NUMBERS, COMBINATIONS OF NUMBERS AND FACTORIALS

The Number Line

The concept of number is perhaps the most fundamental in mathematics. One of the easiest ways of envisaging numbers is by means of a number line. The simplest such line is shown below. It is defined by an arbitrarily chosen point called zero and a series of equally spaced points in each direction representing the other *positive and negative integers.* Zero is regarded as both positive and negative. Any other real number x is represented as a point distant x from the origin (the unit of distance being the distance between adjacent integer points).

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

Comparison Operators

The following are common relationships between two real numbers x and y

$$x = y \qquad x \neq y \qquad x < y \qquad x \leq y \qquad x \geq y \qquad x > y$$

If $x < y$, the set of numbers z such that $x < z < y$ is called the *open interval* (x,y). The set of numbers such that $x \leq z \leq y$ is called the *closed interval* $[x,y]$.

Simple Arithmetic Operators

Two numbers x and y can be combined to form a third number as follows:

$x + y$ $\qquad x - y$ $\qquad xy$ and x/y (provided $y \neq 0$)

Numbers of the form m/n where m and n are integers are called *rational numbers*. Most real numbers are not rational numbers, but any real number can be approximated arbitrarily closely by a rational number.

Exponential Operations

A number x multiplied by itself n-1 times where n is an integer ≥ 1, is *raised to the power n*, written x^n; this definition extends to $n \leq 0$ when multiplication -1 times is interpreted as division.

Any real number y such that $y^n = x$ where n is a positive integer, is called the n^{th} root of x. We write $y = x^{1/n}$. If n is even, there are two equal and opposite real n^{th} roots of x; if n is odd there is only one. Roots of rational numbers are usually not rational.

$x^{1/2}$ is called the square root of x and is often written as $\sqrt{x}$. The n^{th} root of x is sometimes written as $\sqrt[n]{x}$

The result of raising $x^{1/n}$ to the power m is written $x^{m/n}$, thus defining x^r for all rational r, and this definition can be extended to define x^y for all real y. Powers combine as follows:

$x^y x^z = x^{y+z}$ $\qquad$ $x^y / x^z = x^{y-z}$ $\qquad$ $(x^y)^z = x^{xz}$

Unitary Operators

The magnitude of a number x regardless of its sign is called its modulus, $|x|$ i.e.,

$|x| = x$ $\qquad$ if $x > 0$
$|x| = -x$ $\qquad$ if $x \leq 0$

Precedence of Operators

Algebraic expressions often denote several pairwise combinations of numbers. Such expressions are unambiguous only because of the following strict convention regarding the order in which operations are carried out in combining adjacent numbers:

1. raising to a power
2. multiplication
3. division
4. addition and subtraction

Any departure from this convention is indicated by the use of brackets. The expression inside a pair of brackets is reduced to a single number before being combined with an adjacent number.

Factorials

In many areas of mathematics there arises a need to think in terms of rearrangements of a set of objects. The number of ways in which n different objects can be rearranged in a row is

n x (n-1) x (n-2) x …x 3 x 2 x 1.

This quantity is called *factorial n* and is denoted by $n!$

Summations and Products

A common requirement is to work with quantities that are the summation or product of other quantities and we often use a special notation to simplify expressions involving such sums or products. Consider a set of variables $\{x_1, x_2, \ldots x_n\}$ then we use the symbol Σ and Π as a shorthand for summation and multiplication respectively, as follows:

$$x_1 + x_2 + \ldots + x_n = \sum_{i=1}^{n} x_i$$

and

$$x_1 \times x_2 \times \ldots \times x_n = \prod_{i=1}^{n} x_i$$

These conventions are used extensively in the course.