ASEN 5050 Spaceflight Dynamics
Fall 2023

Homework 1 Solutions

Notes: You must use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition") for full credit:

$$G(m_{Sun}) = 1.32712428 \times 10^{11}\ km^3/s^2$$
$$G(m_{Jupiter}) = 1.268 \times 10^{8}\ km^3/s^2$$

See the syllabus for a reminder of the expected components of your working.

Problem 1

The Origins, Spectral Interpretation, Resource Identification, Security-Regolith Explorer (OSIRIS-REx) spacecraft is currently on its way back to Earth to return a sample of the asteroid Bennu; the spacecraft is expected to deliver the sample return capsule to Earth on September 24, 2023. At a specific instant of time on August 30th, 2023, the state of the spacecraft in its heliocentric orbit was partially described by the following information, relative to the Sun:

$$r = 1.6358 * 10^8\ km$$
$$\varepsilon = -385.4054\ km^2/s^2$$
$$v_r = -6.2627\ km/s$$

For this problem, let's assume that a Sun-spacecraft two-body problem is a reasonable approximation. At this instant of time:

$$Assume\ m_{s/c} << m_{Sun} \therefore \mu = G(m_{Sun} + m_{s/c}) \approx Gm_{Sun}$$

a)

Write the position and velocity vectors, $\bar{r}$ and $\bar{v}$, of the spacecraft relative to the Sun in the $\hat{r}$, $\hat{\theta}$, $\hat{h}$; axes.

Solution:

Calculate $\bar{r}$:

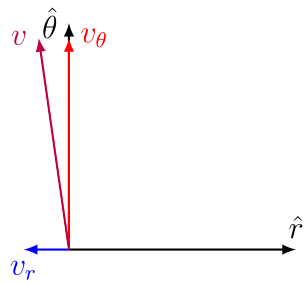
$$\bar{r} = r\hat{r}$$
$$\boxed{\therefore \bar{r} = 1.6358 \times 10^8 \hat{r}\ km}$$

Calculate $\bar{v}$:

Use the specific energy equation to calculate speed:

$$\varepsilon = \left(\frac{v^2}{2} - \frac{\mu}{r}\right)$$
$$v^2 = 2\left(\varepsilon + \frac{\mu}{r}\right) = 851.7889\ km/s$$

From Triangle Geometry:



$$v^2 = v_r^2 + v_\theta^2$$
$$v_\theta = +\sqrt{v^2 - v_r^2}$$
$$^*v_\theta \textit{ is positive due to definition of } \hat{\theta}$$
$$\bar{v} = v_r\hat{r} + v_\theta\hat{\theta}$$
$$\boxed{\therefore \bar{v} = -6.2627\,\hat{r} + 2.8506 \times 10^1\,\hat{\theta}\ km/s}$$

b)

Calculate the following information describing the orbit of the spacecraft relative to the Sun: semi-major axis a, eccentricity e, specific angular momentum h, semi-latus rectum p, true anomaly θ^*, orbit period P, type of conic

Solution:

Calculate a:

From the specific energy equation:

$$a = -\frac{\mu}{2\varepsilon}$$
$$\boxed{\therefore a = 1.7217 \times 10^8\ km}$$

Calculate h:

$$\bar{h} = \bar{r} \times \bar{v}$$
$$h = rv_\theta$$
$$\boxed{\therefore h = 4.6629 \times 10^9\ km^2/s}$$

Calculate e and define conic:

$$e = \sqrt{1 - \frac{h^2}{\mu a}}$$

or

$$e = \sqrt{1 + \frac{2\varepsilon h^2}{\mu^2}}$$

$$\boxed{\therefore e = 0.2200}$$

e < 1, therefore: $\boxed{\text{Orbit is Elliptical}}$

It can be deduced that the conic was not hyperbolic or parabolic by the given ε.
However the eccentricity should be checked to ensure it was not circular.

Calculate p:

$$p = a(1 - e^2)$$

$$\boxed{\therefore p = 1.6384 \times 10^8 \ km}$$

Calculate P:

$$P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$\boxed{\therefore P = 3.8965 \times 10^7 \ s \ = \ 4.5098 \times 10^2 \text{ days} \ = \ 1.2347 \text{ years}}$$

Vallado Defines a Year as 365.25 Days

Calculate θ^*:

From the Conic Equation:

$$\theta^* = \pm\cos^{-1}\left(\frac{p}{re} - \frac{1}{e}\right)$$

Choose $\theta^* < 0$ *because* $v_r < 0$

$$\boxed{\therefore \theta^* = -1.5637 \text{ rad} \ = \ 1.5779 \text{ rad} \ = -89.5932^o \ = \ 270.4068^o}$$

At the same instant of time, the spacecraft is also described by the following truncated position and velocity vectors measured relative to the Sun in an inertial frame with axes $\hat{X}\hat{Y}\hat{Z}$:

$$\bar{R} = 1.504 \times 10^8 \hat{X} \;-\; 5.5874 \times 10^7 \hat{Y} \;-\; 3.1770 \times 10^7 \hat{Z}\ km$$
$$\bar{V} = 5.4410 \hat{X} \;+\; 2.4995 \times 10^1 \hat{Y} \;+\; 1.4050 \times 10^1 \hat{Z}\ km/s$$

c)

Use these position and velocity vectors to calculate h, e, a. (Do not use any of the information previously calculated in parts a-b in these calculations; the goal is to independently verify this information!) Are these newly-calculated values consistent with those previously calculated or given in parts a-b?

Solution:

Calculate h:

$$\bar{h} = \bar{R} \times \bar{V} = 0.0091 \times 10^9 \hat{X} - 2.2860 \times 10^9 \hat{Y} + 4.0633 \times 10^9 \hat{Z}$$
$$h = |\bar{h}|$$
$$\boxed{\therefore h = 4.6622 \times 10^9\ km^2/s}$$

Calculate r:

$$r = |\bar{R}|$$
$$\therefore r = 1.6356 \times 10^8\ km$$

Calculate v:

$$v = |\bar{V}|$$
$$\therefore v = 29.1849\ km/s$$

Calculate a:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -385.4054\, km^2/s^2$$
$$a = -\frac{\mu}{2\varepsilon}$$
$$\boxed{\therefore a = 1.7212 \times 10^8\ km}$$

Calculate e:

$$\bar{e} = \frac{\bar{v} \times \bar{h}}{\mu} - \frac{\bar{r}}{r}$$
$$\bar{e} = 0.0877\,\hat{X} + 0.1760\,\hat{Y} + 0.0988\,\hat{Z}$$
$$e = |\bar{e}|$$
$$\boxed{\therefore e = 0.2201}$$

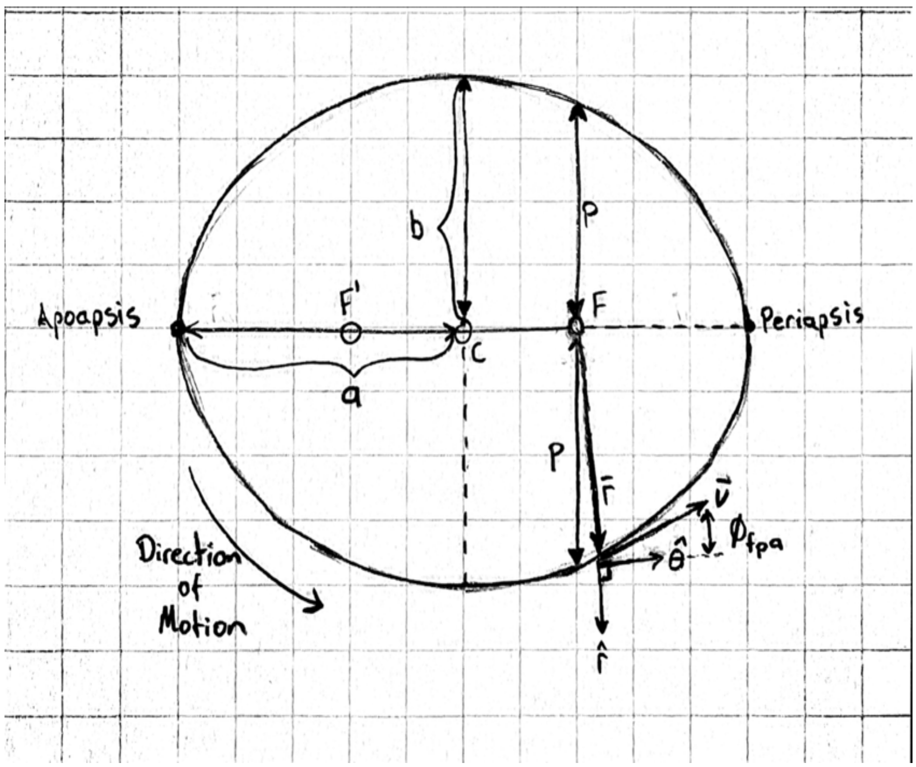
The values computed for e, a, h in parts a) and c) are close: for each quantity, the difference is in the 4th decimal place, consistent with the precision of the provided values.

d)

Sketch a diagram of the orbit (looking down on the orbit plane so that $\hat{h}$ is directed out of the page), indicating the following information where applicable: semi-major axis, semi-minor axis, C, F, F', periapsis, apoapsis, semi-latus rectum, and direction of motion. Do not plot this in a mathematical software package; sketch it by hand. On this diagram, draw the position and velocity vectors for the object at the selected instant of time on this diagram, also indicating the $\hat{r}, \hat{\theta}, \hat{h}$ unit vectors, true anomaly, and flight path angle. Be sure to accurately locate the spacecraft in the correct region of its orbit relative to any known special locations along the orbit.

Solution:

Not to Scale



$\hat{h}$ points out of page to complete right-hand rule with $\hat{r}$ and $\hat{\theta}$

e)

Given the location of the object in its orbit at the selected instant of time, justify in as much detail as possible whether the combination of r and θ^* indicate that you have drawn the spacecraft in the correct location relative to known special locations in the orbit in part d. (Hint: consider any upper and lower bounds on r and θ^* due to conic geometry)

Solution:

> Since v_r is negative, we can deduce that the spacecraft is heading towards periapsis as the distance is decreasing, therefore $-180^o < \theta^* < 0^o$. In addition $r_p < r < p$ so the spacecraft must be located in $-90^o < \theta^* < 0$ (Quadrant IV), between periapsis and the location used to measure the semi-latus rectum in the bottom half of the orbit.

f)

Discuss whether you think the dynamical environment governing the motion of the OSIRIS-REx spacecraft over the next 2 months is well-approximated by the Sun-spacecraft two-body problem.

Solution: When the spacecraft passes by the Earth to dropoff a sample, its gravity field may be significant, decreasing the accuracy of the dynamical model. Solar radiation pressure could incur significant perturbations. The irregular gravity fields (J2) of neighboring bodies and atmospheric drag from the Earth may also perturb the spacecraft trajectory once it is close enough to be affected by these phenomena. Therefore, the Sun-spacecraft relative 2BP is not a precise approximation of the dynamical environment.

Problem 2

The Voyager 1 and 2 spacecraft have been traveling in space for over 40 years, enabling us to learn more about the outer planets before reaching interstellar space. The interplanetary trajectory of each spacecraft incorporated flybys of Jupiter and Saturn. We will cover planetary flybys in far more detail in the middle of the semester. In the meantime, note that during a planetary flyby, the spacecraft follows a hyperbolic path relative to a planet. In this problem, let's study the hyperbolic path of Voyager 1 relative to Jupiter during its flyby of Jupiter in March 1979. This hyperbola is described by the following parameters measured relative to Jupiter:

$$v_\infty = 10.7527\,km/s \quad \theta^*_\infty = 139.3724^o$$

For this problem, let's assume that a Jupiter-spacecraft two-body problem is a reasonable approximation.

$$Assume\ m_{s/c} << m_{Jupiter} \therefore \mu = G(m_{Jupiter} + m_{s/c}) \approx Gm_{Jupiter}$$

a)

Calculate the values of a, e, δ for the hyperbolic trajectory followed by Voyager 1.

Solution:

Calculate a:

$$v_\infty = \frac{\mu}{|a|}$$
$$a = -\frac{\mu}{v_\infty^2}$$
$$a < 0 \text{ by convention for hyperbolic orbits}$$
$$\boxed{\therefore a = -1.0967 \times 10^6\ km}$$

Calculate e:

$$\theta^*_\infty = \pm\cos^{-1}\left(\frac{-1}{e}\right)$$
$$e = -\frac{1}{\cos(\theta^*_\infty)}$$
$$\boxed{\therefore e = 1.3176}$$

Calculate δ:

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right)$$
$$\boxed{\therefore \delta = 98.7448^o \ = \ 1.7234 \text{ rad}}$$

b)

Calculate the periapsis radius r_p and speed v_p of Voyager 1 at periapsis.

Solution:

Calculate r_p:

$$r_p = a(1 - e)$$

$$\boxed{\therefore r_p = 3.4831 \times 10^5 \ km}$$

Calculate v_p:

$$v_p = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}}$$

$$\boxed{\therefore v_p = 29.0468 \ km/s}$$

c)

Discuss whether you think the dynamical environment governing Voyager 1 as it travels along this hyperbola during the Jupiter flyby is well-approximated by the Jupiter-spacecraft relative two-body problem.

Solution: Using the 2BP solution here may give useful first order results, but additional perturbations that are not modeled can have significant consequences. Namely, in this situation the irregular gravitational field of Jupiter and the gravitational influence of any nearby moons as the spacecraft performs the flyby. The Sun may also still have an effect on the trajectory. After comparing the periapsis radius to the radius of Jupiter we can see that the flyby occurs at about 5 times the equatorial radius Jupiter from Jupiter's center placing it out of reach of atmospheric drag effects.