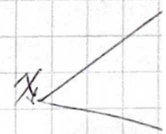
$$\langle q_n | P_n \rangle = \int_{-1}^{1} P(x)\, q(x)\, dx \qquad \langle q_n | P_n \rangle = \int_{0}^{1} P(x)\, q(x)\, dx$$

$$\cos\theta = \frac{\int_a^b dx\, f^*(x)\, g(x)}{\left(\int_a^b dx\, |f(x)|^2\right)^{\frac{1}{2}} \left(\int_a^b dx\, |g(x)|^2\right)^{\frac{1}{2}}}$$

$$\boxed{\text{Vectores} \quad |X_1\rangle = 1 \qquad |X_2\rangle = t \qquad |X_3\rangle = 1 - t}$$

$$|X_1\rangle = 1 \qquad |X_2\rangle = x \qquad |X_3\rangle = 1 - x$$



$$d(|X_1\rangle, |X_2\rangle) = \sqrt{\langle X_2 - X_1 | X_2 - X_1 \rangle}$$

para $\langle q_n | P_n \rangle = \int_{-1}^{1} P(x)\, q(x)\, dx$

$$= \sqrt{\int_{-1}^{1} [x - 1]^2\, dx} = \frac{2\sqrt{2}}{\sqrt{3}}$$

$$d(|X_1\rangle, |X_3\rangle) = \sqrt{\int_{-1}^{1} [(1-x) - 1]^2\, dx} = \sqrt{\frac{2}{3}}$$

$$d(|X_2\rangle, |X_3\rangle) = \sqrt{\int_{-1}^{1} [(1-x) - x]^2} = \sqrt{\frac{14}{3}}$$

$$\theta = \arccos\left[\frac{\langle X_1 | X_2 \rangle}{\sqrt{\langle X_1 | X_1 \rangle}\sqrt{\langle X_2 | X_2 \rangle}}\right] = \arccos\left[\frac{\int_{-1}^{1} (1)(x)\, dx}{\sqrt{\int_{-1}^{1} (1)(1)\, dx}\sqrt{\int_{-1}^{1} x^2\, dx}}\right] =$$

$$\boxed{= \frac{\pi}{2} = 1{,}57079}$$

$$\theta = \arccos \frac{\int_{-1}^{1} (1)(1-x)\, dx}{\sqrt{\int_{-1}^{1} (1)(1)\, dx}\sqrt{\int_{-1}^{1} (1-x)^2\, dx}} = \boxed{\frac{\pi}{6} = 0{,}52359}$$

$$\theta = \arccos \frac{\int_{-1}^{1}(x)(1-x)\,dx}{\sqrt{\int_{-1}^{1}(x)(x)\,dx}\ \sqrt{\int_{-1}^{1}[(1-x)(1-x)]^2\,dx}} = \boxed{\arccos\left(\frac{-\sqrt{15}}{12}\right) = 1{,}89942}$$

Para $\langle q_n | p_n \rangle$,

$$\theta = \arccos \frac{\int_0^1 1\cdot(x)\,dx}{\sqrt{\int_0^1 (1)^2\,dx}\ \sqrt{\int_0^1 x^2\,dx}} = \boxed{\frac{\pi}{6} = 0{,}52359}$$

$$\theta = \arccos \frac{\int_0^1 (1)(1-x)\,dx}{\sqrt{\int_0^1 (1)^2\,dx}\ \sqrt{\int_0^1 (1-x)^2\,dx}} = \boxed{\frac{\pi}{6} = 0{,}52359}$$

$$\theta = \arccos \frac{\int_0^1 (x)(1-x)\,dx}{\sqrt{\int_0^1 (x)^2\,dx}\ \sqrt{\int_0^1 (1-x)^2\,dx}} = \frac{\pi}{3} = 1{,}04719$$

---

b) Encuentre la distancia y el angulo...

1) $|x_1\rangle = 1 \quad |x_2\rangle = x$

distancia

$$\sqrt{\int_{-1}^{1} (1-x)^2\,dx} = \boxed{\frac{2\sqrt{2}}{\sqrt{3}} = 1{,}63299}$$

$$\theta = \arccos \frac{\int_{-1}^{1} (1)(x)\,dx}{\sqrt{\int_{-1}^{1} (1)^2\,dx}\ \sqrt{\int_{-1}^{1} x^2\,dx}} = \boxed{\frac{\pi}{2}}$$

$$\sqrt{\int_0^1 (1-x)^2 dx} = \boxed{\sqrt{\frac{1}{3}}}$$

$$arc.cos. \frac{\int_0^1 1.x\, dx}{\sqrt{\int_0^1 1^2 dx}\ \sqrt{\int_0^1 x^2 dx}} = \frac{\pi}{6}$$

2) $|x_1\rangle = 2x \qquad |x_2\rangle = x^2$

$$\sqrt{\int_{-1}^1 (2x - x^2)^2 dx} = \boxed{\sqrt{\frac{16}{15}}} \qquad arccos \frac{\int_{-1}^1 (2x)(x^2)\, dx}{\sqrt{\int_{-1}^1 (2x)^2 dx}\ \sqrt{\int_{-1}^1 (x^2)^2 dx}} = \boxed{\frac{\pi}{2}}$$

$$\sqrt{\int_0^1 (2x - x^2)^2 dx} = \boxed{\frac{2\sqrt{2}}{\sqrt{15}}} \qquad arccos \frac{\int_0^1 (2x)(x^2) dx}{\sqrt{\int_0^1 (2x)^2 dx}\ \sqrt{\int_0^1 (x^2)^2 dx}} = \boxed{0{,}25268}$$

Ejercicio 3 . Sea E' un subespacio euclidiano de ...

$|v\rangle = |g\rangle + |h\rangle$ donde $|g\rangle \in \mathbf{E'}$ $\quad |h\rangle \perp \mathbf{E'}$

a) $|h\rangle = (5, 2, -2, 2) \qquad |g_1\rangle = (2, 1, 1, -\alpha)$

$|g_2\rangle = (1, \beta, 3, 0)$

$|h\rangle \perp |g_1\rangle, |g_2\rangle \Rightarrow \cos\theta_{AB} = 0 \quad . \theta = \pi/2$

$|\langle h|g_1\rangle| = 0 \quad \wedge \quad |\langle h|g_2\rangle| = 0$

$= 10 + 2 - 2 - 2\alpha = 0$

$= \alpha = 5$

$|\langle h|g_2\rangle| = 5 + 2\beta - 6 + 0 = 0$

$= \beta = \frac{1}{2}$

Entonces

$|v\rangle = |h\rangle + |g_1\rangle + |g_2\rangle$

$\boxed{|v\rangle = (8, 7/2, 2, -3)}$

$$\begin{bmatrix} A & B & C \\ A & B & C \end{bmatrix} \quad \begin{bmatrix} A & B & C \\ C & A & B \end{bmatrix} \quad \begin{bmatrix} A & B & C \\ B & C & A \end{bmatrix} \quad \begin{bmatrix} A & B & C \\ B & A & C \end{bmatrix} \quad \begin{bmatrix} A & B & C \\ C & B & A \end{bmatrix} \quad \begin{bmatrix} A & B & C \\ A & C & B \end{bmatrix}$$

$I \quad \overline{R_j} \circlearrowright \quad \overleftarrow{Ri} \circlearrowleft \quad \chi_c \quad \chi_B \quad \chi_A$

$\cdot$	I	$\overline{R_j}$	Ri	χ_A	χ_B	χ_c
I	I	$\overline{R_j}$	Ri	χ_A	χ_B	χ_c
$\overline{R_j}$	$\overline{R_j}$	$\cancel{Ri}$	$\cancel{RI}$	χ_B	χ_c	χ_A
Ri	Ri	I	$\overline{R_j}$	χ_c	χ_A	χ_B
χ_A	χ_A	χ_c	χ_B	I	Ri	$\overline{R_j}$
χ_B	χ_B	χ_A	χ_c	$\overline{R_j}$	I	Ri
χ_c	χ_c	$\cancel{\chi_B}$	χ_A	Ri	R_j	I

Lunes 4-5

$$G_\Delta = \{ I, Ri, \overline{R_j}, \chi_K \}$$

- ~~Asociatividad~~
- Cerrada respecto a la operación

$\boxed{*}$: $\{ Ri \in G_\Delta, \overline{R_j} \in G_\Delta \} \; \exists \; R_k = R_i \boxed{*} R_j \in G.$

$$Ri * \overline{R_j} = I \quad y \quad I \in G_\Delta$$

$$(I \cdot Ri) * \overline{R_j} = I \cdot (Ri \cdot R_j) \Rightarrow R \cdot R_j = I. \qquad I \cdot I = I$$

- Asociativa

- Neutro $\Rightarrow \exists \; \hat{g} \in G \Rightarrow gi \,\square\, \hat{g} = g_i = \hat{g} \,\square\, g_i$

$$Ri \cdot I = Ri$$

- Inverso $g_i \in G \Rightarrow \exists g_i^{-1} \in G \Rightarrow g_i \,\square\, g_i^{-1} = g_i^{-1} \,\square\, g_i = \hat{g}$

$$Ri \cdot \overline{R_j} = I$$

Ejercicio 10

$$|P_n\rangle \rightleftharpoons p(x) = a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1} = \sum_{i=0}^{n-1} a_i x^i$$

a) $P(x) \oplus Q(x) = (a_0 + a_1x + a_2x^2 + \dots + a_{n-1}x^{n-1}) + (b_0 + b_1x + b_2x^2 + \dots + b_{n-1}x^{n-1})$

$= (a_0 + b_0) + (a_1x + b_1x) + (a_2x^2 + b_2x^2) + (a_nx^n + b_nx^n)$

$\alpha \cdot P(x) = \alpha a_0 + \alpha a_1 x + \alpha a_2 x^2 + \dots + \alpha a_{n-1} x^{n-1}$

$\hookrightarrow$ mult

- $\forall\ |P_i\rangle, |P_j\rangle, |P_k\rangle \in P \Rightarrow (|P_i\rangle + |P_j\rangle) + |P_k\rangle = |P_i\rangle + (|P_j\rangle + |P_k\rangle)$

- Neutro

$|0\rangle + |P_i\rangle = |P_i\rangle + |0\rangle = |P_i\rangle \quad \forall\ |P_i\rangle \in P$