

# Ex 2 code

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```
# -*- coding: utf-8 -*-
```

```
"""
```

```
Created on Sun Mar 10 15:11:19 2019
```

```
@author: naomi
```

```
"""
```

```
#-----
```

```
#import the libraries required in the code
```

```
from __future__ import division
```

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
import time
```

```
from matplotlib import cm
```

```
from scipy.sparse import diags
```

```
import scipy.sparse.linalg
```

```
from mpl_toolkits.mplot3d import axes3d
```

```
import copy
```

```
import random
```

```
import math
```

```
#-----
```

```
#Grid size
```

```
Nx = 30
```

```
Ny = 30

#grid spacing

delta = 1

#initial guess of the matrix

Vguess = random.randint(0,10)


#general Gauss-seidel method

def Gauss_seidel_gen(Nx,Ny,delta,x):

    nx = Nx-1
    ny = Ny-1


    X, Y = np.meshgrid(np.arange(0, Nx), np.arange(0, Ny))


    V = np.empty((Nx, Ny))
    V.fill(Vguess)


    Vtop = 0
    Vbottom = 0
    Vleft = 0
    Vright = 0


    V[(ny):, :] = Vtop
    V[:,1, :] = Vbottom
```

```

V[:, (nx):] = Vright
V[:, :1] = Vleft

it_count=0

#convergence condtion
while True:

    V_old = copy.deepcopy(V)

    for i in range(1, nx, delta):
        for j in range(1, ny, delta):
            V[i, j] = 0.25 * (V[i+1][j] + V[i-1][j] + V[i][j+1] + V[i][j-1])

        it_count += 1

    if abs(np.sum(V_old.flatten()) - np.sum(V.flatten())) < x:
        #print(Iteration number:, it_count)
        break

    return V,it_count,Vtop

#general Jacobi method
def Jacobi_gen(Nx,Ny,delta,x):

    nx = Nx-1
    ny = Ny-1

    X, Y = np.meshgrid(np.arange(0, Nx), np.arange(0, Ny))

    V = np.empty((Nx, Ny))

```

```
V.fill(Vguess)
```

```
Vtop = 0
```

```
Vbottom = 0
```

```
Vleft = 0
```

```
Vright = 0
```

```
V[(ny):, :] = Vtop
```

```
V[:, 1, :] = Vbottom
```

```
V[:, (nx):] = Vright
```

```
V[:, :1] = Vleft
```

```
it_count = 0
```

```
#convergence conditon
```

```
while True:
```

```
    V_new = V.copy()
```

```
    V_old = copy.deepcopy(V)
```

```
    for i in range(1, nx, delta):
```

```
        for j in range(1, ny, delta):
```

```
            
$$V[i, j] = 0.25 * (V\_new[i+1][j] + V\_new[i-1][j] + V\_new[i][j+1] + V\_new[i][j-1])$$

```

```
    it_count += 1
```

```
    if abs(np.sum(V_old.flatten()) - np.sum(V.flatten())) < x:
```

```

        #print(Iteration number:, it_count)

        break

    return V,it_count,Vtop

#standard deviation error for Gauss

def Gauss_gen_error(x):

    V, it_count, Vtop = Gauss_seidel_gen(Nx,Ny,delta,x)

    V_av = np.mean(V)

    std = math.sqrt(np.mean(abs(V - V_av)**2))

    return std

#test to see if the grid points converge to the boundaries

def Gauss_gen_test(x):

    V, it_count, Vtop = Gauss_seidel_gen(Nx,Ny,delta,x)

    V_av = np.mean(V)

    abs_err = (abs(V_av-Vtop))

    return abs_err

#standard deviation error for Jacobi

def Jacobi_gen_error(x):

    V, it_count, Vtop = Jacobi_gen(Nx,Ny,delta,x)

    V_av = np.mean(V)

    std = math.sqrt(np.mean(abs(V - V_av)**2))

    return std

#test to see if the grid points converge to the boundaries

def Jacobi_gen_test(x):

```

```

V, it_count, Vtop = Jacobi_gen(Nx,Ny,delta,x)

V_av = np.mean(V)

abs_err = (abs(V_av-Vtop))

return abs_err

#Capacitor model using the Gauss Seidel method
def Gauss_seidel_cap(Nx,Ny,delta):

    V = np.empty((Nx+1,Ny+1))

    V.fill(Vguess)

    Vtop = 0

    Vbottom = 0

    Vleft = 0

    Vright = 0


    nx = Nx-1

    ny = Ny-1


    V[-1:, :] = Vtop

    V[:, 1] = Vbottom

    V[:, -1:] = Vright

    V[:, :1] = Vleft


    sep = 4

    len_plate=10

```

```

# Boundary condition

halflen_plate=int((len_plate/2))

mid_px=int((Nx/2))
mid_py=int((Ny/2))


plate1=int(mid_px-sep)
plate2=int(mid_px+sep)

min_py = mid_py-halflen_plate
max_py = mid_py+halflen_plate

V[plate1,min_py:max_py] = -100
V[plate2,min_py:max_py] = 100

it_count = 0

while True:

    V_old = copy.deepcopy(V)

    for i in range(1,nx,delta):

        for j in range(1,ny,delta):

            if i == plate1 or i == plate2:

                continue

            if j < min_py and j> max_py:

                continue

            else:

                
$$V[i, j] = 0.25 \cdot (V[i+1, j] + V[i-1, j] + V[i, j+1] + V[i, j-1])$$


                it_count += 1

```



```

    if abs(np.sum(V_old.flatten()) - np.sum(V.flatten())) < 1e-10:

        #print(Iteration number:, it_count)

        break

    return V

#conditions for the furnace and no heat loss
def PhiT_vect_furnace():

    n = 11

    T_H = 1000 + 273

    T_C = 20 + 273

    phiT=np.empty((n,1))

    phiT[0]= T_H

    phiT[1:]= T_C

    return phiT

#conditions for the furnace and the ice bath
def PhiT_vect_ice():

    n = 11

    T_H = 1000 + 273

    T_C = 20 + 273

    T_ice = 273

```

```
phiT=np.empty((n,1))
```

```
phiT[0]= T_H
```

```
phiT[1:]= T_C
```

```
phiT[n-1,0:]= T_ice
```

```
return phiT
```

```
#Diffusion coefficients with dirichlet BC's
```

```
def Tridia_mat_dir():
```

```
    diff = 59
```

```
    spec_heat = 450
```

```
    density = 7900
```

```
    alpha = diff/(spec_heat*density)
```

```
    rodlen = 0.50
```

```
    nx = 10
```

```
    dx = rodlen/(nx-1)
```

```
    dt = 1
```

```
    a = alpha*dt/(dx**2)
```

```
    diagonal = np.zeros(nx+1)
```

```
    lower = np.zeros(nx)
```

```
    upper = np.zeros(nx)
```

```
# Precompute sparse matrix
```

```
diagonal[:] = 1 + 2*a
```

```
lower[:] = -a #1
```

```
upper[:] = -a #1
```

```
# Insert boundary conditions
```

```
diagonal[0] = 1
```

```
upper[0] = 0
```

```
lower[-1] = -a
```

```
diagonal[nx] = 1 + a
```

```
A = scipy.sparse.diags(
```

```
    diagonals=[diagonal, lower, upper],
```

```
    offsets=[0, -1, 1], shape=(nx+1, nx+1),
```

```
    format='csr')
```

```
return A.todense()
```

```
#Diffusion coefficients with dirichlet BC's
```

```
def Tridia_mat_heatloss():
```

```
    diff = 59
```

```
    spec_heat = 450
```

```
density = 7900

alpha = diff/(spec_heat*density)

rodlen = 0.50

nx = 10

dx = rodlen/(nx-1)

dt = 0.1


a = alpha*dt/(dx**2)


diagonal = np.zeros(nx+1)

lower = np.zeros(nx)

upper = np.zeros(nx)


# Precompute sparse matrix

diagonal[:] = 1 + 2*a

lower[:] = -a #1

upper[:] = -a #1

# Insert boundary conditions

diagonal[0] = 1

upper[0] = 0

diagonal[nx] = 1

lower[-1] = 0
```

```
A = scipy.sparse.diags(  
    diagonals=[diagonal, lower, upper],  
    offsets=[0, -1, 1], shape=(nx+1, nx+1),  
    format='csr')  
return A.todense()
```

```
MyInput = '0'  
while MyInput != 'q':  
    MyInput = input('Enter a choice, \n "a)" to explore convergence conditions,\n "b)" to  
plot the capacitor, \n "c)" to explore the diffusion rod, \n "q)" to quit: ')  
    print('You entered the choice: ',MyInput)
```

```
if MyInput == 'a':  
    print('You have chosen to explore the convergence conditions of the Gauss and  
Jacobi methods')  
    print('The respective errors for Jacobi and Gauss Seidel are:')  
    print(Jacobi_gen_test(x=1e-3))  
    print(Gauss_gen_test(x=1e-3))
```

```
X = []
```

```
IT1 = []
```

```
IT2 = []
```

```
for x in np.logspace(-30,-50,10):
```

```
    Nx = 10
```

```
    Ny = 10
```

```
    V1, it_count1,Vtop = Gauss_seidel_gen(Nx,Ny,delta,x)
```

```
    V2, it_count2,Vtop = Jacobi_gen(Nx,Ny,delta,x)
```

```
    IT1.append(it_count1)
```

```
    IT2.append(it_count2)
```

```
    X.append(math.log(x))
```

```
plt.plot(X,IT1, color = "r",marker = "^", label = "Gauss")
```

```
plt.plot(X,IT2, color = "b",marker = "^", label = "Jacobi")
```

```
plt.legend(title = "Method",fontsize = 15)
```

```
plt.xlabel('Convergence tolerance',fontsize = 15)
```

```
plt.ylabel('Number of iterations',fontsize = 15)
```

```
plt.grid(True)
```

```
plt.show()
```

```
def plotofgrid_den(x):
```

```
N = []
```

```
IT1 = []
```

```
IT2 = []
```

```
for Nx in range(0,30,3):
```

```
    for Ny in range(0,30,3):
```

```
        V, it_count1, Vtop = Gauss_seidel_gen(Nx,Ny,delta,x)
```

```
        V, it_count2, Vtop = Jacobi_gen(Nx,Ny,delta,x)
```

```
        IT1.append(it_count1)
```

```
        IT2.append(it_count2)
```

```
        N.append(Nx)
```

```
IT1=np.array(IT1)
```

```
IT1=np.reshape(IT1,(10,10))
```

```
IT2=np.array(IT2)
```

```
IT2=np.reshape(IT2,(10,10))
```

```
N=np.array(N)
```

```
N=np.reshape(N,(10,10))
```

```
plt.plot(N[:,1],IT1[9], color = "r",marker ="^", label = "Gauss")
plt.plot(N[:,1],IT2[9], color = "b",marker ="^", label = "Jacobi")
plt.legend(title = "Limit",fontsize = 15)
plt.xlabel('Sqrt of the no. grid points',fontsize = 15)
plt.ylabel('Number of iterations',fontsize = 15)
plt.grid(True)
```

```
plt.show(plotofgrid_den(x=1e-3))
```

```
print('The standard deviation on the mean of Jacobi method is',
Jacobi_gen_test(x=1e-3))
print('The standard deviation on the mean of Gauss-seidel method
is',Gauss_gen_test(x=1e-3))
```

```
def plotofgrid_sd(x):
```

```
    N = []
```

```
    Jac = []
```

```
    Gau = []
```

```
    for Nx in range(0,30,3):
```

```
        for Ny in range(0,30,3):
```

```
            j = Jacobi_gen_error(x=1e-3)
```



```
g = Gauss_gen_error(x=1e-3)
```

```
Jac.append(j)
```

```
Gau.append(g)
```

```
N.append(Nx)
```

```
plt.plot(N,Jac, color = "r",marker = "^", label = "Gauss")
```

```
plt.plot(N,Gau, color = "b",marker = "^", label = "Jacobi")
```

```
plt.legend(title = "Limit",fontsize = 15)
```

```
plt.xlabel('Sqrt of the no. grid points',fontsize = 15)
```

```
plt.ylabel('Standard deviation',fontsize = 15)
```

```
plt.grid(True)
```

```
plt.show(plotofgrid_sd(x=1e-3))
```

```
def plotofconvergence_sd():
```

```
    X = []
```

```
    Jac = []
```

```
    Gau = []
```

```
    for x in np.logspace(-30,-50,10):
```

```
        j = Jacobi_gen_error(x)
```

```
        g = Gauss_gen_error(x)
```

```
Jac.append(j)

Gau.append(g)

X.append(math.log(x))
```

```
plt.plot(X,Jac, color = "r",marker ="^", label = "Gauss")
plt.plot(X,Gau, color = "b",marker ="^", label = "Jacobi")
plt.legend(title = "Method",fontsize = 15)
plt.xlabel('Convergence tolerance',fontsize = 15)
plt.ylabel('Standard deviation',fontsize = 15)
plt.grid(True)
```

```
plt.show(plotofconvergence_sd())
```

```
elif (MyInput == 'b'):
```

```
    #create 2-d mesh
```

```
    X, Y = np.meshgrid(np.linspace(0,1,Nx + 1), np.linspace(0,1,Ny + 1))
```

```
    start = time.time()
```

```
    V = Gauss_seidel_cap(Nx,Ny,delta)
```

```
    end = time.time()
```

```
print(end-start,'s')

# Set colour interpolation and colour map
colorinterpolation = 50
colourMap = plt.cm.hot #you can try: colourMap = plt.cm.coolwarm

# Configure the contour
plt.title("Contour of Voltage")
plt.contourf(X, Y, Gauss_seidel_cap(Nx,Ny,delta), colorinterpolation,
cmap=colourMap)

# Set Colorbar
plt.colorbar()

plt.show()

Grad = np.gradient(V)
u_val, v_val = np.gradient(Gauss_seidel_cap(Nx, Ny, delta))
u_val = np.zeros((Nx+1, Ny+1)) - u_val
v_val = np.zeros((Nx+1, Ny+1)) - v_val

E = np.sqrt(u_val*u_val + v_val*v_val) #magnitude of electric field

plt.title("Contour of Electric Field")
```

```
plt.contourf(X, Y, E, colorinterpolation, cmap=colourMap)
```

```
# Set Colorbar
```

```
plt.colorbar()
```

```
plt.show()
```

```
vector_u, vector_v = Grad
```

```
vector_u = np.zeros((Nx+1,Ny+1)) - vector_u
```

```
vector_v = np.zeros((Nx+1, Ny+1)) - vector_v
```

```
mag = np.sqrt(vector_u*vector_u + vector_v*vector_v)
```

```
width = 4*mag/mag.max()
```

```
plt.streamplot(X, Y, vector_v, vector_u, color= 'b' ,  
               linewidth=width)
```

```
fig = plt.figure()
```

```
ax = fig.gca(projection='3d')
```

```
surf = ax.plot_surface(X, Y, V, rstride=1, cstride=1, cmap=cm.coolwarm,  
                       linewidth=0, antialiased=False)
```

```
#ax.set_zlim(0, 110)
```

```
ax.set_xlabel('Position x [in m]')  
ax.set_ylabel('Position y [in m]')  
ax.set_zlabel('Potential, V(x,y)')  
plt.title("Plot of Electric Potential V(x,y)")  
  
fig.colorbar(surf, shrink=0.5, aspect=5)  
plt.show()
```

```
elif (MyInput == 'c'):  
    M = Tridia_mat_dir()  
    N = Tridia_mat_heatloss()  
  
    b = PhiT_vect_furnace()  
    c = PhiT_vect_ice()  
  
    phi_prime1 = np.empty_like(b)  
    phi_prime2 = np.empty_like(c)  
  
    nx = 11
```

```

for t in range(1,8000,500):
    for i in range(round(t)):
        phi_prime1 = scipy.sparse.linalg.spsolve(M, b)
        phi_prime1, b = b, phi_prime1

X = np.linspace(0,0.5,nx)

plt.plot(X, phi_prime1, label= str(t) + 's')
plt.xlabel("Postion along rod (m)")
plt.ylabel("Temperature (K)")
plt.legend(bbox_to_anchor=(1.04,1), borderaxespad=0)

plt.show()

for t in range(1,8000,500):
    for i in range(round(t)):
        phi_prime2 = scipy.sparse.linalg.spsolve(N,c)
        phi_prime2, c = c, phi_prime2

X = np.linspace(0,0.5,nx)

plt.xlabel('Postion on the rod\ m')
plt.ylabel('Temperature\ K')

```

```
plt.plot(X, phi_prime2, label= str(t) + 's')

plt.xlabel("Position along rod (m)")

plt.ylabel("Temperature (K)")

plt.legend(bbox_to_anchor=(1.04,1), borderaxespad=0)


plt.show()


elif MyInput != 'q':

    print('This is not a valid choice')


print('You have chosen to finish - goodbye.')
```

# Ex 2 code

## GRADEMARK REPORT

FINAL GRADE

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GENERAL COMMENTS

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