ASEN 5050 – Spaceflight Dynamics

Homework #3

Assigned: Tuesday, September 19, 2023
Due: Tuesday, September 26, 2023 at 9pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Saturn} = 3.794 \times 10^7 km^3/s^2$
  - Equatorial radius of Mars: 3,397.2 km
  - Equatorial radius of Saturn: 60,268 km

Problem 1:

Recall that the Cassini mission ended with the spacecraft descending into Saturn's atmosphere. To explore the end of the mission in a manner that is solvable at this point in the semester, assume that the spacecraft does not break up during atmospheric entry – but, rather, impacts the surface of Saturn. Also assume that Saturn is a perfect sphere with a radius equal to the equatorial radius and that the Saturn-spacecraft two-body problem is a reasonable approximation of the dynamical environment.

At a time t_1, during the final orbit, the position and velocity vectors of the spacecraft are expressed as follows in a Saturn-centered inertial system $(\hat{X}, \hat{Y}, \hat{Z})$ that uses Saturn's equatorial plane to define the reference plane:

$$\bar{r}_1 = -720{,}000\hat{X} + 670{,}000\hat{Y} + 310{,}000\hat{Z} km$$
$$\bar{v}_1 = 2.160\hat{X} - 3.360\hat{Y} + 0.620\hat{Z} km/s$$

a) Calculate the position and velocity vectors of the spacecraft at impact, expressed in the Saturn-centered inertial system.
b) Discuss whether you think the dynamical environment governing the motion of the spacecraft relative to Saturn from t_1 to impact is well-approximated by the Saturn-spacecraft two-body problem.

Problem 2:

Please install either STK or GMAT on your computer following the instructions that are available on Canvas as soon as you can; alternatively, you can access STK or GMAT via the Cloud Computing site setup by the College of Engineering. I recommend that you complete the installation as soon as possible in case you have questions about STK/GMAT. It may take up to an hour to download and install STK, depending on your internet connection. If you use STK, you should be able to use either Enterprise or Premium (Space).

Answer the following questions while working through the instructions in the HW 3 Supplement Document, available on Canvas for each software package. Note that you should use the values listed in the text of the instructions (rather than the screenshots, when applicable); some of the pictures may have been created for slightly different scenarios and are used simply to help you navigate the graphical interface of each software.

Notes:

- Some of the steps, parameters and answers may be different for each type of software.
- Be sure to show your working and answers for any questions that ask you to calculate quantities

a) (ungraded, optional) Why did you choose to use either GMAT or STK?

b) Use the initial state information to calculate on your own, outside of GMAT/STK, the orbital period and periapsis altitude in the two-body problem (in a useful set of units). You will use the orbital period to govern the integration time of the scenario whereas the periapsis altitude of an orbit is often a useful quantity to check.

c) Outside of STK/GMAT, transform the orbital element description of the initial state to Cartesian position and velocity vectors in the specified Mars-centered inertial frame using the transformation matrix approach we have covered in class. Compare the values you computed to the state vector computed by STK or GMAT and, if applicable, speculate on the reason for any differences. To generate this state vector:
    - In STK, open the initial state segment in the MCS and change the "Coordinate type" to Cartesian.
    - In GMAT, open the spacecraft property window and change the "State Type" to Cartesian.

   Once you complete this problem, revert the initial state Coordinate/State type in STK/GMAT to the original Keplerian orbital elements.

d) We can use STK/GMAT to transform the initial condition to other coordinate frames. In STK/GMAT, navigate to the initial condition panel and change the coordinate system to the following:
    - In STK, use "Mars ICRF"
    - In GMAT, use the "MarsICRF" coordinate system you created

   When you change the coordinate system, STK/GMAT will automatically transform the original initial condition to this new coordinate frame; report this new set of orbital elements in the Mars-centered inertial frame that uses the axes of the ICRF. Compare the orbital elements calculated using the axes of the ICRF with the orbital elements calculated in the original Mars inertial coordinate system used in part c) [Note: GMAT and STK may produce slightly different answers to this problem due to slight differences in the inertial frames used in the instructions]. Discuss whether the differences in the orbital elements expressed in these two coordinate frames indicate any differences in the definition of the first axis and/or the reference plane spanned by the first two axes. After completing this problem and before proceeding with the rest of the homework, revert back to the original coordinate system provided in the instructions in STK/GMAT and double-check that the orbital elements of the initial state match the values specified in the instructions.

e) Include two snapshots of the 3D graphics window in your report, displaying only the "MAVEN" spacecraft orbit computed in the Mars point mass dynamical model: one looking down on the orbital plane with the specific angular momentum pointed out of the page, and the other providing a useful three-dimensional perspective.

f) Describe MAVEN's orbit when propagated in the Mars point mass dynamical model in your own words, discussing its geometry, orientation, and characteristics in as much detail as possible using precise terminology and quantities that we have discussed during the lectures.

g) How do the eccentricity and specific angular momentum evolve over time when you run the simulation for the "MAVEN" spacecraft using the Mars point mass dynamical model for 10 full orbital periods? Is this observation consistent with your expectations. In your discussion, be sure to reference any theoretical results we have covered in lectures and display any data/figures you are generating.
   - In STK, comment both on the vector directions via the 3D view and the magnitude via the time history plot.
   - In GMAT, comment on the magnitude only via the time history plot.

h) Include two snapshots of the 3D graphics window in your report, displaying only the trajectory of the "MAVEN" spacecraft, propagated in a dynamical model that reflects a higher-fidelity model of Mars' dynamical environment (in STK, incorporating atmospheric drag; in GMAT, a more complex model of the gravitational field): one looking approximately down on the orbital plane with the specific angular momentum pointed out of the page, and the other providing a useful three-dimensional perspective. Ensure that you use the correct integration time interval specified towards the end of the STK or GMAT instructions.

i) Run the simulation for the "MAVEN" spacecraft and describe its motion in your own words, discussing how the higher fidelity model of Mars' dynamical environment has impacted the geometry, orientation, and characteristics of the trajectory during the integration time interval in as much detail as possible using precise terminology and quantities that we have discussed in lectures.

j) How do the eccentricity and specific angular momentum evolve over time when you run the simulation for "MAVEN" with the higher fidelity dynamical model? Be sure to include any data/figures generated. Is this observation consistent with your expectations? When, in the orbit, do the values change most significantly and why do you think that is?
   - In STK, comment both on the vector directions via the 3D view and the magnitude via the time history plot.
   - In GMAT, comment on the magnitude only via the time history plot.

k) Use the MAVEN website (http://lasp.colorado.edu/home/maven/) to determine the activities being conducted by the MAVEN spacecraft on the epoch corresponding to the initial condition given in the problem statement. Using your research from this website, do you think that the relative two-body problem is a sufficiently good approximation of the dynamical environment to reliably design the trajectory of the MAVEN spacecraft?

Suggestion: Take some time this week to use STK/GMAT to supplement your understanding of the geometry of orbits in three-dimensional space. Try modifying the orbital elements and see how the orbit changes. Or identify a concept that you may be struggling with and try to use the software to investigate further.