GENERAL ELECTRIC
SPACE DIVISION
PHILADELPHIA
PROGRAM INFORMATION REQUEST / RELEASE

PIR NO.	*CLASS. LTR.	OPERATION	PROGRAM	SEQUENCE NO.	REV. LTR.
	U	1K21	UARS	123	A

*USE "C" FOR CLASSIFIED AND "U" FOR UNCLASSIFIED

FROM	TO
P. JASPER - Room M7222 PEJ	R. QUINN - UARS Systems Analysis Mgr.

DATE SENT	DATE INFO. REQUIRED	PROJECT AND REQ. NO.	REFERENCE DIR. NO.
October 28, 1985			

SUBJECT

UARS INSTRUMENT REFERENCE FRAMES

INFORMATION REQUESTED/RELEASED

REVISIONS

This is revision A of this PIR. Corrections and additions are indicated by a vertical line on the right margin of each change page. Besides a few typos, the basic changes are: a) Table 1 has been updated and the instruments numbered from 1 to 10 for reference purposes, b) the page ordering has been changed for clarity, c) Appendix A deriving the geocentric limb tangency point has been added, d) Appendix B deriving the elements of the transfomation matrix between the the Local Vertical Orbital Frame and the Earth Centered Inertial Frame has been added and e) a transformation matrix has been defined for the SSPP Instrument Pointing Frame of reference consistent with PIR U-1R50-UARS-029, "SSPP Coordinate System." In PIR-029, the boresights of the SSPP instruments are defined in terms of the angles α and β. Since α is measured differently for the SSPP instruments than it is for the other instruments, it is referred to as δ in this document to avoid confusion. Angle β remains defined as in PIR-029.

INTRODUCTION

Purpose of this PIR is to lay the groundwork for development of the software pointing algorithms for processing UARS pointing data and also for developing computer simulations of pointing events such as sun, moon, or stellar calibration sightings. We also wish to establish a standard reference coordinate frame for defining the nominal boresight of each instrument. A standard pointing convention for all instruments will greatly simplify the on-board computer software and the ground software and furthermore reduces the likelihood of an error in the mathematical transformations and the software algorithms. In addition, it was felt useful to include the transformation matrices to show how we will relate the boresight unit vector of a given instrument in its own frame of reference to its pointing components in Earth Centered Inertial, ECI, coordinates. Thus, any measurement taken at some time, t, may be directly related to the current position of the earth and the atmosphere at that instant of time.

ACTION REQUIRED

If in reviewing this document, you find any errors or discrepancies, please call me directly and let me know. (Pete Jasper (215)-354-1519). Revisions will be issued as required.

PIR No.

RETENTION REQUIREMENTS: COPIES FOR	MASTERS FOR
1 MO.	3 MOS.
3 MOS.	6 MOS.
6 MOS.	12 MOS.
MOS.	MOS.
	DO NOT DESTROY

TECHNICAL DISCUSSION

Geocentric Limb Tangency Point

There is a requirement that the UARS on-board computer will issue a message to the limb sensors when the when the limb altitude changes by an increment of 2.5 km. See PIR 1K21-UARS-148, "System Level Flight Software Requirements" para 5.7, page 62. The limb altitude computation will be based on an ellipsoidal model of the earth's radius and the distance from the center of the earth to the geocentric limb tangency point. Ths point is defined as the intersection of a nominal limb boresight with a plane which is normal to the boresight vector and which also passes through the center of the earth. Appendix A derives the ECI coordinates of the limb tangency point as well as the corresponding sub-latitude and sub-longitude on the earth.

Instrument Calibration

If the SSPP instruments are to pointed at a celestial object at some time t, such as the sun or moon, then the boresight gimbal angles will be issued in terms of $\delta(t)$ and $\beta(t)$ relative to the Instrument Reference Frame X_R Y_R Z_R as described on page 17. Calibration of the limb instruments may require a spacecraft roll so that a given boresight is raised above the atmosphere. In such a case , the celestial object at time t will be predicted to be at some precomputed value of azimuth α and elevation ε. When the celestial object appears in the instrument's field of view, the actual measured value of α and ε can then be compared with the predicted values for refining the alignment matrices and calibrating the instrument boresight pointing direction.

Definition of Transformation Matrices

In order to determine exactly where a particular instrument is pointing at any instant of time t, there are 6 transformations required between boresight pointing of the instrument relative to its own frame and the resultant unit vector in inertial space. In their order of usage, they are:

F Transformation from Instrument Pointing Frame to Instrument Reference Frame.

E Transformation from Instrument Reference Frame to Observatory Reference Frame

D Transformation from Observatory Reference Frame to MACS Reference Frame. Multi Mission Attitude Control System

C Transformation from MACS Reference Frame to Local Vertical Reference Frame.

B Transformation from Local Vertical Reference Frame to Local Vertical Orbital Reference Frame.

A Transformation from Local Vertical Orbital Reference Frame to Earth Centered Inertial Frame.

The manner in which these transformations are combined to find an instrument's pointing direction in the Earth Centered Inertial Frame is shown on page 3. Once the ECI pointing direction is established, the geocentric limb tangency point can be found for the limb lookers as described in Appendix A.

COMPILATION OF BORESIGHT UNIT VECTOR IN ECI COORDINATES

To obtain the boresight unit pointing vector of instrument i in ECI coordinates, (X Y Z), given that the boresight pointing angles are α and ε (instruments 1-7) or δ and β (instruments 8-10), then:

elements of F change w sun Δ, these don't)

$\begin{pmatrix} X_U \\ Y_U \\ Z_U \end{pmatrix}$ =	A	B	C	D	E	F	$\begin{pmatrix} X_I \\ Y_I \\ Z_I \end{pmatrix}$
	Transformation from Local Vertical Orbital Frame to ECI Frame	Transformation from Local Vertical Frame to Local Vertical Orbital Frame	Transfomation from MACS Frame to Local Vertical Frame	Transformation from Observatory Frame to MACS Frame	Transformation from Instrument Reference Frame to Observatory Frame	Transformation from Instrument Pointing Frame to Instrument Reference Frame	
Boresight unit vector in ECI coordinates.	Based upon current position and velocity of UARS.	Reflects current yaw orientation of UARS. Flying forward or backward.	Based upon current attitude errors of UARS.	Reflects mechanical misalignments between frames.	Reflects mechanical misalignments between frames. There is one matrix for each instrument.	Reflects current α,ε or δ,β for each boresight. There is one matrix for each boresight.	Z_I is boresight pointing direction of instrument i.

$$\begin{pmatrix} X_U \\ Y_U \\ Z_U \end{pmatrix} = (A)(B)(C)(D)(E)(F)\begin{pmatrix} X_I \\ Y_I \\ Z_I \end{pmatrix}$$

FIGURE 1

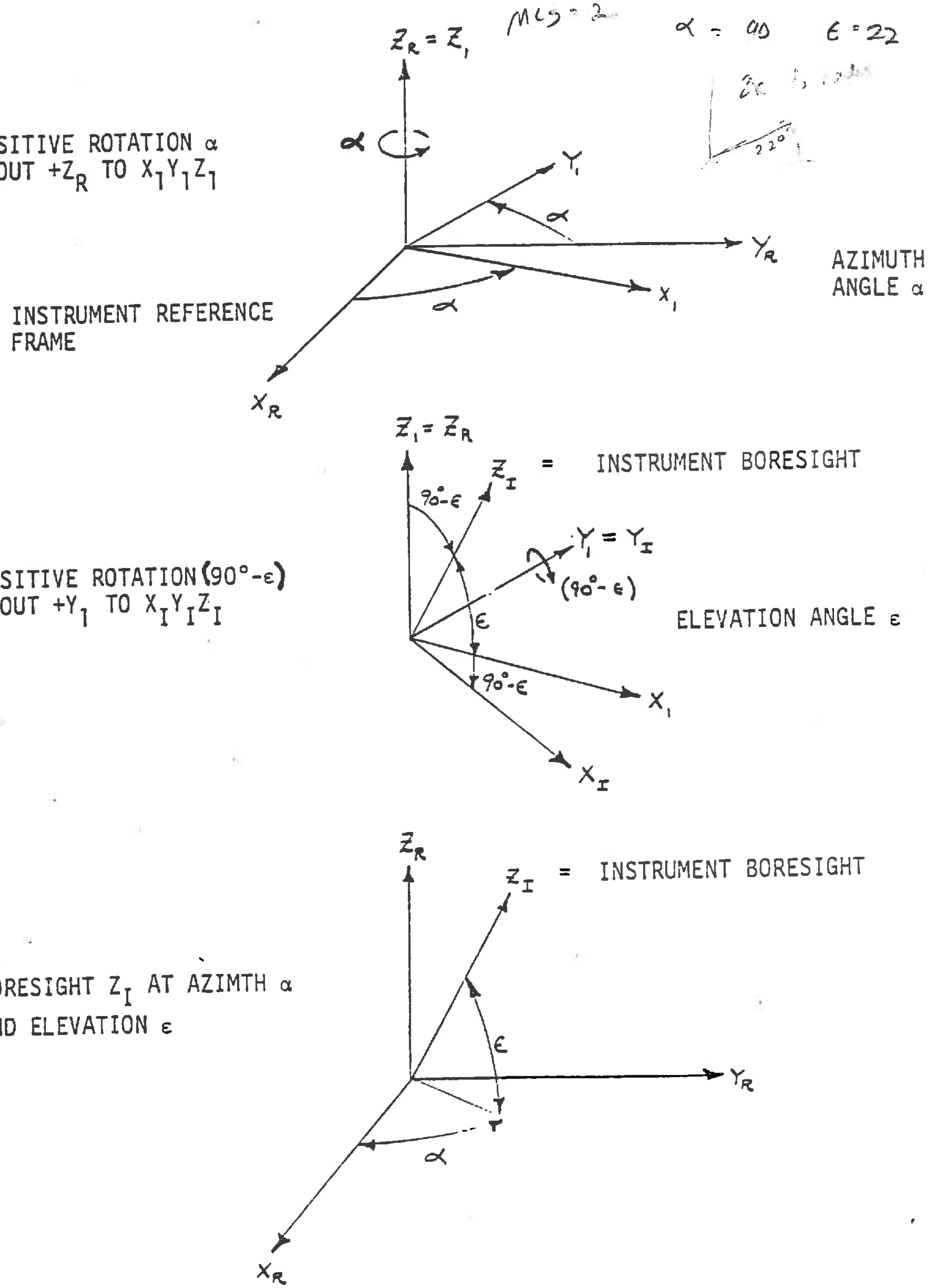
DEFINITION OF INSTRUMENT POINTING FRAME AXES $\hat{X}_I \hat{Y}_I \hat{Z}_I$

(Instruments 1 through 7)

1. POSITIVE ROTATION α ABOUT $+Z_R$ TO $X_1Y_1Z_1$

2. POSITIVE ROTATION (90°-ε) ABOUT $+Y_1$ TO $X_IY_IZ_I$

3. BORESIGHT Z_I AT AZIMTH α AND ELEVATION ε



DERIVATION OF TRANSFORMATION MATRIX BETWEEN INSTRUMENT POINTING FRAME AND INSTRUMENT REFERENCE FRAME

(Instruments 1 through 7)

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R \qquad \text{eq. 1}$$

Positive rotation α about $+Z_R$ to $X_1 Y_1 Z_1$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I = \begin{pmatrix} \cos(90-\varepsilon) & 0 & -\sin(90-\varepsilon) \\ 0 & 1 & 0 \\ \sin(90-\varepsilon) & 0 & \cos(90-\varepsilon) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 \qquad \text{eq. 2}$$

Positive rotation $(90-\varepsilon)$ about $+Y_1$ to $X_I Y_I Z_I$

Using trig identities, Eq. 2 becomes,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I = \begin{pmatrix} \sin(\varepsilon) & 0 & -\cos(\varepsilon) \\ 0 & 1 & 0 \\ \cos(\varepsilon) & 0 & \sin(\varepsilon) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 \qquad \text{eq. 3}$$

Transposing Eq. 1,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 \qquad \text{eq. 4}$$

Transposing Eq. 3,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 = \begin{pmatrix} \sin(\varepsilon) & 0 & \cos(\varepsilon) \\ 0 & 1 & 0 \\ -\cos(\varepsilon) & 0 & \sin(\varepsilon) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I \qquad \text{eq. 5}$$

Substituting equation 5 into equation 4 and multiplying the matrices,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R = \begin{pmatrix} \cos(\alpha)\sin(\varepsilon) & -\sin(\alpha) & \cos(\alpha)\cos(\varepsilon) \\ \sin(\alpha)\sin(\varepsilon) & \cos(\alpha) & \sin(\alpha)\cos(\varepsilon) \\ -\cos(\varepsilon) & 0 & \sin(\varepsilon) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I \qquad \text{eq. 6}$$

INSTRUMENT REFERENCE FRAME | (MATRIX F) | INSTRUMENT POINTING FRAME

MLS (see p18,) well $\varepsilon = 23.3°$, and $22°$

$$F = \begin{pmatrix} 0 & -1 & 0 \\ 0.39555 & 0 & +.91845 \\ -.91845 & 0 & 0.39555 \end{pmatrix}$$

$$\vec{V}_R = F \, \vec{V}_I$$

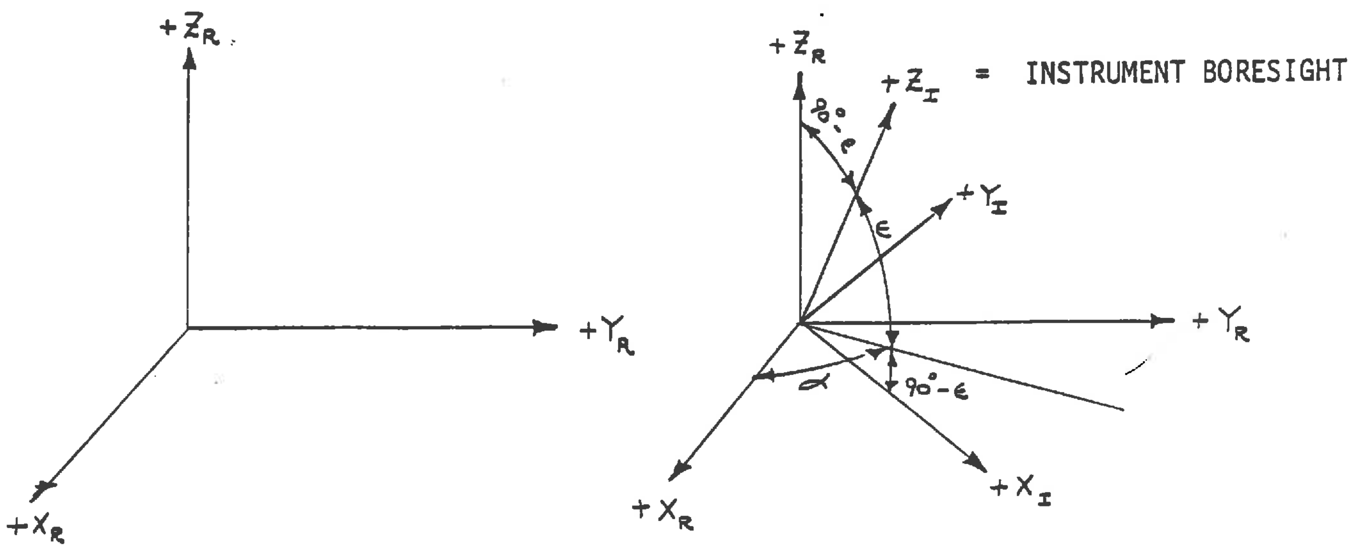
FIGURE 2

INSTRUMENT POINTING FRAME $\hat{X}_I \hat{Y}_I \hat{Z}_I$

TO

INSTRUMENT REFERENCE FRAME $\hat{X}_R \hat{Y}_R \hat{Z}_R$

(Instruments 1 through 7)



INSTRUMENT REFERENCE FRAME $X_R Y_R Z_R$

INSTRUMENT POINTING FRAME $X_I Y_I Z_I$

By definition, $+\hat{Z}_I$ is chosen as the instrument boresight

$$\begin{pmatrix} X_R \\ Y_R \\ Z_R \end{pmatrix} = \begin{pmatrix} \cos\alpha \sin\varepsilon & -\sin\alpha & \cos\alpha \cos\varepsilon \\ \sin\alpha \sin\varepsilon & \cos\alpha & \sin\alpha \cos\varepsilon \\ -\cos\varepsilon & 0 & \sin\varepsilon \end{pmatrix} \begin{pmatrix} X_I \\ Y_I \\ Z_I \end{pmatrix}$$

(MATRIX F)

For a unit pointing vector along the boresight

$X_I = Y_I = 0$ and $Z_I = 1$

∴ Components of a unit vector along the boresight in the instrument reference frame are:

$X_R = \cos\alpha \cos\varepsilon$

$Y_R = \sin\alpha \cos\varepsilon$

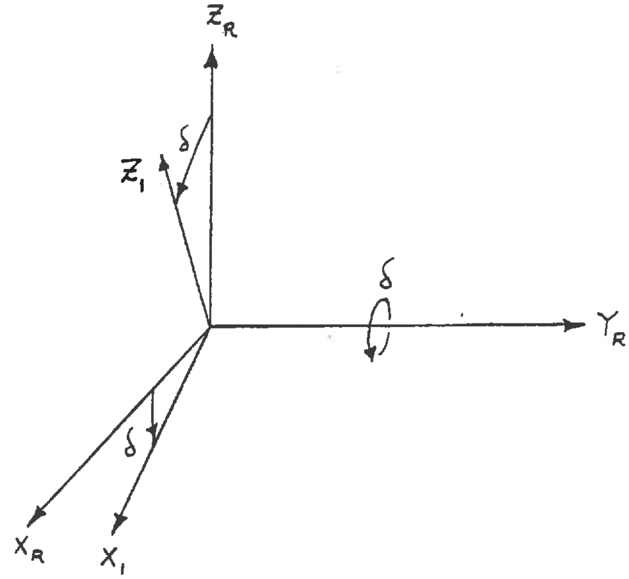
$Z_R = \sin\varepsilon$

FIGURE 3

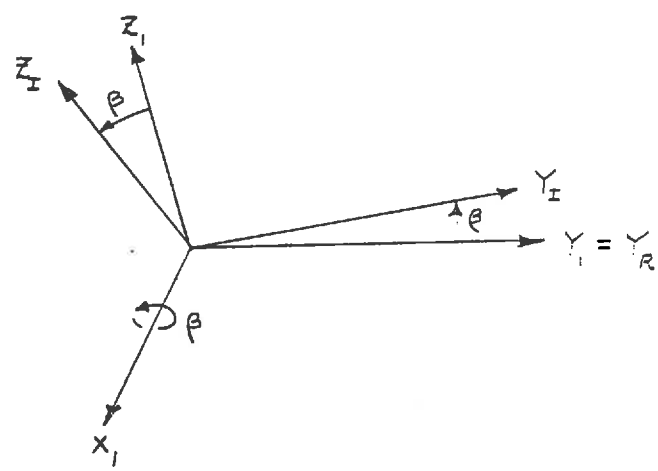
DEFINITION OF INSTRUMENT POINTING FRAME AXES $\hat{X}_I \hat{Y}_I \hat{Z}_I$
(Instruments 8 through 10, SSPP)

NOT MLS

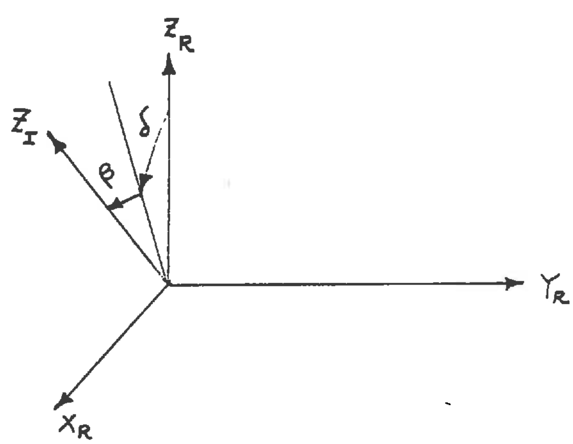
1. POSITIVE ROTATION δ
ABOUT + Y_R TO $X_1 Y_1 Z_1$



2. POSITIVE ROTATION β
ABOUT + X_1 TO $X_I Y_I Z_I$



3. BORESIGHT Z_I
DEFINED BY δ AND β



NO7 MS

DERIVATION OF TRANSFORMATION MATRIX BETWEEN INSTRUMENT POINTING FRAME AND INSTRUMENT REFERENCE FRAME

(Instruments 8 through 10, SSPP)*

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 = \begin{pmatrix} \cos(\delta) & 0 & -\sin(\delta) \\ 0 & 1 & 0 \\ \sin(\delta) & 0 & \cos(\delta) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R \qquad \text{eq. 1}$$

Positive rotation δ about $+Y_R$ to $X_1 Y_1 Z_1$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & \sin(\beta) \\ 0 & -\sin(\beta) & \cos(\beta) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_1 \qquad \text{eq. 2}$$

Positive rotation β ABOUT $+X_1$ to $X_I Y_I Z_I$

combining equations 1 and 2:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & \sin(\beta) \\ 0 & -\sin(\beta) & \cos(\beta) \end{pmatrix} \begin{pmatrix} \cos(\delta) & 0 & -\sin(\delta) \\ 0 & 1 & 0 \\ \sin(\delta) & 0 & \cos(\delta) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R \qquad \text{eq. 3}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I = \begin{pmatrix} \cos(\delta) & 0 & -\sin(\delta) \\ \sin(\beta)\sin(\delta) & \cos(\beta) & \sin(\beta)\cos(\delta) \\ \cos(\beta)\sin(\delta) & -\sin(\beta) & \cos(\beta)\cos(\delta) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R \qquad \text{eq. 4}$$

or, taking the transpose of eq. 4:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_R = \begin{pmatrix} \cos(\delta) & \sin(\beta)\sin(\delta) & \cos(\beta)\sin(\delta) \\ 0 & \cos(\beta) & -\sin(\beta) \\ -\sin(\delta) & \sin(\beta)\cos(\delta) & \cos(\beta)\cos(\delta) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_I \qquad \text{eq. 5}$$

INSTRUMENT REFERENCE FRAME — (MATRIX F) — INSTRUMENT POINTING FRAME

* See "SSPP Coordinate System", PIR U-IR50-UARS-029, F.Mamrol. To avoid confusion with the the other instruments, angle α in PIR-029 is called angle δ in this document. Angle β is the same as described in PIR-029.

NJT Mcs

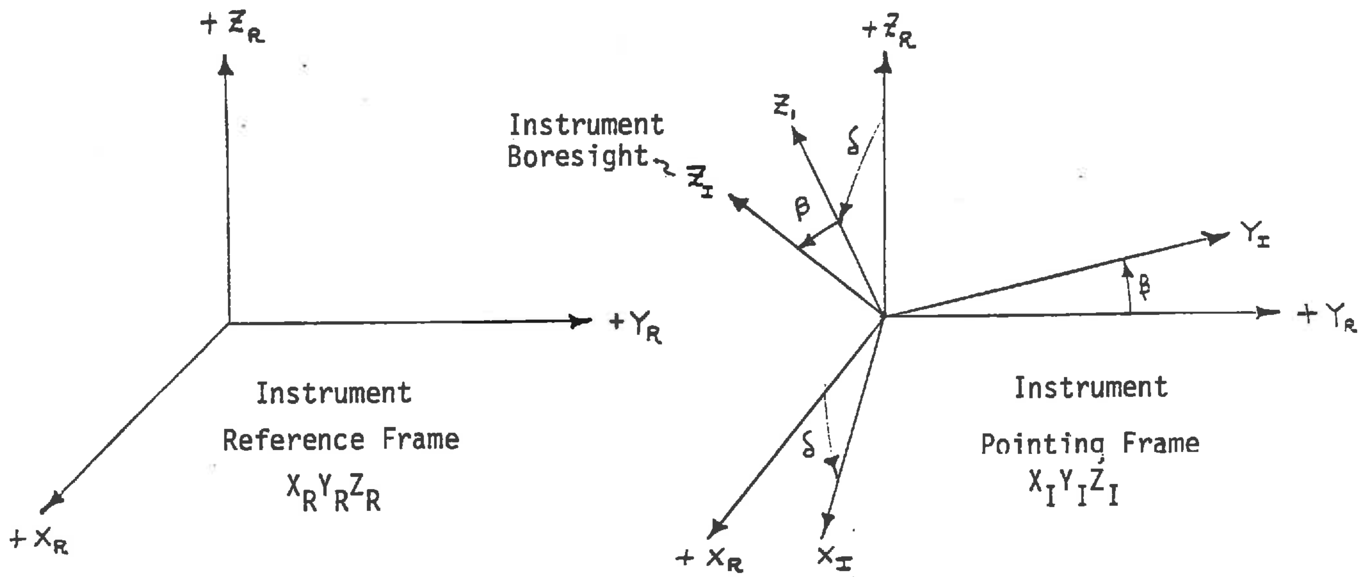
FIGURE 4

INSTRUMENT POINTING FRAME $\hat{X}_I \hat{Y}_I \hat{Z}_I$

TO

INSTRUMENT REFERENCE FRAME $\hat{X}_R \hat{Y}_R \hat{Z}_R$

(Instruments 8-10, SSPP)



By definition, $+\hat{Z}_I$ is chosen as the instrument boresight

$$\begin{pmatrix} X_R \\ Y_R \\ Z_R \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\beta \ \sin\delta & \cos\beta \ \sin\delta \\ 0 & \cos\beta & -\sin\beta \\ -\sin\delta & \sin\beta \ \cos\delta & \cos\beta \ \cos\delta \end{pmatrix} \begin{pmatrix} X_I \\ Y_I \\ Z_I \end{pmatrix}$$

MATRIX F

For a unit pointing vector along the boresight:

$X_I = Y_I = 0$ and $Z_I = 1$

$\therefore$ Components of a unit vector along the boresight in the instrument reference frame are:

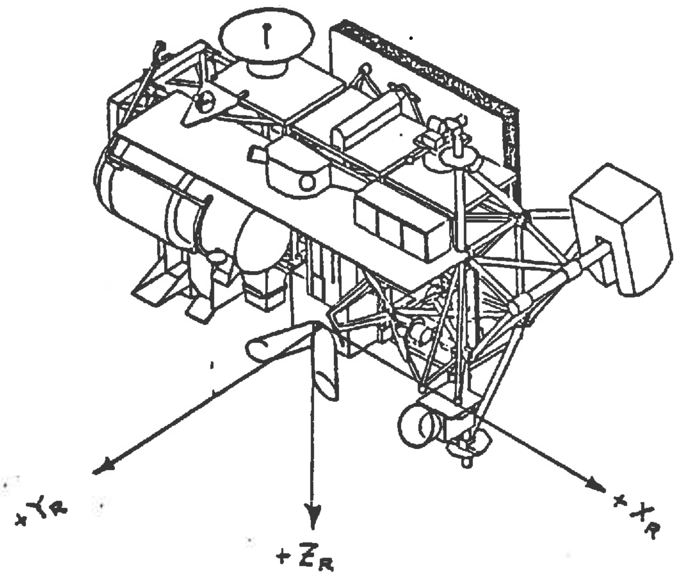
$$X_R = \cos\beta \ \sin\delta$$
$$Y_R = -\sin\beta$$
$$Z_R = \cos\beta \ \cos\delta$$

FIGURE 5

INSTRUMENT REFERENCE FRAME $(\hat{X}_R \hat{Y}_R \hat{Z}_R)_i \quad i = 1, 10$

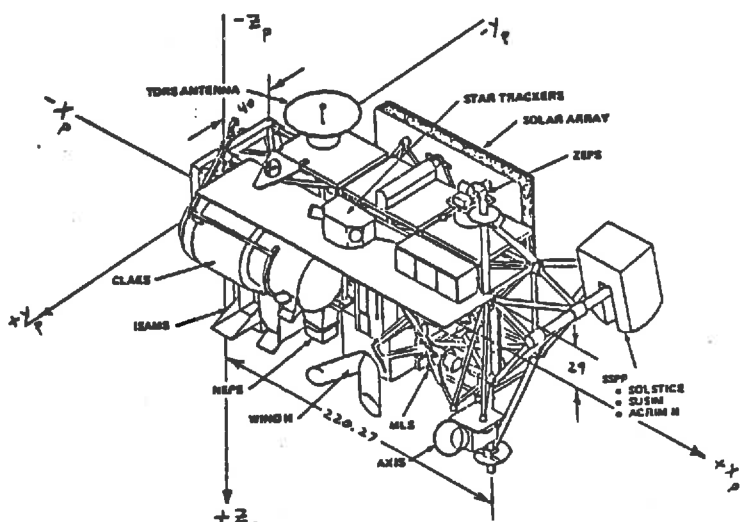
TO

OBSERVATORY REFERENCE FRAME $\hat{X}_P \; \hat{Y}_P \; \hat{Z}_P$



Typical
Instrument
Reference
Frame

1 Reference Frame
for each of
10 Instruments



Observatory
Reference Frame

$$\begin{pmatrix} X_P \\ Y_P \\ Z_P \end{pmatrix} = \underbrace{\overset{\text{Near Unity Matrix}}{\begin{pmatrix} \text{Direction Cosine} \\ \text{Matrix Reflecting} \\ \text{Misalignments} \\ \text{Between Frames} \end{pmatrix}}}_{\text{(MATRIX E)}} \begin{pmatrix} X_R \\ Y_R \\ Z_R \end{pmatrix}$$

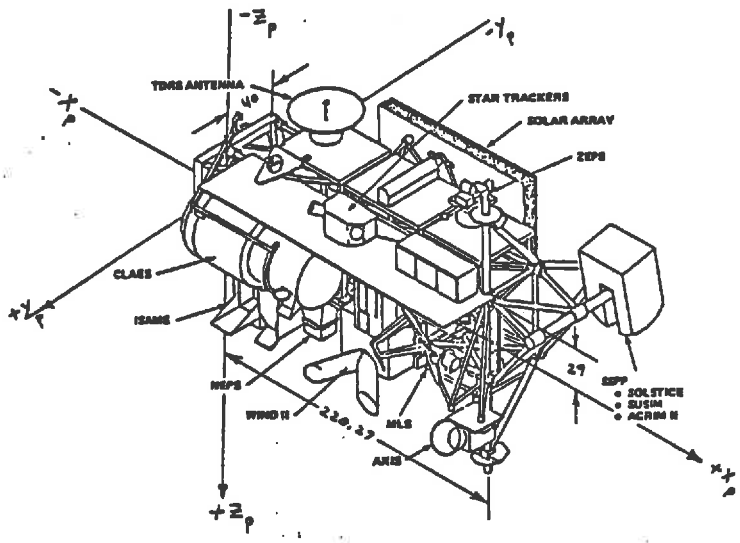
Each instrument will have its own misalignment matrix relative to the Observatory Frame.

FIGURE 6

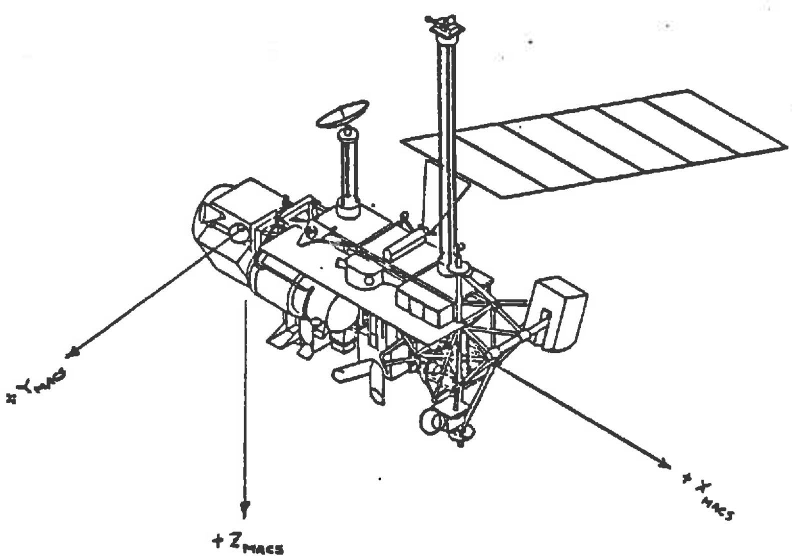
OBSERVATORY REFERENCE FRAME $\hat{X}_p \hat{Y}_p \hat{Z}_p$

TO

MACS REFERENCE FRAME $\hat{X}_{MACS} \hat{Y}_{MACS} \hat{Z}_{MACS}$



OBSERVATORY REFERENCE FRAME



MACS REFERENCE FRAME

For Zero Misalignment Between Frames: $\hat{X}_p \parallel \hat{X}_{MACS}$, $\hat{Y}_p \parallel \hat{Y}_{MACS}$, $\hat{Z}_p \parallel \hat{Z}_{MACS}$

$$\begin{pmatrix} X_{MACS} \\ Y_{MACS} \\ Z_{MACS} \end{pmatrix} = \underbrace{\overbrace{\begin{pmatrix} \text{Direction Cosine} \\ \text{Matrix Reflecting} \\ \text{Misalignments} \\ \text{Between Frames} \end{pmatrix}}^{\text{Near Unity Matrix}}}_{\text{(MATRIX D)}} \begin{pmatrix} X_p \\ Y_p \\ Z_p \end{pmatrix}$$

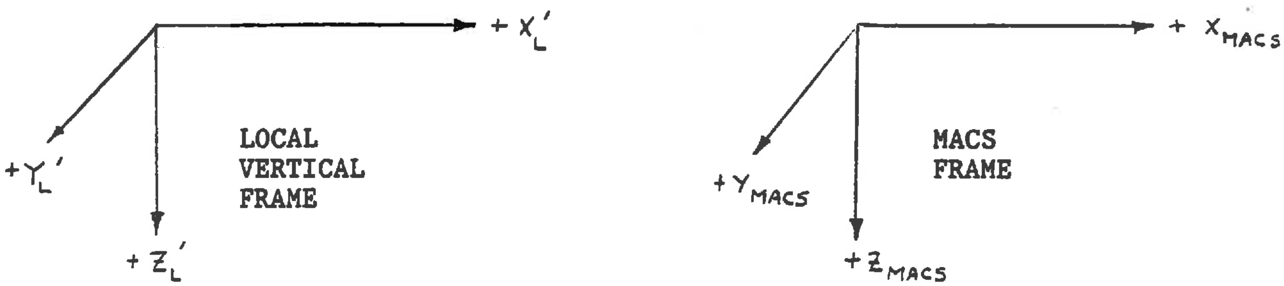
R P Y

FIGURE 7

MACS REFERENCE FRAME $\hat{X}_{MACS}\ \hat{Y}_{MACS}\ \hat{Z}_{MACS}$

TO

LOCAL VERTICAL REFERENCE FRAME $\hat{X}_L'\ \hat{Y}_L'\ \hat{Z}_L'$



For zero attitude errors:

$$\hat{X}_{MACS} \parallel \hat{X}_L'$$
$$\hat{Y}_{MACS} \parallel \hat{Y}_L'$$
$$\hat{Z}_{MACS} \parallel \hat{Z}_L'$$

~~C = R( ) P( ) Y~~

C = Y P R from Evan Fischbein's notes 30 Sept. 1992

For small attitude errors where (< .03 deg/axis)

$\Delta\psi$ = yaw error, rad.
$\Delta\phi$ = roll error, rad.
$\Delta\theta$ = pitch error, rad.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{L'} = \begin{pmatrix} 1 & -\Delta\psi & \Delta\theta \\ \Delta\psi & 1 & -\Delta\phi \\ -\Delta\theta & \Delta\phi & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{MACS}$$

(Matrix C)

$.03° = 5.2 \cdot 10^{-4}$ radians $\approx \sin$

$1 - \cos = 1.37 \cdot 10^{-7}$

error in sine is $2.4 \cdot 10^{-11}$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{L'} = \begin{pmatrix} 1 & -\Delta\psi + (\Delta\theta)\sin(\phi) & (\Delta\psi)\sin(\phi) + \Delta\theta \\ \Delta\psi & 1 & -\sin(\phi) \\ -\Delta\theta & \sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{MACS}$$

for roll ϕ > .03 deg.
pitch θ < .03 deg.
yaw ψ < .03 deg.

(Matrix C)

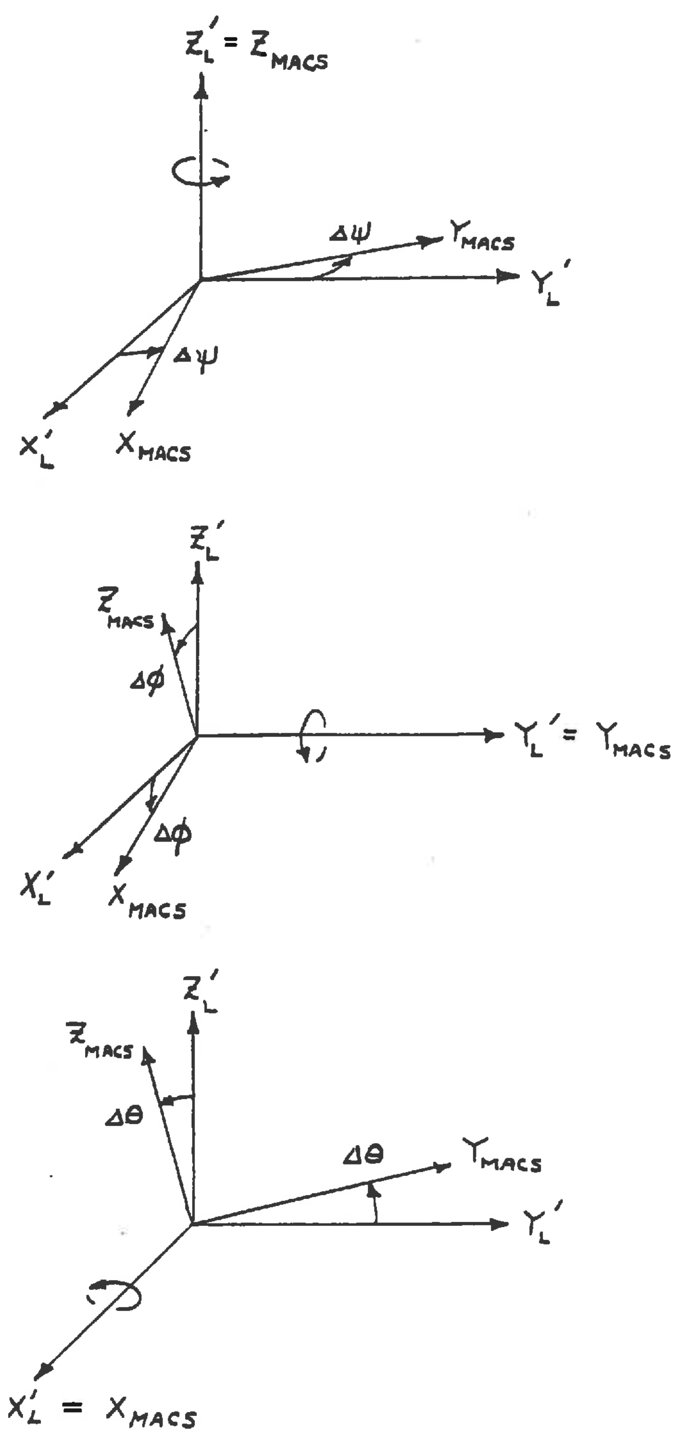
why not $\cos\phi$?

If $\phi = 90°$, $C = \begin{pmatrix} 1 & \Delta\theta - \Delta\psi & \Delta\theta + \Delta\psi \\ \Delta\psi & 1 & -1 \\ -\Delta\theta & 1 & 0 \end{pmatrix}$ not orthogonal! $\|a_2\| = \sqrt{2} + O\,\Delta^2$

FIGURE 8

DEFINITION OF POLARITY OF ATTITUDE ERRORS

RELATIVE TO $\hat{X}_L' \hat{Y}_L' \hat{Z}_L'$ AXES



POSITIVE YAW ERROR, $\Delta\Psi$ AROUND + Z_L'

POSITIVE PITCH ERROR, $\Delta\emptyset$ AROUND $+Y_L'$

POSITIVE ROLL ERROR, $\Delta\Theta$ AROUND $+X_L'$

FIGURE 9

LOCAL VERTICAL REFERENCE FRAME $\hat{X}'_L \hat{Y}'_L \hat{Z}'_L$

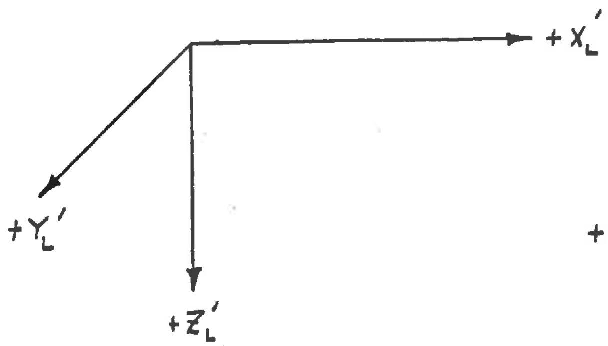
TO

LOCAL VERTICAL ORBITAL REFERENCE FRAME $\hat{X}_L \hat{Y}_L \hat{Z}_L$

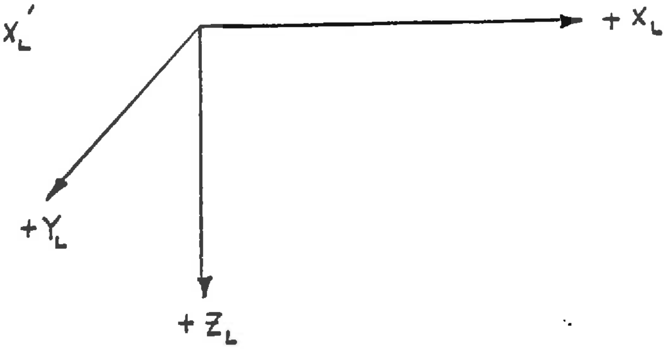
(Accounts for UARS Flying Forward or Backward)

CASE 1

LOCAL VERTICAL FRAME



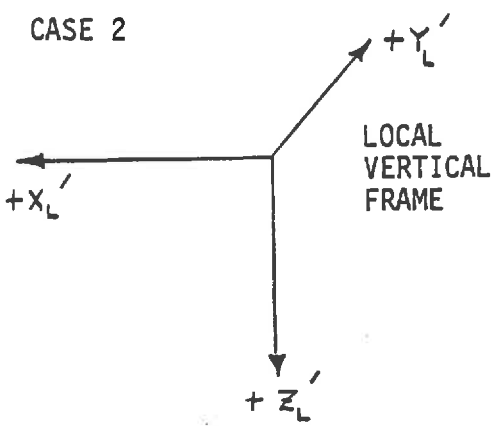
LOCAL VERTICAL ORBITAL FRAME



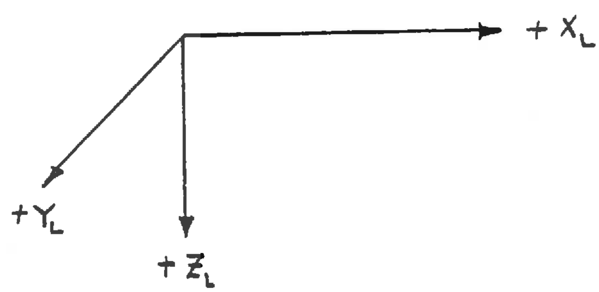
$$\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_L' \\ Y_L' \\ Z_L' \end{pmatrix}$$

USED WHEN:
OBSERVATORY $+X_P$ AXIS IS POINTING IN SAME DIRECTION AS UARS VELOCITY VECTOR. FRAMES ARE IDENTICAL.

CASE 2



LOCAL VERTICAL ORBITAL FRAME



$$\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_L' \\ Y_L' \\ Z_L' \end{pmatrix}$$

MATRIX B

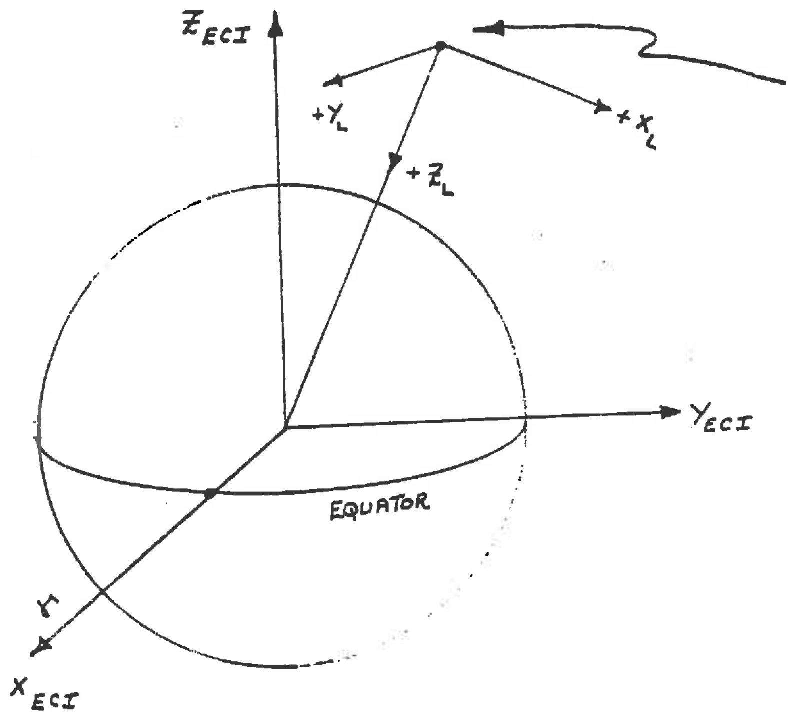
USED WHEN:
UARS IS YAWED 180° SO THAT OBSERVATORY $+X_P$ AXIS IS POINTING IN OPPOSITE DIRECTION TO VELOCITY VECTOR. X AND Y AXES ARE REVERSED BETWEEN FRAMES.

FIGURE 10

LOCAL VERTICAL ORBITAL REFERENCE FRAME $\hat{X}_L \hat{Y}_L \hat{Z}_L$

TO

EARTH CENTERED INERTIAL REFERENCE FRAME $\hat{X}_{ECI} \hat{Y}_{ECI} \hat{Z}_{ECI}$



UARS position and velocity in ECI coordinates $XYZ\ \dot{X}\dot{Y}\dot{Z}$ at time t.

$+\hat{X}_L$ in orbit plane and

$+\hat{Z}_L$ to earth center.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{ECI} = \begin{pmatrix} \dfrac{(\dot{X}Z-\dot{Z}X)Z + (\dot{X}Y-\dot{Y}X)Y}{r^2 V\cos\gamma} & \dfrac{\dot{Y}Z-\dot{Z}Y}{rv\cos\gamma} & \dfrac{-X}{r} \\ \dfrac{(\dot{Y}Z-\dot{Z}Y)Z + (\dot{Y}X-\dot{X}Y)X}{r^2 V\cos\gamma} & \dfrac{\dot{Z}X-\dot{X}Z}{rV\cos\gamma} & \dfrac{-Y}{r} \\ \dfrac{(\dot{Z}Y-\dot{Y}Z)Y + (\dot{Z}X-\dot{X}Z)X}{r^2 V\cos\gamma} & \dfrac{\dot{X}Y-\dot{Y}X}{rV\cos\gamma} & \dfrac{-Z}{r} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{L}$$

where:

$$r = \sqrt{X^2 + Y^2 + Z^2}$$

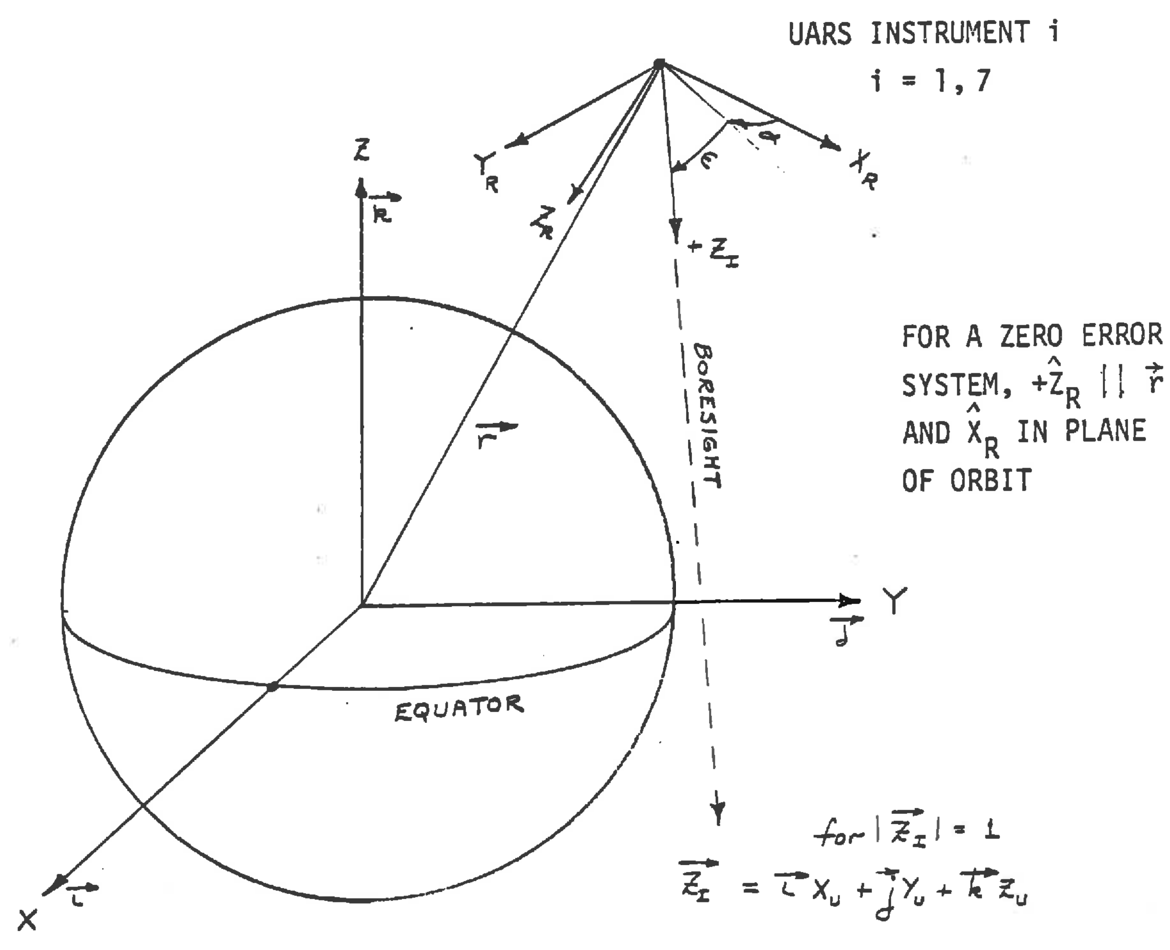
$$V = \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2}$$

$$\cos\gamma = \sqrt{1 - \left(\frac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{rV}\right)^2}$$

(Matrix A)

FIGURE 11

BORESIGHT OF INSTRUMENT i
RELATIVE TO INSTRUMENT REFERENCE FRAME $(\hat{X}_R \hat{Y}_R \hat{Z}_R)_i$
AND ECI FRAME



Computation of Boresight Pointing Angles

If the boresight pointing vector has been determined to be $\begin{pmatrix} X_U \\ Y_U \\ Z_U \end{pmatrix}$ at some time t, then the required pointing direction of the boresight in α, ε or δ, β may be found from:

$$\left\{(A)(B)(C)(D)(E)\right\}^{-1} \begin{pmatrix} X_U \\ Y_U \\ Z_U \end{pmatrix} = (F) \begin{pmatrix} X_I \\ Y_I \\ Z_I \end{pmatrix}$$

For a unit vector along the boresight, $X_I = Y_I = 0$ and $Z_I = 1$.
Then,

$$(F) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha)\ \cos(\varepsilon) \\ \sin(\alpha)\ \cos(\varepsilon) \\ \sin(\varepsilon) \end{pmatrix} \quad \text{or} \quad (F) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\beta)\ \sin(\delta) \\ -\sin(\beta) \\ \cos(\beta)\ \cos(\delta) \end{pmatrix}$$

(Instruments 1-7) (Instruments 8-10)

$$\text{and} \quad \left\{(A)(B)(C)(D)(E)\right\}^{-1} \begin{pmatrix} X_U \\ Y_U \\ Z_U \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

So that the boresight angle pairs, α and ε or δ and β may be found directly from:

$$\begin{aligned} \cos(\alpha)\ \cos(\varepsilon) &= a \\ \sin(\alpha)\ \cos(\varepsilon) &= b \\ \sin(\varepsilon) &= c \end{aligned} \quad \text{or} \quad \begin{aligned} \cos(\beta)\ \sin(\delta) &= a \\ -\sin(\beta) &= b \\ \cos(\beta)\ \cos(\delta) &= c \end{aligned}$$

$$\alpha = \tan^{-1}(b/a) \qquad \delta = \tan^{-1}(a/c)$$

$$\varepsilon = \sin^{-1}(c) \quad \text{or} \quad \beta = -\sin^{-1}(b)$$

(Instruments 1-7) (Instruments 8-10)

Tabulation of Boresight Ranges

In Table 1, the operating ranges are given for the boresight pointing direction of each instrument in α,ε and δ,β with respect to its own Instrument Reference Frame (X Y Z) . Also included is the assumed boresight zero position.

TABLE 1

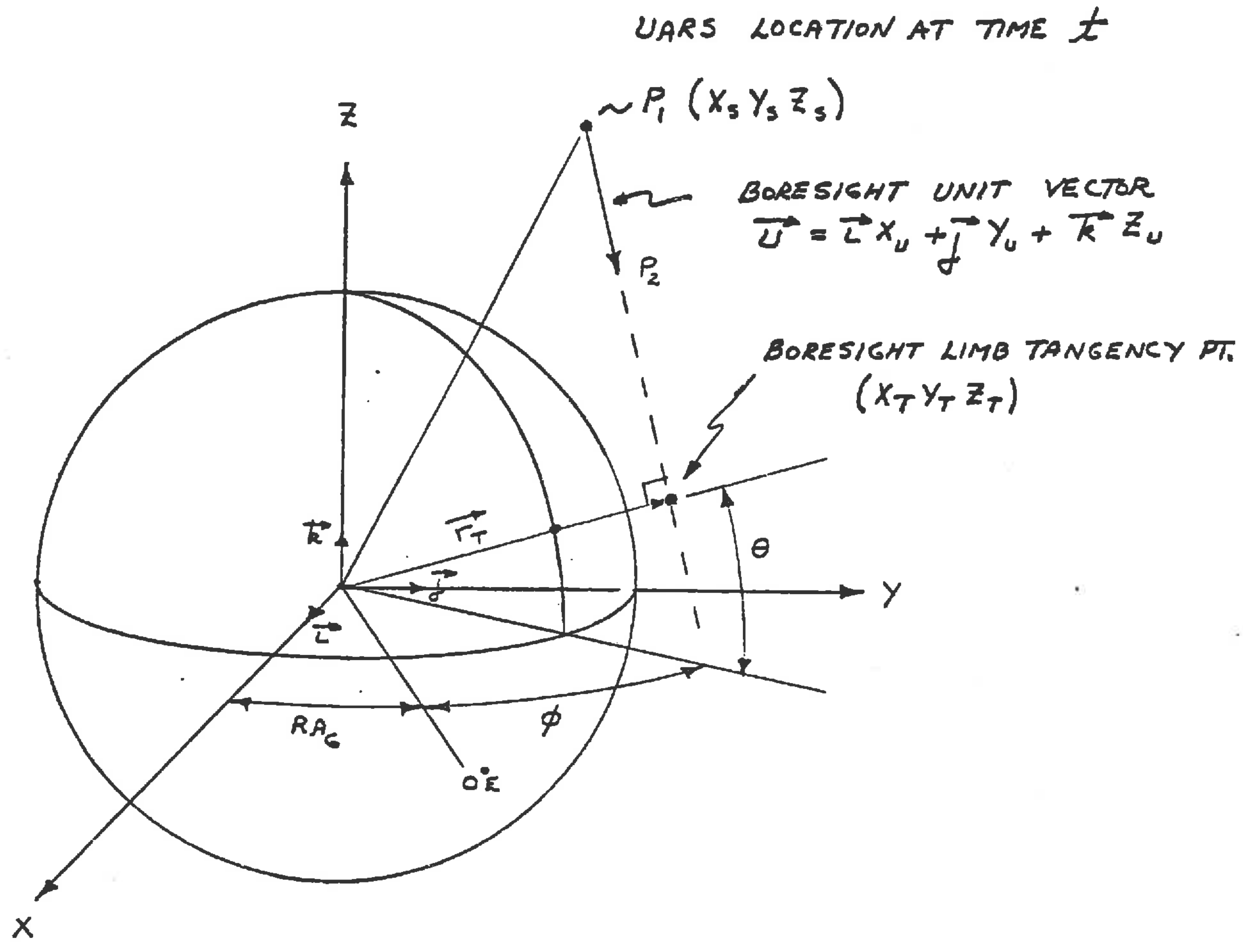
BORESIGHT AZIMUTH AND ELEVATION OF UARS INSTRUMENTS WITH RESPECT TO INSTRUMENT REFERENCE FRAME $X_R Y_R Z_R$

Instrument	Instrument Number	Azimuth	Elevation	Assumed Boresight Zero Position	
		α (deg)	ε (deg)	α (deg)	ε (deg)
CLAES	1	90	23.2±1	90	23.2
MLS	2	90	22.0±4.6	90	22
WINDII	3	45 135	16.1 to 22.1 16.1 to 22.1	45 135	19.1 19.1
ISAMS	4	90 -90	15 to 26 15 to 26	90 -90	23 23
HRDI	5	45±5 135±5 225±5 315±5	15 to 90 15 to 90 15 to 90 15 to 90	45 135 225 315	23 23 23 23
PEM(ZEPS)	6Z	0	-90 to 0	0	-90
PEM(NEPS)	6N	0	90	0	90
PEM(AXIS)	6A	0	67.5	0	67.5
HALOE	7	-180 to 0	-11 to +28	0	0
		δ (deg)	β (deg)	δ (deg)	β (deg)
SOLSTICE*	8	5 to 355	-2 to +90	0	0
SUSIM*	9	5 to 355	-2 to +90	0	0
ACRIM*	10	5 to 355	-2 to +90	0	0

* These instruments are located on the SSPP. See Figures 3 and 4 for definition of δ and β.

APPENDIX A

DERIVATION OF GEOCENTRIC LATITUDE & LONGITUDE OF BORESIGHT LIMB TANGENCY POINT



θ = TANGENCY POINT GEOCENTRIC LATITUDE

ϕ = TANGENCY POINT EAST LONGITUDE AT TIME t

RA_G = RIGHT ASCENSION OF GREENWICH AT TIME t

FIRST, SOLVE FOR $X_T\ Y_T\ Z_T$:

VECTOR $\vec{r}_T$ TERMINATES AT TANGENCY POINT AND IS $\perp$ TO $\vec{U}$
($\vec{U}$ IS THE BORESIGHT UNIT VECTOR)

$\therefore$ SINCE $\vec{r}_T \perp \vec{U}$, THEN $\vec{r}_T \bullet \vec{U} = 0$

SO THAT, $\left(X_T\vec{i} + Y_T\vec{j} + Z_T\vec{k}\right)\bullet\left(X_U\vec{i} + Y_U\vec{j} + Z_U\vec{k}\right) = 0$

THIS YIELDS THE EQUATION OF A PLANE THROUGH THE ORIGIN $\perp\ \vec{U}$

$$X_T X_U + Y_T Y_U + Z_T Z_U = 0 \qquad (1)$$

THE EQUATION OF A LINE $\overline{P_1P_2}$ WHICH PASSES THROUGH THE TANGENCY POINT AT $(X_T Y_T Z_T)$ IS:

$$\frac{X_T - X_S}{X_U} = \frac{Y_T - Y_S}{Y_U} = \frac{Z_T - Z_S}{Z_U} \qquad (2)$$

$$\therefore\ Y_T = \frac{Y_U}{X_U}\left(X_T - X_S\right) + Y_S = \frac{Y_U X_T}{X_U} - \frac{Y_U X_S}{X_U} + Y_S \qquad (3)$$

LET $Y_T = a_1 X_T + a_2$ WHERE $a_1 = \frac{Y_U}{X_U}$ AND $a_2 = Y_S - \frac{Y_U X_S}{X_U}$ (4)

SIMILARLY, $Z_T = \frac{Z_U}{X_U}\left(X_T - X_S\right) + Z_S = \frac{Z_U X_T}{X_U} - \frac{Z_U X_S}{X_U} + Z_S$ (5)

LET $Z_T = b_1 X_T + b_2$ WHERE $b_1 = \frac{Z_U}{X_U}$ AND $b_2 = Z_S - \frac{Z_U X_S}{X_U}$ (6)

SUBSTITUTING EQUATIONS (4) AND (6) INTO EQUATION (1)

$$X_U X_T + Y_U Y_T + Z_U Z_T \tag{1}$$

$$X_U X_T + Y_U \left(a_1 X_T + a_2\right) + Z_U \left(b_1 X_T + b_2\right) = 0$$

$$X_U X_T + Y_U a_1 X_T + Y_U a_2 + Z_U b_1 X_T + Z_U b_2 = 0$$

$$X_T \left(X_U + Y_U a_1 + Z_U b_1\right) + Y_U a_2 + Z_U b_2 = 0$$

$$D = X_U + Y_U a_1 + Z_U b_1$$

$$E = -\left(Y_U a_2 + Z_U b_2\right)$$

$$X_T D - E = 0$$

COORDINATES IN THE ECI FRAME OF THE BORESIGHT LIMB TANGENCY POINT

$$X_T = \frac{E}{D} \tag{7}$$

$$Y_T = a_1 X_T + a_2 \tag{8}$$

$$Z_T = b_1 X_T + b_2 \tag{9}$$

THE CORRESPONDING EARTH GEOCENTRIC LATITUDE OF THE BORESIGHT LIMB TANGENCY POINT IS:

$$\theta = \tan^{-1}\left\{\frac{Z_T}{\sqrt{X_T^2 + Y_T^2}}\right\} \tag{10}$$

THE CORRESPONDING EARTH EAST LONGITUDE OF THE BORESIGHT LIMB TANGENCY POINT IS:

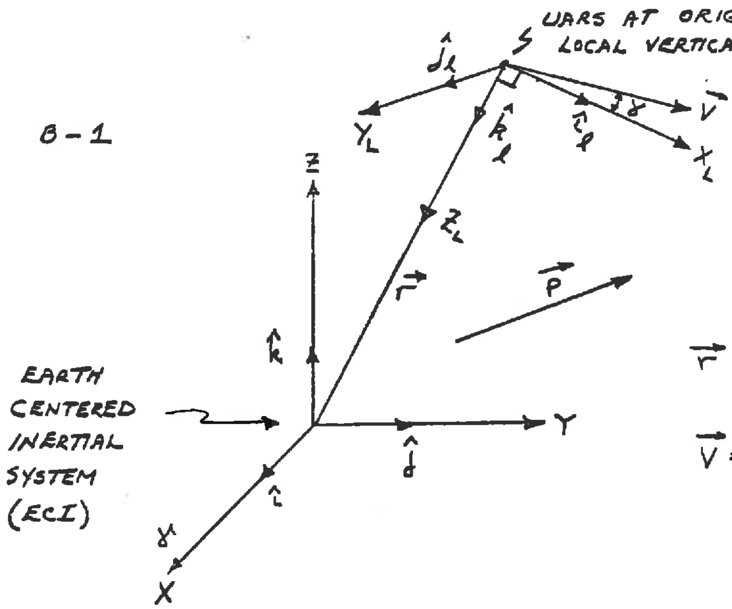
$$\phi = \tan^{-1}\left\{Y_T / X_T\right\} - RA_G \tag{11}$$

$L = RA - RA_G$

APPENDIX B

DERIVATION OF TRANSFORMATION BETWEEN LOCAL VERTICAL ORBITAL COORDINATE SYSTEM $X_L Y_L Z_L$ AND EARTH CENTERED INERTIAL COORDINATE SYSTEM XYZ

FIGURE B-1



$\hat{k}_\ell = -\frac{\vec{r}}{|\vec{r}|}$ TO EARTH CENTER

$\hat{\imath}_\ell$ AND $\hat{k}_\ell$ IN ORBIT PLANE

$\vec{r}$ = SATELLITE LOCATION = $\hat{\imath}X + \hat{j}Y + \hat{k}Z$

$\vec{V}$ = SATELLITE VELOCITY = $\hat{\imath}\dot{X} + \hat{j}\dot{Y} + \hat{k}\dot{Z}$

ASSUME AN ARBITRARY VECTOR $\vec{P}$ IN THE XYZ SYSTEM GIVEN BY:

$$\vec{P} = \hat{\imath}X + \hat{j}Y + \hat{k}Z \qquad (1)$$

THE SAME VECTOR $\vec{P}$ IN THE $X_L Y_L Z_L$ SYSTEM IS GIVEN BY:

$$\vec{P} = \hat{\imath}_\ell X_L + \hat{j}_\ell Y_L + \hat{k}_\ell Z_L \qquad (2)$$

THE RELATIONSHIP BETWEEN THE COMPONENTS $X_L Y_L Z_L$ IN THE $\hat{\imath}_\ell \hat{j}_\ell \hat{k}_\ell$ SYSTEM AND THE COMPONENTS XYZ IN THE $\hat{\imath}\hat{j}\hat{k}$ SYSTEM IS GIVEN BY THE TRANSFORMATION MATRIX:

$$\underbrace{\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}}_{\text{ECI}} = \begin{pmatrix} \hat{\imath}\cdot\hat{\imath}_\ell & \hat{\imath}\cdot\hat{j}_\ell & \hat{\imath}\cdot\hat{k}_\ell \\ \hat{j}\cdot\hat{\imath}_\ell & \hat{j}\cdot\hat{j}_\ell & \hat{j}\cdot\hat{k}_\ell \\ \hat{k}\cdot\hat{\imath}_\ell & \hat{k}\cdot\hat{j}_\ell & \hat{k}\cdot\hat{k}_\ell \end{pmatrix} \underbrace{\begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix}}_{\text{LOCAL VERTICAL ORBITAL}} \qquad (3)$$

SEE "ADVANCED ENGINEERING MATHEMATICS" by ERWIN KREYSIG, JOHN WILEY AND SONS 1962, PAGES 312/313

TO OBTAIN THE ELEMENTS OF THE MATRIX, IT IS NECESSARY ONLY TO FIND THE UNIT VECTORS $\hat{i}_\ell$ $\hat{j}_\ell$ AND $\hat{k}_\ell$ AND THEN DOT THEM WITH $\hat{i}$ $\hat{j}$ $\hat{k}$

FROM FIGURE B-1:

$$\hat{k}_\ell = \frac{-\vec{r}}{r} \tag{4}$$

SINCE $\vec{r} = \hat{i}X + \hat{j}Y + \hat{k}Z$ AND $r = \sqrt{X^2 + Y^2 + Z^2}$ (5)

THEN
$$\boxed{k_\ell = -\hat{i}\frac{X}{r} - \hat{j}\frac{Y}{r} - \hat{k}\frac{Z}{r}} \tag{6}$$

UNIT VECTOR $\hat{j}_\ell$ IS GIVEN BY THE CROSS PRODUCT OF $\vec{V}$ AND $\vec{r}$

$$\vec{V} \times \vec{r} = rV \sin\theta \, \hat{j}_\ell \quad ; \quad V = \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2} \tag{7}$$

(defined as $\hat{i}_\ell$ & $\hat{k}_\ell$) WHERE θ IS THE ANGLE BETWEEN $\vec{V}$ AND $\vec{r}$

$\theta = 90° \pm \gamma$ WHERE γ IS THE PATH ANGLE

SINCE $\sin\theta = \sin(90° + \gamma) = \sin(90° - \gamma) = \cos\gamma$

THEN
$$\hat{j}_\ell = \frac{\vec{V} \times \vec{r}}{rV\cos\gamma} \tag{8}$$

OR
$$\hat{j}_\ell = \frac{1}{rV\cos\gamma} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dot{X} & \dot{Y} & \dot{Z} \\ X & Y & Z \end{vmatrix} \tag{9}$$

$$\boxed{\hat{j}_\ell = \frac{1}{rV\cos\gamma} \left\{ \vec{i}\,(\dot{Y}Z - \dot{Z}Y) + \vec{j}\,(\dot{Z}X - \dot{X}Z) + \vec{k}\,(\dot{X}Y - \dot{Y}X) \right\}} \tag{10}$$

UNIT VECTOR $\hat{i}_\ell$ IS THE CROSS PRODUCT OF $\hat{j}_\ell$ AND $\hat{k}_\ell$

$$\hat{i}_\ell = \hat{j}_\ell \times \hat{k}_\ell \tag{11}$$

$$\therefore \quad \hat{i}_\ell = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\dot{y}z - \dot{z}y}{rV\cos\gamma} & \frac{\dot{z}x - \dot{x}z}{rV\cos\gamma} & \frac{\dot{x}y - \dot{y}x}{rV\cos\gamma} \\ -\frac{x}{r} & -\frac{y}{r} & -\frac{z}{r} \end{vmatrix} \tag{12}$$

$$\boxed{\begin{aligned} \hat{i}_\ell = {} & \hat{i}\left\{\frac{(\dot{x}z - \dot{z}x)z + (\dot{x}y - \dot{y}x)y}{r^2V\cos\gamma}\right\} \\ & + \hat{j}\left\{\frac{(\dot{y}z - \dot{z}y)z + (\dot{y}x - \dot{x}y)x}{r^2V\cos\gamma}\right\} \\ & + \hat{k}\left\{\frac{(\dot{z}y - \dot{y}z)y + (\dot{z}x - \dot{x}z)x}{r^2V\cos\gamma}\right\} \end{aligned}} \tag{13}$$

$\cos\gamma$ IS FOUND FROM THE DOT PRODUCT OF $\vec{r}$ AND $\vec{V}$

$$\vec{r} \cdot \vec{V} = rV\cos(90° \pm \gamma) \tag{14}$$

$$\vec{r} \cdot \vec{V} = \mp rV\sin\gamma$$

$$x\dot{x} + y\dot{y} + z\dot{z} = \mp rV\sin\gamma$$

SINCE $\cos^2\gamma = 1 - \sin^2\gamma$

$$\boxed{\cos\gamma = \sqrt{1 - \left(\frac{x\dot{x} + y\dot{y} + z\dot{z}}{rV}\right)^2}} \tag{15}$$

ELEMENT a_{11} OF THE TRANSFORMATION MATRIX IS GIVEN BY:

$$a_{11} = \hat{i} \cdot \hat{i}_{l} \tag{16}$$

SUBSTITUTING EQ (13) INTO EQ(16)

$$\begin{aligned} a_{11} = {} & \overset{1}{\cancel{\hat{i} \cdot \hat{i}}} \left\{ (\dot{X}Z - \dot{Z}X)Z + (\dot{X}Y - \dot{Y}X)Y \right\} / r^2 V \cos\gamma \\ & + \overset{0}{\cancel{\hat{i} \cdot \hat{j}_0}} \left\{ (\dot{Y}Z - \dot{Z}Y)Z + (\dot{Y}X - \dot{X}Y)X \right\} / r^2 V \cos\gamma \\ & + \overset{0}{\cancel{\hat{i} \cdot \hat{k}}} \left\{ (\dot{Z}Y - \dot{Y}Z)Y + (\dot{Z}X - \dot{X}Z)X \right\} / r^2 V \cos\gamma \end{aligned}$$

$$\therefore \quad a_{11} = \left\{ (\dot{X}Z - \dot{Z}X)Z + (\dot{X}Y - \dot{Y}X)Y \right\} / r^2 V \cos\gamma$$

THE REMAINING ELEMENTS OF THE TRANSFORMATION MATRIX ARE FOUND IN THE SAME WAY BY SUBSTITUTING EQUATIONS (6), (10) AND (13) INTO THE DOT PRODUCTS CONTAINED IN MATRIX (3). THE RESULT IS:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \dfrac{(\dot{X}Z - \dot{Z}X)Z + (\dot{X}Y - \dot{Y}X)Y}{r^2 V \cos\gamma} & \dfrac{\dot{Y}Z - \dot{Z}Y}{rV\cos\gamma} & \dfrac{-X}{r} \\[2ex] \dfrac{(\dot{Y}Z - \dot{Z}Y)Z + (\dot{Y}X - \dot{X}Y)X}{r^2 V \cos\gamma} & \dfrac{\dot{Z}X - \dot{X}Z}{rV\cos\gamma} & \dfrac{-Y}{r} \\[2ex] \dfrac{(\dot{Z}Y - \dot{Y}Z)Y + (\dot{Z}X - \dot{X}Z)X}{r^2 V \cos\gamma} & \dfrac{\dot{X}Y - \dot{Y}X}{rV\cos\gamma} & \dfrac{-Z}{r} \end{pmatrix} \begin{pmatrix} X_L \\ Y_L \\ Z_L \end{pmatrix}$$

ECI

LOCAL VERTICAL ORBITAL

UARS DISTRIBUTION LIST
PROGRAM INFORMATION RELEASES (PIRs) 8/20/85

GE MANDATORY**

W. Andiario	M-7226
J. L. Buckley	M-7235
E. Buerger	M-7235
G. Ehrgott	M-7223
C. Faust	M-7235
T. Horn	M-7223
T. Kelly	M-7226
N. Koepp-Baker	M-7223
P. Kujawski	M-7211
S. Mendelsohn	M-7226
S. Neste	M-7226
R. Pahmeier	U-4018
R. Quinn	M-7226
J. Roberts	M-7226
J. Schumacher	M-7235
S. Siegel	M-7226
E. W. Smith	M-7223
E. Stipandic	M-7223
J. Traver	U-7035
R. Vitz	M-7235
T. Wilks	Bl.300
UARS Lib.(5)*	M-7047

GE DISCRETIONARY***

~~R. Ballert~~	~~M-2700~~
J. Benson	M-7226
~~C. Bosch~~	~~M-2412~~
~~F. Buie~~	~~M-2445~~
~~J. Campbell~~	~~U-4018~~
~~B. Cohen~~	~~M-4018~~
J. Cymerman	M-7223
S. Dietz	M-7223
~~J. DiLello~~	~~M-7223~~
~~W. Downs~~	~~M-4018~~
D. Evans	M-7226
~~A. Farnese~~	~~U-1219~~
~~J. Ferri~~	~~M-8613~~
T. Ford	M-7226
W. Franklin	M-7223
D. Freesland	M-7226
M. Garnek	M-4018
~~W. Hein~~	~~U-7035~~
~~A. Hiltz~~	~~U-4212~~
S. Jaremski	M-2412
P. Jasper (7)	M-7226
~~R. Juner~~	~~M-2515~~
~~W. Kuttruff~~	~~M-2101~~
M. Lamnin	M-7223
~~L. Lux~~	~~M-7223~~
F. Mamrol	M-7235
~~P. Matheson~~	~~M-7226~~
~~C. Mazzocco~~	~~M-2408~~
~~W. McCauley~~	~~M-2421~~
~~H. McNeil~~	~~M-2700~~
~~J. Medved~~	~~M-7226~~
~~R. Meier~~	~~U-4018~~
~~P. Mellusi~~	~~M-4447~~
~~T. Menago~~	~~M-7226~~
R. Pietroski	M-7223
N. Pockell	M-9257
G. Quanbeck	M-7226
~~G. Reinhardt~~	~~M-1211~~
W. Richmond	M-7226
~~R. Roberts~~	~~M-7226~~
~~S. Santore~~	~~M-7226~~
A. Schiefer	M-1223
~~G. Scibilia~~	~~M-1237~~
T. Scollon	M-4018
M. Sedlacek	M-7223
C. Stahle	M-4018
R. Stauffer	M-7223
~~J. Stevens~~	~~M-2221~~
~~E. Strenger~~	~~M-2312~~
R. Talipsky	M-7223
~~H. Thompson~~	~~M-1223~~
J. Thompson	M-7226
~~D. Ulman~~	~~M-2432~~
~~R. Vernot~~	~~M-2408~~
G. Weikel	M-7223
~~W. Woebkenberg~~	~~M-7235~~
G. Yeager	M-7226
D. TANTINO	M-9142
J. MILLER	M-9161
J. BOWES	U-2215

NASA/GSFC, CODE 430 MANDATORY*

P. Burr
F. Brooks(3)Lib.
J. Donley
J. Pandelides
L. Rouser
J. Tomasello

NASA/GSFC, MANDATORY*

A. Azzolini	712.1
R. Coladonato	725.1
H. Culver	711.1
J. Day	711.2
C. DeKramer	716.2
R. Federline	731.3
R. Jackson	727.1
J. Oberright	732.1
E. Painter	733.1
E. Travis	731.0
J. Triolo	732.4
K. Dolan	408.0
D. STUDENICK	408
R. STRAFELLA	554.1
J. RASH	531.1

NASA/GSFC, CODE 430 MANDATORY**

A. Arman
W. Browne
R. Drummond
G. Fleischer
H. Goldberg
E. Johnson
R. Larsen
W. Leavy
T. Michalek
R. Moe
S. Osler
N. Piterski
M. Sing
C. Stouffer
W. Tallant
T. Taylor
C. Trevathan
J. Wonsever

Notes: *COMPLETE copies of ALL PIRs
**Copies of title page, as a minimum, of ALL PIRs
***Author's discretion as to recipients