



MATLAB之高等数学

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多元函数微分学

1. 偏导数与全微分;
 2. 曲面的切平面与法线并绘制图形;
 3. 多元函数的梯度;
 4. 多元函数的极值;
-



diff函数

1. 求函数 $f(x,y)$ 关于 x 的导数

```
>> syms x y
```

```
>> diff(f(x,y), x)
```

2. 求函数 $f(x,y)$ 关于 x 的 n 阶导数

```
>> diff(f(x,y), x, n)
```

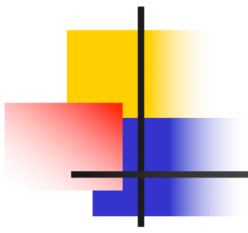
3. 求函数 $f(x,y)$ 关于 x 和 y 的2阶混合偏导数

```
>> diff(f(x,y), x, y)
```

4. 求函数 $f(x,y)$ 的全微分

```
>> syms dx dy
```

```
>> dz = diff(f(x,y), x)*dx + diff(f(x,y), y)*dy
```



$$z = \frac{2xy}{x^2 + y^2}, \text{求} \frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^3 z}{\partial x^2 \partial y}$$

```
>> syms x y;
```

```
>> z = 2*x*y/(x^2 + y^2);
```

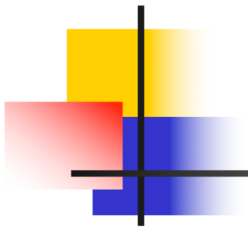
```
>> dzx = diff(z,x)
```

```
>> dzxx = diff(z,x,2)
```

```
>> dzxxy = diff(dzxx,y)
```

命令行窗口

```
>> syms x y;  
>> z=2*x*y/(x^2+y^2);  
>> dzx=diff(z,x)  
  
dzx =  
  
(2*y)/(x^2 + y^2) - (4*x^2*y)/(x^2 + y^2)^2  
  
>> dzxx=diff(z,x,2)  
  
dzxx =  
  
(16*x^3*y)/(x^2 + y^2)^3 - (12*x*y)/(x^2 + y^2)^2  
  
>> dzxxy=diff(dzxx,y)  
  
dzxxy =  
  
(16*x^3)/(x^2 + y^2)^3 - (12*x)/(x^2 + y^2)^2 + (48*x*y^2)/(x^2 + y^2)^3 - (96*x^3*y^2)/(x^2 + y^2)^4  
fx >>
```



$$u = xy^2z^3, \text{求} du$$

```
>> syms x y z dx dy dz;  
>> u = x * y ^ 2 * z ^ 3;  
>> dux = diff(u,x);  
>> duy = diff(u,y);  
>> duz = diff(u,z);  
>> du = dux * dx + duy * dy + duz * dz
```

命令行窗口

```
>> syms x y z dx dy dz;  
>> dux=diff(u,x);  
>> duy=diff(u,y);  
>> duz=diff(u,z);  
>> du=dux*dx+duy*dy+duz*dz
```

du =

```
dx*((32*x^3*y^5)/(x^2 + y^2)^3 - (48*x^5*y^5)/(x^2 + y^2)^4) + dy*((40*x^4*y^4)/(x^2 + y^2)^3 - (48*x^4*y^6)/(x^2 + y^2)^4)
```

求 $z = 2x^2 + y^2$ 在点(1,1)处的切平面、法线方程，并画出他们的图形。

```
>> syms x y z
>> f = 2*x^2+y^2-z;
>> fx = diff(f,x); fy = diff(f,y);
>> x = 1; y = 1; z = 3;
>> fxv = eval(fx), fyv = eval(fy)
```

fxv =

4

fyv =

2

```
>> [x,y] = meshgrid(-2:0.1:3);
z1 = 2*x.^2+y.^2;
z2 = fxv*(x-1)+fyv*(y-1)+3;
t = -1:0.1:1;
x3 = fxv*t+1; y3 = fyv*t+1; z3 = -t+3;
hold on
mesh(x,y,z1)
mesh(x,y,z2)
plot3(x3,y3,z3)
hold off
```

切平面方程

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) = z - z_0$$

法线方程

$$\frac{x - x_0}{f_x(x_0, y_0)} = \frac{y - y_0}{f_y(x_0, y_0)} = \frac{z - z_0}{-1}$$

