Math135 - November 18, 2015

Conversions Between Polar Form and Standard Form - DMT

Polar Form

The polar form for the complex number z is $z = r(\cos\theta + i\sin\theta)$. We usually factor the r, rather than showing it as $z = r\cos\theta + ri\sin\theta$

Polar to Standard

Given r and θ, how do we find z in standard form?

By trig ratio's, we can figure out that $x = r\cos\theta$ and $y = r\sin\theta$.

Ex. If $r = 3$, $\theta = \frac{\pi}{4}$, what is z in standard form?

$x = r\cos\theta$
$x = 3\cos\frac{\pi}{4}$
$x = \frac{3\sqrt{2}}{2}$

$y = r\sin\theta$
$y = 3\sin\frac{\pi}{4}$
$y = \frac{3\sqrt{2}}{2}$

$$z = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$$

Standard to Polar

Suppose we're given $z = x + yi$, and we want to convert to polar form, $z = r(\cos\theta + i\sin\theta)$. We need to figure out r and θ.

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$
$\theta = \arctan\frac{y}{x}$

Ex. If $z = \sqrt{6} + \sqrt{2}i$. what is z in polar form?

$r = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2}$
$r = \sqrt{8}$
$r = 2\sqrt{2}$

$$\theta = \arctan\frac{\sqrt{2}}{\sqrt{6}}$$
$$\theta = \arctan\frac{1}{\sqrt{3}}$$
$$\theta = \frac{\pi}{6}$$

$$z = 2\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

Ex. Write $\cos\frac{15\pi}{6} + i\sin\frac{15\pi}{6}$ in standard form.

By inspection, we see $r = 1$, $\theta = \frac{15\pi}{6}$.

$$\begin{aligned} x &= r\cos\theta \\ &= 1\cos\frac{15\pi}{6} \\ &= 1\cos\frac{5\pi}{2} \\ &= 1\cos\frac{\pi}{2} \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= r\sin\theta \\ &= 1\sin\frac{15\pi}{6} \\ &= 1\sin\frac{\pi}{2} \\ &= 1 \end{aligned}$$

$\therefore z = 0 + 1i = i$

Ex. Convert $z = -3\sqrt{2} + 3\sqrt{6}i$ into polar form.

$$\begin{aligned} r &= \sqrt{18+54} \\ &= \sqrt{72} \\ &= 6\sqrt{2} \end{aligned}$$

$$\begin{aligned} \theta &= \arctan\left(\frac{3\sqrt{6}}{-3\sqrt{2}}\right) \\ &= \arctan(-\sqrt{3}) \\ &= \frac{2\pi}{3} \end{aligned}$$

$z = 6\sqrt{2}(\cos \frac{2\pi}{3} + i \sin \frac{2\imath}{3})$

Polar Multiplication of Complex Numbers (PMCN)

$$\text{If } z_1 = r_1(\cos\theta_1 + i\sin\theta_1) \text{ and } z_2 = 4_2(\cos\theta_2 + i\sin\theta_2), \text{then } z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$$

Ex. Multiply $(\sqrt{6} + \sqrt{2}i)(-3\sqrt{2} + 3\sqrt{6}i)$

$$\begin{aligned} &= \left[2\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right] \cdot \left[6\sqrt{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right] \\ &= 24\left[\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right] \\ &= 24\left[\frac{-\sqrt{3}}{2} + i\frac{1}{2}\right] \\ &= -12\sqrt{3} + 12i \end{aligned}$$

We can see ghat when we multiply complex numbers, geometrically we multiply the distances from the pole and ad the angles from the polar axis.

De Moivre's Theorem

$$\text{If } \theta \in \mathbb{R}, n \in \mathbb{Z}, \text{ then:}$$
$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$$

Ex. Simplify $(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})^{-1000}$

$$\begin{aligned} &= (\cos\frac{-3000\pi}{4} + i\sin -3000\pi 4) \\ &= \cos(-750\pi) + i\sin(-750\pi) \\ &= \cos 0 + i\sin 0 \\ &= 1 + oi \\ &= 1 \end{aligned}$$

Corollary To DMT

$$\text{If } z = r(cos\theta + i\sin\theta) \text{ and } n \text{ is an integer,}$$
$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$