$$\langle q_n | P_n \rangle = \int_{-1} P(x) q(x) dx \qquad \langle q_n | P_n \rangle = \int_0^1 P(x) q(x) dx$$

- $$\cos\theta = \frac{\int_a^b dx\, f^*(x)\, g(x)}{\left(\int_a^b dx\, |f(x)|^2\right)^{\frac{1}{2}} \left(\int_a^b dx\, |g(x)|^2\right)^{\frac{1}{2}}}$$

Vectores $|x_1\rangle = 1 \quad |x_2\rangle = t \quad |x_3\rangle = 1 - t$

$$|x_1\rangle = 1 \quad |x_2\rangle = x \quad |x_3\rangle = 1 - x$$

- $d(|x_1\rangle, |x_2\rangle) = \sqrt{\langle x_2 - x_1 | x_2 - x_1 \rangle}$

para $\langle q_n | P_n \rangle = \int_{-1}^1 P(x) q(x) dx$

$$= \sqrt{\int_{-1}^1 [x-1]^2 dx} = \frac{2\sqrt{2}}{\sqrt{3}}$$

- $d(|x_1\rangle, |x_3\rangle) = \sqrt{\int_{-1}^1 [(1-x)-1)]^2 dx} = \sqrt{\frac{2}{3}}$

- $d(|x_2\rangle, |x_3\rangle) = \sqrt{\int_{-1}^1 [(1-x)-x]^2} = \sqrt{\frac{14}{3}}$

- $$\theta = \arccos\left[\frac{\langle x_1 | x_2 \rangle}{\sqrt{\langle x_1 | x_1 \rangle}\sqrt{\langle x_2 | x_2 \rangle}}\right] = \arccos\left[\frac{\int_{-1}^1 (1)(x)\, dx}{\sqrt{\int_{-1}^1 (1)(1)\, dx}\sqrt{\int_{-1}^1 x^2 dx}}\right] =$$

$$\boxed{= \frac{\pi}{2} = 1{,}57079}$$

- $$\theta = \arccos \frac{\int_{-1}^1 (1)(1-x) dx}{\sqrt{\int_{-1}^1 (1)(1) dx}\sqrt{\int_{-1}^1 (1-x)^2 dx}} = \boxed{\frac{\pi}{6} = 0{,}52359}$$

$$\theta = \arccos . \frac{\int_{-1}^{1}(x)(1-x)\,dx}{\sqrt{\int_{-1}^{1}(x)(x)\,dx}\ \sqrt{\int_{-1}^{1}[(1-x)(1-x)]^2\,dx}} = \boxed{\arccos\left(\frac{-\sqrt{15}}{12}\right) = 1,89942}$$

Para $\langle q_n | p_n \rangle$,

$$\theta = \arccos \frac{\int_0^1 1.(x)\,dx}{\sqrt{\int_0^1 (1)^2 dx}\ \sqrt{\int_0^1 x^2 dx}} = \boxed{\frac{\pi}{6} = 0,52359}$$

$$\theta = \arccos \frac{\int_0^1 (1)(1-x)\,dx}{\sqrt{\int_0^1 (1)^2 dx}\ \sqrt{\int_0^1 (1-x)^2 dx}} = \boxed{\frac{\pi}{6} = 0,52359}$$

$$\theta = \arccos \frac{\int_0^1 (x)(1-x)\,dx}{\sqrt{\int_0^1 (x)^2 dx}\ \sqrt{\int_0^1 (1-x)^2 dx}} = \frac{\pi}{3} = 1,04719$$

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b) Encuentre la distancia y el angulo...

1) $|x_1\rangle = 1 \quad |x_2\rangle = x$

distancia

$$\sqrt{\int_{-1}^{1}(1-x)^2 dx} = \boxed{\frac{2\sqrt{2}}{\sqrt{3}} \approx 1,63299}$$

$$\theta = \arccos \frac{\int_{-1}^{1}(1)(x)\,dx}{\sqrt{\int_{-1}^{1}(1)^2 dx}\ \sqrt{\int_{-1}^{1} x^2 dx}} = \boxed{\frac{\pi}{2}}$$

$$\sqrt{\int_0^1 (1-x)^2\,dx} = \boxed{\sqrt{\frac{1}{3}}}$$

$$\arccos \frac{\int_0^1 1.x\,dx}{\sqrt{\int_0^1 1^2 dx}\ \sqrt{\int_0^1 x^2 dx}} = \frac{\pi}{6}$$

2) $|x_1\rangle = 2x \qquad |x_2\rangle = x^2$

$$\sqrt{\int_{-1}^1 (2x - x^2)^2\,dx} = \boxed{\sqrt{\frac{16}{15}}} \qquad \arccos \frac{\int_{-1}^1 (2x)(x^2)\,dx}{\sqrt{\int_{-1}^1 (2x)^2 dx}\ \sqrt{\int_{-1}^1 (x^2)^2 dx}} = \boxed{\frac{\pi}{2}}$$

$$\sqrt{\int_0^1 (2x - x^2)^2\,dx} = \boxed{\frac{2\sqrt{2}}{\sqrt{15}}} \qquad \arccos \frac{\int_0^1 (2x)(x^2)\,dx}{\sqrt{\int_0^1 (2x)^2 dx}\ \sqrt{\int_0^1 (x^2)^2 dx}} = \boxed{0{,}25268}$$

Ejercicio 3 . Sea E' un subespacio euclidiano de ...

$|v\rangle = |g\rangle + |h\rangle$ donde $|g\rangle \in \mathbf{E'}$ $\quad |h\rangle \perp \mathbf{E'}$

a) $|h\rangle = (5, 2, -2, 2) \qquad |g_1\rangle = (2, 1, 1, -\alpha)$

$|g_2\rangle = (1, \beta, 3, 0)$

$|h\rangle \perp |g_1\rangle, |g_2\rangle \Rightarrow \cos\theta_{AB} = 0 \quad \theta = \pi/2$

$|\langle h|g_1\rangle| = 0 \quad \wedge \quad |\langle h|g_2\rangle| = 0$

$= 10 + 2 - 2 - 2\alpha = 0$

$= \alpha = 5$

$|\langle h|g_2\rangle| = 5 + 2\beta - 6 + 0 = 0$

$= \beta = \frac{1}{2}$

Entonces

$|v\rangle = |h\rangle + |g_1\rangle + |g_2\rangle$

$$\boxed{|v\rangle = (8, 7/2, 2, -3)}$$