Math137 - November 2'nd, 2015

More EVT, Mean Value Theorem

EVT Example:

The fuel efficiency E of a car driven at a speed of r km/h is given by:

$E(v) = \frac{1600v}{6400+v^2}$

If the speed limit is 100, what legal speed maximizes fuel efficiency?

To find the maximum or minimum values of $E(v)$, we take the derivative and find where it equals 0. These 'critical points' represent the functions local minimums and maximums. Then, we can plug in these values, plus our end-points, into the original function to find which value minimizes/maximizes the function value.

Same substitute as yesterday, her solution to this question made no sense. Lovely.

Mean Value Theorem (MVT)

Suppose f is a function such that:

i) f is continuous on the closed interval $[a, b]$

ii) f is differentiable on the open interval (a, b)

Then, there exists a point c in the open interval (a, b) such that

$f'(c) = \frac{f(b)-f(a)}{b-a}$

In other words, at some point c in the open interval (a, b), the tangent line of f at c will equal the slope of the line connecting point $(a, f(a))$ and $(b, f(b))$.

Example:
Two police cars are located at fixed positions 10km apart on a highway where the speed limit is 100 km/h. A car passes the first police car traveling at 95 km/h and 5 minutes later, passes the second police car traveling at 100 km/h. The car is stopped for speeding. Explain why.

Soln:
Let $f(t)$ be the position function of the car at time t.
So, $f'(t)$ is the velocity function of the car at time t.

By MVT, we know there is a point in the open interval $(0, 5)$ (minutes) such that $f'(c) = \frac{(f(b)-f(a)}{b-a} = \frac{10}{\frac{1}{12}} = 120$
$\therefore$ At some point between 0 and 5 minutes, the car was traveling 120 km/h.

Short note again :/ If the prof isn't back next lecture, I'll just go to a different calc lecture.