

Math137 - November 30, 2015

More U-Sub Examples

Ex.
$$\int \frac{1}{x + x \sin^2(\ln x)} dx$$

$$= \int \frac{1}{x(1 - \sin^2(\ln x))} dx$$

Let $u = \ln x$
 $\frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x} dx$

$$\int \frac{1}{x(1 - \sin^2(\ln x))} dx$$

$$= \int \frac{1}{1 - \sin^2(u)} du$$

$$= \int \frac{1}{\cos^2(u)} du$$

$$= \int \sec^2(u) \cdot du$$

$$= \tan(u) + C$$

$$= \tan(\ln x) + C$$

Ex.
$$\int \cot(x) dx$$

Let $u = \sin x$
 $\frac{du}{dx} = \cos x$
 $du = \cos x \cdot dx$

$$\int \frac{\cos x}{\sin x} dx$$

$$= \int \frac{1}{\sin u} \cdot du$$

$$= \ln u$$

$$= \ln(\sin x)$$

Ex.
$$\int_{\frac{3\pi}{4}}^{\pi} \cos^2(2x) \sin(2x) dx$$

Let $u = \cos(2x)$
 $\frac{du}{dx} = -2 \sin(2x)$
 $\frac{du}{2} = \sin(x) \cdot dx$

$$\int_{\frac{3\pi}{4}}^{\pi} \cos^2(2x) \sin(2x) dx$$

$$= \frac{-1}{2} \int_0^1 u^2 \cdot du$$

$$= \frac{-1}{2} \left[\frac{u^3}{3} \right]_0^1$$

$$= \frac{-1}{2} \left[\frac{1}{3} - 0 \right]$$

$$= \frac{-1}{6}$$

Thats it for today, not many examples.