum of a body is $\propto$ the impressed force and it take place in the direction of the force acting on it. Newtons 2nd law gives the eqn. $F = ma$ →

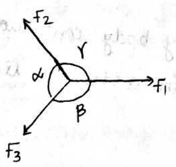
Newtons 3rd law: It states that every action there is an equal and opposite reaction.

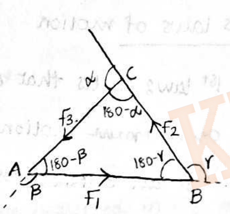
Lami's Theoram

If a particle acted upon by 3 forces remains in equillibrium ~~when~~ then, each force acting on the particle bears the same Propotionality with the sine of the angle between the other 2 forces. Lami's theoram is also known as law of sine.



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$





By sine rule

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

Newtons law of gravitation: 2 bodies will be attracted towards each other along their connecting line with a force which is $\propto$ to the product of their masses and Inversely propotional to Square of distance b/w their centers.

$$F = \frac{GMm}{r^2}$$

G- Universal constant of gravitation $- 6.673 \times 10^{-11}\ m^3/kg\text{-}s^2$

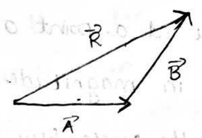
r- distance b/w the particles.

M and m are particle masses.

Triangle law of forces

If 2 forces acting on a body are represented one after another by the sides of triangle, their resultant is represented by the closing side of a triangle taken from first point to the last point.

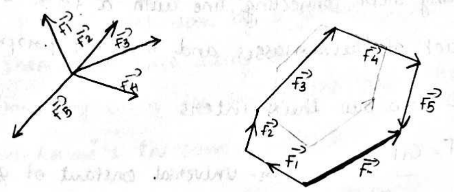
$$\vec{R} = \vec{A} + \vec{B}$$



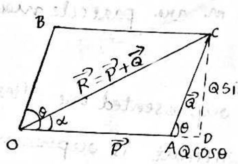
Law of polygon of forces

If a no of forces acting at a point and represented in magnitude and direction by sides of a polygon in order, then the forces are in equillibrium.

The resultant of the forces represented in magnitude and direction by the closing side of polygon takeng in oppo order.



Parellelogram law



If 2 forces acting simultaniously at a point are represented in magnitude and direction by the 2 adjesent side of parellelogram. Then the resultant is represented in by the diagonal of parellelogram which passes through the point of intersection of the 2 sides representing the forces.

Proof

Let 2 forces P, Q act at a point O, These forces P and Q are represented in magnitude and direction by vectors OA and OB. Let the angle b/w 2 forces P and Q is θ and α is the angle made by resultant force w.r.t horizondal axis.

$$OC^2 = OD^2 + CD^2$$

$AD = Q\cos\theta \quad DC = Q\sin\theta$

$$OC^2 = OD^2 + CD^2$$

~~$R^2 = P + Q\cos$~~

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta$$

$$R^2 = P^2 + 2PQ\cos\theta + Q^2$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

Direction of R

Considering $\triangle ODC$.

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

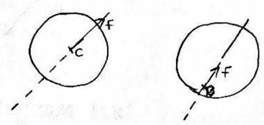
$$\boxed{\alpha = \tan^{-1}\frac{Q\sin\theta}{P + Q\cos\theta}}$$

Equillibrium law: It states that a body is said to remain in state of equillibrium when the resultant of forces acting on the body is 0.

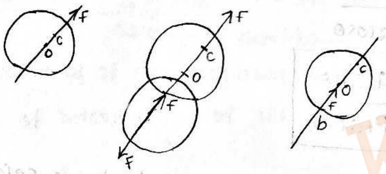
If 2 forces are in equillibrium only if they are equal in magnitude, opposite in direction, and collinear in action.

Law of transmisibility of forces

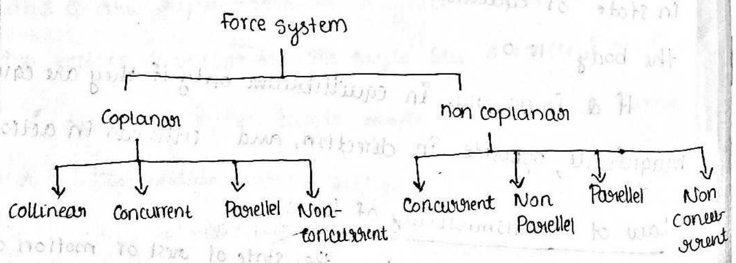
According to this law the state of rest or motion of a rigid body is unaltered if a force acting on the body is replaced by another force of same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.



Law of Superposition: The action of a given system of forces of a rigid body is unaltered by adding or subtracting another system of forces in equilibrium.



Force System



Composition of forces

consider 2 forces F_1, F_2 let the angle b/w 2 forces F_1, F_2 be θ

by using parellelogram law

$$R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$$

$$\alpha = \tan^{-1}\frac{F_2\sin\theta}{F_1+F_2\cos\theta}$$

Special cases

$\theta = 90$.

$$R = \sqrt{F_1^2+F_2^2}$$

$\theta = 0$

$$R = \sqrt{F_1^2+F_2^2+2F_1F_2} = \sqrt{(F_1+F_2)^2}$$
$$= F_1+F_2$$

$\theta = 180$

$R = F_1 - F_2$

? The resultant of 2 forces one of which is double the other is 260 N. If the direction of larger force is reversed and other remains unaltered. The resultant reduced to 180 N. Determine the magnitude of forces and angle b/w forces

I let forces be F_1 and F_2

$F_1 = F \quad F_2 = 2F$

$R = 260$ N

$$260 = \sqrt{F^2+(2F)^2+2F\cdot 2F\cos\theta}$$

$$260 = \sqrt{F^2+4F^2+4F^2\cos\theta}$$

$$= 260 = \sqrt{5F^2+4F^2\cos\theta}$$

$260^2 = 5f^2 + 4f^2 \cos\theta$

$67600 = 5f^2 + 4f^2 \cos\theta$ —(1)

Case 2

$f_1 = f$ $\quad f_2 = -2f$ $\quad R = 180N$

$R^2 = f^2 + (-2f)^2 + 2 \times 2f \times f \cos\theta$

$180^2 = 5f^2 - 4f^2 \cos\theta$

$32400 = 5f^2 - 4f^2 \cos\theta$ —(2)

(1) + (2)

$5f^2 + 4f^2 \cos\theta = 67600 +$

$5f^2 - 4f^2 \cos\theta = 32400$

$10f^2 + 0 = 100000$

$f^2 = \frac{100000}{10}$ $\quad f^2 = 10000$

$F = 100$

(1) - (2)

$5f^2 + 4f^2 \cos\theta = 67600 -$

$5f^2 - 4f^2 \cos\theta = 32400$

$0 + 8f^2 \cos\theta = 35200$

$\cos\theta = \frac{35200}{8 \times 10000} = 0.44$

$\theta = \cos^{-1}(0.44) = 63.89$

Two equal forces acting at a point at an angle of 60° b/w them. The resultant force is $20\sqrt{3}$ N, Find magnitude of each force.

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$20\sqrt{3} = \sqrt{P^2 + P^2 + 2P^2 \cos 60}$

$= \cancel{40 \times 3}$

$400 \times 3 = 2P^2 + 2P^2 \times 1/2$

$400 \times 3 = 3P^2$

$P^2 = \frac{400 \times 3}{3} = 400$ $\quad P = \sqrt{400} = 20N$

$P = 20N$ $\quad Q = 20N$

The resultant of 2 forces ~~at~~ when they act at angle 60° is 14 N. If same forces are acting at right angle resultant is $\sqrt{136}$ N. Determine magnitude of 2 forces.

1st case

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\theta = 60°$

$14^2 = P^2 + Q^2 + 2PQ \times \frac{1}{2}$

$14^2 = P^2 + Q^2 + PQ$

$P^2 + Q^2 + PQ = 196$ —(1)

IInd case

$136 = P^2 + Q^2 + 2 \times P \times Q \cos\pi/4$

$136 = P^2 + Q^2$

$P^2 + Q^2 = 136$ —(2)

$P^2 + Q^2 + PQ = 196 -$

$P^2 + Q^2 = 136$

$0 + 0 + PQ = 60$

$P^2+Q^2 = 136$ — (1)

$PQ = 60$ — (2)

$\cancel{P^2-Q^2 = (P^2+Q^2) - 4PQ}$

$\cancel{P^2-Q^2 = 136 - 4\times 60} = 1$

multiplying equ(2) x 2.

$2PQ = 120$ — (3)

(1) + (3)

$= P^2+Q^2+2PQ = 256$

$(P+Q)^2 = 256$

$P^2+Q^2+2PQ = 256$

$P^2+Q^2-2PQ = 16$

$\therefore \quad P+Q = 16 \quad \boxed{P = 16-Q}$ — (4)

Substitute (4) to equ (3)

$2 \times (16-Q) \times Q = 120$

$2Q(16-Q) = 32Q^2 - 2Q^2 = 120$

$Q^2 - 16Q + 60 = 0 \quad 16Q^2 - Q^2 =$

$Q = 10 \quad Q = 6$

$P = 16-10 = 6$ // $\quad P = 16-6 = 10$ //

$P^2+Q^2+PQ = 196+$

$P^2+Q^2 = 136$

$2P^2+2Q^2 =$

$P^2-Q^2 =$

$\cancel{(P-Q)^2 = 256 - 4\times 60}$

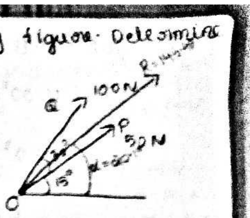
$(P-Q)^2 = 256-240$

$(P-Q)^2 = 16$

$\cancel{P^2+Q^2 = 136}$

$\cancel{(P+Q)^2 = 16^2}$

? 2 forces are acting at a point O showing figure. Determine the resultant in magnitude and direction.



$= P = 50\,N \quad \theta = 30^\circ \quad \alpha = 15^\circ$

$R^2 = 50^2 + 100^2 + 2P\,50 \times 100 \times \cos 30$

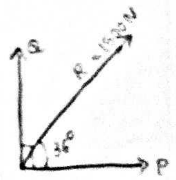
$R = \sqrt{50^2+100^2+2\times 50\times 100 \times \frac{\sqrt{3}}{2}}$

$= \sqrt{50^2+100^2+8660.254038} = 145.46\,N$

$\alpha = \tan^{-1} \frac{100 \sin 30}{50+100\cos 30} \qquad \alpha = \tan^{-1} \frac{100 \times \frac{1}{2}}{50+100\times\sqrt{3}/2}$

$\alpha = \tan^{-1} \frac{50}{50+50\sqrt{3}} \qquad \tan^{-1} \frac{50}{136.6025404} = 20.10$

The resultant of 2 concurrent force is ~~1500 N~~ 1500 N and the angle b/w force is 90°. The resultant makes an angle 36° with one of the force. Find the magnitude of each force.



$1500^2 = P^2+Q^2+2PQ\cos\theta$

$1500^2 = P^2+Q^2$ —(1) $\qquad \alpha = 36^\circ$

$\tan\alpha = \frac{Q \sin 90}{P+Q\cos 90} \Rightarrow \tan\alpha = \frac{Q}{P}$

$Q = P \times 0.726$ — (2)

Substitute (2) in (1)

$$1500^2 = P^2 + (P \times 0.726)^2$$

$$1500^2 = P^2 + P^2 \times 0.527$$

$$P^2 + 0.527P^2 = 1500^2$$

$$P^2(1 + 0.527) = 1500^2$$

$P = 12$

$$P^2 = \frac{1500^2}{1 + 0.527} = 1473477.407$$

$$\underline{P = 1213.86\ N} \quad Q = \underline{881.26\ N}$$

another method:

By Lamis Theoram

$$\frac{P}{\sin(Q \& R)} = \frac{Q}{\sin(R \& P)} = \frac{R}{\sin(Q \& P)}$$

$$90^{\circ} - 36 = 54$$

$$\frac{P}{\sin 54} = \frac{Q}{\sin 36} = \frac{R}{\sin 90}$$

$$= \frac{P}{0.809016} = \frac{Q}{0.587785} = \frac{R}{1}$$

The sum of 2 concurrent force P and Q is 270 N. The resultant is 180 N. The angle b/w the force P and resultant R is 90°. Find the magnitude of each force and angle b/w them.

$P = 75\ N$
$Q = 195\ N$
$\alpha = 112.618\ N$

$$P + Q = 270 \quad \alpha = 90^{\circ}$$

$$R = 180$$

$$Q = 270 - P$$

$$\tan 90 = \frac{Q \sin\theta}{P + Q\cos\theta} \Rightarrow \infty = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$P + Q\cos\theta = 0$$

$$\cos\theta = \frac{-P}{Q} \qquad R = 180$$

$$180 = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$180 = \sqrt{P^2 + Q^2 + 2PQ \times \frac{-P}{Q}} \Rightarrow \sqrt{P^2 + Q^2 - 2P^2}$$

$$180 = \sqrt{P^2 + (270 - P)^2 - 2P^2}$$

$$= \sqrt{P^2 + (270)^2 + P^2 - 540P - 2P^2}$$

$$180 = \sqrt{270^2 - 540P} \Rightarrow 180^2 = 270^2 - 540P$$

$$P = \frac{180^2 - 270^2}{-540} = 75$$

$$Q = \underline{270 - 75} = 195$$

$$\cos\theta = \frac{-P}{Q} = \frac{-75}{195} = \underline{-5/13}$$

$$\theta = \cos^{-1}(-5/13) = \underline{112.619}$$

Different methods of finding resultant from a given set of forces

If ~~give~~ given no of forces are limited to 2.

i) Triangle law of forces.

ii) parallelogram law of forces.

If no of forces are more than 2.

i) Polygon method.

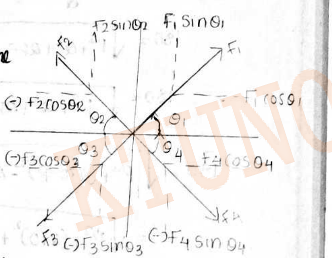
ii) graphical method.

iii) composition of forces by method of resolution.

Resolution of forces (coplanar concurrent forces)

The process of finding the component of force is known as resolution.

considering 4 forces F_1, F_2, F_3, F_4 are making an angle of $\theta_1, \theta_2, \theta_3, \theta_4$ w.r.t corresponding x axis.



Composition of forces by method of resolution

$$\Sigma f_x = F_1\cos\theta_1 - F_2\cos\theta_2 - F_3\cos\theta_3 + F_4\cos\theta_4.$$

$$\Sigma f_y = F_1\sin\theta_1 + F_2\sin\theta_2 - F_3\sin\theta_3 - F_4\sin\theta_4.$$

resultant $\boxed{R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}}$

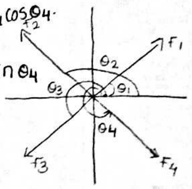
direction $\boxed{\alpha = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}}$

Alternate method

If 4 forces F_1, F_2, F_3, F_4 acting at a point $\theta_1, \theta_2, \theta_3, \theta_4$ are the angle made by the forces w.r.t first quadrant x axis.

$$\Sigma f_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 + F_4\cos\theta_4.$$

$$\Sigma f_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 + F_4\sin\theta_4$$

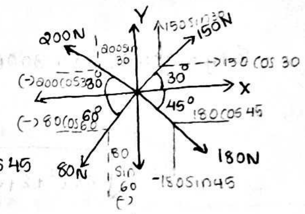


$$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right)$$

? Find the resultant of given system of forces.

$$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}$$



$$\Sigma f_x = 150\cos 30 - 200\cos 30 - 80\cos 60 + 180\cos 45$$

$$150 \times \frac{\sqrt{3}}{2} - 200 \times \sqrt{3}/2 - 80 \times 1/2 + 180 \times \frac{1}{\sqrt{2}}$$

$$= 75\sqrt{3} - 100\sqrt{3} - 40 + 180 \times \frac{1}{\sqrt{2}}$$

$$f_x = 43.97.$$

$$\Sigma f_y = 150 \times \frac{1}{2} + 200 \times \frac{1}{2} - 80 \times \frac{\sqrt{3}}{2} - 180 \times \frac{1}{\sqrt{2}}$$

$$= -21.56\ N.$$

$$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2}$$

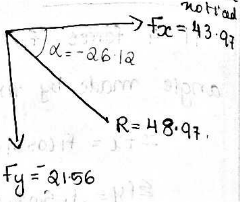
$$= 48.97$$

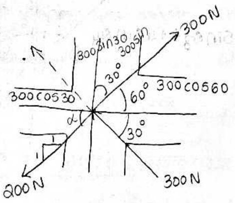
Direction

quadrant must be noticed.

$\alpha = \tan^{-1}\left(\frac{\Sigma fy}{\Sigma fx}\right)$

$= -26.12$





$\tan\alpha = 1$

$\alpha = \tan^{-1} 1 = 45^\circ$

$\Sigma fx = 300\cos 60 - 300\cos 30 - 200\cos 45$

$= -251.22$

$\Sigma fy = 300\sin 60 + 300\sin 30 - 200\sin 45$

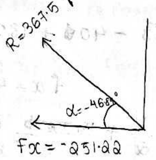
$= 268.386 \Rightarrow 268.39$

$R = \sqrt{\Sigma fx^2 + \Sigma fy^2} = \sqrt{135144.6805} = 367.5$

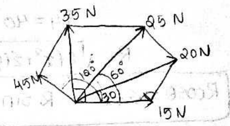
$\alpha = \tan^{-1}\left(\frac{268.39}{-251.22}\right) = -46.89^\circ$

Fx is -ve

Fy +ve



? Forces of 15 N, 20 N, 25 N, 35 N and 45 N act at an angular point of regular hexagon towards the other angular points. Calculate the magnitude and direction of resultant force.



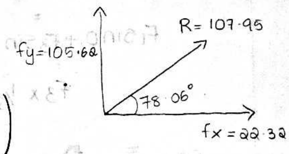
Σfx	Σfy
15 cos0	15 sin0
20 cos30	20 sin30
25 cos60	25 sin60
35 cos90	35 sin90
45 cos120 (-)	45 sin(120)

$\Sigma fx = 22.32$ $\Sigma fy = 105.6217$

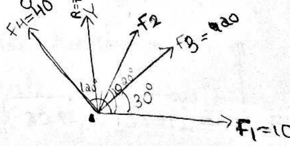
$R = \sqrt{11654.1407} = 107.95$

$\alpha = \tan^{-1}\left(\frac{105.6217}{22.32}\right)$

$\alpha = 78.06$



? The resultant of 4 forces which are acting at a point O is. The resultant is acting along y axis. The magnitude of 4 forces F_1, F_3, F_4 are 10, 20, 40 kN. Find the magnitude and direction of F_2 if the resultant is 72 kN.

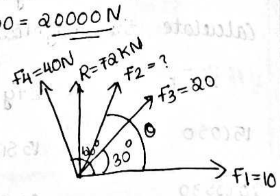


$F_1 = 10\,kN \Rightarrow 10 \times 1000 = 10000\,N$

$F_2 = ?$ $\quad F_3 = 20\,kN \Rightarrow 20 \times 1000 = 20000\,N$

$F_4 = 40\,kN$.

$R = \sqrt{\Sigma fx^2 + \Sigma fy^2}$



$\boxed{R\cos\theta = \Sigma fx \qquad R\sin\theta = \Sigma fy}$

$R\cos 90 = \Sigma fx \quad R\sin 90 = \Sigma fy$.

$0 = \Sigma fx \quad R = \Sigma fy$.

~~F1cos30 + F2cos50~~

$F_1\cos 0 + F_3\cos 30 + F_2\cos \cancel{60}^{\theta_2} + F_4\cos 120 = 0$

$F_1 + F_3\frac{\sqrt{3}}{2} + F_2 \cos\theta_2 + F_4 \times -\frac{1}{2} = 0$

$F_1 + F_3 \times \frac{\sqrt{3}}{2} + F_2 \cos\theta_2 - F_4/2 = 0 \quad —(1)$

$F_1\sin 0 + F_3\sin 30 + F_2\sin\theta_2 + F_4\sin 120 = 72 \sin 90$

$F_3 \times \frac{1}{2} + F_2\sin\theta_2 + F_4 \times \frac{\sqrt{3}}{2} = 72 \times 1 \quad —(2)$

(1) becomes $10 + 20 \times \frac{\sqrt{3}}{2} + F_2\cos\theta_2 - 40/2 = 0$

$10 + 10\sqrt{3} + F_2\cos\theta_2 - 20 = 0$

$F_2\cos\theta_2 = 20 - 10 - 10\sqrt{3} = -7.32$.

(2) becomes

$10 + F_2\sin\theta_2 + 20\sqrt{3} = 72$

$F_2\sin\theta_2 = \cancel{20\sqrt{3} - 10}$

$72 - 10 - 20\sqrt{3} = 27.359$

$\frac{(2)}{(1)} \Rightarrow \frac{F_2\sin\theta_2}{F_2\cos\theta_2} = -3.73$

$\tan\theta_2 = -3.73$

$\theta_2 = \tan^{-1}(-3.73) = -74.992$

$F_2\cos\theta_2 = -7.32$

$F_2 \times 0.258 = -7.32$; $\quad F_2 = \frac{-7.32}{0.258} = -28.372\,kN$

1) The following forces act at a point:

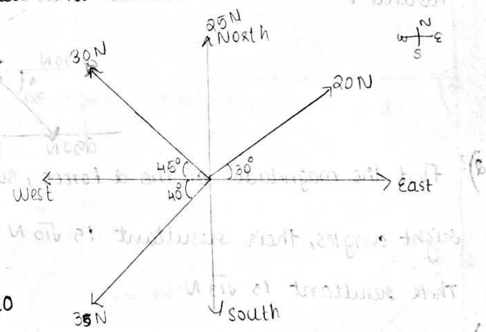
i) 20 N inclined to 30° towards north of east,

ii) 25 N towards North

iii) 30N towards North West and

iv) 35 N inclined at 40° towards South of west

Find the magnitude and direction of the resultant force.



$R = \sqrt{\Sigma fx^2 + \Sigma fy^2}$

$\Sigma fx = 20\cos 30 - 30\cos 45 - 30\cos 40$

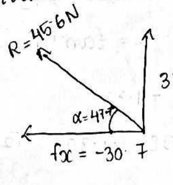
$\Sigma fx = 20\cos 30 + 25\cos 90 - 30\cos 45 - 35\cos 40 = \cancel{-26.87}\ -30.70$

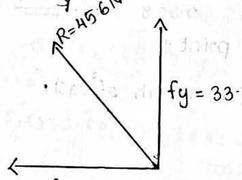
$\Sigma fy = 20\sin 30 + 25\sin 90 + 30\sin 45 - 35\sin 40 = \cancel{36.92}\ 33.71$

~~$R = \sqrt{(26.87)^2 + (36.92)^2} = 45.66$~~ $\quad R = \sqrt{(-30.70)^2 + (33.71)^2} = 45.59$

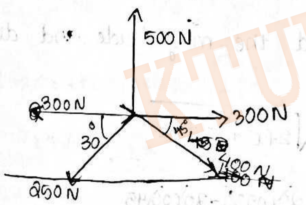
$\alpha = \tan^{-1} \frac{-33.71}{30.70} = \underline{-47.7^\circ} = 47.7^\circ$

actual angle of the resultant $= 180^\circ - 47.7^\circ = \underline{132.3^\circ}$





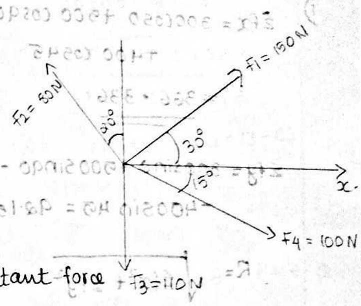
1) Find the resultant of 4 concurrent forces acting on a Particle P



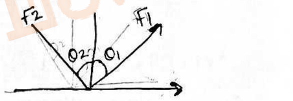
2)? Find the magnitude of the 2 forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60, Their resultant is $\sqrt{13}$ N.

3)? The greatest and least resultants of 2 forces F_1 and F_2 are 17N and 3N respectively. Determine the angle b/w them when their resultant is $\sqrt{149}$ N.

4) Four forces act on bolt A is shown. Determine the resultant of the forces on bolt.

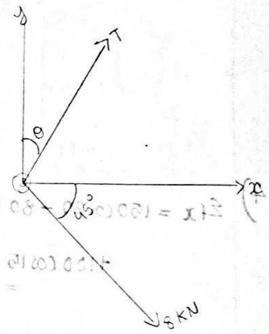


5)? If the magnitude of the resultant force is to be 9 KN directed along the +ve x axis, determine the magnitude of force T acting on the eyebolt and its angle

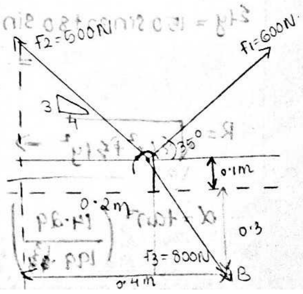


$\Sigma fx = f_1 \sin\theta_1 + f_2 \sin\theta_2$

$\Sigma fy = f_1 \cos\theta_1 + f_2 \cos\theta_2$



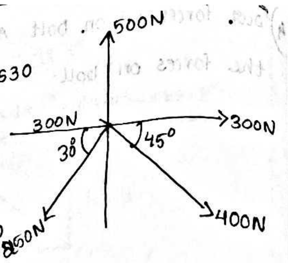
6)? Find R and its directions.



Answers

1) $\Sigma fx = 300\cos 0 + 500\cos 90 - 250\cos 30 + 400\cos 45$

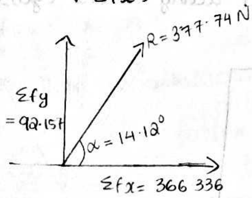
$= \underline{366 \cdot 336\,N}$



$\Sigma fy = 300\sin 0 + 500\sin 90 - 250\sin 30 - 400\sin 45 = 92 \cdot 157\,N$

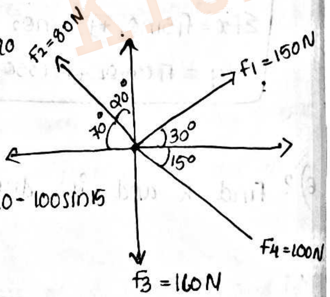
$R = \sqrt{\Sigma fx^2 + \Sigma fy^2} \Rightarrow \sqrt{142694 \cdot 477} = \underline{\underline{377 \cdot 74\,N}}$

$\alpha = \tan^{-1}\left(\frac{\Sigma fy}{\Sigma fx}\right) \Rightarrow \tan^{-1}\left(\frac{92 \cdot 157}{366 \cdot 336}\right) = \underline{\underline{14 \cdot 12^\circ}}$



4) $\Sigma fx = 150\cos 30 - 80\cos 70 - 110\cos 90 + 100\cos 15$

$= \underline{\underline{199 \cdot 13}}$

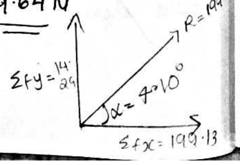


$\Sigma fy = 150\sin 30 + 80\sin 70 - 110\sin 90 - 100\sin 15$

$= \underline{\underline{14 \cdot 29}}$

$R = \sqrt{\Sigma fx^2 + \Sigma fy^2} \rightarrow \sqrt{39856 \cdot 961} = \underline{\underline{199 \cdot 64\,N}}$

$\alpha = \tan^{-1}\left(\frac{14 \cdot 29}{199 \cdot 13}\right) = \underline{\underline{4 \cdot 10^\circ}}$



2) $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Case 1

$\theta = 90, \; R = \sqrt{10}$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90}$

$R^2 = P^2 + Q^2$

$10 = P^2 + Q^2$ — (1)

Case 2

$\theta = 60 \quad R = \sqrt{13}$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R^2 = P^2 + Q^2 + PQ$

$P^2 + Q^2 + PQ = 13$ — (2)

(2) − (1) $\Rightarrow PQ = 3 \Rightarrow 2PQ = 6$ — (3)

(3) + (1) $\Rightarrow P^2 + Q^2 + 2PQ = 16 \quad (P+Q)^2 = 16 \quad P + Q = 4 \quad P = 4 - Q$

$(4 - Q)\,Q = 3 \quad 4Q - Q^2 - 3 = 0 \Rightarrow -Q^2 + 4Q - 3 = 0$

$Q = 3, 1$

$P = 1, 3$

$\Rightarrow Q = 3, 1\,N$

$\underline{\underline{P = 1, 3\,N}}$

3) $P^2 + Q^2 + 2PQ = 17^2$

$P^2 + Q^2 - 2PQ = 3^2$

$R = \sqrt{149}$

$2P^2 + 2Q^2 = 298$

$P^2 + Q^2 = 149$

$2PQ = 140$

$\sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{149}$

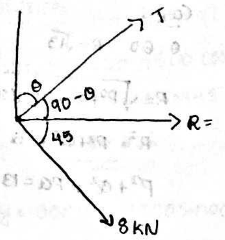
$149 + 140\cos\theta = 149$

$140\cos\theta = 0$

$\cos\theta = 0$

$\theta = \cos^{-1}(0) = 90^\circ$

5)



~~$\Sigma fx = 9\cos\theta = 0$~~

$\Sigma fx = R = 9kN$ $\quad 9\cos\theta = 9$

$\Sigma fy = 9\sin\theta = 0$

T cos ·

$R = \sqrt{\Sigma fx^2 + \Sigma fy^2}$

$\Sigma fx = F_1\sin\theta + F_2\sin\theta$

$9 = T\sin\theta - \frac{8}{\sqrt{2}}$

$\Sigma fy = T\cos\theta - 8\cos 45$

$T\cos\theta = 8/\sqrt{2} = 4\sqrt{2}$

$9 = T\sin\theta - T\cos\theta = T(\sin\theta - \cos\theta)$

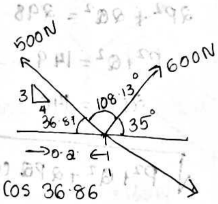
$\theta = \tan^{-1}(2.59) = 68.88$

$T\cos 68.88 = 4\sqrt{2} = \underline{9.42N}$

6)

$\tan\theta = 3/4$

$\theta = \tan^{-1}(3/4) = 36.86°$



$\Sigma fx = 600\cos 35 - 500\cos 36.86 + 800\cos 36.86$

$= 731.52\ N$

$\Sigma fy = 600\sin 35 + 500\sin 36.86 - 800\sin 36.86 = 164.18N$

$R = \underline{749.71}\ N$

Conditions of Equillibrium of coplanar concurrent force system

A number of forces acting on a particle is said to be in equillibrium when the resultant force is 0. If resultant force is not equal to zero, the particle can be brought to rest by applying a force equal and opposite to resultant force. Such force is called equilibriant. resultant and equillibriant are equal in magnitude and opposite direction.

For an equillibrium system R should be zero. both Σfx and Σfy must be zero. Thus, the equillibrium conditions are $\Sigma fx = 0$ and $\Sigma fy = 0$. Equillibrium conditions of coplanar concurrent force system $\Sigma fx = 0$ $\Sigma fy = 0$.

Equations of Equillibrium of coplanar force system

$\Sigma fx = 0$

$\Sigma fy = 0$

$\Sigma M_o = 0$

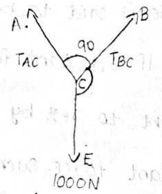
M → momentum about the point

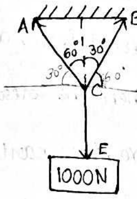
Note

coplanar concurrent force system with only 3 forces we can use lami's Theorem for getting solution.

If no. of forces are more than 3 we can use ~~more~~ equations of equillibrium.

2. Find the tension in the cable AC and BC





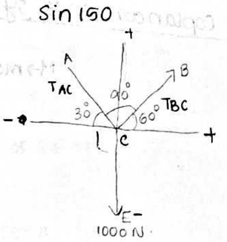
By lami's Theoram

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

$$\frac{1000}{\sin 90} = \frac{T_{AC}}{\sin 150} = \frac{T_{BC}}{\sin 120}$$

$$T_{AC} = \frac{1000 \times \sin 150}{\sin 90} = \underline{\underline{500\ N}}$$

$$\frac{500 \times \sin 120}{\sin 150} = T_{BC} = 866.02\ N$$



left - -ve
right - +ve
up - +ve
down - -ve

$T_{BC}\cos 60 - T_{AC}\cos 30 - E\cos 90 =$

$= \Sigma f_x = T_{BC}\cos 60 - T_{AC}\cos 30 - 0)$

$\Sigma f_y = T_{BC}\sin 60 + T_{AC}\sin 30 - E\sin 90$

$= T_{BC}\sin 60 + T_{AC}\sin 30 - 1000$

$T_{BC} = \underline{\underline{866.02 N}}$ $\quad$ $\underline{\underline{T_{AC} = 500}}$

General Equations of Equillibrium

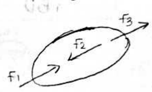
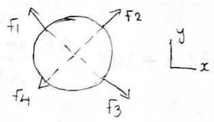
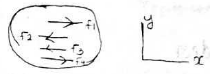
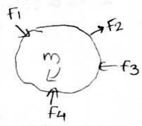
1. The algebraic sum of all forces in a force system is 0.

$R = \Sigma F = 0$

2. The algebraic sum of all moments in a force system is 0.

$M = \Sigma M = 0$

Equations of Equillibrium for coplanar systems

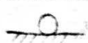
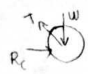
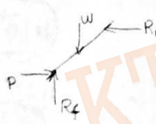
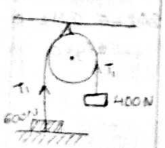
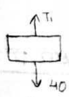
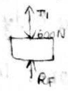
Force system	Freebody diagram	Independant equations
1. collinear		$\Sigma f_x = 0$
2. concurrent at a point		$\Sigma f_x = 0$, $\Sigma f_y = 0$
3. Parellel		$\Sigma f_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma f_x = 0$ $\Sigma M_z = 0$ $\Sigma f_y = 0$

Solving equillibrium problems

1. Draw proper free body diagram.
2. Resolve all the forces into x and y components.
3. Apply equillibrium conditions along the x and y directions.
4. Solve the resultant algebraic equations.

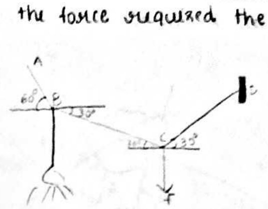
Free body diagram.

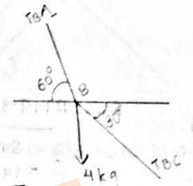
The diagram of a body in which the body under consideration is freed from all the contact surfaces and all the forces acting on it including reaction at contact surfaces is called a free body diagram [FBD]

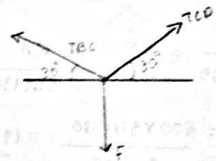
	FBD for	FBD
	Ball	
	Ball	
	Ladder	
	400 N wt	
	600 N, wt	

Q. Determine the force required the hold the 4 kg lamp in position

F = 39.2 N







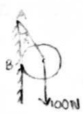
doubt

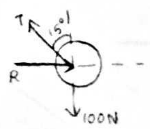
$$\frac{T_{AB}}{\sin 60} = \frac{T_{BC}}{\sin 150} = \frac{\cancel{40}\ 39.2}{\sin 150} \qquad T_{BC} = 40\ N \qquad T_{AB} = \cancel{69.05\ N}\ 67.89\ N$$

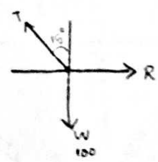
$$\frac{\cancel{40}\ 39.2}{\sin 120} = \frac{T_{CD}}{\sin 120} = \frac{F}{\sin 180} \qquad \cancel{T_{CD} = 40\ N} \qquad \cancel{F = 40\ N}$$

$T_{CD} = 39.2$ $\qquad F = 39.2 \qquad F\ g \rightarrow$

? Find out the tension and reaction.







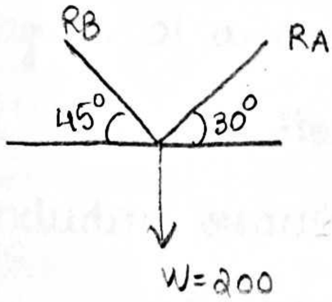
In lamis theorem all forces must be drawn

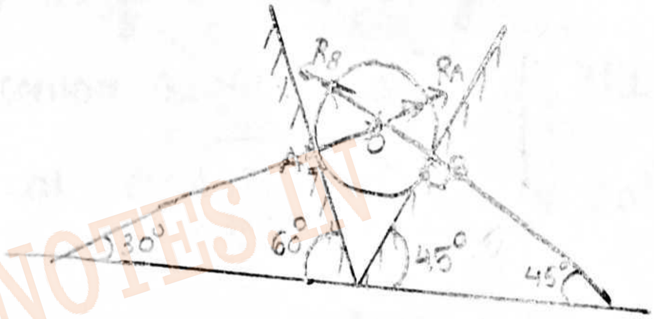
Surface of contact

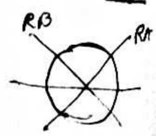
$$\frac{T}{\sin 90} = \frac{100}{\sin 105} = \frac{R}{\sin 165} \qquad \frac{T}{1} = \frac{100}{\sin 105} \qquad T = \frac{100}{\sin 105} = 103.52$$

$$R = \frac{100 \times \sin 165}{\sin 105} = 26.79\ N$$

Find the reaction component at A and B.







$$\frac{200}{\sin 105} = \frac{R_B}{\sin 120} = \frac{R_A}{\sin 135}$$

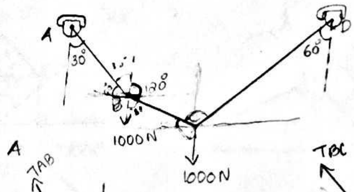
$$R_B = \frac{200 \times \sin 120}{\sin 105} = \underline{\underline{179.31}}\ N$$

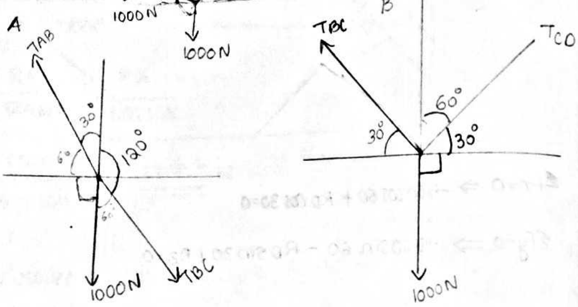
$$R_A = \frac{179.31 \times \sin 135}{\sin 120} = \underline{146.4\ N}$$

Q Find tension in the string AB, BC, CD?

$T_{AB} = 1732 N$

$T_{BC} = 1000 N$





I^{st} case

$$\frac{T_{AB}}{\sin 60} = \frac{1000}{\sin 150} = \frac{T_{BC}}{\sin 150}$$

$$T_{AB} = \frac{1000 \times \sin 60}{\sin 150} = 1732.05 N$$

$$T_{BC} = \frac{1000 \times \sin 150}{\sin 150} = 1000 N$$

II^{nd} case

$$\frac{1000}{\sin 120} = \frac{T_{BC}}{\sin 120} = \frac{T_{CD}}{\sin 120}$$

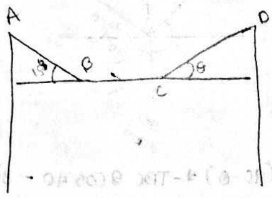
$$T_{CD} = \frac{1000 \times \sin 120}{\sin 120} = 1000 N$$

$$T_{BC} = \frac{1000 \times \sin 120}{\sin 120} = 1000 N$$

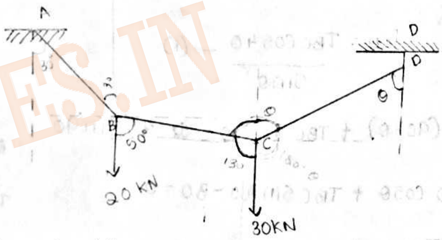
H.W

A) Determine the tension in the cables AB, BC, and CD necessary to support the 10 kg and 15 kg traffic lights at B and C respectively. Also find the angle θ.

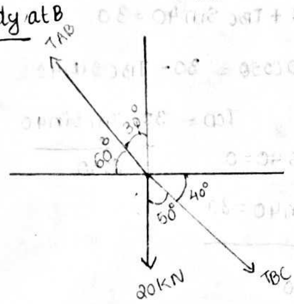
θ unknown → Equations of Equilibrium. more than a variable Lami's theorem



?



Freebody at B

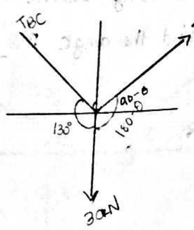


By Lami's Theorem.

$$\frac{T_{AB}}{\sin 50} = \frac{20}{\sin 160} = \frac{T_{BC}}{\sin 150}$$

$$T_{AB} = \frac{20 \times \sin 50}{\sin 160} = 44.79 KN$$

$$T_{BC} = \frac{20 \times \sin 150}{\sin 160} = 29.23 KN$$



$18-\theta-90$

$\Sigma f_x = 0$

$T_{CD} \cos(90-\theta) - T_{BC} \cos 40 - 30 \cos 90$

$= T_{CD} \sin\theta - T_{BC} \cos 40 = 0$ — (1)

$T_{CD} \sin\theta = T_{BC} \cos 40$

$T_{CD} = \dfrac{T_{BC} \cos 40}{\sin\theta}$ — (1)

$\Sigma f_y = 0$

$T_{CD} \sin(90-\theta) + T_{BC} \sin 40 - 30 \sin 90$

$= T_{CD} \cos\theta + T_{BC} \sin 40 - 30 = 0$

$T_{CD} \cos\theta + T_{BC} \sin 40 = 30$

$T_{CD} \cos\theta = 30 - T_{BC} \sin 40$

$T_{CD} = \dfrac{30 - T_{BC} \sin 40}{\cos\theta}$

$\theta = 63.7°$

$T_{CD} = 251N$

$T_{CD} \sin\theta - T_{BC} \cos 40 = 0$

$T_{CD} \cos\theta + T_{BC} \sin 40 = 30$

$T_{CD} \sin\theta = 29.23 \times \cos 40$

$T_{CD} \sin\theta = \underline{\underline{22.39}}$

$T_{CD} \cos\theta = 30 - (29.23 \times \sin 40) = \underline{\underline{11.21}}$

$T_{CD} \sin\theta = 22.39$ — (3)

$T_{CD} \cos\theta = 11.21$ — (4)

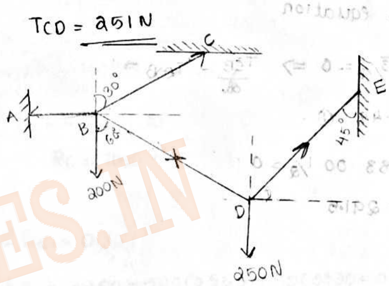
$\dfrac{(3)}{(4)}$ $\tan\theta = 1.997$

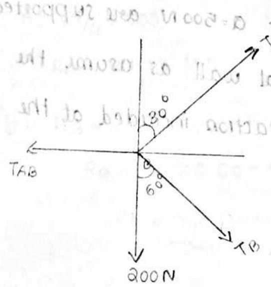
$\theta = \tan^{-1}(1.997) = \underline{\underline{63.4°}} \approx \underline{63.7°}$

$T_{CD} \sin 63.7° = 22.39$

$T_{CD} = \underline{\underline{251N}}$

?





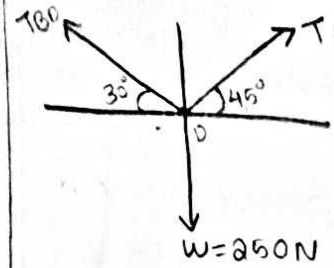
$\Sigma f_x = T_{BC} \cos 60 - T_{AB} \cos 0 - 200 \cos 90 + T_{BD} \cos 30 = 0$

$T_{BC} \times 1/2 - T_{AB} + T_{BD} \times \dfrac{\sqrt{3}}{2} = 0$ — (1)

$\Sigma f_y = T_{BC} \sin 60 + T_{AB} \sin 0 - 200 \sin 90 - T_B \sin 30 = 0$

$\Sigma f_y = T_{BC} \sqrt{3}/2 - 200 - T_B \times 1/2 = 0$

$T_{BC} \sqrt{3}/2 - \dfrac{T_B}{2} = 200$ — (2)



$$\frac{T_{BD}}{\sin 135} = \frac{T_{DE}}{\sin 120} = \frac{W}{\sin 105}$$

$$= \frac{T_{BD}}{0.707} = \frac{T_{DE}}{0.866} = 258.81$$

$T_{DE} = 258.81 \times 0.866 = 224.13$

$T_{BD} = 0.707 \times 258.81 = 183.00$

~~Σx~~ we have the equation

$\frac{T_{CB}}{2} - T_{AB} + 183 \times \sqrt{3}/2 = 0 \Rightarrow \frac{T_{CB}}{2} - \cancel{T_{AB}} =$

$T_{AB} - \frac{T_{CB}}{2} = 158.48$ —(1)

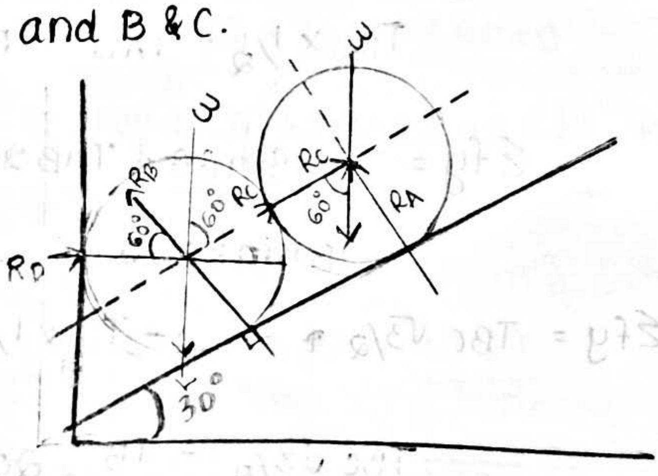
$\Sigma f_y = T_{CB}\sqrt{3}/2 - 200 - 183.00 \, 1/2 = 0$

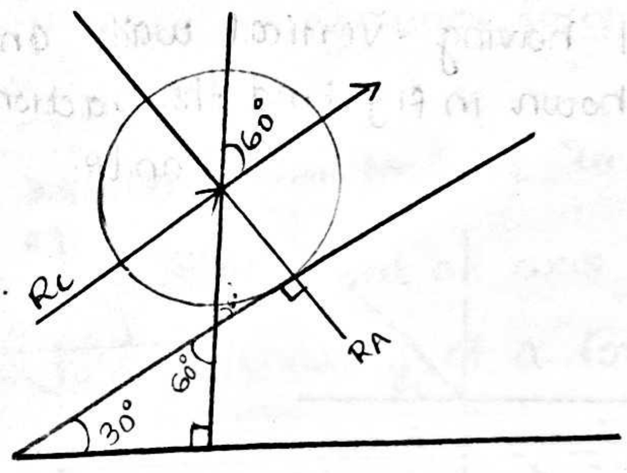
$T_{CB}\sqrt{3}/2 = -291.5$

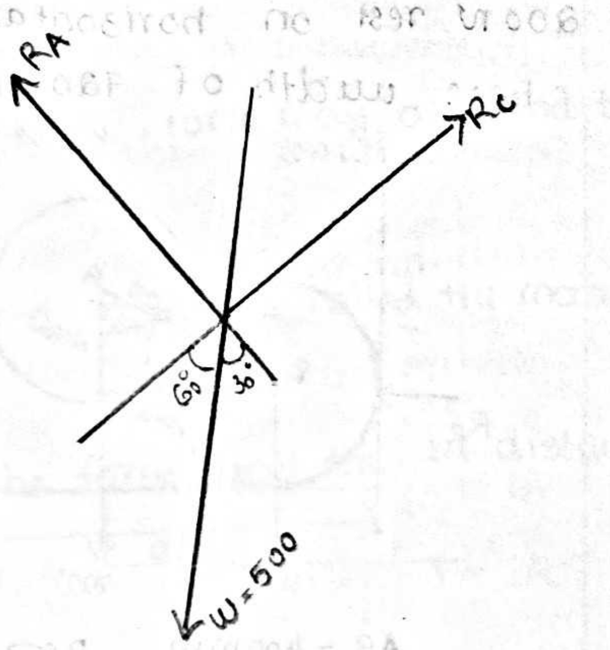
$T_{AB} = 326.8\,N$

$T_{CB} = \underline{\underline{336.5\,N}}$

Two identical roller each of wt $Q = 500\,N$ are supported by an inclined plane and vertical wall as asume the surfaces are smooth. find the reaction included at the Point A and B & C.



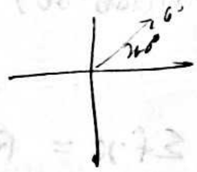




$$\frac{R_A}{\sin 120} = \frac{R_C}{\sin 150} = \frac{500}{\sin 90}$$

$R = 433.01$ N

$R_C = 250$ N



$\Sigma f_x = 0$

$\Sigma f_x = -250 - 500\cos 60 + R_D \cos 30 = 0$

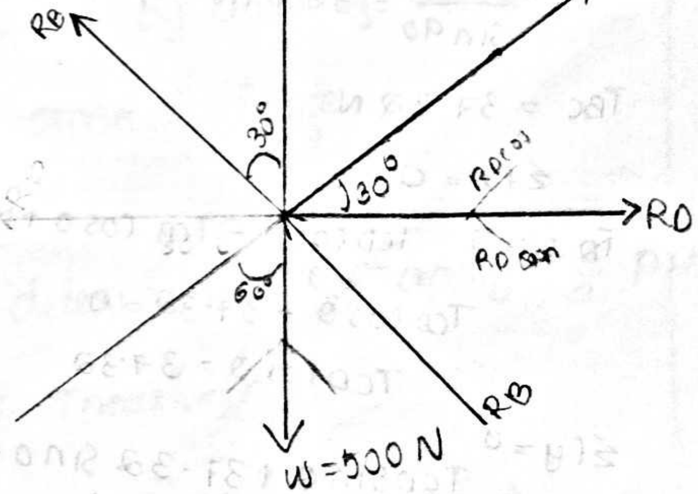
$R_D \sqrt{3}/2 = 500$

$R_D = 577.35$ N

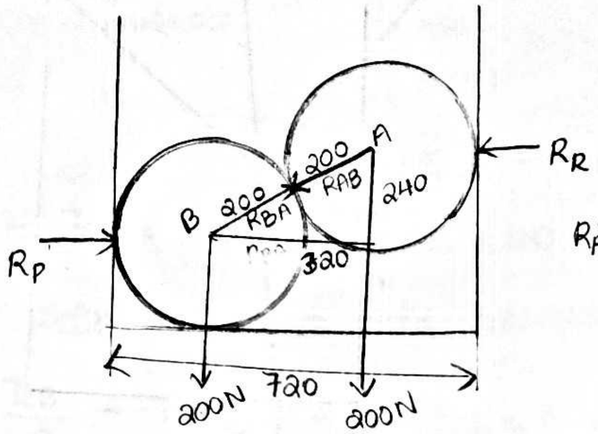
$f_y = R_B - 500 \sin 60 - R_D \sin 30 = 0$

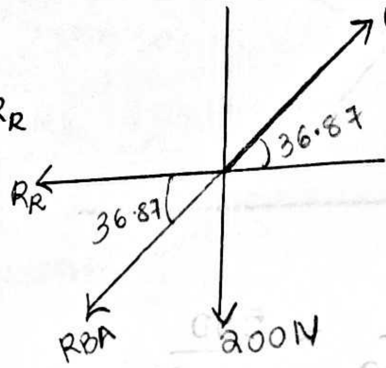
$R_B - 500 \sin 60 - 577.35 \sin 30 = 0$

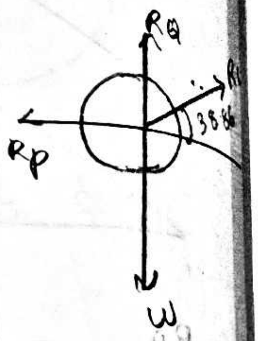
$R_B = 721.68$ N



Two smooth cylinders A and B each of diameter 400 mm and 200 N rest on horizontal channel having vertical walls and base width of 720 mm as shown in fig. Find the reaction of P, Q and R.







$AB = 400\,mm$ $\quad \cancel{BC} \quad \theta = 36.87^{\circ}$

doubt

$$\frac{R_r}{\sin(126.86)} = \frac{200}{\sin 36.86} = \frac{R_c}{\sin 90}$$

$R_r = 266.67\,N$

~~$R_C = 333.41\,N$~~ $R_C = 333.41\,N$

$\Sigma f_x = R_P\cos 0 + R_c \cos 38.86 - R_Q \cos 90 - 200\cos 90 = 0$

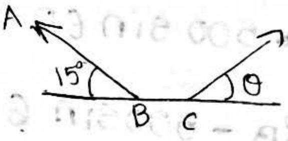
$R_P = -266.76\,N \qquad R_P = \cancel{226}\,266.76\,N$

$\Sigma f_y = R_P \sin 0 + 333.41 \sin 36.86 + 200 \sin 90 - R_Q \sin 90 = 0$

$R_Q = -400\,N \qquad R_Q = 400\,N$

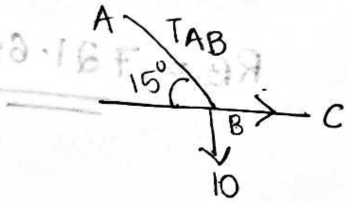
Answer

1) $$\frac{T_{AB}}{\sin 90} = \frac{T_{BC}}{\sin 105} = \frac{10}{\sin 165}$$



$$\frac{T_{AB}}{\sin 90} = 38.64\,N \Rightarrow T_{AB} = 38.64\,N$$

$T_{BC} = 37.32\,N$

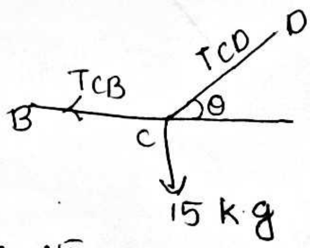


$\Sigma f_x = 0$

$T_{CD}\cos\theta - T_{CB}\cos 0 + 15\cos 90 = 0$

$T_{CD}\cos\theta - 37.32 = 0$

$T_{CD}\cos\theta = 37.32$



$\Sigma f_y = 0$

$T_{CD}\sin\theta + 37.32 \sin 0 - 15 \sin 90 = 0$

$T_{CD}\sin\theta - 15 = 0 \qquad T_{CD}\sin\theta = 15$

$\tan\theta = \frac{15}{37.32} \Rightarrow \theta = \tan^{-1}(0.4019)$