Contents

Homework 1 StatOD

```
addpath('C:\Users\user\OneDrive - UCB-O365\Desktop\Spring 2025\StatOD\repo');
```

Problem 1

```
% part a
syms x y z a mu J2 J3

r = sqrt(x^2 + y^2 + z^2);

U = mu/r - mu*a^2*J2*(3*z^2-r^2)/(2*r^5) - mu*a^3*J3*(5*z^3-3*z*r^2)/(2*r^7);

% take partial of U to get acceleration each component
accelxyz = jacobian(U, [x y z]);

ax_latex = latex(accelxyz(1));
ay_latex = latex(accelxyz(2));
az_latex = latex(accelxyz(3));

% take the partial of the acceleration to build up STM
syms x y z vx vy vz mu J2 J3
stateVec = [x y z vx vy vz mu J2 J3];
Aaccel = jacobian(accelxyz, stateVec);


% final A matrix
% Amat = [zeros(3,3), ones(3,3), zeros(3,3); ...
%     double(subs(Aaccel,stateVec,state)) ; ...
%     zeros(3,9)];


% part c
state = [-0.64901376519124, 1.18116604196553, -0.75845329728369, -1.10961303850152, -0.84555124000780, -0.57266486645795, -0.55868076447397, 0.17838022584977, -0.19
[Amat] = Utility.DynamicsA_J2_J3(state);

A = load('CanvasAproblem1.mat')

Adiff = struct2array(A) - Amat;
```

```
A =

  struct with fields:

    A: [9×9 double]
```

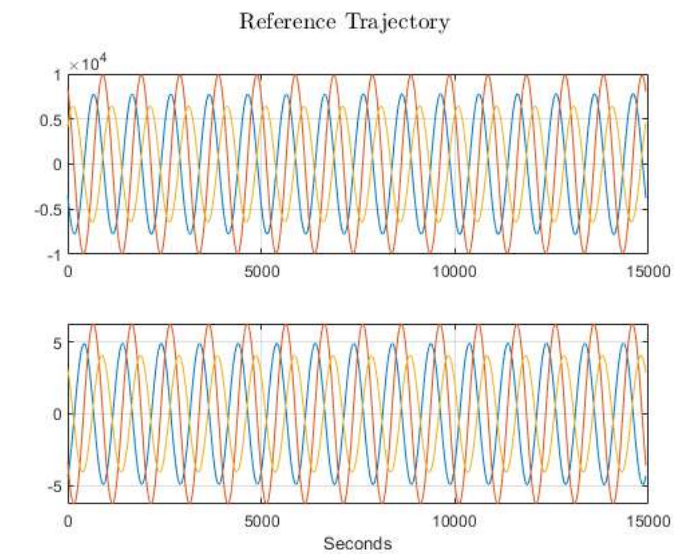
Problem 2

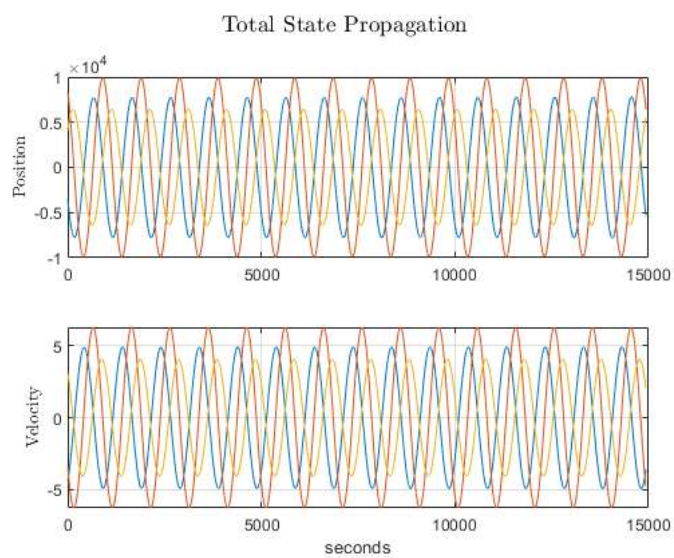
```
SMA   = 10000;  % km
eccen = 0.001;
inc   = 40; % deg
RAAN  = 80; % deg
AOP   = 40; % deg
TA0   = 0; % deg

% Convert from Orbital Elemenets to Cartesian to get initial state vector
[r0,v0] = Utility.OrbCart(SMA,eccen,inc,RAAN,AOP,TA0,Const.OrbConst.muEarth);

% Period of orbit
period = 2*pi*sqrt(SMA^3/Const.OrbConst.muEarth);

% propagate for 15 orbits
t = 0:10:15*period;

% initial state vector
Y0 = [r0;v0];

% use ode45 to propagate for 15 orbits
odeoptions = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
[T,Y] = ode45(@Utility.NumericJ2Prop, t, Y0, odeoptions, Const.OrbConst.muEarth);

% reference trajectory
```

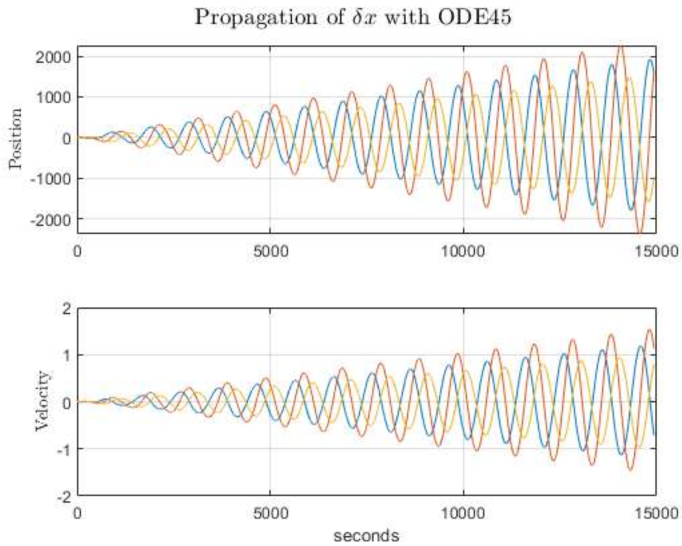
```
refPos = Y(:,1:3);
refVel = Y(:,4:6);

% plotting trajectory
fig = 1;
figure(fig)
subplot(2,1,1)
plot(refPos)
grid on

subplot(2,1,2)
plot(refVel)
grid on
xlabel('Seconds')

sgtitle('Reference Trajectory', 'Interpreter', 'latex')

fig = fig + 1;


% --- Integrate second trajectory by perturbing initial state
pert0 = [1;0;0;0;.01;0];

% perturbed initial state
Y0pert = Y0 + pert0;

% use ode45 to propagate for 15 orbits
odeoptions = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
[T,Ypert] = ode45(@Utility.NumericJ2Prop, t, Y0pert, odeoptions, Const.OrbConst.muEarth);

% reference trajectory
pertPos = Ypert(:,1:3);
pertVel = Ypert(:,4:6);

% Plotting Trajectory
figure(fig)
subplot(2,1,1)
plot(pertPos)
grid on
ylabel('Position', 'Interpreter', 'latex')

subplot(2,1,2)
plot(pertVel)
grid on
xlabel('seconds')
ylabel('Velocity', 'Interpreter', 'latex')

sgtitle('Total State Propagation', 'Interpreter', 'latex')

fig = fig + 1;

% compare reference trajecory with perturbed trajectory
trajDiff = Y - Ypert;

% difference in position
posDiff = trajDiff(:,1:3);

% difference in velocity
velDiff = trajDiff(:,4:6);

% plot the difference
figure(fig)
subplot(2,1,1)
plot(posDiff)
grid on
ylabel('Position', 'Interpreter', 'latex')

subplot(2,1,2)
plot(velDiff)
grid on
xlabel('seconds')
ylabel('Velocity', 'Interpreter', 'latex')


sgtitle('Propagation of $\delta x$ with ODE45', 'Interpreter', 'latex')

fig = fig + 1;
```

```
Tpqw_ijk =

       -0.351900933636988        -0.689527809386471         0.633022221559489
        0.839911542566906        -0.531121287922501        -0.11161889704895
        0.413175911166535         0.492403876506104         0.766044443118978


r =

        -3515.49032703351
         8390.71631024339
         4127.62735255368
```

```
v =

  -4.35767632217815
  -3.35657913876455
   3.1118929278699
```





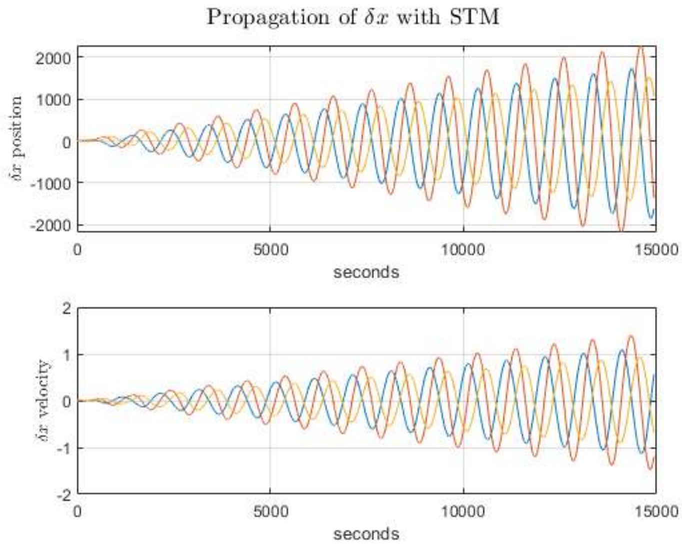


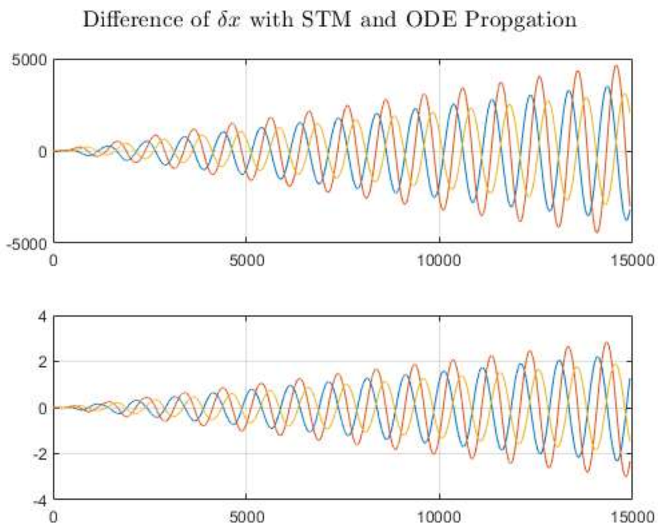
--- propagating state using STM

```
syms x y z vx vy vz

r = sqrt(x^2 + y^2 + z^2);

mu = Const.OrbConst.muEarth;

J2 = 0.00108248;

a = 6378;

U = mu/r - mu*a^2*J2*(3*z^2-r^2)/(2*r^5);

% take partial of U to get acceleration each component
accelxyz = jacobian(U, [x y z]);

stateVector = [x, y, z, vx, vy, vz];

accelWRTState = jacobian(accelxyz, stateVector);


% create a funciton handle for the Jacobian to be evaulted later
% numerically
A_func = matlabFunction(accelWRTState, 'Vars', {x, y, z, vx, vy, vz});

deltaT = 10; % seconds

% set pert0 as the first deltaX
deltaX_old = pert0;

% pre-allocate deltaX
deltaX = zeros(6, length(t));

for i = 1:length(t)
    % Update A based on the current state reference trajectory
    currState = Y(i,:);

    A = [zeros(3,3), eye(3,3); ...
    double(A_func(currState(1), currState(2), currState(3), currState(4), currState(5), currState(6)))];

    % propagate the delta forward in time
    deltaX(:,i) = expm(A * deltaT) * deltaX_old;

    deltaX_old = deltaX(:,i);

end

% plot results for perturbation propagation with STM
figure(fig)

subplot(2,1,1)
plot(deltaX(1:3,:)')
grid on
xlabel('seconds')
ylabel('$\delta x$ position', 'Interpreter', 'latex')

subplot(2,1,2)
plot(deltaX(4:6,:)')
grid on
xlabel('seconds')
ylabel('$\delta x$ velocity', 'Interpreter', 'latex')
```

```
sgtitle('Propagation of $\delta x$ with STM', 'Interpreter', 'latex')
fig = fig + 1;

% --- Validity of using STM to propagate
% veloDiff and posDiff are the perturbations from ODE45

posODEdiffSTM = deltaX(1:3,:)' - posDiff;
velODEdiffSTM = deltaX(4:6,:)' - velDiff;

figure(fig)
subplot(2,1,1)
plot(posODEdiffSTM)
grid on

subplot(2,1,2)
plot(velODEdiffSTM)
grid on

sgtitle('Difference of $\delta x$ with STM and ODE Propgation', 'Interpreter', 'latex')



% --- ode45 STM propagation testing

phi0 = [1; 0; 0; 0; 1; 0; 0; 0; 1]
[T,Y] = ode45(@Utility.NumericJ2Prop, t, Y0, odeoptions, Const.OrbConst.muEarth);
```

```
phi0 =

     1
     0
     0
     0
     1
     0
     0
     0
     1
```





Problem 3 - Measurement Partials

```
% test data
R = [0.42286036448769; 1.29952829655200; -1.04979323447507];
V = [-1.78641172211092; 0.81604308103192; -0.32820854314251];

% spacecraft state
scState = [R;V];

Rs = [-1.21456561358767; 1.11183287253465; -0.50749695482985];
Vs = [-0.00008107614118; -0.00008856753168; 0];

% station State
statState = [Rs; Vs];

% Linearized Sensing matrix function call
[Htilde] = Utility.HtildeSC(scState, statState)

[HtildeStation] = Utility.HtildeStation(scState, statState)
```

```
Htilde =

  Columns 1 through 3

       0.943721609480218         0.108177242829583        -0.312549528769213
      -0.216436065590536         0.563578048868017        -0.458452371651641

  Columns 4 through 6

                       0                         0                         0
       0.943721609480218         0.108177242829583        -0.312549528769213


HtildeStation =
```

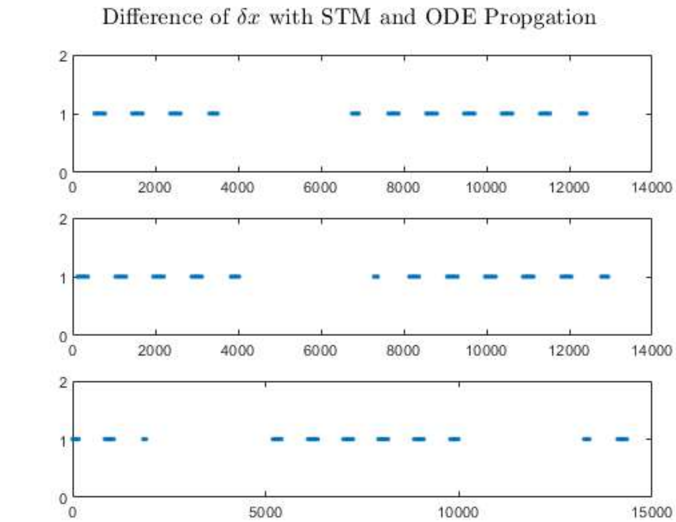
```
   -0.943721609480218        -0.108177242829583         0.312549528769213
    0.216436065590536        -0.563578048868017         0.458452371651641
```

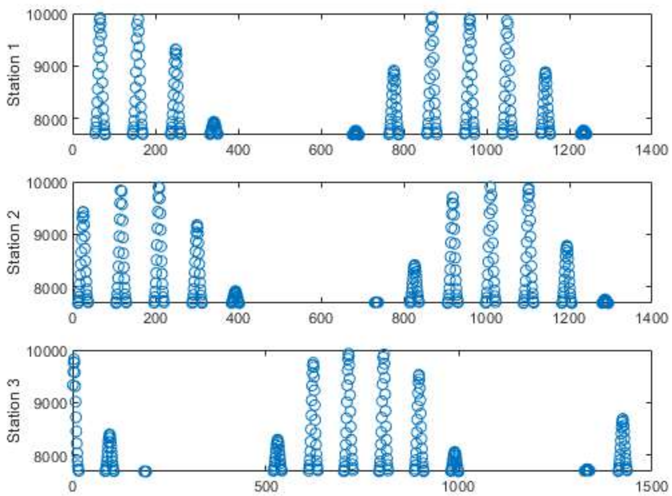
Problem 4 - Simulating Measurements

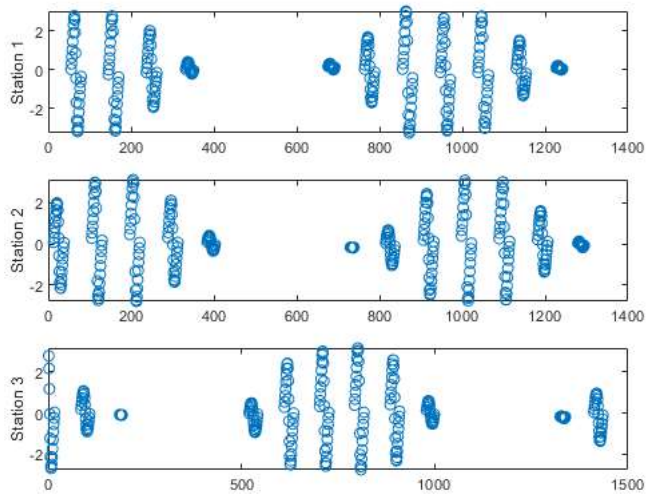
```
% staion lat and long
stat1.lat = -35.398333;
stat1.long = 148.981944;

stat2.lat = 40.427222;
stat2.long = 355.749444;

stat3.lat = 35.247164;
stat3.long = 243.205;

% earth rotation
rotEarth.Deg = 360 / (24*60*60); % deg/sec
rotEarth.rad = (2*pi) / (24*60*60);

% initial rotation of ECEF wrt ECI
Theta0 = 122;

% convert groundstations to cartesian
[stat1.ecef.x, stat1.ecef.y, stat1.ecef.z] = sph2cart(deg2rad(stat1.long), deg2rad(stat1.lat), 6378);
[stat2.ecef.x, stat2.ecef.y, stat2.ecef.z] = sph2cart(deg2rad(stat2.long), deg2rad(stat2.lat), 6378);
[stat3.ecef.x, stat3.ecef.y, stat3.ecef.z] = sph2cart(deg2rad(stat3.long), deg2rad(stat3.lat), 6378);

% build components for ease
stat1ecef = [stat1.ecef.x, stat1.ecef.y, stat1.ecef.z];
stat2ecef = [stat2.ecef.x, stat2.ecef.y, stat2.ecef.z];
stat3ecef = [stat3.ecef.x, stat3.ecef.y, stat3.ecef.z];

% each column is station efec position
statAllecef = [stat1ecef',stat2ecef',stat3ecef'];

%--- compute the velocity of each station

% projection of each station onto XY plane
stat1XYproj = [stat1ecef(1:2)';0];
stat2XYproj = [stat2ecef(1:2)';0];
stat3XYproj = [stat3ecef(1:2)';0];

statAllXYProj = [stat1XYproj, stat2XYproj, stat3XYproj];

for stat = 1:3

    % station velocity magnitude
    statVelMag(stat) = rotEarth.rad * norm(statAllXYProj(:,stat));

    % velocity unit vector
    statVelUnitVec(1:3,stat) = cross([0;0;1], statAllXYProj(:,stat)) / (norm([0;0;1])*norm(statAllXYProj(:,stat)));

    % station velocity vector
    stationVeloVec(1:3,stat) = statVelMag(stat) * statVelUnitVec(1:3,stat);

end

% create a function that rotates about the z axis - ransformation btwn both
% frames!
Rz = @(Theta) [cosd(Theta) -sind(Theta) 0; sind(Theta) cosd(Theta) 0; 0 0 1];

% theta of Earth rotation - to be updated each step!
thetaCurrent = Theta0;

% reference transmit frequency for Doppler
refTransFreq = 8.44*10^9; % Hz

% speed of light
c = 229792; % km/s

% simulate the Earth spinning
for i = 1:length(t)

    % how far the Earth has rotated - EVERY 10 SECONDS!
    thetaCurrent = t(i)*rotEarth.Deg + Theta0;

    % reference trajecotry is satellite position and velocity
    satPos = refPos(i,:);
    satVel = refVel(i,:);

    spacecraftState(:,i) = [satPos'; satVel'];

    for j = 1:3
    % put the station coordinates into ECI
    statECI(:,j) = Rz(-thetaCurrent)*statAllecef(:,j);

    % state of this station
    stationState(:,j) = [statECI(:,j); stationVeloVec(1:3,stat)];
```

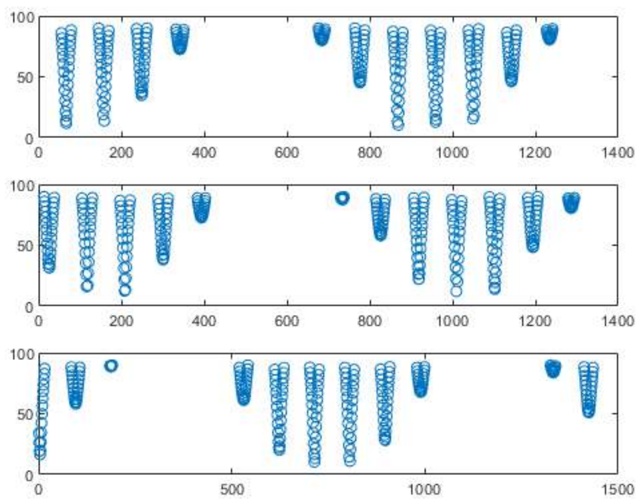
```
    % get LOS for station to satellite
    rho(:,j) = spacecraftState(1:3,i) - statECI(:,j);

    % Dot product Station position with LOS for satllite
    eleAngStat(j) = acosd(dot(statECI(:,j), rho(:,j)) / (norm(statECI(:,j))*norm(rho(:,j))));

    % save the value of the dot produsct for each station
    eleDotStat(j) = dot(statECI(:,j), rho(:,j));


    % --- Check if the station is able to make a measurement
    % The check for observability is if dot > 0 and eleAng > 100

        % for each station
        if eleAngStat(j) > 10 && eleDotStat(j) > 0
            % if the elevation is more than 10 degree elevation.
            % If dot product is positive then measuring the correct angle!

            % mask for all the visibility
            visibiltyMask(j,i) = 1; % measurement made!

            % Determine what the range and range rate is for each
            %rangeMeasurement(j,i)    = norm(LOS(:,j));
            [HtildeSC] = Utility.HtildeSC(spacecraftState(:,i), stationState(:,j));

            % Measurement!
            Measurement = HtildeSC * spacecraftState(:,i);

            % save off rho and rhoDot measurement
            rhoMeas(i,j) = Measurement(1);
            rhoDotMeas(i,j) = Measurement(2);

           % rangeDotMeasurement(j,i) = norm(dot(satPos-statECI(:,j),satVel-stationVeloVec(1:3,j)) / rangeMeasurement(j,i));

            % save the elevation angle for each measurement
            savedEleAng(j,i) = eleAngStat(j);

            % Calculate frequecny shift
           % freqShift(j,i) = -2*rangeDotMeasurement(j,i)/c * refTransFreq;

           % RU(j,i) = (221/749)*(rangeMeasurement(j,i)/c) * refTransFreq;

        else
            % Satellite not seen
            visibiltyMask(j,i) = NaN;

            % save off rho and rhoDot measurement
            rhoMeas(i,j) = NaN;
            rhoDotMeas(i,j) = NaN;

            % Measurement not made
            % rangeMeasurement(j,i)    = NaN;
            % rangeDotMeasurement(j,i) = NaN;

            % save the elevation angle for each measurement
            savedEleAng(j,i) = NaN;

            % frequecny shift for doppler
         %   freqShift(j,i) = NaN;

         %   RU(j,i) = NaN;
        end
    end

end

% plots for each of the stations visibility
 figure(fig)
 subplot(3,1,1)
 plot(visibiltyMask(1,:), '.')

 subplot(3,1,2)
 plot(visibiltyMask(2,:), '.')

 subplot(3,1,3)
 plot(visibiltyMask(3,:), '.')

 fig = fig + 1;


 % plot for each station range measurement

figure(fig)
subplot(3,1,1)
plot(rhoMeas(1:10:end,1), 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(rhoMeas(1:10:end,2), 'o')
ylabel('Station 2')
```

```
subplot(3,1,3)
plot(rhoMeas(1:10:end,3), 'o')
ylabel('Station 3')

fig = fig + 1;


% plot each range dot measurement
figure(fig)
subplot(3,1,1)
plot(rhoDotMeas(1:10:end,1), 'o')
ylabel('Station 1')

subplot(3,1,2)
plot(rhoDotMeas(1:10:end,2), 'o')
ylabel('Station 2')

subplot(3,1,3)
plot(rhoDotMeas(1:10:end,3), 'o')
ylabel('Station 3')

fig = fig + 1;


% elevation angle plot
figure(fig)
subplot(3,1,1)
plot(savedEleAng(1,1:10:end)', 'o')

subplot(3,1,2)
plot(savedEleAng(2,1:10:end)', 'o')

subplot(3,1,3)
plot(savedEleAng(3,1:10:end)', 'o')

fig = fig + 1;

% --- part C - Problem 3

% plot range units and frequency shift
```


Difference of δx with STM and ODE Propgation


Station 1
Station 2
Station 3





BE SURE TO FIX THIS

figure(fig) subplot(3,1,1) plot(freqShift(1,1:10:end)', 'o')

subplot(3,1,2) plot(freqShift(2,1:10:end)', 'o')

subplot(3,1,3) plot(freqShift(3,1:10:end)', 'o')

sgtitle('Frequency Shift')

fig = fig + 1;

% plot Range units figure(fig) subplot(3,1,1) plot(RU(1,1:10:end)', 'o')

subplot(3,1,2) plot(RU(2,1:10:end)', 'o')

subplot(3,1,3) plot(RU(3,1:10:end)', 'o')

sgtitle('Range Units')

fig = fig + 1;

% --- Part D - add noise sigmaNoise = 0.5*10^-6; % km/s rangeDotMeasNoise = rangeDotMeasurement + sigmaNoise * randn(3,14929);

figure(fig) subplot(3,1,1) plot(rangeDotMeasNoise(1,1:10:end)', 'o') ylabel('Station 1')

subplot(3,1,2) plot(rangeDotMeasNoise(2,1:10:end)', 'o') ylabel('Station 2')

subplot(3,1,3) plot(rangeDotMeasNoise(3,1:10:end)', 'o') ylabel('Station 3')

sgtitle('Range Rate with Noise')

fig = fig + 1;

% plot difference btwn noisey and not rangeDotDiff = rangeDotMeasNoise - rangeDotMeasurement;

figure(fig) subplot(3,1,1) plot(rangeDotDiff(1,1:10:end)', 'o') ylabel('Station 1')

subplot(3,1,2) plot(rangeDotDiff(2,1:10:end)', 'o') ylabel('Station 2')

subplot(3,1,3) plot(rangeDotDiff(3,1:10:end)', 'o') ylabel('Station 3')

sgtitle('Range Rate with Noise')

fig = fig + 1;
