UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 2

SECI1013

DISCRETE STRUCTURE

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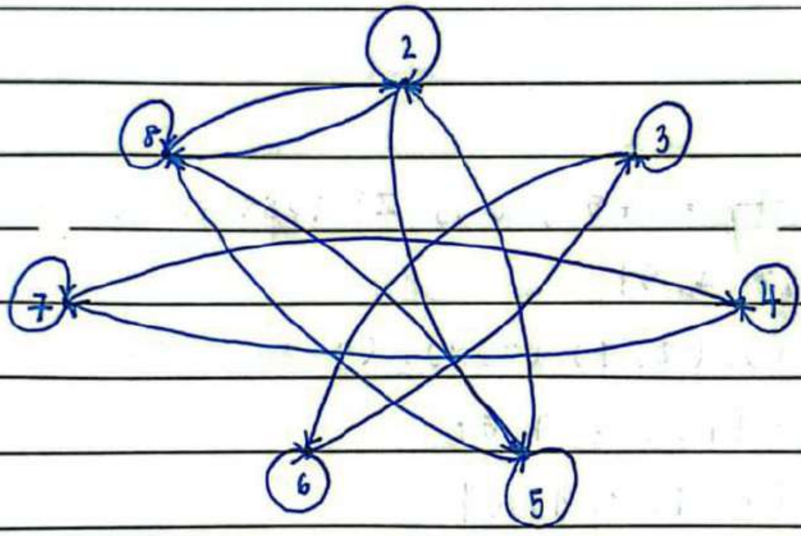
Q1 Relation

1. $A = \{2, 3, 4, 5, 6, 7, 8\}$; xRy if $x - y = 3n$, $n \in Z$

$R = \{(2,2), (2,5), (2,8), (3,3), (3,6), (4,4),$
$(4,7), (5,2), (5,5), (5,8), (6,3), (6,6),$
$(7,4), (7,7), (8,2), (8,5), (8,8)\}$



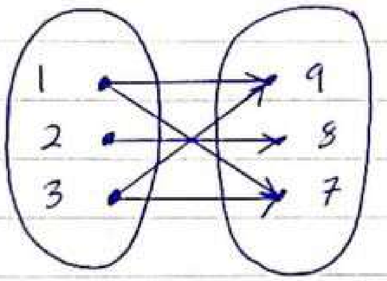
- The relation is reflexive because every vertex have a loop.
- Relation R is symmetric relation
- The relation R is transitive

2. $A = \{1, 2, 3\}$ and $B = \{9, 8, 7\}$

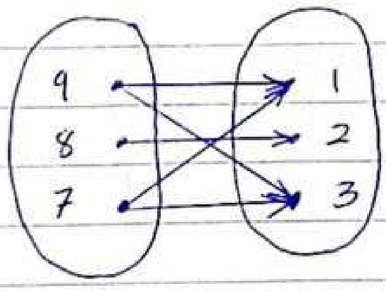
a) $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$

$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$

b) $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$



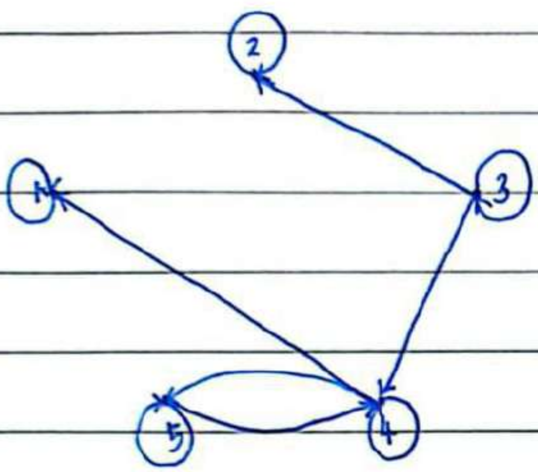
$R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$



c) R^{-1} is the inverse of R. That means R^{-1} is obtained by changing the first element to second and second element to first.

3. $A = \{1, 2, 3, 4, 5\}$

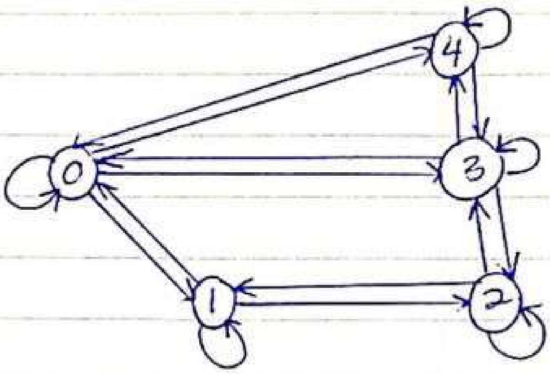
$R = \{(1,1), (2,2), (3,2), (3,3), (3,4),$
$(4,1), (4,4), (4,5), (5,4), (5,5)\}$



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

4. $A = \{0, 1, 2, 3, 4\}$

$R = \{(0,0), (0,1), (0,3), (0,4), (1,0), (1,1), (1,2), (2,1), (2,2), (2,3), (3,0), (3,2), (3,3), (3,4), (4,0), (4,3), (4,4)\}$



$$\begin{array}{c c} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$\overset{x\ y}{(0,1)}, \overset{y\ z}{(1,0)}$ then $(0,0) \in R$

$(2,1), (1,2)$ then $(2,2) \in R$

$\therefore$ R is reflexive

R is not asymetric

R is transitive

5. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

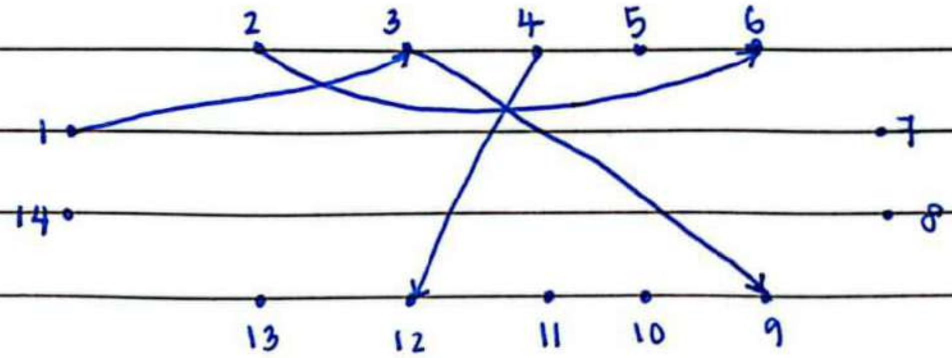
$R = \{(x,y) = 3x - y = 0\}$

$3x - y = 0$

$3x = y$

$x = \frac{y}{3}$

$R = \{(1,3), (2,6), (3,9), (4,12)\}$



a. The relation is irreflexive because there is no loop at every vertex ; $(x,x) \notin R$ for every $x \in A$.

b. The relation is asymmetric because when (x,y) exist , (y,x) doesn't exist. For all $x, y \in A$, when $(x,y) \in R$, then $(y,x) \notin R$.

c. The relation is not transitive because $(1,3)$ and $(3,9)$ exist but $(1,9)$ doesn't exist.

6.

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

a) RS

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b) SR

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

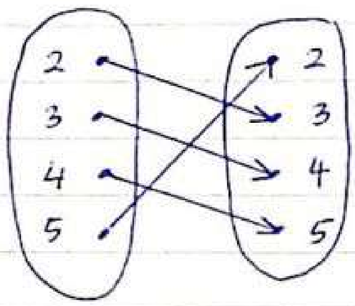
$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Q2 Function

7. Relation can have many outputs from one input while function has a single output for a single input.

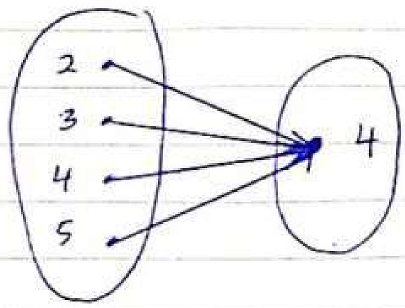
8. $A = \{2, 3, 4, 5\}$

i) $R = \{(2,3), (3,4), (4,5), (5,2)\}$



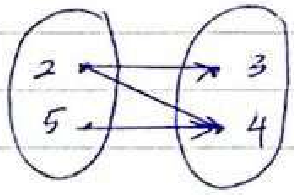
= It's a function because the domain of f is X and each elements in domain is connected with a unique element in codomain.

ii) $R = \{(2,4), (3,4), (5,4), (4,4)\}$



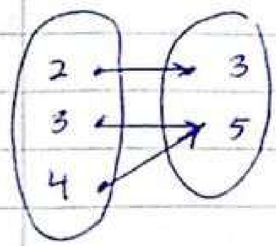
= function because domain of f is X.

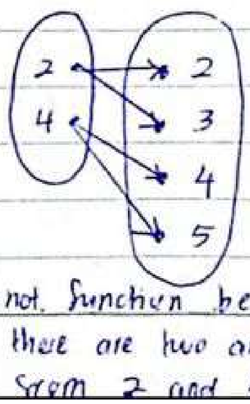
iii) $R = \{(2,3), (2,4), (5,4)\}$



= Not function because (2,3) and (2,4) in R but $3 \neq 4$, and there are two arrows from 2.

iv) $R = \{(2,3), (3,5), (4,5)\} \vee \{(2,2), (2,3), (4,4), (4,5)\}$





= (function) $\vee$ (not function)

= not function

= function

= not function because there are two arrows from 2 and 4

9. $R = \{(x, y) \mid y = x + 5$, x is Z^+ less than $6\}$

$x = \{1, 2, 3, 4, 5\}$

$R = \{(1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

Domain $= \{1, 2, 3, 4, 5\}$

Range $= \{6, 7, 8, 9, 10\}$

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10. v) $f = R - R$, $f(x) = 1 - 2x$

$f(x) = 1 - 2x$

$1 - 2x_1 = 1 - 2x_2$

$\cancel{-2} x_1 = \cancel{-2} x_2$

$x_1 = x_2$ ← one to one

$f(x) = 1 - 2x$

$1 - 2x = y$

$y = 1 - 2x$

$2x = 1 - y$

$x = \frac{1-y}{2}$ ← real number ← onto

∴ It is bijection because it is one to one and onto

vi) $f = R \to R$, $f(x) = 5x^2 - 1$

$f(x_1) = f(x_2)$

$5x_1^2 \cancel{-1} = 5x_2^2 \cancel{-1}$

$\cancel{5}x_1^2 = \cancel{5}x_2^2$

$x_1^{\cancel{2}} = x_2^{\cancel{2}}$

$x_1 = x_2$ ← one to one

$f(x) = y$

$y = 5x^2 - 1$

$5x^2 = y + 1$

$x^2 = \frac{y+1}{5}$

$x = \pm\sqrt{\frac{y+1}{5}}$ ← onto

∴ It is bijection because It's one to one and onto

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vii) $f = R \to R,\ f(x) = x^4$

$$f(x) = x^4$$
$$x_1^4 = x_2^4$$
$$x_1 = x_2 \leftarrow \text{one to one}$$

$$f(x) = y$$
$$y = x^4$$
$$x = \sqrt[4]{y} \leftarrow \text{onto} \leftarrow \text{real number}$$

$\therefore$ It is bijection because one to one and onto

viii) $f = R \to R,\ f(x) = \left(\frac{x-2}{x-3}\right)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$
$$\cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 3x_2 - 2x_1 + \cancel{6}$$
$$-3x_1 - 2x_2 = -3x_2 - 2x_1$$
$$-3x_1 + 2x_1 = -3x_2 + 2x_2$$
$$\cancel{-}x_1 = \cancel{-}x_2$$
$$x_1 = x_2 \leftarrow \text{one to one}$$

$f(x) = y$

$y = \frac{x-2}{x-3}$

$$(x-3)y = (x-2)$$
$$xy - 3y = x - 2$$
$$xy - x = -2 + 3y$$
$$x(y-1) = -2 + 3y$$
$$x = \left(\frac{-2+3y}{y-1}\right) \to y \neq 1 \to$$

not onto because not all values of real number can use

$\therefore$ it's not bijective

11. Find $f[g(x)]$, find value of function if $x:\{0,1,2,3\}$

ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

$f[g(x)] = 3(x^2 - 1) - 1$

$f[g(x)] = 3x^2 - 4$

$x = 0$, $f[g(0)] = 3(0)^2 - 4$

$= -4$

$x = 1$, $f[g(1)] = 3(1)^2 - 4$

$= -1$

$x = 2$, $f[g(2)] = 3(2)^2 - 4$

$= 8$

$x = 3$, $f[g(3)] = 3(3)^2 - 4$

$= 23$

x) $f(x) = x^2$; $g(x) = 5x - 6$

$f[g(x)] = (5x - 6)^2$

$f[g(x)] = 25x^2 - 60x + 36$

$x = 0$, $f[g(0)] = 25(0)^2 - 60(0) + 36$

$= 36$

$x = 1$, $f[g(1)] = 25(1)^2 - 60(1) + 36$

$= 1$

$x = 2$, $f[g(2)] = 25(2)^2 - 60(2) + 36$

$= 16$

$x = 3$, $f[g(3)] = 25(3)^2 - 60(3) + 36$

$= 81$

xi) $f(x) = x - 1$; $g(x) = x^3 + 1$

$f[g(x)] = (x^3 + 1) - 1$

$f[g(x)] = x^3$

$x = 0$, $f[g(0)] = 0^3 = 0$

$x = 1$, $f[g(1)] = 1^3 = 1$

$x = 2$, $f[g(2)] = 2^3 = 8$

$x = 3$, $f[g(3)] = 3^3 = 27$

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12. xii) $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0 = 1$ $a_1 = 6$

$a_2 = 6a_{2-1} - 9a_{2-2}$

$= 6a_1 - 9a_0$
$= 6(6) - 9(1)$
$= 27$

$a_4 = 6a_{4-1} - 9a_{4-2}$

$= 6a_3 - 9a_2$
$= 6(108) - 9(27)$
$= 405$

$a_3 = 6a_{3-1} - 9a_{3-2}$

$= 6a_2 - 9a_1$
$= 6(27) - 9(6)$
$= 108$

$= 1, 6, 27, 108, 405, \ldots$

xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; $a_0 = 2, a_1 = 5, a_2 = 15$

$a_3 = 6a_{3-1} - 11a_{3-2} + 6a_{3-3}$

$= 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_{4-1} - 11a_{4-2} + 6a_{4-3}$

$= 6a_3 - 11a_2 + 6a_1$
$= 6(47) - 11(15) + 6(5)$
$= 147$

$= 2, 5, 15, 47, 147, \ldots$

xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3a_{3-1} - 3a_{3-2} + a_{3-3}$

$= -3a_2 - 3a_1 + a_0$

$= -3(-1) - 3(-2) + 1$

$= 10$

$a_4 = -3a_{4-1} - 3a_{4-2} + a_{4-3}$

$= -3a_3 - 3a_2 + a_1$

$= -3(10) - 3(-1) + (-2)$

$= -29$

$= 1, -2, -1, 10, -29, \ldots$

13. $a_{n+1} = 5a_n - 3$; $a_1 = k$

i) a_4 in terms of k

$a_1 = k$

$a_2 = 5k - 3$

$a_3 = 5(5k-3) - 3$

$= 25k - 18$

$a_4 = 5(25k - 18) - 3$

$a_4 = 125k - 93$

ii) $a_4 = 7$

$a_4 = 125k - 93$

$7 = 125k - 93$

$125k = 100$

$k = 0.8$

$k = \frac{4}{5}$