Problem of the Week

Week 6, due Feb 23th 11.59pm

NAME: _____

NAU Email: _____

Instructor: _____

Please write clean, neat and complete solutions to the problem in order to receive full credit. Your job is to convince me, or really anybody who reads this document, that you understand the problem and are able to communicate what you are thinking about. Please submit your solutions through Gradescope(https://www.gradescope.com/) by the indicated deadline. You might need to create an account with your NAU email. To enroll into the Problem of the Week course use entry code: NYZ56P. Good luck and have fun! NO CHATGPT, etc. IS ALLOWED WHEN WORKING ON THE PROBLEM.

PROBLEM. A positive integer is called *primal* if it is equal to the sum of its divisors different from itself. For example $6 = 1 + 2 + 3$, $28 = 1 + 2 + 4 + 7 + 14$. Prove all three parts:

(1) If $2^n - 1$ is prime for some positive integer n then $2^{n-1}(2^n - 1)$ is primal.
(2) If an even positive integer k is primal then $k = 2^{n-1}(2^n - 1)$ for some positive integer n.
(3) No odd primal number exists.

You are allowed to look up the Fundamental Theorem of Arithmetic , but NO CHATGPT, etc. IS ALLOWED WHEN WORKING ON THE PROBLEM.