



# KTU **NOTES**

The learning companion.

**KTU STUDY MATERIALS | SYLLABUS | LIVE  
NOTIFICATIONS | SOLVED QUESTION PAPERS**

## Module - 2.

Elementary Concept of Magnetic Materials

- \* A magnetic is a material or object that produces a magnetic field.

e.g. Iron, steel, nickel, cobalt

- \* Classified into 3 :

- Diamagnetic material
- Paramagnetic material
- Ferromagnetic material

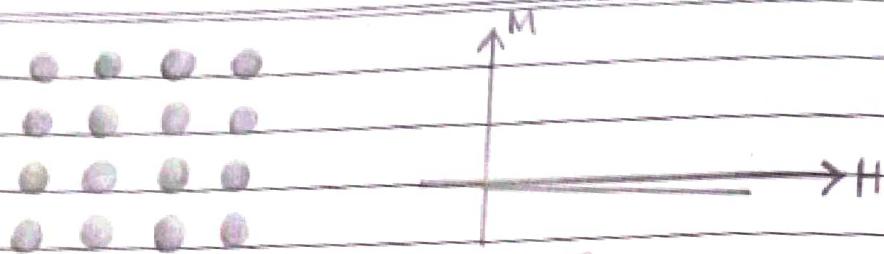
- \* Diamagnetic materials

• Diamagnetic substances are composed of atoms which have no net magnetic moments.

• i.e. all the orbital shells are filled and there are no unpaired electrons.

• When exposed to a field, a negative magnetization is produced and thus the susceptibility is negative.

• Susceptibility is measure of how much a material will become magnetized in an applied magnetic field.



Magnetization and magnetic field intensity.

### \* Paramagnetic materials

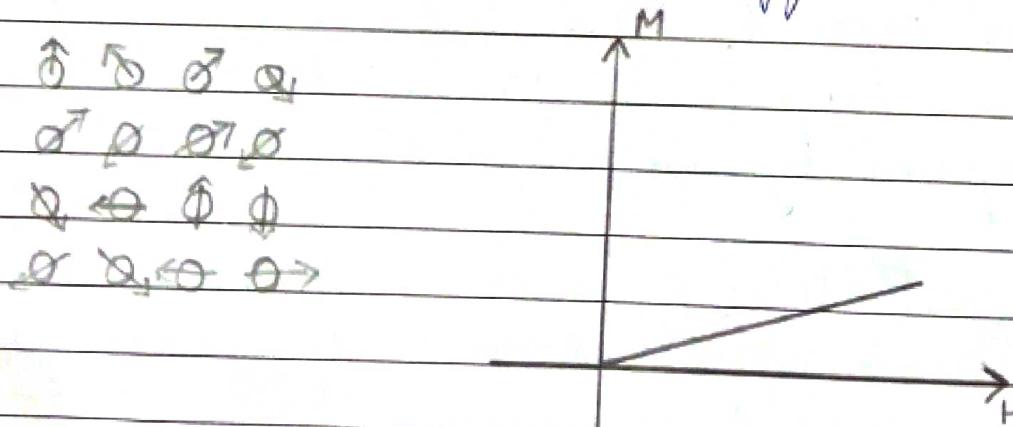
Some of the atoms or ions in the materials have a net magnetic moment due to unpaired electrons.

When placed in a magnetic field, magnetic field within the material gets enhanced.

When placed in a non uniform MF, it tends to move from low to high field region.

Have permanent dipole moment.

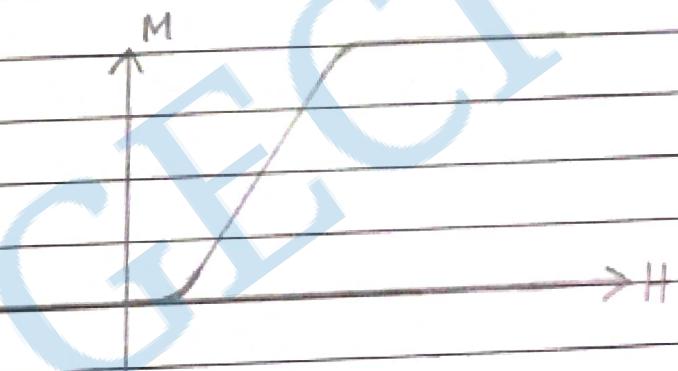
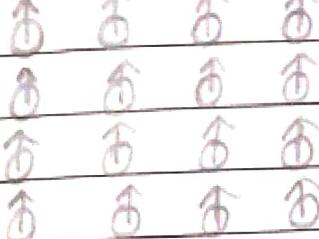
e.g. Aluminium, Sodium, Copper chloride.



## Ferromagnetic Materials

- \* Gets magnetized even by weak magnetic field.
- \* Ferromagnetic materials exhibit parallel alignment of moments resulting in large net magnetization even in the absence of a magnetic field.

e.g. Fe, Ni, Co.



## Classification of magnets

- \* Permanent magnet -  
• Retain magnetism even after removing the external magnetic field.
- An retain their magnetism and magnetic properties for a longer time. Strongly magnetized hard materials make up permanent magnets.
- Hardened steel and alloy of steel can be transformed into permanent

## magnets - artificial Magnets.

- A permanent magnet does not require a continuous supply of electrical energy to maintain its magnetic field.

### \* Electromagnet

- Magnetization is done by passing electric current in a coil surrounding the material is called electromagnet.
- Strength of electromagnet varies according to the flow of electric current into it.
- An electromagnetic magnet only displays magnetic properties when an electric current is applied to it.
- An electromagnet's magnetic field can be rapidly manipulated over a wide range by controlling the amount of electric current supplied to the electromagnet.

### \* Magnetic Induction

- Phenomenon of changing magnetic substance to magnet.
- Two properties :
  - 1) Attraction

## 2) Repulsion

- Magnet has 2 end-poles
- North pole
- South pole.

## \* Magnetic field

Space around a magnet where magnetic effect can be detected.

## \* Magnetic flux

- Represents total number of magnetic lines of force in a magnetic field.

- Denoted by  $\phi$

- Unit : Weber (Wb)

## \* Magnetic Flux Density

- Flux passing per a unit area.

- B

- Wb/m<sup>2</sup> or Tesla (T)

- $B = \phi/A$ .

## \* Permeability / Absolute Permeability

- Ability of a material to pass/conduct magnetic flux.

$$\mu = 4\pi \times 10^{-7} \text{ H/m}$$

### Relative permeability

Permeability with resp. to free space

$\mu_r$

$$\mu_r = \mu/\mu_0$$

Relative permeability of air is 1.

### Magnetic Field Intensity

Force experienced in a unit north pole placed at that point.

$H$

N/Wb

$$B = \mu H \quad (\mu = \mu_0 \mu_r)$$

### Magneto motive force (mmf)

Magnetic pressure that sets up magnetic flux in a magnetic circuit.

Mmf - no of turns ~~in~~ of coil  $\times$  current in it  
unit AT (ampere turns).

### Reluctance

Opposition offered to magnetic lines of force in a magnetic circuit.

$S$

AT/Wb

$b/ua$ 

- $b$  - length of magnetic path
- $A$  - cross sectional area.

Permeance

- Reciprocal of reluctance
- $Wb/AT$

## \* Electric and Magnetic Circuit - comparison

### Electric circuit

### Magnetic circuit

- |                               |                                  |
|-------------------------------|----------------------------------|
| Path traced by the current    | Path traced by the magnetic flux |
| is known as electric cur-     | is called as magn-               |
| ent.                          | etic circuit.                    |
| EMF is the driving force      | MMF is the driving force         |
| in the electric circuit.      | in the magnetic circuit.         |
| The unit is volts.            | The unit is ampere turns.        |
| There is a current $I$ in     | There is a flux $\phi$ in the    |
| the electric circuit which    | magnetic circuit which           |
| is measured in amperes.       | is measured in weber.            |
| The flow of electrons         | The number of magne-             |
| decides the current in the    | ties of force de-                |
| conductor.                    | -des the flux.                   |
| Resistance ( $R$ ) oppose the | Reluctance ( $S$ ) is opposed    |
| flow of the current.          | by magnetic path to the          |
| The unit is Ohm.              | flux. The unit is Atm/Wb         |
| $R = \rho \frac{L}{A}$        | $\therefore S = b/ua$            |
| Directly proportional to $L$  | $S = b/a$ Directly propor-       |
| Inversely proportional to $A$ | tional to $b$ . Inversely        |
| Depends on nature of          | proportional to $u - NAs$        |
| material.                     | Inversely proportional to $A$ .  |

- The current  $I = \text{MMF}/\text{resistance}$

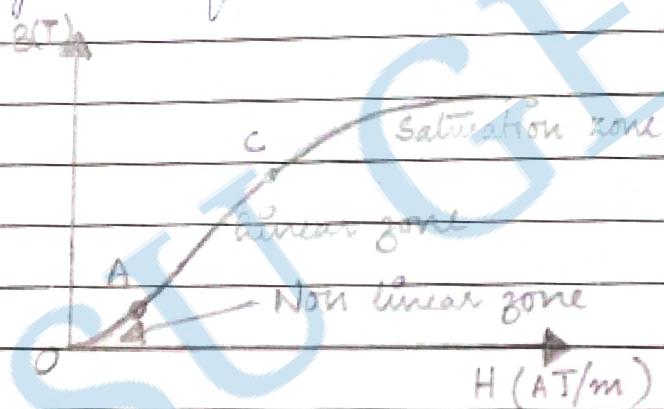
$\text{The flux} = \text{MMF}/\text{Reluctance}$

- The current density
- Kirchhoff current law
- Faraday's law is applicable to the electric circuit.

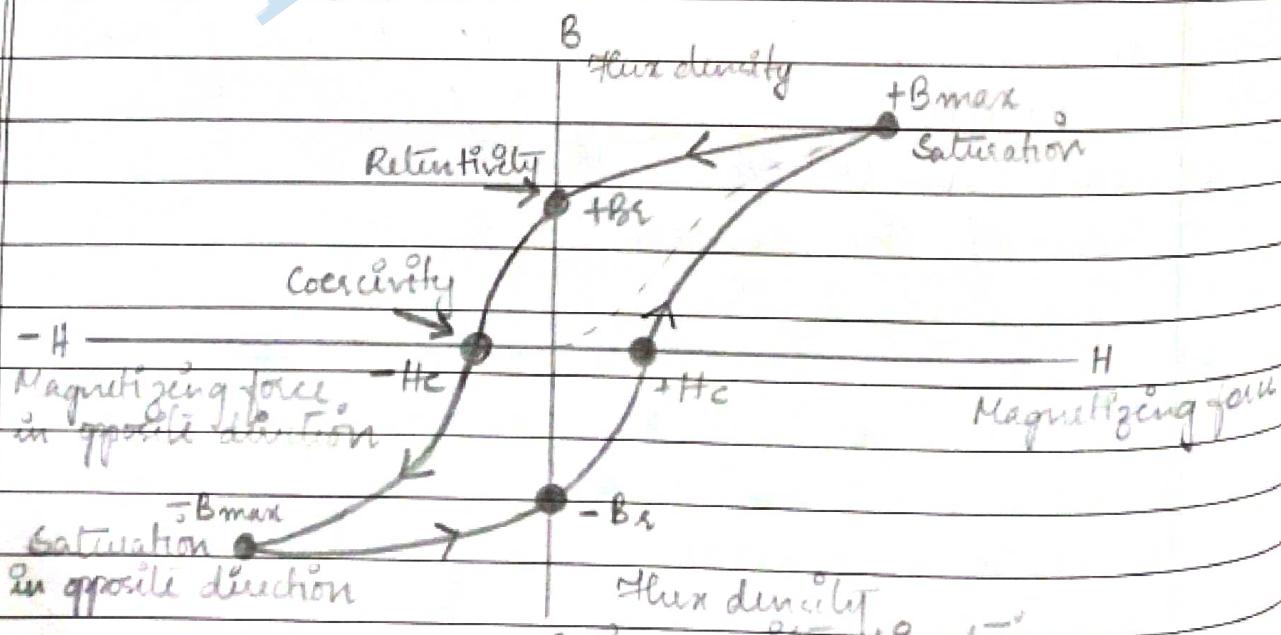
The flux density  
Kirchhoff mmf law & flux law is applicable to the magnetic flux.

## \* Magnetization curve (BH Curve)

- Graph between magnetic flux density ( $B$ ) and magnetic force ( $H$ ).



## \* Magnetic Hysteresis

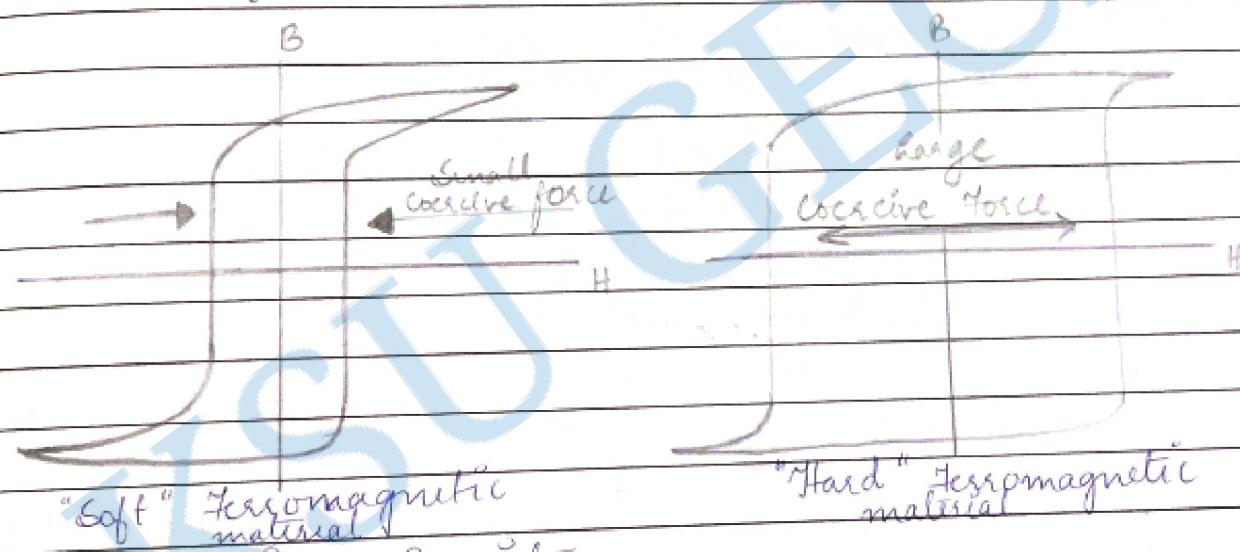


## \* Relativity

It is defined as the degree to which a magnetic material gains its magnetism after magnetizing force ( $H$ ) is reduced to zero.

## \* Coercivity

The amount of reverse driving field required to Demagnetize it is called its coercivity.



## \* Magnetic Circuit

Closed path followed by magnetic lines of force.

$$\mathbf{B} = \frac{l}{\mu_0 A} = \frac{l}{4\pi \times 10^{-7} \text{ Henry/metre}}$$

- $l$  is the length in 'm'.
- $\mu_0$  is the permeability of vacuum, equal to  $4\pi \times 10^{-7}$  henry metre.

- $\mu_r$  is the relative permeability of the material
- $\mu$  is the permeability of the material ( $\text{N} \cdot \text{A}^{-1}$ )
- $A$  is the cross-sectional area of the circuit in square metres.

### \* Composite magnetic circuit

$$S = \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_g}{\mu_0 \mu_{Ag} A_g}$$

total MMF = flux  $\times$  reluctance ( $S$ )

$$= \phi \times \left[ \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_g}{\mu_0 \mu_{Ag} A_g} \right] \quad \left\{ \begin{array}{l} \mu_{Ag} = 1 \\ (\text{air}) \end{array} \right\}$$

Magnetic flux density,

$$B = \frac{\phi}{A}$$

$$\therefore \text{total MMF} = \frac{B_1 l_1}{\mu_0 \mu_{r1}} + \frac{B_2 l_2}{\mu_0 \mu_{r2}} + \frac{B_g l_g}{\mu_0} =$$

(i) An iron ring having cross-sectional area of  $400 \text{ mm}^2$  and mean circumference of  $500 \text{ mm}$  carries a coil of 250 turns wound uniformly around it. Calculate

- Reluctance of the ring.
- Current required to produce a flux of  $1000 \text{ mWb}$  in the ring.

Relative permeability of iron is 400.

$$m) i) S = l$$

$$4 \cdot 10^{-3} A$$

$$l = 500 \times 10^{-3} m$$

$$A = 400 \times 10^{-6} m^2$$

$$n = 250$$

$$M_s = 400$$

$$M_0 = 4\pi \times 10^{-7}$$

$$S = 500 \times 10^{-3}$$

$$4\pi \times 10^{-7} \times 400 \times 400 \times 10^{-6}$$

$$= \frac{500 \times 10^{-3}}{16 \times 4 \times 10^4 \times \pi} \times 10^6$$

$$= \frac{500}{64\pi} \times \frac{10^{10}}{10^4}$$

$$= \frac{500}{64\pi} \times 10^6$$

$$\underline{S = 2.48 \times 10^6 AT/wh}$$

b)  $I = ?$

$$\phi = 1000 \text{ mwb} = 1000 \times 10^{-6} \text{ wb}$$

$$mmf = \phi \times S$$

$$NI = \phi \times S$$

$$I = \frac{\phi \times S}{N}$$

$$= \frac{1000}{1000} \times 10^{-6} \times 2.48 \times 10^6$$

$$= 2.48 \times 10^0$$

$$= 2.48 A$$

$$= \underline{9.92 A}$$

92) A mild steel ring has a mean circumference of 500 mm and a uniform cross-sectional

area of  $300 \text{ mm}^2$ . Calculate the mmf required to produce a flux of  $500 \mu\text{wb}$ ? Assume  $\mu_r = 1200$ .

$$\begin{aligned}\text{Ans}) \quad l &= 500 \text{ mm} \\ &= 500 \times 10^{-3} \text{ m} \\ A &= 300 \text{ mm}^2 \\ &= 300 \times 10^{-6} \text{ m}^2 \\ \mu_r &= 1200 \\ \mu_0 &= 4\pi \times 10^{-7}\end{aligned}$$

$$\begin{aligned}S &= \frac{l}{\mu_0 \mu_r A} \\ &= \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 1200 \times 300 \times 10^{-6}} \\ &= \frac{500 \times 10^{-3} \times 10^7 \times 10^6}{4\pi \times 10^4} \\ &= \frac{500}{4\pi} \times 10^6 \\ &= 1.10 \times 10^6 \text{ AT/wb}\end{aligned}$$

$$\begin{aligned}\text{mmf} &= \phi \times S \\ &= 500 \times 10^{-6} \times 1.1 \times 10^6 \\ &= 550 \text{ AT}\end{aligned}$$

- 23) An iron ring of mean length 60 cm has an air gap of 2 mm and a winding of 300 turns. If the relative permeability of iron used in the ring is 400 when a circuit current of 1.5 A flows through it, find

the flux density?

$$l = 60\text{cm}$$

$$= 60 \times 10^{-2} \text{m}$$

$$l_2 = 2\text{mm}$$

$$= 2 \times 10^{-3} \text{m}$$

$$n = 300$$

$$U_A = 400$$

$$I = 1.5\text{A}$$

$$B = ?$$

$$\text{mmf} = Hl$$

$$\text{mmf}_1 = H_1 l_1$$

$$B = \mu_0 H$$

$$H = \frac{B}{\mu_0 l_1}$$

$$\therefore \text{mmf}_1 = \frac{B}{\mu_0 l_1} \times l_1$$

$$\text{mmf}_2 = \frac{B l_2}{\mu_0 l_1} \quad \left\{ U_A = 1\text{e.m.f.} \right\}$$

$$= \frac{B l_2}{\mu_0}$$

$$\text{total mmf} = \text{mmf}_1 + \text{mmf}_2$$

$$N \times I = \frac{B l_1}{\mu_0 l_1} + \frac{B l_2}{\mu_0}$$

$$300 \times 1.5 = \frac{B}{\mu_0} \quad \left\{ l_1 + l_2 \right\} \quad \mu_0$$

$$300 \times 1.5 \times \mu_0 = B \quad \left\{ \frac{60 \times 10^{-2}}{400} + \frac{2 \times 10^{-3}}{} \right\}$$

$$450 \times 4\pi \times 10^{-7} = B \quad \left\{ \frac{60 \times 10^{-2}}{400} + \frac{8 \times 10^{-1}}{} \right\}$$

$$450 \times 400 \times 4\pi \times 10^{-7} = B$$

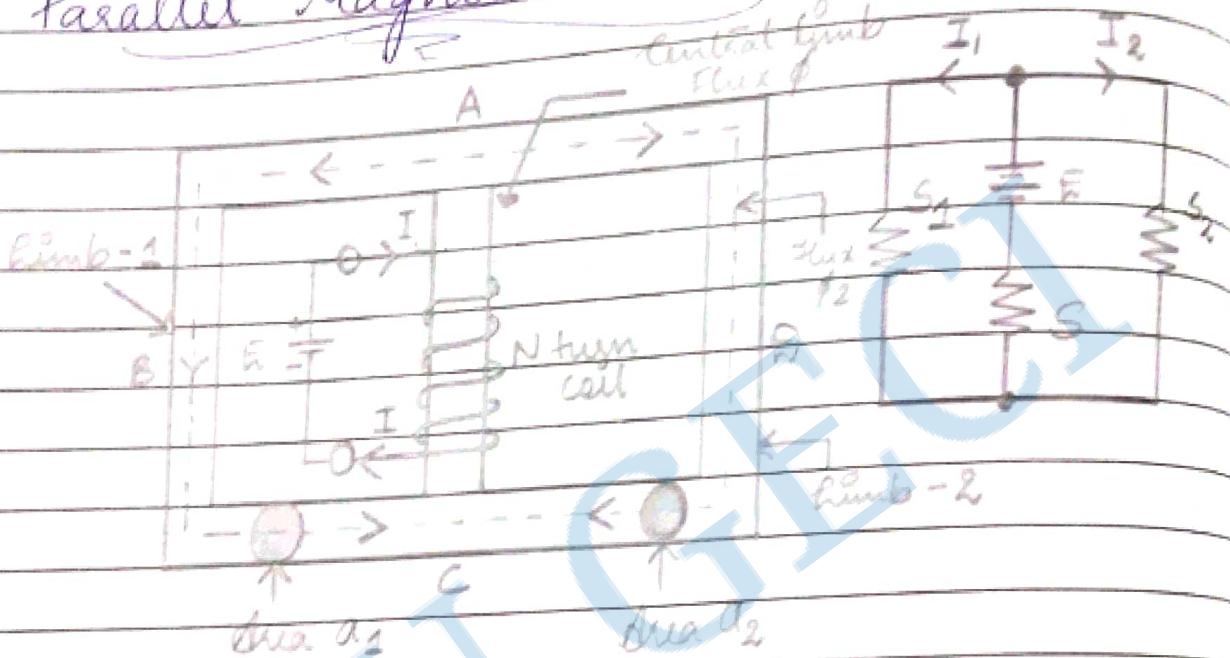
$$\frac{60 \times 10^{-2} + 8 \times 10^{-1}}{400} = B$$

$$B = \frac{2260800 \times 10^{-7}}{1.4}$$

$$= 161.4854 \cdot 10^{-7}$$

$$= 0.1615 \text{ mT}$$

## \* Parallel Magnetic Circuit



The total m.m.f. produced by the coil of  $N$  turns is,

$$\text{Total mmf} = N \times I \text{ (AT)}$$

Total m.m.f. can also be expressed as,

$$\text{total m.m.f.} = \text{Total reluctance} * \text{Total flux}$$

m.m.f. for path ABCA :  $F = \text{MMF of path ABC} + \text{MMF of path AC}$

Thus total MMF = MMF of central limb +  
MMF of limb - 1 or 2

$$\therefore S \times \phi = NI = \phi_s s_e + I \phi_1 s_1 \text{ or } \phi_2 s_2$$

The reluctances  $S_1, S_2$ , and  $S_c$  are given by

$$S_1 = \frac{l_1}{\mu a_1}, S_2 = \frac{l_2}{\mu a_2}, S_c = \frac{l_c}{\mu a_c}$$

Assuming the cross sectional area of the three limbs to be same i.e

$$a_1 = a_2 = a_c = a,$$

the expression for  $S_1, S_2, S_c$  gets modified as

$$S_1 = \frac{l_1}{\mu a}, S_2 = \frac{l_2}{\mu a}, S_c = \frac{l_c}{\mu a}$$

Substituting these values in equations

Total MMF,

$$F = \phi_1 \times \frac{l_1}{\mu a} + \phi_c \times \frac{l_c}{\mu a}$$

$$\therefore F = \frac{B_1 l_1}{\mu} + \frac{B_c l_c}{\mu}$$

$$= \frac{B_1 l_1 + B_c l_c}{\mu}$$

$$\text{and } F = \frac{B_2 l_2}{\mu} + \frac{B_c l_c}{\mu}$$

$$= \frac{B_2 l_2 + B_c l_c}{\mu}$$

$$\text{But } (\mu) = H$$

$$\therefore \text{for loop ABCA, MMF}(F) = H_1 l_1 + H_c l_c$$

$$\text{and for loop ADCA, MMF}(F) = H_2 l_2 + H_c l_c$$

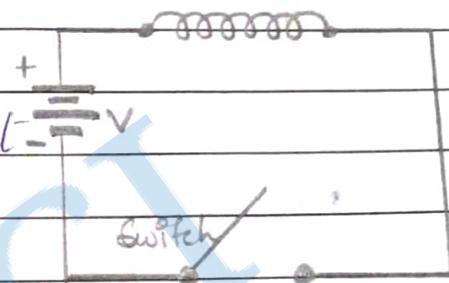
## \* Energy stored in a magnetic field

$$e = L \frac{di}{dt}$$

$$V = iR + L \frac{di}{dt}$$

Multiplying through out by  $i \cdot dt$

$$Vi \cdot dt = i^2 R dt + L i di$$



Energy absorbed by the magnetic field during time  $dt$

$$= L i di \text{ Joules}$$

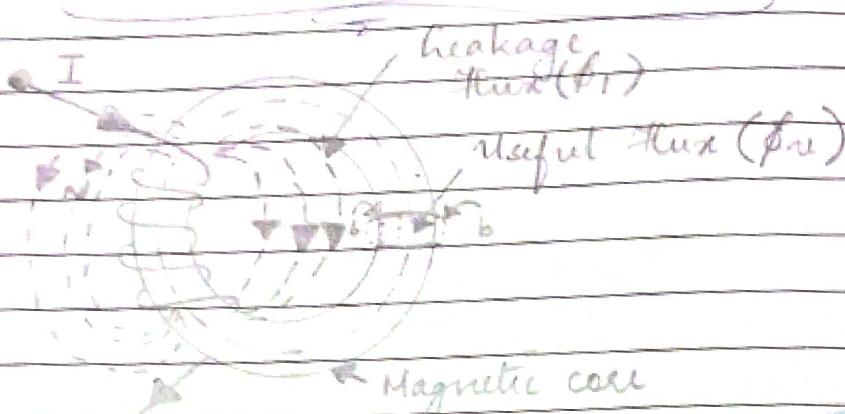
$$\text{Total energy} = \int_0^1 L i \cdot di$$

$$= L \int_0^1 i di$$

$$= L \left[ \frac{i^2}{2} \right]_0^1$$

$$= \frac{1}{2} L i^2$$

- Leakage and fringing in magnetic circuit



- Total flux = useful flux + leakage flux

- Leakage factor

$$\lambda = \frac{\text{total flux}}{\text{useful flux}}$$

- Force experienced by a current-carrying conductor in a magnetic field

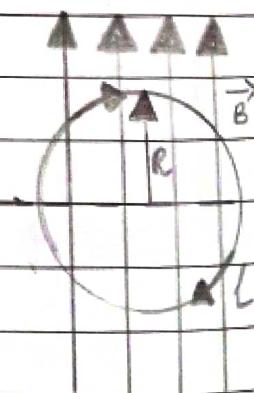
- $F = B i l$  (newton);  $F = B i l \sin \theta$

$B \rightarrow$  flux density

$i \rightarrow$  current through the conductor

(a)

$l \rightarrow$  conductor length



- (Q1) A circular iron ring having cross sectional area of  $20 \text{ cm}^2$  and length 30cm in iron has an airgap of 2mm made by saw cut.

Relative permeability of iron is 900. The ring is wound with a coil of 2500 turns and current in the coil is 3A. Determine air gap flux. Given leakage coefficient is 1.1.

$$A = 20 \text{ cm}^2$$

$$= 20 \times 10^{-4} \text{ m}$$

$$l = 30 \text{ cm}$$

$$= 30 \times 10^{-2} \text{ m}$$

$$l_a = 2 \text{ mm}$$

$$= 2 \times 10^{-3} \text{ m}$$

$$\mu_r = 900$$

$$n = 2500$$

$$I = 3 \text{ A}$$

$$\lambda = 1.1$$

$$\lambda = \frac{\text{total flux}}{\text{useful flux}}$$

$$\lambda = \frac{\phi_{\text{total}}}{\phi_g}$$

$$\phi_g = \frac{\phi_{\text{total}}}{\lambda}$$

$$NI = \phi_{\text{total}} \times S_{\text{total}}$$

$$S_{\text{total}} = S_I + S_A$$

$$= \frac{l_I}{\mu_0 \mu_r A} + \frac{l_A}{\mu_0 A}$$

$$= 30 \times 10^{-2} + 2 \times 10^{-3} \times 900$$

$$4\pi \times 10^{-7} \times 900 \times 20 \times 10^{-4}$$

$$= \frac{30 \times 10^{-2} + 18 \times 10^{-1}}{4 \times 9 \times 2 \times 3.14 \times 10^{-11}}$$

$$= \frac{0.30 + 1.8}{22608 \times 10^{-11}}$$

$$S = \frac{2.1 \times 10^{-11}}{22608}$$

~~So effective flux =  $S \times A$~~

$$\phi_{\text{total}} = NI$$

$N_{\text{total}}$

$$= \frac{2500 \times 3 \times 22608 \times 10^{-11}}{9.288 \times 10^{-16}} = 2.1$$

$$= \frac{2500 \times 3 \times 10^{16}}{9.288} = 804428571.4 \times 10^{-11}$$

$$\phi_g = \frac{\phi_{\text{total}}}{1.1}$$

$$= \frac{804428571.4 \times 10^{-11}}{1.1}$$

$$= 734025974 \times 10^{-16}$$

$$\phi_g = 7.340 \times 10^{-3} \text{ wb}$$

### \* Induced e.m.f.

- Two types :
- Dynamically induced e.m.f
- Statically induced e.m.f.
- Dynamically Induced emf

By moving a conductor in a uniform magnetic field and e.m.f produced in this way is known as dynamically induced e.m.f.

Area swept by conductor =  $l \cdot da$

Flux cut by the conductor = flux density  $\times$   
area  
=  $B \cdot l \cdot dx$  (Weber)

According to Faraday's law

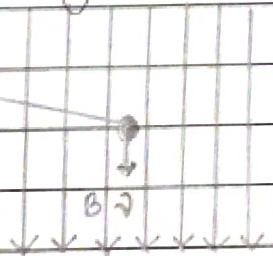
$e$  = ratio of change of flux linkage

$$= \frac{B l \cdot dx}{dt}$$

conductor

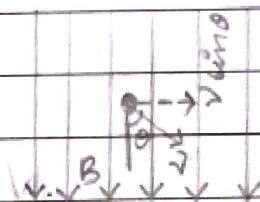
$$= BlV \text{ (volt)}$$

where  $V = \frac{dx}{dt} \Rightarrow$  velocity.



Dynamically induced e.m.f =  $BlV$  (volt)

$$e = BlVs \sin \theta \text{ (volt)}$$



\* Statically Induced e.m.f

By changing the magnetic flux

e.g. Transformer

Magnitude and direction can be changed

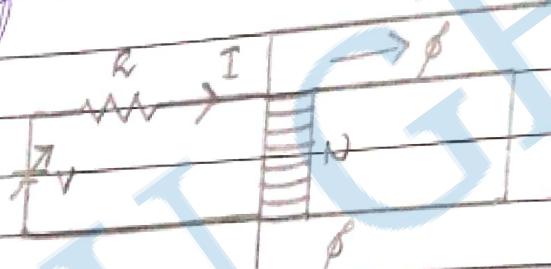
- Two types

- Self induced e.m.f

- Mutually induced e.m.f

- \* Self induced e.m.f

- Self-induced e.m.f is the e.m.f induced in the coil due to the change of its own flux linked with it.



- The property of coil, which opposes a change of current or flux through it, is called its self inductance,  $h$ .

- \* Self inductance of the coil

$$e = -N \frac{d\phi}{dt} \quad \text{--- (1)}$$

$$= -N \frac{d(N\Phi)}{dt}$$

$$= -N^2 \frac{d\Phi}{dt}$$

$$e = -L \frac{di}{dt} \quad \text{--- (2)} \quad L \text{ is the self inductance}$$

- Unit of inductance - Henry

$$L = -\frac{e}{i}$$

$$1 \text{ Henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second.}}$$

1 henry is the amount of inductance of a coil in which rate of change of current of one ampere induces an e.m.f. of one volt.

Comparing eqns ① and ②

$$N \frac{d\phi}{dt} = - L \frac{dI}{dt}$$

Integrating

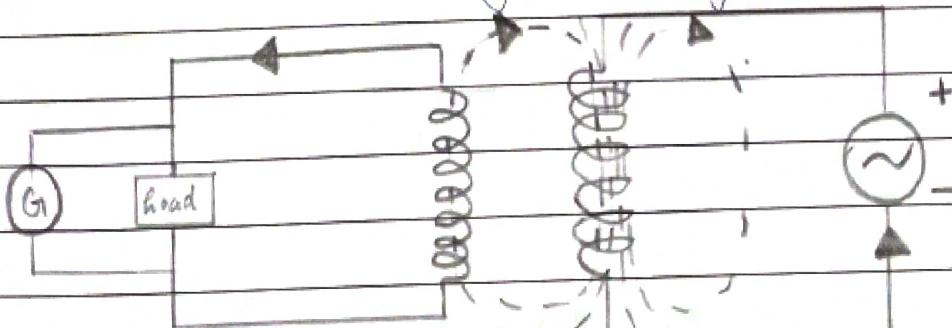
$$N\phi = L I$$

$$L = \frac{N\phi}{I}$$

Self inductance of the coil is the flux linkage per ampere.

Mutually induced e.m.f

Mutual induction : Generation of induced emf in a circuit by changing the current in the neighbouring circuit.





$$e_2 = -N_2 \frac{d\phi}{dt} \quad \text{--- (1)}$$

But  $\frac{d\phi}{dt} \propto \frac{dI_1}{dt}$

$$e_2 \propto -\frac{dI_1}{dt}$$

$$e_2 = -M \frac{dI_1}{dt} \quad \text{--- (2)}$$

Equating (1) and (2)

$$N_2 \frac{d\phi}{dt} = M \frac{dI_1}{dt}$$

Integrating,

$$N_2 \phi = MI_1$$

$$M = \frac{N_2 \phi}{I_1}$$

Similarly,

$$M = N_1 \phi$$

## Coefficient of coupling.

$$\phi_2 = k_1 \phi_1$$

$$H = \frac{N_2 \phi_2}{I_1} = N_2 k_1 \frac{\phi_1}{I_1} - ①$$

$\phi_2$  produced in second coil.

$$M = \frac{N_1 \phi_1}{I_2} = N_1 k_2 \frac{\phi_2}{I_2} - ②$$

Multiplying ① & ②

$$M^2 = \frac{N_2 k_1 \phi_1 \times N_1 k_2 \phi_2}{I_1 \times I_2}$$

$$M^2 = k_1 k_2 n_1 n_2$$

$$\{k = \sqrt{k_1 k_2}\}$$

$$k = \frac{M}{\sqrt{n_1 n_2}}$$

$\{k$  is called coefficient  
of coupling. $\}$

## Self Inductance of a solenoid

$l \rightarrow$  length of the solenoid

$N \rightarrow$  number of turns

$I \rightarrow$  current through the solenoid

$A \rightarrow$  Area of cross-section of the solenoid

$h \rightarrow$  Self inductance of the solenoid.

$$\text{Flux } (\phi) = \frac{\text{MMF}}{\text{Reluctance}}$$

$$\phi = \frac{N_2 I}{R / M_{max}} - ①$$

$$\text{But } h = N\phi$$

$$\phi = \frac{LI}{N} - \textcircled{2}$$

Comparing equations  $\textcircled{1}$  and  $\textcircled{2}$

$$\frac{hI}{N} = \frac{NI}{l/A(\text{molla})}$$

$$h = \frac{N^2 A(\text{molla})}{l}$$

$$\text{But reluctance } (S) = \frac{l}{A(\text{molla})}$$

$$\therefore h = \frac{N^2}{S}$$

Q) Derive the expression for effective inductance when 2 coils are connected in

- 1) Series
- 2) parallel

Ans) Q) Consider induced emf across each conductor

a) aiding

$$V = h_1 \frac{dI}{dt} - \textcircled{1}$$

$$V_1 = h_1 \frac{dI}{dt} + M \frac{dI}{dt} - \textcircled{2}$$

$$V_2 = h_2 \frac{dI}{dt} + M \frac{dI}{dt} - \textcircled{3}$$

Total voltage =  $V_1 + V_2$

$$(2) + (3) = h_1 \frac{dI}{dt} + M \frac{dI}{dt} + h_2 \frac{dI}{dt} + M \frac{dI}{dt}$$

$$N = (h_1 + h_2 + 2M) \frac{dI}{dt}$$

∴ From ①

$$h' = h_1 + h_2 + 2M$$

Effective inductance when 2 coils are connected in series.

b) opposing

$$V = h'' \frac{dI}{dt} - ①$$

$$N_1 = h_1 \frac{dI}{dt} - M \frac{dI}{dt} - ②$$

$$N_2 = h_2 \frac{dI}{dt} - M \frac{dI}{dt} - ③$$

$$\begin{aligned} (2) + (3) &= h_1 \frac{dI}{dt} + h_2 \frac{dI}{dt} - 2M \frac{dI}{dt} \\ &= (h_1 + h_2 - 2M) \frac{dI}{dt} \end{aligned}$$

∴ From ①

$$h'' = h_1 + h_2 - 2M$$

Effective inductance when 2 coils are connected in series.

iii) Parallel connection

$$V = h_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} - \textcircled{1}$$

$$V = h_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} - \textcircled{2}$$

From  $\textcircled{1}$

$$V - M \frac{dI_2}{dt} = \frac{dI_1}{dt} - \textcircled{3}$$

$h_1$

Sub  $\textcircled{3}$  in  $\textcircled{2}$

$$V = h_2 \frac{dI_2}{dt} + M \left( V - M \frac{dI_2}{dt} \right) \frac{h_1}{h_1}$$

$$V = h_1 h_2 \frac{dI_2}{dt} + M V - M^2 \frac{dI_2}{dt}$$

$h_1$

$$V h_1 - M V = (h_1 h_2 - M^2) \frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \frac{V(h_1 - M)}{h_1 h_2 - M^2} - \textcircled{4}$$

Sub  $\textcircled{4}$  in  $\textcircled{3}$

$$\frac{dI_1}{dt} = V - M \times \frac{V(h_1 - M)}{h_1 h_2 - M^2}$$

$h_1$

$$= \frac{V(h_1 h_2 - M^2) - M V h_1 + M^2 V}{h_1 (h_1 h_2 - M^2)}$$

$$= V h_1 h_2 - V M^2 - M V h_1 + M^2 V$$

$$= V h_1 h_2 - V M^2 - M V h_1 + M^2 V (h_1 h_2 - M^2)$$

$$= \frac{V B_1 (h_2 - M)}{B_1 (h_1 h_2 - M^2)}$$

$$\frac{dI_1}{dt} = \frac{V (h_2 - M)}{h_1 h_2 - M^2}$$

$$\therefore \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$= \frac{V (h_2 - M)}{h_1 h_2 - M^2} + \frac{V (h_1 - M)}{h_1 h_2 - M^2}$$

$$\frac{dI}{dt} = \frac{V (h_1 + h_2 - 2M)}{h_1 h_2 - M^2}$$

$$V = \frac{h_1 h_2 - M^2}{h_1 + h_2 - 2M} \frac{dI}{dt}$$

$$N = h_p \frac{dI}{dt}$$

$$h_p = \frac{h_1 h_2 - M^2}{h_1 + h_2 - 2M}$$

Effective inductance when 2 coils are connected in parallel.

- a) Two coupled coils connected in series have an equivalent inductance of 0.725H when aiding and 0.425H when opposing. Find self and mutual inductance when  $K = 0.42$ .

Ans)  $h' = 0.725H ; h'' = 0.425H$

$$K = 0.42$$

$$\underline{M} = 0.42$$

$$\sqrt{h_1 h_2}$$

$$0.725 = h_1 + h_2 + 2M \quad -\textcircled{1}$$

$$0.425 = h_1 + h_2 - 2M \quad -\textcircled{2}$$

$$\textcircled{1} + \textcircled{2}$$

$$0.575 = h_1 + h_2 \quad -\textcircled{3}$$

$$\textcircled{2} - \textcircled{1}$$

$$0.03 = 2M$$

$$M = 0.015$$

$\therefore$  Sub  $M = 0.015$  and  $\textcircled{3}$  in  $K$

$$0.42 = \frac{0.015}{\sqrt{(0.575 - h_2) h_2}}$$

$$h_2^2 - 0.575 h_2 + 0.03188 = 0$$

$$\Delta = b^2 - 4ac = \sqrt{(-0.575)^2 + 4(1)(0.03188)} \\ = \sqrt{0.203055} = 0.4506$$

$$h_2 = \frac{-b \pm \Delta}{2a}$$

$$h_2 = \frac{-(-0.575) + 0.4506}{2}; h_2 = \frac{0.575 - 0.4506}{2}$$

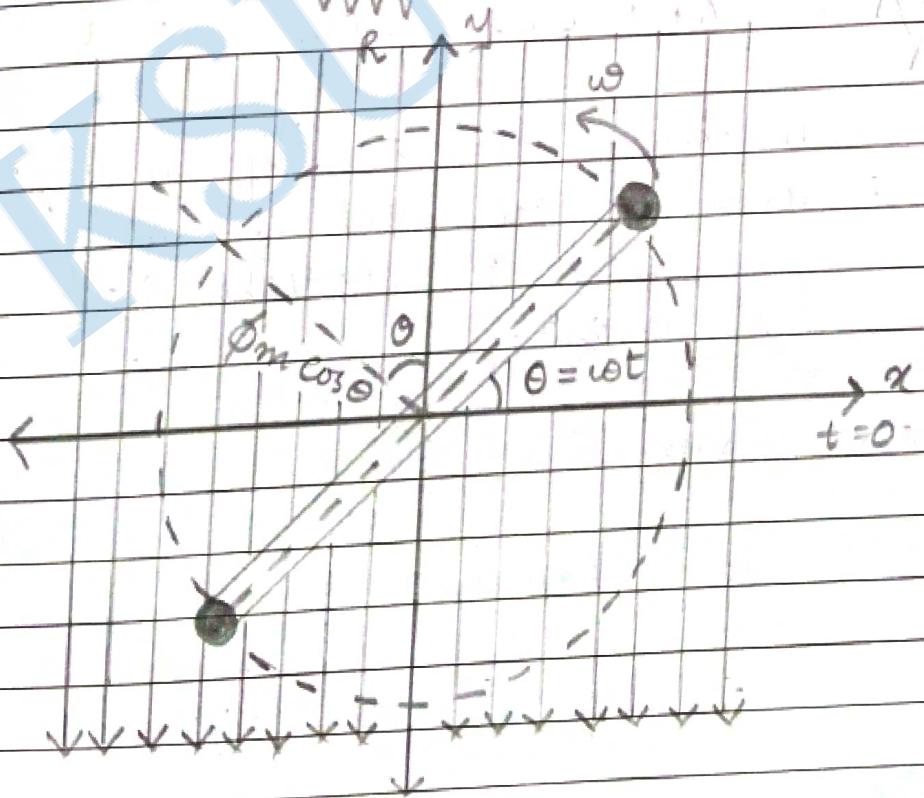
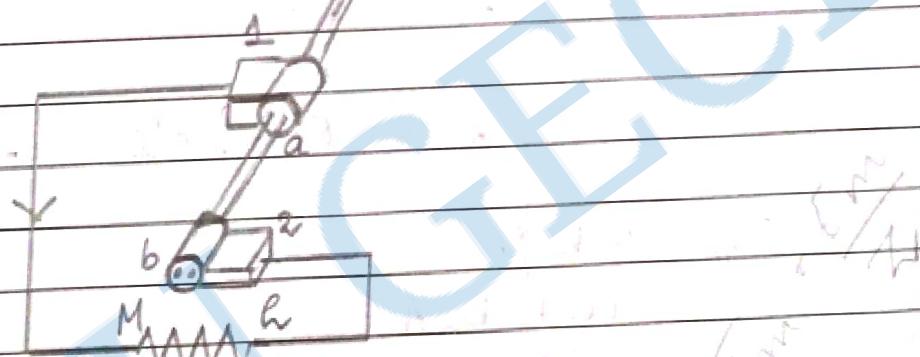
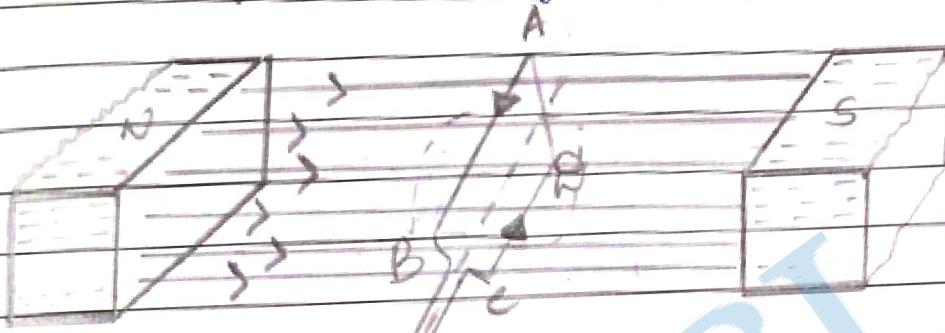
$$\therefore \text{when } h_2 = 0.5126; h_1 = 0.0624$$

$$\text{and when } h_2 = 0.0624; h_1 = 0.5126$$

$$M = 0.015$$

## Alternating E.m.f.

### Production of alternating emf.



$$\theta = \omega t - \Theta$$

$$\phi = \Phi \cos \omega t$$

$$N\phi = N\phi_m \cos \theta$$

{from ①}

$$e = -d(N\phi) / dt$$

$$= -d(N\phi_m \cos \theta) / dt$$

$$= -N d(\phi_m \cos \theta) / dt$$

$$= -N\phi_m \omega (-\sin \theta) \text{ volt}$$

$$= N\omega \phi_m \sin \theta$$

$$e = N\omega \phi_m \sin \theta \text{ volt.} \quad - \textcircled{2}$$

$$E_m = \omega N \phi_m$$

$$= \omega N B_m A$$

$$= 2\pi f N B_m A \text{ volt} \quad - \textcircled{3}$$

$$\left\{ \begin{array}{l} \omega = 2\pi f \\ \phi_m = B_m A \end{array} \right.$$

$$\left\{ \begin{array}{l} \phi_m = B_m A \\ \omega = 2\pi f \end{array} \right.$$

$B_m$ : maximum flux density in  $\text{Wb/m}^2$

$A$ : area of the coil in  $\text{m}^2$

$f$ : frequency of rotation of the coil in  $\text{rev/second.}$

∴ from ② & ③

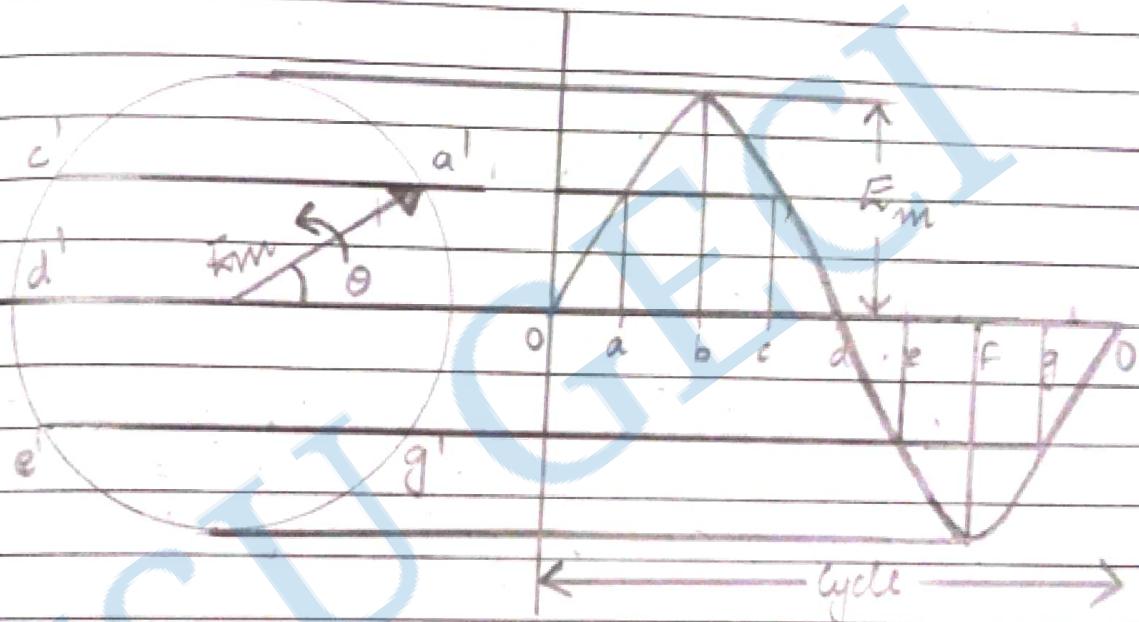
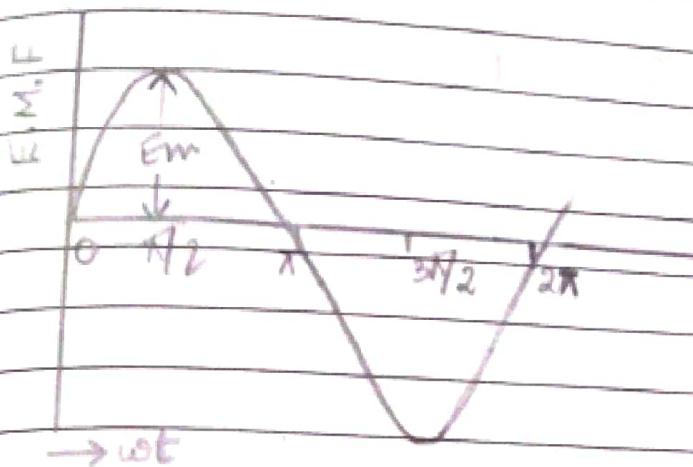
$$e = E_m \sin \theta$$

$$e = E_m \sin \omega t \quad \{ \text{from ①} \}$$

Now,

$$i = I_m \sin \omega t$$

$$\left\{ \begin{array}{l} I_m = \bar{I}_m \\ \omega = \frac{2\pi}{T} \end{array} \right.$$



\* Cycle

• One complete set of positive and negative values of alternating quantity is known as cycle.

• One complete cycle is said to spread over  $360^\circ$  or  $2\pi$

\* Time period

The time taken by an alternating

quantity to complete one cycle is called its time period. ( $T$ )

e.g. 50 Hz alternating current has a time period of  $\frac{1}{50}$  seconds.

\* Frequency

The number of cycles/second is known as frequency.

unit hertz ;  $F = \frac{1}{T}$

\* Amplitude

The maximum value, positive or negative, of an alternating quantity is known as amplitude.

\* Instantaneous value

It is the value at a particular instant.

\* Average value

It is the arithmetic mean of the ordinates at equal interval over a half cycle of a wave.

Methods :

D) Mid-ordinate method : graphical method  
2) Analytical method.

Mid-ordinate method

Analytical method

$$\text{avg - Area} = \frac{1}{\pi} \int_0^\pi i \, d\omega \quad i = I_m \sin \omega t$$

RMS Value (Root mean square value)

RMS Value  $\equiv$  That value of DC current which when flows through a given conductor produces same amount of heat as that produced by the alternating current passing through the same conductor for the same time.

$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{\pi} \int_0^\pi i^2 \, d\omega}$$

$$\text{RMS value of sine wave} = I_m$$

Average value of sine wave =  $\frac{2I_m}{\pi}$

\* Form Factor

Form factor =  $\frac{\text{RMS value}}{\text{Avg value}}$

\* Peak factor

Peak factor =  $\frac{\text{Max. value}}{\text{RMS value}}$

Form factor of sine wave =  $\frac{I_m/\sqrt{2}}{2I_m/\pi}$

$$= \frac{\pi}{2\sqrt{2}} \\ = 1.11$$

Peak factor of sine wave =  $\frac{I_m}{I_m/\sqrt{2}}$

$$= \frac{1}{\sqrt{2}}$$