

- Let  $O$  be the matrix of optimally combined (OC) data, of shape  $(v, t)$  (where  $v$  is the number of voxels in the brain mask and  $t$  is the number of timepoints in the run).
- Let  $M$  be the mixing matrix from the MEICA decomposition, of shape  $(c, t)$ , where  $c$  is the number of components.
- Let  $n$  be the index of all non-ignored components in  $M$ , i.e., all accepted (BOLD-like) and rejected (non-BOLD) components.
- Let  $a$  be the index of accepted (BOLD-like) components in  $M$ .

1. The voxel-wise means ( $\mu_O$ ) and standard deviations ( $\sigma_O$ ) of the optimally data are computed.
2. The optimally combined data are z-normalized.

$$O_z = \frac{O - \mu_O}{\sigma_O} \quad (1)$$

3. The betas  $B$  for the MEICA mixing matrix  $M$  fitted to the normalized data  $O_z$  are computed.

$$O_z = B \cdot M + \epsilon \quad (2)$$

4. The residuals  $R$  using only the non-ignored components (indexed in  $n$ ) are computed using the normalized data  $O_z$ , the betas  $B$ , and the mixing matrix  $M$ .

$$R = O_z - (B[:, n] \cdot M[n, :]) \quad (3)$$

5. The time series of BOLD-like components  $P$  is constructed from the betas  $B$ , the mixing matrix  $M$ , and the index of accepted components  $a$ .

$$P = B[:, a] \cdot M[a, :] \quad (4)$$

6. The voxel-wise minimum map of the T1-like effect  $T$  (of shape  $v$ ) is computed from the BOLD time series  $P$ , and is then mean-centered.

$$T_i = \min_{j \in J} x_{ij}, i = 1, \dots, m \quad (5)$$

$$T = T - \mu_T \quad (6)$$

7. The volume-wise global signal timeseries  $g$  based on the T1-like effect is computed with least squares regression.

$$O_z = Tg + \epsilon \quad (7)$$

8. This global signal is removed from the BOLD timeseries  $P$  with least squares regression to create  $F$ , which is multiplied by  $g$  to create a predicted T1-like BOLD timeseries.

$$P^T = g^T F + \epsilon \quad (8)$$

$$P_c = P - (F \cdot g) \quad (9)$$

9. The BOLD timeseries (ME-HK+MIR,  $H$ ), scaled to match the real data, is constructed from  $P_c$  and  $\sigma_O$ .

$$H = \sigma_O P_c \quad (10)$$

10. The denoised timeseries (ME-DN+MIR,  $D$ ) is constructed from  $P_c$ ,  $\sigma_O$ ,  $\mu_O$ , and  $R$ .

$$D = \sigma_O(P_c + R) + \mu_O \quad (11)$$

Symbol	Shape	Description
$v$		Number of voxels after masking
$t$		Number of volumes
$c$		Number of components from MEICA
$O$	v, t	Optimally combined data
$O_z$	v, t	Z-normalized optimally combined data
$M$	c, t	Mixing matrix (component timeseries) from MEICA
$n$	<c	Index of all non-ignored (accepted and rejected) components in $M$
$a$	<c	Index of accepted, BOLD-like components in $M$
$B$	v, c	Voxel-wise beta values for components in $M$ fitted to $O_z$
$R$	v, t	Data not accounted for by accepted and rejected MEICA components fitted to $O_z$
$P$	v, t	Reconstructed BOLD-like component timeseries
$T$	v	Map of T1-like effect
$g$	t	Time series of T1-like global signal
$P_c$	v, t	Predicted T1-like BOLD timeseries
$H$	v, t	High-Kappa (aka BOLD) timeseries, minus impact of T1-like effect
$D$	v, t	Denoised (BOLD + ignored, but no rejected) timeseries, minus effect of T1-like effect