Math136 - February 1'st, 2016

Matrix Properties

Second Definition of Matrix-Vector Multiplication

If $A = [\vec{a}_1 \ \dots \ \vec{a}_n] \in M_{m\times n}(\mathbb{R})$ and $\vec{x} \in \mathbb{R}^n$ is an $n \times 1$ matrix then $A\vec{x} = x_1\vec{a}_1 + \cdots + x_n\vec{a}_n$

Theorem 3.1.3

If $\vec{e}_i$ is the ith standard basis vector and $A = [\vec{a}_1 \ \dots \ \vec{a}_n]$, then $A\vec{e}_i = \vec{a}_i$

Theorem 3.1.5 - Matrices Equal Theorem

if $A, B \in M_{m\times n}(\mathbb{R})$ and $A\vec{x} = B\vec{x} \, \forall \, \vec{x} \in \mathbb{R}^n$, then $A = B$

Identity Matrix

The **identity matrix** in $M_{m\times n}(\mathbb{R})$ is $I_n = \begin{bmatrix} e_1 & e_2 & \dots & e_n \end{bmatrix}$

E.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Theorem 3.1.6

$AI = IA = A$ where $A \in M_{n\times n}(\mathbb{R})$ and $I = M_{n\times n}(\mathbb{R})$ is the identity matrix. This characterizes the identity matrix as the multiplicative identity for matrix multiplication, the only matrix with this property.

Remember, even though in this example, $AI = IA$, matrix multiplication is not guaranteed to be commutative. In most cases, $AB \neq BA$.

Block Form

The definition we learned in class is super confusing and I'm pretty sure Shalom wrote some of the index's wrong so it doesn't even make sense. Look this up in the course notes or abuse a different friend :)