Math138 - January 27'th, 2016

The Comparison Theorems and Intro to Differential Equations

Comparison theorems give us a way to determine if an improper integral converges or diverges without integrating!

The Theorems:

Suppose f and g are continuous functions where $f(x) \geq g(x) \geq 0$

1. If $\displaystyle\int_a^\infty f(x)\,dx$ converges, then $\displaystyle\int_a^\infty g(x)\,dx$ also converges.
2. If $\displaystyle\int_a^\infty g(x)\,dx$ diverges, then $\displaystyle\int_a^\infty f(x)\,dx$ also diverges.

Examples

Determine if the following converge or diverge.

Recall: $\int_1^\infty \frac{1}{x^p}$ converges iff $p > 1$ and $\int_0^\infty e^{-x}$ converges.

1. $\int_0^\infty e^{-x^2}\,dx$, note that $0 \leq e^{-x^2} \leq e^{-x}$ eventually $(x \geq 1)$ and $\int_0^\infty e^{-x}\,dx$ converges. So, $\int_0^\infty e^{-x^2}\,dx$ converges by comparison.
2. $\int_1^\infty \frac{x}{(x^2+2)^2}\,dx$, note if $x \geq 1$, then $0 \leq \frac{x}{(x^2+2)^2} < \frac{x}{x^4} = \frac{1}{x^3}$ and $\int_1^\infty \frac{1}{x^3}\,dx$ converges because $p > 1$. So, $\int_1^\infty \frac{x}{(x^2+2)^2}\,dx$ converges by comparison.

Recall

If you split up the region, you only need one divergent integral for the whole thing to diverge.

But, if you split up the integrand, you must check all of them. A positive divergence and a negative divergence can cancel out.

Introduction to Differential Equations

An equation that contains derivatives of a dependent variable or function $y = f(x)$ is called a **differential equation** (DE).

(Actually an ordinary differential equation (ODE) when working with one variable).

The **order** of an ODE is the order of the highest derivative that appears.

An ODE is called **linear** if it contains only linear functions if y, y', y'', etc.

The **general solution** of an ODE is the collection of all possible solutions including arbitrary constants.

A **particular solution** is a solution where we solve for all arbitrary constants.

To get a particular solution, we need extra information like the value of y, y', y'' etc. at certain points. These values are called **initial conditions**.

An ODE plus initial conditions is called an **Initial Value Problem** (IVP).