ASEN 5050 – Spaceflight Dynamics
Homework #8

Assigned: Tuesday, November 14, 2023
Due: Tuesday, November 28, 2023 at 9pm MT

Notes:

- This homework is designed to be completed in 1 week before and after Fall Break; you are not expected to work on this homework over Fall Break.
- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - Gravitational parameters:
    - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
    - $Gm_{Earth} = 3.986004415 \times 10^5 km^3/s^2$
  - Semi-major axes relative to the Sun:
    - $a_{Earth} = 1.0000010178\ AU$
    - 1 AU = 149,597,870.7 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Consider a scientific satellite orbiting an unknown planet and observed to have the following orbit characteristics: periapsis radius = 7500 km, apoapsis radius = 8500 km, inclination = 105°, and orbital period =110 minutes. The radius of the planet is 6500 km. The planet is at a constant distance of 2.25 AU from the Sun, and the satellite is in a sun-synchronous orbit.

Determine the mass (M_P) and 2nd degree zonal gravitational coefficient ($J_{2,P}$) of the planet given this information. Be sure to list all assumptions that you use in your answer.

Problem 2:

Recall that a special perturbation approach to studying the impact of perturbations focuses on generating and characterizing a single point solution. One important component of the special perturbations approach (and simply generating a trajectory in a complex dynamical model) is numerical integration. In this problem, you will explore the accuracy of numerically integrating the path of a spacecraft relative to the Earth and in the two-body problem, using the following initial position and velocity vectors at t_0:

$$\bar{R}_0 = [-6402, -1809, 1065]\ km \qquad \bar{V}_0 = [0.999, -6.471, -4.302] km/s$$

expressed in the Geocentric Celestial Reference Frame (GCRF).

a) In Matlab, write a script to numerically integrate the equations of motion for the two-body problem, using ode45, a variable time-step Runge-Kutta Method. (I strongly recommend using Matlab for this question, but if you use a coding/computational environment other than Matlab, please identify a numerical integration scheme with similar properties to ode45 and describe the scheme you used in your writeup.) In ode45, use a relative tolerance of 1×10^{-12} and an absolute tolerance of 1×10^{-12} for now (we will change these values later). Include a copy of your script in your submission. In your writeup, describe the following:

- the numerical integration scheme that ode45 implements.
- the state vector and first-order differential equations you are providing to ode45
- the definition of relative and absolute tolerances
- the approach used by the integrator to determine whether the error at each time step is acceptable using these two tolerances.

b) Using the expressions derived this semester to solve the two-body problem analytically, calculate the state vector of the spacecraft at $t_1 = t_0 + 100$ hr. At this epoch, calculate the associated values of the specific energy and the magnitude of the specific angular momentum vector.

c) Using the script you constructed in part a), generate the trajectory of the spacecraft from the provided initial state at t_0 to t_1. List the state vector of the spacecraft at the end of this time interval, as well as the associated values of the specific energy and the magnitude of the specific angular momentum vector. Compare the results of your numerical integration with the state vector calculated in part b) and discuss the reasons for any differences.

d) In this section, you will examine the impact of the tolerances on the accuracy of the integrated solution. In this problem, set the absolute and relative tolerances to identical values. The tolerance will be set to one of the following values: 1×10^{-4}, 1×10^{-6}, 1×10^{-8}, 1×10^{-10}, 1×10^{-12}. For each value, numerically integrate the trajectory of the spacecraft from the specified initial condition at t_0 to t_1 and report the state vector at the end of this time interval. Consider the "truth" value of the state vector at t_1 equal to the vector computed analytically in part b). Then, for each of the five numerical integrations performed in this question, calculate the quantities ΔR and ΔV, representing the magnitude of the difference in the position and velocity components at the end of the numerically generated trajectory and the "truth" data. Also calculate $\Delta \mathcal{E}$ and Δh, representing the difference in the specific energy and specific angular momentum between the states at time t_0 and t_1 along the numerically-integrated solution. Summarize this information in a table resembling the following:

	ΔR	ΔV	$\Delta \mathcal{E}$	Δh
Tol = 1×10^{-4}				
Tol = 1×10^{-6}				
Tol = 1×10^{-8}				
Tol = 1×10^{-10}				
Tol = 1×10^{-12}				

e) Discuss the results summarized in your table. Also discuss the tradeoff you would perform to select an appropriate tolerance when numerically generating solutions in the two-body problem via ode45.

Problem 3:

The goal of this problem is to construct scenarios using a more complex dynamical environment than in the two-body problem and study the impact of each perturbation on specific trajectories. Follow the instructions for GMAT/STK and answer the following questions when indicated.

When discussing your observations of the impact of each perturbation on the trajectory, be sure to use terminology we have covered in class throughout the semester that describes their characteristics and geometry.

a) **STK:** Open the “Earth Point Mass” propagator in the “Component Browser” and navigate to the “Numerical Integrator” tab. Report the integrator used in this propagator function as well as the absolute and relative tolerances used. Using either the summary or report function, discuss whether you think that this combination of integrator and tolerances has recovered a solution that is close to the true solution in the two-body problem. (Hint: Which quantities will you use to assess the accuracy of the solution?)
GMAT: Open the “EarthPM” propagator and navigate to the “Integrator” panel. Report the integrator used in this propagator function as well as the “accuracy” value used. Using either the summary or report function, discuss whether you think that this combination of integrator and tolerance has recovered a solution that is close to the true solution in the two-body problem. (Hint: Which quantities will you use to assess the accuracy of the solution?)
b) Describe the impact of the Earth J2 perturbation on each of the orbital elements for this trajectory, including your five plots of the orbital elements in your report. Discuss in as much detail as possible whether these time history plots are consistent with your expectations based on concepts we have covered in lectures.
c) Include both a three-dimensional plot and a groundtrack plot of the satellite motion when a J2 perturbation is included in the dynamical model. Describe the impact of the Earth’s oblateness on the entire spacecraft orbit using these plots as a reference.
d) Rerun the scenario using the “Earth Higher Order” propagator. Describe, in as much detail as possible, the impact of the higher-order gravitational model on each of the orbital elements for this trajectory, including your five plots of the orbital elements in your report. Discuss, in as much detail as possible, how the time evolution of these orbital elements differs from the time history of the orbital elements when subject only to a perturbation from J2. If you were designing a trajectory for this spacecraft, could a preliminary analysis in either the two-body problem, or the two-body problem with a J2 perturbation sufficiently predict the evolution of the orbit?
e) Describe the impact of the Moon’s third-body perturbation on each of the orbital elements, including your five plots of the orbital elements over 100 days in your writeup (you may show additional zoomed-in views if you consider them necessary). Also include a three-dimensional plot of the spacecraft motion. Discuss whether you think these observations are generalizable beyond this single trajectory.
f) Describe how adding the gravitational influence of the Sun impacts the motion of the spacecraft. Use plots and, if needed, the output of the reports/summary function to justify your answer.