Math138 - January 4'th, 2016

Introduction to Calculus II

Administrative Stuff

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Assignments Due:	Fridays at 2PM
First Assignment Due:	January 15'th

This Week:

- Review Integration (Chapter 5)
- Integration by Parts (Chapter 7.1)
- Trig Integrals (Chapter 7.2)

Integration

We already know lots of integrals (anti-derivatives). Ones we should know include powers of x, $\frac{1}{x}$, e^x, basic trig, trig inverses and hyperbolic trig functions.

Recall: Definite integrals represent the area below the curve from $x = a$ to $x = b$, shown as $\displaystyle\int_a^b f(x)dx$

We use the Fundamental Theorem of Calculus (FTC) to evaluate.

Fundamental Theorem of Calculus

If $f(x)$ is continuous and...

Part 1: ...$F(x) = \int_a^x f(t)dt$ then $F'(x) = f(x)$

Part 2: ...If $f(x)$ is any anti-derivative of $f(x)$ (so $F'(x) = f(x)$), then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

Ex. $$\int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} = \frac{1}{3}$$

We can also use FTC tp find the derivative of integral functions.
Suppose $P(x) = \int_{g(x)}^{h(x)} f(t)dt$
We want to solve for $P'(t)$

Let $F(x)$ be an anti-derivative of $f(x)$. Then by FTC, $P(x) = F(h(x)) - F(g(x))$

$$\begin{aligned} P'(x) &= F'(h(x)) \cdot h'(x) - F'(g(x)) \cdot g'(x) \\ &= f(h(x)) \cdot h'(x) - f(g(x)) \cdot g'(x) \end{aligned}$$

U-Substitution (Reverse Chain Rule)

$\int f(g(x)) \cdot g'(x)dx = \int f(u)du$ where $u = g(x)$

But how do we pick u?

- If you see a function together with its derivative, let u be this function.
- Let $u =$ the base of an ugly power or denominator.
- Let $u =$ whats inside a function like sin, log, etc.

U-Sub Examples

a) $\displaystyle \int \frac{\ln x}{x} dx$

$\displaystyle = \int u \, du$

$\displaystyle = \frac{u^2}{2} + c$

Side Work
Let $u = \ln x$
$x \, du = dx$

b) $\displaystyle \int_0^1 \frac{x^3}{1+x^4} \, dx$

$\displaystyle = \int_1^2 \frac{x^3}{u} \cdot \frac{du}{4x^3}$

$\displaystyle = \int_1^2 \frac{1}{4u} \, du$

$\displaystyle = \frac{1}{4} \int_1^2 \frac{1}{u} \, du$

$\displaystyle = \frac{1}{4} \left[\ln |u|\right]_1^2$

$\displaystyle = \frac{1}{4}(\ln 2 - \ln 1)$

$\displaystyle = \frac{\ln 2}{4}$

Side Work
Let $u = 1 + x^4$
$\frac{du}{dx} = 4x^3$
$\frac{du}{4x^3} = dx$
If $x = 0$, $u = 1$
If $x = 1$, $u = 2$