Math137 - November 18, 2015

Anti-Derivatives

Notice:

From this point out, all notes will be taken in Mike Edens class.

Anti-Derivatives

An anti-derivative is the opposite of a derivative. You will be given a derivative and asked to find the original function that gave that dertivative.

Ex. Given $f'(x)$, find $f(x)$.

a) $f'(x) = 6$
$f(x) = 6x + C$ (C is some constant)

b) $f'(x) = 2x$
$f(x) = x^2 + C$

c) $f'(x) = 3x$
$f(x) = \frac{3}{2}x^2 + C$

d) $f'(x) = -3x^5$
$f(x) = \dfrac{-3x^6}{6} + C$
$f(x) = \dfrac{1}{2}x^6 + C$

When we differentiate powers of x, we:

1) Multiply by the exponent.
2) Subtract 1 from the exponent.

When anti-differentiate powers of x, we:

1) Add 1 to the exponent.
2) Divide by the new exponent.
3) Add a constant C

Notation: We denote the anti-derivative of $f(x)$ as $F(x)$

Anti-Derivatives We Know

$f(x)$	x^p $(p \neq 1)$	$x^{-1} = \frac{1}{x}$	e^x	a^x	$\sin x$	$\cos x$	$\sec^2 x$	$\sec x \tan x$	$\dfrac{1}{\sqrt{1-x^2}}$
$F(x)$	$\dfrac{x^{p+1}}{p+1} + C$	$\ln\lvert x\rvert + C$	$e^x + C$	$\dfrac{a^x}{\ln a} + C$	$-\cos x + C$	$\sin x + C$	$\tan x + C$	$\sec x + C$	$\arcsin x + C$

$f(x)$	$\dfrac{1}{1+x^2}$	$\sinh x$	$\cosh x$
$F(x)$	$\arctan x + C$	$\cosh x + C$	$\sinh + C$

Ex.

a) $f(x) = \frac{1}{2}e^{3x} + \sin(3x)$ $F(x) = \frac{1}{2}e^{3x} \cdot \frac{1}{3} + (-\cos(3x)) \cdot \frac{1}{3} + C$
$F(x) = \frac{1}{6}e^{3x} - \frac{1}{3}\cos(3x) + C$

b) $f(x) = \dfrac{\sqrt{x} - x^2}{x^3}$
$f(x) = x^{\frac{1}{2}-3} - x^{2-3}$
$f(x) = x^{-5}2 - x^{-1}$
$F(x) = \dfrac{x^{-3}2}{\frac{-3}{2}} - \ln|x| + C$

$F(x) = \dfrac{-2}{3}x^{\frac{-3}{2}} - \ln|x| + C$

c) $f(x) = 2\sec^2(\frac{2}{\pi})$
$F(x) = \dfrac{2\tan(\frac{x}{\pi})}{\frac{1}{\pi}}$
$F(x) = 2\pi\tan(\frac{x}{\pi}) + C$

d) $f(x) = \dfrac{2x^2 - 1}{1 + x^2}$
$f(x) = 2 + \dfrac{-3}{1 + x^2}$
$F(x) = 2x - \arctan(x) + C$