Contents

Homework 1 StatOD

```
addpath('C:\Users\user\OneDrive - UCB-O365\Desktop\Spring 2025\StatOD\repo');
```

Problem 1

```
% part a
syms x y z a mu J2 J3

r = sqrt(x^2 + y^2 + z^2);

U = mu/r - mu*a^2*J2*(3*z^2-r^2)/(2*r^5) - mu*a^3*J3*(5*z^3-3*z*r^2)/(2*r^7);

% take partial of U to get acceleration each component
accelxyz = jacobian(U, [x y z]);

ax_latex = latex(accelxyz(1));
ay_latex = latex(accelxyz(2));
az_latex = latex(accelxyz(3));

% take the partial of the acceleration to build up STM
syms x y z vx vy vz mu J2 J3
stateVec = [x y z vx vy vz mu J2 J3];
Aaccel = jacobian(accelxyz, stateVec);


% final A matrix
% Amat = [zeros(3,3), ones(3,3), zeros(3,3); ...
%     double(subs(Aaccel,stateVec,state)) ; ...
%     zeros(3,9)];


% part c
state = [-0.64901376519124, 1.18116604196553, -0.75845329728369, -1.10961303850152, -0.84555124000780, -0.57266486645795, -0.55868076447397, 0.17838022584977, -0.19
[Amat] = Utility.DynamicsA_J2_J3(state);

A = load('CanvasAproblem1.mat');

Adiff = struct2array(A) - Amat
```

```
Adiff =

  Columns 1 through 3

                       0                         0                         0
                       0                         0                         0
                       0                         0                         0
      1.2516975402832e-05      2.6702880859375e-05     -6.96182250976563e-05
      2.6702880859375e-05     -2.38418579101563e-05     0.000131607055664063
    -6.96182250976563e-05     0.000131607055664063      1.62124633789063e-05
                       0                         0                         0
                       0                         0                         0
                       0                         0                         0

  Columns 4 through 6

                       0                         0                         0
                       0                         0                         0
                       0                         0                         0
                       0                         0                         0
                       0                         0                         0
                       0                         0                         0
```

```
                     0                         0                         0
                     0                         0                         0
                     0                         0                         0

  Columns 7 through 9

                     0                         0                         0
                     0                         0                         0
                     0                         0                         0
    2.24113464355469e-05      1.80443748831749e-09     -7.05718994140625e-05
   -4.29153442382813e-05     -3.49245965480804e-09        0.0001220703125
    5.81741333007813e-05       1.9557774066925e-08      -5.340576171875e-05
                     0                         0                         0
                     0                         0                         0
                     0                         0                         0
```

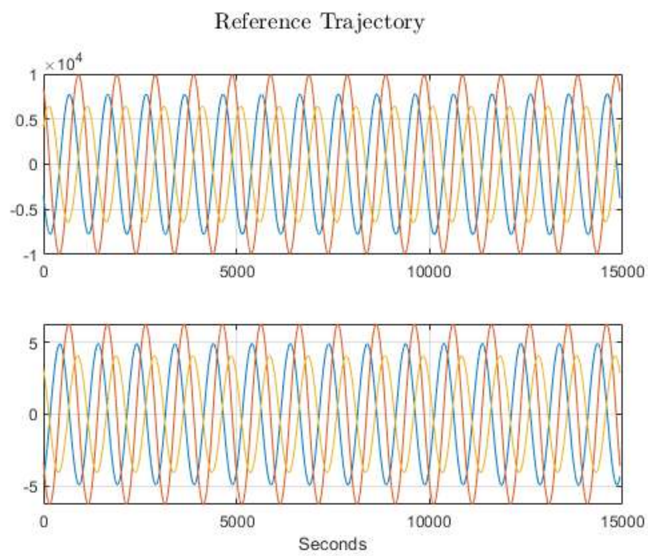
Problem 2 - State Transition Matrix

```
SMA   = 10000;  % km
eccen = 0.001;
inc   = 40; % deg
RAAN  = 80; % deg
AOP   = 40; % deg
TA0   = 0; % deg

% Convert from Orbital Elemenets to Cartesian to get initial state vector
[r0,v0] = Utility.OrbCart(SMA,eccen,inc,RAAN,AOP,TA0,Const.OrbConst.muEarth);

% Period of orbit
period = 2*pi*sqrt(SMA^3/Const.OrbConst.muEarth);

% propagate for 15 orbits
t = 0:10:15*period;

% initial state vector
Y0 = [r0;v0];

J2 = 0.00108248;
Re = Const.OrbConst.EarthRadius;
```

```
Tpqw_ijk =

        -0.351900933636988        -0.689527809386471         0.633022221559489
         0.839911542566906        -0.531121287922501         -0.11161889704895
         0.413175911166535         0.492403876506104         0.766044443118978


r =

         -3515.49032703351
          8390.71631024339
          4127.62735255368


v =

         -4.35767632217815
         -3.35657913876455
           3.1118929278699
```

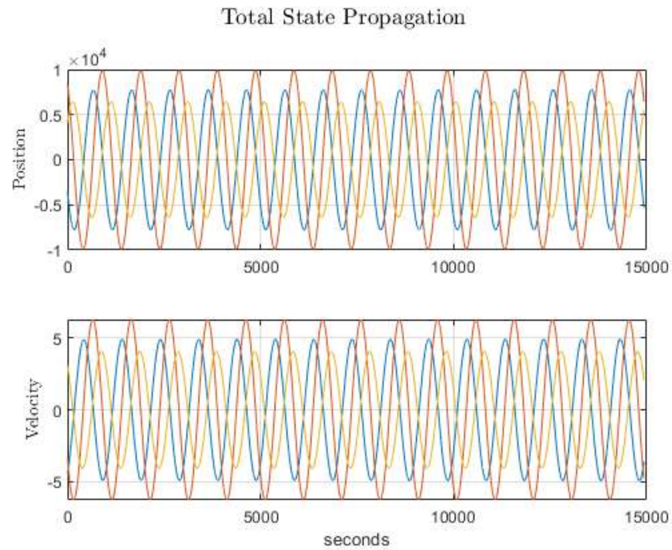
ode45 to propagation of reference - 15 orbits

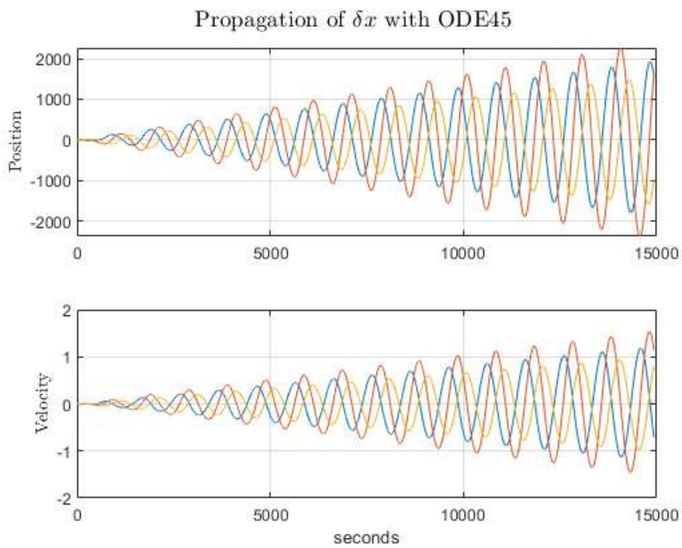
```
odeoptions = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
[T,Y] = ode45(@Utility.NumericJ2Prop, t, Y0, odeoptions, Const.OrbConst.muEarth, J2, Re);

% reference trajectory
refPos = Y(:,1:3);
refVel = Y(:,4:6);

% plotting trajectory
fig = 1;
figure(fig)
subplot(2,1,1)
plot(refPos)
grid on

subplot(2,1,2)
plot(refVel)
grid on
xlabel('Seconds')

sgtitle('Reference Trajectory', 'Interpreter', 'latex')

fig = fig + 1;
```



Integrate second trajectory by perturbing initial state

```
pert0 = [1;0;0;0;.01;0];

% perturbed initial state
Y0pert = Y0 + pert0;

% use ode45 to propagate for 15 orbits
odeoptions = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
[T,Ypert] = ode45(@Utility.NumericJ2Prop, t, Y0pert, odeoptions, Const.OrbConst.muEarth, J2, Re);

% reference trajectory
pertPos = Ypert(:,1:3);
pertVel = Ypert(:,4:6);

% Plotting Trajectory
figure(fig)
subplot(2,1,1)
plot(pertPos)
grid on
ylabel('Position', 'Interpreter', 'latex')

subplot(2,1,2)
plot(pertVel)
grid on
xlabel('seconds')
ylabel('Velocity', 'Interpreter', 'latex')

sgtitle('Total State Propagation', 'Interpreter', 'latex')

fig = fig + 1;

% compare reference trajecory with perturbed trajectory
trajDiff = Y - Ypert;

% difference in position
posDiff = trajDiff(:,1:3);

% difference in velocity
velDiff = trajDiff(:,4:6);

% plot the difference
figure(fig)
subplot(2,1,1)
plot(posDiff)
grid on
ylabel('Position', 'Interpreter', 'latex')

subplot(2,1,2)
plot(velDiff)
grid on
xlabel('seconds')
ylabel('Velocity', 'Interpreter', 'latex')


sgtitle('Propagation of $\delta x$ with ODE45', 'Interpreter', 'latex')

fig = fig + 1;
```



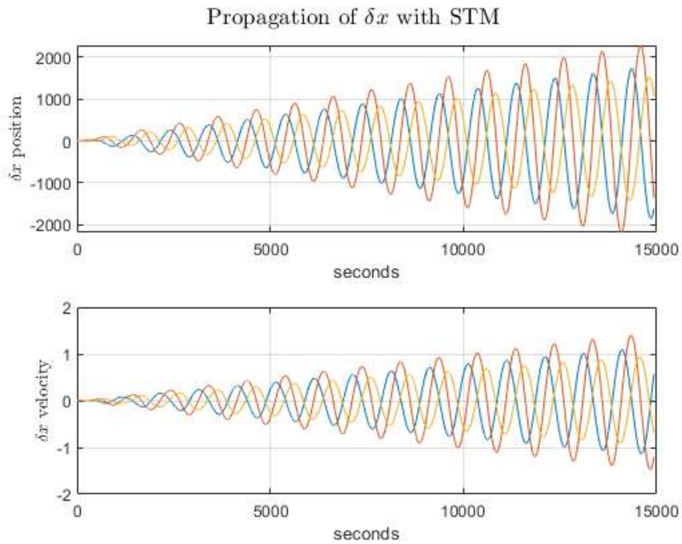


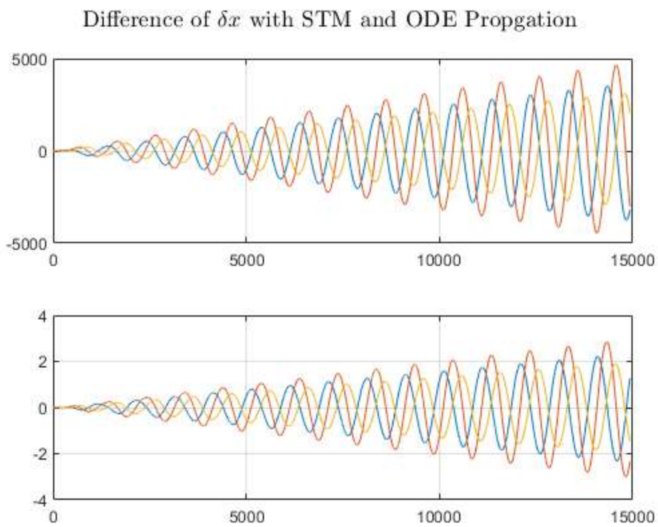
Propagating state using STM

```
syms x y z vx vy vz a J2 mu

r = sqrt(x^2 + y^2 + z^2);

 mu = Const.OrbConst.muEarth;

 J2 = 0.00108248;

a = 6378;

U = mu/r - mu*a^2*J2*(3*z^2-r^2)/(2*r^5);

% take partial of U to get acceleration each component
accelxyz = jacobian(U, [x y z]);

stateVector = [x, y, z, vx, vy, vz];

accelWRTState = jacobian(accelxyz, stateVector);

simplify(accelWRTState)

% create a funciton handle for the Jacobian to be evaulted later
% numerically
A_func = matlabFunction(accelWRTState, 'Vars', {x, y, z, vx, vy, vz});

deltaT = 10; % seconds

% set pert0 as the first deltaX
deltaX_old = pert0;

% pre-allocate deltaX
deltaX = zeros(6, length(t));

for i = 1:length(t)
```

```
    % Update A based on the current state reference trajectory
    currState = Y(i,:);

    A = [zeros(3,3), eye(3,3); ...
    double(A_func(currState(1), currState(2), currState(3), currState(4), currState(5), currState(6)))];

    % propagate the delta forward in time
    deltaX(:,i) = expm(A * deltaT) * deltaX_old;

    deltaX_old = deltaX(:,i);

end

% plot results for perturbation propagation with STM
figure(fig)

subplot(2,1,1)
plot(deltaX(1:3,:)')
grid on
xlabel('seconds')
ylabel('$\delta x$ position', 'Interpreter', 'latex')

subplot(2,1,2)
plot(deltaX(4:6,:)')
grid on
xlabel('seconds')
ylabel('$\delta x$ velocity', 'Interpreter', 'latex')

sgtitle('Propagation of $\delta x$ with STM', 'Interpreter', 'latex')
fig = fig + 1;

% --- Validity of using STM to propagate
% veloDiff and posDiff are the perturbations from ODE45

posODEdiffSTM = deltaX(1:3,:)' - posDiff;
velODEdiffSTM = deltaX(4:6,:)' - velDiff;

figure(fig)
subplot(2,1,1)
plot(posODEdiffSTM)
grid on

subplot(2,1,2)
plot(velODEdiffSTM)
grid on

sgtitle('Difference of $\delta x$ with STM and ODE Propgation', 'Interpreter', 'latex')

fig = fig + 1;
```

```
ans =

[(13695806883309290*x^6 + 20543710324963935*x^4*y^2 + 20543710324963935*x^4*z^2 + 1809246700193654636544*x^4 + 1356935025145240977408*x^2*y^2 - 12212415226307168796
[                                                                                                                              (15*x*y*(1369580688330929*x^4 + 273916
[                                                                                                                              (15*x*z*(1369580688330929*x^4 + 273916
```





Problem 2 part b - Implement STM Computation

```
% load in canvas data to check code
X0 = load('X0canvasprob2.mat');
Phi0 = load('Phi0canvasprob2.mat');

% initial state
initState = [struct2array(X0)'; reshape(struct2array(Phi0), [49,1])];

Re = 6378;
mu = Const.OrbConst.muEarth;

% put the state into my code
[ydot] = Utility.NumericJ2Prop(t,initState,mu,initState(7),Re);

% get just the state
outputState = ydot(1:7)

% get just the phiDot
outputPhiDot = reshape(ydot(8:end), [7,7])

% subtract from what I am supposed to get
PhidotCanvas = load('Phidotcanvasprob2.mat');
XdotCanvas = load('XdotCanvasprob2.mat');

% subtract to see how close I am
StateDiffernece = outputState - struct2array(XdotCanvas)'
phiDotDifference = outputPhiDot - struct2array(PhidotCanvas)
```

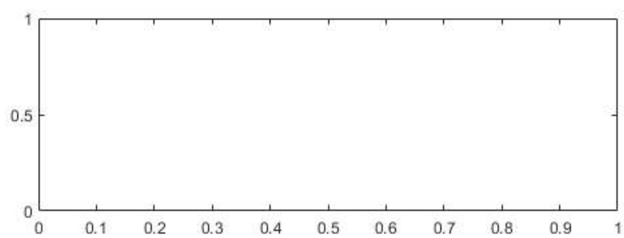
```
outputState =

          0.87587414783453
         -0.24278953633334
           0.1668134394535
```

```
          3413686851177.17
         -3207050208768.91
          357532131195.697
                         0


outputPhiDot =

  Columns 1 through 3

         -0.27515724067569          1.40216228633781          0.78840921622743
          0.60365844582581         -1.36774699097611          0.92873604681331
           1.7812518932425         -0.29253499915187         -0.49079037626976
          8623316544092.63         -13270636762467.4         -2377141237137.59
          3594622546175.42          13304253840432.5          4640092972826.98
           -12213869495081         -8797861337559.08          1951695586029.15
                         0                         0                         0

  Columns 4 through 6

          -0.5352479677759          0.92621639416896          0.19974026229838
         -0.15508038549279         -1.48167521167231          0.42586431913121
          0.61212237077216         -0.55805780868504         -1.27004345059705
          3041231990889.75         -9252589372824.85         -10647863464404.4
         -8821801281771.42          3101266557663.67            17084383676013
         -476668803634.497         -749665500163.869          1645281687361.35
                         0                         0                         0

  Column 7

          0.04073081174943
          0.28297017716199
           0.0635612193025
         -9133367252776.52
          4716460510738.95
          9473313663562.51
                         0


StateDiffernece =

                         0
                         0
                         0
             0.02978515625
              -0.013671875
         -0.00152587890625
                         0


phiDotDifference =

  Columns 1 through 3

                         0                         0                         0
                         0                         0                         0
                         0                         0                         0
          12372683570244.5         -11066484881512.4          4751367886281.35
               0.013671875               0.123046875               0.013671875
          6914588918890.83         -6184607672082.13          2655345992608.04
                         0                         0                         0

  Columns 4 through 6

                         0                         0                         0
                         0                         0                         0
                         0                         0                         0
         -6693812199159.54         -12333028897677.9          2725804604346.97
              -0.04296875               0.029296875                 0.0703125
         -3740899005027.34         -6892427537493.35          1523341173745.71
                         0                         0                         0

  Column 7

                         0
                         0
                         0
         -2910735574169.31
              -0.017578125
         -1626691560703.49
                         0
```

ode45 STM Propagation

```
% starting STM is identity
InitStatePartC = [Y0; J2; reshape(eye(7,7), [49,1])];

% use ode45 to propagate for 15 orbits
odeoptions = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
```

```
[T,YPartC] = ode45(@Utility.NumericJ2Prop, t, InitStatePartC, odeoptions, Const.OrbConst.muEarth, J2, Re);

% Get the STM history
STMhist = YPartC(1:end, 8:end);

% starting perturbation state
pert_prev = [pert0; J2];

% multiply the perturbation by each STM in time
for i = 1:14929

    STM = reshape(STMhist(i,:), [7,7]);

    pert_next(1:7, i) = STM * pert_prev(1:7, i);

    pert_prev(1:7, i+1) = pert_next(1:7, i);

end

figure(fig)

subplot(2,1,1)
plot(pert_prev(1:3,end))
```



Problem 3 - Measurement Partials

```
% test data
R = [0.42286036448769; 1.29952829655200; -1.04979323447507];
V = [-1.78641172211092; 0.81604308103192; -0.32820854314251];

% spacecraft state
scState = [R;V];

Rs = [-1.21456561358767; 1.11183287253465; -0.50749695482985];
Vs = [-0.00008107614118; -0.00008856753168; 0];

% station State
statState = [Rs; Vs];

% Linearized Sensing matrix function call
[Htilde] = Utility.HtildeSC(scState, statState)

[HtildeStation] = Utility.HtildeStation(scState, statState)
```

```
Htilde =

  Columns 1 through 3

        0.943721609480218         0.108177242829583        -0.312549528769213
       -0.216436065590536         0.563578048868017        -0.458452371651641

  Columns 4 through 6

                        0                         0                         0
        0.943721609480218         0.108177242829583        -0.312549528769213


HtildeStation =

       -0.943721609480218        -0.108177242829583         0.312549528769213
```

```
    0.216436065590536    -0.563578048868017    0.458452371651641
```

Problem 4 - Simulating Measurements

```
% staion lat and long
stat1.lat = -35.398333;
stat1.long = 148.981944;

stat2.lat = 40.427222;
stat2.long = 355.749444;

stat3.lat = 35.247164;
stat3.long = 243.205;

% earth rotation
rotEarth.Deg = 360 / (24*60*60); % deg/sec
rotEarth.rad = (2*pi) / (24*60*60);

% initial rotation of ECEF wrt ECI
Theta0 = 122;

% convert groundstations to cartesian
[stat1.ecef.x, stat1.ecef.y, stat1.ecef.z] = sph2cart(deg2rad(stat1.long), deg2rad(stat1.lat), 6378);
[stat2.ecef.x, stat2.ecef.y, stat2.ecef.z] = sph2cart(deg2rad(stat2.long), deg2rad(stat2.lat), 6378);
[stat3.ecef.x, stat3.ecef.y, stat3.ecef.z] = sph2cart(deg2rad(stat3.long), deg2rad(stat3.lat), 6378);

% build components for ease
stat1ecef = [stat1.ecef.x, stat1.ecef.y, stat1.ecef.z];
stat2ecef = [stat2.ecef.x, stat2.ecef.y, stat2.ecef.z];
stat3ecef = [stat3.ecef.x, stat3.ecef.y, stat3.ecef.z];

% each column is station efec position
statAllecef = [stat1ecef',stat2ecef',stat3ecef'];

%--- compute the velocity of each station

% projection of each station onto XY plane
stat1XYproj = [stat1ecef(1:2)';0];
stat2XYproj = [stat2ecef(1:2)';0];
stat3XYproj = [stat3ecef(1:2)';0];

statAllXYProj = [stat1XYproj, stat2XYproj, stat3XYproj];

for stat = 1:3

    % station velocity magnitude
    statVelMag(stat) = rotEarth.rad * norm(statAllXYProj(:,stat));

    % velocity unit vector
    statVelUnitVec(1:3,stat) = cross([0;0;1], statAllXYProj(:,stat)) / (norm([0;0;1])*norm(statAllXYProj(:,stat)));

    % station velocity vector
    stationVeloVec(1:3,stat) = statVelMag(stat) * statVelUnitVec(1:3,stat);

end

% create a function that rotates about the z axis - ransformation btwn both
% frames!
Rz = @(Theta) [cosd(Theta) -sind(Theta) 0; sind(Theta) cosd(Theta) 0; 0 0 1];

% theta of Earth rotation - to be updated each step!
thetaCurrent = Theta0;

% reference transmit frequency for Doppler
refTransFreq = 8.44*10^9; % Hz

% speed of light
c = 229792; % km/s

% simulate the Earth spinning
for i = 1:length(t)

    % how far the Earth has rotated - EVERY 10 SECONDS!
    thetaCurrent = t(i)*rotEarth.Deg + Theta0;

    % reference trajecotry is satellite position and velocity
    satPos = refPos(i,:);
    satVel = refVel(i,:);

    spacecraftState(:,i) = [satPos'; satVel'];

    for j = 1:3
    % put the station coordinates into ECI
    statECI(:,j) = Rz(-thetaCurrent)*statAllecef(:,j);

    % state of this station
    stationState(:,j) = [statECI(:,j); stationVeloVec(1:3,stat)];

    % get LOS for station to satellite
```

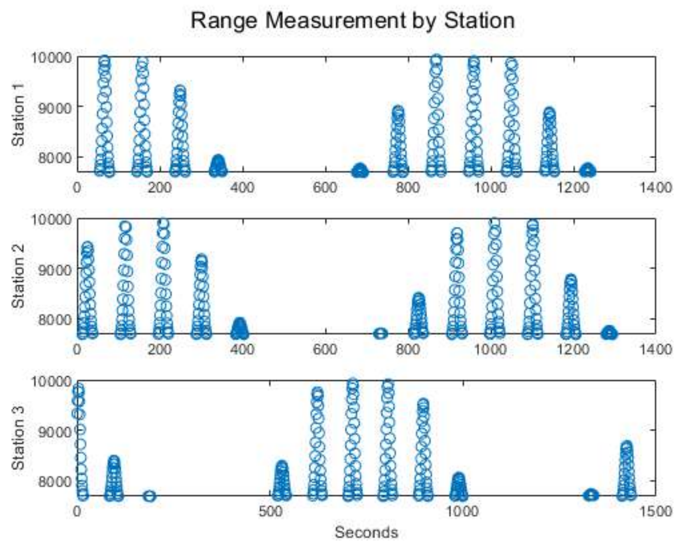
```
    rho(:,j) = spacecraftState(1:3,i) - statECI(:,j);

    % Dot product Station position with LOS for satllite
    eleAngStat(j) = acosd(dot(statECI(:,j), rho(:,j)) / (norm(statECI(:,j))*norm(rho(:,j))));

    % save the value of the dot produsct for each station
    eleDotStat(j) = dot(statECI(:,j), rho(:,j));


    % --- Check if the station is able to make a measurement
    % The check for observability is if dot > 0 and eleAng > 100

        % for each station
        if eleAngStat(j) > 10 && eleDotStat(j) > 0
            % if the elevation is more than 10 degree elevation.
            % If dot product is positive then measuring the correct angle!

            % mask for all the visibility
            visibiltyMask(j,i) = 1; % measurement made!

            % Determine what the range and range rate is for each
            %rangeMeasurement(j,i)    = norm(LOS(:,j));
            [HtildeSC] = Utility.HtildeSC(spacecraftState(:,i), stationState(:,j));

            % Measurement!
            Measurement = HtildeSC * spacecraftState(:,i);

            % save off rho and rhoDot measurement
            rhoMeas(i,j) = Measurement(1);
            rhoDotMeas(i,j) = Measurement(2);

            % save the elevation angle for each measurement
            savedEleAng(j,i) = eleAngStat(j);

            % Calculate frequecny shift
            freqShift(j,i) = -2*rhoDotMeas(i,j)/c * refTransFreq;

            RU(j,i) = (221/749)*(rhoMeas(i,j)/c) * refTransFreq;

        else
            % Satellite not seen
            visibiltyMask(j,i) = NaN;

            % No rho and rhoDot measurement
            rhoMeas(i,j) = NaN;
            rhoDotMeas(i,j) = NaN;

            % save the elevation angle for each measurement
            savedEleAng(j,i) = NaN;

            % frequecny shift for doppler
            freqShift(j,i) = NaN;

            RU(j,i) = NaN;
        end
    end

end
```

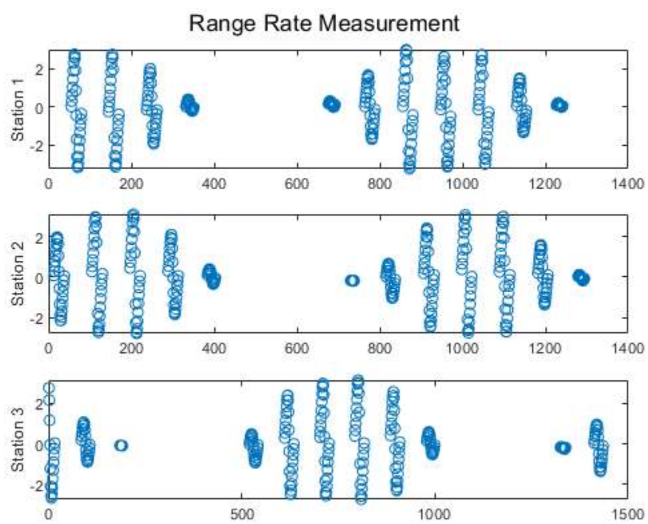
Plots for Range and Range Rate

plot for each station range measurement

```
figure(fig)
subplot(3,1,1)
plot(rhoMeas(1:10:end,1), 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(rhoMeas(1:10:end,2), 'o')
ylabel('Station 2')

subplot(3,1,3)
plot(rhoMeas(1:10:end,3), 'o')
ylabel('Station 3')
xlabel('Seconds')

sgtitle('Range Measurement by Station')

fig = fig + 1;


% plot each range dot measurement
figure(fig)
subplot(3,1,1)
plot(rhoDotMeas(1:10:end,1), 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(rhoDotMeas(1:10:end,2), 'o')
```

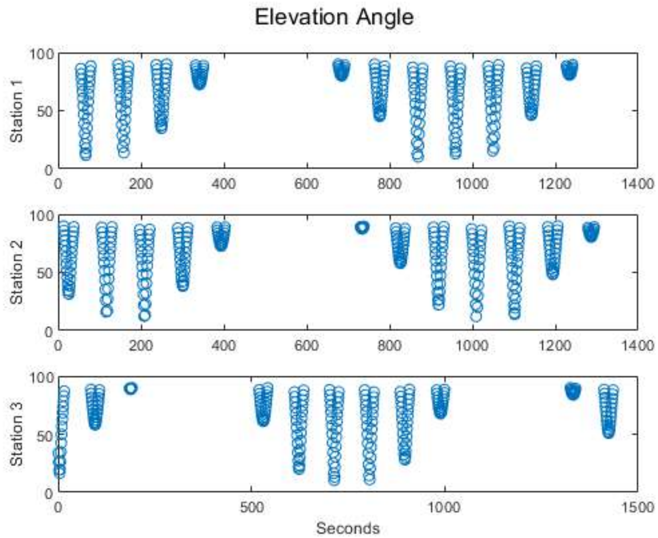
```
ylabel('Station 2')

subplot(3,1,3)
plot(rhoDotMeas(1:10:end,3), 'o')
ylabel('Station 3')

sgtitle('Range Rate Measurement')

fig = fig + 1;
```





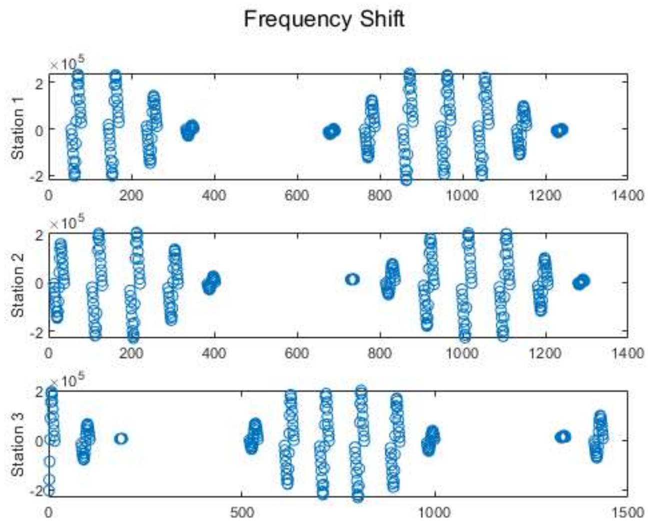
Elevation angle plot

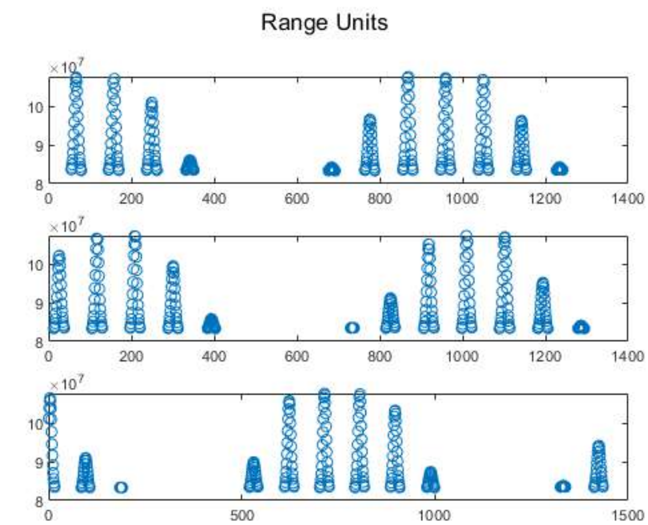
```
figure(fig)
subplot(3,1,1)
plot(savedEleAng(1,1:10:end)', 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(savedEleAng(2,1:10:end)', 'o')
ylabel('Station 2')

subplot(3,1,3)
plot(savedEleAng(3,1:10:end)', 'o')
ylabel('Station 3')
xlabel('Seconds')

sgtitle('Elevation Angle')

fig = fig + 1;
```



problem 3 part C - Doppler Shift

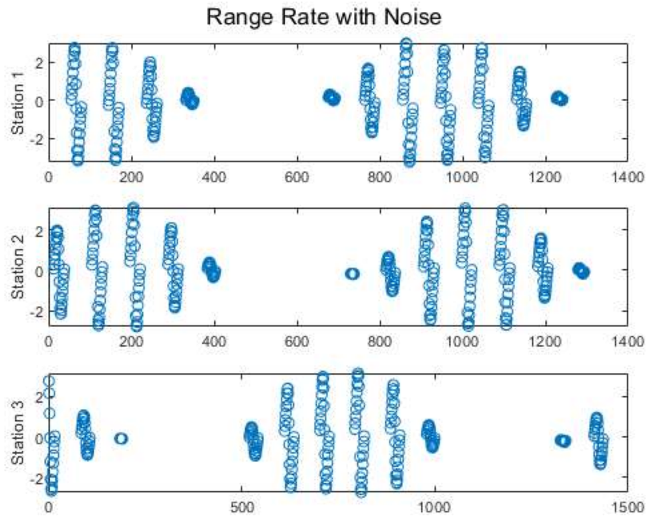
```
% plot range units and frequency shift

figure(fig)
subplot(3,1,1)
plot(freqShift(1,1:10:end)', 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(freqShift(2,1:10:end)', 'o')
ylabel('Station 2')

subplot(3,1,3)
plot(freqShift(3,1:10:end)', 'o')
ylabel('Station 3')

sgtitle('Frequency Shift')

fig = fig + 1;

% plot Range units
figure(fig)
subplot(3,1,1)
plot(RU(1,1:10:end)', 'o')

subplot(3,1,2)
plot(RU(2,1:10:end)', 'o')

subplot(3,1,3)
plot(RU(3,1:10:end)', 'o')

sgtitle('Range Units')

fig = fig + 1;
```





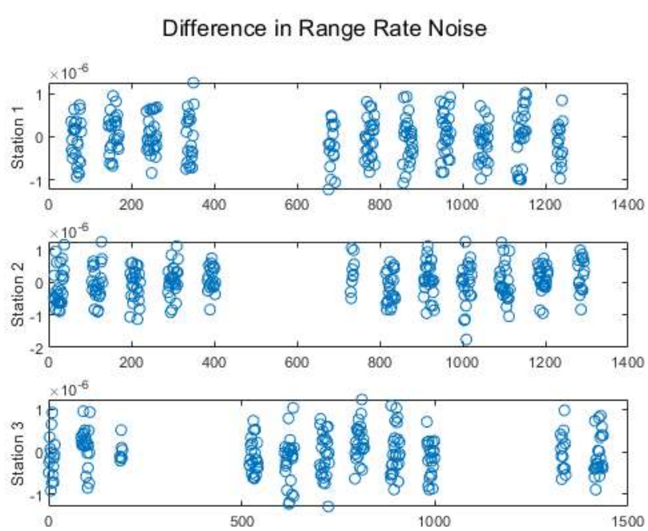
Part D - Range Rate Noise

```
sigmaNoise = 0.5*10^-6; % km/s
rangeDotMeasNoise = rhoDotMeas' + sigmaNoise * randn(3,14929);

figure(fig)
subplot(3,1,1)
plot(rangeDotMeasNoise(1,1:10:end)', 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(rangeDotMeasNoise(2,1:10:end)', 'o')
ylabel('Station 2')

subplot(3,1,3)
plot(rangeDotMeasNoise(3,1:10:end)', 'o')
ylabel('Station 3')

sgtitle('Range Rate with Noise')

fig = fig + 1;
```



Noise Difference

```
rangeDotDiff = rangeDotMeasNoise - rhoDotMeas';

figure(fig)
subplot(3,1,1)
plot(rangeDotDiff(1,1:10:end)', 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(rangeDotDiff(2,1:10:end)', 'o')
ylabel('Station 2')

subplot(3,1,3)
plot(rangeDotDiff(3,1:10:end)', 'o')
ylabel('Station 3')

sgtitle('Difference in Range Rate Noise')

fig = fig + 1;
```



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