ASEN 5050 – Spaceflight Dynamics

Homework #5

Assigned: Tuesday, October 10, 2023
Due: Thursday, October 19, 2023 at 9pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
  - $Gm_{Mars} = 4.305 \times 10^4 km^3/s^2$
  - $Gm_{Moon} = 4902.799 km^3/s^2$
  - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
  - $Gm_{Saturn} = 3.794 \times 10^7 km^3/s^2$
  - Equatorial radius of Mars: 3397.2 km
  - Equatorial radius of the Moon: 1738 km
  - Semi-major axis of Earth's heliocentric orbit: 1.0000010178 AU
  - Semi-major axis of Saturn's heliocentric orbit: 9.554909595 AU
  - 1 AU = 149,597,870.7 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

A spacecraft is currently in a lunar orbit described by the following orbital elements:

a_1 = 7,045 km $\qquad$ e_1 = 0.23

At a true anomaly of -142 degrees, an impulsive maneuver is applied. Assume that this maneuver does not change the orientation of the orbit plane.

a) Calculate the velocity vector $\bar{v}_1$ at the specified true anomaly immediately before the maneuver and express it in the $(\hat{r}, \hat{\theta}, \hat{h})$ axes.
b) A maneuver, $\Delta\bar{v} = 0.30\hat{r} - 0.10\hat{\theta}\ km/s$, is then applied. Draw the velocity vector $\bar{v}_1$ immediately before the maneuver, the $\Delta\bar{v}$ vector, and the velocity vector $\bar{v}_2$ immediately after the maneuver. Also add the $\hat{r}$ and $\hat{\theta}$ unit vectors to this diagram. Do not add the orbit or any additional information to this diagram.
c) Calculate the velocity vector $\bar{v}_2$ immediately after the maneuver and express it in the $(\hat{r}, \hat{\theta}, \hat{h})$ axes.
d) After the maneuver, calculate the orbital elements a_2 and e_2 of the new orbit. Also calculate the true anomaly along the new orbit, immediately after the maneuver.
e) Calculate the change in the argument of periapsis due to the maneuver.
f) Consider the LRO spacecraft. Consistent with the example covered in class, assume that this spacecraft possesses a mass of 1,224 kg. At this time, the LRO spacecraft only has 156 kg of propellant mass remaining. Also assume that the propulsion system is characterized by a specific impulse of 212 s.* Could the LRO spacecraft, when described by these parameters, implement the maneuver specified in part b)?

* Quantities derived from the following source: Houghton, M.B., Tooley, C.R., Saylor, R.S., 2007, "Mission Design and Operations Considerations for NASA's Lunar Reconnaissance Orbiter", IAC-07-C1.7.06

Problem 2:

A spacecraft is currently in orbit around Mars and must adjust the size and shape of its orbit for an upcoming mission extension. To change its state and orbit, the spacecraft will apply a single impulsive maneuver $\Delta\bar{v}$.

Immediately prior to the maneuver, the following information about the location of the spacecraft and the size and shape of Orbit 1 is known:

At t_1: $r_1 = 6{,}500\ km$ and $E_1 = \frac{\pi}{2}$ rad Orbit information: $r_{p,1} = 5{,}915\ km$

Immediately after the maneuver, Orbit 2 is described by the following information:

$r_{p,2} = 5{,}712\ km$ $r_{a,2} = 7{,}888\ km$

and the spacecraft will be moving from periapsis to apoapsis.

Assume that the maneuver does not change the orientation of the orbit plane.

a) Calculate the velocity vector $\bar{v}_1$ of the spacecraft in the $(\hat{r}, \hat{\theta}, \hat{h})$ axes immediately before the maneuver.
b) Calculate the magnitude of the planned impulsive maneuver $\Delta\bar{v}$ that adjusts the spacecraft state and orbit as described in this problem.

Problem 3:

a) Calculate the total Δv (to five significant figures) and time of flight required for a spacecraft to complete a Hohmann transfer in the Sun-spacecraft two-body problem between an approximate circular orbit for Earth and an approximate circular orbit for Saturn. Assume the semi-major axes of each orbit is equal to the value provided.
b) Calculate the initial relative phase angle (between the spacecraft and Saturn) that is required for the spacecraft to rendezvous with Saturn after completing the Hohmann transfer.
c) Assuming an intermediate radius (r_B) of 11 AU, calculate the total Δv (to five significant figures) and time of flight required for a bi-elliptic transfer between Earth's and Saturn's assumed circular orbits.
d) Compare the total Δv and time of flight of this bi-elliptic transfer with the total Δv and time of flight of the Hohmann transfer calculated in part a). Justify why this comparison is consistent with your expectations based on the concepts you have learned in class.
e) Do you think the total Δv values you computed in 3a) and 3c) are large for a spacecraft to implement? Justify.

Problem 4:

The goal of this short problem is to learn a new feature of the modeling software we use: modifying stopping conditions for numerical integration, enabling customization of the mission scenario. We will find this particularly useful when we create complex transfers next time!

Follow the instructions for GMAT/STK, available on the Canvas page in the HW 5 module and answer the following questions when indicated.

a) At the initial state, use the reported values to record the magnitude of the specific angular momentum, specific energy, and eccentric anomaly. Also record the corresponding date in UTC modified Julian date format (recall that the modified Julian date represents the days past a reference epoch) from the initial condition input panel. Use the initial conditions provided in the instructions to calculate on your own (outside of GMAT/STK) the values of $h, \varepsilon, E(t_0)$. Compare your calculated values to those reported by GMAT/STK.

b) Propagate the specified initial condition forward in time in GMAT/STK using the constructed dynamical model until the spacecraft reaches an altitude of 500 km. At the end of the trajectory, use the report function to list the true anomaly, eccentric anomaly, and altitude (to check if you correctly implemented the stopping condition). Use the epoch at this altitude and the initial epoch to straightforwardly calculate the time of flight along the propagated trajectory segment.

c) Use the epochs reported by GMAT/STK before and after the second propagate segment to calculate the time the spacecraft spends in its orbit below an altitude of 500 km. Then, perform the following calculation on your own outside of GMAT/STK: use the eccentric anomaly reported by GMAT/STK at the 500 km altitude from part b), as well as the orbital elements, to verify this flight time using Kepler's equation. Compare the results generated with each of these two approaches.

d) Take two snapshots of your orbit in three-dimensions: one looking down on the orbital plane, and one different but useful view. Also take a snapshot of the two-dimensional view of the ground-track of the spacecraft (you may need to update the orbit colors for clarity in this 2D view). The ground-track essentially describes the projection of the spacecraft's location onto the surface and we will learn more about this concept very soon. Go to https://trek.nasa.gov/mars (which functions a lot like Google Maps for Mars and locates named regions on Mars' surface). Assume that one of the sensors onboard the spacecraft can only take observational data at altitudes below 500 km. Refer to the orbit you modeled in GMAT or STK – during the segment of the orbit that the spacecraft remains below an altitude of 500km, visually identify a region of the Mars surface that the spacecraft may be able to observe. Report the name of one of these regions.