ASEN 5050 – Spaceflight Dynamics

Homework #2

Assigned: Tuesday, September 12, 2023
Due: Tuesday, September 19, 2023 at 9pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2022, "Fundamentals of Astrodynamics and Applications, 5th Edition"):
  - $Gm_{Earth} = 3.986004415 \times 10^5 km^3/s^2$
  - $Gm_{Moon} = 4.902799 \times 10^3 km^3/s^2$
  - Equatorial radius of Earth: 6378.1363 km
  - Equatorial radius of the Moon: 1738.0 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

The Lunar Reconnaissance Orbiter (LRO) mission was launched in June 2009. Since then, the LRO spacecraft has studied the Moon's terrain and environment to support future missions. At a specific time instant on the day that this homework is assigned, the state of LRO in its lunar orbit is described by the following position and velocity vectors, expressed in a Moon-centered inertial frame with axes $\hat{X}\hat{Y}\hat{Z}$:

$$\bar{r} = -3.02073 \times 10^2 \hat{X} + 2.25688 \times 10^1 \hat{Y} + 1.80166 \times 10^3 \hat{Z}\ km$$
$$\bar{v} = 1.31575\hat{X} - 0.95477\hat{Y} + 0.20835\hat{Z}\ km/s$$

a) At this instant of time, calculate the following orbital elements in the Moon-centered inertial frame: $a, e, i, \Omega, \omega, \theta^*$
b) At this instant of time, write the position and velocity vectors of the spacecraft relative to the Moon and expressed using the $\hat{r}\hat{\theta}\hat{h}$ axes using two methods: 1) a transformation via the direction cosine matrix, and 2) by directly computing the components of each vector using the analytical expressions from the first set of notes that focused on the two-body problem. Compare the results that you recover using both approaches.
c) At a later instant of time, the spacecraft is located at periapsis. Calculate the position and velocity vectors of the spacecraft in the inertial frame at this location.

Problem 2:

A spacecraft is currently in an elliptical orbit around the Earth. This orbit is described by the following vectors, expressed in an Earth-centered inertial frame with axes $\hat{X}\hat{Y}\hat{Z}$:

$$\hat{n} = 0.3746\hat{X} + 0.9272\hat{Y} + 0\hat{Z}$$
$$\hat{h} = 0.7595\hat{X} - 0.3069\hat{Y} + 0.5736\hat{Z}$$
$$\bar{e} = 0.0999\hat{X} - 0.2515\hat{Y} - 0.2668\hat{Z}$$

a) Calculate the following orbital elements in the Earth-centered inertial frame: e, i, Ω, ω
b) When the spacecraft is located at the ascending node, r = 19,148 km. Calculate the semi-major axis, a, of the orbit.

c) Sketch by hand the orbit of the spacecraft in three-dimensional space; do not use a calculator or mathematical software to construct this plot. On this plot, draw the following information: the inertial frame axes $\hat{X}, \hat{Y}, \hat{Z}$; the focus F; periapsis and apoapsis; the eccentricity vector; the line of nodes; the specific angular momentum unit vector; the inclination; the right ascension of the ascending node; and the argument of periapsis. Hint: while your orbit may not be to scale or look like a perfect ellipse, you can use the provided information and/or angular quantities in the orbital element set to sufficiently capture the orientation of the orbit and the correct quadrant for any vectors or angles in your sketch.