MODULE - 1

Basic concepts regarding Electrical

1) charge - Electrical property of a substance
2) Current
3) Voltage
4) power
5) Energy

1) <u>Charge</u> (Q)

Electrical property of a substance.

unit coulomb (C) $\boxed{Q = It}$

2) <u>Current</u> (I)

It is the directed flow of free e^-s. Its direction is just opposite to the flow of e^-.

unit Ampere (A) $\boxed{I = q/t}$

3) <u>Voltage /emf / potential</u> (V)

workdone per charge (work/charge)

unit : J/C or Volt. $\boxed{V = w/q}$

A (+5) —— B (+3)

Pd b/n A & B

$V_{AB} = V_A - V_B$

$= 5 - 3 = 2V$

4) **power (P)**

Rate of doing work is called as power

$\boxed{P = VI}$

unit = J/sec or watts

5) **Energy (E)**

Capacity to do work

$\boxed{E = P \cdot t}$

unit = J / wh (watthour) / kwh

1unit = 1kwh.

Circuit

It is a closed path. It is the interconnection of Various circuit elements.

Circuit elements can be of 2 types

1) passive elements :- element that doesn't generate energy. Eg : Resistor, inductor & capacitor.
2) Active elements

↓

capable of generating energy

Eg: Battery, Generator etc.

RESISTOR

Resistance (R) :-

It is a property of a substance by which it opposes the flow of current

Unit = ohm (Ω)

Resistance of a material depends upon

1) length , $R \propto L$
2) Area , $R \propto 1/A$
3) Nature of the material
4) Temp.

$R \propto L/A$

$$\boxed{R = \rho L/A}$$

ρ = Resistivity / specific Resistance of the materia

$\rho = \frac{RA}{L} = \Omega m^2/m$

$$\boxed{\rho = \Omega m}$$

Conductance (G) :-

It is the reciprocal of Resistance

$$\boxed{G = 1/R}$$

Ohm's Law

It states that the ratio of p.d (V) applied ~~of~~ across the ends of the conductor to the current flowing through the conductor (I) is always a constant.

OR

ohm's law states that the p.d applied across the ends of ~~the~~ a conductor is directly proportional to the current flowing through it.

$$\frac{V}{I} = \text{a const} \Rightarrow \frac{V}{I} = R$$

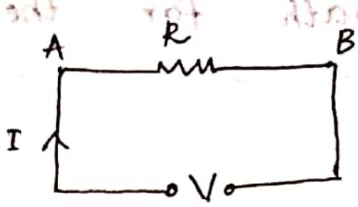
$$V \propto I$$

$$\boxed{V = IR}$$

$P = VI$
$P = I^2R$
$P = \frac{V^2}{R}$

$$P = IR \cdot I = I^2R$$

$$P = V \cdot V/R = \frac{V^2}{R}$$



DC CIRCUIT

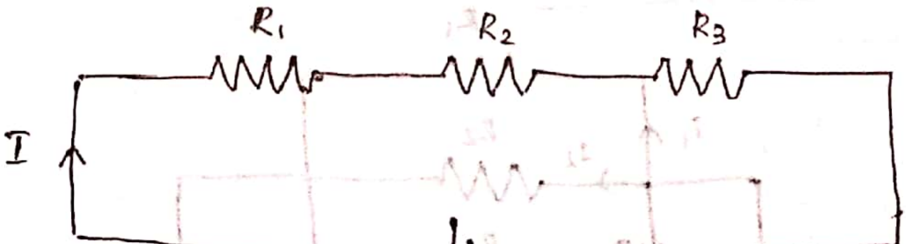
A circuit consist of the following components

1) Source of power
2) conductor for carrying current
3) load.

There are 2 types of DC circuit

1) DC series circuit
2) DC parallel circuit

Dc Series Circuit



In a Series circuit each resistance are connected end to end such that there is only one path for the current through the circuit.

Let V_1, V_2, & V_3 be the Voltages across R_1, R_2 & R_3 respectively. Then

$$V = V_1 + V_2 + V_3$$

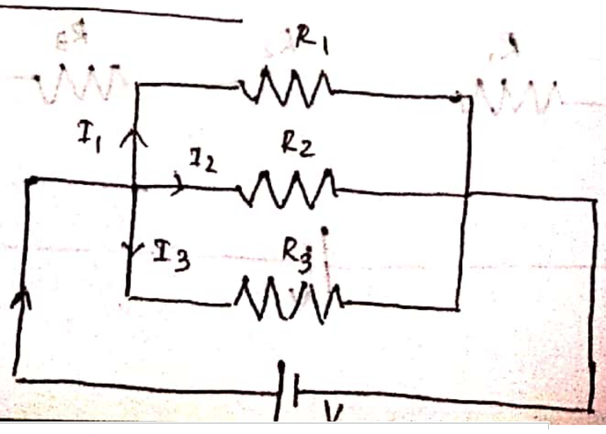
$$IR = IR_1 + IR_2 + IR_3$$

$$\boxed{R_s = R_1 + R_2 + R_3}$$

where R is the total or equivalent resistance of the Series Circuit.

$$\text{power } (P) = P_1 + P_2 + P_3$$

Dc parallel circuit



In ||el circuit one end of each resistor is connected to a common point & other end to another common point so, that current in the circuit will have different paths.

Voltage across each element is same in a parallel circuit.

Total current $(I) = I_1 + I_2 + I_3$

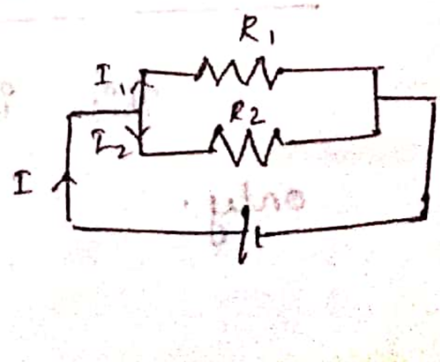
$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\boxed{\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

ie; Reciprocal of Total resistance in a ||el circuit equal to the sum of the reciprocals of individual resistances.

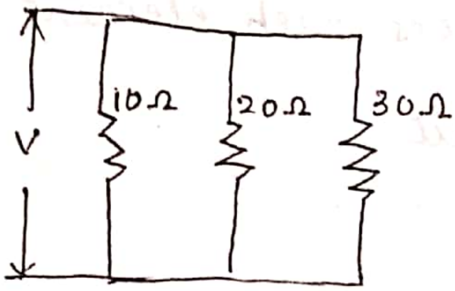
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$* \quad \boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$



$$\boxed{P = P_1 + P_2 + P_3}$$

* Find the equivalent resistance of the given ||el network



→

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{10} + \frac{1}{20} + \frac{1}{30}$$

$$= 0.1 + 0.05 + 0.0333$$

$$\frac{1}{R} = \cancel{0.18\,\Omega} \quad 0.1833$$

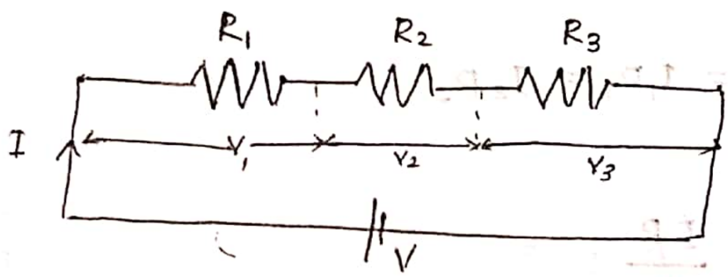
$$\therefore R = 5.45\cancel{45}\ \Omega$$

{Equivalent resistance is less than the smallest resistance in ||el circuit}

VOLTAGE DIVISION RULE

This is applicable in series circuits only.

if we have 2 resistances R_1 & R_2 Connected in series and V be the total Voltage applied across the combinatⁿ. Let I be the current flowing through the circuit. According to Voltage divider rule



$$V_1 = IR_1$$

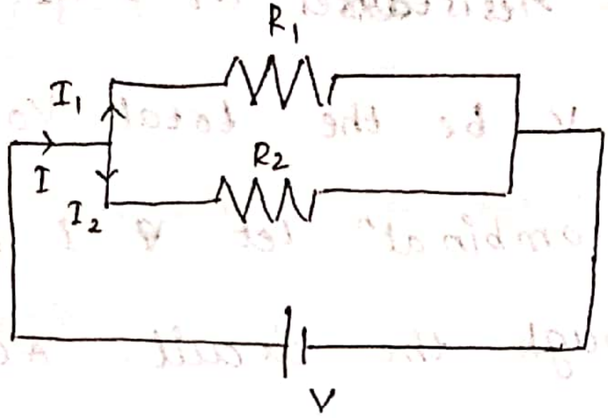
$$V_1 = \frac{V}{R} R_1$$

$$\boxed{V_1 = \left(\frac{V}{R_1+R_2}\right) R_1} = \frac{\text{Total Voltage}}{\text{Total Resist}} \times R_1$$

$$V_2 = \left(\frac{V}{R_1+R_2}\right) R_2$$

<u>CURRENT</u> <u>DIVISION RULE</u>

It is applicable only in ‖el circuit.



$$V = V_1 = V_2$$

$$IR = I_1R_1 = I_2R_2$$

$$I_1 = \frac{IR}{R_1}$$

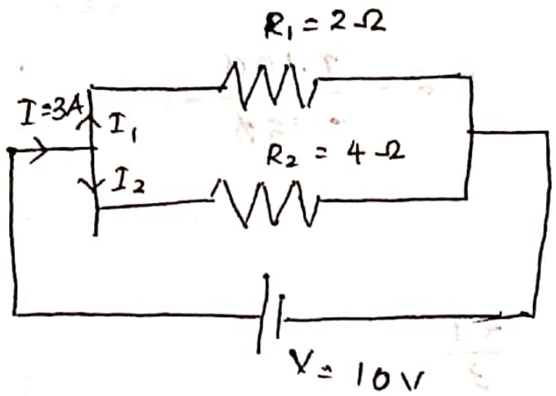
where R is the equivalent resistance of the ||el combination.

$$I_1 = \frac{I\,\frac{R_1R_2}{R_1+R_2}}{R_1} = \frac{I\,\cancel{R_1}R_2}{\cancel{R_1}(R_1+R_2)}$$

$$\boxed{I_1 = \frac{IR_2}{R_1+R_2}}$$

$$I_2 = \frac{IR_1}{R_1+R_2}$$

*



$I_1 = ?$

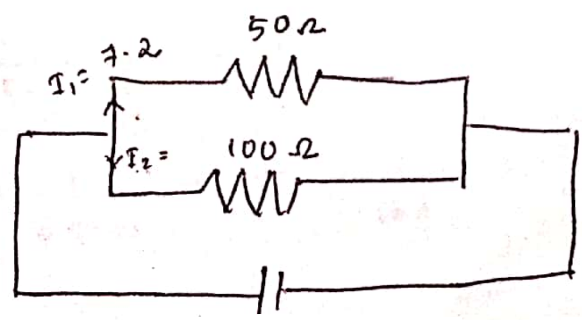
$I_2 = ?$

→ $I_1 = \dfrac{I R_2}{R_1 + R_2} = \dfrac{3 \times 4}{6} = \dfrac{12}{6} = \underline{\underline{2}}\ A$

$I_2 = \dfrac{I R_1}{R_1 + R_2} = \dfrac{3 \times 2}{6} = \dfrac{6}{6} = \underline{\underline{1}}\ A$

* A 50Ω resistor is in ||el with a 100Ω resistor. current in 50Ω is 7.2 A. what is the value of third resistance to be added in ||el to this circuit to make total current 12.1A

→



$\dfrac{5000}{150} = 33.3$

$12.1 - 10.8 = 1.3$

$$I_1 = \frac{I R_2}{R_1 + R_2} = \quad 7.2 = \frac{I \times 100}{150}$$

$$7.2 = \frac{\overset{2}{\cancel{10}} I}{\cancel{15} 3} = \frac{2I}{3}$$

$$\frac{21.6}{2} = I = \underline{\underline{10.8}} \text{ A.}$$

$$I_2 = \frac{I R_1}{R_1 + R_2} = \frac{10.8 \times 50}{150} = \frac{10.8}{3} = \underline{\underline{3.6 \text{ A}}}$$

$$I_3 = \frac{I R_{12}}{R_{12} + R_3} = \frac{12.1 \times \cancel{150}\, 33.3}{\underset{33.3}{\cancel{150}} + R_3} \qquad \& \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

$$= 33.3$$

$$8.5 = \frac{\cancel{1815}\; 402.93}{\underset{33.3}{\cancel{150}} + R_3}$$

$$\underset{33.3}{\cancel{150}} + R_3 = \frac{\overset{402.93}{\cancel{1815}}}{8.5} = 213.5$$

$$R_3 = \cancel{213.5} - \cancel{150}$$

$$47.40 - 33.3$$

$V = IR$

$V = V_1 = V_2 = V_3$

$V_1 = I_1 R_1$

$V = IR$

$R = V/I$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$I_R = \frac{IR_1}{R_1+R_2}$$

$$= \frac{12.1 \times 50}{50+100} = 4.034\ A$$

$I_1 = I_1 + I_2 + I_3$

$12.1 = 7.2 + 4.03 + I_3$

$= 0.87$

$$I_1 = \frac{IR}{R_1} \qquad 7.2 = \frac{12.1R}{50}$$

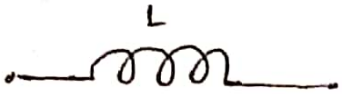
$$R = \frac{72 \times 5}{12.1} = 29.75$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_3} = \frac{1}{29.75} - \frac{1}{50} - \frac{1}{100} = \frac{3.61}{1000} = 277\Omega$$

INDUCTOR

A conductor is twisted to form a basic inductor. This usually denoted by the letter L



(mag field produce ചെയ്യുന്നു)

when a current 'I' flows through the inductor a mag field will be set up around that inductor

$$\therefore \boxed{\text{Inductance } 'L' = \frac{N\phi}{I}} \Rightarrow \text{Henry (unit)}$$

N = no. of turns of the coil

I = Current

ϕ = mag flux in webber

when current through the inductor ↑ses mag field expands & when the current ↓ses mag field will reduce.

ie; whenever there is a change in current, there will be a change in mag flux. According to Faraday's laws of electromagnetic Induction,

An emf is (voltage) is induced in the inductor. ie; Voltage across the inductor be proportional to $\frac{di}{dt}$

$$V \propto \frac{di}{dt}$$

$$\boxed{V = L\frac{di}{dt}} \quad \text{--- (1)}$$

⇒ <u>power absorbed in the Inductor</u>

$$P = Vi$$

$$= L\frac{di}{dt}i$$

$$\boxed{P = Li\frac{di}{dt}} \quad \text{--- (2)}$$

⇒ <u>Energy</u> ~~abo~~ (E)

$$E = Pt$$

$$E = \int_0^t P\,dt$$

$$E = \int_0^{i} L i \frac{di}{dt} \, dt$$

$$E = Li^2/2$$

$$\boxed{E = \frac{1}{2} Li^2}$$

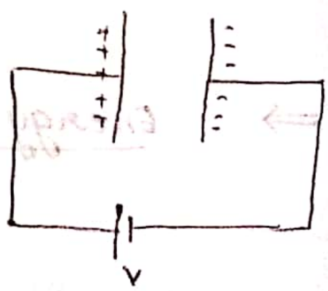
CAPACITOR

Two conducting plates are seperated by a dielectric medium ⇒ capacitor.

It is used to store electric charge.

Let Q be the charge stored on the capacitor & V be the potential applied across the capacitor

$$C = \frac{\text{charge}}{\text{voltage}} = \frac{Q}{V}$$



$\boxed{C = \frac{Q}{V}}$ ⇒ Faraday (unit)

⇒ Current through the capacitor

$$I = \frac{dQ}{dt}.$$

$$\boxed{I = C\frac{dV}{dt}}$$

$$C = \frac{Q}{V}$$

$$Q = CV$$

⇒ power

$$P = Vi$$

$$= \frac{Q}{C} i \quad V C \frac{dV}{dt}$$

$$\boxed{P = CV\frac{dV}{dt}}$$

⇒ Energy

$$E = Pt$$

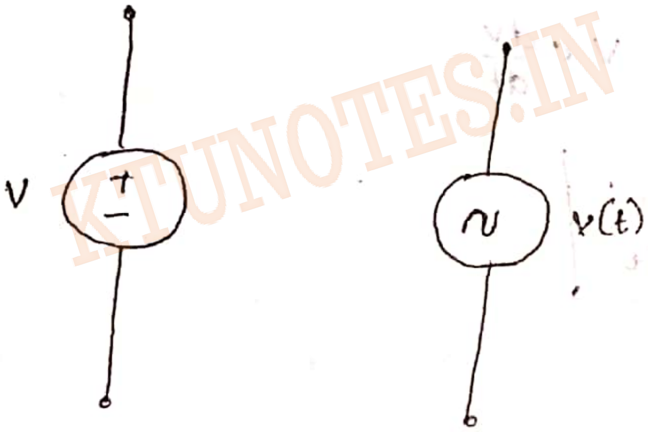
$$= \int_0^V Pt \; = \int_0^V CV\frac{dV}{dt}dt$$

$$= \frac{CV^2}{2} \; = \frac{1}{2}CV^2$$

$$\boxed{E = \frac{1}{2}CV^2}$$

CONSTANT VOLTAGE SOURCE (Ideal Voltage Source)

It is defined as the energy source whose terminal voltage is independent of the current through it. It means that when the current drawn the source varies from zero to infinity, the terminal voltage of the source remains unaffected. An Ideal Voltage source has zero internal resistance



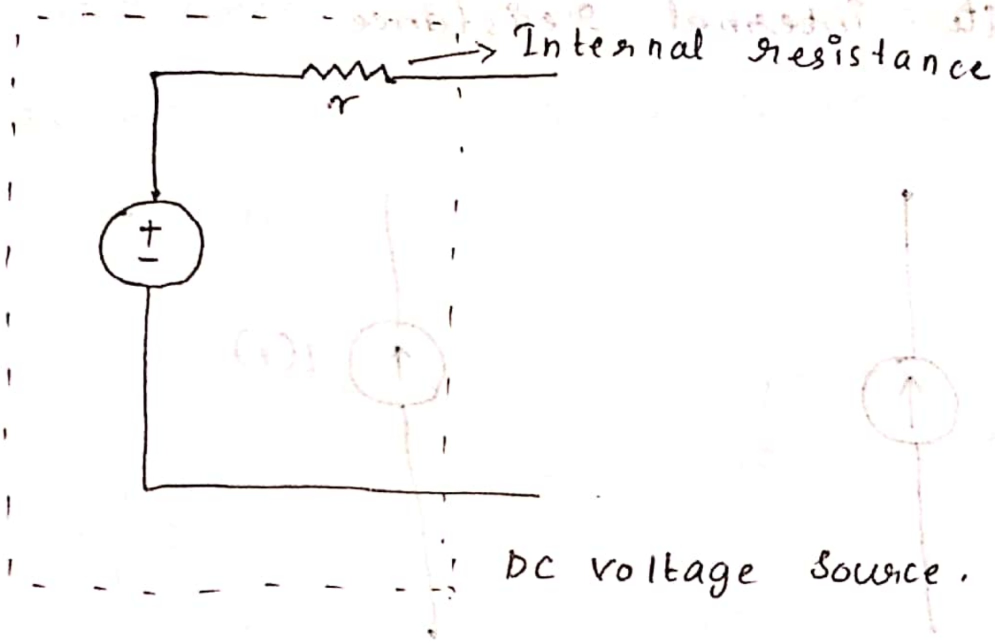
Const. dc Voltage source

Const. AC Voltage source.

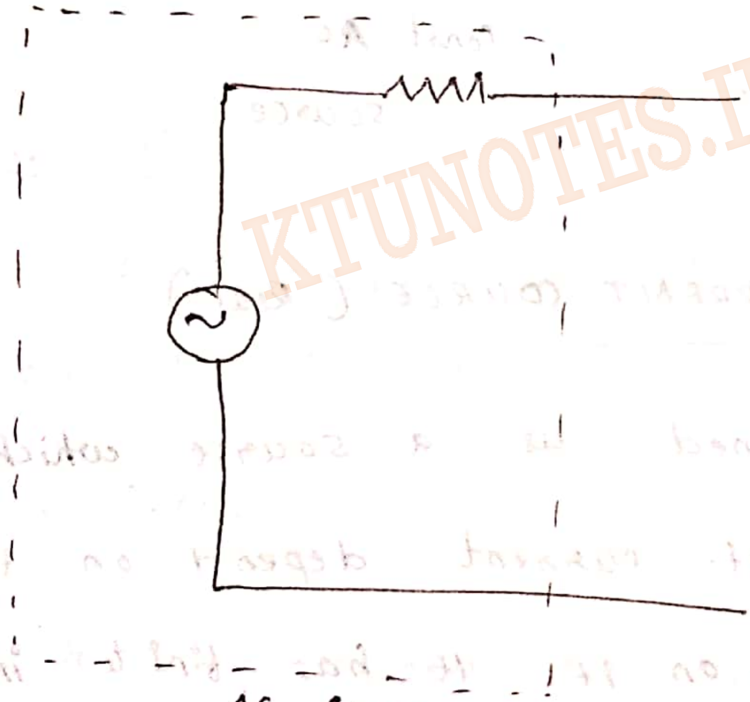
PRACTICAL VOLTAGE SOURCE (Real Voltage source)

It has low but finite internal resistance that causes its terminal voltage to $\downarrow$se when the load current is $\uparrow$sed &

vice versa. It can be represented by an ideal voltage source in series with a resistance.



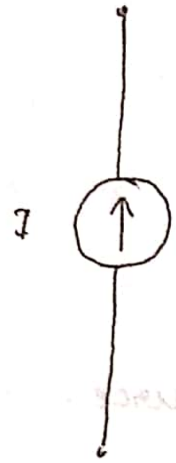
DC voltage source.



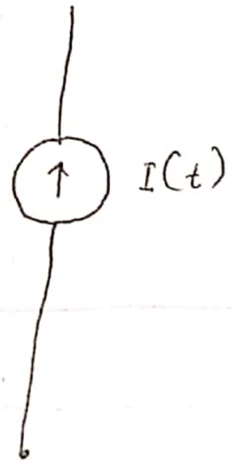
AC source

CONSTANT (IDEAL) CURRENT SOURCE

It is defined as a source which delivers constant current independent of the load placed on it. Thus the output current of such a source remains unaltered from zero to infinity load. It has infinite internal resistance



const. DC source

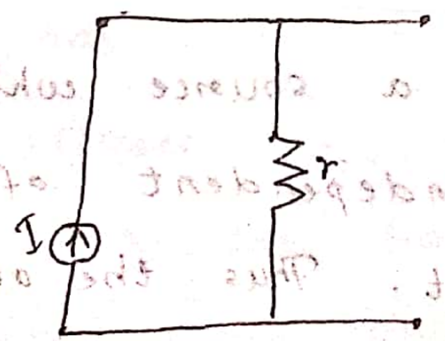


const. AC source.

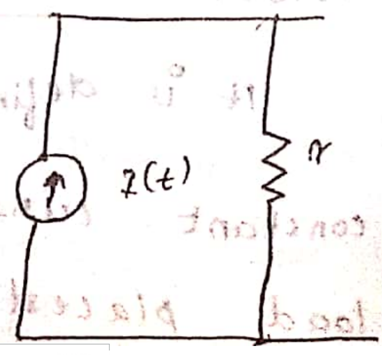
PRACTICAL CURRENT SOURCE (Real)

It defined as a source which delevers const. current depend on the load placed on it. It has finite internal resistance

const DC source

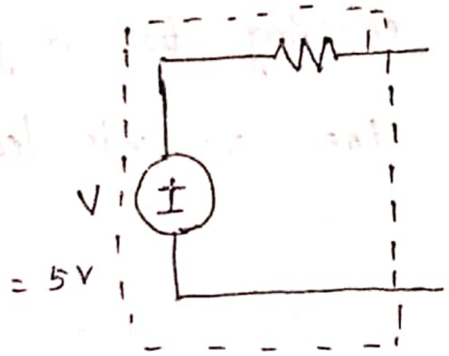


const ac source.

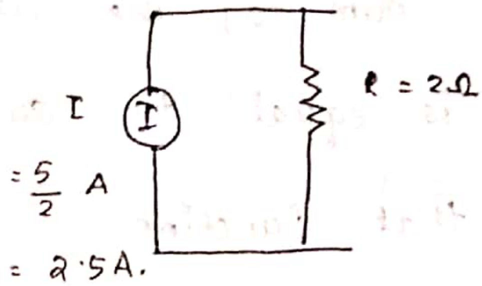


SOURCE CONVERSION

If we have a Voltage source in series with a resistance can be converted into an equivalent current source in ||el with a resistor and vice versa.



$\Rightarrow$



NODE (Junction)

It is a point in a circuit where two or more elements meet

LOOP (Mesh)

It is a closed path in a circuit

KIRCHOFF'S LAW

~~First~~ There are 2 law

First law : kirchoff's current law (KCL) or Junction Rule.

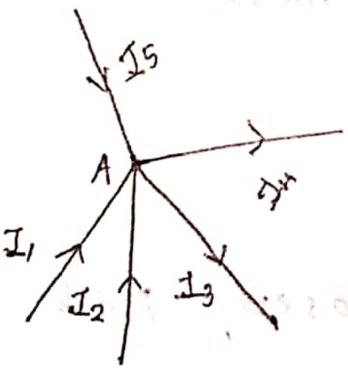
It is associated with current in a circuit.
The law states that the algebraic sum of the currents meeting at a point (node) is always zero.

OR

Sum of the currents entering to a junction is equal to sum of the currents leaving that junction.

The incoming currents can be taken as +ve & outgoing currents can be taken as -ve.

$$\boxed{\Sigma I = 0}$$



$$I_1 + I_2 - I_3 - I_4 + I_5 = 0$$

$$I_1 + I_2 + I_5 = I_3 + I_4$$

II^{nd} Law : Kirchoff's Voltage Law (KVL)

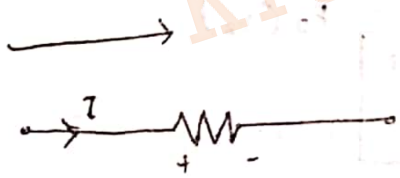
It is associated with Voltages in a loop

$$\boxed{\overbrace{\sum IR}^{\text{Voltage drop}} + \overbrace{\sum Emf}^{\text{Source}} = 0}$$

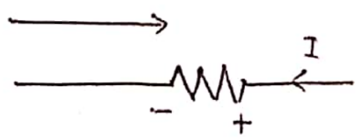
KVL states that algebraic sum of Voltage drop across the resistors plus algebraic sum of emf in a closed mesh (loop) is always Zero.

Sign Convention

(1)



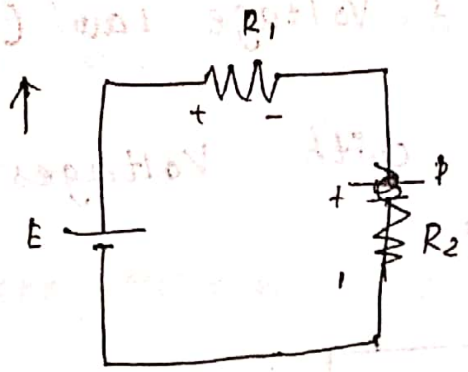
$+IR \Rightarrow$ Rise in potential.



(2)

$-IR \Rightarrow$ fall in potential.

③.

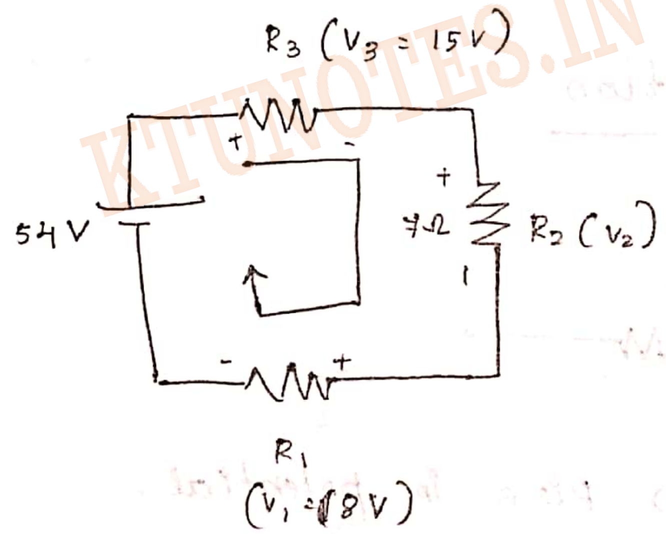


$$+IR_1 + IR_2 = E$$

$$IR_1 + IR_2 - E = 0$$

$$IR_1 + IR_2 = E$$

Q.



Determine,

① V_2 using KVL

② current

③ R_1 & R_3

$\rightarrow \quad IR_3 + IR_2 + IR_1 - E = 0$

$IR_3 + IR_2 + IR_1 = E$

$I(R_1 + R_2 + R_3) = 54V$

$18 + V_2 + 15 = 54V$

$V_2 = 54 - 15 - 18$

$= \underline{\underline{21V}}$

$15 + 18 = 33$

$54 - 33 = 21$

$IR_2 = 21V$

$I \times 7 = 21V$

$\underline{\underline{I = 3A}}$

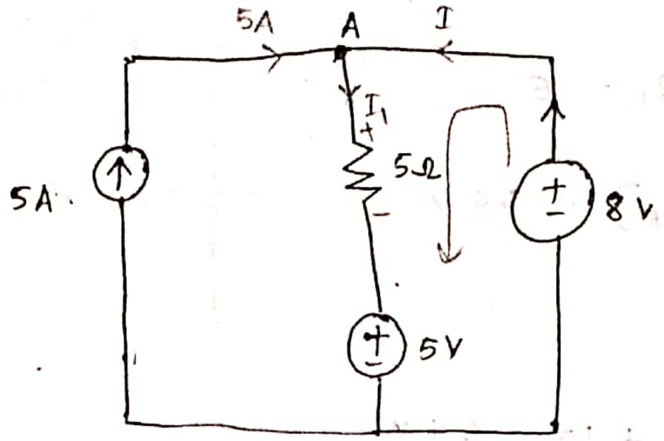
$IR_1 = 3 \times R_1$

$IR_1 = 18$

$R_1 = \frac{18}{3} = \underline{\underline{6}}\,\Omega$

$IR_3 = 15$

$R_3 = \frac{15}{3} = \underline{\underline{5}}\,\Omega$



Find I.

→ KCl at node A

$+5A + I - I_1 = 0$

$5 + I = I_1$ —— (1)

KVL to 2nd mesh

$5I_1 + 5 - 8 = 0$

$5I_1 = 3$

$I_1 = \frac{3}{5}$ —— (2)

(2) in (1)

$5 + I = \frac{3}{5}$

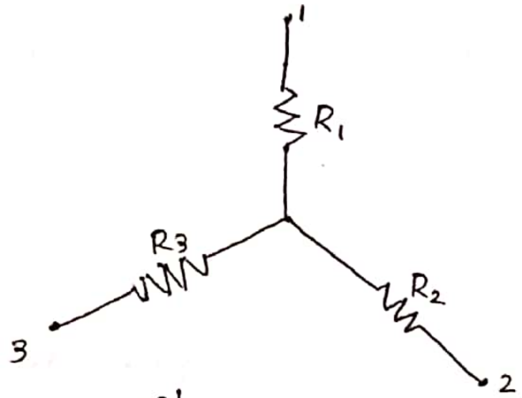
$\frac{3}{5} - 5 = I$

$\frac{3-25}{5} = I$

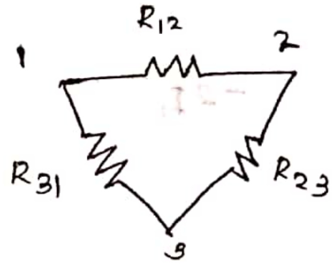
$= -\frac{22}{5} = \underline{-4.4\text{ A}}$ given direc^n is opposite to the actual direction.

Star to Delta & Delta to star conversion

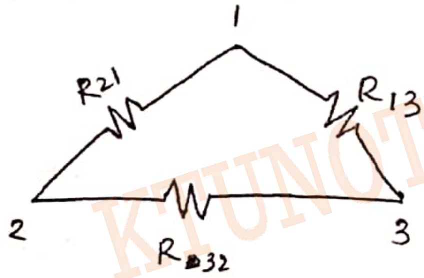
Y-Δ & Δ-Y



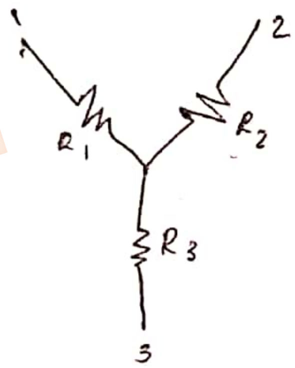
Star



Delta

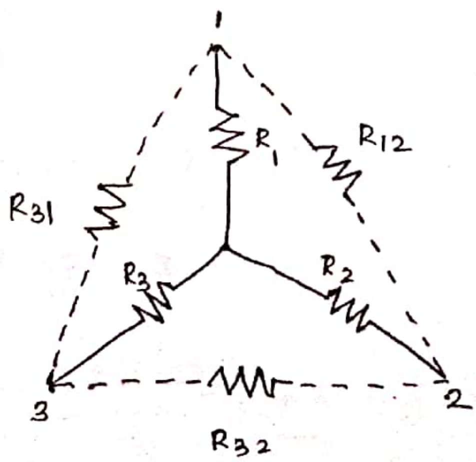


Delta



star

Star to Delta conversion

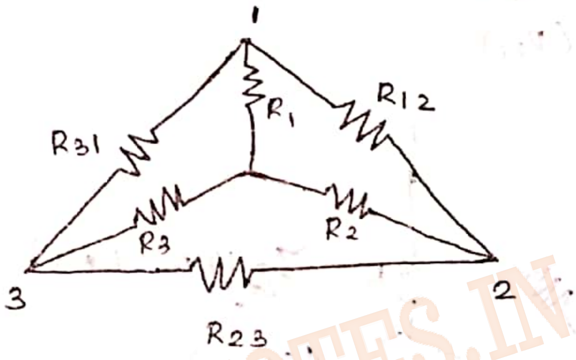


$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{32} = R_3 + R_2 + \frac{R_3 \cdot R_2}{R_1}$$

$$R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

Delta to star conversion

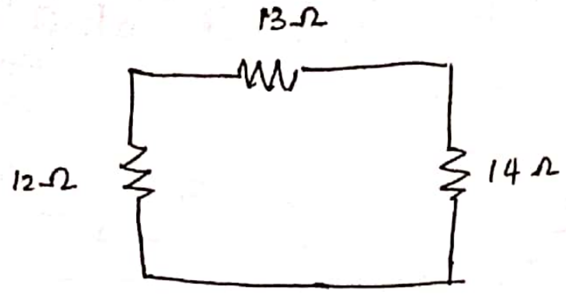


$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{23} + R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} + R_{31}}{R_{12} + R_{23} + R_{31}}$$

* obtain the star equivalent of Δ connected network as given.



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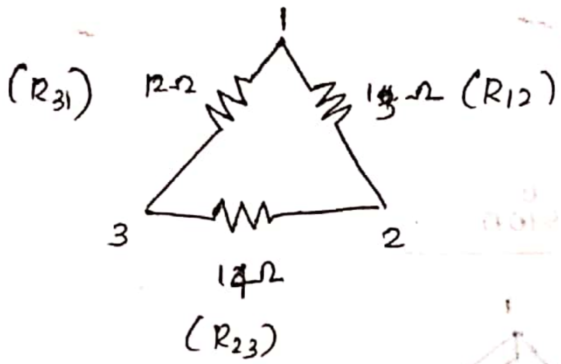
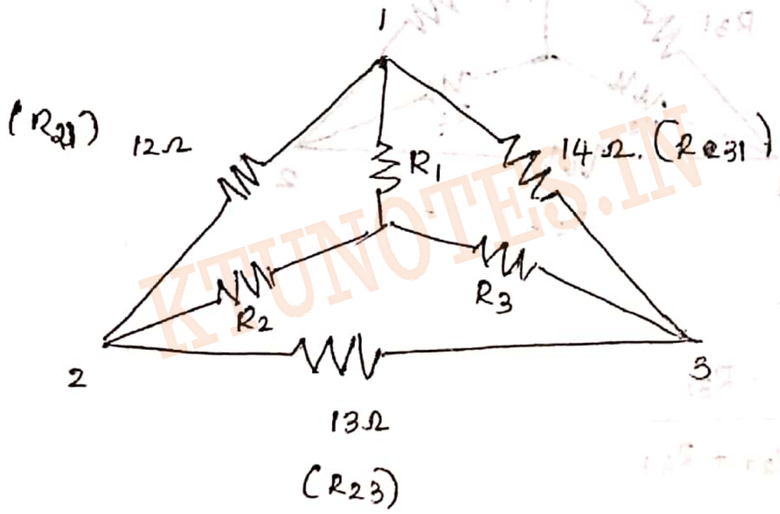


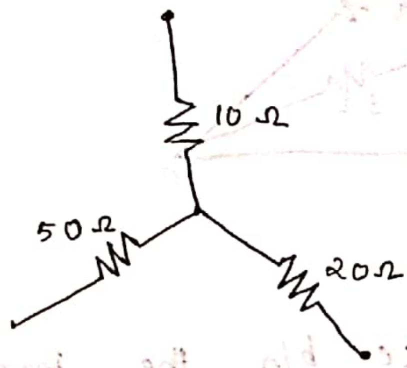


$$R_1 = \frac{R_{21} * R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{12 \times 14}{12 + 14 + 13} = \frac{168}{39} = 4.3$$

$$R_2 = \frac{R_{21} + R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{12 \times 13}{12 + 14 + 13} = \frac{156}{39} = 4$$

$$R_3 = \frac{R_{31} + R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{14 \times 13}{12 + 14 + 13} = \frac{182}{39} = 4.6$$

* obtain the Δ equivalent of the Y network



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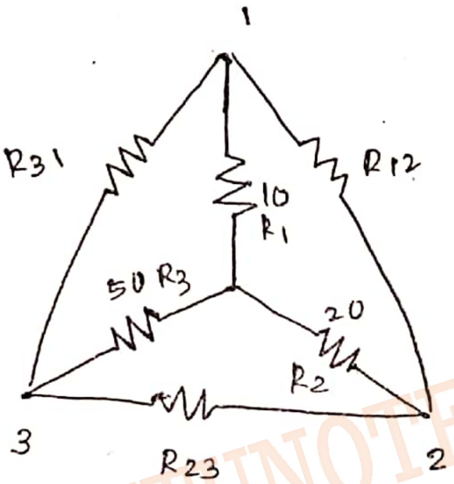


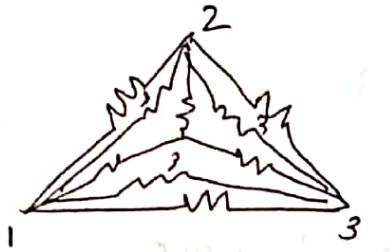
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} = \cancel{50 \times 10} \quad 50 + 10 + \frac{50 \times 10}{20}$$

$$= \underline{\underline{85\,\Omega}}$$

$$R_{12} = 10 + 20 + \frac{10 \times 20}{50} = \underline{\underline{34\,\Omega}}$$

$$R_{23} = 20 + 50 + \frac{20 \times 50}{10} = \underline{\underline{170\,\Omega}}$$

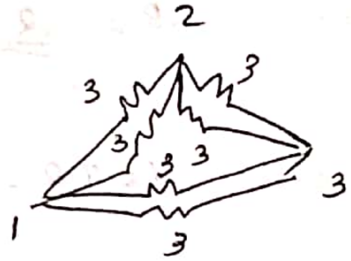
→



$$R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

$$= 1 + 1 + \frac{1}{1}$$

$$= \underline{\underline{3}}.$$



$$R_{32} = 3$$

$$R_{13} = 3.$$

$$\frac{1}{R_{12}} = \frac{3}{3} + \frac{3}{3} = \frac{6}{9}$$

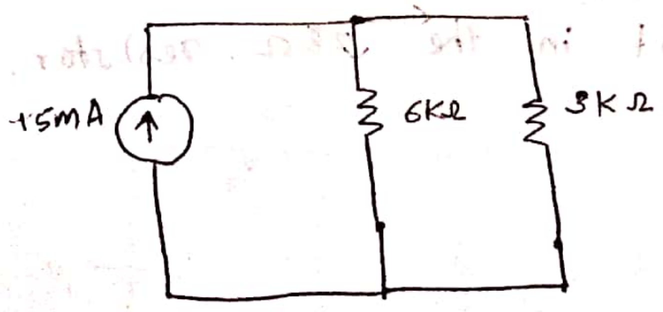
$$= \cancel{\frac{2+1}{2}} = \cancel{\frac{3}{2} = 1.5}$$

~~R_{32}~~

$\cancel{R_{12} = 0.67}$ $\quad R_{12} = \frac{9}{6} = \frac{3}{2} = \underline{\underline{1.5}}.$

$$R_{AB} = \frac{1.5 \times 3}{1.5 + 3} = \underline{\underline{1}}.$$

Q)



Find current in 6KΩ resistor.

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3}$$

$$= \frac{3+6}{18} = \frac{9}{18} = \frac{1}{2} = \underline{\underline{0.5}}$$

$$\therefore R = \underline{\underline{2}}$$

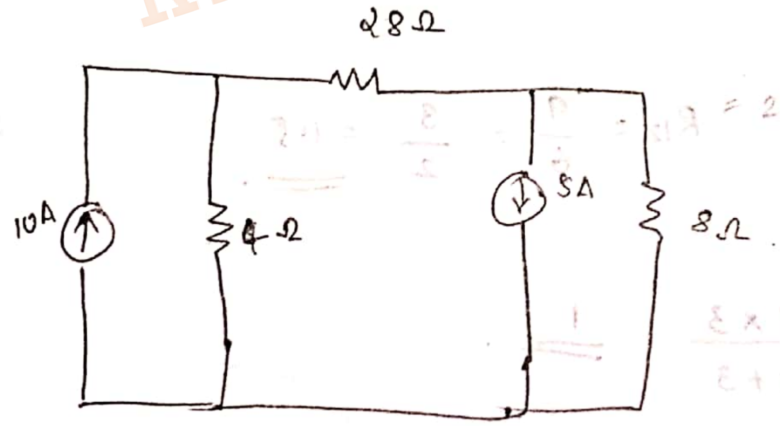
$$V = IR = 15 \times 10^{-3} \times 2$$

$$= \underline{\underline{30 \times 10^{-3}}}.$$

$$I = \frac{V}{R_1} = \frac{\overset{5}{\cancel{30}} \times 10^{-3}}{\cancel{6}}$$

$$= \underline{\underline{5\,mA}}.$$

9)



Find the current in the 28Ω resistor.

$V_1 \, / \, \frac{I}{R} = \frac{105}{42} = \frac{5}{2}$

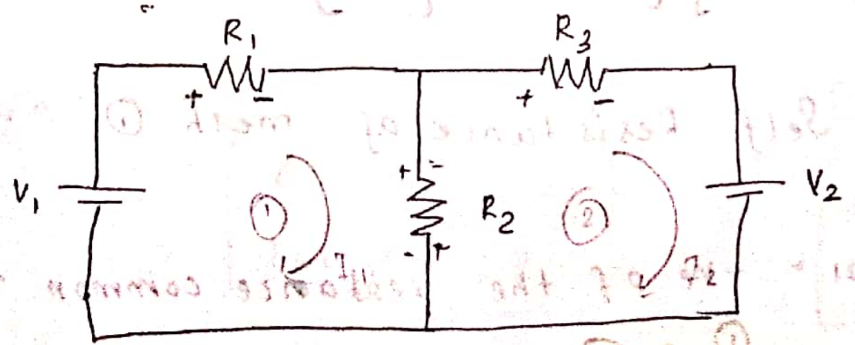
$V_1 = I_1 R_1 = 10 \times 4 = 40V$

$V_2 = I_2 R_2 = 5 \times 8 = 40V$

MESH CURRENT METHOD (Maxwell's loop current method)

This method is based on KVL. In this method 1st of all we have to identify the meshes. Then current in each mesh is assigned in the clockwise direction.

Eg:



Apply KVL in mesh ①

$$I_1R_1 + I_1R_2 - I_2R_2 - V_1 = 0$$

$$I_1(R_1+R_2) - I_2R_2 = V_1 \quad \text{——— (1)}$$

Apply KVL in mesh ②

$$I_2R_2 + I_2R_3 - I_1R_2 + V_2 = 0$$

$$-I_1R_2 + I_2(R_2+R_3) + V_2 = 0$$

$$-I_1R_2 + I_2(R_2+R_3) = -V_2 \quad \text{——— (2)}$$

$$\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} +V_1 \\ -V_2 \end{bmatrix}$$

In general $[R][I] = [V]$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

R_{11} = Self Resistance of mesh ① (sum of all resistance in mesh ①)

$R_{12} = R_{21}$ = -ve of the resistance common to meshes ① & ②

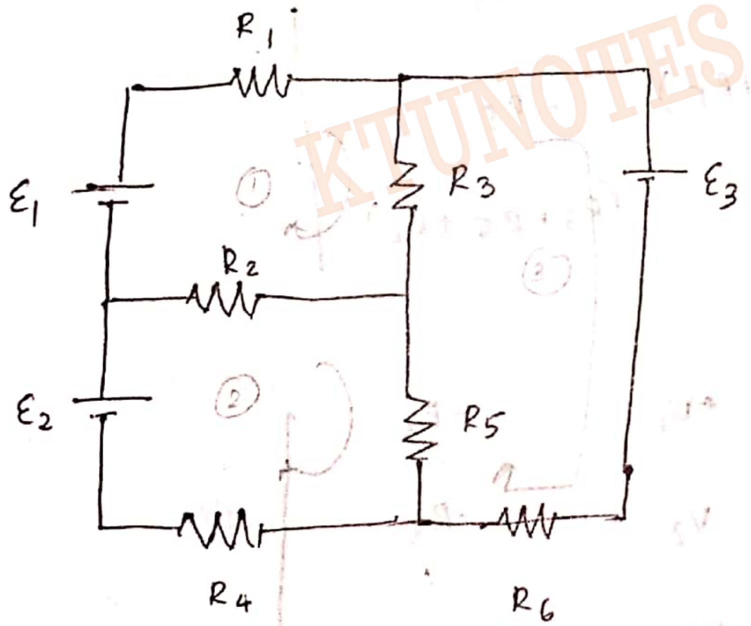
R_{22} = self-resistance of mesh ②

$$I_1 = \frac{\Delta_1}{\Delta} \; ; \; I_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix} \qquad \Delta_1 = \begin{vmatrix} V_1 & R_{12} \\ V_2 & R_{22} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} R_{11} & V_1 \\ R_{21} & V_2 \end{vmatrix}$$

*



→

$$[R][I] = [V]$$

$$[R] = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \quad I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad V = \begin{bmatrix} V_1 \\ V_2 \\ +V_3 \end{bmatrix}$$

$$\begin{bmatrix} (R_1+R_2+R_3) & -R_2 & -R_3 \\ -R_2 & (R_2+R_4+R_5) & -R_5 \\ -R_3 & -R_5 & (R_3+R_5+R_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

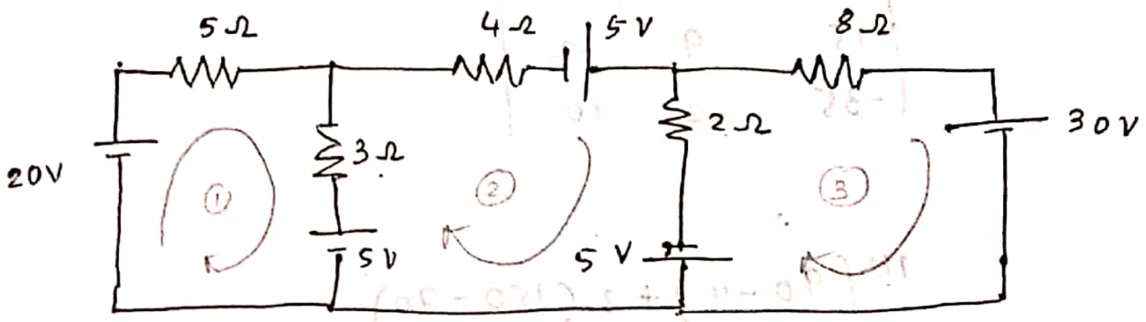
$$\Delta = \begin{vmatrix} (R_1+R_2+R_3) & -R_2 & -R_3 \\ -R_2 & (R_2+R_4+R_5) & -R_5 \\ -R_3 & -R_5 & (R_3+R_5+R_6) \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} V_1 & -R_2 & -R_3 \\ V_2 & (R_2+R_4+R_5) & -R_5 \\ -V_3 & -R_5 & (R_3+R_5+R_6) \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} R_1+R_2+R_3 & V_1 & -R_3 \\ -R_2 & V_2 & -R_5 \\ -R_3 & -V_3 & R_3+R_5+R_6 \end{vmatrix}$$

$$\Delta_3 \begin{vmatrix} R_1+R_2+R_3 & -R_2 & V_1 \\ -R_2 & R_2+R_4+R_5 & V_2 \\ -R_3 & -R_5 & -V_3 \end{vmatrix}$$

* Find currents in each mesh using mesh current method.



→ $[R][I] = [V]$

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{bmatrix} 5+3 & -3 & 0 \\ -3 & 3+4+2 & -2 \\ 0 & -2 & 2+8 \end{bmatrix}$$

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -5+20 \\ +5+5+5 \\ -30-5 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix} = 8(90-4) + 3(-30+0)$$

$$= 8 \times 86 - 90 = 598$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix}$$

$= 15(90-4) + 3(150-70)$

$= 15 \times 86 + 3 \times 80$

$= 1530$

$I_1 = \frac{1530}{598} = \underline{\underline{2.5}}$

$$I_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} 8 & 15 & 0 \\ -3 & 9 & -2 \\ 0 & 2 & 10 \end{vmatrix}$$

$= 8(90+4) - 15(-30-0)$

$= 8 \times 86 + 450$

$= 1138$

$I_2 = \frac{1138}{598} = \underline{\underline{1.9}}$

$$\Delta_3 = \begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 9 \\ 0 & -2 & 2 \end{vmatrix}$$

$$= 8(18+18) + 3(-6+0) + 15(6+0)$$

$$= -18 + 90$$

$$= \underline{\underline{72}}$$

$$I_3 = \frac{72}{598} = \underline{\underline{0.12}}$$

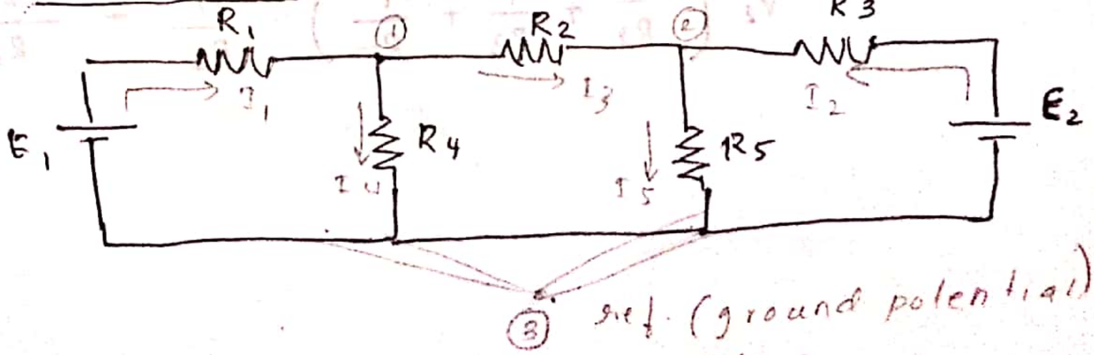
NODE VOLTAGE METHOD (Nodal Analysis)

This method is based on KCL

1. Identifying the nodes, one of the nodes can be taken as the reference node which is at ground potential.

2. if there are 'n' no. of nodes then we have to solve $(n-1)$ equations.

Eg: Analysis with Voltage source



ref. (ground potential) = zero

$n = 3$

$n-1 = 3-2 = 2$ eq^n are need to solve.

KCL at node ① $\Rightarrow I_1 - I_3 - I_4 = 0$

$$\left(\frac{E_1 - V_1}{R_1}\right) - \left(\frac{V_1 - V_2}{R_2}\right) - \frac{V_1}{R_4} = 0$$

$$\frac{E_1}{R_1} - \frac{V_1}{R_1} - \frac{V_1}{R_2} + \frac{V_2}{R_2} - \frac{V_1}{R_4} = 0$$

$$-V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) + \frac{V_2}{R_2} = -\frac{E_1}{R_1}$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_2}{R_2} = \frac{E_1}{R_1} \quad \text{——} \ ①$$

KCL at node ② $\Rightarrow I_3 + I_2 - I_5 = 0$

$$\frac{V_1 - V_2}{R_2} + \frac{E_2 - V_2}{R_3} - \frac{V_2}{R_5} = 0$$

$$\frac{V_1}{R_2} - \frac{V_2}{R_2} + \frac{E_2}{R_3} - \frac{V_2}{R_3} - \frac{V_2}{R_5} = 0$$

$$-V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) + \frac{V_1}{R_2} = -\frac{E_2}{R_3}$$

$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_1}{R_2} = \frac{E_2}{R_3} \quad \text{——} \ ②$$

From ① & ② =>

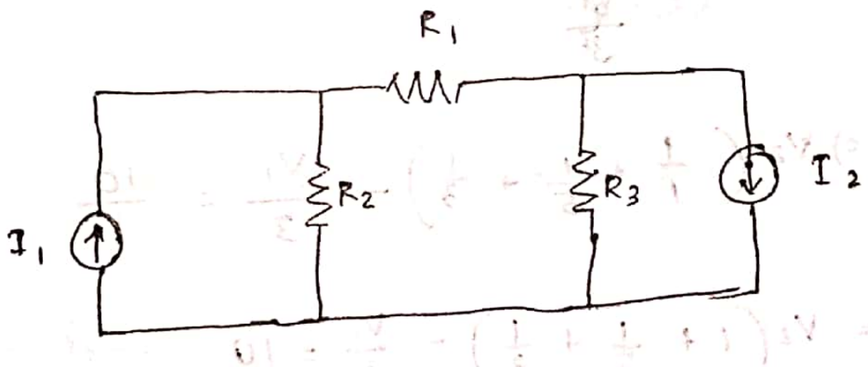
$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right) & \frac{-1}{R_2} \\ \frac{-1}{R_2} & \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} E_1/R_1 \\ E_2/R_2 \end{bmatrix}$$

$$V_1 = \Delta 1/\Delta$$

$$V_2 = \Delta 2/\Delta$$

$$\Delta = \begin{vmatrix} \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right) & \frac{-1}{R_2} \\ \frac{-1}{R_2} & \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right) \end{vmatrix}$$

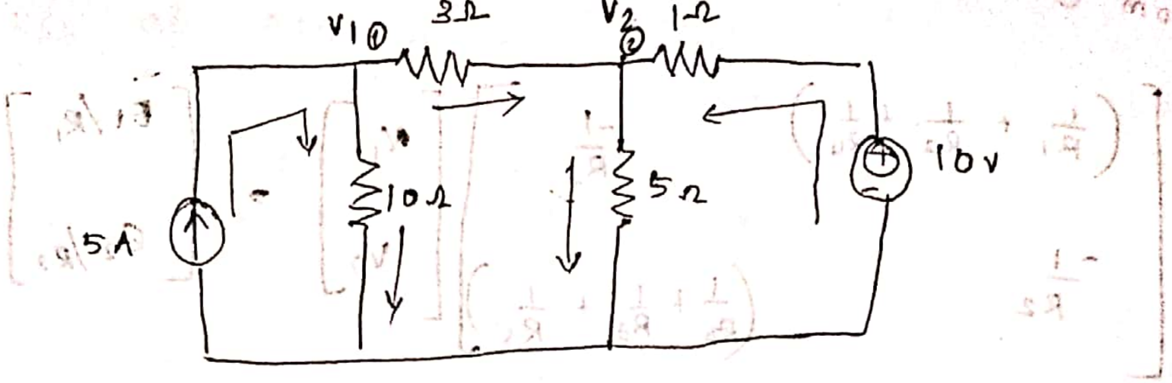
Analysis with current sources



Node ① => $V_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \frac{V_2}{R_1} = I_1$

Node ② => $V_2\left(\frac{1}{R_1}+\frac{1}{R_3}\right) - \frac{V_1}{R_1} = I_2$

Q)



Write the node voltage eq^{ns} and determine the current in each branch for the network shown.

→ Node ① ⇒ $V_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{V_2}{R_1} = I_1$

$V_1\left(\frac{1}{3}+\frac{1}{10}\right)-\frac{V_2}{3} = 5$

$= \frac{\cancel{4}}{3}$

Node ② ⇒ $V_2\left(\frac{1}{1}+\frac{1}{5}+\frac{1}{3}\right)-\frac{V_1}{3} = \frac{10}{1}$

$= V_2\left(1+\frac{1}{5}+\frac{1}{3}\right)-\frac{V_1}{3} = 10$

$= V_2\,\frac{23}{15}-\frac{V_1}{3} = 10$

$$\begin{bmatrix} +\frac{13}{30} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{23}{15} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\frac{23}{15}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\Delta = 0.05633 = \frac{83}{150}$$

$$\Delta_1 = \begin{vmatrix} 5 & -1/3 \\ 10 & 23/15 \end{vmatrix} = 11$$

$$\Delta_2 = \begin{vmatrix} 13/30 & 5 \\ -1/3 & 10 \end{vmatrix} = 6$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{11}{83/150} = \frac{11}{0.55} = \cancel{20}\ 19.87V$$

$$V_2 = \frac{6}{0.55} = 10.9V$$

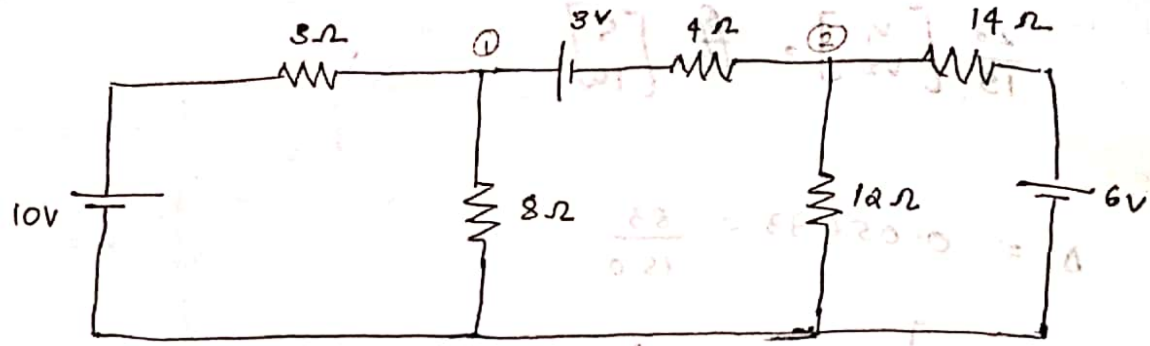
$$I_1 = \frac{V_1}{10} = \frac{19.87}{10} = 1.987 A$$

$$I_2 = \frac{V_1 - V_2}{3} \quad \frac{19.87 - 10.84}{3} = 3.01 A$$

$$I_3 = \frac{V_2 - 0}{5} = \frac{10.84}{5} = 2.168 A$$

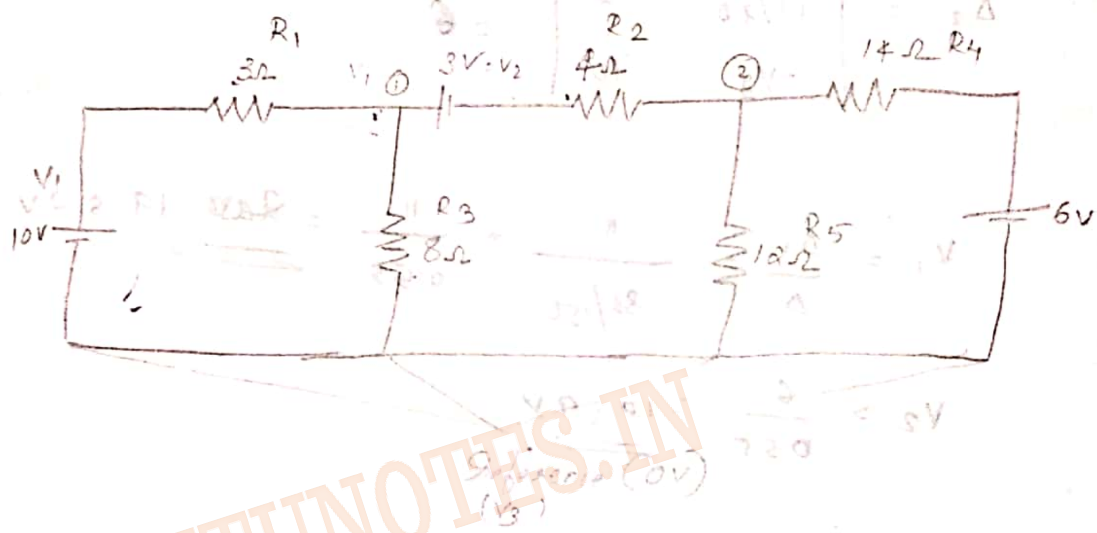
$$I_4 = \frac{10 - V_2}{10} = \frac{10 - 10.84}{10} = -0.087 A$$

*



Find current through 8Ω & 12Ω resistor.

→



$n = 3$

$n-1 = 3-2 = 2\ eq^n$

Node ① $\Rightarrow V_1\left(\frac{1}{3} + \frac{1}{8} + \frac{1}{4}\right) - \frac{V_2}{4} = \frac{10}{3}$

Node ② $\Rightarrow V_2\left(\frac{1}{4} + \frac{1}{12} + \frac{1}{14}\right) - \frac{V_1}{4} = \frac{6}{12}$

$$\begin{bmatrix} \frac{1}{3} + \frac{1}{8} + \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} + \frac{1}{12} + \frac{1}{14} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{10}{3} + \frac{3}{4} \\ \frac{6}{14} - \frac{3}{4} \end{bmatrix}$$

$$\begin{bmatrix} 0.708 & -0.25 \\ -0.25 & 0.404 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.33+\frac{3}{4} \\ 0.42-\frac{3}{4} \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0.708 & -0.25 \\ -0.25 & 0.404 \end{vmatrix} = \begin{vmatrix} 17/24 & -1/4 \\ -1/4 & 17/42 \end{vmatrix}$$

$(0.708 \times 0.404) - (-0.25 + 0.25)$
$= 0.286032$

$$= \frac{47}{336} = 0.224$$

$$\Delta_1 = \begin{vmatrix} 3.33+\frac{3}{4} & -1/4 \\ 0.42-\frac{3}{4} & 17/42 \end{vmatrix} = \quad \cancel{0.793} \quad 1.57$$

$(4.08 \times 0.404) - (-0.25 + 0.3\ldots$
$= \cancel{1.6548} = 0.0952$

$$\Delta_2 = \begin{vmatrix} 17/24 & 10/3+\frac{3}{4} \\ -1/4 & 6/14-\frac{3}{4} \end{vmatrix} = \frac{1585}{1008} = \cancel{1.57}\ \frac{533}{672} = 0.7$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{0.793}{\frac{47}{336}} = 5.67$$

$$V_2 \quad \frac{\Delta_2}{\Delta} = \frac{1.57}{0.139} = 11.22$$

$$I_1 = \frac{V_1}{\quad}$$

$$V_1 = \frac{1.57}{0.139} = 7.01$$

$$V_2 = \frac{0.793}{0.124} = 3.5\,A\,V$$

$$I_1 = \frac{V_1}{8} = \frac{7.01}{8} = 0.876\text{ A}$$

$$I_2 = \frac{V_2}{12} = \frac{3.54}{12} = 0.2946\text{ A}$$

Q Find power in each resistor