Math136 - January 22'th, 2016

Solving Systems of Equations

Equivalence

Two systems of linear equations which have the same solution sets are **equivalent**.

(Augmented) Coefficient Matrix

Suppose we have a system of m equations in n variables:

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ &a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ &\vdots \\ &a_{m1}x_1 + a_{m2}x_n + \cdots + a_{mn}x_n = b_n \end{aligned}$$

The coefficient matrix of this system is:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

And the augmented coefficient matrix is:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array}\right]$$

For now, we can think of the augmented matrix as a convenient way to represent our system of equations. Later, we'll see that we get a lot more out of this by developing matrix operations.

Elementary Row Operations

The elementary row operations are:

1. Multiplying a row by a non-zero scalar
2. Adding a multiple of one row to another
3. Swapping two rows

Note: All of these operations are reversible.

As we will see, if we start with a system of equations, then perform elementary row operations on the augmented matrix, we end up with the augmented matrix of an equivalent system of equations.

Row Equivalent

Two matrices A and B are row-equivalent if one can be obtained from the other by a sequence of elementary row operations.

Theorem 2.2.1

If the augmented matrices $[A_1 \mid \vec{b}_1]$ and $[A \mid \vec{b}]$ are row equivalent, then the associated systems of linear equations are equivalent.

Reduced Row Echelon Form

A matrix R is in reduced row echelon form (RREF) if:

1. All rows containing a non-zero entry are above rows which contain only zero's.
2. The first non-zero entry in each row is 1 (The leading 1)
3. The leading in each non-zero row is to the right of the leading row in any row above
4. A leading one is the only non-zero entry in it's column.

If A is row equivalent to R which is in RREF, then R is the RREF of A.