Math138 - February 3'rd, 2016

Linear First Order ODEs

Recall

First order: y' appears, no higher derivative. Linear: only linear functions in y and y'

A general form of a linear first order ODE is

$A(x)y' + B(x)y = c(x)$

We can manipulate to get $y' + P(x)y = Q(x)$

Preliminary Example

Suppose we want to solve $\frac{dy}{dx} + \frac{1}{x} \cdot y = 1$. This ODE is not separable. The trick is to multiply both sides of the function by x.

$x\frac{dy}{dx} + y = x$

Notice the left side of this function is really just the product rule! Whoa! We can rewrite as:

$\frac{d}{dy}(xy) = x$

Integrating gives:

$xy = \frac{x^2}{2}$
$y = \frac{x}{2} + \frac{c}{x}$

Finding the Trick

In this last example, we multiplied both sides by x and the left side became the product rule. This allowed us to simplify the ODE very easily. Unfortunately, multiplying by x won't always work. Luckily, there is a closed form solution to find this "trick" function.

Suppose we have the ODE $\dfrac{dy}{dx} + P(x)y = Q(x)$. Call the function we want $\mu(x)$

We do a bunch of fancy math, and at the end of it, we find:

$\mu = e^{\int P(x)dx}$

Yeah thats an integral in the exponent. How awesome is that?