Math138 - January 20'th, 2016

Plane Projections, Intro to Linear Systems

Example

Find $\text{proj}_{\vec{v}}(\vec{u})$ and $\text{perp}_{\vec{v}}(\vec{u})$ where:

$\vec{u} = \begin{bmatrix}1\\2\\3\end{bmatrix} . \vec{v} = \begin{bmatrix}-1\\0\\1\end{bmatrix}$ in $\mathbb{R}^3$

$$\begin{aligned}
\text{Proj}_{\vec{v}}(\vec{u}) &= \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}\\
&= \frac{1(-1)+2(0)+3(1)}{(-1)^2+0^2+1^2}\begin{bmatrix}-1\\0\\1\end{bmatrix}\\
&= \frac{2}{2}\begin{bmatrix}-1\\0\\1\end{bmatrix} = \begin{bmatrix}-1\\0\\2\end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\text{perp}_{\vec{v}}(\vec{u}) &= \vec{u} - \text{proj}_{\vec{v}}(\vec{u})\\
&= \begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}-1\\0\\1\end{bmatrix}\\
&= \begin{bmatrix}2\\2\\2\end{bmatrix}
\end{aligned}$$

Projection onto a Plane

To project onto a plane, we will first find the vertical component in the direction of the normal vector.

So first, we find $\text{proj}_{\vec{n}}(\vec{u})$, then

$\text{proj}_p(\vec{u}) = \vec{u} - \text{proj}_{\vec{n}}(\vec{u}) = \text{perp}\vec{n}(\vec{u})$

Where p is the plane(through $\vec{0}$), $\vec{n}$ is the normal to the plane and $\vec{u}$ is the vector being projected onto p.

Recall: The scalar equation for a plane p will be:

$\vec{x}\cdot\vec{n} = 0$ OR

$n_1x_1 + n_2x_2 + x_3x_3 = 0$

Ex. Consider the plane p in $\mathbb{R}^3$ with vector equation:

$$\vec{x} = c_1\begin{bmatrix}1\\2\\3\end{bmatrix} + c_2\begin{bmatrix}1\\0\\-1\end{bmatrix}, \quad c_1, c_2 \in \mathbb{R}$$

Find $\text{proj}_p\vec{u}$ where $\vec{u} = \begin{bmatrix}7\\2\\3\end{bmatrix}$

First, find the normal vector by taking the cross product.

$$\vec{n} = \begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}1\\0\\-1\end{bmatrix}$$
$$= \begin{bmatrix}-2\\4\\-2\end{bmatrix}$$

Next, find $\text{proj}_{\vec{n}}\vec{u}$

$$\begin{aligned}\text{proj}_{\vec{n}}\vec{u} &= \left(\frac{\vec{n}\cdot\vec{u}}{\vec{n}\cdot\vec{n}}\right)\vec{n}\\ &= \frac{(-2)7 + 4\cdot 2 + (-2)(3)}{(-2)^2 + 4^2 + (-2)^2}\vec{n}\\ &= \frac{-1}{2}\begin{bmatrix}-2\\4\\-2\end{bmatrix}\\ &= \begin{bmatrix}1\\-2\\1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\text{proj}_{p}\vec{u} = \text{perp}_{\vec{n}}\vec{u} &= \vec{u} - \text{proj}_{\vec{n}}\vec{u}\\ &= \begin{bmatrix}7\\2\\3\end{bmatrix} - \begin{bmatrix}1\\-2\\1\end{bmatrix}\\ &= \begin{bmatrix}6\\4\\2\end{bmatrix}\end{aligned}$$

Systems of Linear Equations

An equation in n variables $x_1, x_2, \ldots, x_n$ of the form $a_1x_1 + \cdots + x_nx_n = b$ where $a_1, \ldots, a_n, b$ are constant scalars is called a **Linear Equation**

The constants are the coefficients and b is often referred tp as the **Right hand side**

A set of m linear equations in the variable $x_1, \ldots, x_n$is called a **System of m linear equations in n variables**

Ex. $x_1 - 2x_2 = 0$
$3x_1 + x_2 = 4$

Is a system of 2 linear equations in 2 variables.
We can solve for the variables using old high-school substitution.

A system of linear equations with at least one solution is called **Consistent**.

Ex. $x_1 + x_2 + x_3 = 0$
$x_1 - x_2 = 0$
$2x_1 + x_3 = 1$

We can try to solve this, but we'll get a contraction! So no solution exists and this system of linear equations is **Inconsistent**.