第一型曲面积分

设S的方程为

$$S: z = z(x,y), \quad (x,y) \in D_{xy}$$

第一型曲面积分的计算公式

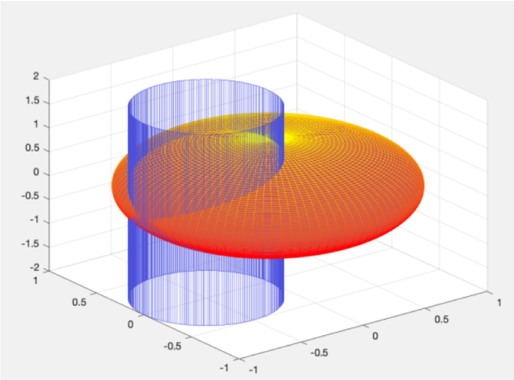
$$\iint_S f(x,y,z)dS = \iint_{D_{xy}} f(x,y,z(x,y))\sqrt{1+z_x^2(x,y)+z_y^2(x,y)}dxdy$$

第一型曲面积分

例：求球面 $x^2+y^2+z^2=1$ 含在柱面 $x^2+y^2=x$ 内部的曲面S面积。即求第一型曲面积分 $\iint\limits_{S} dS$

确定曲面S

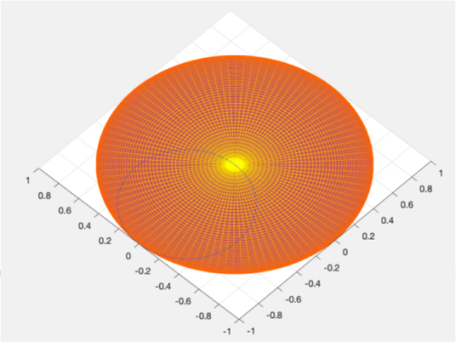
```
>> [x,y,z] = sphere(50);
h1 = mesh(x,y,z); h1.EdgeColor = rand(1,3);
hold on
[x,y,z] = cylinder(1/2,100);
x = x-1/2;
z = 2*z; colormap(hot)
h2 = surf(x,y,z); h2.FaceColor = rand(1,3);
h3 = surf(x,y,-z); h3.FaceColor = h2.FaceColor;
hold off
hidden off
```



确定投影平面，转化为二重积分

$$D_{xy}: r=\cos\theta,\quad -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$$

$$\iint\limits_{S} dS = \iint\limits_{D_{xy}} \sqrt{1+z_x^2(x,y)+z_y^2(x,y)}\,dxdy$$



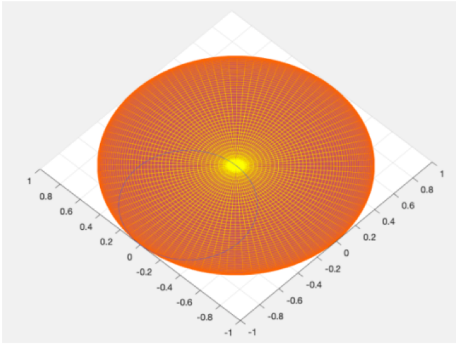
第一型曲面积分

例：求球面 $x^2+y^2+z^2=1$ 含在柱面 $x^2+y^2=x$ 内部的曲面S面积。即求第一型曲面积分 $\iint\limits_S dS$

二重积分计算

```
>> syms x y
>> z = sqrt(1-x^2-y^2);
>> zx = diff(z,x);
>> zy = diff(z,y);
>> f = sqrt(1+zx^2+zy^2);
```

$$\iint\limits_S dS = \iint\limits_{D_{xy}} \sqrt{1+z_x^2(x,y)+z_y^2(x,y)}dxdy$$



$$\iint\limits_D f(x,y)dxdy=\iint\limits_D f(r\cos t, r\sin t)rdrdt$$

```
>> syms r t
>> newf = subs(f,[x,y],[r*cos(t),r*sin(t)]);
>> f_final = simplify(newf);
```

```
>> area = int(int(f_final*r,r,0,cos(t)),t,-pi/2,pi/2)
```

$$\iint\limits_S dS = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^{\cos\theta} \frac{1}{\sqrt{1-r^2}} rdr = \text{pi-2}$$