Math136 - January 25'th, 2016

Solving Systems in Reduced Row Echelon Form

Theorem 2.2.2

If A is a matrix, then A has a unique RREF R.

Terminology

Try to solve this system of linear equations:

$x_1 + x_2 + x_3 = 0$
$2x_1 - x_2 - x_3 = 1$
$3x_1 = 1$

After a number of elementary row operations, you'll manage to get the augmented coefficient matrix to this state:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 1 & -1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Since the columns corresponding to x_1 and x_2 have leading ones, we cal them **leading variables**. Since the column corresponding to x_3 does not, it is a **free variable**.

What we note is that to get a solution to our system: $x_1 = -1/3,\ \ x_2 + x_3 = -1/3,\ \ 0 = 0$, we can assign any value to our free variables and then the leading variables can have their values determined.

Say $x_3 = s$ for some $s \in \mathbb{R}$

Then: $x_1 = -1/3,\ \ x_2 = -1/3 - s$

So,all solutions to our system of equations are:

$x_1 = -1/3,\ \ x_2 = -1/3 - s,\ \ x_3 = s$

Algorithm to solve a System of Linear Equations

1. Write the augmented matrix
2. Use ERO's to get RREF
3. Write the system corresponding to the RREF
4. If the system has an equation "0=1" the system in inconsistent, STOP!
5. Otherwise, each free variable (corresponding to a clumn without a leafing 1) is assigned a value which is a **free parameter**)
6. Move the free variables to the right hand side to determine the corresponding values for the leading parameters.

If we have k free variables in a consistent system, then the set of solutions forms a k-flat.