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Assignment – 1 Maths

Ques-1 Prove that $\lim\limits_{x \to y} \dfrac{x^2 y}{x^4+y^4}$ does not exists.

Given $f(x,y) = \lim\limits_{x,y \to (0,0)} \dfrac{x^2 y}{x^4+y^4}$

Let $(x,y) \to (0,0)$ Along the path $x^2 = my$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \frac{my \cdot y}{m^2 y^2 + y^2} = \frac{my^2}{y^2(m^2+1)}$$

$$\boxed{f(x,y) = \frac{m}{1+m^2}}$$

which is independent of 'x, y' but dependent of 'y' 'm'

f(x,y) does not exists.

f(x) is not continuos.

Ques-3 if $v = r^m$ where $r^2 = x^2+y^2+z^2$ show that $v_{xx} + v_{yy} + v_{zz} = 0$

$v = r^m$ $\qquad r^2 = x^2+y^2+z^2$

$\qquad\qquad\qquad \cancel{2r}\dfrac{dr}{dx} = \cancel{2}x + 0 + 0$

$$\frac{\delta v}{\delta x} = m r^{m-1} \frac{dr}{dx}$$

$$\frac{\delta v}{\delta x} = m r^{m-1} \times \frac{x}{r}$$

$$\frac{d^2 v}{dx^2} = m\left[r^{m-2} + (x)(m-2) r^{m-3} \frac{x}{r}\right] \quad \text{—— (1)}$$

Similarly.

$$\frac{\delta^2 v}{\delta y^2} = m r^{m-2} + m y^2 (r^{m-4})(m-2) \text{ — (2)}$$

$$\frac{\delta^2 v}{\delta z^2} = m r^{m-2} + m z^2 r^{(m-4)}(m-2) \text{ —— (3)}$$

By adding eqⁿ ① , ② & ③

$3mr^{m-2} + mr^{m-4}(m-2)(x^2+y^2+z^2)$

$3m\,r^{(m-2)} + m r^{(m-4)}(m-2)\,r^2$

$r^{(m-2)}\left[3m + m^2 - 2m\right]$

$r^{(m-2)}\,(m^2+m)$

$$\boxed{V_{xx} + V_{yy} + V_{zz} = m(m+1)\,r^{(m-2)}}$$

③ find the first & the second derivative

$\log z = x+y+z.$

$\frac{1}{z}\frac{dz}{dx} = 1 + \frac{dz}{dx}$

$\frac{dz}{dx}\left(\frac{1}{z}-1\right) = 1$

$\frac{dz}{dx} = \frac{z}{1-z}.$

$\log z = x+y+z$

$\frac{1}{z}\frac{dz}{dy} = 0 + 1 + \frac{dz}{dy}$

$\frac{dz}{dy}\left[\frac{1}{z}-1\right] = 1$

$\frac{dz}{dy} = \frac{z}{1-z}$

Partially again w.r.t 'x'

$\frac{\delta^2 z}{\delta x \delta y} = \frac{(1-z)\frac{dz}{dx} + z\frac{dz}{dx}}{(1-z)^2}$

$\frac{(1-z)\left(\frac{z}{1-z}\right) + \frac{z^2}{(1-z)}}{(1-z)^2}$

$\frac{z(1-z) + z^2}{(1-z)^3}$

$= \frac{z - z^2 + z^2}{(1-z)^3}$

$\frac{\delta^2 z}{\delta x \delta y} = \frac{z}{(1-z)^3}$

$\frac{\delta^2 x}{\delta x^2} = \frac{z}{(1-z)^3}$

$\frac{\delta^2 z}{\delta y^2} = \frac{z}{(1-z)^3}$

Query Examine for Extreme value

$f(x,y) = x^2 + y^2 + 6x + 12.$

$f_x = 2x + 6$

$f_x = 0$

$x = -3$

$f_{(xx)} = 2$

$f(x,y) = 0$

$f_y = 0 + 2y + 0$

$f_{(yy)} = 2$

$f_y = 0$

$y = 0$

Extreme pt $(-3, 0)$

A.T.Q

$rt - s^2$

$2 \times 2 - 0$

4

$rt - s^2 > 0$

local minima

$f(-3, 0) = 9 + 0 + 6 \times -3 + 12$

$= 3.$

local minima at pt. $(3, 0)$ having value 3.

Q5.

$u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$. find the value of

$x^2 \frac{\delta^2 u}{\delta x^2} + 2xy \frac{\delta^2 u}{\delta x \delta y} + y^2 \frac{\delta^2 u}{\delta y^2} = ?$

Put $(hx, hy) = h^2 [f(x,y)]$

Hence $f(x)$ is homogeneous degree '2'

$n = 2$

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$u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$

$\frac{\delta u}{\delta x} = x^2 \cdot \frac{1}{1+\left(\frac{y^2}{x^2}\right)} \times \frac{-1}{x^2} - 2x\tan^{-1}\left(\frac{y}{x}\right) - y^2 \frac{1}{1+\left(\frac{x^2}{y^2}\right)} \times \frac{1}{y}$

$= \frac{-x^2 y}{x^2+y^2} + 2x\tan^{-1}\left(\frac{y}{x}\right) - \frac{y^3}{x^2+y^2}$

$x\frac{du}{dx} = \frac{-x^3 y}{x^2+y^2} + 2x\tan^{-1}\left(\frac{y}{x}\right) - \frac{xy^3}{x^2+y^2}$

Similarly

$y\frac{du}{dy} = \frac{y^3 x}{x^2+y^2} - 2y^2 \tan^{-1}\left(\frac{x}{y}\right) + y\frac{x^3}{x^2+y^2}$ — 2

$x\frac{du}{dx} + y\frac{du}{dy} = nu = 2u$

$x\frac{du}{dx} + y\frac{du}{dy} = 2\left(x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)\right) = 2u$

$$\boxed{x^2\frac{\delta^2 u}{\delta x^2} + 2xy\frac{\delta^2 u}{\delta x \delta y} + y^2\frac{\delta^2 u}{\delta y^2} = 2u}$$

Que 6 find the minimum distance from the pt $(1,2,0)$ to the cone $z^2 = x^2+y^2$

Distance b/w the pt. $(1,2,0)$ $x^2+y^2-z^2=0$

$D = \sqrt{(x-1)^2+(y-2)^2+(z-0)^2}$

$f(x,y,z) = (x-1)^2+(y-2)^2+z^2$ — 1

$\phi = x^2+y^2-z^2 = 0$

So Lagranges f(n)

$F = f + \lambda\phi$

~~for~~. $F = \left((x-1)^2+(y-2)^2+z^2\right) + \lambda\left(x^2+y^2-z^2\right)$

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for Stationary pt

$$\frac{\delta f}{\delta x} + \lambda \frac{\delta \phi}{\delta x} = 0, \quad \frac{\delta f}{\delta y} + \lambda \frac{\delta \phi}{\delta y} = 0 \quad \frac{\delta f}{\delta z} + \lambda \frac{\delta \phi}{\delta z} = 0$$

$$\frac{\delta f}{\delta x} = 2(x-1)$$

$$\frac{\delta f}{\delta y} = 2(y-2)$$

$$\frac{\delta f}{\delta z} = 2z$$

$$2(y-2) + \lambda(2y) = 0$$

$$2y(y-2) + 2\lambda y^2 = 0$$

$$2y^2 - 4y + 2\lambda y^2 = 0$$

$$2z + (-2z\lambda) = 0$$

$$2z^2 - 2\lambda z^2 = 0 \quad - (5)$$

$$2(x-1) + \lambda 2x = 0$$

$$2x(x-1) + 2x^2\lambda = 0$$

$$2x^2 - 2x + 2\lambda x^2 = 0$$

$$2(x-1) + 2\lambda x = 0$$

$$(x-1) + \lambda x = 0$$

$$x(1+\lambda) = 1$$

$$x = \frac{1}{1+\lambda}$$

$$2(y-2) + 2\lambda y = 0$$

$$y = \frac{2}{1+\lambda}$$

$$2z - 2\lambda z = 0$$

$$2z = 2\lambda z$$

$$\boxed{\lambda = 1}$$

$$x^2 + y^2 - z^2 = 0$$

$$\left(\frac{1}{1+\lambda}\right)^2 + \left(\frac{2}{1+\lambda}\right)^2 - z^2 = 0$$

$$\frac{5}{(1+\lambda)^2} = z^2 \quad \text{if } z = 0$$

$$\lambda = \infty, z = 0$$

($x = 0, y = 0, z = 0$)

(So the minimum distance from the cone is zero)

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Ques. find the dimention of the rectangular box open at the top of maximum capacity whose surface is 432 sq. cm?

Sol?

Let (x, y, z) be the edges of the box capacity $= xyz$

and surface area $= xy + 2yz + 2zx = 432$

$f(x,y,z) = xyz$

$\phi(x,y,z) = xy + 2yz + 2zx - 432 = 0$

$F = f + \lambda\phi$

$f = xyz + \lambda(xy + 2yz + 2zx - 432)$

for Stationary pt.

$\frac{\delta f}{\delta x} + \lambda\frac{\delta \phi}{\delta x} = 0$ | $\frac{\delta \phi}{\delta x} = y + 2z$

$\frac{\delta f}{\delta x} = yz$ | $\frac{\delta \phi}{\delta y} = x + 2z$

$\frac{\delta f}{\delta y} = xz$ | $\frac{\delta \phi}{\delta z} = 2y + 2x$

$\frac{\delta f}{\delta z} = xy$

$yz + \lambda(y + 2z) = 0$ —— (1) $xz + \lambda(x + 2z) = 0$

$xyz + \lambda x(y + 2z) = 0$ $xyz + \lambda y(x + 2z) = 0$ —— (2)

$xy + \lambda(2y + 2x) = 0$

$xyz + \lambda z(2y + 2x) = 0$ (3)

① + ② + ③

$$3xyz = -\lambda[2yz + 2zx + xy + 2xz + xy + 2yz)$$
$$= -\lambda[4yz + 4zx + 2xy]$$

$$\lambda = \frac{-3xyz}{4yz + 4zx + 2xy}$$
$$= \frac{-3xyz}{2(xy + 2y + 2zx)}$$
$$= \frac{-3xyz}{2 \times 432} = \frac{-xyz}{288}$$
$$\lambda = \frac{f}{288}$$

$\therefore$ $xyz + \lambda x(y + 2z) = 0$

$$f = \frac{fx(y+2z)}{288}$$
$$288 = xy + 2xz$$
$$288 - xy = 2zx$$
$$2zx = 2yz$$
$$\boxed{x = y}$$

$$-xyz = \lambda y(x + 2z)$$
$$-f = \frac{-f}{288} y(x + 2z)$$
$$288 = xy + 2yz$$
$$288 - xy = 2yz$$
$$y^2 + 2yz = 288$$

$$\boxed{x = 2z = y}$$

$$-f = \frac{-fz}{288}(2y + 2x)$$
$$\Rightarrow 288 = 2(zy + zx)$$
$$144 = yz + xz$$
$$288 = 2yz + 2xz$$
$$-288 = -xy + 2xz$$
$$+ \quad + \quad -$$
$$2yz = xy$$
$$x = 2z$$

$$xy + 2yz + 2xz = 432$$
$$4z^2 + 4z^2 + 4z^2 = 432$$
$$4 \times 3z^2 = 432$$
$$4z^2 = 36$$
$$z^2 = 36$$
$$\boxed{z = 6}$$
$$2z = x$$
$$x = 2 \times 6 = \boxed{12 = x}$$
$$\boxed{y = 12}$$

(8)

Que- Given $f(x,y,z) = \dfrac{5xyz}{x+2y+4z}$ find the value of (x,y,z) for which $f(x,y,z)$ is a maximum subject to condition $xyz=8$

$$f = \frac{40}{x+2y+4z}$$

$$\phi = xyz - 8 = 0$$

$$F = f + \lambda \phi$$

$$F = \frac{40}{x+2y+4z} + \lambda(xyz-8)$$

for stationary pt. $dF = 0$

$$\frac{\delta f}{\delta x} = \frac{-40}{(x+2y+4z)^2} \qquad \frac{\delta \phi}{\delta x} = yz$$

$$\frac{\delta f}{\delta y} = \frac{-2\times 40}{(x+2y+2z)^2} \qquad \frac{\delta \phi}{\delta y} = xz$$

$$\frac{\delta f}{\delta z} = \frac{-4\times 40}{(x+2y+4z)^2} \qquad \frac{\delta \phi}{\delta z} = xy$$

$$\frac{-40}{(x+2y+4z)^2} + \lambda yz = 0$$

$$\frac{-40x}{(x+2y+4z)^2} + \lambda xyz = 0.$$

$$\frac{-40\times 2}{(x+2y+4z)^2} + \lambda xyz = 0$$

$$\frac{-40\times 2\times y}{(x+2y+4z)^2} + \lambda xyz = 0$$

eq^n 3 = 4 + 5 then

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$$\frac{-40}{(x+2y+4z)^2}\left[x+2y+4z\right) = -3xyz$$

$$\frac{-40}{x+2y+4z} = -3\times 8\times \lambda$$

$$3\lambda = \frac{5}{x+2y+4z}$$

$$3\lambda = \frac{f}{8}$$

$$\lambda = \frac{f}{24}$$

$$\frac{-40}{(x+2y+4z)^2} + \frac{fyz}{24} = 0$$

$$\frac{-1}{(x+2y+4z)} = \frac{-yz}{24}$$

$$\frac{x}{x+2y+4z} = \frac{8}{24} = \frac{1}{3}$$

$$x-y-2z=0 \quad \text{— ⑥}$$

$$\frac{-80y}{(x+2y+4z)^2} + \lambda xzy = 0$$

$$\frac{-80y}{(x+2y+4z)^2} = \frac{f}{24}$$

$$\frac{fy}{x+2y+4z} = \frac{f}{3}$$

$$x-4y+4z=0 \quad \text{— ⑦}$$

$$\frac{-160z}{(x+2y+4z)^2} = -\lambda xyz$$

$$-\frac{fz}{(x+2y+4z)} = -\frac{f\times 8}{24}$$

$$\frac{z}{x+2y+4z} = \frac{1}{3}$$

$$x+2y-8z=0$$

By the eqn ⑥, ⑦, ⑧

$y=2z$, $x=4z$.

$$xyz=8$$

$$4z\times 2z\times z=8$$

$$8z^3=8$$

$$z^3=1$$

$$\boxed{z=1}$$

$$y=2z$$

$$\boxed{y=2}$$

$$x=4z$$

$$\boxed{x=4}$$

$$\boxed{\begin{matrix}x=4\\ y=2\\ z=1\end{matrix}}$$