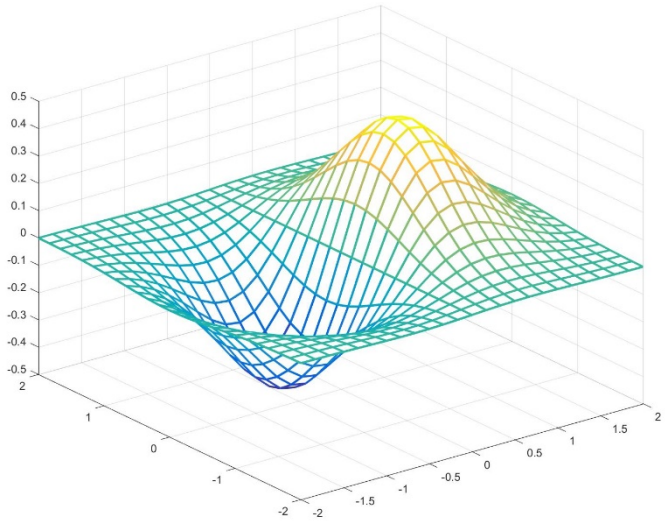
等高线和梯度场

作函数 $f(x,y)=xe^{-x^2-y^2}$ 的等高线和梯度线，并观察梯度线与等高线的关系。

命令行窗口

```
>> x1=linspace(-2,2,25);
>> y1=linspace(-2,2,25);
>> [x,y]=meshgrid(x1,y1);
>> z=x.*exp(-x.^2-y.^2);
>> mesh(x,y,z)
>>
```



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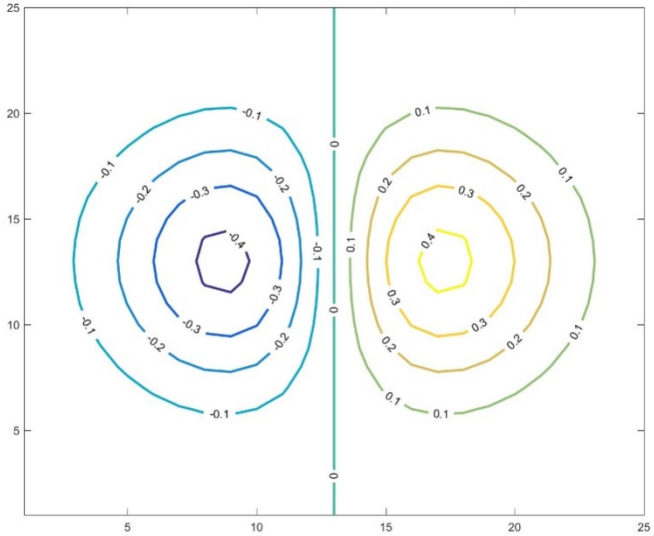
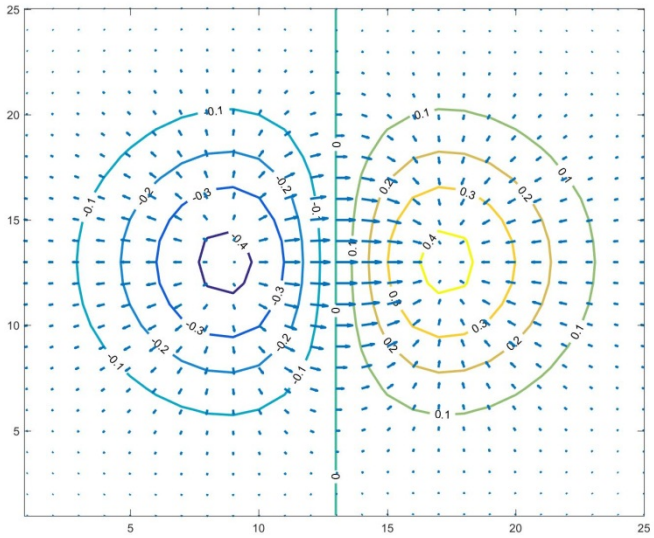
>> y1=linspace(-2,2,25);
>> [x,y]=meshgrid(x1,y1);
>> z=x.*exp(-x.^2-y.^2);
>> h=contour(z);
>>
```



命令行窗口

```
>> x1=linspace(-2,2,25);
>> y1=linspace(-2,2,25);
>> [x,y]=meshgrid(x1,y1);
>> z=x.*exp(-x.^2-y.^2);
>> h=contour(z);
>> [dx,dy]=gradient(z,0.2,0.2);
>> hold on
>> quiver(dx,dy);
>>
```



约束条件极值

抛物线 $z = x^2 + y^2$ 被平面x+y+z=1截成一个椭圆，求这个椭圆到原点的最长和最短距离。

数学模型

$$f(x,y,z) = x^2 + y^2 + z^2$$

$$s.t. \quad \begin{cases} z = x^2 + y^2 \\ x + y + z = 1 \end{cases}$$

构造Lagrange函数

```
>> syms x y z u v
>> f = x^2+y^2+z^2+u*(z-x^2-y^2)+v*(x+y+z-1);
```

求稳定点

```
>> fx = diff(f,x);
>> fy = diff(f,y);
>> fz = diff(f,z);
>> fu = diff(f,u);
>> fv = diff(f,v);
>> eqn = [fx==0,fy==0,fz==0,fu==0,fv==0];
>> vars = [x y z u v];
>> S = solve(eqn, vars);
```

约束条件极值

求最值 $f\left(\frac{-1\pm\sqrt{3}}{2},\frac{-1\pm\sqrt{3}}{2},2\mp\sqrt{3}\right)=9\mp5\sqrt{3}$

```
>> double(subs(x^2+y^2+z^2,vars,[S.x(1) S.y(1) S.z(1) S.u(1) S.v(1)]))

ans =

  17.660254037844386

>> double(subs(x^2+y^2+z^2,vars,[S.x(2) S.y(2) S.z(2) S.u(2) S.v(2)]))

ans =

  -0.250000000000000

>> double(subs(x^2+y^2+z^2,vars,[S.x(3) S.y(3) S.z(3) S.u(3) S.v(3)]))

ans =

  -0.250000000000000

>> double(subs(x^2+y^2+z^2,vars,[S.x(4) S.y(4) S.z(4) S.u(4) S.v(4)]))

ans =

   0.339745962155614
```