Math137 - November 26, 2015

Examples: Solving Definite Integrals

Ex. 1 The function g is defined by

$$g(x) = \int_0^x (t - t^2)dt$$

Calculate $g'(1)$ and find the location of any inflection points of g.

$g'(x) = x - x^2$ (By FTC1)
$g'(1) = 1 - 1^2 = 0$

For inflection points, we use the second derivative as usual.

$g''(x) = 1 - 2x$
$g''(x)_{=}0@x = \frac{1}{2}$
$\therefore$ POI at $x = \frac{1}{2}$

	$0 < x < \frac{1}{2}$	$x > \frac{1}{2}$
$1 - 2x$	+	-
$g(x)$	Conc. Up	Conc. Down

Ex. 2 Determine $\frac{dy}{dx}$ given that:

$$y = \int_{1+3x^2}^4 \frac{1}{2+e^t}dt$$

Let $u(x) = 1 + 3x^2$ and $f(t) = \dfrac{1}{2+e^t}$

$$y = F(u) = \int_u^4 f(t)dt$$
$$y = F(u) = -\int_4^u f(t)dt$$
$$\frac{dy}{du} = F'(u) = -f(u)$$

$$\begin{aligned}\therefore \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= -f(u) \cdot \frac{d}{dx}(1+3x^2) \\ &= \frac{-1}{2+e^u} \cdot 6x \\ &= \frac{-6x}{2+e^{1+3x^2}}\end{aligned}$$

<u>In General:</u>

$$y = \int_c^{g(x)} f(t)dt \quad \text{, then}$$
$$\frac{dy}{dx} = f(g(x)) \cdot g'(x)$$

Ex. 3 Evaluate each of the definite integrals.

a) $\int_1^3 (u + \frac{u}{2})du$

Suppose $f(x) = (u + \frac{u}{2})$
By FTC2, $\int_1^3 (u + \frac{2}{u})du = F(3) - F(1)$

$$F(x) = \left[\frac{u^2}{2} + 2\ln \mid u \mid\right]_1^3 \quad \text{(By FTC2)}$$
$$= (\frac{3^2}{2} + 2\ln 3) - (\frac{1^2}{2} + 2\ln 1)$$
$$= 4 + \ln 9$$

Note: When solving a definite integral, we don't write the $+c$ when we anti-differentiate since it cancels out later anyways.

b) $\int_0^{\ln 8} (2e^{-2t})dt$

$$= \left[-e^{2t}\right]_0^{\ln 8} \quad \text{(By FTC2)}$$
$$= -(e^{-2\ln 8} - e^{-2(0)})$$
$$= -(e^{\ln 8^{-2}} - 1)$$
$$= -(8^{-2} - 1)$$
$$= \frac{-63}{64}$$

c) $\int_0^1 (2u+1)^2 du$

We don't have a power rule, so we'll expand.

$$\int_0^1 (4u^2 + 4u + 1)du$$
$$= \left[\frac{4u^3}{3} + \frac{4u^2}{2} + u\right]_0^1 \quad \text{(By FTC2)}$$
$$= \left[\frac{4u^3}{3} + 2u^2 + u\right]_0^1$$
$$= \frac{3}{4} + 2 + 1 - 0$$
$$= \frac{13}{3}$$

d) $\int_{\frac{-1}{2}}^0 (\cos(\pi x))dx$

$$\left[\frac{\sin(\pi x)}{\pi}\right]_{\frac{-1}{2}}^0$$
$$= \frac{\sin 0}{\pi} - \frac{\sin(\frac{\pi}{2})}{\pi}$$
$$= \frac{1}{\pi}$$