ASEN 5050 – Spaceflight Dynamics
Homework #7

Assigned: Thursday, November 2, 2023
Due: Tuesday, November 14, 2023 at 9pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
    - Gravitational parameters:
        - $Gm_{Sun} = 1.32712428 \times 10^{11} km^3/s^2$
        - $Gm_{Earth} = 3.986004415 \times 10^{5} km^3/s^2$
        - $Gm_{Saturn} = 3.794 \times 10^{7} km^3/s^2$
        - $Gm_{Jupiter} = 1.268 \times 10^{8} km^3/s^2$
    - Semi-major axes relative to the Sun:
        - $a_{Earth} = 1.0000010178\ AU$
        - $a_{Saturn} = 9.554909595\ AU$
        - $a_{Jupiter} = 5.202603191\ AU$
        - 1 AU = 149,597,870.7 km
    - Radius of celestial bodies:
        - $r_{Saturn} = 60{,}268\ km$
        - $r_{Jupiter} = 71{,}492\ km$
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Consider a mission to Jupiter with the goal of placing a spacecraft into an orbit about Jupiter for long-term observations of its surface and environment.

a) Let's consider a Hohmann transfer from the Earth to Jupiter, first assuming the orbits of both bodies are circular and coplanar and modeling only solar gravity. Calculate the semi-major axis and eccentricity of the transfer as well as the required time of flight.

Consider a scenario where the spacecraft propulsion system malfunctions during the heliocentric transfer and a Jupiter orbit insertion maneuver does not occur. In this case, the spacecraft performs a Jupiter flyby. During the flyby, the closest approach distance is 1,000,000 km from the center of Jupiter and the spacecraft passes on the 'Sun-side' of Jupiter.

b) Incorporate the gravity field of Jupiter to calculate the turning angle along the hyperbolic orbit during the arrival segment and the heliocentric velocity magnitude (v_{out}) after the flyby. Did the flyby increase or decrease the spacecraft energy in this case?
c) Draw a vector diagram with only the following vectors: heliocentric spacecraft velocities before and after the flyby, Jupiter's velocity vector, the velocity vectors of the spacecraft before and after the flyby relative to Jupiter, and the $(\hat{r}, \hat{\theta})$ axes. In a second diagram, draw the hyperbolic arc during the flyby in a Jupiter-centered view, including: incoming and outgoing $\bar{v}_\infty$ vectors, r_p, the planet's velocity vector, and the turning angle. Note: even if you use these diagrams to solve the previous sub-problem, please draw and submit them in this sub-problem.

Problem 2:

Consider a spacecraft in a large orbit around Saturn, described by a periapsis radius of 600,000 km and an apoapsis radius of 1,800,000 km – and lying in the same orbit plane as Titan. Assume that Titan is in a circular orbit of radius 1,221,830 km, possesses a mass of 1.3455x10^{23} kg and is modeled as a sphere with a radius equal to the equatorial radius, 2,575 km.

a) As the spacecraft travels from apoapsis to periapsis in its orbit, what is the value of the true anomaly when it intersects Titan's orbit? At this value of the true anomaly, write the velocity vector, $\bar{v}_{in}$, relative to Saturn in the $(\hat{r}\hat{\theta}\hat{h})$ axes. Also calculate the velocity vector of the spacecraft relative to Titan, i.e., $\bar{v}_{\infty,in}$. Draw a vector diagram of these two velocity vectors along with Titan's velocity vector and the $(\hat{r}\hat{\theta})$ axes.
b) If the spacecraft performs a flyby of Titan with a closest approach distance of 3,000 km, and passes ahead of Titan, calculate the turning angle. Draw a diagram of the hyperbolic orbit in a Titan-centered view, including: incoming and outgoing $\bar{v}_{\infty,in}$ vectors, r_p, Titan's velocity vector, and the turning angle.
c) Add the turning angle and post-flyby relative velocity vector to your diagram in part a). Use the vector diagram to calculate the velocity vector, $\bar{v}_{out}$, relative to Saturn in the $(\hat{r}\hat{\theta}\hat{h})$ axes.
d) Determine the orbital elements $(a_{out},\ e_{out}, \theta^*_{out})$ of the Saturn-centered orbit after the flyby and describe the impact of the gravity assist in your own words. Did the flyby increase or decrease the spacecraft energy?
e) Calculate the equivalent impulsive maneuver, $\Delta\bar{v}_{eq}$, that would be required to change the Saturn-centered velocity of the spacecraft from $\bar{v}_{in}$ to $\bar{v}_{out}$ if a gravity assist was not used.

Problem 3:

The goal of this problem is to learn how to model a trajectory in STK/GMAT using different dynamical models along different segments. Follow the instructions for GMAT/STK available on the Canvas page in the HW 7 assignment and answer the following questions when indicated.

a) Take a screenshot of the MCS (STK) or mission sequence (GMAT). Also take two clear screenshots of the trajectory: one that is a heliocentric view; and one Jupiter-centered view displaying the Jupiter flyby from above the orbit plane. Describe, in your own words, the characteristics of the trajectory in each of the heliocentric view and the Jupiter-centered view of the flyby; in this description, use accurate terminology and concepts that we have covered in class throughout the entire semester. Note: we will examine the Saturn-centered hyperbola in a later question.
b) Instructions based on software used:
    i. STK: Use the summary function to report the periapsis altitude for the hyperbolic arc describing the Jupiter flyby, using the "Prop to Jupiter Periapsis" segment and listed as the "Rad. Peri" item. In addition, list the epoch in Gregorian TDB date format at periapsis, the inclination, and the v-infinity along the hyperbola (listed as "Excess Vel" in the summary) all relative to Jupiter and in the Jupiter inertial frame. Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola as well as the turning angle.

ii. GMAT: Use the report to list the periapsis altitude for the hyperbolic arc describing the Jupiter flyby. In addition, list the epoch in Gregorian TDB date format at periapsis, the inclination, and the v-infinity along the hyperbola (compute this from the Incoming C3) all relative to Jupiter and in the Jupiter inertial frame. Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola as well as the turning angle.

c) Instructions based on software used:

   i. STK: Use the summary function to report the periapsis altitude for the hyperbolic arc describing the Saturn approach, using the "Prop to Saturn Periapsis" segment. In addition, list the epoch in Gregorian TDB date format at periapsis, the inclination, and the v-infinity along the hyperbola (listed as "Excess Vel" in the summary) all relative to Saturn and in the Saturn inertial frame. Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola as well as the turning angle.

   ii. GMAT: Use the report to determine the periapsis altitude for the hyperbolic arc describing the Saturn approach arc. In addition, list the epoch in Gregorian TDB date format at periapsis, the inclination, and the v-infinity along the hyperbola (compute this from the Incoming C3) all relative to Saturn and in the Saturn inertial frame. Use these quantities to calculate on your own the semi-major axis and eccentricity of the hyperbola as well as the turning angle.

d) The true Cassini trajectory reached periapsis at Saturn on July 1, 2004 with a radius of 81,000 km. Compare this information to your answer in part c), and speculate why the simulation you constructed produces the observed differences in the trajectory at its Saturn arrival.

e) After running the targeter, list the new values of the components of the initial state vector that achieve the goal of a periapsis radius relative to Saturn that is equal to 81,000 km. How many iterations did the targeter require to reach this solution? How much did the position and velocity vectors change to achieve the prescribed change in the periapsis radius at Saturn? Use the summary (STK) or report (GMAT) function to list the periapsis radius of the Saturn approach arc and verify that it is, indeed, equal to 81,000 km. Include a screenshot of the heliocentric view of the complete trajectory, as well as the Saturn-centered view of the Saturn approach arc. Why do you think such a small change in the initial state achieved such a significant change in the periapsis radius at Saturn?