# Math138 - January 29'th, 2016

## Intro to Mathematical Modeling

**Examples of Modeling Real World Problems with ODEs**

**Ex.** Newton's Law of Cooling: A hot/cold object's temperature changes proportionally to the difference to the surrounding temperature.

$\frac{dT}{dt} = -k(T - T_{room})$

**Mixing Tank Problems**

**Ex.** A tank has 80L of fresh water. At $t = 0$, a salt solution of 0.24kg/L flows into the tank at 8L/min. Liquid drains out of the tank at 12L/min. Construct an ODE for the mass of salt $x(t)$ in the tank at time $t$.

$$\begin{aligned}\frac{dx}{dt} &= \text{Rate of change of salt mass in the tank.}\\ &= \text{Rate of salt going in - Rate of salt going out}\\ &= (0.25)\cdot(8) - (\text{concentration}\cdot 12)\end{aligned}$$

What is the concentration in the tank??

$$\begin{aligned}\text{conc.} &= \frac{\text{Amount of Salt}}{\text{Volume of Liquid}}\\ &= \frac{x}{80 - 12t - 8t}\\ &= \frac{x}{80 - 4t}\end{aligned}$$

Sooo, we get:

$$\begin{aligned}\frac{dx}{dt} &= 2 - \frac{x}{80 - 4t}(12)\\ &= 2 - \frac{3x}{20 - t}, \quad x(0) = 0 \quad 0 \leq t \leq 20\end{aligned}$$

**Direction Fields**

Lost (most) ODEs can't be solved using elementary functions.

**Ex.** $\frac{dy}{dx} = y - x^2$

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Lets look at a graphical approach to solve ODEs. Consider $\frac{dy}{dx} = f(x, y)$, this tells us what the slope of the tangent looks like at each point.

**Comparsion**

| Analytical | Geometric |
|---|---|
| $y' = f(x, y)$ | Direction Field |
| $y(x)$ =soln to ODE. | Solution Curve |

Think of the $x - y$ plane sprinkled with iron filings. $f(x, y)$ is the magnetic field that aligns with these filings.

The solution curve is a curve that is tangent to one of these filings at each point.