Supplementary data

Mathematical Modelling of Photonic-Phononic Scattering Suppression in Nanophoxonic Waveguide for MWIR frequency range

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I. The Complex Phonon frequency

Starting with the optomechanical damping rate (τ_{OM}) associated with the brillouin nanophotonics as the basis[21]. Referring to the phoxonic structure interacting with a midwave infrared (MWIR) ambience as shown in Fig. 2(c). The MWIR radiating photon field (E) can be represented as the laterally varying thermal density perturbation propagating along the vertically oriented sensor thickness (t_s) across the entire active sensing area (A_s). A phonon displacement field (U) causes an elastic potential of $\left(\frac{1}{2}\ A_s \mathrm{P}\ (\partial_Z U)E^2\right)$ where P being the spring constant per unit area. In general sense, the momentum transfer from the incident optically radiating photons field to the laterally vibrating phonons field can be represented as the kinetic energy relation $\frac{\hat{\Pi}^2(z)}{P} = \left(\frac{1}{2}\ A_s\ \rho\ \dot{U}^2\right)$, where ρ is the mass density that is assumed to be changing as a result of thermal density perturbations. We thus modified the interactions with the photo-thermal field (E_{th}) as$\left(\frac{1}{2}\ A_{OPT}\ \gamma\ (\partial_Z U){E_{th}}^2\right)$, where γ is the electro-restrictive constant and A_{OPT} is the area of the optical frequency mode. Further, assuming that the MWIR radiations are uniformly distributed across the whole sensing cross-section. The phoxonic interaction to a large extent is governed by the law of heat-convention under the discrete boundary conditions while getting coupled to the underlying waveguide platform. Here we rewrite the k-mode field component of the phonon displacement field as $\tilde{u}_k$ at the moving boundaries in terms of the heat-flux (Ξ) and thermal temperature ($\lambda T =$

0.2898 $cm\ degree\ Kelvin$) as $\left(\frac{\Xi}{\rho C_P T}\right)$ where C_P is the specific heat of the material. To match the phase-matching condition for obtaining desired brillouin gain, the interaction area (A_s) should be taken as twice that of optical mode area (A_{OPT})[2,19]. The simple explanation to this could be the fact that the mechanical phonon field cross-section area (A_m) is much smaller than that of optical photon field cross-section area. The equation (A.1) finally formulates the classical picture of Hamiltonian with $\widehat{\Pi}^2(z) = A_s\rho$ representing the momentum density function across the moving boundaries.

$$\widehat{\mathcal{H}} = \int_0^{t_s} dz \left[\frac{\widehat{\Pi}^2(z)}{2A_s\rho} + \frac{A_s P}{2}\left((\partial_z \tilde{u}_k)\right)^2 + \frac{A_{OPT}\,\gamma\,(\partial_z \tilde{u}_k) E_{th}{}^2}{2} + \cdots\cdots\right] \tag{A.1}$$

The omitted part being the field inside the medium. As per the Maxwellian theory, for the case of purely electromagnetic phenomenon, energy can be considered as a continuous spatial phenomenon. Before the understanding of energy as a finite number of energy quanta localized in space, the case of a blackbody irradiating infrared waves was for a long time assumed to be a discontinuously distributed beam of light from a point source. The supposition that radiations can be considered being completely random in nature that led to the derivation of Herr Plank's idea of dynamic equilibrium found the average energy density$\langle\widetilde{E_{th}}\rangle = c^8 \frac{\rho_V(r,t)}{2\omega^2}$, where c being velocity of light. Unlike for the case of conventional spectrum, let us consider the **thermal energy density per unit volume** ($\rho_V = \rho dV$) for the radiations oscillating between $[\omega, \omega \pm \Delta(\Omega\ or\ \chi)]$ to be $\left[\rho_V = \alpha\omega^2 e^{-\beta\Omega/P}\right]$. The work still relies on values derived by Plank's radiation formulae and its derived temperature-dependent constants given by $\alpha = 6.1 \times 10^{-56}$; $\beta = 4.866 \times 10^{-11}$. Important to note that the Ω represents a purely real phononic mode frequency for the case of lossless photon-phonon scattering waveguide (parity-time symmetry) while $\chi = \Omega - \mathrm{j}\frac{\Gamma}{2}$ is a complex phononic mode frequency of Lossy scattering waveguide (broken parity-time symmetry) with a non-radiative decay rate Γ; both cases being two different scenarios. The final electromagnetic field wave propagating in z-direction of the form ($\widetilde{E_{th}}(z) = \sum_k \tilde{E}_k\left[\hat{a}_k e^{ikz} + H.c\right]$, summed over desired optical wave-vector (k) and zero-point amplitude, $\tilde{E}_k$ (accounting for vacuum fluctuations) is compared here by average energy density, $\langle\tilde{E}\rangle$ for static and dynamic case respectively,

$$\rho_V = k^2\left[\frac{\hbar\omega}{A_{OPT}L\epsilon}\right]^{0.5} \tag{A.2}$$

$$\Delta\rho_V = q^2\left[\frac{\hbar\omega}{A_{OPT}L\epsilon}\right]^{0.5} \tag{A.3}$$

Here, $k = \omega/c\ and\ q = \Omega/v_a$ are the photon and phonon wave-vector respectively. The later of course accounts for the zero-point fluctuation as a result of phoxonic coupling. Also to mention the common assumption for defining the total energy of electric and magnetic fields as $(\frac{\hbar\omega}{4})$ per mode for the ground or vacuum state. This work compares the above equations to energy density per unit volume of a cavity length, L which brings to following expression for phonon frequency;

$$\Omega = -\frac{1}{\beta T}\ln\sqrt{\frac{2\hbar\omega}{(\alpha c^2)^2 A_{OPT}L\epsilon}} \tag{A.4}$$

Also, note that ϵ is the material permittivity and the whole idea of PhoXonics revolves around change in permittivity due to change in the stiffness of the material which is function of material density fluctuations,

$$\gamma = \epsilon = \rho\frac{\partial\epsilon}{\partial\rho} \tag{A.5}$$

The longitudinal acoustic waves associate with phononic mode on the other hand is usually quantized as ($\tilde{u}(z) = \sum_k \tilde{u}_k\left[\hat{b}_k e^{ikz} + H.c\right]$, referred from literature. But for considering the thermally generated phonons restricted by the discretized periodic lattice boundaries[22] in the direction of irradiation (mentioned earlier), incoming flux can be discretely approximated as the heat by convection for the phoxonic interaction. The equation for phonon fields thus formulates to,

$$(\tilde{u}(z) = \sum_k (\tilde{u}_k + \tilde{u}_{Th})[\hat{b}_k e^{ikz} + H.c] \quad (A.6)$$

Where $\widetilde{u}_{Th} = \frac{\Xi}{\rho C_P T}$. Substituting Eq. (A.6) into Eq. (A.1) and solving for the coefficients $(\hat{b}_k{}^{\dagger}\hat{b}_k)$ which represents number of phonon modes as per the famous Dirac's formalism, we get the expression,

$$\tilde{u}_k{}^2 = \xi^2(\rho\Omega_k)^{-1} \quad (A.7)$$

Here, $\xi^2 = \frac{\hbar\omega}{A_{OPT}L}$ is a dimensionless figure of merit depending on incident photon energy, optomechanical mode area (A_{OPT}), photon-phonon coupling length (L). Also knowing that phonon velocity ($v_{pn} = \sqrt{P/\rho}$) ; the heat-flux (Ξ) and phonon-field relation can be reformulated as,

$$\Xi^2 = (\rho C_P T)^2 \tilde{u}_k{}^2 \quad (A.8)$$

Substituting Eq. 1.7 in 1.8, we obtain the expression for complex phonon frequency,

$$\Omega_k = \rho \left(\frac{\xi C_P T}{\Xi}\right)^2\Big]_k = |\mathrm{k}| v_{pn} \quad (A.9)$$

II. QuTiP Code

https://github.com/NANOPHOTONIC-RESEARCH-SOCIETY-AT-PEC/PHD18307002_Supplementry_Data_Anurag-Sharma-/blob/main/QUTIP%20code%20.py

List of Symbols

Symbols	**Formulae**	**Description**
Ω_m or Ω	-	Angular frequency of the mechanical resonator
Γ_m or Γ	-	Damping rate of the mechanical resonator
Q_m	-	Quality-factor of the mechanical resonator
ω_{cav}	-	Angular frequency of a resonating cavity
ω_{source}	-	Angular frequency of the pump Laser
Δ	-	Detuning between cavity and source ($\omega_{source} - \omega_{cav}$)
κ	-	Cavity decay rate
κ_{ex}	-	Cavity decay rate from input coupling
κ_o	-	Cavity decay rate from intrinsic losses
x_{zpf}	$x_{zpf} = \sqrt{\hbar/2m_{eff}\Omega_m}$	Mechanical zero-point fluctuation amplitude
g_o		Single photon coupling strength
g	$= g_o\sqrt{\tilde{n}_{cav}}$	Light-enhanced Single photon coupling strength for the linearized regime
$\hat{\mathcal{H}}_{int}$	$= -\hbar g_o\, \hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger)$	The interaction Hamiltonian
G	$\partial\omega_{cav}\,/\partial x$	Optical frequency shift
$\hat{a}$	-	The Photon annihilation operator with $\hat{a}^\dagger\hat{a}$ as the circulating photon number
$\hat{b}$	-	The Phonon annihilation operator with $\hat{b}^\dagger\hat{b}$ as the phonon number
$\tilde{n}$	$\tilde{n} = \langle\hat{b}^\dagger\hat{b}\rangle$	The average number of phonons stored in the mechanical resonator
$\tilde{n}_{cav}$	$\tilde{n}_{cav} = \langle\hat{a}^\dagger\hat{a}\rangle$	The photon number circulating inside the cavity
$\tilde{n}_{th}$	$\tilde{n}_{th} = \left(e^{(\hbar\Omega_m/k_BT)} - 1\right)^{-1}$	The average phonon number in thermal equilibrium

P		Incoming source power
C_{qu}	$C/\tilde{n}_{th}$	Quantum cooperativity
χ_{OPT}	-	Optical susceptibility of the cavity
χ_M	-	Mechanical susceptibility
Γ_{OM}	-	Optomechanical damping rate
U	-	Phonon displacement field
E	-	The radiating photon field
ρ	-	The mass density of the mechanical resonator
ρ_V	ρdV	The thermal energy density per unit volume
E_{th}		Photo-thermal field
γ	$\approx \Gamma_m \tilde{n}_{th}$	Thermal decoherence rate
A_{OPT}		The optical frequency mode area
Ξ	$\frac{\Xi}{\rho C_P T}$	The thermally generated heat-flux with a specific heat (C_P) of the matter
χ	$\Omega - j\frac{\Gamma_m}{2}$	The complex phononic mode frequency
S	-	Stokes frequency shift
AS	-	Anti-Stokes frequency shift