ASEN 5050 – Spaceflight Dynamics

Homework #6

Assigned: Thursday, October 19, 2023
Due: Thursday, November 2, 2023 at 9pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
  - 1 AU = 149,597,870.7 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Let's design a transfer for a spacecraft to travel from the Earth to Venus. In this problem, use the geometric approach to solving Lambert's problem as presented in class – not the universal variable formulation for solving Lambert's problem that appears in the optional textbook.

At a Julian date of 2460012.5 TDB, the state of the Earth is described in a Sun-centered inertial frame by the following position and velocity vectors:

$$\bar{R}_1 = -1.451526 \times 10^8 \hat{X} + 2.872782 \times 10^7 \hat{Y} + 1.245389 \times 10^7 \hat{Z}\ km$$
$$\bar{V}_1 = -6.766088\hat{X} - 26.810318\hat{Y} - 11.620955\hat{Z}\ km/s$$

At a later Julian date of 2460146.5 TDB, the state of Venus is described in a Sun-centered inertial frame by the following position and velocity vectors:

$$\bar{R}_2 = 2.454556 \times 10^7 \hat{X} - 9.611903 \times 10^7 \hat{Y} - 4.480278 \times 10^7 \hat{Z}\ km$$
$$\bar{V}_2 = 33.882657\hat{X} + 7.872920\hat{Y} + 1.398796\hat{Z}\ km/s$$

a) Write a script to implement an iterative numerical method for the single step of solving **Lambert's equation.** The goal is to solve for the value of *a* that produces a specified value of the TOF given the information only about the space triangle, transfer angle, and gravitational parameter. This script may not include additional steps to precompute the values of quantities that do not change between iterations. Write this method only for arcs along elliptical orbits. Describe the setup of your numerical method, the stopping condition/s, and initial guess used – in your own words. Attach your code to this problem. (Hint: fsolve is a useful root-finding function in Matlab)

b) We will use Lambert's problem to calculate a transfer with a transfer angle of **less than** 180 degrees that connects these two position vectors at the specified epochs. Calculate the values of *a* and *e* along the heliocentric transfer in the Sun-spacecraft two-body problem. Except for using the script from part a) to solve **Lambert's equation**, you must show all your working by hand or typed up with mathematical notation and discuss the procedure you use to solve this problem.

c) What is the lower bound on the semi-major axis for any elliptical transfer to connect these two position vectors with any time of flight?

d) At both the beginning and end of this transfer, calculate the true anomaly and velocity vectors in the Sun-centered inertial coordinate frame. Use these velocities to calculate the maneuver magnitudes Δv_1 and Δv_2 required for the spacecraft to simply match the velocities of Earth and Venus at the beginning and end of the transfer.

Problem 2:

Let's increase the complexity of the transfer design problem to incorporate a variety of potential departure and arrival epochs around the single combination of epochs examined in Problem 1: when the epoch changes, so too do the states associated with each planetary body. We can recover a variety of transfers within a subset of the design space and visualize them on a porkchop plot. Let's also consider a slightly different scenario: trajectories that deliver a spacecraft from the Earth to Mars.

The two text files available in this module contain the heliocentric state vectors for Earth and Mars at various selected epochs in a Sun-centered inertial frame. This data is generated from the JPL HORIZONS webpage. Import this data to create matrices containing the epoch and heliocentric state components for each body. Hint: the "Import Data" feature in Matlab simplifies this process significantly!

a) Convert the entire procedure that you used to solve Problem 1 into a numerical script that you can run to produce a transfer with a transfer angle that is less than 180 degrees given the following inputs: the initial and final state vectors and the time of flight. Attach your code.
b) For every possible combination of the provided initial and final states and epochs for the Earth and Mars (where the final epoch occurs after the initial epoch, of course), use your script to compute a transfer from the Earth to Mars with a transfer angle that is less than 180 degrees. For each transfer, calculate the v-infinity magnitude at each of Earth departure and Mars arrival; discuss in your writeup how you calculated this quantity. Then, create two porkchop plots: 1) displaying the v-infinity at Earth departure and 2) displaying the v-infinity at Mars arrival. For each porkchop plot, display the epoch associated with the Earth's state vector on the horizontal axis (i.e., the departure date) and display the epoch associated with Mars' state vector on the vertical axis (i.e., the arrival date).
c) Analyze and discuss the recovered subset of the transfer design space for trajectories from the Earth to Mars across the provided range of departure and arrival dates using the porkchop plots you have constructed. How could the transfer design space be expanded further in future analysis?

Problem 3:

The goal of this problem is to learn how to create a transfer sequence in GMAT/STK and to use the software to help you calculate these quantities. Before starting this problem, **you must have completed Problem 1.**

Follow the instructions for GMAT/STK, available on the Canvas page in the HW 6 module and answer the following questions when indicated.

a) Provide a screenshot of the transfer generated in STK/GMAT, along with the orbits of Earth and Venus, in a heliocentric view looking down on the xy-plane of the Sun-centered inertial frame (with the angular momentum vector of the planets in their heliocentric orbits directed out of the page). Indicate the direction of motion along each arc using arrows added to the screenshot or state in words the direction of motion.

b) Use the summary or reporting function described in the instructions to list the state of the spacecraft in the Sun-centered inertial coordinate system after the second maneuver has been applied. Compare the state components to the $\bar{R}_2$ and $\bar{V}_2$ vectors provided in Problem 1. If there are any differences between the quantities, discuss a potential reason for this difference.

c) Provide a screenshot of the corrected transfer (i.e., computed by the targeter in GMAT/STK), along with the orbits of Earth and Venus, in a heliocentric view looking down on the ecliptic. Indicate the direction of motion along each arc using arrows. Use the summary or reporting function described in the instructions to list the state of the spacecraft in the Sun-centered inertial coordinate system after the second maneuver has been applied – does this state vector lie within the specified tolerance of the desired state vector? Also list the $\Delta\bar{v}_1$ and $\Delta\bar{v}_2$ in the Sun-centered inertial coordinate system as computed by the targeter in STK/GMAT. Compare these vectors to the quantities you reported in Problem 3a). How many iterations did the targeter require to recover this solution?