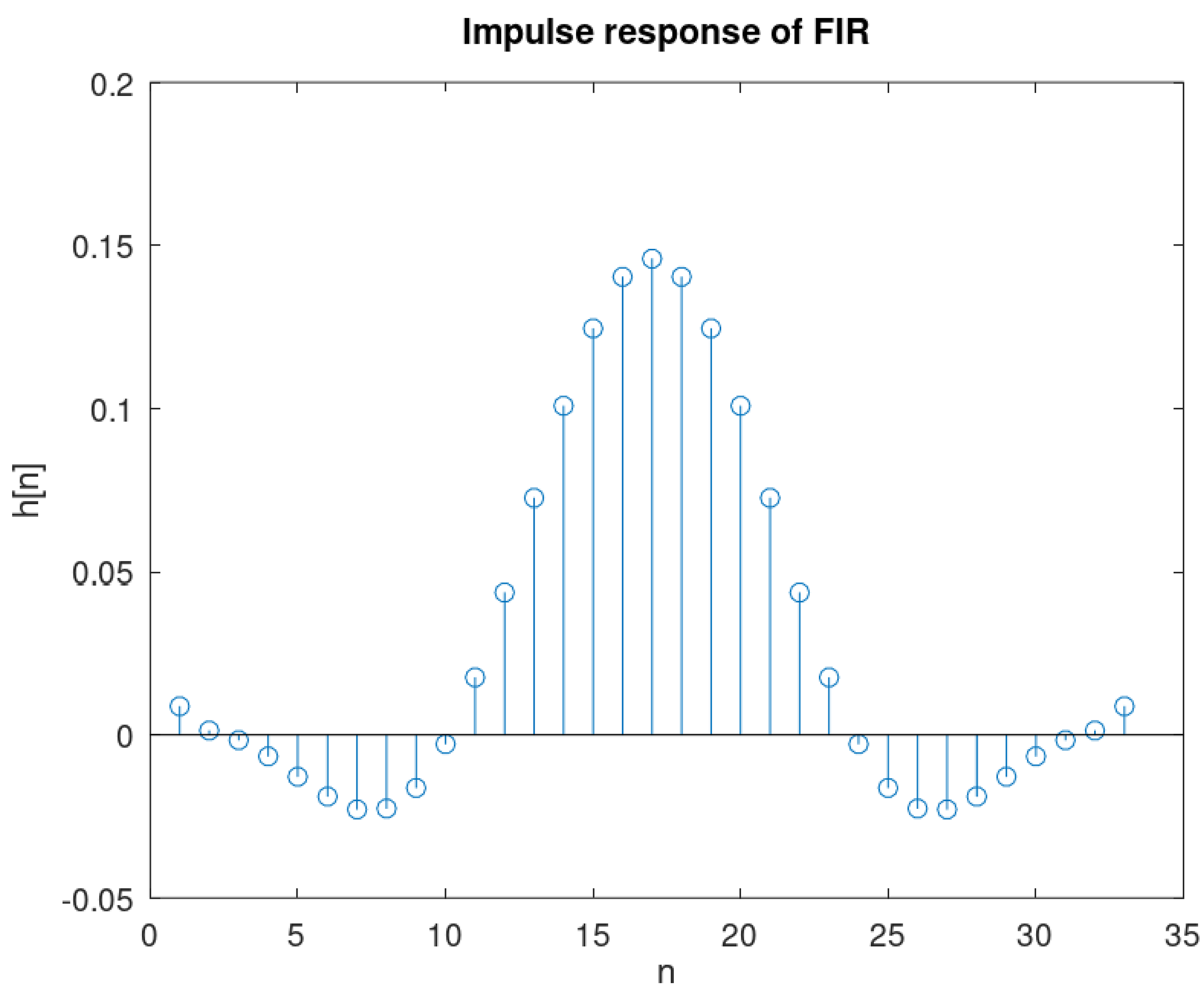
Digital Signal Processing SS 2022: 6. Assignment

Analog Signals and Systems

Group number 34

1. Task

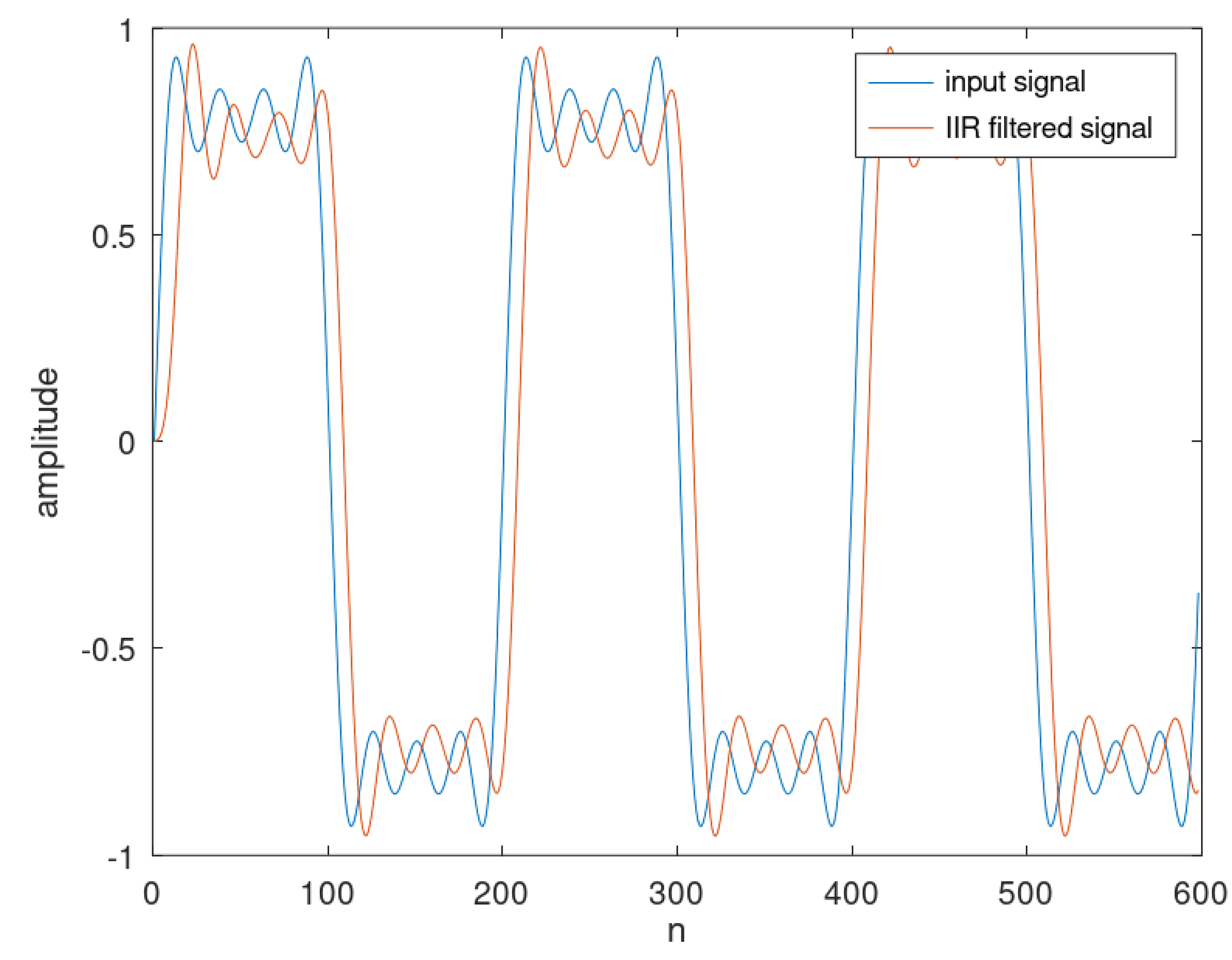
Generate three periods of the signal and then filter it with FIR and IIR filters.



In the case of FIR the b coefficents define the impulse response (DSP04, p. 37).
In DSP10, p. 13 for FIR filter it was explained, when the impulse response is symmetric then the group delay would be a constant what means that we would have NO group distortions.
As you can see in the plot above we have a axial symmetry of impulse response here. To guarantee symmetry we calculated the middle and compared the first half of h[n] values with the second half with following code:

```
middle_floor = floor(length(b1)/2)
all(b1(1:middle_floor) == fliplr(b1((middle_floor+2):length(b1))))
```

The result is 1 what means we have symmetry here. That means we have NO group distortions!
In FIR all poles are in the center of pole-zero diagram which shows us that the FIR filter ist stable (DSP10, p. 12).



In the picture you can see the original signal and its IIR filtered version.

Generally, IIR filters lead to group distortions (DSP10, p. 32). Also in the plot above you can see that the filtered signal did not keep the original form. Single hills of the signals are no longer symmetric to the relating ones. The first hill on hill reaches an higher value than the relating one of the original signal and the last hill on hill a lower value. This could be a sign of group distortion. When some frequencies were differently delayed than the other ones the signal should be distorted in frequency domain what has a direct impact on the form of signal in time domain. So, yes, we have group distortion here with IIR!

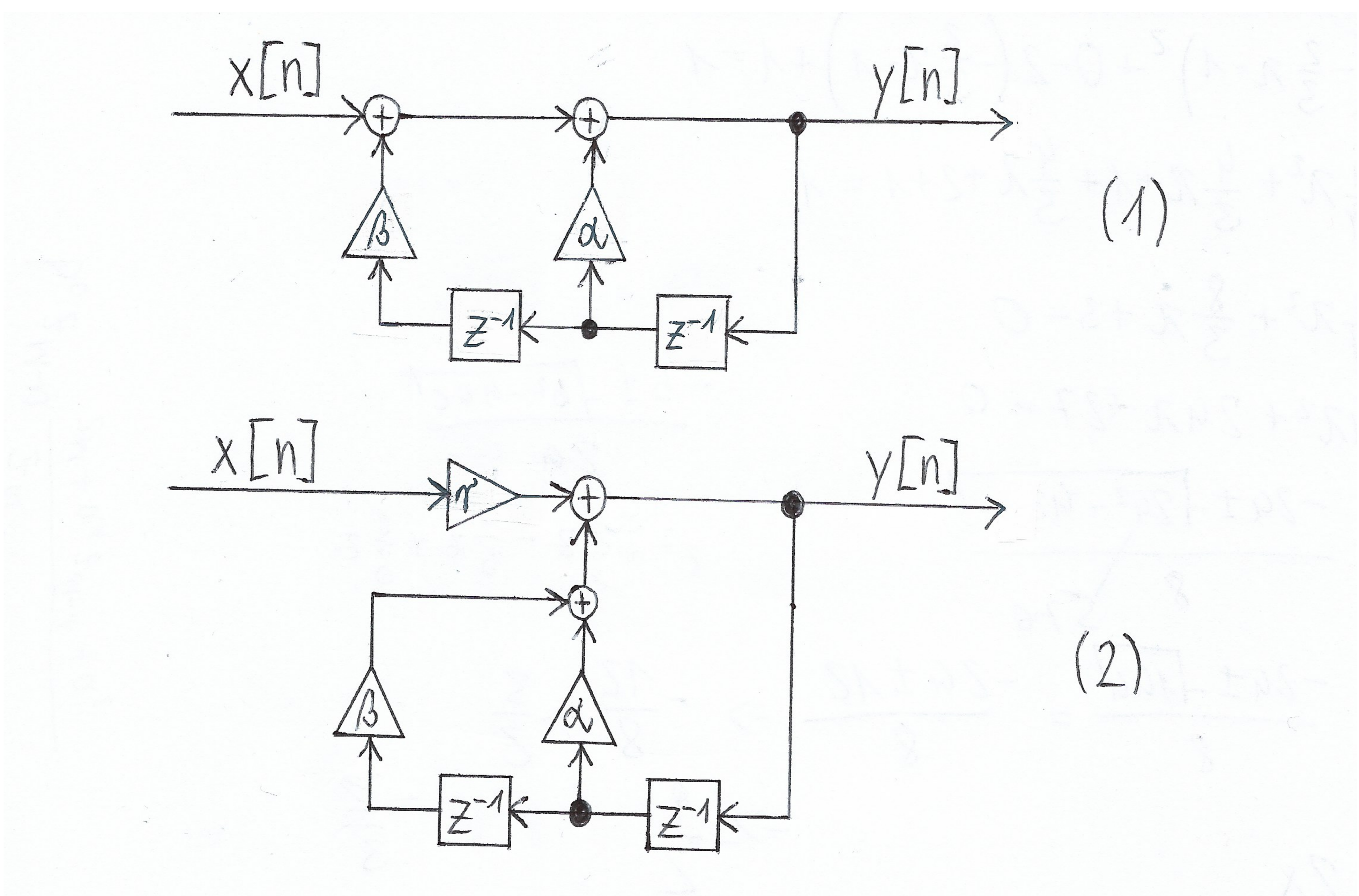
2. Task

Z-Transform

a) Given the difference equation

$$
\begin{aligned}
y[n] &= x[n] - \frac{1}{15} \cdot y[n-1] + \frac{2}{5} \cdot y[n-2] \\
\longleftrightarrow Y(z) &= X(z) - \frac{1}{15} \cdot Y(z) \cdot z^{-1} + \frac{2}{5} \cdot Y(z) \cdot z^{-2} \,,
\end{aligned}
$$

draw a sketch of the LTI system. The figure below features two block diagrams depicting the said LTI system. While the upper diagram (1) is a simplified version, the bottom one (2) has been extended to separate inputs from previous outputs in the summation step. The coefficients are given by $\alpha = -\frac{1}{15}$, $\beta = \frac{2}{5}$, and $\gamma = 1$.



b) Determine the filter type of the LTI system. The difference equation is in the form

$$
y[n] = -\sum_{i=1}^{M} a_i \cdot y[n-i] + \sum_{i=0}^{N} b_i \cdot x[n-i] \,,
$$

with $M = 2$, $a_1 = \frac{1}{15}$, $a_2 = -\frac{2}{5}$, $N = 0$, and $b_0 = 1$, which suggests a recursive procedure. The output signal depends on previous outputs, i.e. it is continuously fed back to the system. This behavior is a clear indication of an IIR system, as FIR systems, in general, are non-recursive.

c) Compute the transfer function $H(z)$. Using

$$
\begin{aligned}
x[n] &= y[n] + \frac{1}{15} \cdot y[n-1] - \frac{2}{5} \cdot y[n-2] \\
\longleftrightarrow X(z) &= Y(z) + \frac{1}{15} \cdot Y(z) \cdot z^{-1} - \frac{2}{5} \cdot Y(z) \cdot z^{-2} \\
&= Y(z) \cdot \left(1 + \frac{1}{15} z^{-1} - \frac{2}{5} z^{-2}\right),
\end{aligned}
$$

we obtain the transfer function

$$
H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + \frac{1}{15} z^{-1} - \frac{2}{5} z^{-2}} = \frac{b_0}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2}} \,.
$$

d) The transfer function may be rewritten as

$$
H(z) = z^2 \cdot \frac{1}{z^2 + \frac{1}{15} z - \frac{2}{5}} = b_0 \cdot z^{M-N} \cdot \frac{z^N}{z^M + a_1 \cdot z^{M-1} + a_2} \,.
$$

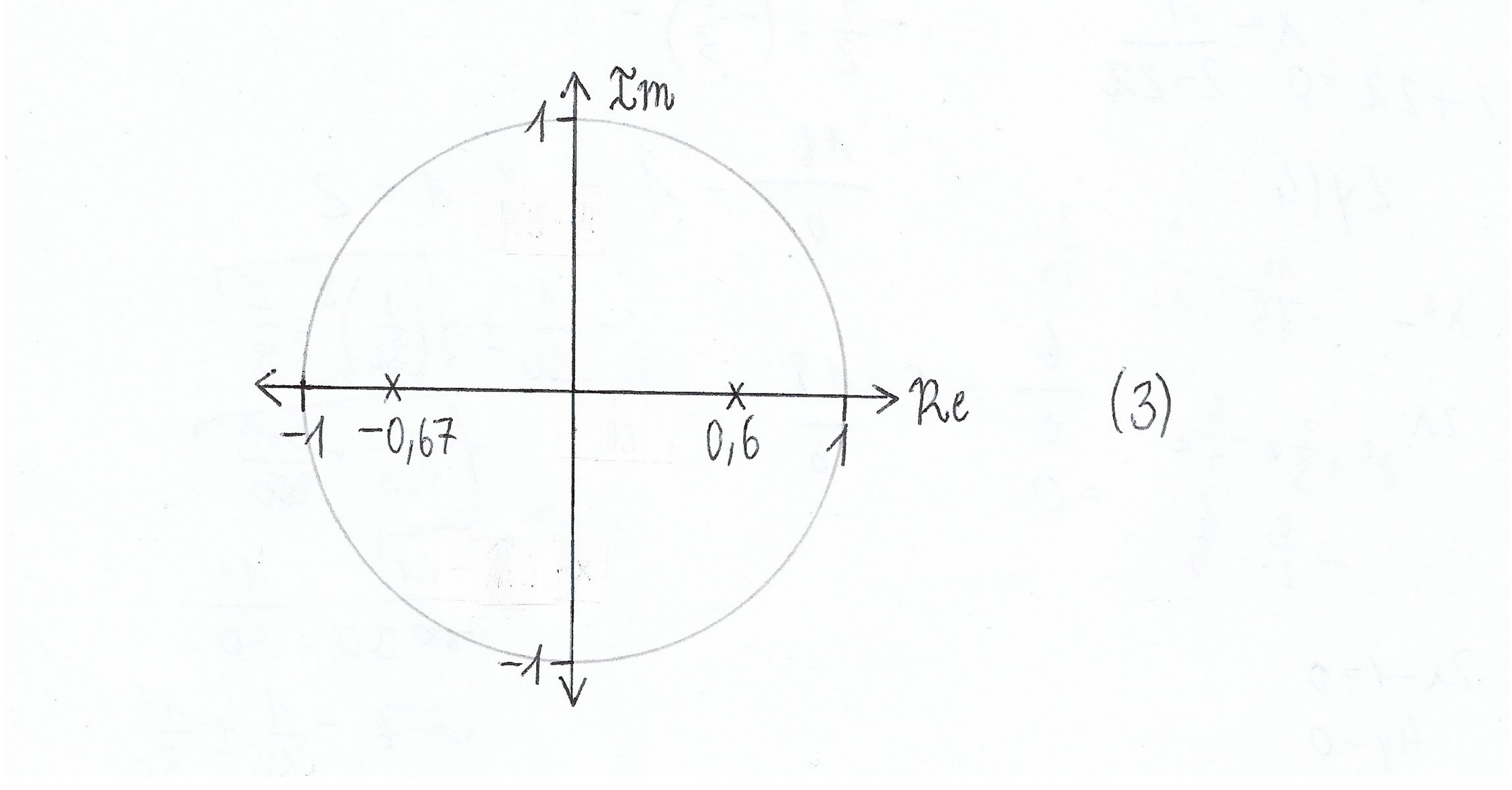
Solving the quadratic equation

$$
z^2 + \frac{1}{15} z - \frac{2}{5} = 0 \longleftrightarrow z = -\frac{1}{30} \pm \frac{19}{30} \,,
$$

we obtain the poles $p_1 = -\frac{2}{3}$, and $p_2 = \frac{3}{5}$, which results in the following transfer function:

$$
H(z) = z^2 \cdot \frac{1}{\left(z + \frac{2}{3}\right) \cdot \left(z - \frac{3}{5}\right)} = b_0 \cdot z^{M-N} \cdot \frac{z^N}{(z - p_1) \cdot (z - p_2)} \,.
$$

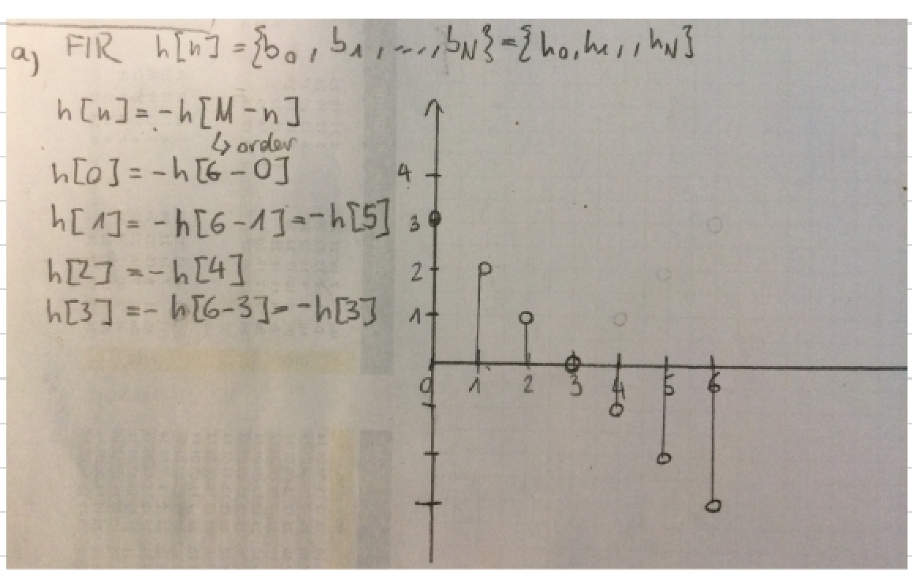
Note that there are no zeros to consider (apart from the origin), as $N = 0$. The figure (3) below shows the pole–zero diagram of the IIR system.



e) A time-discrete LTI system is BIBO stable if and only if for all poles p_i of the transfer function we have that $|p_i| < 1$ (DSP 9, p. 20). This inequality indeed holds, since $|p_1| = \frac{2}{3}$, and $|p_2| = \frac{3}{5}$. Thus, the system is stable.

3. Task

Working with symmetric impulse response which yields linear phase response.



b) Get $R(\Omega)$ as weighted sinusoids:

$$
H(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\Omega n} = \sum_{n=0}^{M} h[n] e^{-j\Omega n} = \sum_{n=0}^{\frac{M}{2}-1} h[n] e^{-j\Omega n} (+0+) \sum_{n=\frac{M}{2}+1}^{M} h[n] e^{-j\Omega n}
$$

$$
= \sum_{n=0}^{\frac{M}{2}-1} h[n] e^{-j\Omega n} + \sum_{n=0}^{\frac{M}{2}-1} -h[n] e^{-j\Omega(M-n)} = \sum_{n=0}^{\frac{M}{2}-1} h[n] \left(e^{-j\Omega n} - e^{-j\Omega(M-n)}\right)
$$

$\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$ as aim

$$
H(e^{j\Omega}) = \sum_{n=0}^{\frac{M}{2}-1} h[n] \left(e^{-j\Omega n} - e^{-j\Omega M} \cdot \overset{\text{twice}}{e^{+j\Omega n}}\right) = \sum_{n=0}^{\frac{M}{2}-1} h[n] e^{-j\Omega \frac{M}{2}} \left(e^{j\Omega \frac{M}{2}} e^{-j\Omega n} - e^{-j\Omega \frac{M}{2}} e^{-j\Omega n}\right)
$$

Pull out half of term out of equation

$$
= \sum_{n=0}^{\frac{M}{2}-1} h[n] e^{-j\Omega \frac{M}{2}} \underbrace{\left(e^{j\Omega(\frac{M}{2}-n)} - e^{-j\Omega(\frac{M}{2}-n)}\right) \cdot \frac{2j}{2j}}_{\text{sin}} = j \underbrace{e^{-j\Omega \frac{M}{2}}}_{\text{Phase}} \underbrace{\sum_{n=0}^{\frac{M}{2}-1} h[n] \cdot 2 \sin(\Omega(M-n))}_{R(\Omega)\ \text{Magnitude}}
$$

real

$e^{-j(\Omega \frac{M}{2} + \frac{\pi}{2})}$

$\arg(H(e^{j\Omega}))$

$= -\Omega \frac{M}{2} + \frac{\pi}{2}$

c) Getting N for given M

$N = -\frac{M}{2}$

In [ ]: