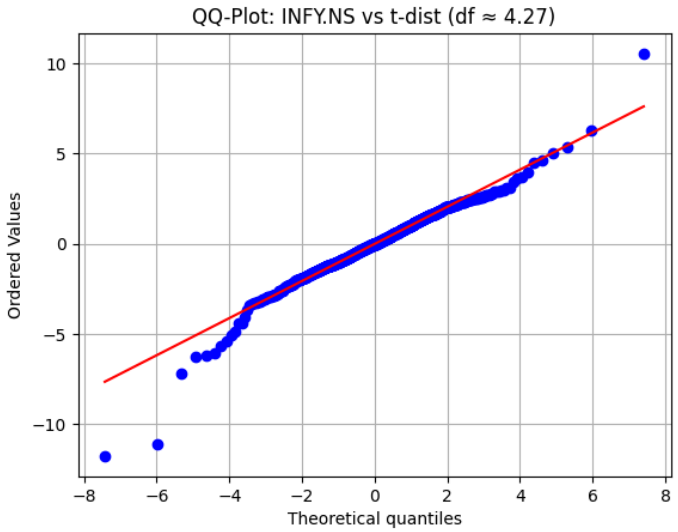
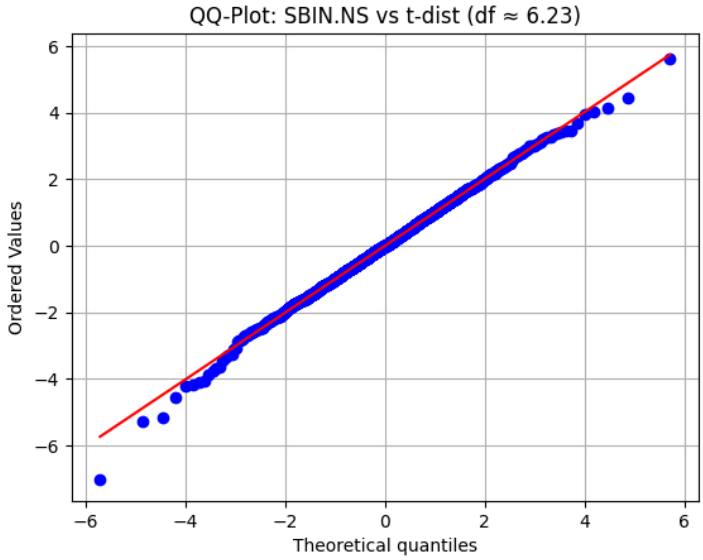
I will first share the plots etc related to model trained on data from 2000 to 2015 for sbi stock infosys stock before backtesting :

Plot on standardised residuals got from GJR ARMA GARCH :
hill estimate was taken on that data where gpd is fitted.



Right Tail: ξ = 0.4589, implied t-df ≈ 2.18
Left Tail: ξ = 0.4501, implied t-df ≈ 2.22

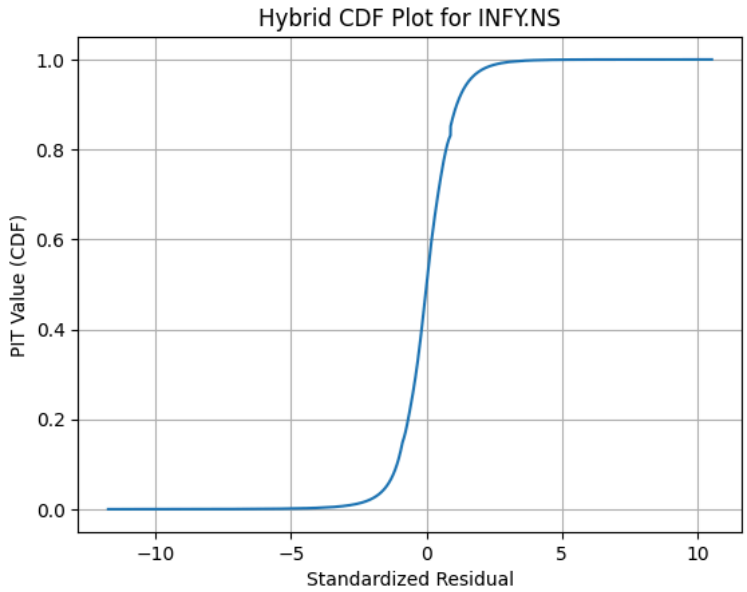


Right Tail: ξ = 0.3936, implied t-df ≈ 2.54
Left Tail: ξ = 0.4334, implied t-df ≈ 2.31

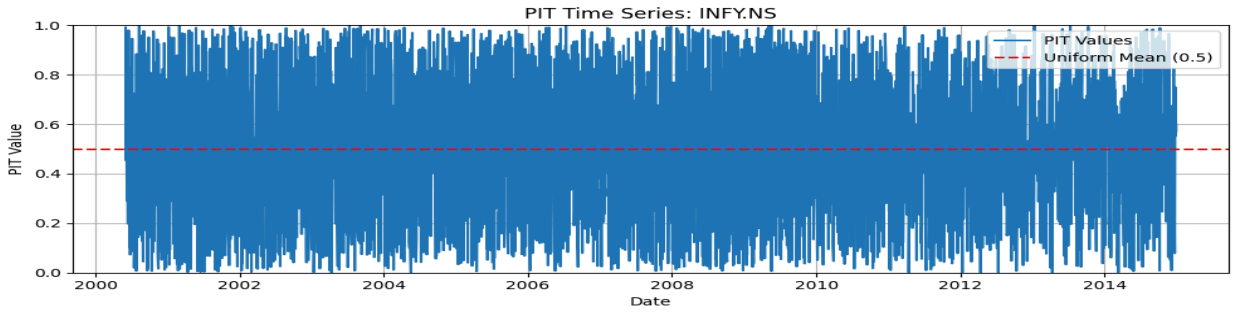
Thresholds : selected by considering the threshold for which there are more exceedences among those which satisfy p values conditions and shape greater than zero

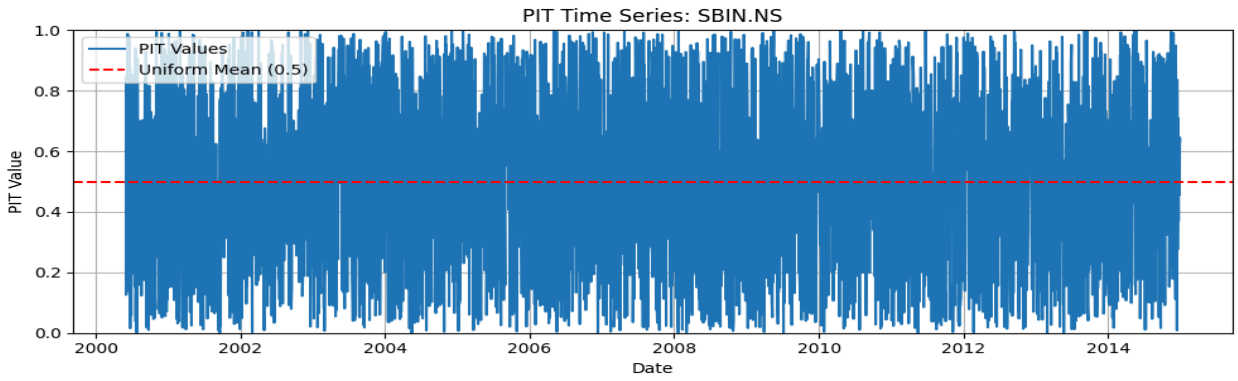
	Stock	Tail	Threshold (u)	Shape	Loc	Scale	KS Statistic	KS p-value	adpvale	AD Statistic	Num Exceedances
0	INFY.NS	upper	0.888134	0.082975	0	0.569218	0.029843	0.700872	0.513333	0.815261	549
1	INFY.NS	lower	-0.880525	0.245557	0	0.481040	0.044728	0.215527	0.176667	1.556100	549
2	SBIN.NS	upper	1.060755	0.004230	0	0.596688	0.021404	0.982632	0.963333	0.234607	454
3	SBIN.NS	lower	-0.935413	0.068135	0	0.561687	0.033040	0.575110	0.516667	0.814327	549

CDF plot after applying gpd and kde :



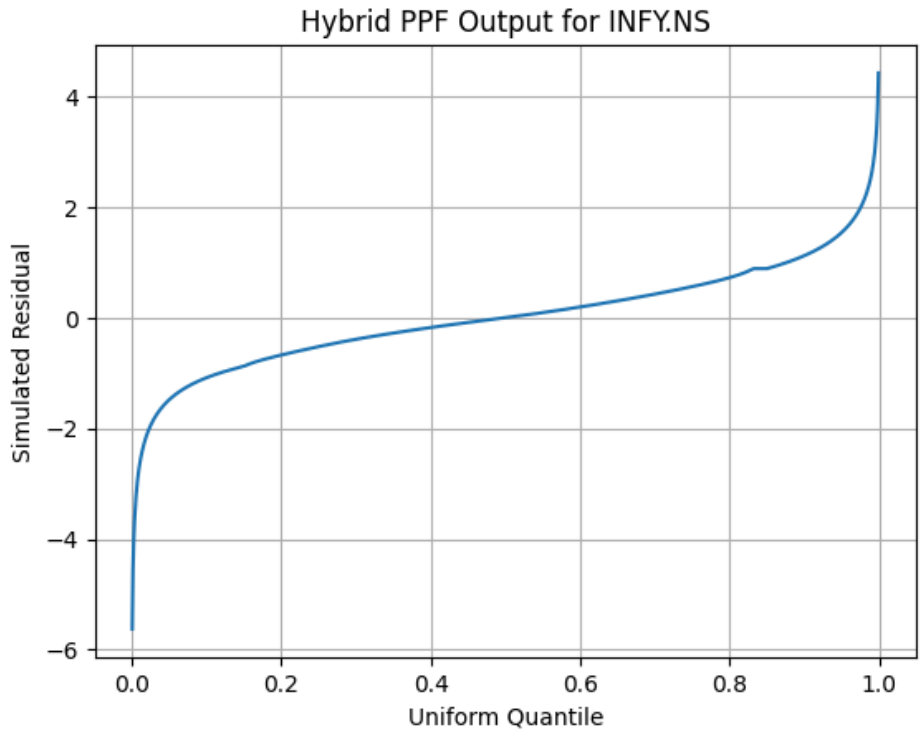
Pit values :



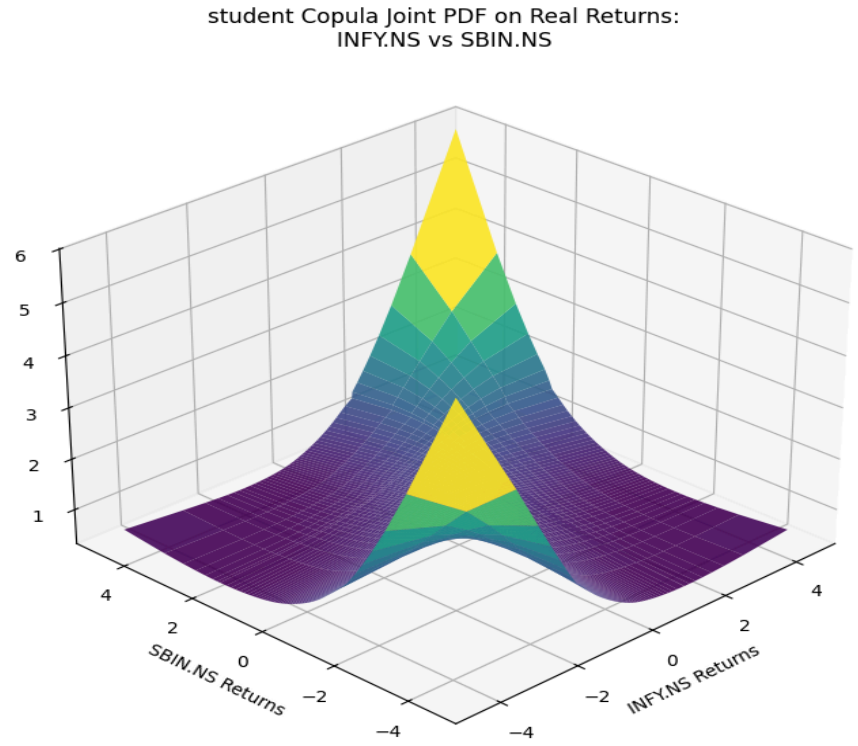


Pit uniformity test :
INFY.NS: KS statistic = 0.0190, p-value = 0.1414
SBIN.NS: KS statistic = 0.0198, p-value = 0.1104

Inverse function for pit to residuals:



Joint probability distribution :



Tail dependency :
By considering bootstrapping 5000 times.
95% CI for λ_L: (np.float64(0.2872923154193872), np.float64(0.3642735176382287))
95% CI for λ_U: (np.float64(0.21052386552974614), np.float64(0.28102189781021897))

The actual tail dependencies of simulated and empirical pit values are :
empirical_lambda_L:0.3248638838475499 , empirical_lambda_U:0.2459016393442623
sim_lambda_L:0.32717190388170053,sim_lambda_U: 0.28807339449541286

Sim_lambda_u is little bit outside to 95 percent confidence interval .

Copula gof test :
For t copula
Cramér–von Mises statistic: 0.000073
Global GoF p-value: 0.136

Backtesting:

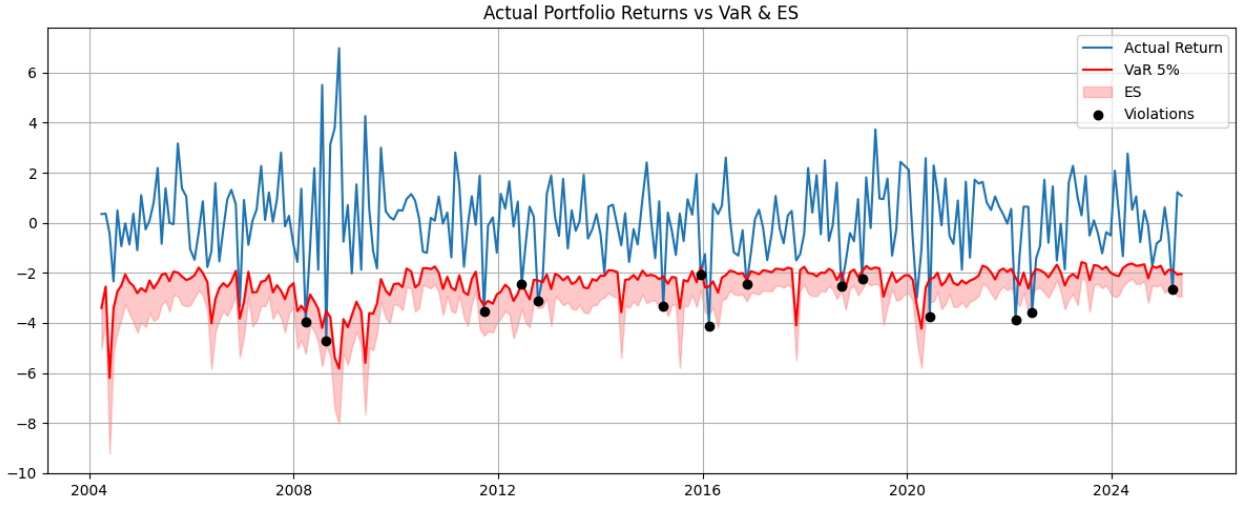
I have backtested by taking a rolling window of 1000 points and i have done this for data from 2000 to 2025 by moving a window by step size of 20 . at every rolling window i estimated return for one day and able to get around 260 days var for entire period. During backtesting there are some scenarios where no tests are passing for gpd fit then i did gpd fit to only those that passes gpd tests and for remaining data i have used kde. If left tail passes i did gpd to left tail but for remaining data i have used kde for center and right tail .

VaR violations: 15 / 262 (5.73%)
Kupiec Test p-value: 0.6134
Christoffersen p-value: 0.1761
Acerbi–Szekely ES stat: 2.0949
Acerbi–Szekely crit value: 1.6107
Acerbi–Szekely ES p-value: 0.6753
Mean Absolute ES 0.5568

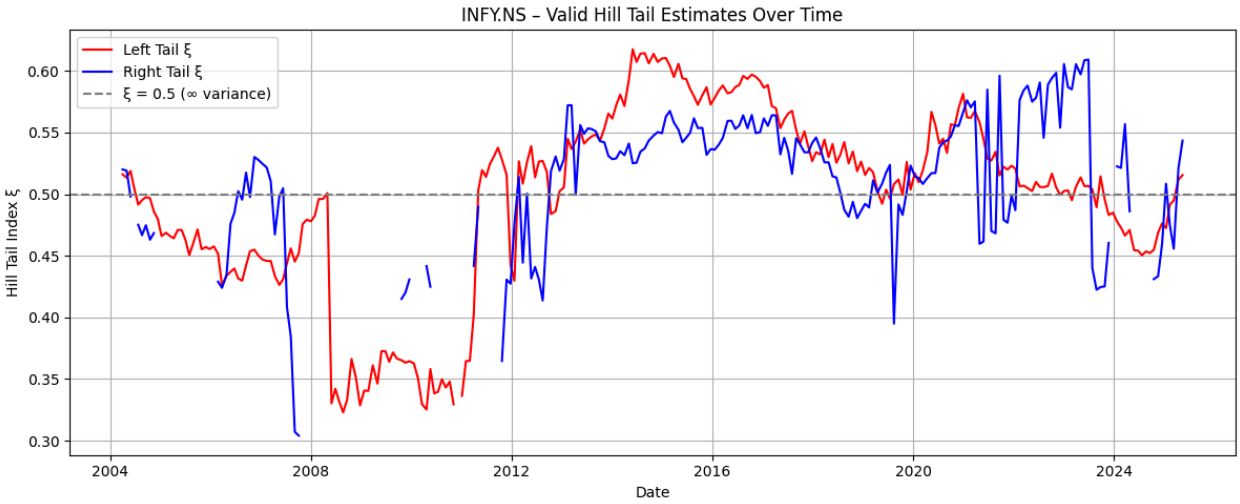
Empirical ES : -1.39

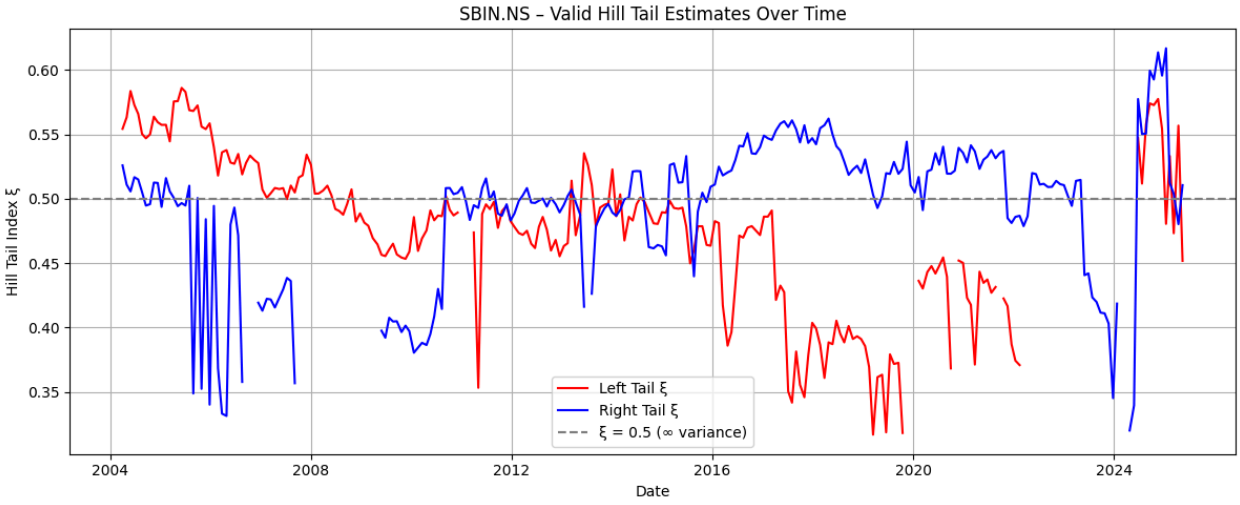
Empirical es is calculated by considering mean of actual returns on violation days.

Plot of actual returns(log(returns)*100) and var on those days when it is calculated :



Hill estimates plot on those days when gpd fit is done with dates :
Hill estimate is calculated on date where gpd is fitted .

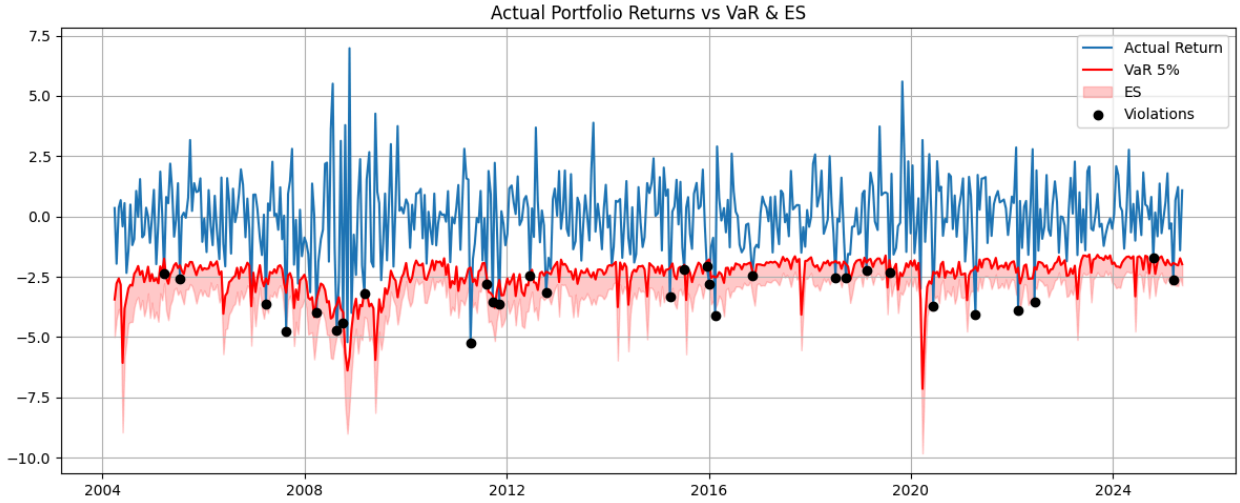


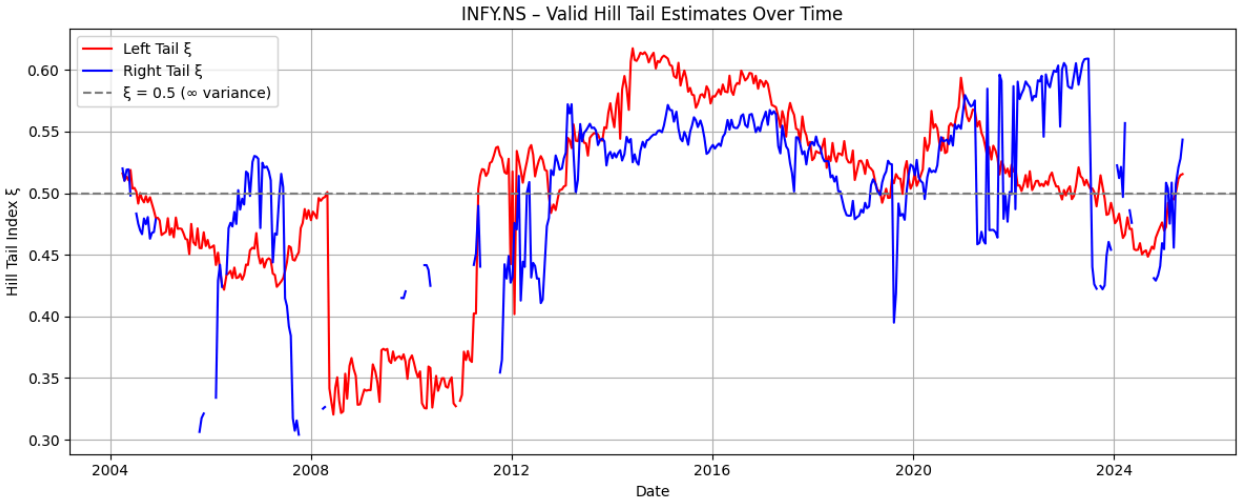


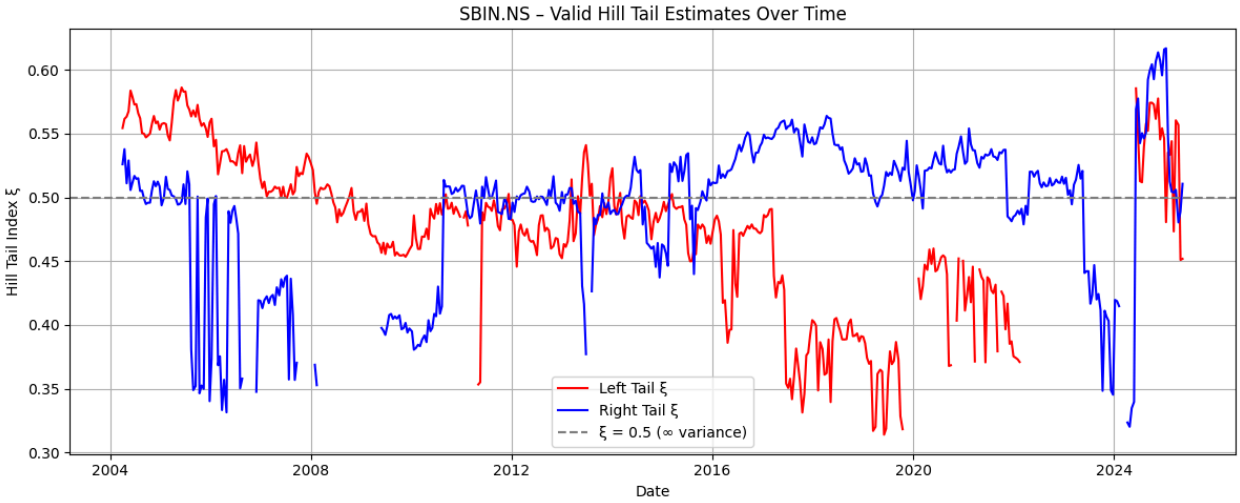
The results for step size of 10 are :

VaR violations:	30 / 523 (5.74%)
Kupiec Test p-value:	0.4691
Christoffersen p-value:	0.5288
Acerbi-Szekely ES stat:	2.1169
Acerbi-Szekely crit value:	1.6883
Acerbi-Szekely ES p-value:	0.7592
Mean Absolute ES Error:	0.5764

Empirical ES : -1.39


Actual Portfolio Returns vs VaR & ES
Actual Return
VaR 5%
ES
Violations
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
2004
2008
2012
2016
2020
2024


INFY.NS – Valid Hill Tail Estimates Over Time
Left Tail ξ
Right Tail ξ
ξ = 0.5 (∞ variance)
Hill Tail Index ξ
0.60
0.55
0.50
0.45
0.40
0.35
0.30
2004
2008
2012
2016
2020
2024
Date

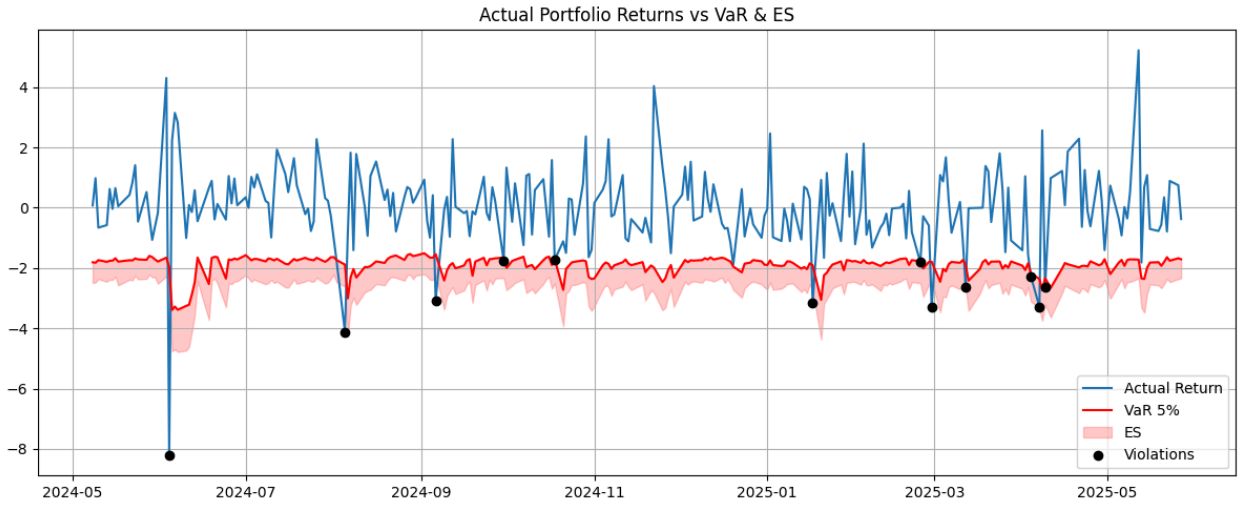

SBIN.NS – Valid Hill Tail Estimates Over Time
Left Tail ξ
Right Tail ξ
ξ = 0.5 (∞ variance)
Hill Tail Index ξ
0.60
0.55
0.50
0.45
0.40
0.35
0.30
2004
2008
2012
2016
2020
2024
Date

Results with window stepsize of one from 2020 to 2025 :

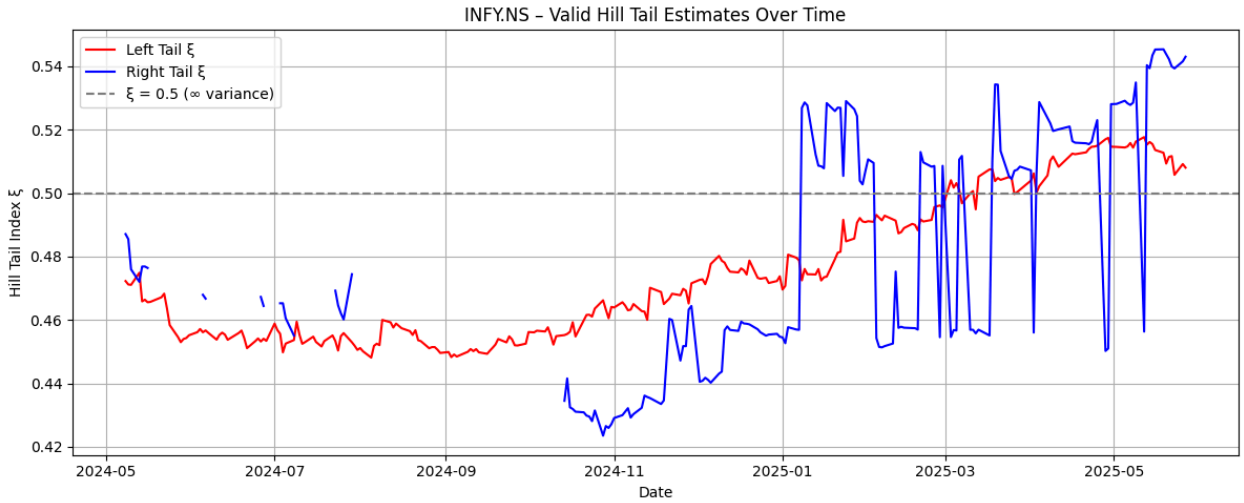
VaR violations: 12 / 261 (4.60%)
Kupiec Test p-value: 0.7563
Christoffersen p-value: 0.5683
Acerbi–Szekely ES stat: 2.0899
Acerbi–Szekely crit value 5 %: 1.6130
Acerbi–Szekely ES p-value: 0.6708
Mean Absolute ES Error: 1.0589
Empirical ES in % : -3.12%,
Acerbi-szekely critl val 95% : 2.455

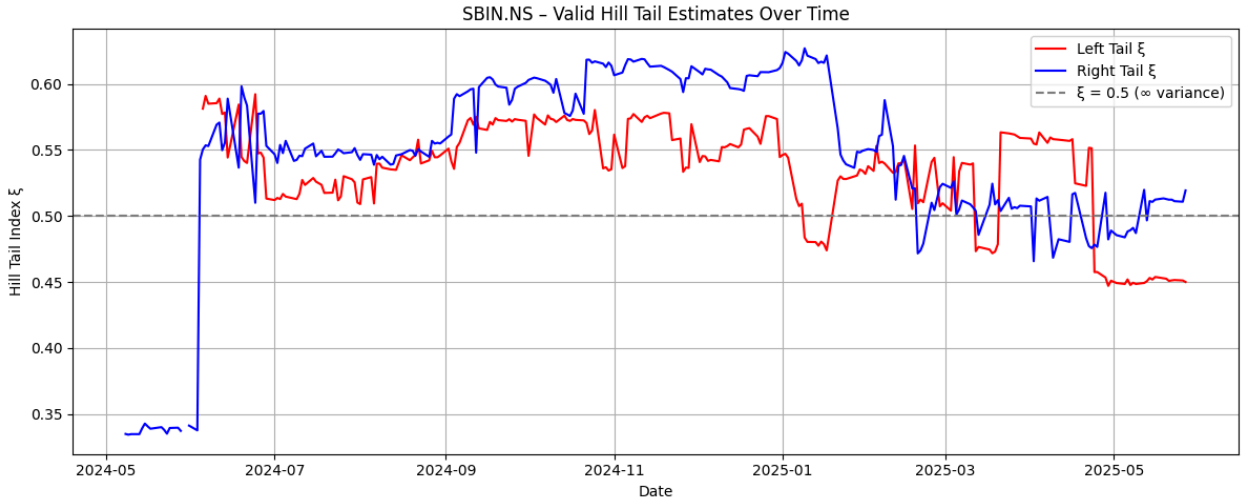
The statistic value is greater than critical value(acerbi szekely test) which is valid as this critical value is calculated at 5 % .it means we are not underestimating risk .the p value is greater than 0.05 so it is acceptable. The statistic value is lesser than critical value calculated at 95 percent .

The plot of actual returns vs var and ES :



Hill plots :





Colab link for modeling :
https://colab.research.google.com/drive/12cH_uzLyoL2TzS0BABUgMJJkPTtMmYo6?usp=sharing

Colab link for backtesting :
https://colab.research.google.com/drive/1gmZtV9yIsKJ0UTsSm2te1POnyQcwsCJd?usp=sharing