Math136 - February 5'rd, 2016

More Theorems Related to Linear Mappings

Range

Let $L : \mathbb{R}^n \to \mathbb{R}^m$ be a linear mapping. The range of l is $\text{Range}(L) = \{L(\vec{x}) \in \mathbb{R}^m \mid \vec{x} \in \mathbb{R}^m\}$

Theorem 3.3.2

If $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear mapping, then Range(L) is a subspace of $\mathbb{R}^m$

Kernel

The kernel of a linear mapping $L : \mathbb{R}^n \to \mathbb{R}^m$ is $\text{Ker}(L) = \{\vec{x} \in \mathbb{R}^n \mid L(\vec{x}) = \vec{0}\}$

Note: $\text{Range}(L) \subseteq \mathbb{R}^m$ is contained in the codomain $\mathbb{R}^m$, while Ker(L) is contained in the domain $\mathbb{R}^n$

Fact: $\vec{0} = L(\vec{0})$, so $\vec{0} \in$Ker(L) for any L.

Theorem 3.3.3

Ker(L) is a subspace of $\mathbb{R}^n$

Next, we will discuss concepts analogous to kernel and range, but with relation to a matrix.

Null Space

The null space of A is $\text{Null}(A) = \{\vec{x} \in \mathbb{R} \mid A\vec{x} = \vec{0}\}$

Note: If $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear mapping, then $[L] \in M_{m\times n}(\mathbb{R})$, and Ker($L$) =Null($[L]$)

Column Space

Let $A = \begin{bmatrix}\vec{a}_1 & \dots & \vec{a}_n\end{bmatrix}$ be an $m\times n$ matrix with columns $\vec{a}_1, \dots, \vec{a}_n$. Then the column space of A is Col(A) =span$\{\vec{a}_1, \dots, \vec{a}_n\}$

Note: If $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear mapping, then Range(L) =col($[L]$)