Session regression I: simple linear regression

Learning outcomes

- understand simple linear regression model incl. terminology and mathematical notations
- estimate model parameters and their standar error
- use model for checking the association between x and y
- use model for prediction
- assees model accuracy with RSE and R^2
- check model assumptions
- to be able to use `lm` function in R for model fitting, obtaining confidence interval and predictions

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Introduction

- Quiz: What do we already know about `simple linear regression`?

Description

- Simple linear regression is a statistical method that allows us to summarize and study relationships between two continuous (quantitative, numerical) variables
  - one variable, denoted `x` is regarded as the *predictor*, *explanatory*, or *indepedent variable*, e.g. body weight (kg)
  - the other variable, denoted `y`, is regarded as the *response*, *outcome*, or *dependent variable*, e.g. plasma volume (l)
- It is used to estimate the best-fitting straight line to describe the association

Used for to answer questions such as:

- is there a relationship between `x` exposure (e.g. body weight) and `y` outcome (e.g. plasma volume)?
- how strong is the relationship between the two variables?
- what will be a predicted value of the `y` outcome given a new set of exposure values?
- how accurately can we predict the outcome?

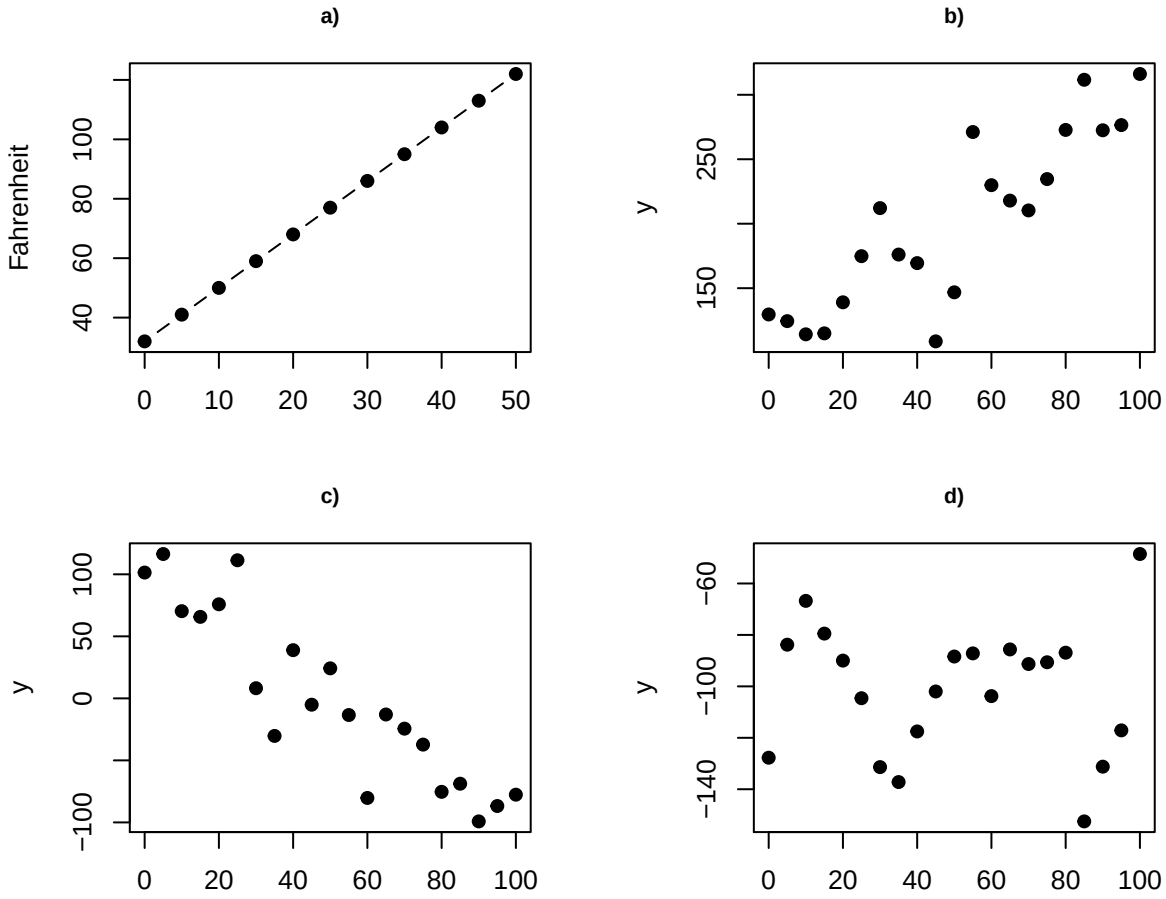


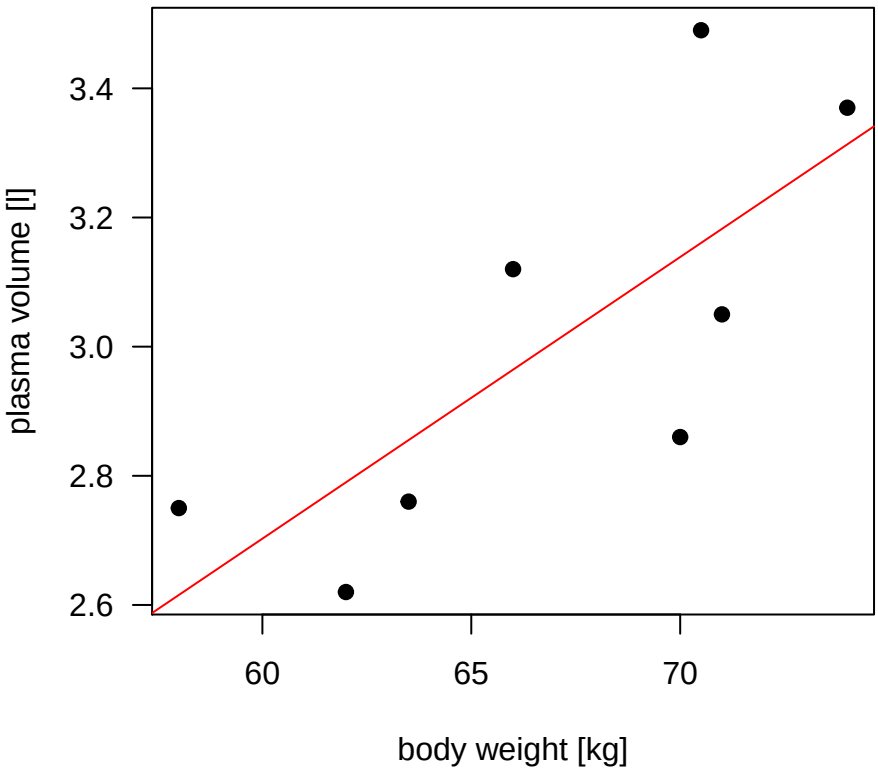
Figure 1: **Determinisitc vs. statistical relationship**: a) deterministic: equation exactly describes the relationship between the two variables e.g. $Fahrenheit = 9/5 * Celcius + 32$; b) statistical relationship between x and y is not perfect (increasing), c) statistical relationship between x and y is not perfect (decreasing), d) random signal

Example data

Example data contain the body weight and plasma volume for eight healthy men.

```
weight <- c(58, 70, 74, 63.5, 62.0, 70.5, 71.0, 66.0) # body weight (kg)
plasma <- c(2.75, 2.86, 3.37, 2.76, 2.62, 3.49, 3.05, 3.12) # plasma volume (liters)
```

Scatter plot of the data shows that high plasma volume tends to be associated with high weight and *vice verca*. Linear regrssion gives the equation of the straight line that best describes how the outcome changes (increase or decreases) with a change of exposure variable (in red)



The equation of the regression line is:

$$y = beta_0 + beta_1 x$$

Estimating the Coefficients

Assessing the Accuracy of the Coefficient Estimates

Asesssing the Accuracy of the Model

```
## [1] 0.7591266
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.27880 -0.14178 -0.01928  0.13986  0.32939
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.08572    1.02400   0.084   0.9360
## x            0.04362    0.01527   2.857   0.0289 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2188 on 6 degrees of freedom
```

```
## Multiple R-squared:  0.5763, Adjusted R-squared:  0.5057
## F-statistic:  8.16 on 1 and 6 DF,  p-value: 0.02893
```

```
##          x
## 0.01526836
```

```
##        x
## 1.023998
```

Next: Regression session II: multiple linear regression