ASEN 5050 Fall 2023 HW 7 Solutions

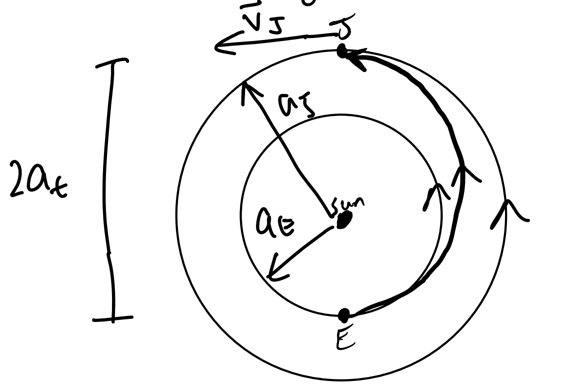
Problem 1:

a) - Assume Earth → Jupiter, circular + coplanar orbits about Sun

$\mu_{sun} = 1.32712428 \cdot 10^{11}$ km³/s²   1 AU = 149,597,870.7 km

$r_E = a_E = 1.00000010178$ AU $\mu_J = 1.268 \cdot 10^{8}$ km³/s²

$r_J = a_J = 5.202603191$ AU

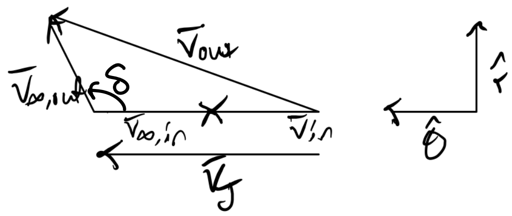


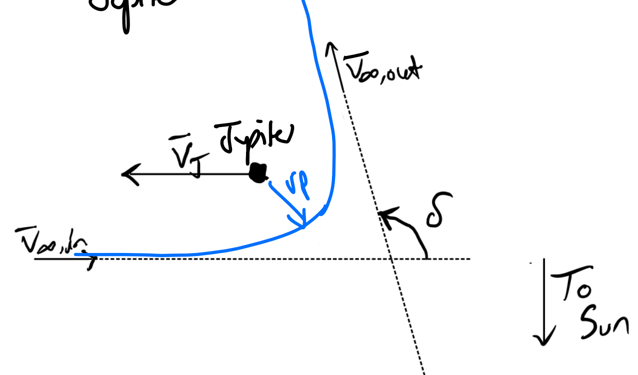
$a_t = \frac{1}{2}(a_E + a_J) = \boxed{4.6395 \cdot 10^{8} \text{ km}}$

$e_t = \frac{r_{at} - r_{pt}}{r_{at} + r_{pt}} = \frac{a_J - a_E}{a_J + a_E} = \boxed{0.6776}$

$TOF = \pi \sqrt{\frac{a_t^3}{\mu_{sun}}} = \boxed{997.435 \text{ days}}$

b. $r_p = 1{,}000{,}000$ km, sun-side





$\bar{V}_J = \sqrt{\frac{\mu_S}{r_J}} = 13.0582\hat{\theta}$ km/s

$\bar{V}_{in} = \sqrt{\frac{2\mu_S}{r_{at}} - \frac{\mu_S}{a_t}} = 7.4150\hat{\theta}$ km/s

$\hat{r}, \hat{\theta}$ defined in Sun-s/c 2BP

$\bar{V}_{\infty,in} = \bar{V}_{in} - \bar{V}_J = -5.6432\hat{\theta}$ km/s

$\mathcal{E}_h = \frac{V_{\infty,in}^2}{2} = 15.9228$ km²/s²

$a_h = -\frac{\mu_J}{2\mathcal{E}_h} = -3.9817 \cdot 10^{6}$ km

$e_h = 1 - \frac{r_{ph}}{a_h} = 1.2511$

Calculating properties of hyperbola in Jupiter-s/c 2BP

$\delta = 2\sin^{-1}\left(\frac{1}{e_h}\right) = \boxed{106.12^\circ}$

No quadrant check needed as $\left|\frac{\delta}{2}\right| \leq 90^\circ$

$|\bar{V}_{\infty,out}| = |\bar{V}_{\infty,in}| = 5.6432$ km/s

$V_{out}^2 = V_J^2 + V_{\infty,out}^2 - 2V_{\infty,out}V_J\cos\delta \Rightarrow V_{out} = \boxed{15.5975 \text{ km/s}}$

$\varepsilon_{before} = \frac{v_{in}^2}{2} - \frac{\mu_s}{r_{sat}} = -143.0251 \ km^2/s^2$

$\varepsilon_{after} = \frac{V_{out}^2}{2} - \frac{\mu_s}{r_{sat}} = -48.8750 \ km^2/s^2$

- Increased s/c energy due to flyby.

Problem 2

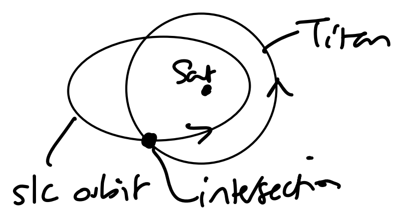
2a. Around Saturn, same plane as Titan, $\mu_{sat} = 3.794 \cdot 10^7 \ km^3/s^2$

$r_p = 600{,}000 \ km \qquad r_a = 1{,}800{,}000 \ km \qquad M_T = 1.3455 \cdot 10^{23} \ kg$

- Titan in circular orbit $r = 1{,}221{,}830 \ km \qquad r_T = 2{,}575 \ km$

Before flyby:

$a_{in} = \frac{1}{2}(r_p + r_a) = 1.2 \cdot 10^6 \ km$



$e_{in} = 1 - \frac{r_p}{a_{in}} = 0.500$

$p_{in} = a_{in}(1 - e_{in}^2) = 900{,}000 \ km$

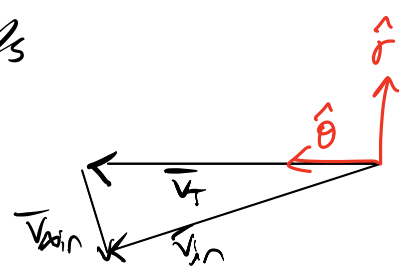
$\theta_{in}^* = \pm \cos^{-1}\left(\frac{p_{in} - r}{r e_{in}}\right) = \boxed{-121.8°}$ apoapsis → periapsis, $\theta_{in}^* = [-180, 0]$

$h_{in} = \sqrt{\mu_{sat} p_{in}} = 5.8435 \cdot 10^6 \ km^3/s^2$

$V_{r,in} = \frac{\mu_{sat}}{h_{in}} e_{in} \sin(\theta_{in}^*) = -2.7594 \ \hat{r} \ km/s$

$V_{\theta,in} = \frac{\mu_{sat}}{h_{in}} (1 + e_{in} \cos(\theta_{in}^*)) = 4.7825 \ \hat{\theta} \ km/s$

$\bar{V}_{in} = -2.7594 \ \hat{r} + 4.7825 \ \hat{\theta} \ km/s$



$\bar{V}_T = \sqrt{\frac{\mu_{sat}}{r}} = 5.5724 \ \hat{\theta} \ km/s$

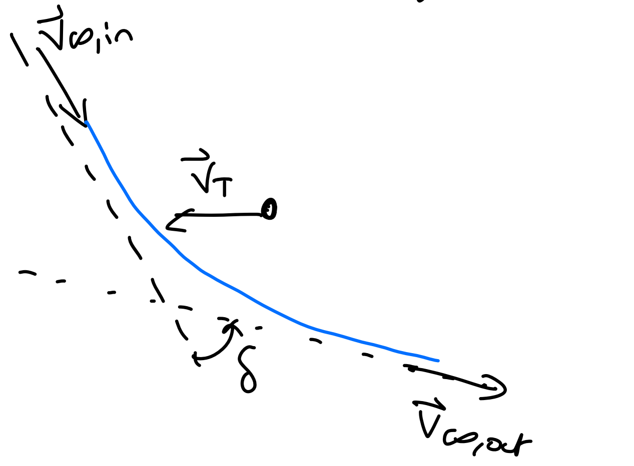
Relative to Titan:

$\bar{V}_{\infty,in} = \bar{V}_{in} - \bar{V}_T = -2.7594 \ \hat{r} - 0.7899 \ \hat{\theta} \ km/s$

$|\bar{V}_{\infty,in}| = \boxed{2.8702 \ km/s}$

b. $r_{ph} = 3{,}000$ km $\qquad \mu_T = GM_T = 6.673\cdot 10^{-20} \cdot 1.3455 \cdot 10^{23} = 8978.5 \ \text{km}^3/\text{s}^2$

- passes ahead of Titan



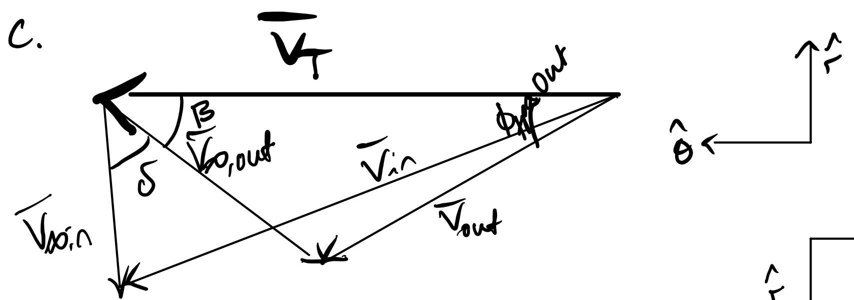
$$\mathcal{E}_h = \frac{v_{\infty,in}^2}{2} = 4.119 \ \text{km}^2/\text{s}^2$$

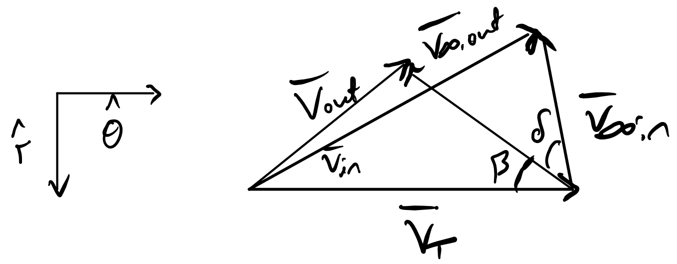
$$a_h = -\frac{\mu_T}{2\mathcal{E}_h} = -1{,}089.9 \ \text{km}$$

$$e_h = 1 - \frac{r_{ph}}{a_h} = 3.7526$$

$$\delta = 2\sin^{-1}\left(\frac{1}{e_h}\right) = \boxed{30.9^\circ}$$

c.





$$\bar{V}_{out} = \bar{V}_T + \bar{V}_{\infty,out}$$

$$V_{in}^2 = V_{\infty,in}^2 + V_T^2 - 2V_{\infty,in}V_T\cos(\beta+\delta) \Rightarrow \beta = 43.1^\circ$$

$$V_{out}^2 = V_{\infty,out}^2 + V_T^2 - 2V_{\infty,out}V_T\cos(\beta) \Rightarrow$$

$$V_{out} = 3.9924 \ \text{km/s}$$

$$V_{\infty,out}^2 = V_{out}^2 + V_T^2 - 2V_{out}V_T\cos(\phi_{fpa,out}) \Rightarrow$$

$$\phi_{fpa,out} = \cos^{-1}\left(\frac{V_{\infty,out}^2 - V_{out}^2 - V_T^2}{-2V_T V_{out}}\right) = 29.4^\circ$$

$$V_{r,out} = V_{out}\sin(\phi_{fpa,out}) = -1.9617 \ \hat{r} \ \text{km/s}$$

$$V_{\theta,out} = V_{out}\cos(\phi_{fpa,out}) = 3.4773 \ \hat{\theta} \ \text{km/s}$$

$$\bar{V}_{out} = \boxed{-1.9617 \ \hat{r} + 3.4773 \ \hat{\theta} \ \text{km/s}}$$

d. $$\mathcal{E}^+ = \frac{V_{out}^2}{2} - \frac{\mu_{sat}}{r} = -23.082 \ \text{km}^2/\text{s}^2$$

$$a^+ = -\frac{\mu_{sat}}{2\mathcal{E}} = \boxed{8.2185\cdot 10^5 \ \text{km}}$$

$h^+ = r V_{out_\theta} = 4.2486 \cdot 10^6 \ km^3/s^2$

$e^+ = \sqrt{1 + \frac{2h^2 \mathcal{E}}{\mu_{sat}^2}} = \boxed{0.6489}$

$\theta^{*+} = \pm \cos^{-1}\left(\frac{a^+(1-e^{+2}) - r}{r e^+}\right) = \boxed{-160.2^\circ}$ $\ominus \rightarrow V_{out} \cdot \hat{r} < 0$

- Flyby has **decreased** the specific energy. This makes sense for a flyby passing ahead. Also increased the size + eccentricity of orbit, while shifting the periapsis location forwards.

e. $\Delta \bar{V}_{eq} = \bar{V}_{out} - \bar{V}_{in}$

$$\boxed{\Delta \bar{V}_{eq} = 0.7977 \ \hat{r} - 1.3053 \ \hat{\theta} \ \ km/s}$$

$\rightarrow$ Significant savings when using a gravity assist!