Math136 - January 29'th, 2016

Matrix Multiplication

Transpose

The **transpose** of a $m \times n$ matrix A is the $n \times m$ matrix A^T where the ijth entry is the jith entry of A. That is: $(A^T)_{ij} = (A)_{ji}$

Theorem 3.1.2 If $A, B \in M_{m\times n}(\mathbb{R})$ then:

1) $(A^T)^T = A$
2) $(A+B)^T = A^T + B^T$
3) $(cA)^T = cA^T$

Note: We will often view vectors in $\mathbb{R}^n$ as n matrices.

Matrix - Vector Multiplication

Suppose A is a $m \times n$ matrix with rows $(\vec{a}_1)^T, (\vec{a}_2)^T, \ldots, (\vec{a}_m)^T$ for $\vec{a}_i \in \mathbb{R}^n$. Then for $\vec{x} \in \mathbb{R}^n$, we define $A\vec{x}$ by:

$$A\vec{x} = \begin{bmatrix} \vec{a}_1 \cdot \vec{x} \\ \vdots \\ \vec{a}_m \cdot \vec{x} \end{bmatrix}$$

And this defines the multiplication of a $m \times n$ matrix by a vector (or equivalently, an $n \times 1$ matrix)

The result is an $m \times 1$ matrix.

So ($m \times n$ matrix) times ($n \times 1$ matrix) gives ($\times 1$ matrix)

Matrix - Matrix Multiplication

Suppose A is an $m \times n$ matrix, and B is a $n \times p$ matrix with columns $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_p$

Then $AB = A\begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \ldots & \vec{b}_p \end{bmatrix} = \begin{bmatrix} A\vec{b}_1 & A\vec{b}_2 & \ldots & A\vec{b}_p \end{bmatrix}$

Matrix Manipulation Theorem

If A, B, C are matrices so that all following products are defined, we have:

1) $A(B+C) = AB + BC$
2) $t(A+B) = (tA)B$
3) $A(BC) = (AB)C$
4) $(AB)^T = B^T A^T$