10. We have,

$$\text{L.H.S.} = \frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = \frac{\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}}{\sin\theta\cos\theta} = \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta} - \frac{\cos^2\theta}{\sin^2\theta\cos^2\theta} = \sec^2\theta - \operatorname{cosec}^2\theta = \text{R.H.S.}$$

30. Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \quad \sin\theta = \cos\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \quad \tan\theta = 1$ $\quad \left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \quad \tan\theta = \tan 45°$ $\quad [\because \tan 45° = 1]$

$\Rightarrow \quad \theta = 45°$

$\therefore \quad \sin^4\theta + \cos^4\theta = \sin^4(45°) + \cos^4(45°)$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = 2\left(\frac{1}{\sqrt{2}}\right)^4 = 2\left(\frac{1}{4}\right) = \frac{1}{2}$$

31. $\dfrac{5}{\cot^2 30°} + \dfrac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°$

$$= \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - (1)^2 + 2(1)^2 = \frac{5}{3} + \frac{4}{3} - 1 + 2 = 4$$

32. Given, $\sin\theta = \cos\theta$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan\dfrac{\pi}{4}$ $\quad [\theta \text{ is acute}]$

$\therefore \quad \theta = \dfrac{\pi}{4}$

So, $\tan^2\theta + \cot^2\theta - 2$

$$= \tan^2\left(\frac{\pi}{4}\right) + \cot^2\left(\frac{\pi}{4}\right) - 2 = 1 + 1 - 2 = 0$$

40. (a) : We have, $\frac{x}{3} = 2\sin A \Rightarrow x = 6\sin A$

and $\frac{y}{3} = 2\cos A \Rightarrow y = 6\cos A$

Now, $x^2 + y^2 = (6\sin A)^2 + (6\cos A)^2$

$= 36\sin^2 A + 36\cos^2 A$

$= 36(\sin^2 A + \cos^2 A) = 36(1)$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= 36$

73. We have $\sin^6 A + 3\sin^2 A\cos^2 A = 1 - \cos^6 A$

Rewriting and arranging the given equation as

$\sin^6 A + \cos^6 A = 1 - 3\sin^2 A\cos^2 A$

Now taking L.H.S. of equation (i), we get

$\sin^6 A + \cos^6 A = (\sin^2 A)^3 + (\cos^2 A)^3$

{By using $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b) \Rightarrow a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)$, here $a = \sin^2 A$ and $b = \cos^2 A$}

$\therefore \quad \sin^6 A + \cos^6 A = (\sin^2 A + \cos^2 A)^3 - 3 \sin^2 A \cos^2 A\,(\sin^2 A + \cos^2 A)$

$= 1^2 - 3 \sin^2 \cos A\,(1) = \text{R.H.S.}$ $\qquad [\because \sin^2 A + \cos^2 A = 1]$

$\Rightarrow \quad$ L.H.S. = R.H.S

Hence, proved.

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9.) Prove:

(i) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta.\cos^2\theta = 1$

$\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta.\cos^2\theta$

L.H.S $\Rightarrow (\sin^2\theta)^3 + (\cos^2\theta)^3 = (\sin^2\theta+\cos^2\theta)^3 - 3\sin^2\theta.\cos^2\theta \times (\sin^2\theta + \cos^2\theta)$

$[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$

$\Rightarrow (1)^3 - 3\sin^2\theta.\cos^2\theta \times (1)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow 1 - 3\sin^2\theta.\cos^2\theta =$ R.H.S

L.H.S = R.H.S

Hence, proved

10.) Prove:

(ii) $\cos^4\theta - \cos^2\theta = \sin^4\theta - \sin^2\theta$

L.H.S $\Rightarrow \cos^2\theta(\cos^2\theta - 1) = 1 - \sin^2\theta[-(1-\cos^2\theta)]$

$\Rightarrow 1 - \sin^2\theta(-\sin^2\theta)$

$\Rightarrow -\sin^2\theta + \sin^4\theta$

$\Rightarrow \sin^4\theta - \sin^2\theta =$ R.H.S

L.H.S = R.H.S

Hence, proved

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(iv) $\cot^4\theta - 1 = \csc^4\theta - 2\csc^2\theta$

$\cot^4\theta = \csc^4\theta + 1 - 2\csc^2\theta$

$\cot^4\theta = (\csc^2\theta - 1)^2$ [$\because a^2+b^2-2ab = (a+b)^2$]

$\cot^4\theta = (\cot^2\theta)^2$ [$\because \csc^2\theta - 1 = \cot^2\theta$]

$\cot^4\theta = \cot^4\theta$

L.H.S = R.H.S

Hence, proved

18) $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$x\sin\theta = y\cos\theta$

$x\sin\theta = y\cos\theta$

Let $\dfrac{x}{\cos\theta} = \dfrac{y}{\sin\theta} = k$

$x = \dfrac{y\cdot\cos\theta}{\sin\theta} = k\cos\theta$ —— ①

$y = \dfrac{x\cdot\sin\theta}{\cos\theta} = k\sin\theta$ —— ②

$x\sin^3\theta + y\cos^3\theta \Rightarrow$ L.H.S

Using ① & ②,

$k\cos\theta\sin^3\theta + k\sin\theta\cos^3\theta \Rightarrow$ L.H.S

$k\cos\theta\sin\theta(\sin^2\theta + \cos^2\theta) \Rightarrow$ L.H.S

L.H.S $\Rightarrow$ $k\cos\theta\sin\theta$ = R.H.S = $\sin\theta\cos\theta$

so, $\boxed{k=1}$

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$x = k\cos\theta$

$x = 1(\cos\theta)$ $[K=1]$

$y = k(\sin\theta)$

$y = 1(\sin\theta)$ $[K=1]$

$x^2+y^2 = \cos^2\theta + \sin^2\theta = 1$ $[\sin^2\theta + \cos^2\theta = 1]$

Therefore, $x^2+y^2=1$

Hence, proved