Math138 - February 1'st, 2016

Solving Differentials - Separable ODEs

Separable ODEs

A separable Ordinary Differential Equation is an equation where we can get a function of x on one side and a function of y on the other side. If we can achieve this, we can integrate both sides like normal.

Separable ODE Definition

A separable ODE is a first-order ODE that can be written as $\frac{dy}{dx} = g(y) \cdot h(x)$. IE, we can separate into a product of functions, one containing only y, the other containing only x

To solve these, move $g(y)$ to the LHS and integrate.

$$
\begin{aligned}
\frac{1}{g(y)}\frac{dy}{dx} &= h(x) \\
\int \frac{1}{g(y)}\frac{dy}{dx} dx &= \int h(x)\ dx \\
\int \frac{1}{g(y)}\ dx &= \int h(x)\ dx
\end{aligned}
$$

Note: Normally you cannot treat the differential as a fraction and cancel out dy or dx in this example, we actually use a substitution and skip a step, so we've not broken any rules.

Ex. Find the particular solution to the IVP $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1$
Notice we can separate this into functions of x and functions of y!

$$
\begin{aligned}
2(y-1)dy &= 3x^2 + 4x + 2 \\
\int 2y - 2\ dy &= \int 3x^2 + 4x + 2\ dx \\
2y^2 - 2y &= x^3 + 2x^2 + 2x + c
\end{aligned}
$$

$$
\begin{aligned}
(-1)^2 - 2(-1) &= 0 + 0 + 0 + c \\
-3 &= c
\end{aligned}
$$

So, we plug in c and try to solve for y

$$
\begin{aligned}
y^2 - 2y &= x^3 + 2x^2 + 2x + 3 \\
(y-1)^2 - 1 &= x^2 + 2x^2 + 2x + 3 \\
y - 1 &= \pm\sqrt{x^3 + 2x^2 + 2x + 4} \\
y &= 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}
\end{aligned}
$$

But only one of these satisfies $y(0) = -1$
So $y = 1 - \sqrt{x^3 + 2x^2 + 2x + 4}$

WARNING: Watch out for dividing by 0! If you create a possible divide by 0 with y, then do two separate cases.