ASEN 5050 – Spaceflight Dynamics
Homework #9

Assigned: Tuesday, December 5, 2023
Due: Tuesday, December 12, 2023 at 9pm MT

Notes:
- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - Gravitational parameters:
    - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
    - $Gm_{Earth} = 3.986004415 \times 10^5 km^3/s^2$
    - $Gm_{Mars} = 4.305 \times 10^4 km^3/s^2$
  - Semi-major axes relative to the Sun:
    - $a_{Earth} = 1.0000010178\ AU$
    - $a_{Mars} = 1.52367934\ AU$
    - 1 AU = 149,597,870.7 km
  - Body properties:
    - Mars J_2: 0.001964
    - Mars equatorial radius: 3397.2 km
    - Mars period of rotation: 24.622962 hours
- See the syllabus for a reminder of the expected components of your working.

A large spacecraft is in a circular orbit of radius 10,000km around the Earth. This spacecraft is also carrying two CubeSats. Let's explore this scenario in Problems 1 and 2.

Problem 1:

The objective of the first CubeSat mission is to demonstrate proximity operations. Accordingly, the CubeSat must follow a trajectory that, relative to the primary spacecraft, is bounded over time.

After deployment, the CubeSat is described by an initial relative state vector $[x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]$ expressed using radial, along-track and cross-track components. However, only two components of this vector are known: $x_0 = 5\ m,\ \dot{x}_0 = 0$. In addition, the following characteristics of the CubeSat's trajectory relative to the primary spacecraft are known:
- The CubeSat and primary spacecraft remain in the same orbit plane
- The relative trajectory exhibits only bounded, oscillatory motion
- Along the relative trajectory, the maximum absolute value of the deviation in the along-track direction is 15 m.

a) Determine a combination of the remaining initial relative state components that satisfy these characteristics.
b) Draw by hand (i.e., do not use a computer to plot) the relative trajectory, representing the radial component on the vertical axis and the along-track component on the horizontal axis. Mark on the plot the initial condition, direction of motion, any extrema of x(t) and y(t), and the mean offset, if applicable, of the relative trajectory. Also locate the primary spacecraft on this plot.

Problem 2:

The goal of the second CubeSat is to achieve a precise relative position with a relative velocity of zero at a specified time. In this problem, you will design a transfer to deliver the CubeSat to this location via two impulsive maneuvers.

After deployment, at a time $t_0=0$, the relative state of the CubeSat is expressed using the radial, along-track and cross-track unit vectors as:

$$[x_0, y_0, z_0] = [2\ ,2\ ,0]m \qquad [\dot{x}_0, \dot{y}_0, \dot{z}_0] = [-0.03, +0.01, 0.05]m/s$$

The goal is for the CubeSat to reach the following relative position at a time, t_1, equal to half the orbit period of the primary spacecraft:

$$[x_1, y_1, z_1] = [-2\ ,2\ ,0]m$$

a) To design a transfer between these two relative position vectors at time t_0 and time t_1, compute the relative velocity required at t_0. Use this relative velocity and the actual velocity of the CubeSat at t_0 to calculate the magnitude of the impulsive maneuver that must be applied for the CubeSat to begin the designed transfer.
b) Compute the relative velocity of the spacecraft at the end of the transfer, i.e. at time t_1. Use this relative velocity to find the magnitude of the second impulsive maneuver that must be applied for the CubeSat to achieve a relative velocity of zero at t_1.
c) Plot the relative trajectory along the transfer, representing the radial component on the vertical axis and the along-track component on the horizontal axis. Add the direction of motion. Mark the initial and final CubeSat locations and the velocities before and after each maneuver. Qualitatively describe the motion of the CubeSat in its Earth-centered orbit along the transfer and compare it with the motion of the primary spacecraft.