ASEN 5050 – Spaceflight Dynamics

Homework #1

Assigned: Tuesday, September 5, 2023
Due: Tuesday, September 12, 2023 at 9pm MT

Notes:

- You must use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition") for full credit:
    - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
    - $Gm_{Jupiter} = 1.268 \times 10^{8} km^3/s^2$
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

The Origins, Spectral Interpretation, Resource Identification, Security-Regolith Explorer (OSIRIS-REx) spacecraft is currently on its way back to Earth to return a sample of the asteroid Bennu; the spacecraft is expected to deliver the sample return capsule to Earth on September 24, 2023. At a specific instant of time on August 30th, 2023, the state of the spacecraft in its heliocentric orbit was partially described by the following information, relative to the Sun:

$r = 1.6358 \times 10^{8}\ km \qquad \mathcal{E} = -385.4054\ km^2/s^2 \qquad v_r = -6.2627 km/s$

For this problem, let's assume that a Sun-spacecraft two-body problem is a reasonable approximation. At this instant of time:

a) Write the position and velocity vectors, $\bar{r}$ and $\bar{v}$, of the spacecraft relative to the Sun in the $\hat{r}, \hat{\theta}, \hat{h}$ axes.
b) Calculate the following information describing the orbit of the spacecraft relative to the Sun: semi-major axis a, eccentricity e, specific angular momentum h, semi-latus rectum p, true anomaly θ^*, orbit period P, type of conic

At the same instant of time, the spacecraft is also described by the following truncated position and velocity vectors measured relative to the Sun in an inertial frame with axes $\hat{X}\hat{Y}\hat{Z}$:

$$\bar{R} = 1.5043 \times 10^{8}\hat{X} - 5.5874 \times 10^{7}\hat{Y} - 3.1770 \times 10^{7}\hat{Z}\ km$$

$$\bar{V} = 5.4410\hat{X} + 2.4995 \times 10^{1}\hat{Y} + 1.4050 \times 10^{1}\hat{Z}\ km/s$$

c) Use these position and velocity vectors to calculate h, e, a. (Do not use any of the information previously calculated in parts a-b in these calculations; the goal is to independently verify this information!) Are these newly-calculated values consistent with those previously calculated or given in parts a-b?

Finally, let's consider the orbit of the spacecraft relative to the Sun:

d) Sketch a diagram of the orbit (looking down on the orbit plane so that $\hat{h}$ is directed out of the page), indicating the following information where applicable: semi-major axis, semiminor axis, *C*, *F*, *F'*, periapsis, apoapsis, semi-latus rectum, and direction of motion. Do not plot this in a mathematical software package; sketch it by hand. On this diagram, draw the position and velocity vectors for the object at the selected instant of time on this diagram, also indicating the $\hat{r}, \hat{\theta}, \hat{h}$ unit vectors, true anomaly, and flight path angle. Be sure to accurately locate the spacecraft in the correct region of its orbit relative to any known special locations along the orbit.

e) Given the location of the object in its orbit at the selected instant of time, justify in as much detail as possible whether the combination of r and θ^* indicate that you have drawn the spacecraft in the correct location relative to known special locations in the orbit in part d) (Hint: consider any upper and lower bounds on r and θ^* due to conic geometry)

f) Discuss whether you think the dynamical environment governing the motion of the OSIRIS-REx spacecraft over the next 2 months is well-approximated by the Sun-spacecraft two-body problem.

Problem 2

The Voyager 1 and 2 spacecraft have been traveling in space for over 40 years, enabling us to learn more about the outer planets before reaching interstellar space. The interplanetary trajectory of each spacecraft incorporated flybys of Jupiter and Saturn. We will cover planetary flybys in far more detail in the middle of the semester. In the meantime, note that during a planetary flyby, the spacecraft follows a hyperbolic path relative to a planet.

In this problem, let's study the hyperbolic path of Voyager 1 relative to Jupiter during its flyby of Jupiter in March 1979. This hyperbola is described by the following parameters measured relative to Jupiter:

$$v_\infty = 10.7527\ km/s \qquad \theta^*_\infty = 139.3724°$$

For this problem, let's assume that a Jupiter-spacecraft two-body problem is a reasonable approximation.

a) Calculate the values of a, e, δ for the hyperbolic trajectory followed by Voyager 1.

b) Calculate the periapsis radius r_p and speed v_p of Voyager 1 at periapsis.

c) Discuss whether you think the dynamical environment governing Voyager 1 as it travels along this hyperbola during the Jupiter flyby is well-approximated by the Jupiter-spacecraft relative two-body problem.