Math138 - January 20'th, 2016

Volumes of Revolution - Washers, Disks, Cylinders

Washers / Disks

Summary: If the cross-section is:

A disk:	Find the radius and use $a = \pi r^2$
A Washer:	Find inner outer radius and use $A = \pi(\text{outer}^2 - \text{inner}^2)$

Examples

Find the volume of the solid obtained by rotating:

1) $y = \sqrt{x-1}$ about the x axis from $x = 1$ to $x = 5$.

$A(x) = \pi(x-1)$

$$\begin{aligned}\text{Volume} &= \int_1^5 \pi(x-1)\,dx \\ &= \pi\int_1^5 x-1\ \ dx \\ &= \vdots \\ &= 8\pi\end{aligned}$$

2) The area between $y = x^2$ and $y = \sqrt{x}$ about $y = 1$.

This is a washer question. Inner radius $= 1 - x^2$, outer radius $= 1 - \sqrt{x}$.

$$\begin{aligned}A &= \pi((1-x^2)^2 - (1-\sqrt{x})^2) \\ &= \pi(x^4 - 2x^2 - x + 2\sqrt{x})\end{aligned}$$

$$\begin{aligned}V &= \int_0^1 A(x)\,dx \\ &= \pi\int_0^1 x^4 - 2x^2 - x + 2\sqrt{x} \\ &= \vdots \\ &= 11\pi/30\end{aligned}$$

Method # 2: Cylinder Shells

Cylinder shells are good for rotating functions of x around vertical lines or functions of y around horizontal lines.

Ex. Rotate $y = 2x^2 - x^3$ about the y axis from $x = 0$ to $x = 2$.

Radius: x, Height $= 2x^2 - x^3$

$A = 2\pi x(2x^2 - x^3)$

$$A = 2\pi \int_0^2 2x^3 - x^4$$

$$= \vdots$$

$$= \frac{16\pi}{5}$$

Ex. Rotate $y = x^2$ and $y = 6x - 2x^2$ about the y-axis.

Radius $= x$, Height $= -3x^2 + 6x$

$$v = 2\pi \int_0^2 x(-3x^2 + 6x)\, dx$$

$$= \vdots$$

$$= 8\pi$$