Thermal model for analysis of Mars infrared mapping

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Abstract. The KRC numerical thermal model has been used in the analysis of observations from virtually all Mars missions with infrared sensors and for the selection of virtually all Mars' landing sites. It uses a physics-based one-layer atmosphere gray at solar and thermal wavelengths to determine the radiative effect of a dusty atmosphere. One gas component may condense to form a seasonal polar cap and affect the global surface pressure. The atmosphere may be omitted entirely to model airless bodies. KRC uses layers that become thicker geometrically with depth and it uses repeated time-step doubling. The surface may be homogeneous or have two zones of material properties, each zone may have temperature-dependent thermal conductivity and specific heat. Surface slopes or depressions are modeled to first order. Here KRC is described in detail and used to compute globally the annual average surface temperature accounting for albedo, thermal inertia, elevation, slope at 3 km resolution and zonal climate. Comparisons with three other thermal models are discussed. The model is available for general use.

1. Introduction

This paper describes a numerical model used extensively for computing planetary surface temperatures. The KRC numerical model has evolved over a period of five decades and has been used for a variety of planet, satellite and comet problems, however use has concentrated on Mars. The model uses a one-layer atmosphere but does allow condensation and global pressure variation; the model can output surface kinetic and planetary (nadir view from space) bolometric temperatures, along with a variety of parameters related to subsurface-layer and atmosphere temperatures, seasonal polar cap mass, heat-flow and numerical performance parameters.

The program is designed to compute surface and subsurface temperatures for a global set of latitudes at a full set of seasons, with enough depth to capture the annual thermal wave, and to compute seasonal condensation mass. For historic reasons (it originated in the era of kilo-Hz processors) the code has substantial optimization. It allows sloped surfaces and two zones of different sub-surface materials. There are generalities that allow this code set to be used for any ellipsoid with any spin vector, in any orbit (around any star); with or without an atmosphere (including condensation); this is also the source of some of the complexity.

In response to an oft-asked question, the acronym KRC is simply K for conductivity, R for "rho" (ρ) for density, and C for specific heat; the three terms in thermal inertia I. KRC uses explicit forward finite differences and is coded in FORTRAN (with some utility C routines). Model development began 1968, and was used to support the Viking mission with a total of 3 cases in an era when computing a single case for 19 latitudes at 40 seasons with a 2-year spin-up took an hour on a large university unshared main-frame computer. For this reason, the code is highly optimized for speed and uses layer thickness increasing exponentially downward and time steps that increase by factors of two deeper into the subsurface where stability criteria are met. The code is modularized based on time-scale and function, and there is extensive use of Commons. The version used for Mariner 9 and Viking was described briefly in **?**. The KRC model was used in many analyzes of the Viking Infrared Thermal Mapper (IRTM) data; derivatives were used to study sublimating comets **?** and ring and satellite eclipses **???**,. The code has undergone step-wise revision, a major change being a 2002 replacement of a down-going steady IR flux equivalent to fixed fraction of the noon insolation with the atmosphere described here. This newer version has been the basis for analysis of THEMIS and Mars Exploration Rover (MER) Mini-TES results. As of 2009, the code allows temperature-dependent thermal conductivity and specific-heat.

Although KRC has many loops to compute temperatures as a function of time-of-day, latitude, season and a multitude physical parameters, a key characteristic is that at its core it only computes one surface temperature at a time. A number of first-order approximations are used to accommodate slopes and circular depressions without an appreciable decrease in speed.

For THEMIS, a "one-point" (OnePoint) capability was included that allows input of a set of points defined by season, latitude, hour and a few major physical parameters; KRC will produce the surface kinetic temperature and planetary brightness temperature for these points; see §5.6.

The Datasets that constitute the on-line auxiliary material include all source code, guides for installation and running KRC, supporting documentation, sample input and output files. The full source code and documentation will be maintained and available at the KRC website: http://krc.mars.asu.edu .

1.1. Use for recent missions

Determination of thermal inertia using the KRC model has been used in selecting all landing sites on Mars; Viking: [**?**, p.352+], Pathfinder: **?**, MER: **??**, Phoenix: **?**, Mars Science Laboratroy) MSL: **??**. Rock abundances have been computed using the KRC model **?** and post-landing assessment has shown the estimates based on IR observations to be close **???**.

Standard data reduction of the Odyssey Thermal Emission Imaging System (THEMIS) uses the KRC model **????**. This involved generation of a large set of models on a grid of thermal inertia I, surface albedo A, elevations and visual dust opacities τ_0 with output of surface kinetic temperature

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T_s and top-of-atmosphere bolometric temperature T_b at a uniform set of latitudes, Hours (H, 1/24 of the planets day) and seasons. The 7-dimensional model set is interpolated first in season (to correspond to a specific Odyssey orbit), then at the latitude and hour of each observation using the elevation corresponding to the observation longitude; interpolation in opacity and albedo are based on prior or current observations. This leaves T_s and T_b as a function of thermal inertia, which may not be monotonic; this relation is interpolated linearly in T, using either T_b or opacity-corrected T_s, and logarithmically in I to get the thermal inertia. KRC was used in analysis and surface thermal observations by Mini-TES, usually with a similar scheme for T_s only. **??**.

KRC thermal modeling has been used for study of general nature of the Martian surface **?????**, Chapter 9 in **?**; and detailed sites: **????????**.

KRC has been used in many analyses of TES data, e.g., **??**.

KRC models are the basis for the surface temperature estimate used for the black-body emission correction to Mars Reconnaissance Orbiter (MRO) Compact Reconnaissance Imaging Spectrometer for Mars (CRISM) reflection spectra, **?**.

KRC **?** and derivatives **??** have been used in study of seasonal slab ice. The capability to model temperatures at the bottom of conical depressions was added to study the potential volatile sublimation in freshly exposed trenches to be dug by the Phoenix mission; this geometric capability can as well be applied to the floors of craters.

In an extreme case, a movie of a thermal day on Mars was made by computing surface temperatures at 0.017 Hour intervals with a "spin-up" of 5 sols at $L_s = 90°$. The measured albedo, thermal inertia and elevation with 0.25 deg resolution in longitude and latitude was used. 1.45 billion instances of KRC OnePoint mode were run on a cluster of 128 CPU's for 48 hours over a weekend. The video can be viewed at
http://mars.asu.edu/ phil/ls90_32_full_5_part_movie.mov [personal communication from Phil Christensen]. A version of KRC specifically modified later for this application increased the speed by a factor of 3800.

1.2. Some other thermal models used for Mars

The Martian atmosphere has a significant effect on surface temperature, both in the physical temperature of the surface being influenced by the dusty atmosphere's modification of the insolation that reaches the surface and the down-going infrared radiation from the atmosphere, and on the apparent temperature measured remotely by infrared radiometry **?**. Thermal models which treat the atmosphere in detail, such as a dusty radiative/convective column **?** or that include lateral heat transport such in a General Circulation Model (GCM) **??**, generally take two to several orders of magnitude longer than KRC to run.

The thermal models for the Mars Global Surveyor (MGS) Thermal Emission Spectrometer (TES) standard data processing were based on the Mellon-Jakosky-Haberle model [hereafter simply the Mellon model], **??**, The Mellon model has direct heritage from KRC and uses a similar subsurface; it has a multi-layer conductive/convective atmosphere with radiative properties based on calculations by Jim Pollack whose basis has been lost [community oral history]. Extensive comparison of the Mellon model and KRC was done in development of the MGS TES production code; see §3.4.3. This model has been used in detailed global mapping of TES observations **?**.

Schorghofer and Aharonson developed a model in which treatment of the atmosphere and slopes is based on KRC. This model uses the Crank-Nicolson scheme and includes vapor diffusion in the soil; it has been used to study the distribution of ice in current and prior climates **???**. Helbert and Benkhoff developed a model that allows detailed layering of properties and vapor diffusion **?**.

A finite-difference thermal model used for estimating depth to liquid water stability **?** was made publicly available. A derivative of this model and KRC was used to study ground ice stability **?**.

A model similar to KRC in subsurface representation was used largely for Mars' polar studies **????**.

1.3. Notation use here

Program and routine names are shown as **PROGRM**. Code variable names are shown `VARIAB`. Input parameters are shown as INPUT. File names are shown as *file*.

For convenience, some physical parameter default values are shown within square brackets at their point of mention and some are listed in Tables **??** and **??**, which contain symbols, variable names, and some indication of how often various terms are calculated.

All units are SI, except the use of days [86400 seconds] for orbital motion. The sample input file, Dataset 8, includes all input parameters. The symbols T_s and T_a are used in the text for kinetic temperature of the surface and the one-layer atmosphere.

2. Physical representation

KRC solves the heat diffusion equation (Eq. 15) with an upper boundary condition (Eq. 13) that includes periodic insolation through a dusty (and cloudy) atmosphere, infrared radiative transfer with the atmosphere, and condensation and accumulation of diurnal and seasonal frosts. The primary outputs are kinetic surface temperature T_s and bolometric 'top-of-the-atmosphere" temperature T_b as might be measured by a remote observer, both as functions of 'hour' (local time of day), season and latitude; it can also produce subsurface layer temperatures, atmosphere temperature and frost amounts.

KRC is designed to do this for a spinning ellipsoid (e.g., Mars) in an elliptical orbit for periods of a few 'sols' (body rotation periods) to many 'years' (orbital periods). All the planetary (or satellite or asteroid) values are specified by a set of about 100 input parameters, described in the Datasets 7 and 8; some are in Table **??**, none are hard-coded.

2.1. Planetary Orientation and Orbit

KRC can accept either fixed heliocentric range and subsolar latitude, or Keplerian orbital elements and a fixed pole-orientation (direction of the spin axis); in both cases, "seasons" are at uniform increments of time. An associated Planetary ORBit program set, **PORB**, main program **porbmn**, accesses files containing the elements for all the planets **?** and a few comets and asteroids (straight-forward to add more); this program set pre-calculates the orbital elements for any epoch, converts them into rotation matrices for the chosen epoch and creates a plain-text parameter set that is then incorporated into the input file for KRC. This plain-text parameter set is described in Dataset 4 under PORBCM.INC; the PORB system is available at the KRC website. For TES and the THEMIS modeling done at the start of those investigations, the Martian elements (with respect to Earth equator and equinox) were evaluated for epoch 1999; Mars' spin-axis orientation was based on pre-Viking data, and differed from the current best estimates **?** by about 0.3°. Within KRC, the orbital position of Mars is computed for each "season", yielding the heliocentric range, the sub-Solar latitude, and the seasonal indicator L_s.

Planetary spin-axis orientations have been updated to **?** and mean elements have been updated to **?**. Short-term perturbations by the planets are ignored, which can yield errors up to 0.03° in L_s for Mars,[?, Figure 2].

The length of a season step is input (units of days) and generally will be a sub-multiple of a planets year; it can be as short as one sol (details of how to do this are in Datasets 7 and 8).

2.2. Atmosphere

A goal of the KRC model has been to account for the first-order effects of the atmosphere, while preserving the speed and flexibility to deal with surface effects such as layered materials and sloping surfaces. A complicating factor in treating the atmosphere more fully is that the opacity of Mars' atmosphere can vary considerably in space and time ??; although seasonal- and latitude-dependent opacity can be specified for KRC, only GCM's with surface dust interaction can begin to reactively model this.

KRC uses a one-layer atmosphere that is gray in the visible and thermal wavelength ranges. Radiative exchange with the Sun, space and the surface determines the model atmosphere energy balance and its temperature variation. The columnar mass (and the surface pressure) can vary with season and surface elevation. A uniformly-mixed dust loading is allowed to modify the visual and thermal opacity. Direct and diffuse illumination are computed using a double-precision 2-stream Delta-Eddington model, with single scattering albedo ϖ and Henyey-Greenstein asymmetry parameter G_H. The thermal opacity due to dust is a constant factor C_2 times the visual dust opacity; this factor is also applied to ice clouds, if considered [a refinement would be to use a separate factor for ice aerosols; the factor is a strong function of particle size for both dust and ice ?]. An option allows an extension of twilight past the geometric terminator.

The current local visible-wavelength (solar) atmospheric opacity of dust can vary with atmospheric pressure; the total (normal) "solar" opacity is $\tau_v = \tau_0 \cdot P/P_0 + \tau_i/C_2$ where τ_0 is the unity air-mass visible dust opacity defined for surface pressure P_0, P is the current local surface pressure and τ_i is the infrared ice-cloud opacity. τ_0 and τ_i may vary with season and latitude; KRC can ingest a climate model for these two parameters as a function of season and latitude.

2.2.1. Delta-Eddington 2-stream

A Delta-Eddington model ?? is used for atmosphere scattering and fluxes (**deding2.f**); output parameters are normalized to unit solar irradiance along the incident direction at the top of atmosphere; so they must be multiplied by S_M, the solar irradiance at the current heliocentric range, to get flux.

Scattering parameters used are the aerosol single scattering albedo ϖ and the Henyey-Greenstein scattering asymmetry parameter ?; both are input constants. The computed values include:

Planetary (atmosphere plus surface system) albedo: `BOND`

Direct beam at the bottom, includes both collimated and aureole: $F_\| =$ `COLL`

Diffuse irradiances: $I_{i,j}$

i: 1 = isotropic, 2 = asymmetric

j: 1 = at top of atmosphere, 2 = at bottom of atmosphere

The net diffuse flux is $F_\ominus = \pi[I_{1j} \pm \frac{2}{3} I_{2j}]$ where + is down, $F_\ominus^\downarrow$; - is up, $F_\ominus^\uparrow$. [?, eq. 8]

The total down-going solar flux at the bottom of the atmosphere is

$$S_t' = S_M \left(\mu_0 \ F_\| + F_\ominus^\downarrow\right) \quad (1)$$

where $\mu_0 \equiv \cos i_0$, i_0 is the incidence angle onto a horizontal plane and the diffuse component is $F_\ominus^\downarrow = \pi\left(I_{1,2} + \frac{2}{3} I_{2,2}\right)$.

Solar heating of the atmosphere, by conservation of energy, is

$$H_V = S_M \left(\mu_0 - F_\ominus^\uparrow(0) - (1-A)\left[\mu_0 \ F_\| + F_\ominus^\downarrow(\tau_v)\right]\right) \quad (2)$$

where A is the surface albedo.

2.2.2. Twilight

Twilight is allowed to account for a turbid atmosphere. It is implemented as having the diffuse downward illumination depend upon an incidence angle scaled to go to 90° when the Sun is `TWILI`$=\eta$ below the geometric horizon. I.e., the incidence angle input to the Delta-Eddington routine for diffuse downward illumination is $\frac{90}{90+\eta} i_0$

Because of the twilight extension, a small negative energy balance near twilight can remain. Physically, this is lateral scattering and does not strictly fit a one-layer model. There is no solar heating of the atmosphere during twilight.

2.2.3. Atmospheric IR radiation

The IR opacity is approximated as $\tau_R = P/P_0 \cdot (C_1 + C_2\tau_0) + \tau_i$ where C_1 represents the opacity of the "clear" (no dust or ice aerosols) atmosphere, primarily due to the 15μm CO_2 band, and C_2 is the IR/visual opacity ratio for dust(e.g., ?).

To estimate the down-going radiation from a clear atmosphere, a synthetic transmission spectrum of the Mars atmosphere with a nominal amount of water vapor (provided by David Crisp, 700 Pa column) was multiplied by blackbody spectra for a range of temperatures to determine the fraction of radiation blocked, see Figure ??. A coefficient of $C_1 = 0.11 \pm 0.004$ covers the range from 187K to 293K; a first-order correction for other surface pressures is to scale this input parameter linearly with the chosen total pressure P_0.

The fractional thermal transmission of the atmosphere at zenith is roughly $e^{-\tau_R}$. The fractional absorption is $\beta \equiv 1. - e^{-\tau_R}$.

The fractional transmission of planetary (thermal) radiation in a hemisphere is:

$$e^{-\tau_e} \equiv \int_0^{\pi/2} e^{-\tau/\cos\theta} \cos\theta \sin\theta \ d\theta \quad (3)$$

Numerical integration shows that the effective hemispheric opacity is, within about 0.05 in the factor,

$$\tau_e \sim [1.0 < (1.50307 - 0.121687 * \ln \tau_R) < 2.0]\, \tau_R \ ; \quad (4)$$

this is used in the effective absorption $\beta_e \equiv 1. - e^{-\tau_e}$.

The hemispheric downward (and upward) emission from a gray slab atmosphere is: $R_\Downarrow = \sigma T_a^4 \beta_e$; σ is the Stephan-Boltzmann constant. The IR heating of the atmosphere is:

$$H_R = \epsilon\sigma T_s^4 (1. - e^{-\tau_e}) - 2R_\Downarrow \quad = \quad \sigma\beta_e(\epsilon T_s^4 - 2T_a^4) \quad (5)$$

where ϵ is the surface emissivity.

2.2.4. Atmospheric temperature

The atmospheric temperature is assumed to follow radiative energy conservation:

$$\frac{\partial T_a}{\partial t} = \frac{H_R + H_V}{C_p M_a} \quad (6)$$

where $M_a = P/g$ is the mass of the atmosphere and C_p is its specific heat at constant pressure.

Because the atmospheric temperature variation has significant time lag relative to the surface (typically about 1/4 sol, as will be seen in Figures 5 and 6), KRC uses the surface temperature from the prior time step (typically 1/384 of a sol) to evaluate H_R with little error [effect on T_a:, average: $< 0.2K$, amplitude: <0.5%, phase: $< 1°$].

If the computed atmospheric temperature at midnight drops below the CO_2 saturation temperature ["frost point"] at one scale height above the local surface, it is bounded at this value and the excess energy loss is converted to snow. If there is frost on the ground, this snow mass is added to the frost; otherwise it is ignored in the heat budget, which strictly does not conserve energy. [Occurs near the edge of North cap, typically for one season step. Maximum rate about ΔM 0.5 kg/m^2 per sol, equivalent to ΔT_a 2.5 K change in atmosphere temperature.]

$$\Delta M = \Delta T_a c_p M_a / L \tag{7}$$

The nadir planetary brightness temperature is given by

$$\sigma T_P^4 = \epsilon \sigma T_S^4 (e^{-\tau_R}) + \sigma T_a^4 (1 - e^{-\tau_R})$$
$$\implies T_P = \left[\epsilon(1. - \beta) T_S^4 + \beta T_a^4\right]^{1/4}. \tag{8}$$

2.3. Geometry and Starting Conditions

2.3.1. Geometry

The diurnal variation of insolation onto the surface at the bottom of the atmosphere is computed for the current season and latitude. The incidence angle from zenith onto the horizontal plane (i_0) or sloped surface (i_2) [rising above an extended plane] is computed by:

$$\cos i_0 = \mathbf{N} \cdot \mathbf{H} \qquad \text{OR} \qquad \cos i_2 = \mathbf{N}_2 \cdot \mathbf{H} \tag{9}$$

Where $\mathbf{N}$ is the ellipsoid normal, $\mathbf{N_2}$ is the local surface normal and $\mathbf{H}$ is the vector from Mars to the Sun. For these vector operations, a coordinate system with Z along the planet's north spin axis and X in the meridian of the virtual surface point (the meridian at the appropriate hour from midnight) is used. Slope is specified by dip and down-slope azimuth (going eastward from north) [Prior to June 2012 the relation $\cos i_2 = \sin\delta \sin(\theta + s_N) - \cos\delta \cos(\theta + s_N) \cos(\phi + s_E)$ was used, where δ is the solar declination, θ is latitude, ϕ is hour angle from midnight, s_N is the north component of surface slope,and s_E is the east component of surface slope.]

Direct (collimated) insolation is computed for the local surface, which may be sloped in any direction and has incidence angle i_2;

Direct insolation is zero when either i_0 or i_2 is $> 90°$. Diffuse illumination is based on i_0, with the optional extension into twilight (see Section 2.2.2). For a sloped surface, the solid angle of skylight is reduced and light reflected off the regional surface (presumed Lambert and of the same albedo) is added; the Delta-Eddington downward diffuse radiance is multiplied by `DIFAC` $= 1 - \alpha + \alpha A$, where $\alpha = (1 - \cos i_2)/2$ is the fraction of the upper hemisphere obscured by ground. For the bottom of conical depressions, $\alpha = \sin^2\left(\frac{\pi}{2} - s\right)$ where s is the slope up to the apparent horizon. See §2.5.2.

If a sloping surface (or pit) is used, the regional horizontal surface (or pit wall) is assumed to be an IR source of solid angle $\alpha 2\pi$ at the same temperature, T_s; this becomes a poor approximation for steep slopes.

The incident flux at the top of atmosphere is: $I = S_M \cos i_0$, where $S_M \equiv \frac{S_o}{U^2}$, S_o is the solar constant and U is heliocentric range of Mars in Astronomical Units.

2.3.2. Starting conditions: Diurnal-average equilibrium

For the first season, the atmosphere temperature is set based on the equilibrium for no net heating of the atmosphere or surface, using the diurnal average of insolation (see Eq. 2 and Eq. 5):

$$\langle H_V \rangle + \langle H_R \rangle = 0 \tag{10}$$

Surface radiation balance, from Eq. 13 for a flat surface with no net sub-surface heat flow:

$$\epsilon\sigma\langle T_s^4\rangle = (1. - A)\langle S'_{(t)}\rangle + \epsilon\sigma\beta_e\langle T_a^4\rangle \tag{11}$$

Expansion of $\langle H_R\rangle$ using Eq. 5 and a combination of Eqs. 10 and 11, yields;

$$\langle T_a^4\rangle = \frac{\langle H_V\rangle/\beta_e + (1 - A)\langle S'_{(t)}\rangle}{\sigma(2 - \epsilon\beta_e)} \tag{12}$$

For computational simplicity, the average top-of-atmosphere insolation is used as an approximation for $\langle S'_{(t)}\rangle$; this slightly over-estimates the temperature of the atmosphere at the start of the first season. $\langle T_s\rangle$ is then derived using 11 to get all layer starting temperatures for the first season, unless starting temperatures are specified by the input parameters.

For the first season, input value `TATM` is used to get a scale height for surface pressure calculations; thereafter, the diurnal average of T_a for the prior season at the current latitude is used. All atmosphere-related approximations are quickly attenuated during spin-up.

2.4. $\mathbf{CO_2}$ Frost condensation and Sublimation

The local frost condensation temperature `TFNOW` may be either fixed at an input value `TFROST`, or derived from the local surface partial pressure at the current season.

The relation between condensation/sublimation temperature and partial pressure is taken to follow an approximation adequate for Martian surface conditions: $\ln P_c = a - b/T$, in **CO2PT** with a=27.9546, b=3182.48 K^{-1} and P_c in Pascal, as given in **?**.

If frost is present, $E = W \cdot \Delta t$ energy is used to modify the amount of frost M (kg/m^2), W is the net heating at the surface defined in Eq. 13 and Δt the duration of an single time step; $\Delta M = -E/L$, where L is the latent heat of sublimation. The frost albedo may depend upon insolation, and there may be an exponential attenuation of the underlying ground albedo; see §2.4.1. Both frost albedo and ΔM are computed at each time step. The amount of frost at each latitude is carried (asymptotic prediction, see §3.2.7) to the next season.

2.4.1. Effective Albedo

A thick frost deposit can have a constant albedo, or be linearly dependent upon the insolation as described by **??**. It should be noted that it is now known that regions of the seasonal caps can have virtually constant low albedo, **??**.

As the seasonal frost thins (or thickens), the effective albedo of the surface can continuously approach that of underlying soil. $A = A_f + (A_s - A_f)e^{-M/M_e}$ where A_f is the albedo of a thick frost, A_s is the albedo of the underlying surface and M_e is the frost required, kg m^{-2}, for unity scattering attenuation. This avoids an unrealistic discontinuity in surface energy balance for a tiny amount of frost.

2.4.2. Global and local pressure

The total amount of atmosphere is set by the annual mean surface pressure at the reference elevation, P_0, input as PTOTAL.

The current global pressure P_g =`PZREF`, can be any of the following: based on the setting of KPREF:

0) constant at P_0

1) P_0 times the normalized Viking Lander pressure curve computed in **VLPRES** and based on the average of the three seasonal curves in **?**.

2) Based on depletion of atmospheric CO_2 by growth of frost caps; P_0 minus the total frost mass at the end of the prior season. In this case, the input value PTOTAL is not the annual-average pressure at zero elevation but the global average of the atmosphere plus cap system. This option requires that a reasonable number of polar latitudes be included; KRC allows this option only if the number of latitudes NLAT>8.

The initial partial pressure of CO_2 at zero elevation is $P_{c0} = P_0 \cdot (1.-\text{non-condensing fraction})$ =PCO2M. The current CO_2 partial pressure at zero elevation is $P_{cg} = P_{c0} + (P_g - P_0)$ =PCO2G.

Both the current local total pressure and CO_2 partial pressure scale with surface elevation and scale height: $P \propto e^{-z/\mathcal{H}}$. The scale height is: $\mathcal{H} = T_a\mathcal{R}/\mathcal{M}g$; where T_a is the mean atmospheric temperature over the prior day (or season), $\mathcal{R}$ is the universal gas constant, $\mathcal{M}$ is the mean molecular weight of the atmosphere (43.46), and g is the Martian gravity.

Local current dust opacity scales with local total pressure: $\tau = \tau_0 \cdot P/P_0$ under the assumption that dust is well-mixed in the atmosphere. The atmospheric saturation temperature is evaluated at one scale height above the local surface; if T_a would fall below this value, condensation is assumed to take place to provide the energy required to prevent this, and the snow is added to the surface frost budget.

2.5. Boundary conditions

2.5.1. Level Surface

The surface condition for a frost-free level surface is :

$$W = (1. - A)S'_{(t)} + \Omega\epsilon R_{\Downarrow} + k\frac{\partial T}{\partial z}_{(z=0)} - \Omega\epsilon\sigma T^4 \quad (13)$$

where W is the heat flow into the surface, A is the current surface albedo, $S'_{(t)}$ is the total solar radiation onto the surface as in Eq. 1, $R_{\Downarrow}$ is the down-welling thermal radiation (assumed isotropic), T is the kinetic temperature of the surface, k is the thermal conductivity of the top layer. Ω is the visible fraction of the sky, ϵ is the surface emissivity and σ the Stefan-Boltzmann constant. In the absence of frost, the boundary condition is satisfied when $W = 0$. Most constant terms are precomputed, see Table **??**.

When frost is present, the values in Eq. 13 are replaced with ϵ_F, A_F, and T_F, where subscript F indicates the frost values, and no iteration is done; leaving W as a non-zero quantity to change the mass of frost on the ground as discussed in §2.4.

2.5.2. Slopes and Conical Depressions

A single 1-dimensional thermal model has a modest ability to account for non-flat geometry. The solar and thermal radiation fields are modified for a planar sloped surface or for the bottom of a circular depression (here termed a "pit", although the geometry applies to any scale). The collimated incident beam is treated rigorously, intensities of the diffuse solar and thermal fields are modified by the fraction of sky visible, and the average reflectance and emittance of the surrounding surface (absent in the level case) are approximated as: the brightness of level terrain with the same albedo, and material having the same temperature as the target surface, respectively; this last approximation accentuates the diurnal surface temperature variation with increasing slope. Then

$$S'_t = S_M \left[F_{\parallel} \cos i_2 + \Omega F^{\downarrow}_{\ominus} + \alpha A(G_1 F_{\parallel} + \Omega F^{\downarrow}_{\ominus}) \right] \quad (14)$$

$F_{\parallel}$ is the collimated beam in the Delta-Eddington model and $F^{\downarrow}_{\ominus}$ is the down-going diffuse beam. $\Omega \equiv 1 - \alpha$ here and in Eq. 13. G_1 is the fraction of the visible surrounding surface which is illuminated. Within the brackets in Eq. 14,

the first term is the direct collimated beam, DIRECT

the second is the diffuse skylight directly onto the target surface, DIFFUSE

the third term is light that has scattered once off the surrounding surface, BOUNCE

For a sloped surface, G_1 is taken as unity. As a first approximation, for depressions $G_1 = (90-i)/s < 1$) where s is the slope to the lip of the depression (the apparent horizon). For the flat-bottom of a depression, $i_2 = i_0$ when the sun is above this slope, and $\cos i_2 = 0$ when below.

2.5.3. Physical properties, layering of materials and sub-surface scaling

For a homogeneous, semi-infinite material with temperature-independent properties under a periodic insolation of angular frequency ω, the amplitude of the surface temperature variation is proportional to $1/\sqrt{\omega} \cdot 1/\sqrt{k\rho C}$ [**?**, §2.6]; k is the thermal conductivity [W/(K m)], ρ is the density [kg/m^3] and C the volume specific heat [J/(kg K)]. $\sqrt{k\rho C}$ is called the thermal inertia I of a material and has units [J m^{-2}s$^{-1/2}$K^{-1}]. The variation is attenuated into the material as $e^{-z/D}$ where $D = \sqrt{\frac{P}{\pi} \cdot \frac{k}{\rho C}}$ is the skin depth and $P = 2\pi/\omega$ is the diurnal period in seconds. The thermal diffusivity is $\kappa = \frac{k}{\rho C}$.

All physical properties are specified by parameters in the input file. Nominal planetary parameters for Mars are the mean solar day, 1.0275 days, and the surface gravity, 3.727 m s^{-2}. Properties of the upper-layer material are specified by the thermal inertia, density and specific heat; these in turn set the conductivity (unless temperature-dependent properties are) used. Beginning with layer IC, all lower layers can have their conductivity, density and specific heat reset to COND2, DENS2, and SPHT2 respectively. If LOCAL is set true, then the physical thickness of these layers scales with the local thermal diffusivity; otherwise, the geometric increase of physical layer thickness continues downward unaltered.

2.5.4. Base of model

Normally, the base of the model is treated as insulating. However, there are also options for it to be held at a fixed temperature, which is useful to model subsurface H_2O ice.

2.6. Relation of thermal inertia to particle size

The relation of I to particle diameter is based on laboratory measurements of thermal conductivity: $k = \left(c_1 P^{0.6}\right) d^{-0.11 \ln(P/c_2)}$, **?**; where $c_1 \sim 0.0015$ and $c_2 \sim 8.1 \times 10^4$ Torr [1.08E7 Pa] are constants. The relation for typical Martian conditions is shown in Figure **??**. Also shown in that figure is a histogram of the thermal inertia determined from TES global map data, **?**, although that map used a different thermal model. The I source data are available at http://www.boulder.swri.edu/inertia/2007/ ; these data were weighted by area. Most areas are in I range of 100:500 J m^{-2}s$^{-1/2}$K^{-1}; values above about 200 are increasingly affected by a rock population or real bedrock, which has $I \sim 2000$ J m^{-2}s$^{-1/2}$K^{-1}.

3. Numerical Methods

3.1. Basic Method

KRC uses layers that increase geometrically in thickness B_i by a factor RLAY [this parameter can be set to 1.0 to obtain uniform layer thickness]. In order to simplify the innermost code loops, KRC places the physical radiating surface between the first and second model layers. The layer index i, increases downward. Below, subscript +(-) is shorthand for $i + (-)1$; i+.5 is the lower boundary of the layer.

Basic differential equation of heat diffusion is :

$$\frac{\partial T}{\partial t} = \frac{-1}{\rho c}\frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}\right) \quad = \quad \frac{k}{\rho c}\frac{\partial^2 T}{\partial z^2} + \left[\frac{1}{\rho c}\frac{\partial k}{\partial z}\frac{\partial T}{\partial z}\right] \quad (15)$$

where t = time, T = temperature and z is the vertical coordinate..

The term in brackets is assumed to be zero within each layer, as is strictly the case for temperature-independent conductivity; the approximation for temperature-dependent k is discussed in §3.2.1.

Expressed for numerical calculations:

$$\frac{\Delta T_i}{\Delta t} = -\frac{H_{i+.5} - H_{i-.5}}{B_i \rho_i C_i} \tag{16}$$

where B_i = is the layer thickness and H = heat flow at the boundary.

The heat flow at interface between two layers is: $H = -k\frac{\Delta T}{\Delta x}$

$$H_{i+.5} = -\frac{T' - T_i}{B_i/2} k_i \quad \text{or} \quad T' - T_i = -\frac{H_{i+.5} B_i}{2k_i} \tag{17}$$

where T' is the temperature at the interface.

$$\text{Similarly} \quad T_{i+1} - T' = -\frac{H_{i+.5} B_{i+1}}{2k_{i+1}}$$

$$\text{Thus} \quad T_{i+1} - T_i = -\frac{H_{i+.5}}{2}\left(\frac{B_i}{k_i} + \frac{B_{i+1}}{k_{i+1}}\right)$$

$$\text{or} \quad H_{i+.5} = -\frac{2(T_{i+1} - T_i)}{\frac{B_i}{k_i} + \frac{B_{i+1}}{k_{i+1}}}$$

$$\text{Similarly} \quad H_{-.5} = -2\frac{T - T_-}{\frac{B}{k} + \frac{B_-}{k_-}}$$

For uniform layer thickness in uniform material, the standard form of explicit forward difference is

$$\frac{\Delta T_i}{\Delta t} = \frac{\kappa}{B^2}\left[T_+ - 2T_i + T_-\right]. \tag{18}$$

3.2. Finite difference scheme for exponential layer thickness

For variable layer thickness: Eq.18 becomes

$$\frac{\Delta T_i}{\Delta t_i} = \frac{2}{B_i \rho_i C_i}\left[\frac{T_+ - T_i}{\frac{B_i}{k_i} + \frac{B_+}{k_+}} - \frac{T_i - T_-}{\frac{B_i}{k_i} + \frac{B_-}{k_-}}\right] \tag{19}$$

For KRC, formulate this as

$$\Delta T_i = F_{1_i}\left[T_+ + F_{2_i} T_i + F_{3_i} T_-\right] \tag{20}$$

KRC defines intermediate constants for each layer:

$$F_{1_i} = \frac{2\Delta t_i}{B_i \rho_i C_i} \cdot \frac{1}{\frac{B_i}{k_i} + \frac{B_+}{k_+}} \equiv \frac{2\Delta t_i}{\rho_i C_i B_i^2} \cdot \frac{k_i}{1 + \frac{B_+}{B_i}\frac{k_i}{k_+}} \tag{21}$$

and

$$F_{3_i} = \left(\frac{B_i}{k_i} + \frac{B_+}{k_+}\right) \cdot \frac{1}{\frac{B_i}{k_i} + \frac{B_-}{k_-}} \equiv \frac{1 + \frac{B_+}{B_i}\frac{k_i}{k_+}}{1 + \frac{B_-}{B_i}\frac{k_i}{k_-}} \tag{22}$$

and

$$F_{2_i} = -(1 + F_{3_i}) \tag{23}$$

Then the inner-most loop, one time-step for one layer, is Eq. 20 followed by

$$T_i = T_i + \Delta T_i \tag{24}$$

The input parameter `FLAY` specifies the thickness of the top "virtual" layer in units of the diurnal skin-depth `SCALE`, so that the scaled thickness of the uppermost layer in the soil is `FLAY*RLAY`, and the physical depth of its center in meters is 0.5*`FLAY*RLAY*SCALE`. Normally, (`LP2` set true) a table of layer thickness, depth, (both scaled and in meters), overlying mass, and numerical convergence factor is printed at the start of a run.

3.2.1. Extension to temperature-dependent properties

The thermal conductivity of many geologic materials decreases with temperature over all Martian surface temperatures; exceptions are very basic materials (Anorthosite, Obsidian, gabbro) and glasses and fused silica, all of which have k that is relatively low and can increase with temperature (survey of **?????????**). Published analytic fits to measurements of bulk materials commonly are in the form $k = 1/(a + bT)$ or some algebraic equivalent **?????**.

The surface of much of Mars and many bodies without an atmosphere is a particulate material whose grains are composed of minerals and glasses. The effective conductivity of these particulates is strongly dependent upon the particle size and increases strongly with gas pressure over a range where the mean-free-path transitions from larger to smaller than the particle (or void) size **???**.

Because some Martian surface conditions have such low pressure that the gas mean free path can exceed the particle size (e.g., m.f.p $> 50\ \mu$m in soil mid-day at top of Olympus Mons), and because it is desirable for KRC to be able to address vacuum conditions, a form including T^3 is advantageous to treat radiative transfer.

Specific heat increases with temperature for geologic materials and Martian conditions. Theoretical models range from the classic model of Debye **?** or [**?**, p.136] to the comprehensive formulation of S.W. Kieffer **?**. There are several few-term empirical relations , e.g. $\frac{A}{B+T}$, $\left(\frac{T}{T_c}\right)^\beta$, polynomial in $\frac{T-T_c}{T_c}$ **?**, $c_1 - c_2/\sqrt{T} - c_3 T^{-2} + c_4 T^{-3}$ where all coefficients are positive **?**, and others in **?**. However, it was found that over the full range of Martian temperatures a cubic polynomial would fit geologic materials with error $< 1\%$.

An informal description of a literature search on thermal properties and the development of code to generate the cubic-polynomial coefficients is contained in **?**. A separate study of the theoretical variation of the effective thermal conductivity of particulate materials as a function of grain-size, cementing and temperature, inspired by the early versions of the numerical modeling of **??** was done **?**. Both of these are available at the KRC website.

The addition of temperature-dependent properties is a significant variation to the constant-conductivity version as these properties must be evaluated at each layer and time-step. The layer setup described in the prior section remains based on the values of the physical properties at a reference temperature, chosen to be 220 K, which is the approximate midpoint of the full range of surface temperatures on Mars.

KRC use the logical flag `KOFT` to enable temperature-dependence of both conductivity and specific heat, and both are implemented as third-degree polynomials, which minimizes the complexity of the code; any of the first, second or third degree terms can be left as zero. To minimize round-off problems, the polynomials uses a scaled independent variable $T' = (T - T_{off})T_{mul} = (T - 220.) * 0.01$. Because KRC allows two materials, the combination of $k_{(T)}$ and $C_{(T)}$ requires a total of 16 coefficients [Implemented 2010 Feb]. If `KOFT` is set false, these coefficients are ignored, the equations of §3.2 are implemented and KRC executes about twice as fast.

Because conductivities and layer thicknesses appear largely as ratios, KRC calculates these as infrequently as possible. With temperature-dependence enabled, KRC computes once per model:

$$F_{C_i} = 2\Delta t_i / \left(\rho_i B_i^2\right)$$

$$\text{and} \qquad F_{B_i} = B_+ / B_i$$

. Then, for each time step and for each layer compute $T_i' = (T_i - \texttt{XOFF})*\texttt{XMUL}$, then

$$k_i = ((c_3 T' + c_2)T' + c_1)T' + c_0$$

and similarly C_i with its coefficients. If two materials are involved, the coefficients for the second material are used for the layers below the material contact.

KRC computes once per time step: $F_{k_i} = k_{i(T)}/k_{+(T)}$ Then Eqs. 21 and 22 become

$$F_{1_i} = F_{C_i} \cdot \frac{k_{i(T)}/C_{(T)}}{1 + F_{B_i} F_{k_i}} \tag{25}$$

and

$$F_{3_i} = \frac{1 + F_{B_i} F_{k_i}}{1 + 1/\left(F_{k\_} F_{B\_}\right)}. \tag{26}$$

Eq. 23 remains the same, but must be evaluated for every layer and time-step.

The approximation associated with ignoring the term in brackets in Eq. 15 was estimated by running comparative models in which only the layer thickness and number of layer was changed, keeping the depth to the center of the bottom layer identical; these runs indicate that the difference in T_s between T-independent and T-dependent conductivity cases appears to have error $< 5\%$ for realistic materials and conditions.

3.2.2. Solving the upper boundary condition

When there is no surface frost, the net energy into the upper boundary must be zero. From Eq. 13, find

$$\frac{\partial W}{\partial T} = -k/X_2 - 4\Omega\epsilon\sigma T^3 \tag{27}$$

where X_2 is the depth to the center of the first soil layer. Note that this includes the normal finite-difference assumption that the temperature gradient in top half of layer 2 is linear. If considering temperature-dependent thermal conductivity, then k is approximated as that of the top material layer at the end of the prior time-step.

The surface kinetic temperature for a balanced boundary condition, Eq. 13, is iterated with Newton convergence until the change in T, $\delta \equiv \frac{W}{\partial W/\partial T}$, is < GGT

If $\mid \delta \mid /T > 0.8$, it is assumed that the model has gone unstable and it is terminated.

if $\mid \delta \mid /T > 0.1$, then δ is reduced by 70% before the next iteration to improve stability

If frost is present, the unbalanced energy W is applied to condensation or sublimation.

After determining the surface temperature, the virtual layer ($i = 1$) temperature is set to yield the proper heat flow between the surface and the top physical layer ($i = 2$);

$$(T_s - T_1)\,\frac{\kappa_1}{B_1/2} = (T_2 - T_s)\,\frac{\kappa_2}{B_2/2}$$

$$\Longrightarrow \qquad T_1 = T_2 - (1 + 1/\texttt{RLAY})\,(T_2 - T_s) \tag{28}$$

where the diffusivity of the virtual layer is treated as identical to that of the top physical layer.

3.2.3. Stability and time doubling

The convergence stability criterion is $\frac{\Delta t}{(\Delta Z)^2}\kappa < \frac{1}{2}$, equivalent to $B^2 > 2\Delta t\kappa$. A convergence safety factor is defined as $B_i/\sqrt{2\Delta t_i \cdot \kappa_i}$. The code was found to be numerically unstable if this factor is less than about 0.8. The routine will stop with an error message if the safety factor is anywhere less than one. As the layer thickness increases with depth, the routine will repeatedly double the time interval for deeper layers if all the following conditions are met:

The safety factor is larger than 2
The layer is at least the 3rd down
The remaining time intervals are divisible by 2
No more than MAXBOT time doublings will be done

To handle potential large jumps in diffusivity that are allowed between two materials, an initial calculation of the safety factor for the upper layer of the lower material is made without time-doubling. If this does not meet the input convergence factor CONVF, then the thickness of this and all lower layers is increased to be stable with this safety factor. If the thickness of this key layer is overly conservative, then the number of allowed time-doubling in the upper materials is set accordingly.

The numerous input parameters that control the time-depth grid and convergence are based upon extensive testing done during the code development.

3.2.4. Starting conditions

For the first season, the model starts at 18 Hours with the surface temperature normally set to the equilibrium surface temperature of a perfect conductor as calculated in Eq. 11. The bottom temperature is also normally set to this value. The input parameter IB allows the option of setting the initial bottom temperature to TDEEP or also the surface temperature to this value; the latter case is useful for studying details of the disappearance of seasonal frost.

Once the top and bottom temperatures are set, all intermediate layer temperatures are set by linear interpolation with depth. The initial atmosphere temperature is always set to the equilibrium values using Eq. 12.

3.2.5. Jump perturbations

In order to make model "spin-up" more efficient, the bottom layers can be "jumped" so that their average temperature is the same as the surface average, the condition for no net heat flow with temperature-independent thermal conductivity. A logical flag LRESET is normally false. It is set True on day NRSET or later of the first season if the lower boundary is adiabatic, but never on the last day of calculation in a season or if the lower boundary temperature is fixed.

On a day when LRESET is true, the summation for average layer temperatures, $\langle T_i \rangle$, is restarted. At the end of that day, all layer temperatures are offset by $\langle T_s \rangle - \langle T_i \rangle$ so as to yield no net heat flow.

To help in situations where both diurnal and seasonal temperatures are being addressed, there is an option to instead perturb temperatures based on a linear plus fractional quadratic function of depth between the diurnal average surface and diurnal average bottom temperatures: if DRSET is not zero, then the layer temperature offsets are:

$$\Delta T_i = (\langle T_s \rangle - \langle T_n \rangle)\,(x + \mathrm{DRSET} \cdot x(1-x))$$

where $x = z_i/z_n$ and $_n$ is the bottom layer: some experimentation can help in selecting an effective value of DRSET

3.2.6. Convergence criteria and parameters

At each time step, if there is not frost, the surface boundary equation is iterated until the change in surface temperature is less than GGT.

The test for continuing full computations each day into a season is based upon Δ_T, defined as the RMS change of layer temperatures at midnight, including the virtual layer, from midnight the prior day; this is stored at the end of each day in DTMJ.

The test for making the next day the last is: either the temperature change over the last two days is nearly constant, or the temperature change is small; i.e.:

$$| 1 - \frac{\Delta_{T,j}}{\Delta_{T,j-1}} | \leq \mathsf{DDT} \qquad \text{or} \qquad \Delta_T \leq \mathsf{DTMAX}$$

where $\Delta_{T,j-1}$ is forced to be at least 10^{-6} . Normally, DDT = 0.002, GGT = 0.1 K and DTMAX = 0.1 K.

After computation of the last day, there is a final check to confirm that convergence has continued: the temperature change has decreased or it is still small; i.e.:

$$\Delta_T \leq \Delta_{T,j-1} \qquad \text{or} \qquad \Delta_T \leq \mathsf{DTMAX}$$

If these tests fail, and there are days left in the season, then daily calculations are resumed.

3.2.7. Prediction to next season

Calculations run from midnight to midnight. When convergence has been reached, commonly in fewer days than separate seasons, the results at the last 3 midnights, y_1, y_2, y_3, are used to forecast asymptotically the model result at the end of the season, $y = b_0 + b_1 r^x$ where x is the number of sols remaining in the season. Normally, this will use a fit over the last 3 midnights; for convenience reformulated as

$$y = y_3 + c_1 \left(1. - r^x\right) \tag{29}$$

where $r = \frac{y_3 - y_2}{y_2 - y_1}$ is the ratio of the last two changes, and $c_1 = \frac{y_3 - y_2}{(1/r) - 1}$. If the fit is not asymptotic (e.g., if $r \geq 1$), or if the forecast distance (from the last computed midnight) is less than 0.9 sols, the routine will do a linear prediction using the most recent two points. In addition, lower and upper limits can be specified, e.g., to keep a temperature from falling below a frost point.

3.3. Effect of spin-up time, depth and bottom conditions

Common challenges for large numerical models are initialization and specification of boundary conditions. This section is meant to provide an introduction to this issue, a few specific examples, and to increase awareness of what should be stated in describing a KRC model run.

The default atmosphere has a time constant of a few sols, so that its initial state has little effect on model conditions for runs a modest factor longer than this; e.g., the default OnePoint mode has a 15 sol effective spin-up . Generally, specification of bottom conditions has the largest effect on surface temperature.

KRC has three lower boundary condition options; the first is the default.

IB=0 All layers start at the equilibrium temperature for the starting season. The boundary is insulating and after a few sols all layers are "jumped" to have the same average as the surface.

IB=1 The top layer starts at the equilibrium temperature and the bottom at TDEEP, intermediate layer temperatures are linear with depth. The bottom boundary is insulating and no jump is done.

IB=2 All layers start at TDEEP and the lower boundary is held at this value. This is useful for spring frost recession details. The effect on T_s compared to setting IB=1 diminishes with a time-constant of the model total thickness.

Effects on T_s related to bottom conditions generally are largest predawn, when insolation has the least influence. An example is shown in Figure **??**, which illustrates the difference in T_s of a 3 year spin-up relative to the OnePoint mode; both models run with IB=0; the differences at dawn are about twice the amount at midday.

For three latitudes (VL-1, equator and 25 S) at $L_s = 100°$, short and long spin-up time were tried; 20 sols and $3\frac{1}{4}$ Mars years, with three sets of lower boundary conditions and 10, 13, 16, 19 and 29 layers, corresponding to total model thicknesses of 4.5, 8.5, 15.6, 27.7 and 177 diurnal skin-depths; the last two correspond to 1.1 and 6.8 annual skin-depths. TDEEP was set to 180 K, intentionally about 20 K below the annual average of T_s; with this large initial offset there is some residual effect for deep models with IB=2 even after 4 years. The seasonal excursions of T_s at 25 S, with an zenith noon Sun near perihelion, are about triple those at VL-1. The diurnal results are shown in Figure **??**.

In these examples, the effect of deep-layer memory on T_s near midday and in the atmosphere at all hours is less than at pre-dawn, both by roughly a factor of 0.6.

If realization of annual effects is desired, a reasonable choice is IB=0, with a total thickness of about 25 diurnal skin-depths (1 annual skin-depth), starting near an equinox, and a spin-up of about 2 years.

3.4. Comparison to other thermal models

All planetary thermal models involve some approximations and assumptions. A comparison between their results provides an estimate of these approximations. All models discussed here produce surface kinetic temperature. KRC and the Mellon model **?** are designed for use with remote IR observations and routinely produce net up-going radiation at the top of the atmosphere. The Vasavada model **?** is designed to define the surface environment and the Ames model **?** is a full Global Circulation Model (GCM). Time shifts of up to 1/2 hour may result from precisely how the various models generate the comparison products.

Surface temperatures T_s of the Vasavada and Mellon models agree closely with KRC; the AMES GCM T_s is generally cooler due to a deep subsurface starting at below the annual average temperature that would require many model years to reach equilibrium.

Mellon models provide perhaps the best indication of difference in T_s results between one-layer and multi-layer atmosphere; Mellon T_s is generally a few K cooler than KRC during the day and about 7K cooler predawn.

The down-going thermal radiation has smaller diurnal variation in KRC than in more detailed atmosphere models; this IR radiation is generally about an order of magnitude smaller than the insolation (note factor of 10 in auxiliary axis scales for Figures **??** and **??**).

KRC atmosphere temperatures have similar phase and somewhat larger diurnal amplitude as the mass-weighted product of the Ames GCM. The phase and amplitude of down-going IR flux are also similar, but fluxes are considerably larger in KRC unless a lower infrared/visual opacity ratio is used. More detailed discussion is in the following sub-sections.

3.4.1. Comparison to Ames GCM

As a check on atmosphere temperatures and down-going radiance, a specific test case was chosen for comparison of the KRC one-layer atmosphere with the multi-layer radiative, conductive and convective-coupled atmosphere of a full Global Circulation Model (GCM), **?**; the Viking-1 landing site, Latitude 22° N, elevation -3.1 km, $L_s = 100°$, $\tau_v = 0.3$, visible/IR opacity ratio 1.0, surface pressure of 7 millibar, bolometric albedo of 0.25, thermal inertia 270 $\mathrm{J\,m^{-2}s^{-1/2}K^{-1}}$, soil density 1600 kg/m^3, soil specific heat 630 J/kg, model depth 40 m. This special GCM run inhibited lateral atmospheric dynamics and output a mass-weighted atmosphere temperature; it started with an isothermal profile at 180K and was "spun up" for 20 sols before the output date (data kindly provided by Robert Haberle).

The resulting temperatures and fluxes are shown in Figure ??, along with those for three KRC runs. The KRC base model used the parameters shown in Dataset 8, apart from the values listed above, use of 29 layers, and having the bottom at 8 m held at 180 K to approximate the effect of the deeper GCM sub-surface; the diurnal skin-depth is 45 mm

The KRC and GCM atmospheric temperatures have similar mean, variation, and phase, with minima near 8H and maxima near 17H; however, the KRC down-going infrared radiance lags the GCM slightly, as expected because the GCM near-surface atmospheric layers dominate the down-going flux and they track the surface temperature more closely than the KRC one-layer atmosphere.

The KRC atmosphere down-going infrared radiances are similar in diurnal behavior but larger than the GCM; use of the KRC default value of 0.5 for IR/visible opacity ratio results in values closer to the GCM, with modest changes in T_s and T_a (lower set of curves in Figure ??) .

The GCM surface temperatures are lower than base KRC model by 5-9 K, due in part to initializing all layers at a temperature about 40K below the surface average. A KRC model with realistic deep temperatures has T_s and T_a a few K higher and 5% higher IR flux (upper set of curves in Figure ??).

3.4.2. Comparison to Vasavada model

Ashwin Vasavada has developed a model used at the Jet Propulsion Laboratory (JPL) for Martian surface environments. It incorporates temperature-dependent heat capacity and has the ability to model sloped surfaces. The subsurface portion is based on ? and the atmosphere interaction is based on a one-dimensional version of the radiative transfer and boundary layer physics from the Geophysical Fluid Dynamics Laboratory (GFDL) Mars GCM, circa 2004 ??? and includes basic CO_2 condensation. There is no geothermal heat flux.

Vasavada provided his results for Holden Crater, a candidate landing site. A=0.13, I=350 (near 200K), the thermal emissivity is 0.98. The model assumes a constant 10 m/s wind for the boundary layer. After a few years of spin-up and equilibration, the model output values at 15 min time-steps for a Martian year. Surface kinetic temperature, down-going solar and down-going thermal radiation at one-hour intervals were supplied. Comparisons are shown for two seasons in Figure ??; T_s and solar flux compare closely. The Vasavada down-going thermal radiation is greater and has greater amplitude than KRC, the phase relation to KRC is similar to that for the Ames GCM

3.4.3. Comparison to Mellon model

For TES standard processing, Mellon models were generated at 8 sol intervals and 5° latitude spacing for 10 thermal inertia's spaced logarithmically; for 3 sets of albedo, 3 sets of dust opacity, and 3 sets of average surface pressure. Mellon uses a multi-layer radiative-convective atmosphere ? and his model takes about two orders of magnitude longer to run than KRC (M.T. Mellon, personal communication, 2011). KRC models were generated on the same grid; the same values were used for all physical parameters identified in the Mellon model file headers. Mellon models were spun-up for two years before the output year ? and KRC for 3 years before the output year. The diurnal surface temperature curves for three thermal inertias and three latitudes are shown in Figure ??. KRC T_s is a few degrees warmer, the greatest at night and for low thermal inertia. A seasonal comparison of T_b is shown in Figure ??; the models track each other closely except for the lowest thermal inertia at 30S near L_s=90°, when CO_2 frost forms at night in only the Mellon model. T_b for the two models are even closer, and this is the value normally compared to remotely observed temperatures. Mellon T_b are generally slightly higher than KRC at midday and 0 to 4 K cooler predawn. Differences are largest for the lowest thermal inertia and have smooth variation with season unless CO_2 frost forms.

4. Effect of T-dependent properties

Temperature-dependent properties have been considered as contributing to some of the "anomalous" thermal behavior observed on Mars, but not quantified ?. Here, the effect of T-dependent properties on Mars surface temperature is assessed using realistic properties; the effect was found to be at most a few K. Although it can be assessed in detail with KRC, this complication may rarely be needed.

A test of the temperature-dependent code is to invoke it when the properties have no temperature dependence. The effect on T_s was measured for latitudes 0 and 40S (which has large seasonal variation) over 40 seasons after a 3-year spin-up. For a homogeneous model the mean absolute difference was less than 0.3 milli-Kelvin, for two-materials (IC=7, 1.6 diurnal skin-depths, 3.5 cm), 0.03 milli-Kelvin.

The effect of temperature-dependent materials on surface kinetic temperature was assessed for homogeneous and two-material cases. The base case has A=0.25, k =0.013 W m^{-1} K^{-1}, ρ =1600 kg m^{-3} , and C =630 J kg^{-1} K^{-1} yielding I=114.5 J m^{-2}s$^{-1/2}$K^{-1}, the lower material has k =2.7868, ρ =928, and C =1710.65 yielding I=2103.3 .

For the homogeneous case, and the upper material in the layered case, the conductivity is that of sediments in ? and the specific heat the chlorite of ?; both scaled slightly to exactly match the above values at T=220K. These materials have relatively large T-dependence. The lower material is pure H_2O ice Ih with T-dependent conductivity and specific heat ?

For latitude 0, the diurnal behavior is similar all year; the season-averaged results are shown in Figure ?? for temperature-dependent k and c individually and together. The effects of k, which generally decreases with temperature, and c, which increases with temperature, somewhat offset each other. The results with a lower layer of ice starting at 1.6 diurnal skin-depths (closer to the surface than generally expected for Mars) are little different from the homogeneous case.

5. KRC architecture and use

KRC is structured so that the main program calls routines to handle the input and setup the output style; it calls a hierarchy of routines to do the calculations for each season, each latitude, and each day (until convergence). This architecture is described in Dataset 1. An index of routines is in Dataset 2 and the code is contained in Datasets 3, 4 and 5.

All the KRC runs for this paper were compiled with the GNU compiler 4.1.2 under a Linux operating system (CentOS 5.4) with 64-bit hardware; a makefile for this system is in Dataset 6. A users guide is available in the Dataset 7.

For normal runs, the user will be prompted for the name of the input file and the name of a print file. All actions are controlled by the input file. Users unfamiliar with KRC or similar models are encouraged to run comparative models with different layer and time-step sizes, spin-up times and different convergence parameters to obtain an estimate of numerical accuracy.

5.1. Setting the starting date

To run KRC with unrecorded "spin-up" seasons:

Choose the L_s of the first output season

Convert that to a full Julian date

Subtract the number of spin-up seasons times the delta-days between seasons

i,e, -(JDISK-1)*DELJUL

Subtract 2440000; use this as the starting date DJUL

5.2. Input parameters

The normal print files list for each case all the current changes, then the resulting full set of parameters (excepting the orbital matrix) and the set of latitudes and elevations. Its format is similar to the input file, shown in Dataset 8.

5.3. Sample layer table

Normally (LP2 true) a layer table is printed at the beginning of each case. This lists the layer thickness and center-depth in both meters and diurnal skin-depth. It also includes the column mass above the center of each layer and the safety factor beyond classical numerical stability. If seasonal memory is not required, then a scaled center depth of order five is adequate. Mars annual skin-depth is 25.85 times the diurnal skin-depth; if the effect of seasonal memory is desired, then the bottom depth should exceed this. An example is shown in Dataset 9.

5.4. Print output

A record of changes and optional notification of season progress appears on the monitor. A separate print file, default name *krc.prt*, is generated for which there are many options, described in the help-list; Dataset 7 . Voluminous output is possible; it is best to start with the defaults in the sample input file, Dataset 8, then experiment with the options for small cases.

5.5. Linked Runs

KRC has the ability to continue from the vertical temperature profile at the end of a prior case, as long as the physical distribution of the layers is not changed. It can also start with the conditions at any season in a prior run stored with `K4OUT`=-1. These can be useful for [at least] two purposes:

- By continuing from memory and incrementing the total number of seasons, it is possible to continuously change parameters in addition to the atmospheric opacity and surface albedo (for which seasonal tables may be specified).

- Details related to seasonal frost appearance/disappearance. A run-up of a few years with about 40 seasons per year can be used to establish a frost budget and deep temperature profile. Then, the season interval can be set to 1 sol, and events followed in detail.

5.6. One-point version: An alternate input

To support some detailed THEMIS studies, an interface to the KRC system was built that computes the temperature for a single condition. Two input files are involved:

- A "master" file specifying all general parameters for a single case. The last line processed must contain the name of the "point" file.

- A "point" file containing formatted lines that each specify the time and conditions at one point; any number of lines are allowed. These values will override those for corresponding items from the master file

The underlying model is the full version of KRC and each point is run as an independent case, so the order of input points is arbitrary.

The default OnePoint master file, shown in Dataset 10, has parameters similar to the KRC defaults (Dataset 8). It specifies one latitude and a layer extending to about 5 diurnal skin depths, so there is virtually no seasonal memory. Thus, it does not treat the seasonal frost properly, and results near the edge of the polar cap are likely to be unreliable. It sets the KRC system into a reasonable mode for one-point calculations with a spin-up of 15 sols. Many parameters in this file could be safely modified. The values for starting date, latitude, elevation, albedo, thermal inertia, dust opacity, slope and azimuth are over-ridden by the values in the OnePoint file.

The fields in the OnePoint input are:

Ls	L_S season, in degrees
Lat	Aerographic latitude in degrees
Hour	Local time, in 1/24'ths of a Martian Day
Elev	Surface elevation (relative to the aeroid), Km
Alb	Bolometric Albedo, dimensionless
Inerti	Thermal Inertia, in SI units
Opac	Atmospheric dust opacity in the visible
Slop	Regional slope, in degrees from horizontal
Azim	Azimuth of the down-slope direction, Degrees East of North.

The two additional columns in the output file are:

TkSur	Surface kinetic temperature
TbPla	Planetary bolometric brightness temperature

Execution time is about 3.5 millisec per point.

6. Examples of use

6.1. Annual average surface temperature

A variety of small-scale morphological features on Mars are similar to terrestrial features whose formation involves flow of liquids. These features are geologically young and some are active **?????**. While some explanations propose dry flow for angle-of-repose slopes, other explanations for both angle of repose and lower slopes invoke obliquity variations, freezing-point suppression for brines, confinement or protection by snow and ice, local slope orientation and shelter; all these are strongly dependent upon the subsurface temperature.

Average annual surface temperature is a strong constraint on subsurface temperature. Typical Martian dry surface conditions have $I \sim 200\ \mathrm{J\,m^{-2}s^{-1/2}K^{-1}}$, $\rho \sim 1000\ \mathrm{kg\,m^{-3}}$, $C \sim 603\ \mathrm{J\,kg^{-1}K^{-1}}$, yielding $k = 0.063\ \mathrm{W\,m^{-1}K^{-1}}$; for regolith $k \sim 1.5$; for H_2O ice near 170K, $k = 3.4$. Estimates of Mars geothermal heat flow range from 8 mW/m^2 at the North polar cap (NPC) [**?**, Supporting Online Material, section 3] through an upper limit of 19 mW/m^2 under the NPC **?** to a global value $\sim$ 30 mW/m^2 **?**. For the highest of these heat-flow estimates, the geothermal gradients corresponding to the above three materials would be roughly 0.5, 0.02 and 0.01 K/m respectively. Thus, for a conservative case of 1 m of $I = 200$ material underlain by regolith, the mean subsurface temperature would be no more than 1 K above the surface annual mean to a depth of at least 26m.

An example of the use of KRC is to compute the mean annual soil temperature $\overline{T_s}$ at MGS-TES resolution of about 3 km. Maps of thermal inertia and albedo with 1/20° resolution were derived from maps described in **?** and available at http://www.boulder.swri.edu/inertia/2007/ . The square-root of the product of day and night thermal inertias was used; day-night differences rise steeply within 12° of the south pole and 17° of the north pole suggesting that they are not quantitatively reliable. For albedo, the average of Mars Year 24, 25 and 26 was used, avoiding the sections of years 24 and 26 that were duplicates of year 25. For both I and A, data are null within 3° of the pole, 0.14% of the surface area. Elevation at 1/20° resolution was obtained by rigorous resolution change (every input pixel used exactly once in total) from the MOLA

1/32° resolution data set; file available at http://pds-geosciences.wustl.edu/missions/mgs/megdr.html .

Centered slopes in the cardinal directions were obtained by differencing adjacent pixels in the N-S direction and pixels separated proportional to secant of latitude E-W; these slopes have 1.85 km posting. The median slope over 3.7 km is about 0.0057 radians [.0326°]; about 0.32% are larger than 0.1 and 0.66% larger than 0.2 radian [5.73 and 11.46 °, respectively]. The slope maps were converted to 1/20° resolution ($\sim$ 3 km) by rigorous resolution change.

The seasonal variation of dust and ice-cloud opacity as a function of latitude derived by THEMIS (digital data provided by M. Smith, see **?**) covering 5.3 Mars years was averaged into one year, smoothed slightly, and any remaining gaps filled by interpolation; the winter polar regions were filled with $\tau_0 = .01$ and $\tau_i = .001$. This provided climate maps with 5° resolution in latitude and L_s which were then linearly interpolated by KRC. The seasonal pressure variation followed the average of the 3 Viking Lander sites.

KRC was used to generate a model set with level terrain at 5° latitude spacing (except +/- 88° rather than the true pole) with 8-sol season spacing, output 48 times of day, for all combinations of three albedos (A) [0.15, 0.25, 0.35], 15 thermal inertias (I) spaced uniformly in logarithm [11, 16, 24, 35, 52, 77, 114, 168, 249, 367, 542, 800, 1181, 1744, 2575 $\mathrm{J\,m^{-2}s^{-1/2}K^{-1}}$] and three surface elevations [-7.422, -1.921, and 5.544 km] (PTOTAL was 502 Pa and T_a=210, so that these correspond to surface pressures at $L_s = 0°$ of 1000, 600 and 300 Pa). The KRC system could produce such maps for any slope and azimuth and at current or prior Mars orbital values (obliquity, eccentricity, and longitude of perihelion), although the climate model used here would not be justified.

To accommodate slopes, an additional set of 72 models (omitting every other thermal inertia level) was run with a slopes of 0.1 and 0.2 radians in each of the cardinal directions.

W-E slopes have a small negative effect on $\overline{T_s}$, generally less than 0.5K and roughly quadratic with slope. $\Delta\ \overline{T_s}$ has a minimum near 300 $\mathrm{J\,m^{-2}s^{-1/2}K^{-1}}$. There is little dependence on surface pressure, and an inverse linear relation with A. S-N slopes have a larger effect, roughly linear with slope magnitude and negative if poleward, changes over -70:+70° latitude are up to 9K for 0.1 radian slope.

Changes of annual mean temperature from the level model set were used to fit a quadratic form to the E-W and N-S slopes separately. The N-S effect is approximately linear with latitude, being zero near +5° (due to orbit asymmetry); $\Delta\ \overline{T_s}$ is accentuated and erratic over latitudes poleward of $\sim$45° where associated with the seasonal polar cap edge.

For both polar zones, a simple linear interpolation with latitude and with tilt was used for all I, A, and P combinations. This fit was done to all models after combining all four N:S tilt models normalized by a factors of .5 for the 0.2 tilts runs and by -1 for the south tilt runs. This linear model was applied to one less latitude zone than fit, in order to provide a reasonably continuous extension.

The resulting global map is shown at reduced resolution in Figure **??**. The full binary file is available in Dataset 11. Map values for A and I are invalid poleward of ±87°, excluding these regions the area-weighted mean $\overline{T_s}$ is 206. K; omitting regions poleward of ±70°, where results are strongly influenced by the seasonal caps, the mean $\overline{T_s}$ is 209.2 . Northern polar latitudes are largely warmer than southern by several K, as shown in Figure **??**.

6.2. MSL landing site

The MSL landing site is at 4.5S, 137.4E on the floor of the crater Gale north-west of the large central mound. The average values for a 15 km square area in this area are A=0.215, I=313 $\mathrm{J\,m^{-2}s^{-1/2}K^{-1}}$, elev=-4440 m (with standard deviations of .014, 52 and 60 respectively). The average horizon is elevated at about 3.5°. Using these values, a homogeneous model with emissivity 0.97 was calculated for a Martian year, using a 3-year spin-up; see Figure **??**. The elevated horizon increases the average daily surface temperature over somewhat more than half the year, with a maximum of 1.1 K near $L_s = 100°$; diurnal average is decreased over $L_s = 190°$ to 350° by up to 0.6 K

The atmosphere dust and ice opacity in the KRC model followed the THEMIS climate, as described in §6.1. The effects of opacity variation can be seen as high-frequency perturbations on the smooth seasonal trends; Martian climate is highly variable from year to year so the rapid variations are only indicative of the general nature of dust-storm and ice-cloud effects.

6.3. Examples of polar frost budgets

Developing thermal models or GCMs that match the details of seasonal polar caps, let alone the seasonal surface pressure variation, has been challenging. Wood and Paige,**?** used a model based on an early version of KRC, but with no topography or aerosol opacity, to match the Viking pressure variation. However, the derived thermal inertia of the surface under the seasonal cap and the derived albedo and emissivity of seasonal frost for various best fits are substantially different from observed values. In a companion paper,**?** they included variation of dust opacity and effective frost emissivity to address inter-annual differences in the seasonal surface pressure.

Recent GCM's include spatial variation of polar surface and frost properties and closely reproduce the Viking pressure measurements, **???** and S.L. Lewis, personal communication 2012. These models are far more detailed than KRC, but each run of them is a substantial commitment. KRC, with its seasonal cap modeling ability, could be used to easily study the effect of small changes in seasonal-cap parameters.

KRC was run using 18 latitudes in each hemisphere from 45° to near the pole, with zonal-average elevations, 80 seasons with a 3-year spin-up. The atmospheric dust and ice opacity followed the annual variation measured by THEMIS, averaged over about 5 Mars years (digital data from M. Smith)**?**. The frost emissivity was 0.95; other surface and frost parameters are close to the observed average values (see Figure **??** caption). By combining the seasonal frost budgets for the south and north hemispheres from different KRC runs (each with globally uniform properties), KRC seasonal frost models can approximate the Viking pressure curves **?**, adjusted to the mean surface elevation of Mars, see Figure **??**. These runs indicate that the total inventory of gas participating in the seasonal cycle is equivalent to a mean annual pressure at the mean surface level of about 741 Pa; **?** obtained 771 to 798 Pa in their best fits.

Use of uniform thermal inertia and frost albedo relations across each seasonal cap is a significant over-simplification for Mars; yet the basic features are captured by the KRC model. The time around $L_s = 330°$ is not well matched, probably due to the great variation of solid CO_2 albedo in the southern spring **????**. None-the-less, KRC can be used to quickly estimate the differential effect of changing cap-related parameters.

7. Summary

KRC provides an efficient and effective way to compute planetary surface temperature and top-of-atmosphere bolometric temperatures useful for remote sensing. Its physics-based one-layer gray atmosphere provides a reasonable approximation of the radiative effects of a dusty atmosphere;