Math137 - November 20, 2015

Integration Applications - Intro to First Principles

Integral Applications:

Note for non-physics people: An object falling close to the earths surface has upward acceleration of -9.8m/s^2.

Ex. A stone is dropped from the Space Deck of the CN Tower 450m above the ground.

a) Find the height h of the stone above the ground at time t.

We know $a(t) = -9.8$
$v(t) = -9.8t + c$
The stone is dropped, so its initial velocity is 0.

$v(0) = 0 = -9.8(0) + c$
$\therefore c = 0$
$\therefore v(t) = -9.8(t)$

$$h(t) = \frac{-9.8t^2}{2} + c_1$$

The stone was dropped at $h = 450$.
$h(0) = 450 \implies \dfrac{-9.8(0)}{2} + c_1 = 450$
$c_1 = 0$
$\therefore h(t) = -4.9t^2 + 450$

b) How long does it take the stone to hit the ground?

We need to find t when $h(t) = 0$

$$0 = -4.9t^2 + 450$$
$$t^2 = \frac{450}{4.9}$$
$$t = +\sqrt{\frac{450}{4.9}} \quad \text{(Time cannot be negative in this application)}$$
$$t \approx 9.58s$$

c) What is the stones impact velocity?

We need to find $v(9.58)$

$$\begin{aligned} v(t) &= -9.8t \\ v(9.58) &= -9.8(9.58) \\ &= -93.9m/s \end{aligned}$$

First Principles:

Note: in this section I reference 'Area Under a Curve', a pdf available under Mike Eden's section on Learn.

1. **What would be another way to estimate the total distance Caroline travelled?**
   We could have used the right endpoint of each interval instead of the left. (Denoted R_6).
2. **How could we make the estimate better?**
   We could have use the midpoint speed of each interval instead of left or right.
   We could decrease the width of our intervals (i.e. increase the number of intervals)
3. **How is this related to area under a curve?**
   The antiderivative of velocity is distance.
   The area under the velocity curve gives distance over an interval $[t_1, t_2]$.

   Conclusion: The anti-derivative of velocity is the area under the velocity curve.