ASEN 5050 Fall 2023 HW 5 Solutions

Problem 1:

Given: ∘ $a_1 = 7045$ km ∘ $e_1 = 0.23$ ∘ $\theta_1^* = -142°$
∘ $\mu_{moon} = 4902.799$ km³/s² ∘ $ER_{moon} = 1738$ km

Assume: 2BP Assumptions, $\mu_{moon} = \mu_L$

a) Calculate $\bar{V}_1$ at θ_1^* in $(\hat{r}, \hat{\theta}, \hat{h})$

$\mathcal{E}_1 = -\mu_L / 2a_1 = -0.3480$ km²/s² ← Can be skipped

$h_1 = \sqrt{\mu_L a_1 (1-e_1^2)} = 5.7195 \cdot 10^3$ km²/s

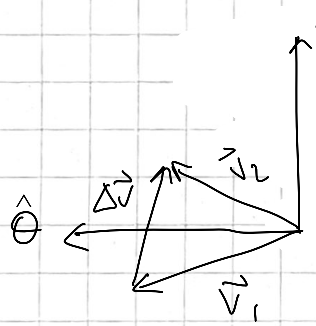
$V_r = \mu_L / h_1 (e_1 \sin(\theta_1^*)) = -0.1214$ km/s

$V_\theta = \mu_L / h_1 (1 + e_1 \cos(\theta_1^*)) = 0.7018$ km/s

$\boxed{\therefore \bar{V}_1 = -0.1214\hat{r} + 0.7018\hat{\theta} \text{ km/s}}$

b) Illustrate $\bar{V}_2$ after $\Delta\bar{V}$ is applied

$\Delta\bar{V} = 0.30\hat{r} - 0.10\hat{\theta}$ km/s



c) Calculate $\bar{V}_2$

$\bar{V}_2 = \bar{V}_1 + \Delta\bar{V}$

$\boxed{\bar{V}_2 = 0.1786\hat{r} + 0.6018\hat{\theta} \text{ km/s}}$

d) Calculate a_2, e_2 & θ_2^*

$r = (h_1^2/\mu_L) / (1 + e_1 \cos(\theta_1^*)) = 8.1493 \cdot 10^3$ km $\therefore \bar{r} = 8.1493 \cdot 10^3 \hat{r}$ km

$\bar{h}_2 = \bar{r} \times \bar{V}_2 = 4.9046 \cdot 10^3 \hat{h}$ km²/s $\therefore h_2 = 4.9046 \cdot 10^3$ km²/s

$V_2 = |\bar{V}_2| = 0.6278$ km/s

$\mathcal{E}_2 = V_2^2/2 - \mu_L / r = -0.4046$ km²/s²

$\boxed{\therefore a_2 = -\mu_L / 2\mathcal{E}_2 = 6059.4 \text{ km}}$

$\boxed{\therefore e_2 = \sqrt{1 - h_2^2/(\mu_L a_2)} = 0.4362}$

$P_2 = a_2(1-e_2^2) = 4.9064 \cdot 10^3$ km

$\therefore \theta_2^* = \pm\cos^{-1}((P_2/r - 1)/e_2)$ $\theta_2^* = [0, 180°]$ because $V_{2,r} > 0$

$\boxed{\therefore \theta_2^* = 155.82°}$

e) Calculate $\Delta\omega$

$$\Delta\omega = \omega_2 - \omega_1 = \theta_1^* - \theta_2^* = -297.82° = 62.18°$$

Problem 1 (cont.):

f) Could the s/c implement the maneuver?

$g_0 = 9.81 \cdot 10^{-3}$ km/s² ; $m_p = 156$ kg

$I_{sp} = 212$ s ; $m_i = 1224$ kg

Rearranging the Ideal Rocket Equation:

$\Delta m = m_i\,(1 - e(-\Delta V/(I_{sp}\, g_0)))$

$\boxed{\therefore \Delta m = 172.6541 \text{ kg} > m_p \quad \text{There is not enough fuel for the maneuver}}$

Problem 2:

Given: $\mu_{mars} = \mu_m = 4.305 \cdot 10^4$ km³/s² ; $r_1 = 6500$ km, $E_1 = \pi/2$ rad, $r_{p1} = 5915$ km

$r_{p2} = 5712$ km, $r_{a2} = 7888$ km (* s/c $0° < \theta_2^* < 180°$)

a) Calculate $\bar{v}_1$ in $(\hat{r}, \hat{\theta}, \hat{h})$

At $E_1 = \pi/2$ the s/c is at the semi-minor axis b

$\therefore a_1 = r_1 = 6500$ km

$r_{a1} = 2a_1 - r_{p1} = 7085$ km

$e_1 = (r_{a1} - r_{p1})/(r_{a1} + r_{p1}) = 0.09$

$\theta_1^* = 2\tan^{-1}(\tan(E_1/2)\sqrt{(1+e_1)/(1-e_1)}) = 95.16°$

$h_1 = \sqrt{\mu_m a_1 (1-e_1^2)} = 16{,}660$ km²/s

$V_r = (\mu_m/h_1)(e_1 \sin(\theta_1^*)) =$

$V_\theta = (\mu_m/h_1)(1 + e_1\cos(\theta_1^*))$

$\boxed{\therefore \bar{V}_1 = 0.2316\,\hat{r} + 2.5631\,\hat{\theta} \text{ km/s}}$

b) Calculate $|\Delta \bar{v}|$

$r_2 = r_1 = r = 6500$ km

$e_2 = (r_{a2} - r_{p2})/(r_{a2} + r_{p2}) = 0.16$

$a_2 = (r_{a2} + r_{p2})/2 = 6800$ km

$h_2 = \sqrt{a_2(1-e_2^2)\mu_m} = \sqrt{p_2 \mu_m} = 16{,}889$ km²/s

$\theta_2^* = \pm\cos^{-1}(a_2(1-e_2^2)/(r_2 e_2) - 1/e_2)$

$\Rightarrow \theta_2^* = 1.4494$ rad $= 83.0457°$ *s/c is moving from periapsis to apoapsis $\therefore 0° < \theta_2^* < 180°$)

$V_r = (\mu_m/h_2)(e_2 \sin(\theta_2^*)) = 0.4048$ km/s

$V_\theta = (\mu_m/h_2)(1 + \cos(\theta_1^*)) = 2.5983$ km/s

$\bar{V}_2 = 0.4048\,\hat{r} + 2.5983\,\hat{\theta}$ km/s

$\Delta\bar{V} = \bar{V}_2 - \bar{V}_1 = 0.1732\,\hat{r} + 0.0353\,\hat{\theta}$ km/s

$\boxed{\therefore |\Delta \bar{v}| = 0.1768 \text{ km/s}}$

PROBLEM 3

given: $a_{\oplus} = 1.00000010178$ AU $\quad$ 1 AU = 149597870.7 km

$a_{sat} = 9.554909595$ AU $\quad$ $\mu_{sun} = 1.32712428 \times 10^{11}$ km^3/s^2

Part a)

$$a_t = \frac{1}{2}(r_{p,t} + r_{a,t}) = \frac{1}{2}(a_i + a_f) = 7.8950 \times 10^8 km$$

$$\Delta\vec{v}_1 = \left(\sqrt{\frac{2\mu_\odot}{a_i} - \frac{\mu_\odot}{a_t}} - \sqrt{\frac{\mu_\odot}{a_i}}\right)\hat{\theta} = 10.2922\hat{\theta} \text{ km/s}$$

$$\Delta\vec{v}_2 = \left(\sqrt{\frac{\mu_\odot}{a_f}} - \sqrt{\frac{2\mu_\odot}{a_f} - \frac{\mu_\odot}{a_t}}\right)\hat{\theta} = 5.4412\hat{\theta} \text{ km/s}$$

$$\Delta v_{total} = |\Delta\vec{v}_1| + |\Delta\vec{v}_2| = 15.7335 \text{ km/s}$$

$$TOF = \frac{P_t}{2} = \pi\sqrt{\frac{a_t^3}{\mu}} = 1.913 \times 10^8 s = 6.0661 \text{ yrs}$$

Part b)

Saturn traces out angle α during this TOF: $\alpha = \sqrt{\frac{\mu}{a_{sat}^3}} \cdot TOF$

The relative initial phase angle is $\phi = \pi - \alpha$

$\alpha = 1.2896$ rad

$$\phi = \pi - \alpha = 1.852 \text{ rad} = 106°$$

part c)

$r_B = 11$ AU

$a_{t1} = \frac{1}{2}(a_{earth} + r_B) = 8.9759 \times 10^8$ km

$\Delta\vec{V}_1 = \left(\sqrt{\frac{2\mu_\odot}{a_\oplus} - \frac{\mu_\odot}{a_{t1}}} - \sqrt{\frac{\mu_\odot}{a_\oplus}}\right)\hat{\theta} = 10.544\,\hat{\theta}$ km/s

$a_{t2} = \frac{1}{2}(r_B + a_{sat}) = 1.5375 \times 10^9$ km

$\Delta\vec{V}_2 = \left(\sqrt{\frac{2\mu_\odot}{r_B} - \frac{\mu_\odot}{a_{t2}}} - \sqrt{\frac{2\mu_\odot}{r_B} - \frac{\mu_\odot}{a_{t1}}}\right)\hat{\theta} = 4.99278\,\hat{\theta}$ km/s

$\Delta\vec{V}_3 = \left(\sqrt{\frac{\mu_\odot}{a_{sat}}} - \sqrt{\frac{2\mu_\odot}{a_{sat}} - \frac{\mu_\odot}{a_{t2}}}\right)\hat{\theta} = -0.333\,\hat{\theta}$ km/s

$\Delta V_{total} = |\Delta\vec{V}_1| + |\Delta\vec{V}_2| + |\Delta\vec{V}_3| = 15.8697$ km/s

$TOF = \frac{P_1}{2} + \frac{P_2}{2} = \pi\sqrt{\frac{a_{t1}^3}{\mu}} + \pi\sqrt{\frac{a_{t2}^3}{\mu}} = 23.839$ yrs.

part d)

	Hohmann	Bielliptic
ΔV	15.73 km/s	15.9 km/s
TOF	6 yrs	24 yrs

this is consistent w/ expectations that the Bi-elliptic transfer is less efficient in this case. This can be proved by calculating $a_f/a_i = 9.5549$, which by using the graph by Curtis in the lecture notes, places the transfer in the region where ΔV_{tot} is lowest for a Hohmann transfer

part e)

using data from the Cassini mission and the ideal rocket eqn, or other propulsion systems + mass combinations we can calculate the mass ratio.

$$\frac{m_i - m_f}{m_i} = 1 - e^{-\Delta V / I_{sp} \cdot g_0}$$

The larger this mass ratio, the more propellant mass required.

For the spacecraft and propulsion system you selected, what mass ratio did you calculate? And does this seem to comprise a large fraction of the initial spacecraft mass?