Math136 - February 3'rd, 2016

Linear Mappings

A function $f : \mathbb{R}^n \to \mathbb{R}^m$ given by $f(\vec{z}) = A\vec{x}$(for $A \in M_{m\times n}(\mathbb{R})$) is called a **matrix mapping**. We will use two notations for convenience.

$f(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}) = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$

$f(x_1, \dots, x_n) = (y_1, \dots, y_m)$

Theorem 3.2.1

If A is an $m \times n$ matrix and $f : \mathbb{R}^n \to \mathbb{R}^m$ is defined by $f(\vec{x}) = A\vec{x}$, then for all $\vec{x}, \vec{y} \in \mathbb{R}^n, b, c \in \mathbb{R}$, we have

$f(b\vec{x} + c\vec{y}) = bf(\vec{x}) + cf(\vec{y})$

Linear Mapping

A **linear mapping** $L : \mathbb{R}^n \to \mathbb{R}^m$ satisfies $L(b\vec{x} + c\vec{y}) = bL(\vec{x}) + cL(\vec{y}) \; \forall \; b, c \in \mathbb{R}, \vec{x}, \vec{y} \in \mathbb{R}^n$

A linear mapping $L : \mathbb{R}^n \to \mathbb{R}^n$ is a linear operation.

Theorem 3.2.2

Every linear mapping $L : \mathbb{R}^n \to \mathbb{R}^m$ can be represented as a matrix mapping whose ith column is the image of the ith standard basis vector of $\mathbb{R}^n$ under L for all $i \in \{1, \dots, n\}$. That is, $L(\vec{x}) = [L]\,\vec{x}$ where $[L] = [L(\vec{e}_1), \dots, L(\vec{e}_n)]$

Short note because I'm super confused and couldn't explain this stuff if I wanted too -_-