ASEN 5050 – Spaceflight Dynamics

Homework #4

Assigned: Tuesday, September 26, 2023
Due: Tuesday, October 3, 2023 at 9pm MT

Notes:

- Use the following planetary constants (from Vallado, D., 2013, "Fundamentals of Astrodynamics and Applications, 4th Edition"):
    - $Gm_{Saturn} = 3.794 \times 10^{7} km^{3}/s^{2}$
    - $Gm_{Jupiter} = 1.268 \times 10^{8} km^{3}/s^{2}$
    - Equatorial radius of Saturn: 60,268 km
    - Equatorial radius of Jupiter: 71,492 km
- See the syllabus for a reminder of the expected components of your working.

Problem 1:

Recall Problem 1 of HW 3, where you studied the conditions associated with impact for the Cassini spacecraft in a simplified model. Let's extend this analysis to explore the time to impact.

Recall that at a time t_1, during the final orbit, the position and velocity vectors of the spacecraft are expressed as follows in a Saturn-centered inertial frame with axes $\hat{X}\hat{Y}\hat{Z}$ using Saturn's equatorial plane to define the $\hat{X}\hat{Y}$-plane:

$$\bar{r}_1 = -720{,}000\hat{X} + 670{,}000\hat{Y} + 310{,}000\hat{Z} km$$
$$\bar{v}_1 = 2.160\hat{X} - 3.360\hat{Y} + 0.620\hat{Z} km/s$$

Calculate t_i-t_1, the duration of time that has elapsed between t_1 and the instant t_i at which impact occurs. For this problem, assume that the motion of the spacecraft is governed by the two-body problem. Note: You may reference information you computed in HW 3 without repeating those specific calculations for the orbital elements and the state vector at impact, but you may want to refer to the solutions (once available) to confirm their accuracy.

Problem 2:

The Juno spacecraft launched in 2011 and reached the Jovian system in 2016. At that time, it began its primary mission of supporting new scientific insights into Jupiter, its atmosphere, and its magnetosphere; this primary mission continued until August 2021 when the spacecraft began its extended mission.

At a time t_1 on the date that this homework is assigned, the spacecraft is described by the following state information, measured relative to Jupiter and in an inertial frame with axes $\hat{X}\hat{Y}\hat{Z}$ that use Jupiter's equatorial plane to define the $\hat{X}\hat{Y}$plane:

$$\bar{R}_1 = 1.89737 \times 10^{6}\hat{X} - 4.08985 \times 10^{6}\hat{Y} - 4.59509 \times 10^{6}\hat{Z} km$$
$$\bar{V}_1 = 0.133694\hat{X} - 0.546336\hat{Y} + 0.369530\hat{Z} km/s$$

Assume that the Jupiter-spacecraft two-body problem is a sufficient approximation of the dynamical environment governing the spacecraft.

a) Calculate the true anomaly θ_1^* and eccentric anomaly E_1 of the spacecraft in its current orbit at t_1.
b) Consider a later instant of time t_2 that occurs the first time that the spacecraft crosses Jupiter's equatorial plane after t_1. Calculate the true anomaly θ_2^* and eccentric anomaly E_2 of the spacecraft in its current orbit at t_2. (Hint: a conceptual diagram looking down on the orbit plane may be quite helpful here!)
c) Calculate the time that has elapsed from t_1 to t_2, i.e., t_2-t_1. Report this quantity in days.
d) Implement an iterative numerical procedure to solve Kepler's equation for the eccentric anomaly, given the following quantities: a time past periapsis, semi-major axis, eccentricity, and system gravitational parameter. Discuss the method you used, noting: 1) the equation used to update the eccentric anomaly at each iteration, 2) the approach you will use to generate an initial guess for the eccentric anomaly, 3) any stopping conditions, and 4) any tolerances. In addition to this discussion, include your code in your submission for this part of the homework.
e) At a time t_3 that occurs 25 days after t_1 in the current orbit, calculate the eccentric anomaly of the Juno spacecraft (accurate to four decimal places). At t_3, what is the <u>altitude</u> of the Juno spacecraft relative to Jupiter and is Juno moving towards or away from periapsis?