



KTU
NOTES
The learning companion.

**KTU STUDY MATERIALS | SYLLABUS | LIVE
NOTIFICATIONS | SOLVED QUESTION PAPERS**

30/10/2017

MODULE IV

DYNAMICS

Dynamics deals with the motion of bodies under the action of forces. It has two distinct parts - kinematics and kinetics.

Equations of Kinematics

$$S = ut + \frac{1}{2}at^2$$

$$V^2 = u^2 + 2as$$

$$V = u + at$$

Kinetics:

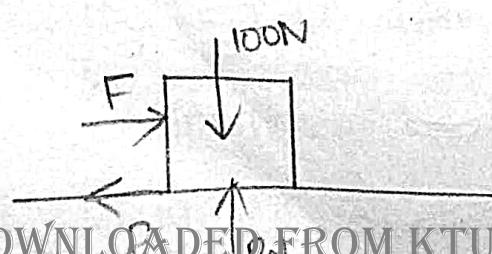
Kinetics is the study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body.

Equations of motion (D'Alembert Principle/Newton's II law)

$$F = ma$$

$$F - ma = 0$$

- Q: A block weighing 100 N rests on a horizontal plane. Find the magnitude of force required to give the box an acceleration of 2.5 m/s^2 . The coefficient of kinetic friction between the block and the plane is 0.25.



~~object will move up~~ $a = 2.5 \text{ m/s}^2$ ~~object moves up towards~~

$$R_N = 100 \text{ N}$$

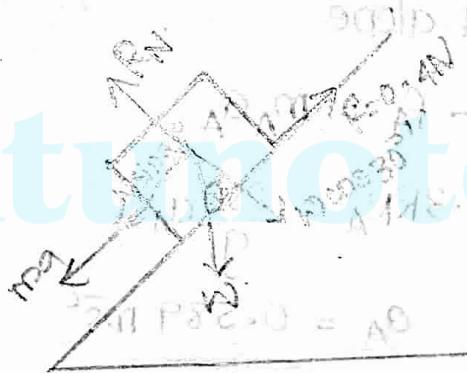
$$F - f = ma$$

$$F - \mu R_N = ma$$

$$F - 0.25 \times 100 = \frac{100}{9.81} \times 2.5$$

$$\therefore F = 50.1 \text{ N}$$

Q: A body of mass 50kg slides down a rough inclined plane inclined 30° to horizontal. Coefficient of friction between plane and body is 0.4. Determine acceleration of the body.



$$-f + w \sin 30^\circ = ma$$

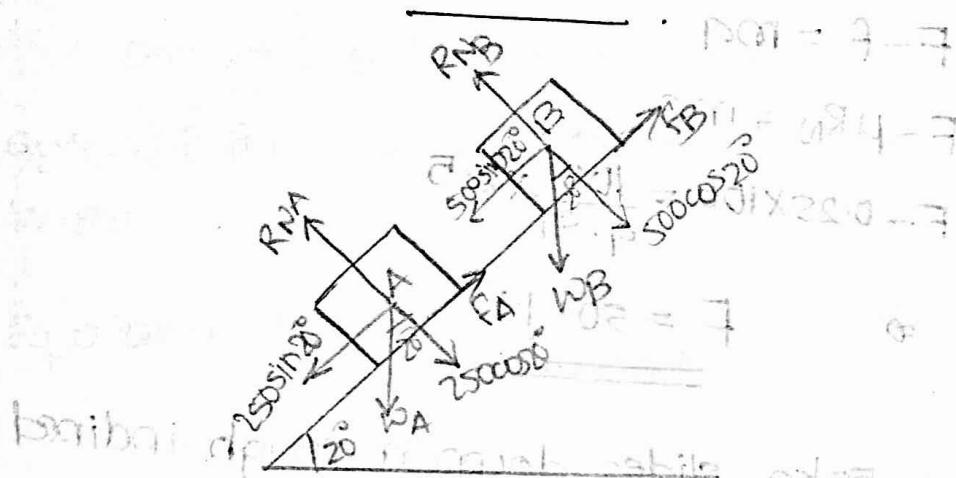
$$-0.4 \times 50 \times 9.81 \cos 30^\circ + 50 \times 9.81 \sin 30^\circ = 50 a$$

$$a = \frac{75.336}{50}$$

Q: Two bodies A and B weighing 250N and 500N respectively are held stationary 10m apart and plane on a 20° inclined plane. Coefficient of friction between A and plane is 0.3 while it is 0.2 between B and plane. If they are released simultaneously, calculate the time taken and the distance

travelled by each block before they are at the verge of collision.

$$100 = 4t$$



Required condition for motion of block A to prevent it from sliding down the incline

$$R_{NA} = 250 \cos 20^\circ = 234.923 \text{ N}$$

$$R_{NB} = 500 \cos 20^\circ = 469.846 \text{ N}$$

Consider motion of A alone

$$250 \sin 20^\circ - f_A = m_A a_A$$

$$250 \sin 20^\circ - 0.3 R_{NA} = \frac{250}{9} a_A$$

$$a_A = 0.589 \text{ m/s}^2$$

Consider motion of B alone

$$500 \sin 20^\circ - 0.2 R_{NB} = \frac{500}{9} a_B$$

$$a_B = 1.51 \text{ m/s}^2$$

Let 'x' be the distance travelled by A in 't' seconds.

Distance travelled by B in 't' sec is $(x+10)$.

$$s = ut + \frac{1}{2}at^2$$

$$x = \frac{1}{2} \times 0.589 t^2$$

$$x+10 = \frac{1}{2} \times 1.51 \times t^2$$

$$\frac{x}{x+10} = \frac{0.589}{1.51}$$

$$1.51x = 0.589x + 5.89$$

$$0.921x = 5.89$$

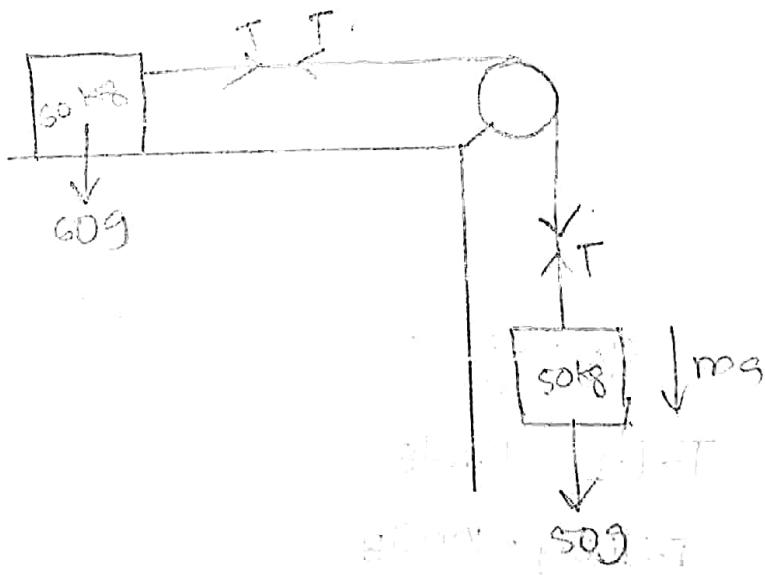
$$x = 6.4 \text{ m}$$

$$6.4 = \frac{1}{2} \times 0.589 t^2$$

$$t = \underline{\underline{4.66 \text{ s}}}$$

Q: A mass of 60kg lies on a smooth horizontal plane. It is connected to a fine string passing through a smooth pulley at the edge of table to a mass 50kg hanging freely. Find tension in the string and acceleration of the system.

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$$60g - T = ma$$

$$50g - T = ma = 50a$$

$$T = 50(g-a) \rightarrow \textcircled{1}$$

$$T = ma = 60a \rightarrow \textcircled{2}$$

Sub ② in ①,

$$\frac{F_{\text{ext}}}{100} = \frac{N}{M + m}$$

$$60a = 50g - 50a$$

$$110a = 50g$$

$$a = \frac{5g}{11} = 4.46 \text{ m/s}^2$$

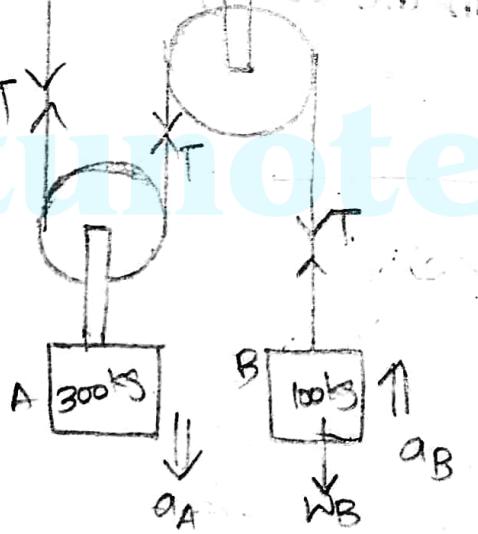
$$T = 60 \times 4.46$$

$$= \underline{\underline{267.55 \text{ N}}}$$

Q: Determine tension in string and acceleration of two bodies of 300 kg and 100 kg connected by a string (by a frictionless, smoothless pulley).

Two bodies A and B are suspended from a ceiling by a string passing over a pulley.

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$$a_A = \frac{a_B}{2}$$

$$T - w_B = m_B a_B$$

$$T - 100g = 100a_B$$

$$T = 100(g + a_B) \rightarrow ①$$

$$300g - 2T = m_A a_A$$

$$2T = 300g - 300a_A$$

$$T = 150(g - a_A) \rightarrow ②$$

$$10g \sin 30^\circ - T = ma_1 \Rightarrow 10a_1 = 10g \sin 30^\circ$$

$$T - 5g \sin 20^\circ = ma_2 \Rightarrow 5a_2 = 5g \sin 20^\circ$$

$$a_1 = a_2 = a_{12} = g_{12} = g_{02}$$

$$10g - T = 10a \Rightarrow 10a = 10g - T$$

$$T - 5g \sin 20^\circ = 5a$$

$$5g - 10a = 5a + 5g \sin 20^\circ$$

$$10a = 5g - 5g \sin 20^\circ$$

$$3a = g - g \sin 20^\circ$$

$$\underline{a = 2.15 \text{ m s}^{-2}}$$

$$T = 5g - 10a$$

$$= 27.55 \text{ N}$$

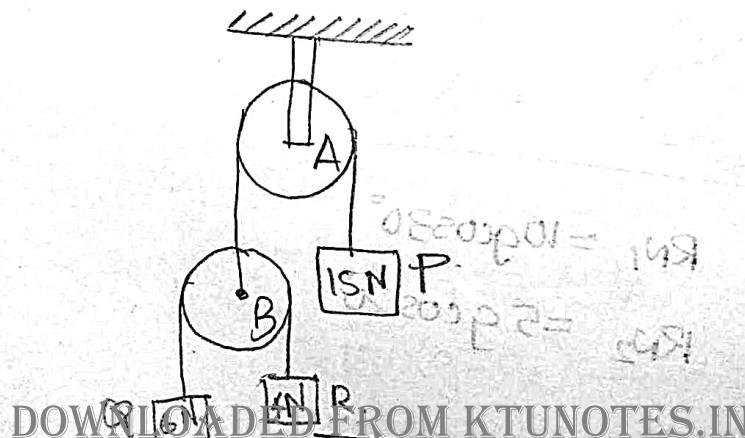
D'Alembert's Principle

It states that resultant of a system of force acting on a body is in dynamic equilibrium with inertia force.

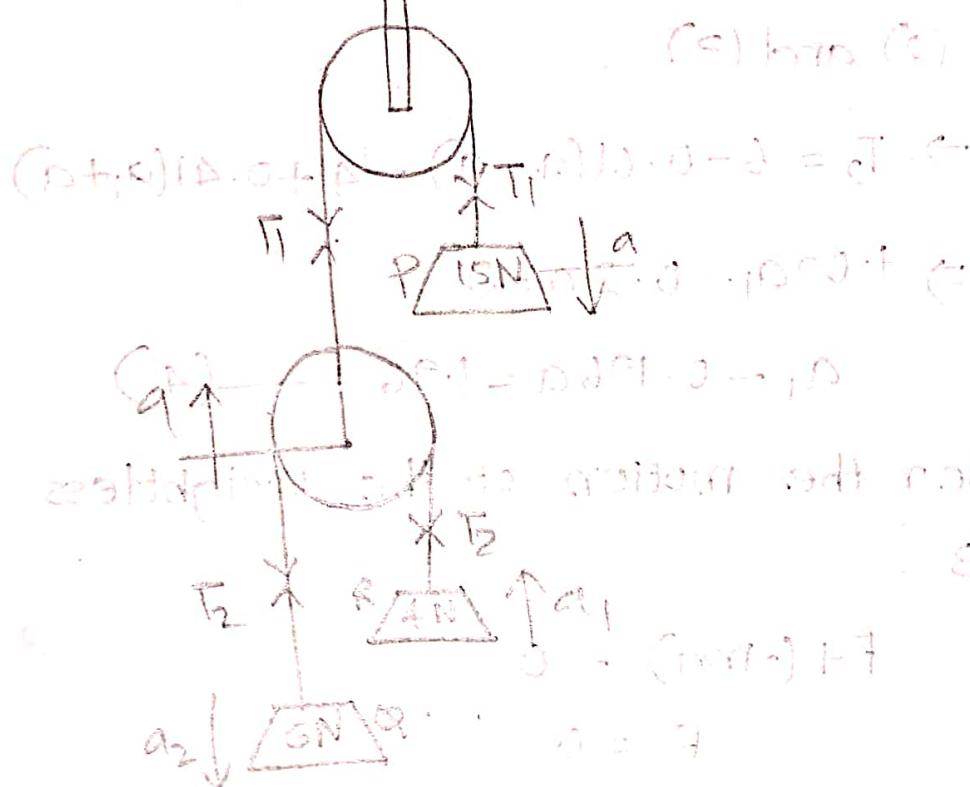
$$F = ma$$

$$F + (-ma) = 0$$

Q: Find the acceleration of weights P, Q and R using D'Alembert's Principle.



(e) ~~11. A pulley of mass m and radius r is attached to a string which hangs vertically over a pulley of negligible mass. A block of mass M hangs from the free end of the string. If the system is released from rest, find the tension in the string and the acceleration of the block.~~



Consider the downward motion of P.

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$$F + (-ma) = 0 \quad (1) \text{ (Ans)} \quad (1)$$

$$15 - T_1 - \frac{15}{9.81} \times a = 0$$

$$T_1 = 15 - 1.53a \quad (1)$$

(2) $\rightarrow 15 - T_1 - 2 \cdot 1 = 0$

consider the downward motion of Q.

$$15 - T_1 - F + (-ma) = 0$$

$$6 - T_2 - \frac{6}{9.81} \times (a_1 - a) = 0$$

$$(2) \rightarrow 6 - T_2 - 0.61(a_1 - a) = 0$$

$$T_2 = 6 - 0.61(a_1 - a) \quad (2) \quad \text{position}$$

Consider the upward motion of R.

$$F + (-ma) = 0$$

$$T_2 - 4 - \frac{4}{9.81} (a_1 + a) = 0$$

$$T_2 = 4 + 0.41(a_1 + a) \quad (3)$$

Eqn.s (2) and (3)

$$\Rightarrow T_2 = 6 - 0.61(a_1 - a) = 4 + 0.41(a_1 + a)$$

$$\Rightarrow 1.02a_1 - 0.2a = 2$$

$$a_1 - 0.196a = 1.96 \quad (4)$$

consider the motion of the weightless pulley B.

$$F + (-ma) = 0$$

$$F = 0$$

$$1.412T_2 - T_1 = 0$$

$$T_1 = 2T_2$$

From eqn.(1),

$$2T_2 = 15.8 - 1.53a$$

$$T_2 = 7.5 - 0.765a \quad (5)$$

Eqn.s. (2) and (5)

$$\Rightarrow 6 - 0.61(a_1 - a) = 7.5 - 0.765a$$

$$-a_1 + 2.25a = 2.46 \quad (6)$$

Adding (4) and (6)

$$2.054a = 4.42$$

$$a = 2.15 \text{ m/s}^2$$

From eqn. (4)

Ans

MOTION ALONG A

$$a_1 - 0.196a = 1.96$$

$$a_1 = 1.96 + 0.196 \times 2.15$$

$$= 2.38 \text{ m/s}^2$$

Acceleration of P = $a = 2.15 \text{ m/s}^2$

$$\text{Acceleration of Q} = a_1 - a = 2.38 - 2.15$$

$$= 0.23 \text{ m/s}^2$$

Acceleration of R = $a_1 + a$

$$= 2.38 + 2.15$$

$$r_1 = 14.53 \text{ m/s}^2$$

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6/11/2019

ANGULAR MOTION

Initial velocity ω_0

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

Final velocity ω

$$\omega = \omega_0 + \alpha t$$

Acceleration α

$$\alpha = \text{constant}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$a = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

RECTILINEAR MOTION

u

v

a

$$v = \frac{ds}{dt}$$

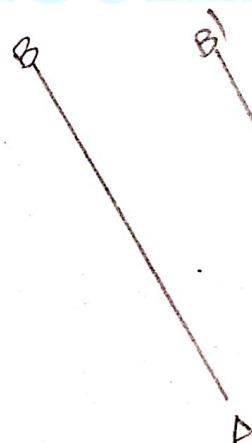
$$a = \frac{dv}{dt}$$

$$v = u + at$$

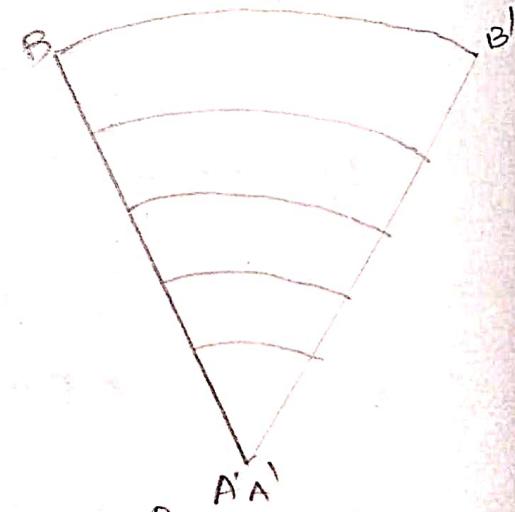
$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2} at^2$$

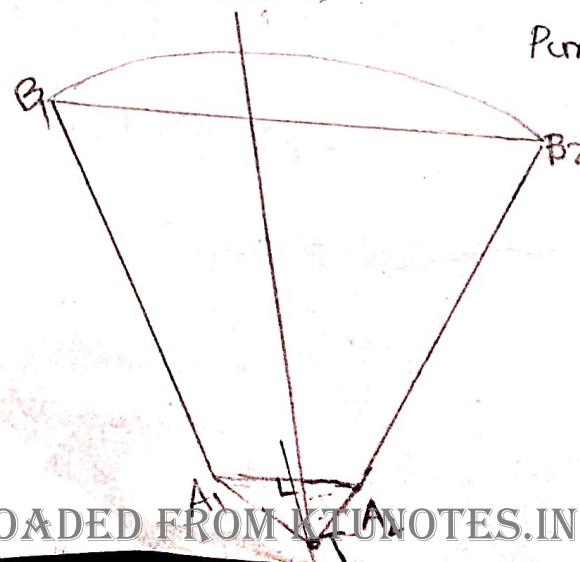
Combined Motions of Translation and Rotation.



Pure translation



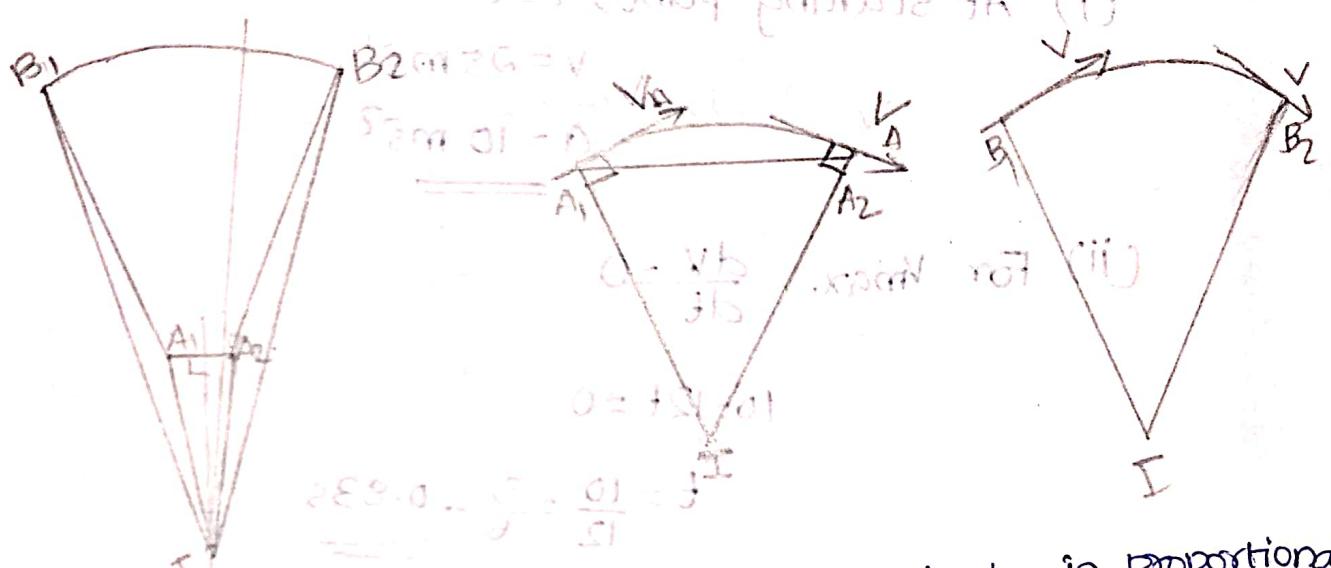
Pure rotation



combined motion of translation and rotation can be considered as a pure rotation about a single point at an instant. That point is called instantaneous centre.

$$\omega_1 = \omega_2 = \omega$$

Q-3. Explain particle A (i)



- The magnitude of velocity of any body is proportional to distance from instantaneous centre and is equal to angular velocity times the distance.

- Direction of velocity is tan to line joining point and instantaneous centre.

$V_A = \omega r_A$

Rectilinear Motion:

- Q: Motion of a particle along a straight line is given by $s = 25t + 5t^2 - 2t^3$, where, s is in m and t in sec. Find:

(i) velocity and acceleration at starting point.

(ii) time the particle reaches maximum velocity and the maximum velocity of the particle.

$$s = 25t + 5t^2 - 2t^3$$

Velocity is $v = \frac{ds}{dt} = 25 + 10t - 6t^2$

$$a = 10 - 12t$$

(i) At starting point, $t=0$

$$v = 25 \text{ m s}^{-1}$$
$$a = 10 \text{ m s}^{-2}$$

(ii) For $v_{\max.}$, $\frac{dv}{dt} = 0$

$$10 - 12t = 0$$

$$t = \frac{10}{12} = \frac{5}{6} = 0.83 \text{ s}$$

$$v_{\max.} = 25 + 10(0.83) - 6(0.83)^2$$

$$= 25 + 8.3 - 4.13$$

$$\therefore v_{\max.} = 29.17 \text{ m s}^{-1}$$

Q: A point is moving in a straight line with acceleration given by $a = 15t - 20$. It passes through a reference point at $t=0$ and another point 30m away after an interval of 5sec. Calculate displacement, velocity and acceleration of the point after a further interval of 5 sec.

Using position $s = \frac{1}{2}at^2 + v_0 t + s_0$

$$s = \frac{1}{2}(15t - 20)t^2 + 0 + 0$$
$$s = 15t^3 - 20t^2$$
$$\therefore \frac{ds}{dt} = 15t^2 - 40t$$

$$dV = 15t dt \quad (\text{and } 20dt \text{ is } 2 \cdot t = V)$$

$$V = \frac{15t^2}{2} - 20t + C$$

$$\frac{dx}{dt} = \frac{15t^2}{2} - 20t + C$$

$$x = \frac{15t^3}{2 \times 3} - \frac{20t^2}{2} + Ct + D$$

$$x = \frac{5t^3}{2} - 10t^2 + Ct + D$$

$$\text{At } t=0, x=0$$

$$\underline{D=0}$$

$$\text{At } t=5, x=30$$

$$30 = \frac{5(5)^3}{2} - 10(5)^2 + (5)C$$

$$30 = \frac{625}{2} - 250 + 5C$$

$$280 \times 2 = 625 + 10C$$

$$10C = -65$$

$$C = -6.5$$

$$\text{Therefore, } x = 2.5t^3 - 10t^2 - 6.5t$$

$$\Rightarrow \text{Total distance covered} = 17.5t^2 - 20t - 6.5$$

$$\text{When } t=10, x = 2.5(10)^3 - 10(10)^2 - 6.5(10)$$

$$= 1435 \text{ m}$$

The time of travel is one hour from 10 to 11 hours

$$V = 7.5(10)^2 - 20(10) + 5.5 \quad \text{m/s} = V_f$$

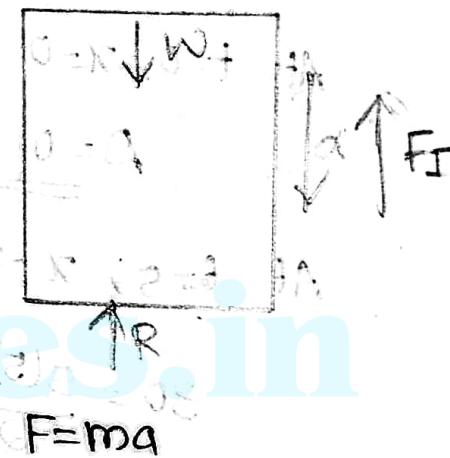
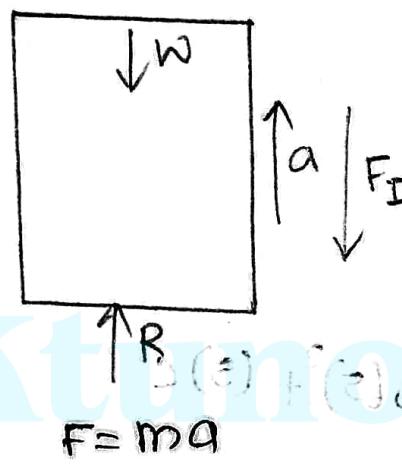
$$= 543.5 \text{ m/s}$$

$$a = 15t - 20$$

$$= 15(10) - 20$$

$$= 130 \text{ m/s}^2$$

MOTION OF LIFT



$$F + F_I = 0$$

$$F + (ma) = 0$$

$$(R - W) - \frac{Wa}{g} = 0$$

$$R = W \left[1 + \frac{a}{g} \right]$$

$$F + F_I = 0$$

$$F + (ma) = 0$$

$$(W - R) - \frac{Wa}{g} = 0$$

$$R = W \left[1 - \frac{a}{g} \right]$$

- Q: A lift has an upward acceleration of 1.2 m/s^2 . What force will a man weighing 750 N exert on the floor of the lift? Also find the force exerted if it is moving with a downward acceleration 1.2 m/s^2 . Find the upward acceleration of lift which cause a weight to exert a

force 900N on the floor.

$$R = 750 \left[1 + \frac{1.2}{9.8} \right]$$

$$= \frac{750 \times 11}{9.8} = \frac{8250}{9.8}$$

$$= \underline{\underline{841.8 \text{ N}}}$$

$$R = 750 \left[1 - \frac{1.2}{9.8} \right]$$

$$= \frac{750 \times 8.6}{9.8}$$

$$= \underline{\underline{658.16 \text{ N}}}$$

$$900 = 750 \left[1 + \frac{a}{9.8} \right]$$

$$\frac{900}{750} - 1 = \frac{a}{9.8}$$

$$\frac{150}{750} = \frac{a}{9.8}$$

$$a = \frac{3 \times 9.8}{15} = 19.6 \text{ m s}^{-2}$$

Q: Calculate the work done in pulling up a block weighing 20 kN for a length of 5m on a smooth plane inclined 20° with horizontal.

$$W = F \cdot S$$

$$= 20 \times 10^3 \times 5 \sin 20^\circ$$

$$= 34.2 \times 10^3 \text{ Nm}$$

Impulse - Momentum.

$$Ft = mV_2 - mV_1$$

Q: An automobile weighing 25 kN is moving at a speed of 60 km/hr. When the brakes are fully applied causing all four wheels to speed up, determine the time required to stop the automobile. Coefficient of friction between road and tyre is 0.5.

$$V_1 = 60 \text{ km/hr} = \left(60 \times \frac{5}{18}\right) \text{ ms}^{-1}$$

$$V_2 = 0$$

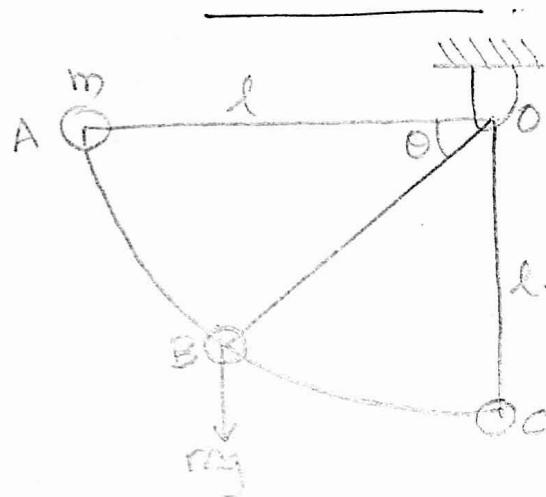
$$F = \mu R_N = 0.5 \times 25 \times 10^3 \text{ N}$$

$$0.5 \times 25 \times 10^3 \times t = \frac{25}{9} \left(60 \times \frac{5}{18}\right)$$

$$t = \frac{300}{18 \times 9.8 \times 0.5 \times 10^3}$$

$$= \underline{\underline{3.4 \times 10^3 \text{ s}}}$$

Q: A simple pendulum is released from rest at A with the strings horizontal and swings downward. Express the velocity of ball as a function of angle ' θ '. Also obtain the expression for angular velocity of ball when the string is in vertical position.



$$A-B \text{ work done} = mgls\sin\theta$$

$$\text{change in K.E} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$$v_B = \sqrt{2gls\sin\theta}$$

$$v_C = \sqrt{2gls\sin 90^\circ}$$

$$= \underline{\underline{\sqrt{2gl}}}$$

$$\omega = \frac{v}{r}$$

$$= \frac{\sqrt{2gl}}{l}$$

$$= \underline{\underline{\sqrt{\frac{2g}{l}}}}$$

24/9/2019
Tuesday

MODULE V

$$\sum \vec{F} = \vec{G}$$

MECHANICAL VIBRATIONS

Vibration of a mechanical system results when a system is displaced from its position of stable equilibrium. The system tends to return to its equilibrium position due to the action of restoring force.

Mechanical vibrations

Forced vibration

Free vibration

Free vibration:

If a disturbing force is applied just to start the motion and is then removed from the system leaving it to vibrate by itself, the system is said to undergo free vibration.

Forced vibration:

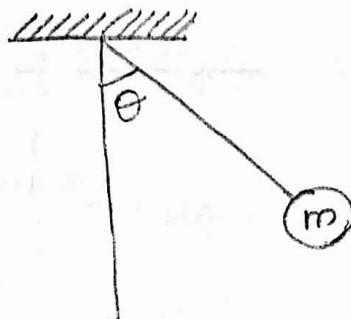
If the disturbing force acts at periodic intervals on the system, the system is said to undergo forced vibration.

Degrees Of Freedom:

It is the number of independent coordinates required to define the configuration of the system. A rigid body in space has six degrees of freedom.

Examples of Single Degree of Freedom System.

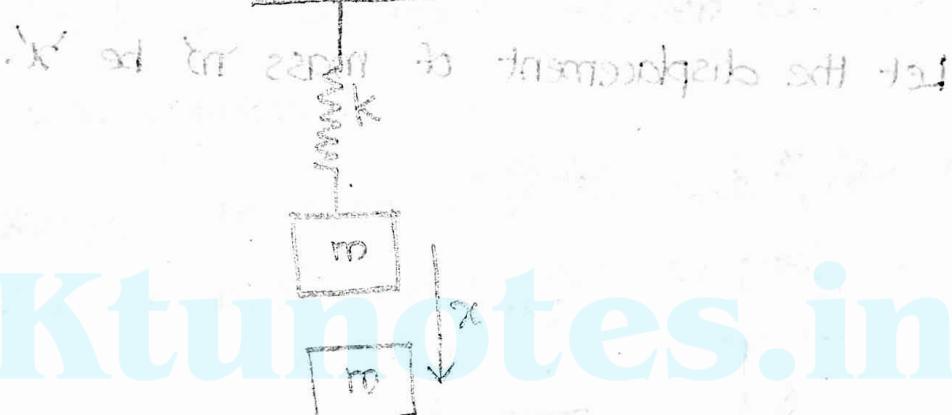
1. Simple Pendulum



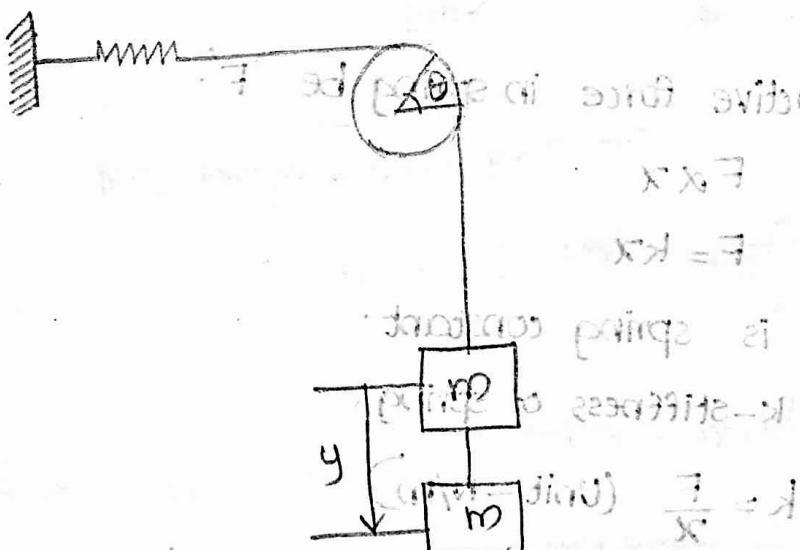
2. Spring-mass model

Mechanical Engineering

UNPUBLISHED FREE ABBREVIATIONS



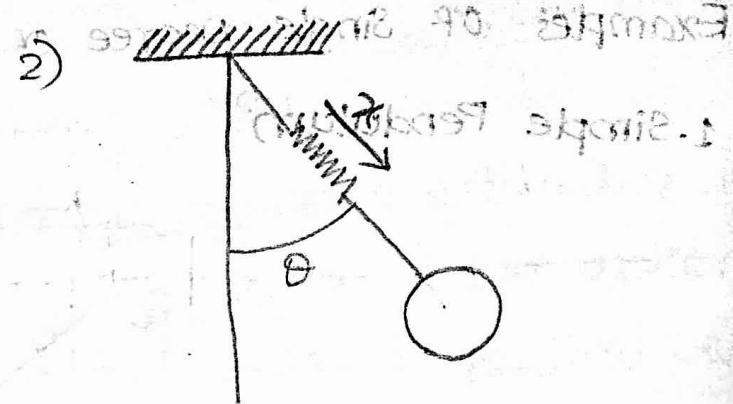
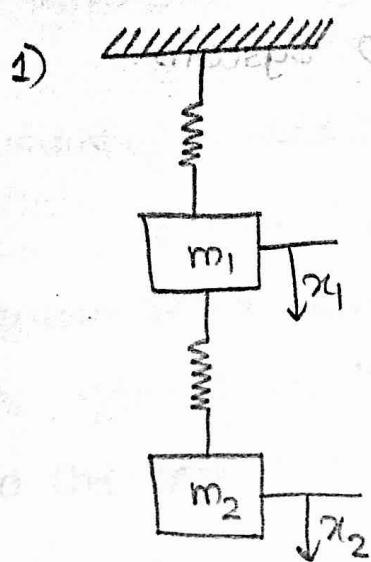
3.



Examples for Two Degrees of Freedom System

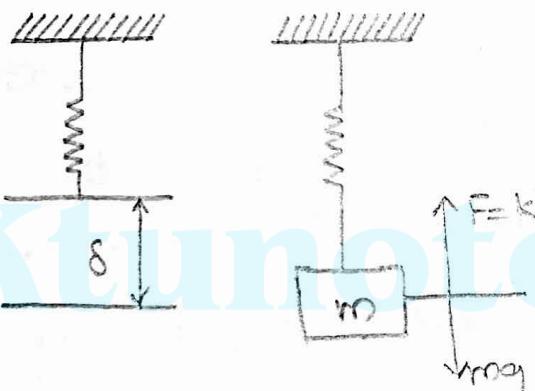
$$\theta_1 = 7 - \text{pm}$$

$$\theta_2 = 84 - \text{pm}$$



UNDAMPED FREE VIBRATIONS OF SPRING-MASS SYSTEM

Let the displacement of mass m be x' .



Let the reactive force in spring be 'F'.

$$F \propto x$$

$$F = kx$$

where, k is spring constant.

k -stiffness of spring

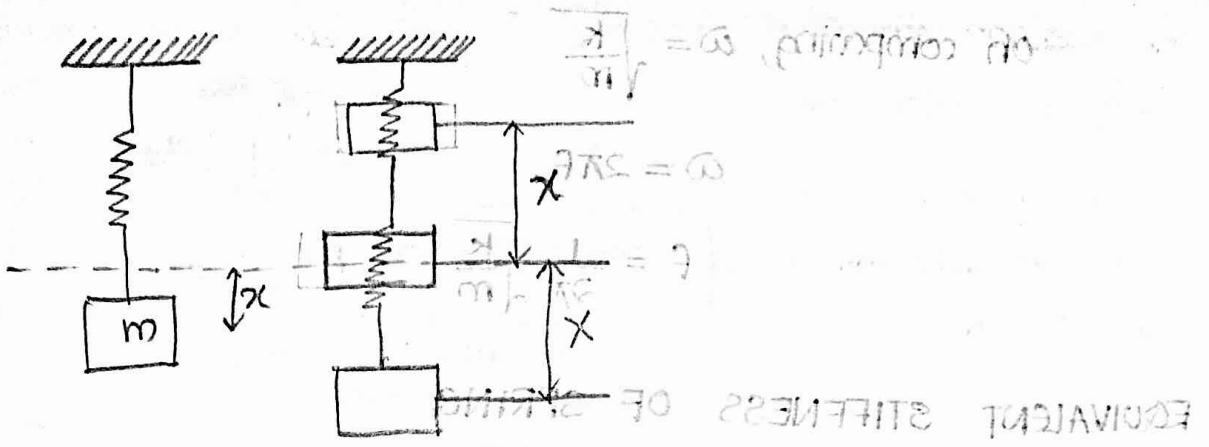
$$k = \frac{F}{x} \text{ (Unit - N/m)}$$

Let the displacement of spring be 's'.

Spring force, $F = ks$

$$mg - F = 0$$

$$mg - ks = 0$$



When an external force is applied, the body is displaced by an amount x' . When the external force is removed, the body will vibrate between two extreme positions with amplitude x' .

Consider the position of the body when it is at a distance x below the equilibrium position.

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$$\text{Net force} = mg - k(\delta + x) = ma$$

$$mg - k\delta - kx = m \frac{d^2x}{dt^2}$$

$$-kx = m \frac{d^2x}{dt^2}$$

This is the equation of motion of free vibration.

This is the equation of motion of free vibration.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

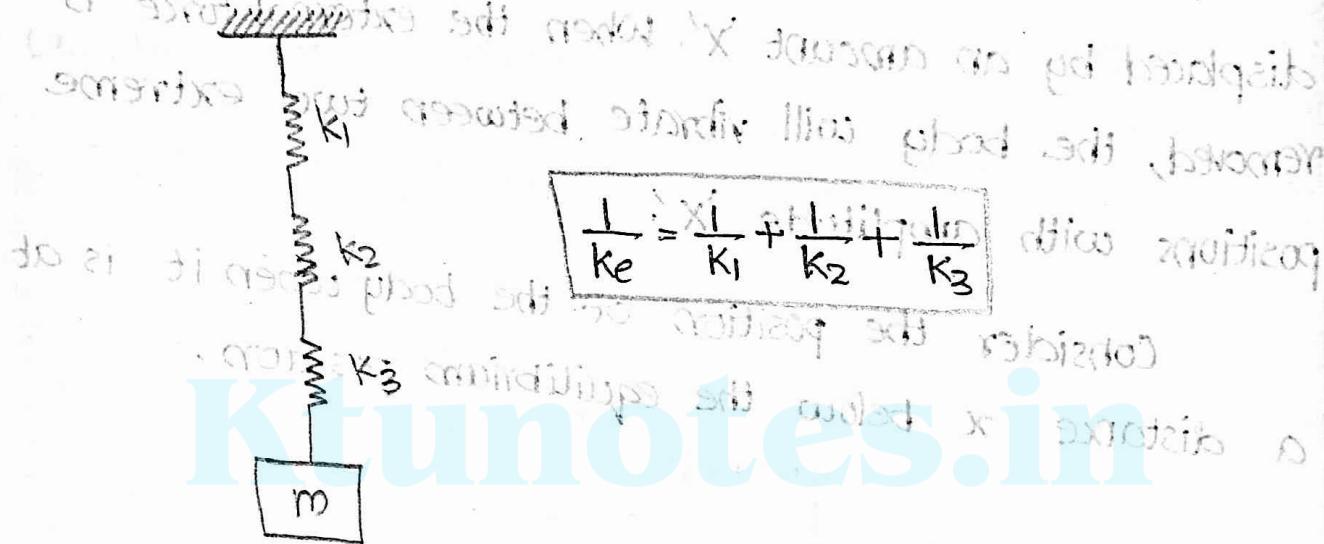
On comparing, $\omega = \sqrt{\frac{k}{m}}$

$$\omega = 2\pi f$$

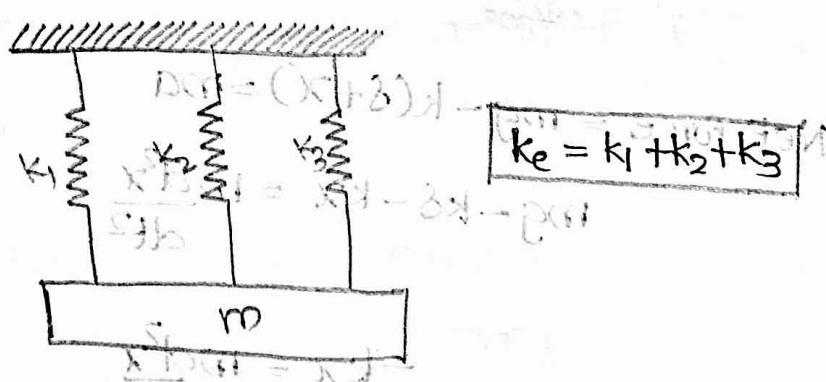
$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

EQUIVALENT STIFFNESS OF SPRING

1. Spring-Spring Series



2. Spring-Spring Parallel



26/9/2019

- Q: A 80 N weight is hung on the end of a helical spring and its end vibrating vertically. The weight makes four oscillations per second. Determine the stiffness of the spring.

$$\omega = \sqrt{k/m}$$

extending $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ more balanced now to follow A

so $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ is able to shift focus on other problems

frequency $f^2 = \frac{1}{4\pi^2} \frac{k}{m}$ is able to form a pointy graph

and so $k = 4\pi^2 f^2 m$ is able to form a straight line

$$= 4\pi^2 \times \frac{80}{9.81} \times 16$$

$$= 5151.108 \text{ N/m}$$

- a: A weight of 50N suspended from a spring vibrates vertically with an amplitude of 8cm and a frequency of 1 oscillation/sec. Find (a) stiffness of the spring (b) the maximum tension induced in the spring (c) the

maximum velocity of the weight.



$$m = \frac{50}{9.81}$$

$$F = 1 \text{ Hz}$$

$$k = 4\pi^2 f^2 m$$

$$= 201.215 \text{ N/m}$$

$$x = 8 \text{ cm}$$

$$= 0.08 \text{ m}$$

Maximum tension,

$$T = kx$$

$$= 201.215 \times 0.08$$

$$= 16.097 \text{ N} \approx 16.1 \text{ N}$$

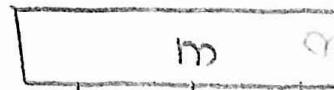
$$V = A\omega$$

$$= 0.08 \times 2\pi \times 1$$

$$= 0.16\pi = 0.502 \text{ ms}^{-1}$$

Q: A tray of mass 'm' is mounted on springs as shown in figure. The period of vibrations of empty tray is 0.5 sec. After placing a mass of 1.5kg on the tray the period was observed to be 0.6 sec. Find the mass of the tray and stiffness of each spring.

$$T = \frac{2\pi}{\sqrt{\frac{m}{k}}} \quad \text{or} \quad T^2 = \frac{4\pi^2 m}{k}$$



$$\text{In series, } \frac{1}{K_e} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \quad \text{period increased}$$

$$K_e = \frac{k}{2}$$

$$\text{In parallel, } K_e = \frac{k}{2} + \frac{k}{2} + \frac{k}{2} = \frac{3k}{2} = 1.5k$$

$$K_e = 1.5k$$

$$T = \frac{2\pi}{\sqrt{\frac{m}{k}}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$0.5 = 2\pi \sqrt{\frac{m}{k}}$$

$$0.6 = 2\pi \sqrt{\frac{m+1.5}{k}}$$

$$\text{Set } \frac{0.5}{0.6} = \sqrt{\frac{m}{k_e}} \times \sqrt{\frac{k_e}{m+1.5}}$$

$$\left(\frac{5}{6}\right)^2 = \frac{m}{m+1.5}$$

$$25(m+1.5) = 36m$$

$$25m + 37.5 = 36m$$

$$11m = 37.5$$

$$m = \frac{37.5}{11} = \underline{\underline{3.41 \text{ kg}}}$$

$$0.5 = 2\pi \sqrt{\frac{3.41}{k_e}}$$

$$k_e = \frac{4\pi^2 \times 0.25 \times 3.41}{m}$$

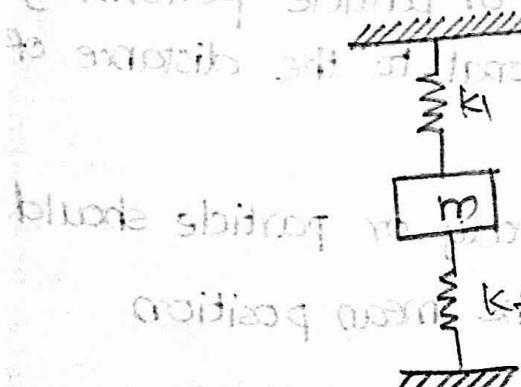
$$= \underline{\underline{538.49 \text{ N/m}}}$$

Stiffness of each spring, $k = \frac{538.49 \times 2}{3}$

$$= 358.99$$

$$= \underline{\underline{359 \text{ N/m}}}$$

- a: A spring of stiffness 6 kN/m is cut into two halves and fixed to a mass m as shown in figure. If the system vibrates with frequency 3 Hz , determine the mass m .



Stiffness of spring is inversely proportional to the number of coils.

$$k = 6 \text{ kN/m}$$

$$k_1 = k_2 = 2 \times 6 = 12 \text{ kN/m}$$

$$f = 3 \text{ Hz}$$

$$K_e = K_1 + K_2$$

$$= 12 + 12$$

$$= \underline{\underline{24 \text{ kN/m}}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k_e}{m}}$$

$$m = \frac{k_e}{4\pi^2 f^2}$$

$$m = \frac{24 \times 10^3}{4\pi^2 \times 9}$$

$$= \underline{\underline{67.55 \text{ kg}}}$$

1/10/2019

Simple Harmonic Motion

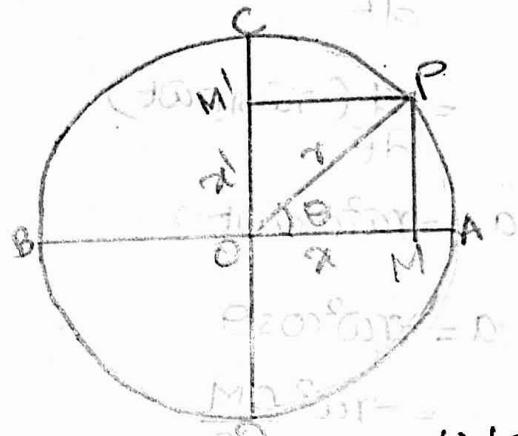
Periodic Motion

Any motion which repeats after equal intervals of time is called periodic motion. For a periodic motion to be simple harmonic, it should satisfy two general conditions:

1. the acceleration of the body or particle performing periodic motion should be proportional to the distance of the body from mean position.

2. the acceleration of the body or particle should always be directed towards the mean position.

Consider a particle moving along the circumference of a circle of radius 'r' with a uniform angular velocity ω rad/s. Let 'P' be the position of the particle after t seconds from the start of motion from the position A.



M is the projection of particle on the horizontal diameter AB. Point M is in SHM.

$$\text{After time } t, \theta = \omega t$$

$$\text{Time period} = T_p, \theta = \omega T_p$$

$$\text{For one oscillation, } \theta = 2\pi$$

$$\omega T_p = 2\pi$$

$$T_p = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi r}{\omega}$$

Now displacement of M from mean position,

$$OM = OP \cos \theta$$

$$x = r \cos \theta$$

$$x = r \cos \omega t$$

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d(r \cos \omega t)}{dt} = -r \omega \sin \omega t$$

$$v = -r \omega \sin \omega t$$

$$\text{Magnitude of velocity, } v = r \omega \sin \theta$$

$$= r \omega \frac{PM}{OP}$$

$$\text{Derivation of SHM} \quad v = r\omega \sqrt{r^2 - x^2}$$

Given a rotating disc of radius 'r' rotates about its axis with angular velocity ω . Then the linear velocity at a distance 'x' from the axis of rotation is given by

Acceleration, $a = \frac{dv}{dt}$ to find out what happens?

$$= \frac{d}{dt} (-r\omega \sin \omega t)$$

$$a = -r\omega^2 \cos \omega t$$

$$a = -r\omega^2 \cos \theta$$

$$\text{Proved that } a = -r\omega^2 \frac{\theta}{OP}$$

$$= -r\omega^2 \frac{\theta}{OP}$$

At the end of time t , the angle made by the radius vector with the vertical is $\theta = \omega t$

$$a = -\omega^2 x$$

The maximum velocity is at $x=0$, i.e., mean position.

$$v_{\max} = \omega r$$

The maximum acceleration is at $x=r$,

$$a_{\max} = -\omega^2 r$$

- Q: A body moving with SHM has an amplitude of 1m and period of oscillation 2sec. Find the velocity and acceleration of the body at $t=0.4$ sec when time is measured from
 (i) the mean position (ii) the extreme position.

(case 1): When time is measured from mean position.

$$T = 1 \text{ sec} \rightarrow \omega$$

$$\text{Given } T_p = 2 \text{ sec} \rightarrow \text{Time to complete one cycle}$$

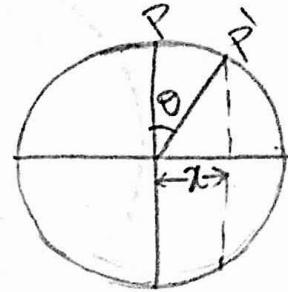
$$T_p = 2 \text{ sec}$$

$$V = \omega \sqrt{r^2 - x^2}$$

$$a = \omega^2 x$$

$$\frac{2\pi}{\omega} = t_p = 2$$

$$\underline{\underline{\omega = \pi}}$$



$$x = r \sin \omega t = 1 \sin(0.4\pi)$$

$$= \underline{\underline{0.95}} \quad \text{(in rad)}$$

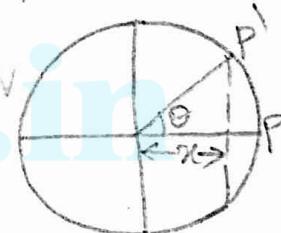
~~0.95 rad = 54 degrees~~

$$V = \pi \sqrt{1 - (0.95)^2} = \underline{\underline{0.981 \text{ m s}^{-1}}}$$

$$a = \omega^2 x = \pi^2 \times 0.95 = \underline{\underline{9.38 \text{ m s}^{-2}}}$$

Case 2: When time is measured from extreme position

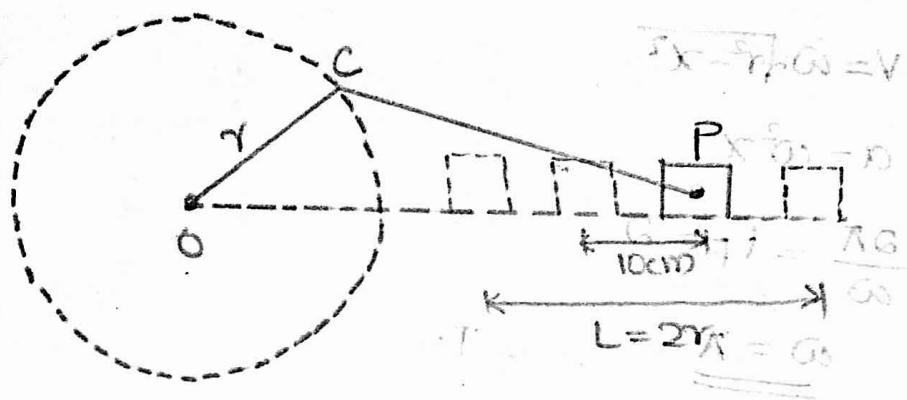
$$\begin{aligned} x &= r \cos \omega t \\ &= 1 \cos(0.4\pi) \\ &= \underline{\underline{0.309}} \end{aligned}$$



$$V = \pi \sqrt{1 - (0.309)^2} = \underline{\underline{2.987 \text{ m s}^{-1}}}$$

$$a = \omega^2 x = \pi^2 \times 0.31 = \underline{\underline{3.06 \text{ m s}^{-2}}}$$

- a: The piston of an IC Engine moves with SHM. The crank rotates at 420 rpm and its stroke length is 40cm. Find the velocity and acceleration of the piston when it is at a distance of 10cm from the mean position.
-



$$L = 40 \text{ cm}$$

$$N = 420 \text{ rpm}$$

$$r = \frac{L}{2} = \frac{40}{2} = 20 \text{ cm}$$

$$V = \omega \sqrt{r^2 - x^2} \quad \text{or} \quad \omega \cdot r \times N = \omega \cdot x \quad \Rightarrow \quad V = \omega \cdot x \cdot N$$

$$\text{rotating smyies} \quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 420}{60} = 43.98 \text{ rad/s}$$

$$V = 43.98 \sqrt{(0.2)^2 - (0.1)^2} \\ = 7.62 \text{ ms}^{-1}$$

$$a = \omega^2 x \\ = (43.98)^2 \times 0.1 \\ = 193.42 \text{ ms}^{-2}$$

and the radius around which the mass is rotating is
the sum of the radius of the pulley and the radius of the belt.
The total radius is 10 + 0.1 = 10.1 m.