Math135 - November 25, 2015

Properties of Integrals - Calculating Definite Integrals

Note:

Again, today Mike Eden taught from a couple of sheets he uploaded to learn. I'll upload these sheets. These notes were NOT made by me, but by Mike Eden. I'm only uploading so that my website can be a source of all course material. (Except the first 2 months I didn't typeset).

MATH 137 Properties of Integrals Fall 2015

1. $\displaystyle\int_a^b c \text{ dx} = c(b-a),$ where c is any constant

2. $\displaystyle\int_a^b [f(x) \pm g(x)] \text{ dx} = \int_a^b f(x) \text{ dx} \pm \int_a^b g(x) \text{ dx}$

3. $\displaystyle\int_a^b c \cdot f(x) \text{ dx} = c \cdot \int_a^b f(x) \text{ dx},$ where c is any constant

4. $\displaystyle\int_a^b f(x) \text{ dx} = \int_a^c f(x) \text{ dx} + \int_c^b f(x) \text{ dx}$

5. If $f(x) \geq 0$ for $a \leq x \leq b,$ then $\displaystyle\int_a^b f(x) \text{ dx} \geq 0$

6. If $f(x) \geq g(x)$ for $a \leq x \leq b,$ then $\displaystyle\int_a^b f(x) \text{ dx} \geq \int_a^b g(x) \text{ dx}$

7. If $m \leq f(x) \leq M$ for $a \leq x \leq b,$ then $\displaystyle m(b-a) \leq \int_a^b f(x) \text{ dx} \leq M(b-a)$

8. If f is an even function, $\displaystyle\int_{-a}^a f(x) \text{ dx} = 2\int_0^a f(x) \text{ dx}$

9. If f is an odd function, $\displaystyle\int_{-a}^a f(x) \text{ dx} = 0$

10. $\displaystyle\int_a^b f(x) \text{ dx} = -\int_b^a f(x) \text{ dx}$

MATH 137 The Fundamental Theorem of Calculus Fall 2015

FTC - Part 1

If f is continuous on the interval $[a, b]$, the integral function $g(x)$ defined by

$$g(x) = \int_a^x f(t)\ \mathrm{dt}, \quad a \le x \le b$$

is continuous on $[a, b]$ and differentiable on (a, b) and

$$g'(x) = f(x)$$

FTC - Part 2

If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\ \mathrm{dx} = F(b) - F(a),$$

where F is the antiderivative of f (ie. $F' = f$)

Proof:

Let $g(x) = \displaystyle\int_a^x f(t)\ \mathrm{dt}$. We know from FTC 1 that $g'(x) = f(x)$; that is, g is an antiderivative of f. If F is any other antiderivative of f on $[a, b]$, then we know that

$$F(x) = g(x) + c$$

for $a < x < b$ by the corollary of the Constant Function Theorem.

Since both F and g are continuous on $[a, b]$, we see that $F(x) = g(x) + c$ also holds when $x = a$ and $x = b$ by taking one-sided limits (as $x \to a^+$ and $x \to b^-$).

Evaluating $F(b) - F(a)$, we have

$$\begin{aligned} F(b) - F(a) &= [g(b) + c] - [g(a) + c] \\ &= g(b) - g(a) \\ &= \int_a^b f(t)\ \mathrm{dt} - \int_a^a f(t)\ \mathrm{dt} \\ &= \int_a^b f(t)\ \mathrm{dt} - 0 \\ &= \int_a^b f(t)\ \mathrm{dt} \qquad \square \end{aligned}$$

Example 1

The function g is defined by $g(x) = \int_0^x (t - t^2)\ \mathrm{dt}$, for all $x > 0$.
Calculate $g'(1)$ and find the location of any inflection points of g.

Example 2

Determine $\frac{dy}{dx}$ given that $y = \int_{1+3x^2}^{4} \frac{1}{2+e^t}\ \mathrm{dt}$.

Example 3

Evaluate each of the following definite integrals.

(a) $\displaystyle\int_1^3 \left(u + \frac{2}{u}\right) \, \mathrm{du}$

(b) $\displaystyle\int_0^{ln8} \left(2e^{-2t}\right) \, \mathrm{dt}$

(c) $\displaystyle\int_0^1 (2u+1)^2 \, \mathrm{du}$

(d) $\displaystyle\int_{-\frac{1}{2}}^0 (\cos \pi x) \, \mathrm{dx}$