

# Introduction to biological network analysis

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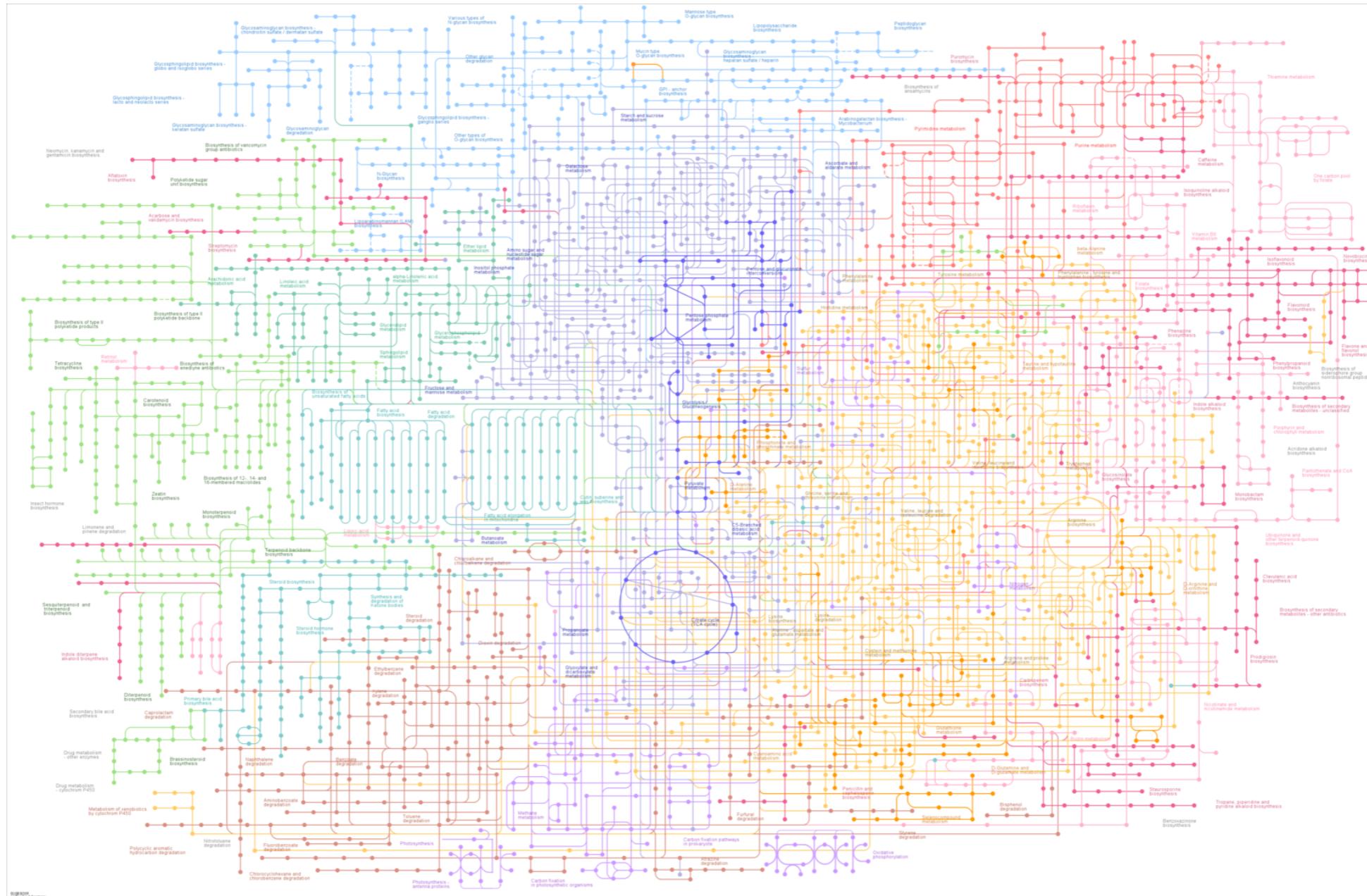
# Overview

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- 1. Introduction to network analysis**
- 2. Terminology**
- 3. Network construction**
- 4. Key network properties**
- 5. Community analysis**

Original sources of images provided as reference and hyperlinks, where applicable.

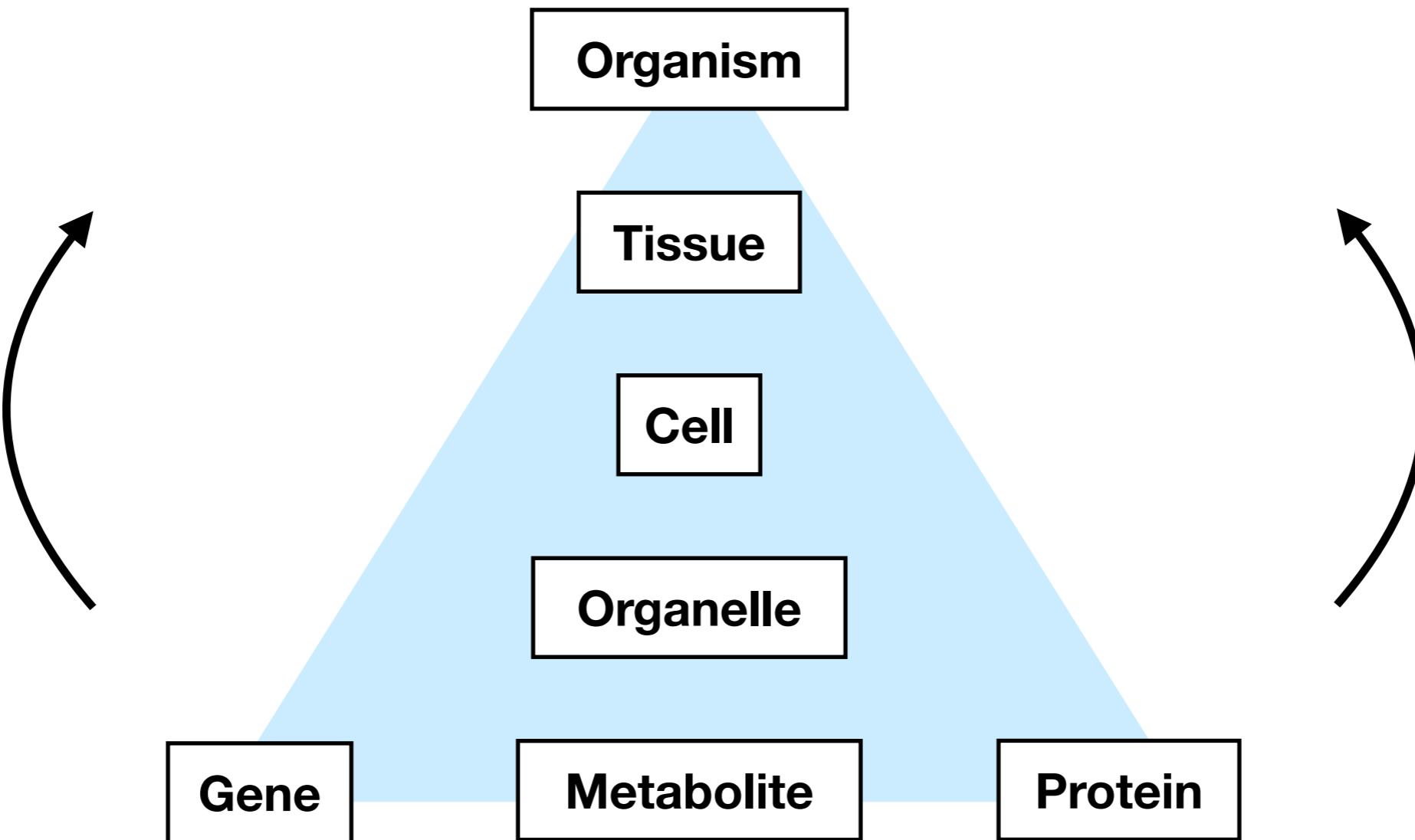
# How to tackle biological complexity?



**Focus: feature-feature relationships**

# How to tackle biological complexity?

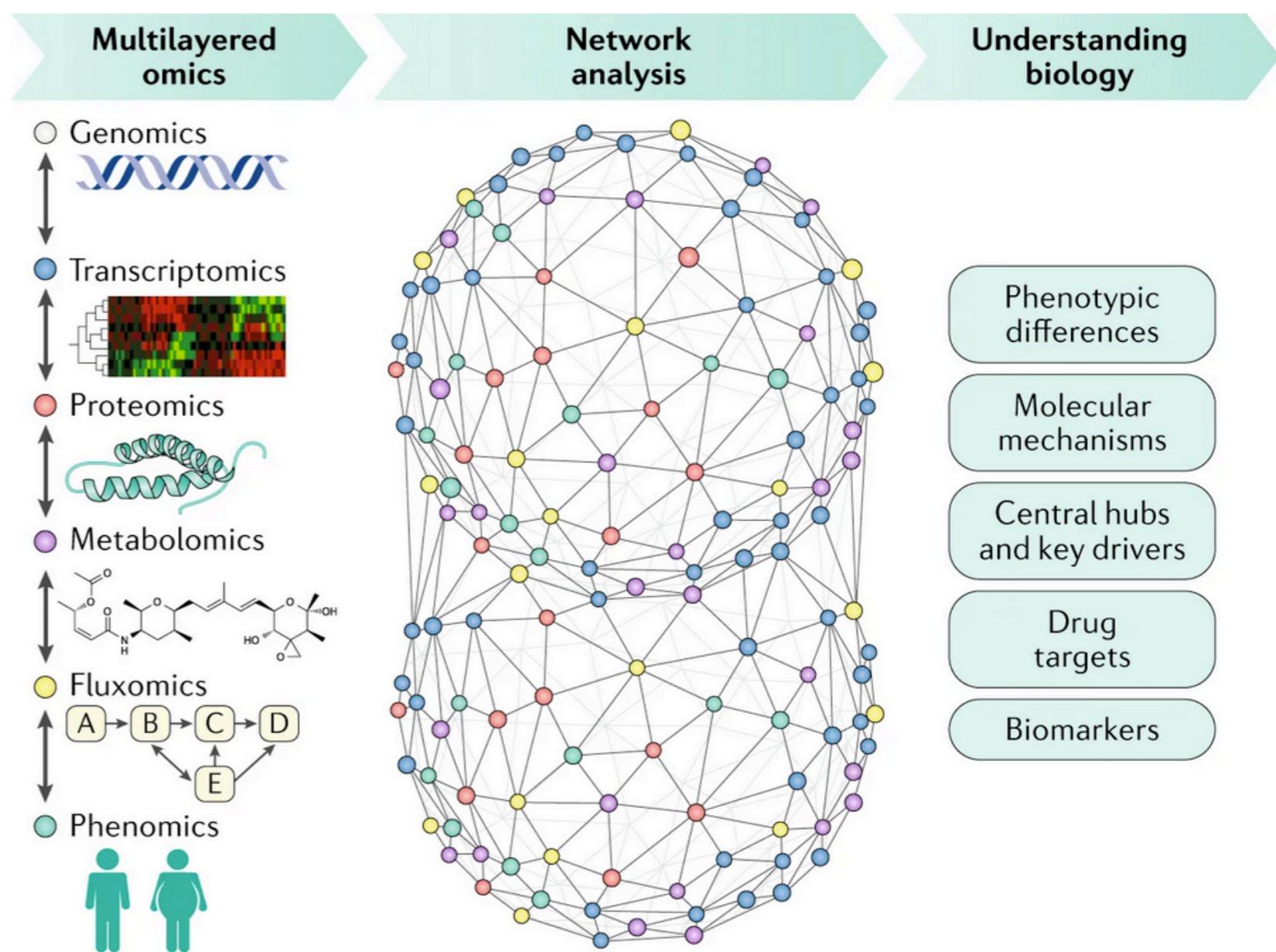
Moving from reductionist approaches towards global characterisations



# How to tackle biological complexity?

Integrative approaches, and global patterns

- Feature association
- Network analysis
- Modeling  
(Genome-scale metabolic modeling)



# What are networks?

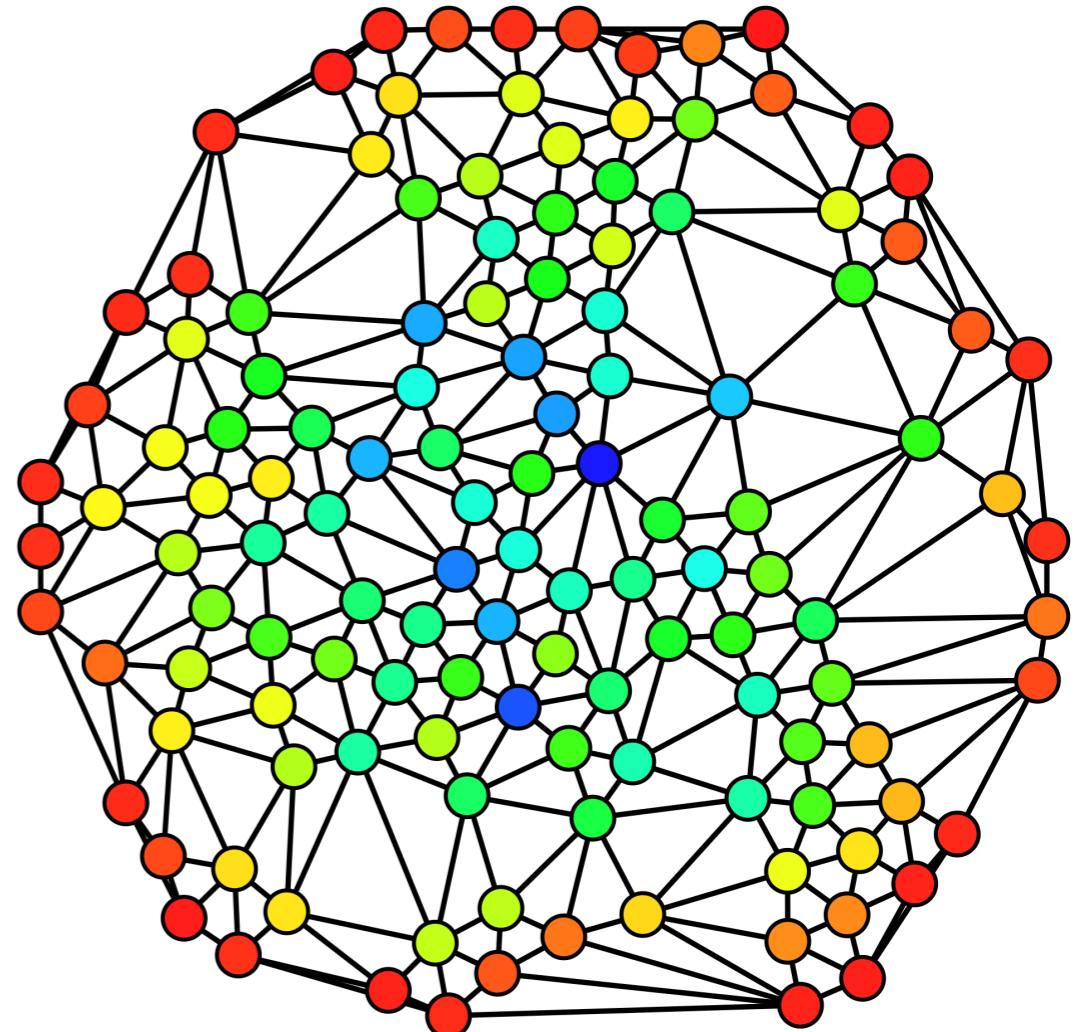
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Networks are representations of complex systems

Permit defining and studying global properties of interacting components

Give us insight not easily achieved by other approaches:

- Comprehensive
- Coordinated



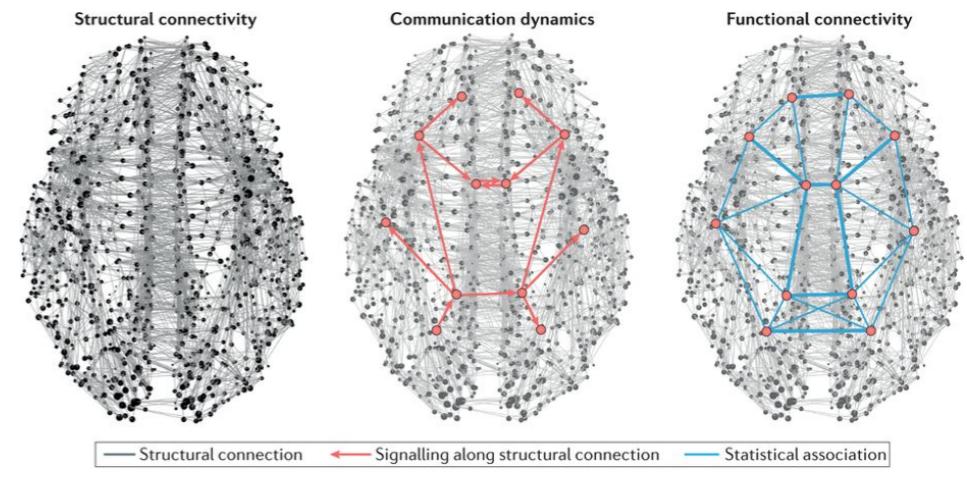
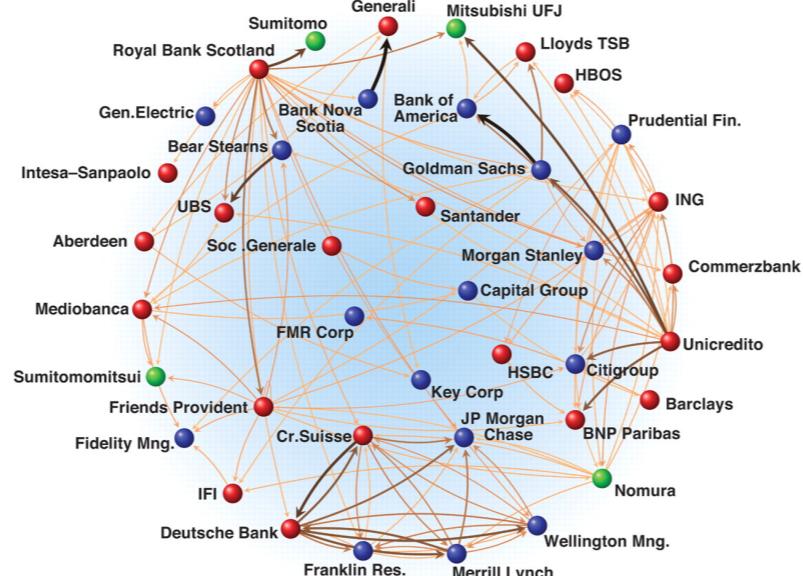
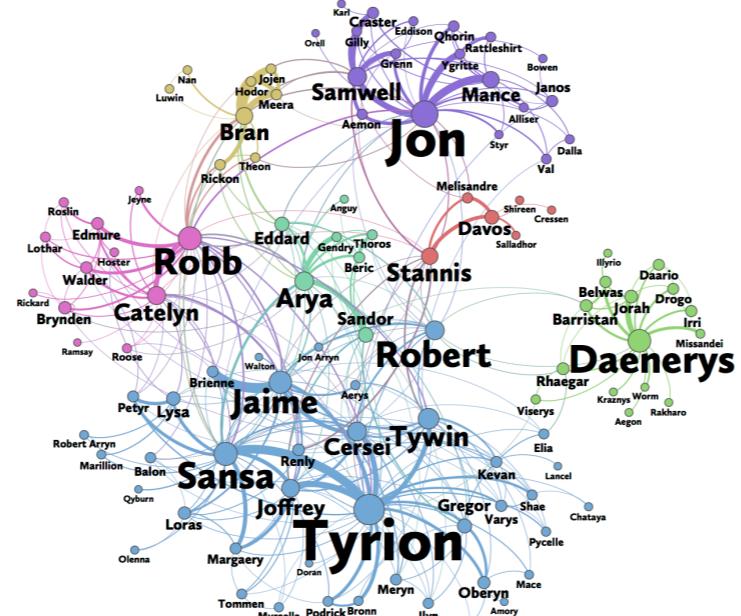
# What are networks?

Social

Economic

Communication

Neuronal



Nature Reviews | Neuroscience

Beveridge & Shan 2017  
Helen Knight MIT News 2013  
Schweitzer et al 2009  
Avena-Koenigsberger et al 2018

# What are biological networks?

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Protein - Protein interaction (PPI) networks

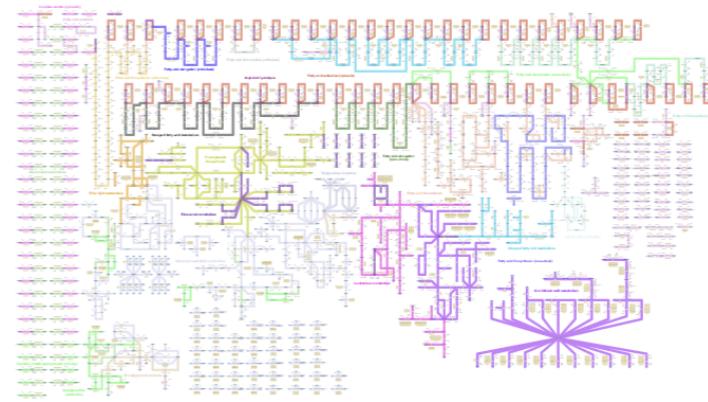
Transcription-factor regulatory networks

Gene - gene co-expression networks

Signal transduction networks

# What are biological networks?

Metabolite - Enzyme - Signal - Genes (GEMs)

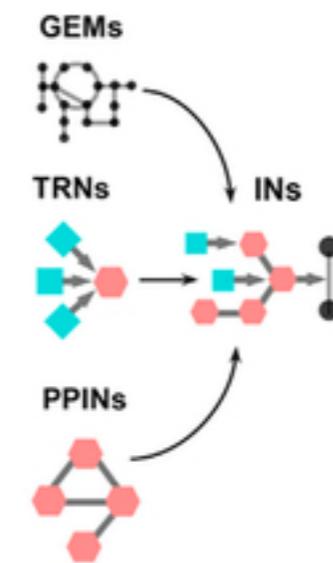
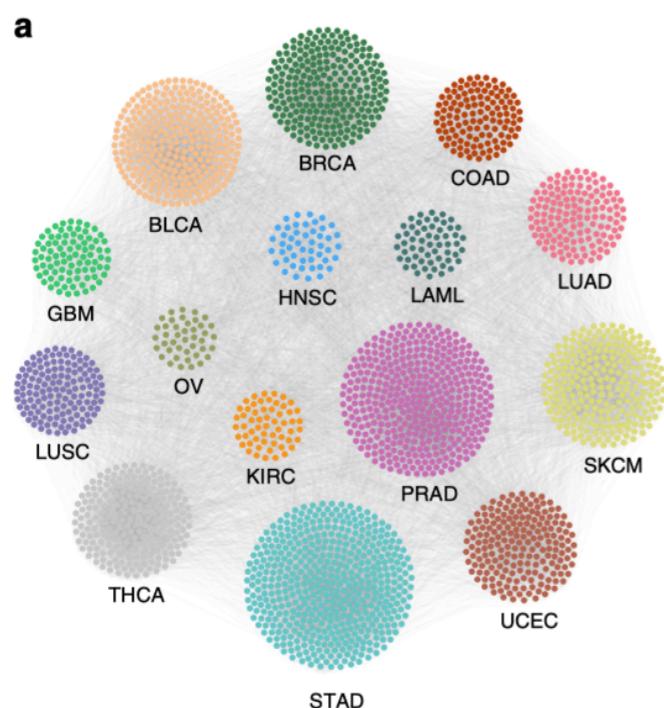
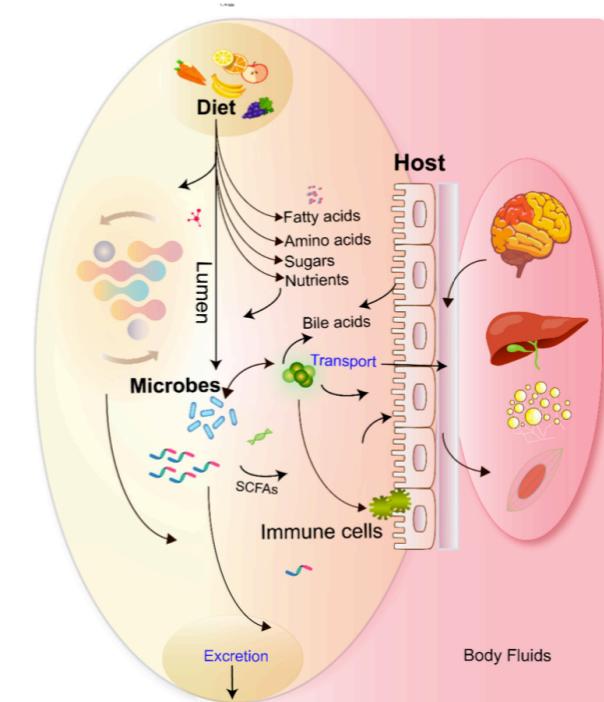
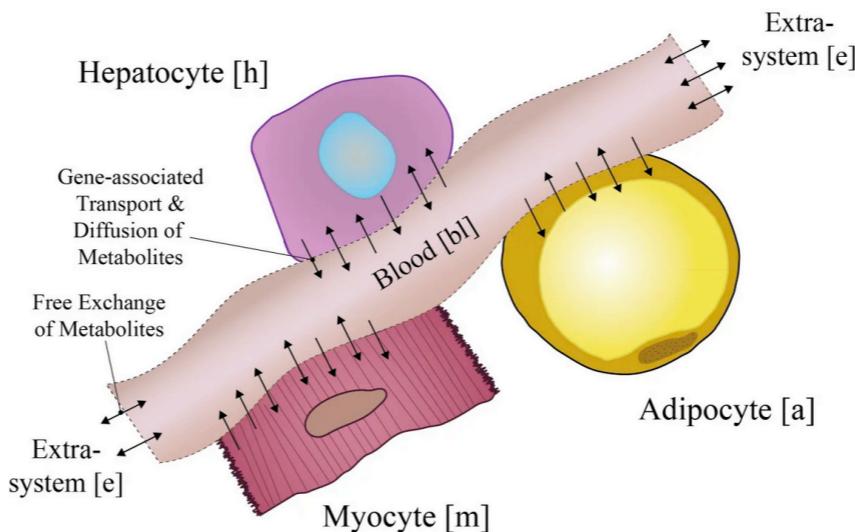


Multi-tissue networks

Multi-species networks

Disease networks

Integrated networks

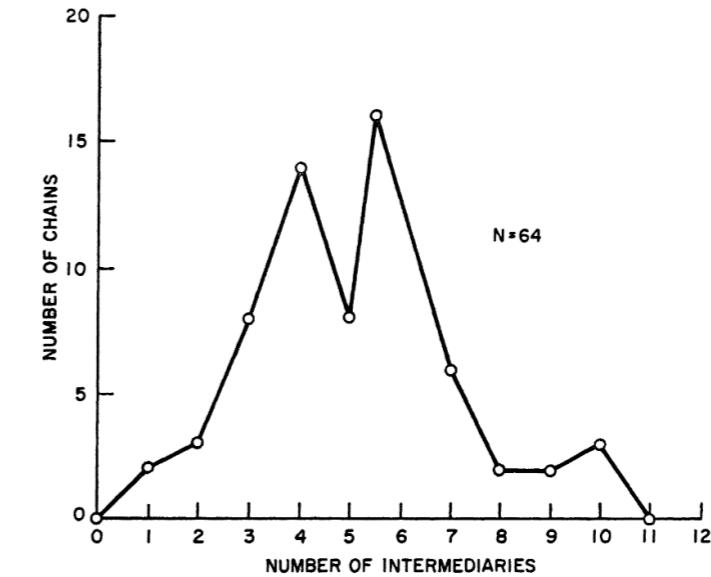


<https://metabolicatlas.org/>  
Bordbar et al 2011  
Sen & Oresic 2019  
Cheng et al 2019  
Lee et al 2016

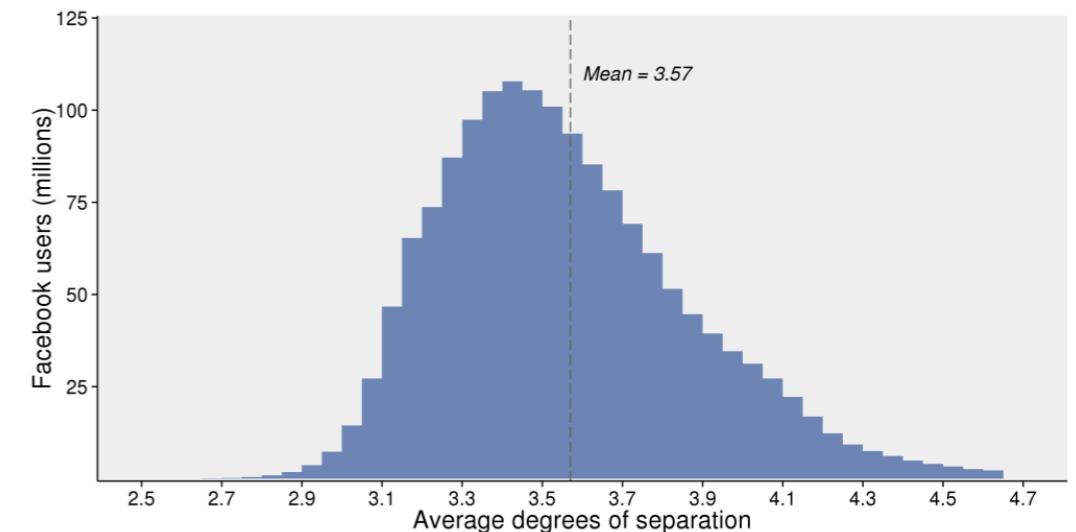
# Small world

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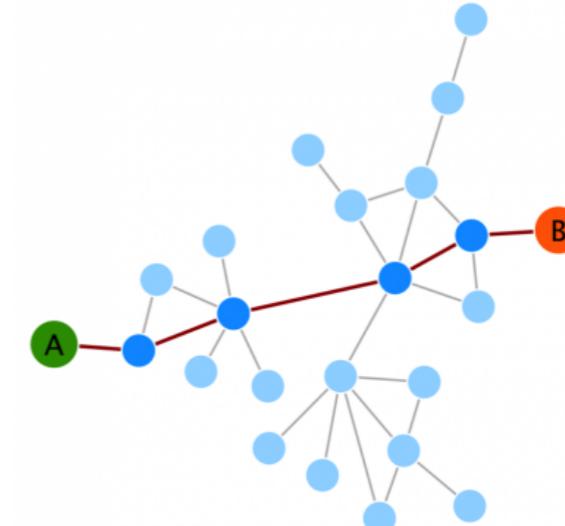
Stanley Milgram (1967) - 6 degrees  
("6 degrees of Kevin Bacon")



Backstrom et al. (2016) - 3.6 degrees



Biological Networks



# Why look at network topology?

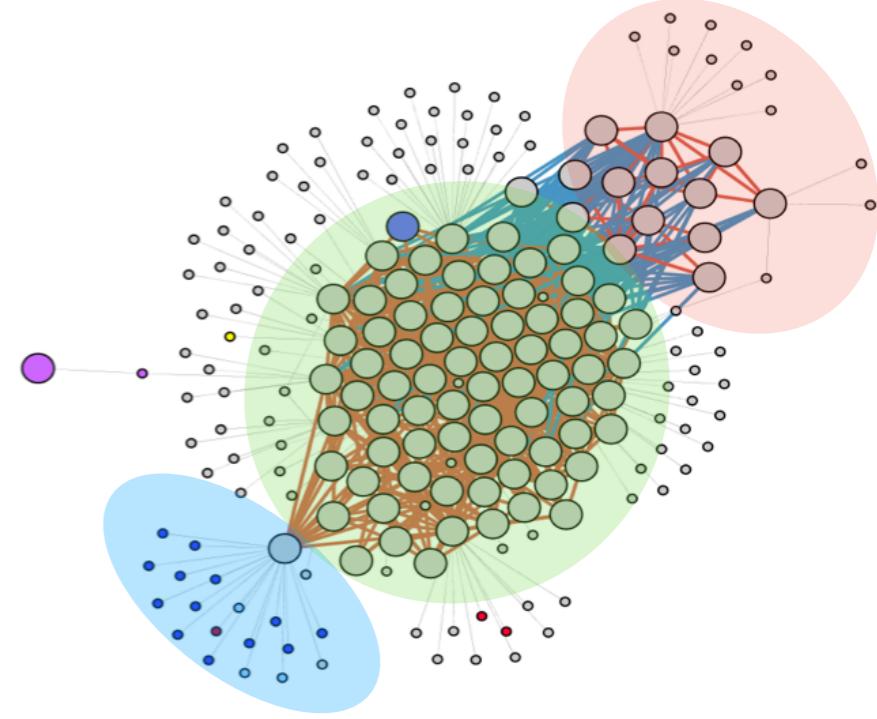
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Use networked systems to:

- Identify global / local patterns
- Identify functional properties
- Make predictions

Examples:

- How associated are the elements of my network?
- What are its first-hand associated elements?
- What are the groups of closely-associated elements in my network?  
What are their functional relationships?
- What are the “key” elements in my network?
- What are the “weakest” links in the network?



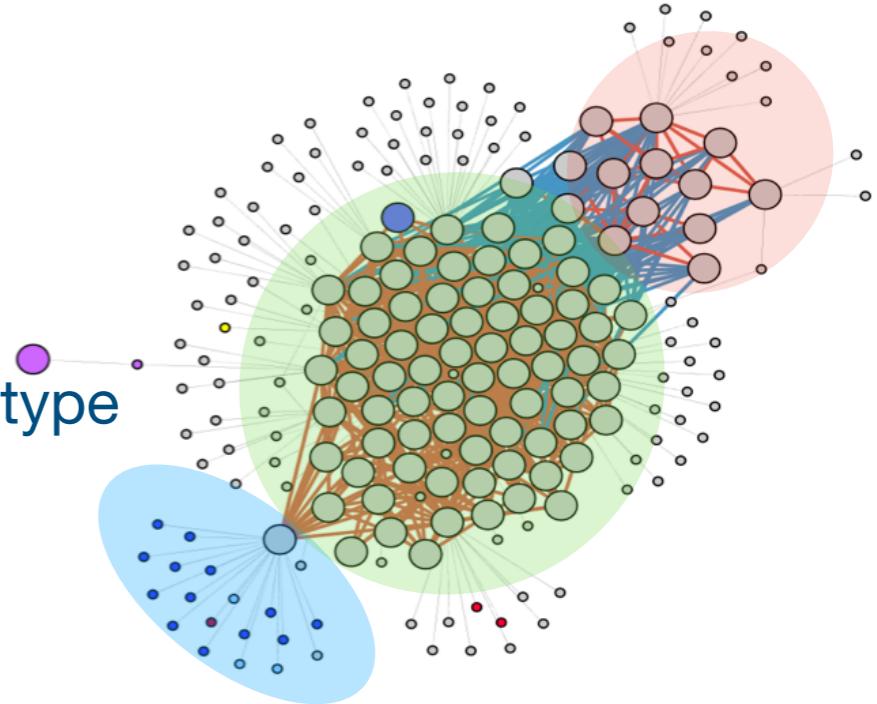
# What is my biological network?

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Any distance matrix may be translated to a network format

Many standard analyses may be employed regardless of data type

...but care must be taken in generating the network



## Limitations:

- Some of the functional analyses depend on annotation
- Sample size
- Effect size
- False discovery

# Overview

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1. Introduction to network analysis

## **2. Terminology**

3. Network construction

4. Key network properties

5. Community analysis

Original sources of images provided as reference and hyperlinks, where applicable.

# Graphs, nodes, edges

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Graph G consists of a set of **nodes** ( $V$ ) interconnected by **edges** ( $E$ )

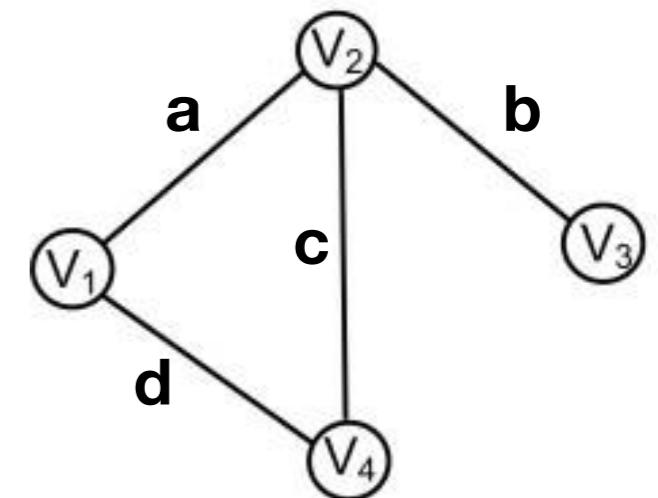
$$G = (V, E)$$

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{a, b, c, d\}$$

**Nodes** sometimes called **vertices**

Two connected nodes are called **neighbours**, **adjacent**, or **end-nodes**



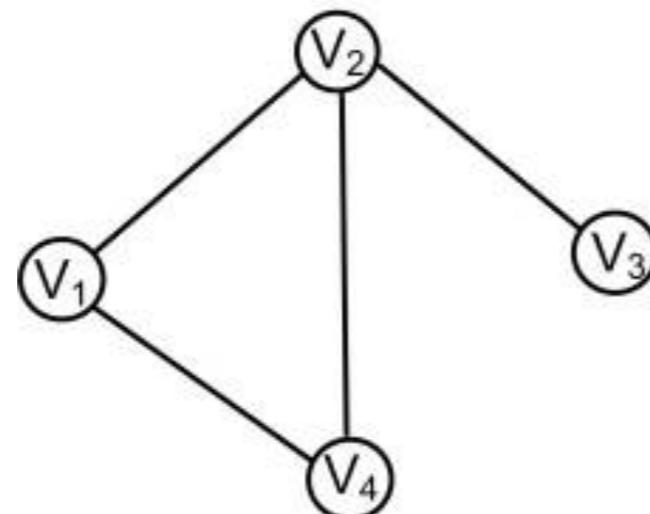
# Simple vs multigraphs

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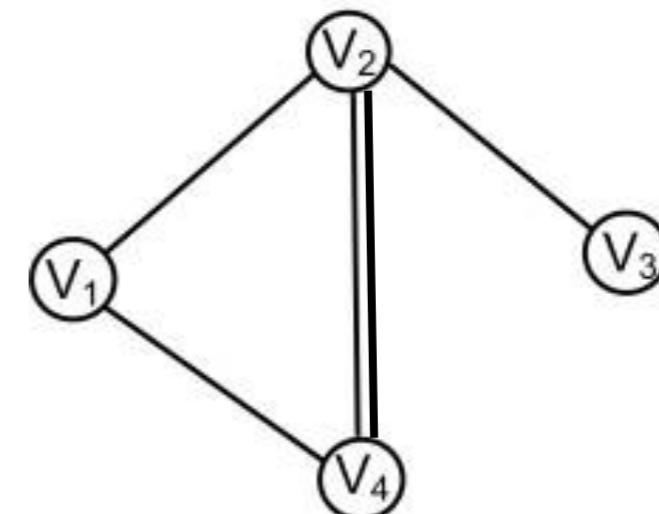
**Multigraphs** contain parallel edges

**Multi-edged** connections indicate different properties

**Simple**

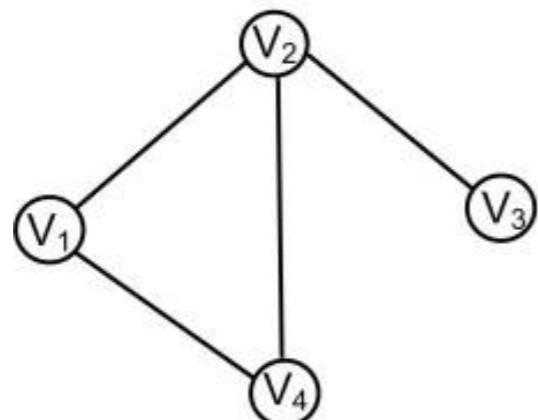


**Multigraphs**



# Directed vs undirected graphs

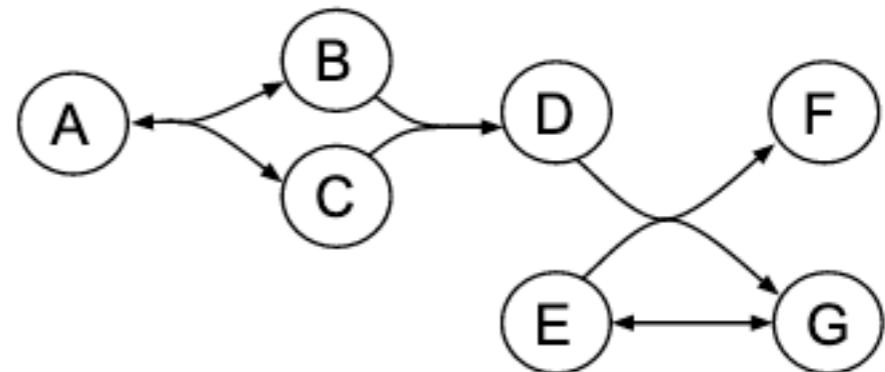
**Undirected graphs:**  
co-expression networks



**Directed graphs:**  
metabolic networks

**Reaction 1:** A → B + C  
**Reaction 2:** B + C → D  
**Reaction 3:** D + E → F + G  
**Reaction 4:** E → G  
**Reaction 5:** B + C → A  
**Reaction 6:** G → E

(a) Reaction network

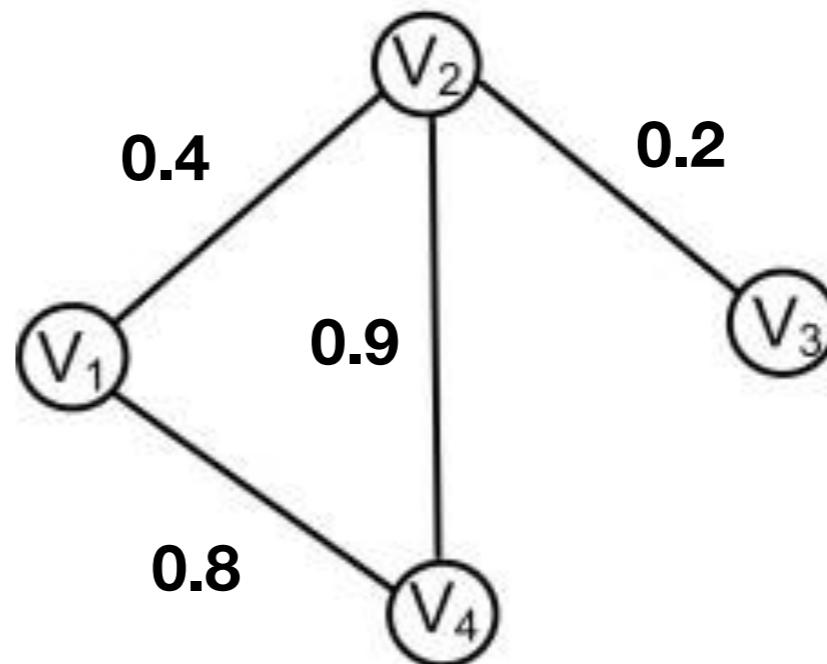


# Weighted vs unweighted graphs

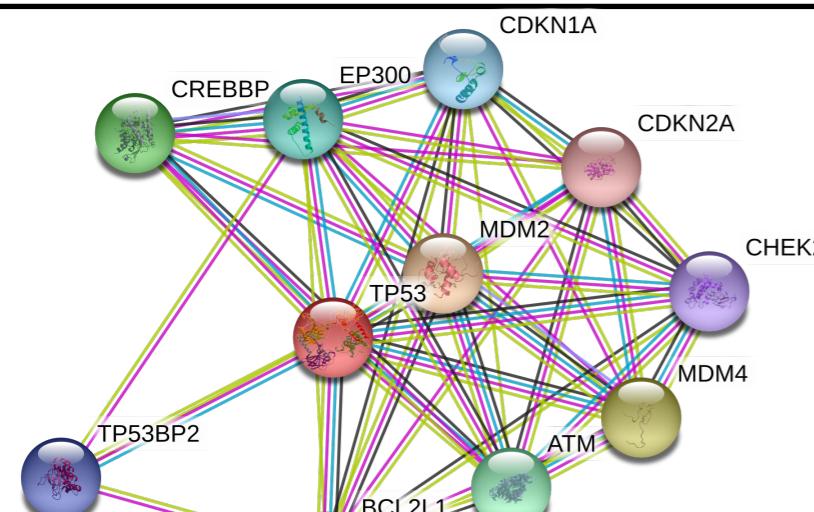
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**Weighted edges** associate a value to an interaction between two nodes. Usually give the confidence in the interaction.

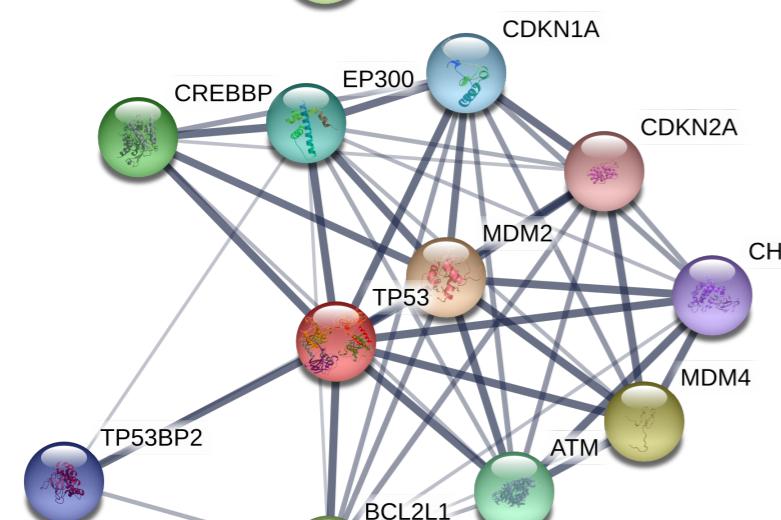
E.g. weighted co-expression networks



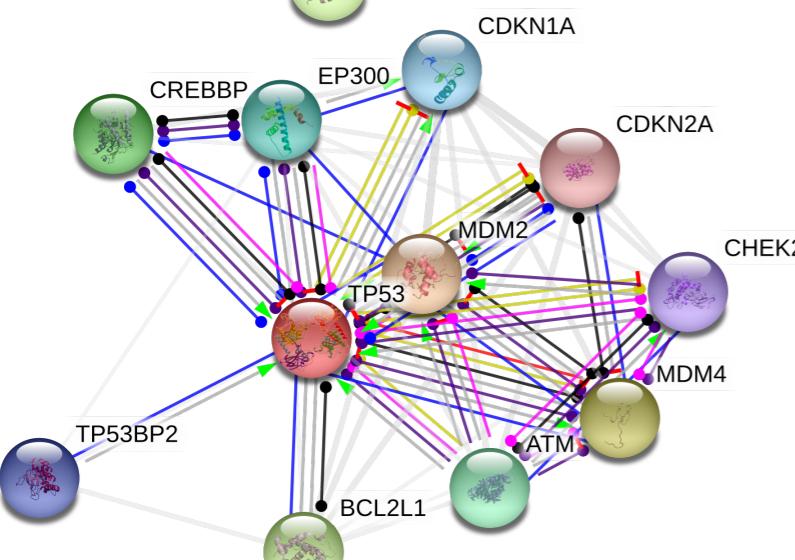
# STRING-db.org: TP53



Multi-edged



Weighted multi-edged



Multi-edged directed



# Bipartite graphs

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A graph

$$G=(V,E)$$

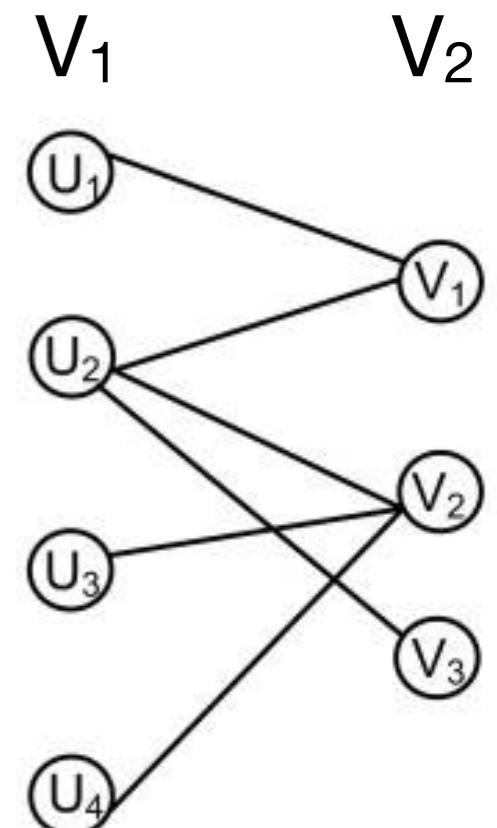
may be partitioned into two sets of nodes ( $V_1$ ,  $V_2$ )  
such that

$$u \in V_1 \text{ and } v \in V_2$$

All  $e_i$  has end-nodes in  $V_1$ ,  $V_2$

A **subgraph** of  $G$  will thus be given by

$$G_1 = (V_1, E_1)$$



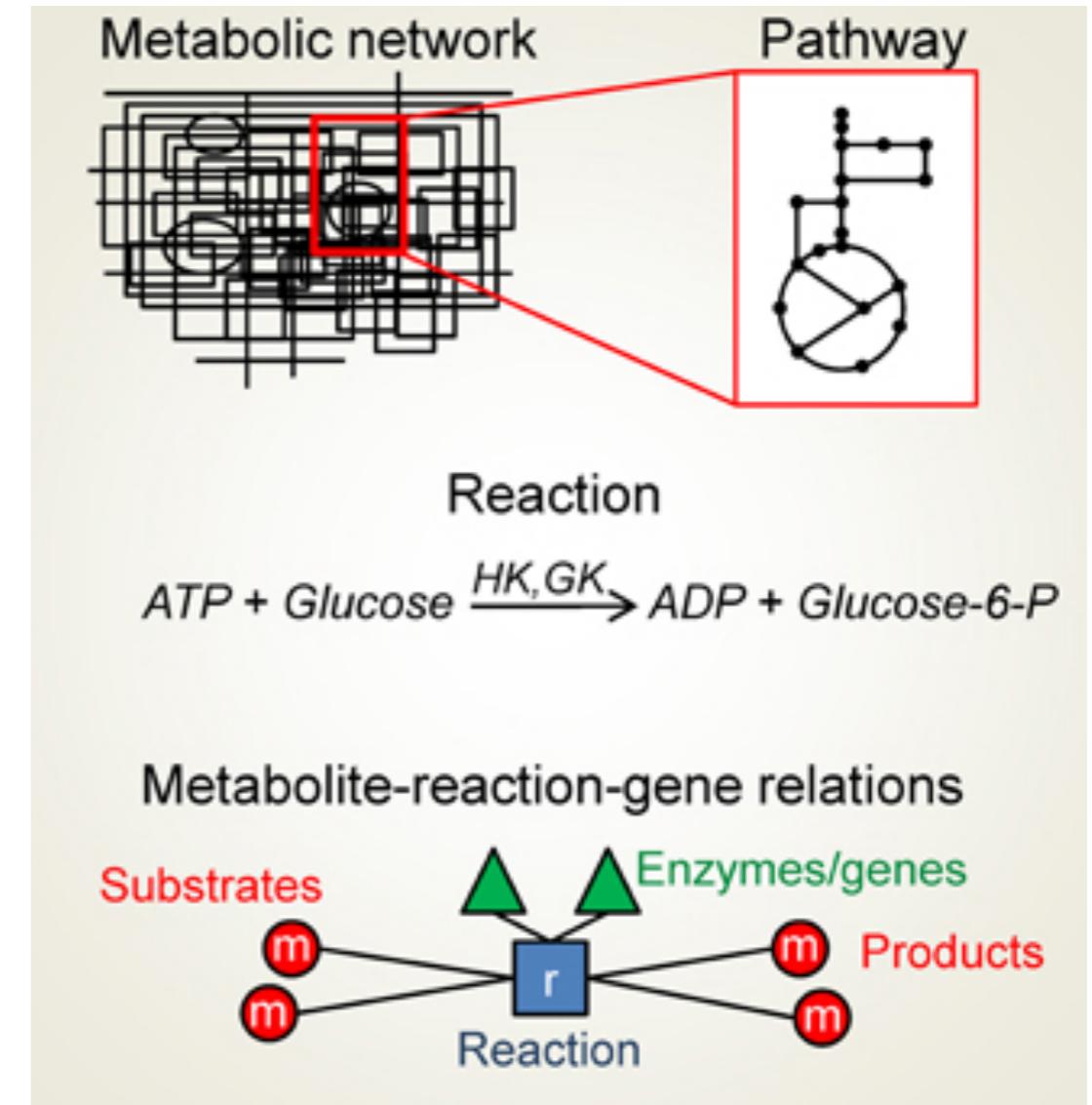
# Bipartite and $k$ -partite graphs

Example of  $k$ -partite graph:

Enzyme - Reaction

Metabolite - reaction - enzyme

$k$ -partite graphs display  $k$ -types of nodes

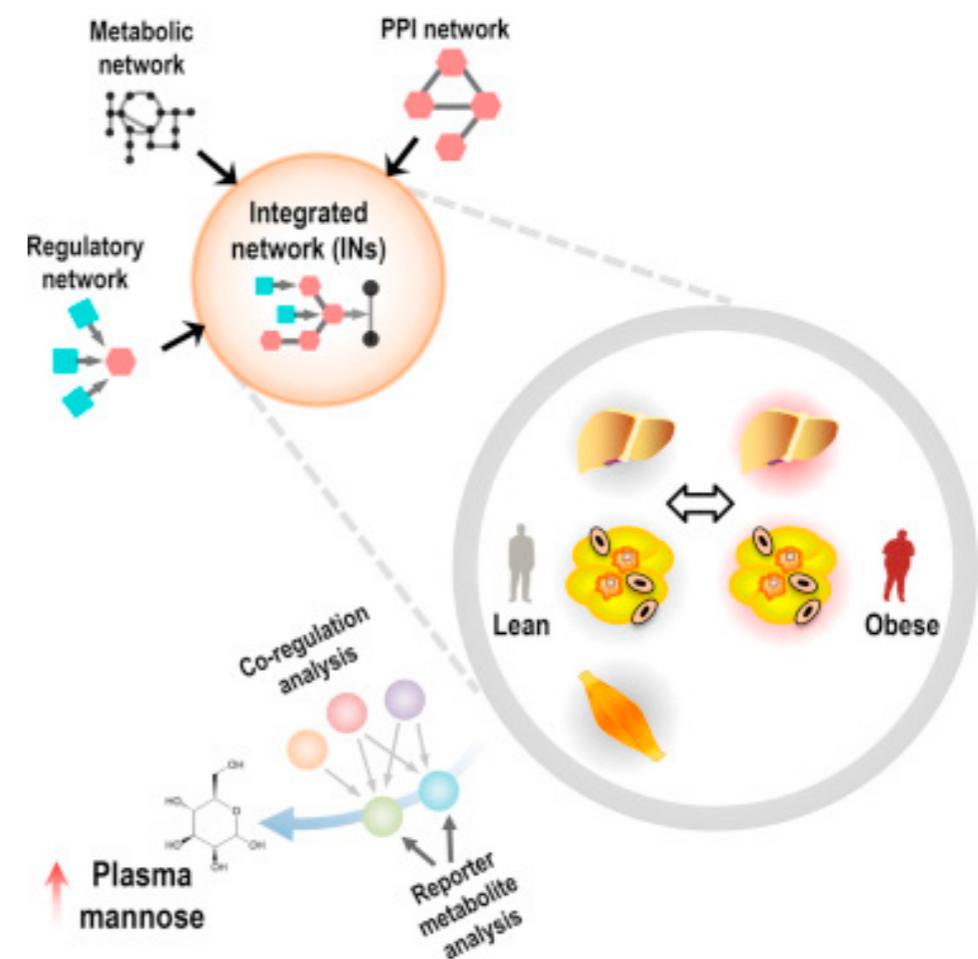


# *k*-partite graphs

Multi-modal (*k*-partite) networks may be generated from different sources

- Transcription-factor - Gene (DNAseq)
- Gene-gene (Co-expression, PPI, GEMs)
- Gene-metabolite (GEM)
- Metabolite-metabolite (GEM)

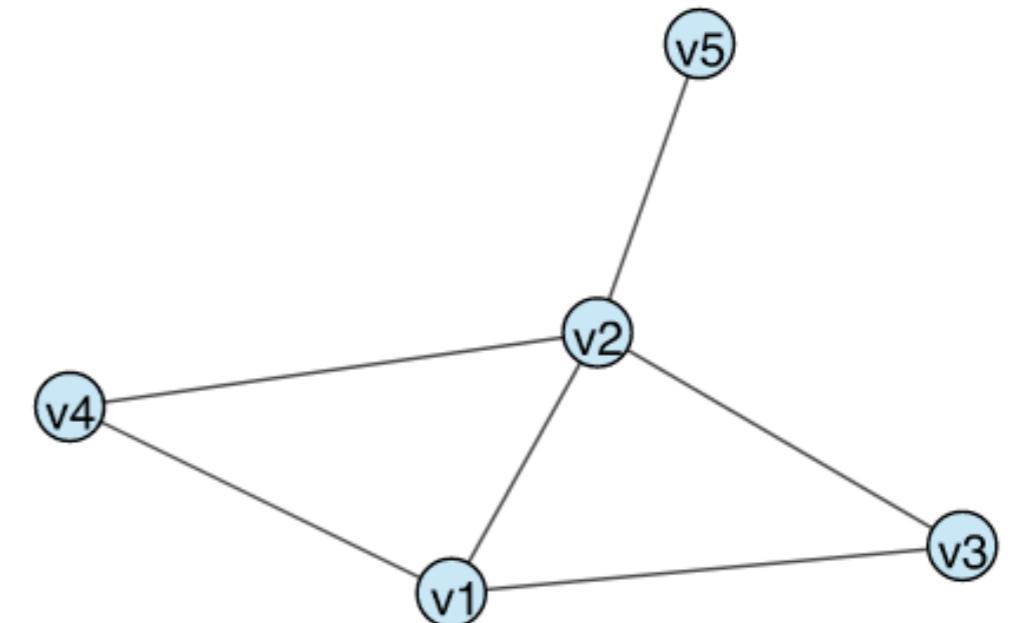
## Integrated Networks



# Adjacency matrix (undirected graphs)

**Vertex association  
(undirected network)**

| n1 | n2 |
|----|----|
| v1 | v2 |
| v1 | v4 |
| v2 | v4 |
| v2 | v3 |
| v2 | v5 |
| v1 | v3 |



**Adjacency matrix is symmetric**

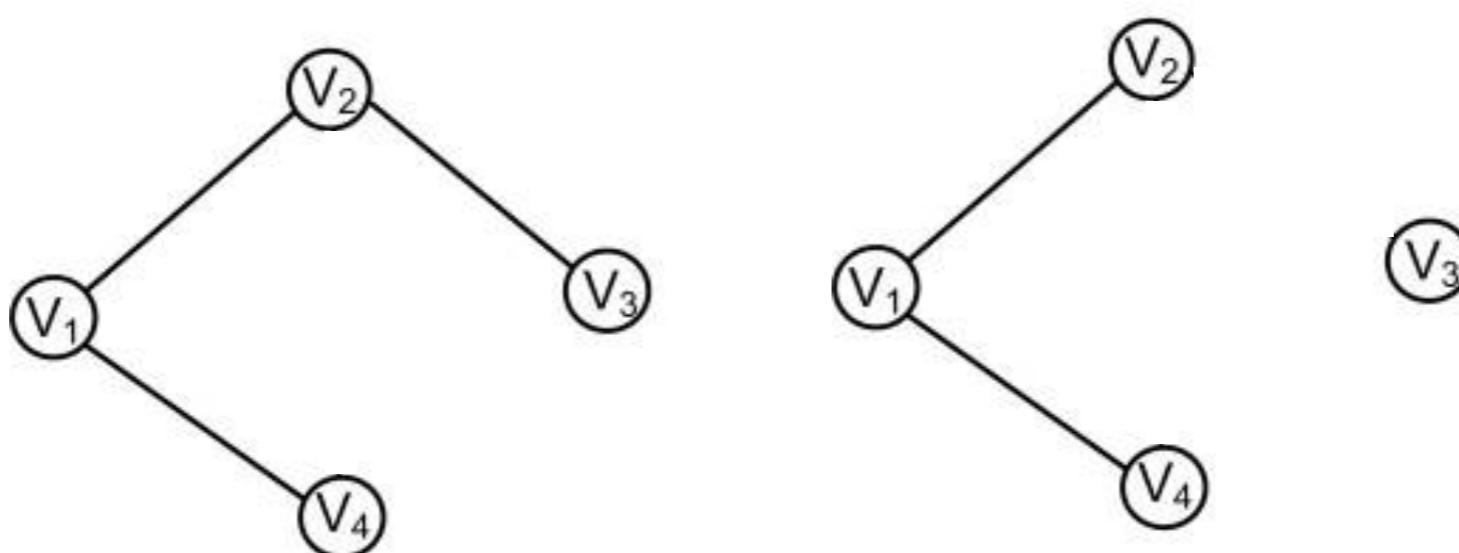
|    | v1 | v2 | v3 | v4 | v5 |
|----|----|----|----|----|----|
| v1 | 0  | 1  | 1  | 1  | 0  |
| v2 | 1  | 0  | 1  | 1  | 1  |
| v3 | 1  | 1  | 0  | 0  | 0  |
| v4 | 1  | 1  | 0  | 0  | 0  |
| v5 | 0  | 1  | 0  | 0  | 0  |

# Connected vs disconnected networks

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**Connected network:** there is at least 1 path connecting all nodes in a network

**Disconnected network:** some of the nodes are unreachable

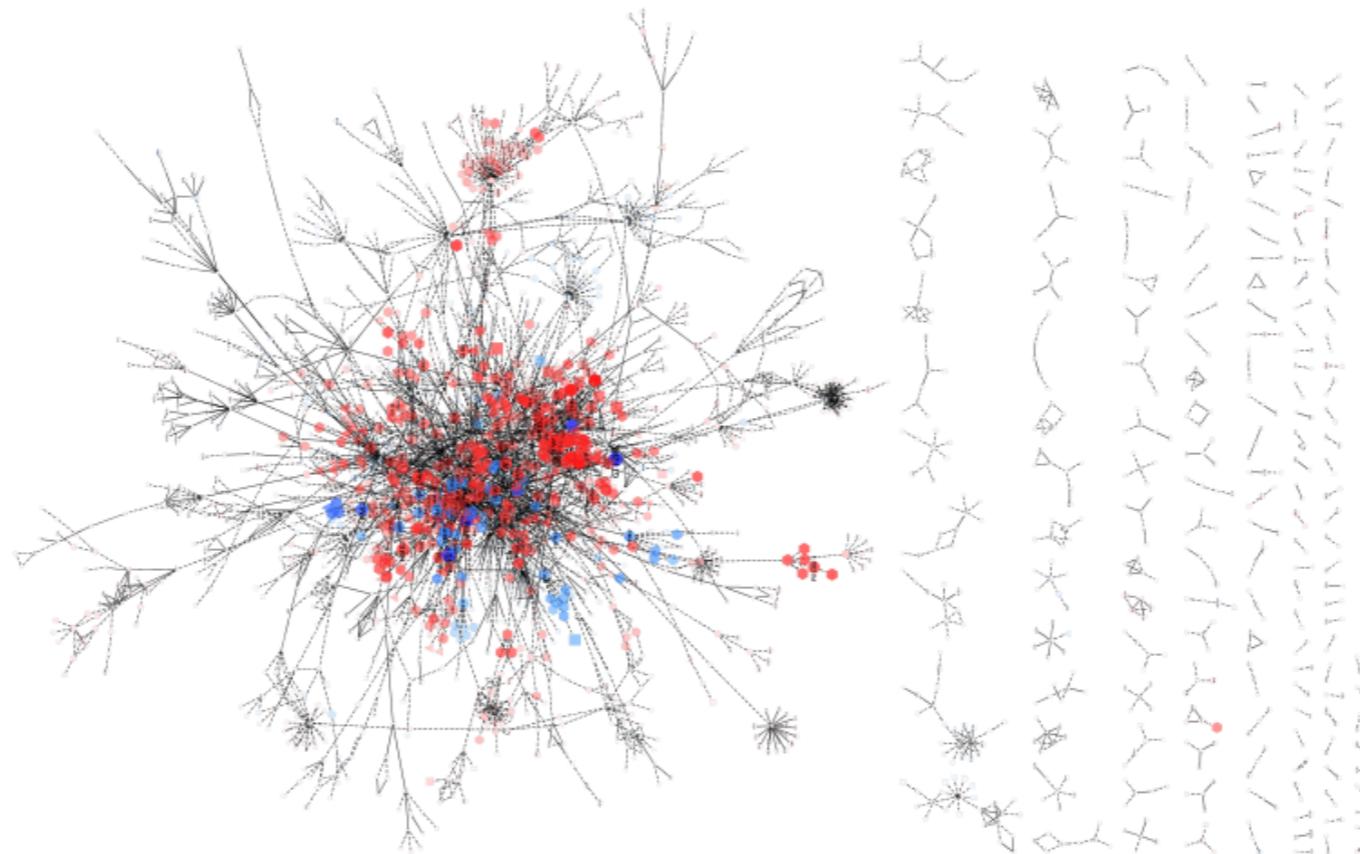


# Connected components

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**Connected components** are those where all nodes of each subgraph are connected.

In biological networks, often the most insightful properties come from the **largest connected component(s)**



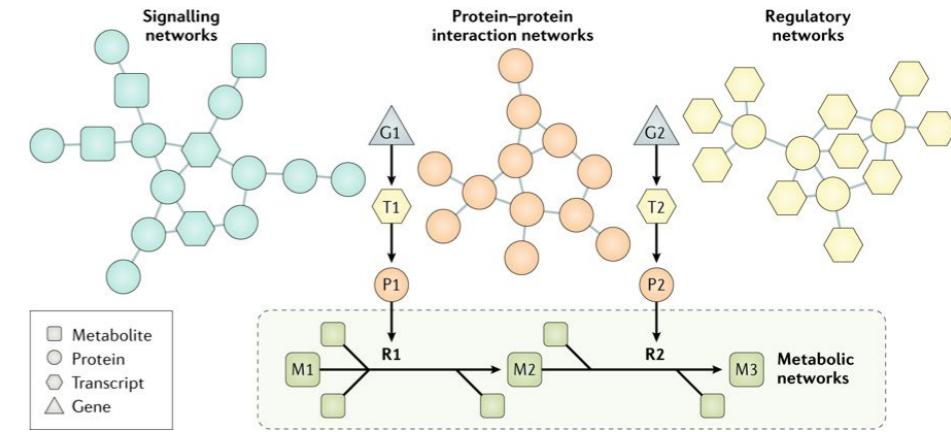
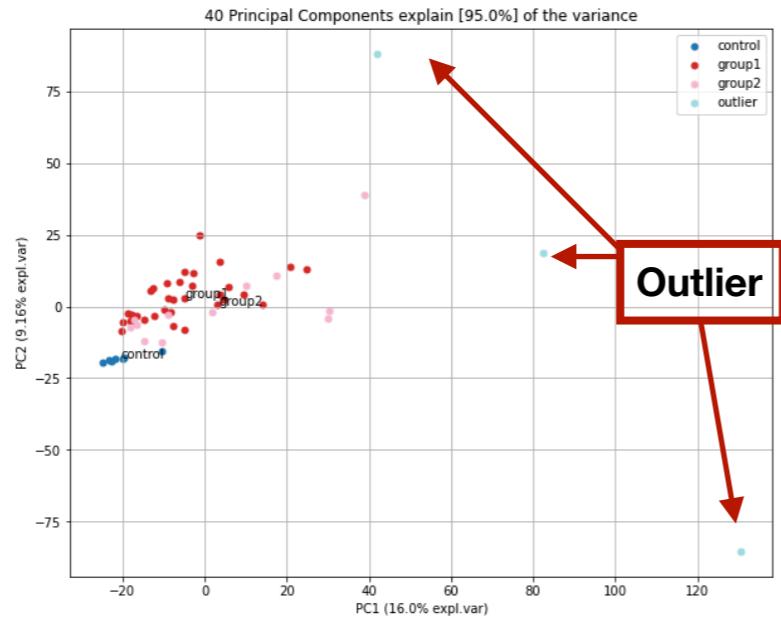
# Overview

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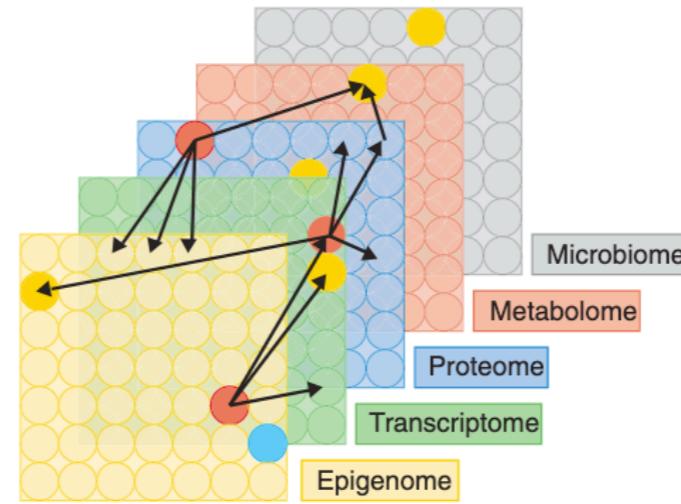
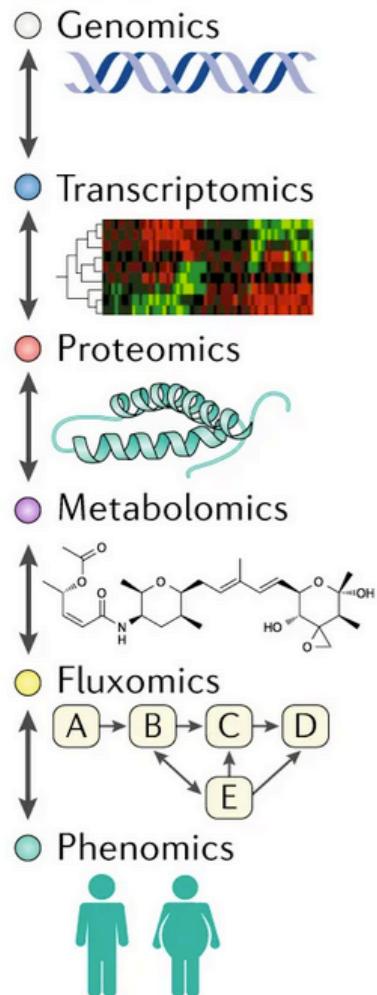
1. Introduction to network analysis
2. Terminology
- 3. Network inference**
4. Key network properties
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# Building networks



Raw → Pre-processing → Distance calculation → Graph analysis



Hasin 2017

Piening 2018

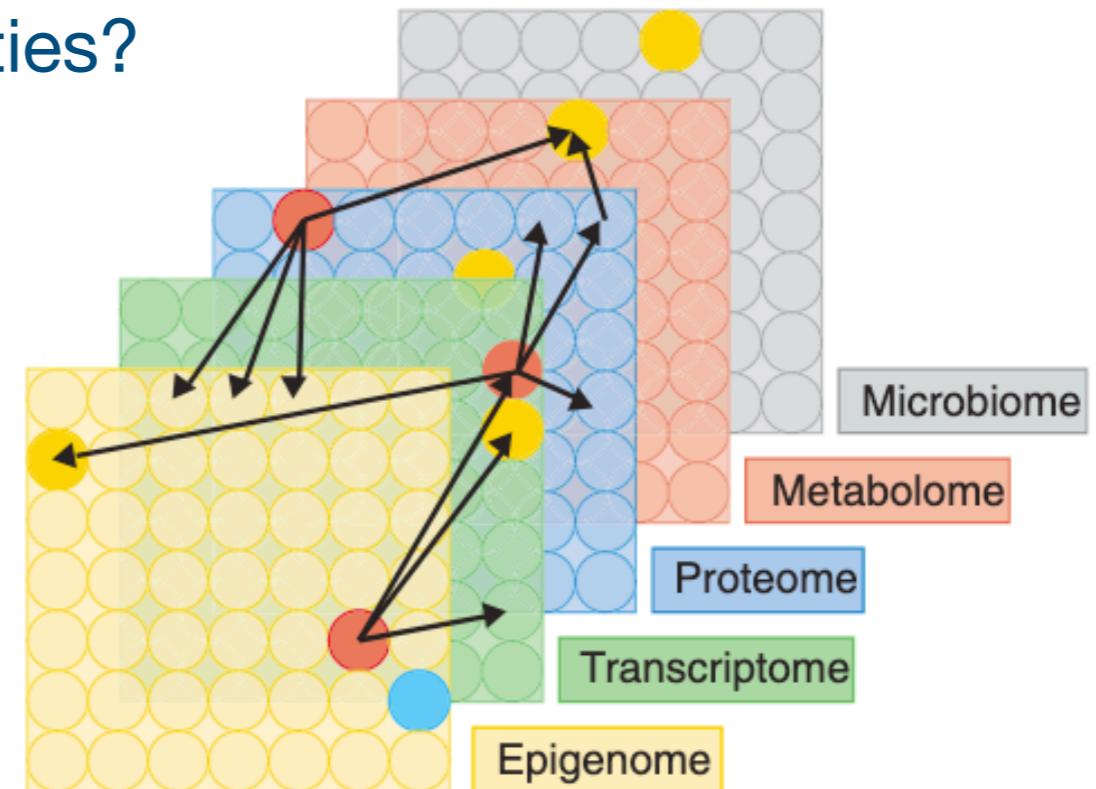
Mardinoglu 2018

# Interomic vs Intraomic networks

Networks may be build for individual omics or for their integration

What is my biological question?

- Do I want to analyse vertical relationships between features?
- Biological motivation for integrating omics with different coverage (e.g. transcriptomic and proteomic)
- Do I want to extract functional properties?



# Different approaches for network inference

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- 1. Feature association
  - 2. K-nearest neighbour graph (k-NNG) construction
  - 3. Pathway-based
  - 4. Genome-scale metabolic models
  - 5. Network deconvolution
- |  |                                       |
|--|---------------------------------------|
|  | <b>No prior graph structure</b>       |
|  | <b>Based on available information</b> |
|  | <b>Filter indirect effects</b>        |

# 1. Association analysis

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Balanced dataset for group sizes

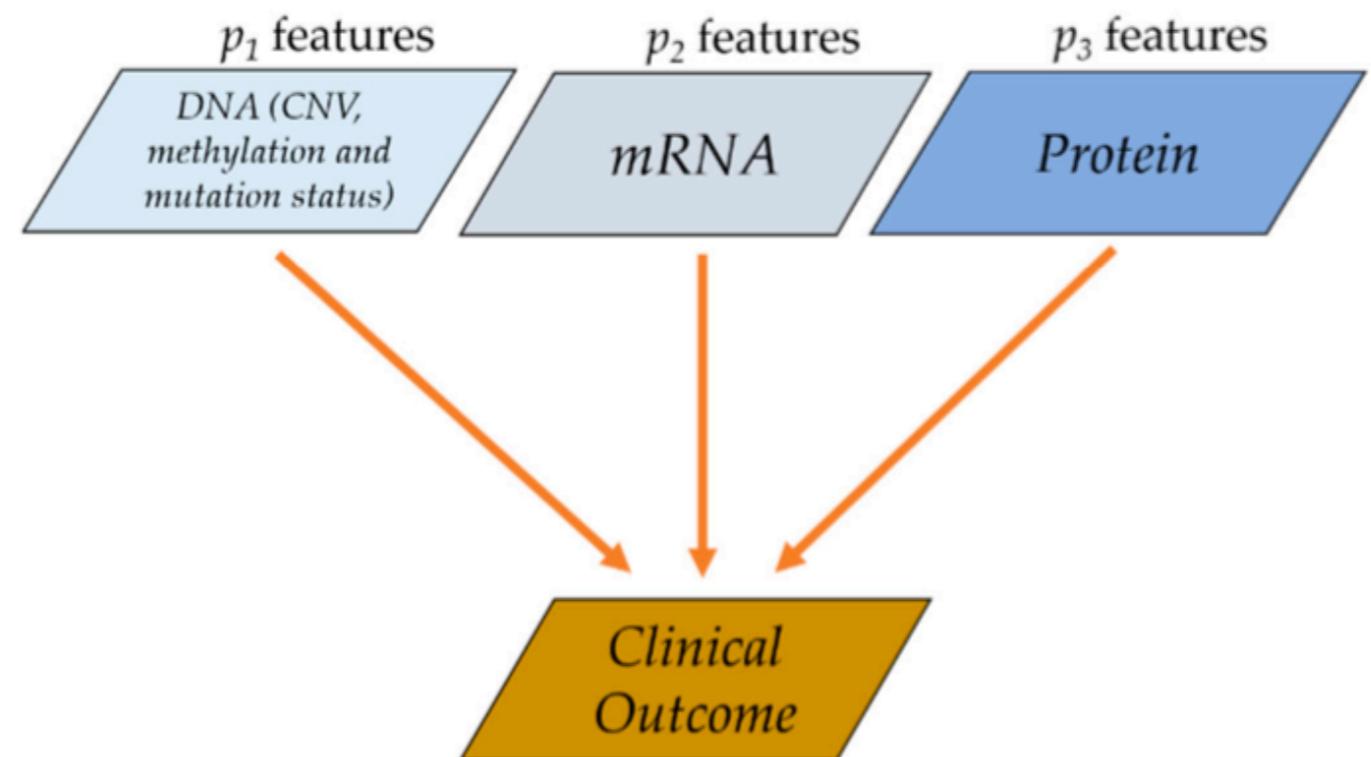
GroupA (80 samples) vs GroupB (20 samples)

GroupA (50 samples) vs GroupB (50 samples)

Common approach: compute correlations between different features

- Spearman
- Pearson

Extend known associations



# 1. Association analysis

Easy to interpret

Unweighted vs weighted ( $-1 \leq \rho \leq 1$ )

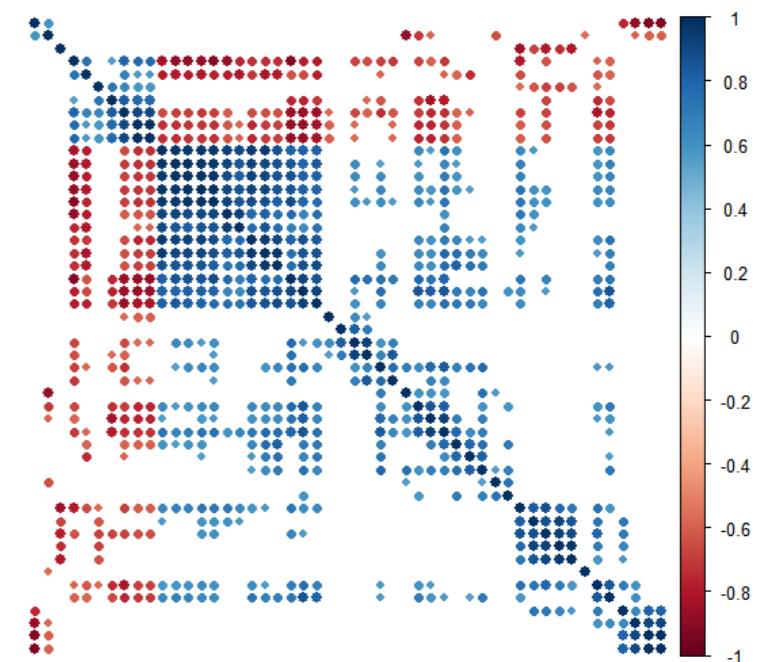
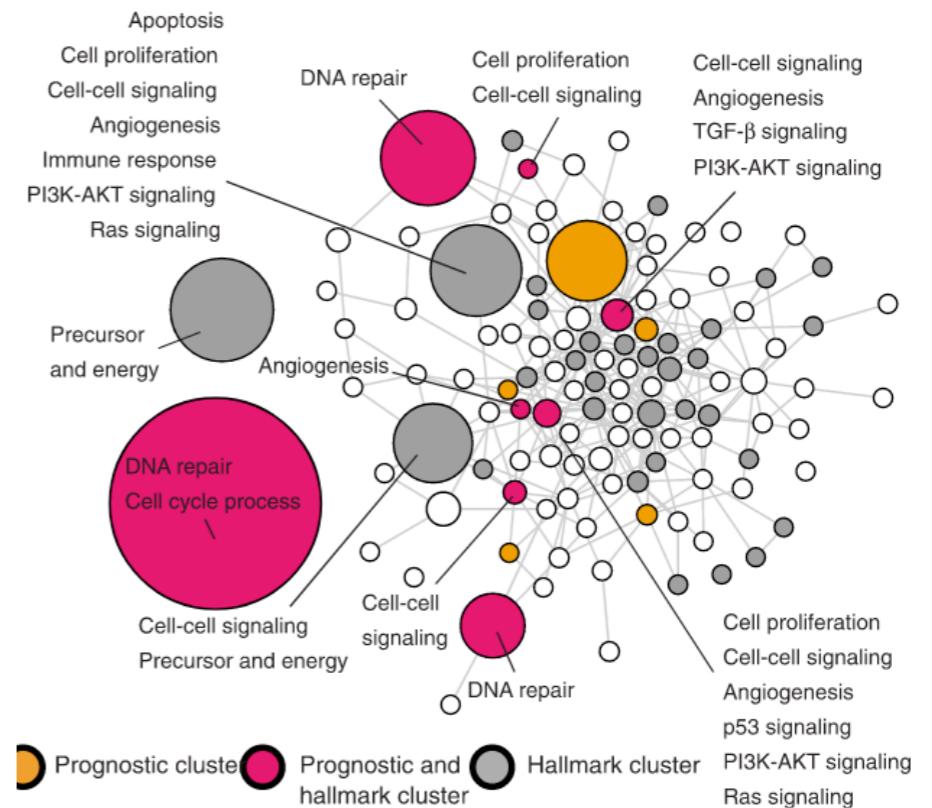
Unbalanced networks

Prone to type I errors

Filtering

- FDR vs Bonferroni
- Effect size cut-off

Need adjustment to possible confounding factors

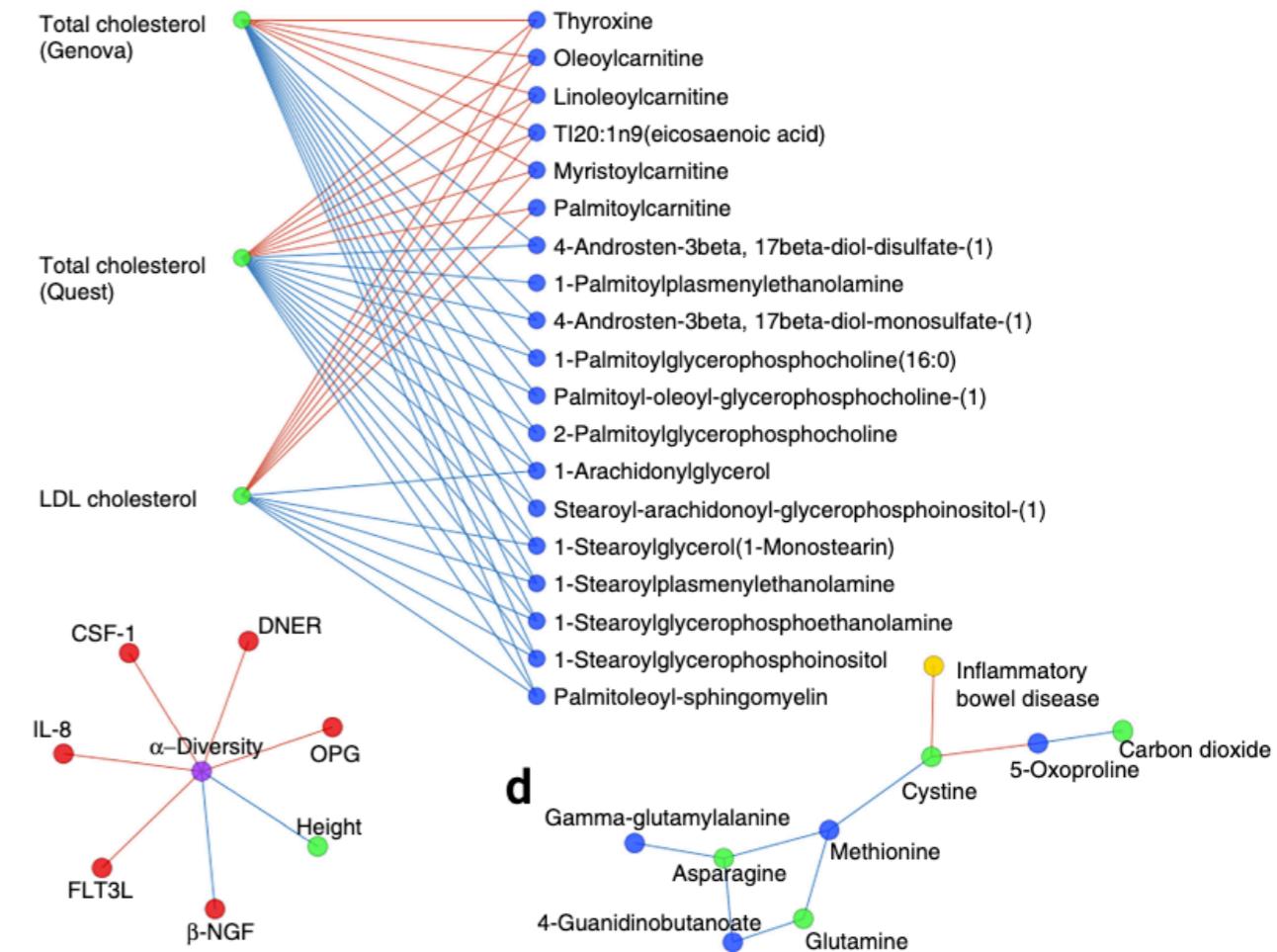
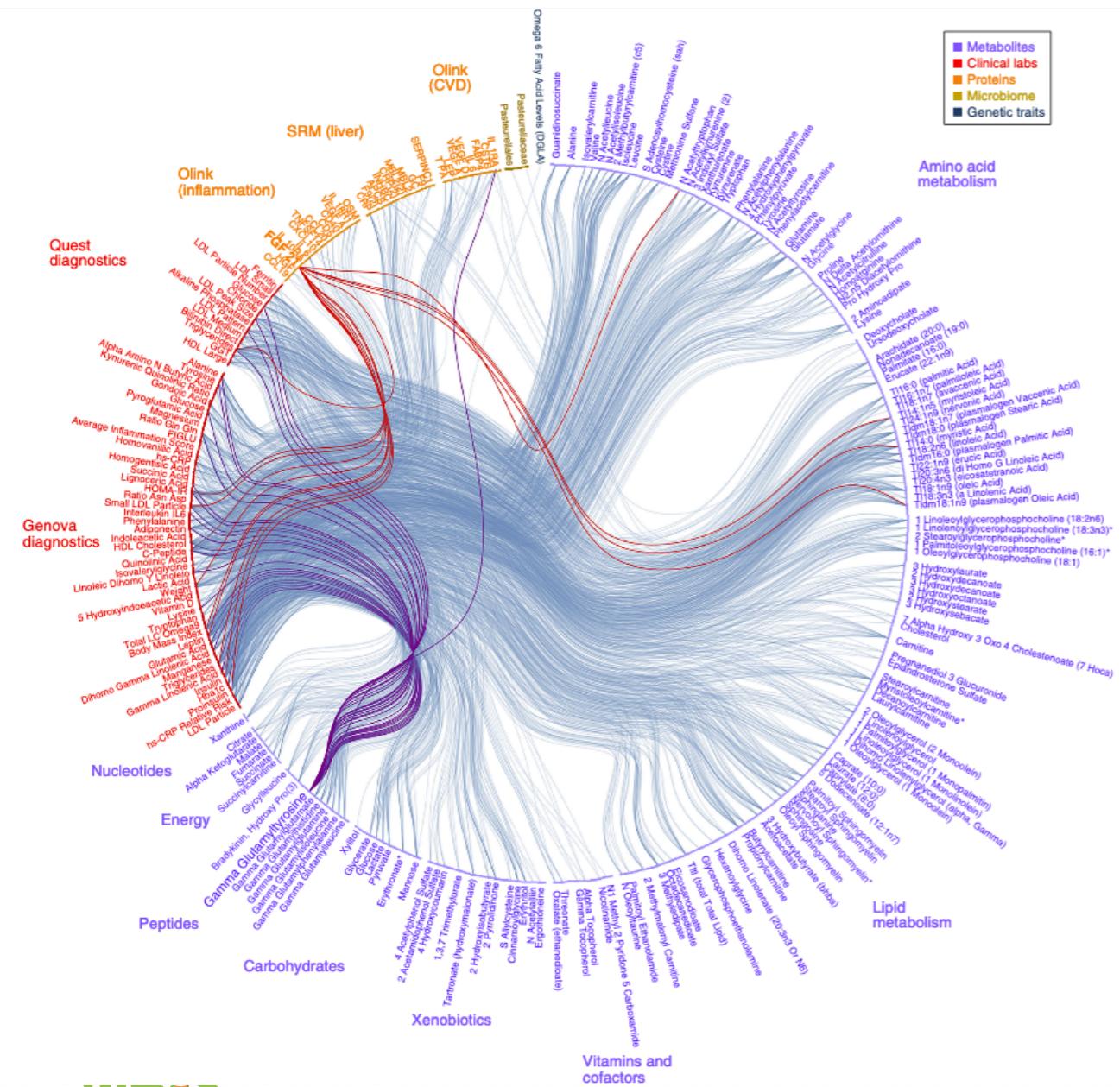


# 1. Association analysis

Adjusting for confounding factors: partial correlation analysis

Below:

- gender and age are known confounding factors
- feature regression on confounding factors, followed by correlation on the residuals of each model



# Overview

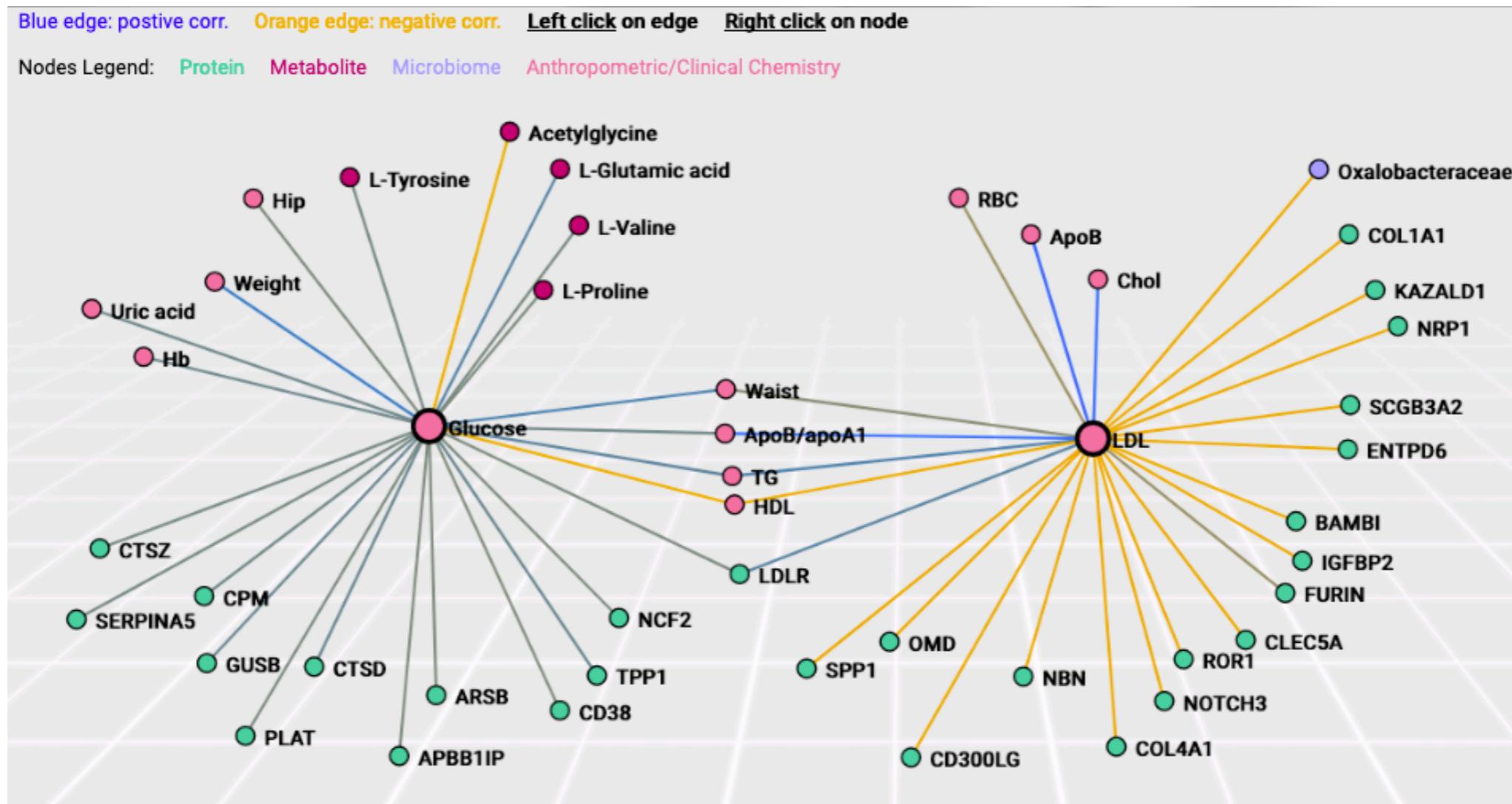
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1. Introduction to network analysis
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Original sources of images provided as reference and hyperlinks, where applicable.

# Motivation

You have built an association network (e.g. PPI, multi-omic).  
How to identify pivotal features, their organization, and biological characteristics?



# Key network properties

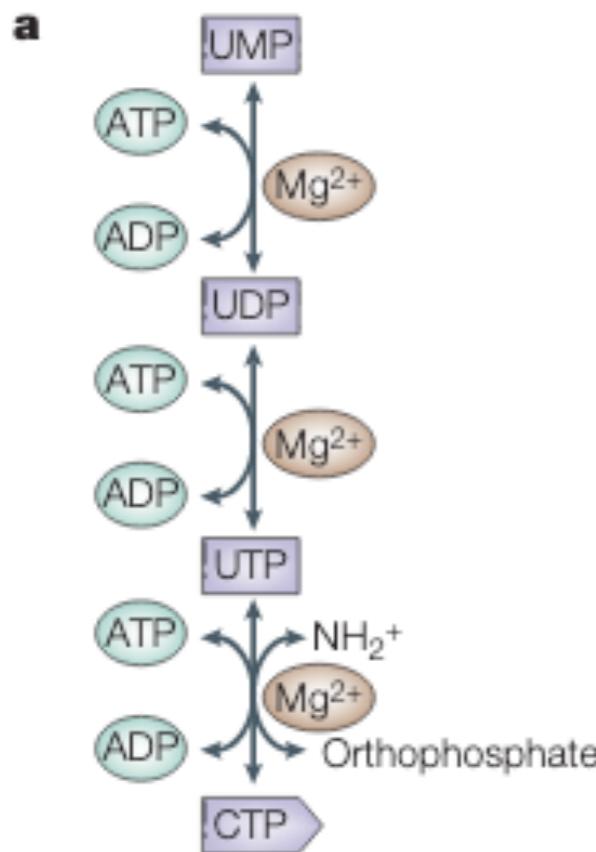
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- 1. Network representations**
- 2. Network density**
- 3. Paths**
- 4. Centrality**
- 5. Clustering coefficient**
- 6. Degree and connectivity distributions**

# 1. Network representations

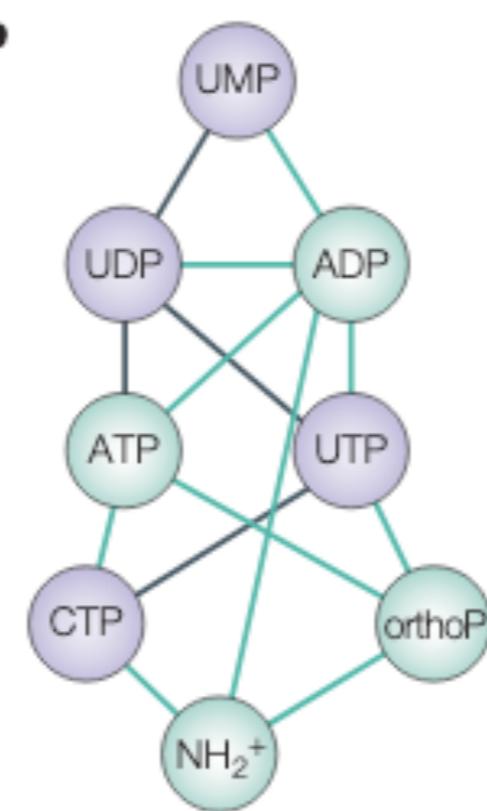
## Representations of a metabolic network: pyrimidine metabolism

### Metabolism



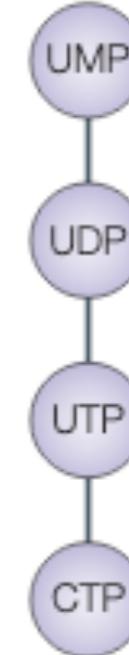
(directed graph)

Graph representation:  
metabolites and co-factors



(undirected graph)

metabolite-metabolite  
association



(undirected graph)

Other representations: Protein-Protein, Protein-Metabolite

## 2. Network density

A **dense graph** is a graph where the number of edges approximates the maximum possible number of edges for the given node number.

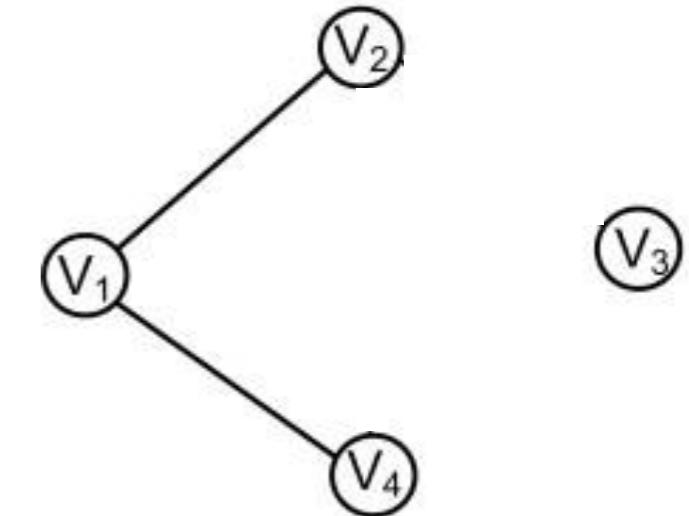
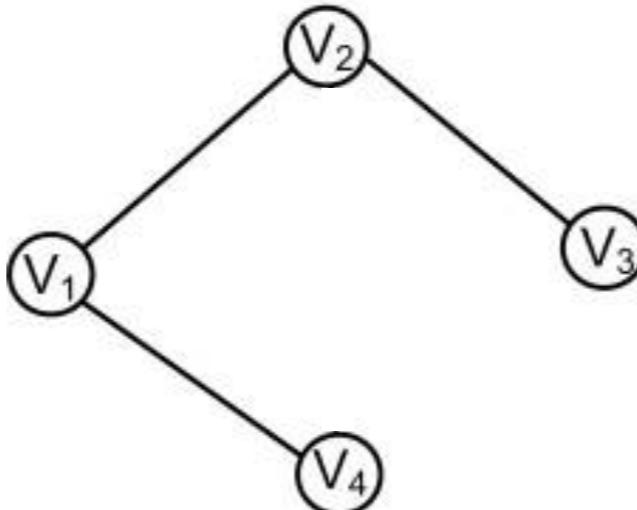
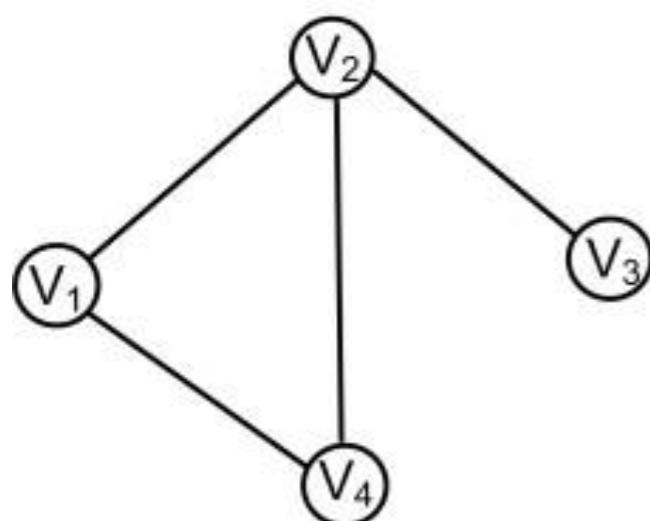
We can thus compute the network **density** (or **global connectivity**) as

**Undirected graphs:**  $D = \frac{2 * E}{V \cdot (V - 1)}$

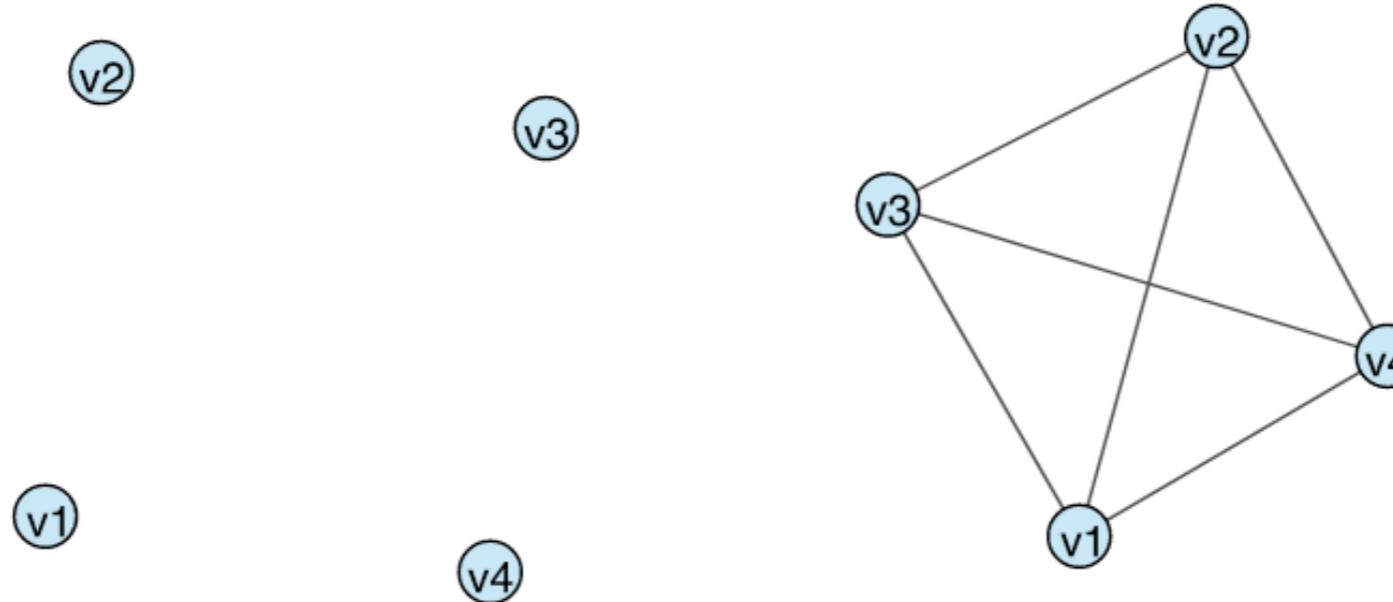
$E$  : number of edges

$V$  : number of vertices

Possible edges =  $\frac{V \cdot (V - 1)}{2}$

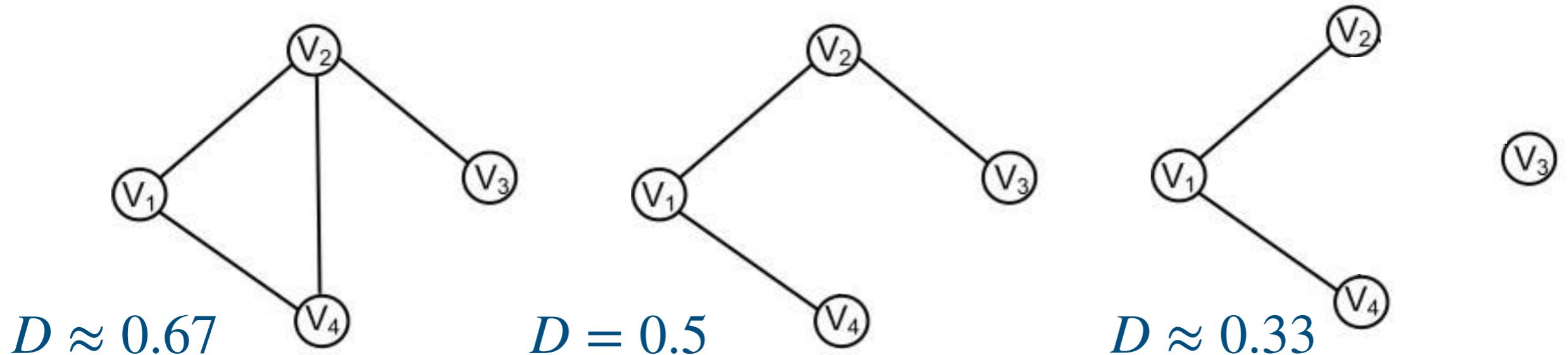


## 2. Network density



$$0 \leq D \leq 1$$

Higher density indicates higher associations in the network, which implies lower resilience to changes.



## 2. Biological network density

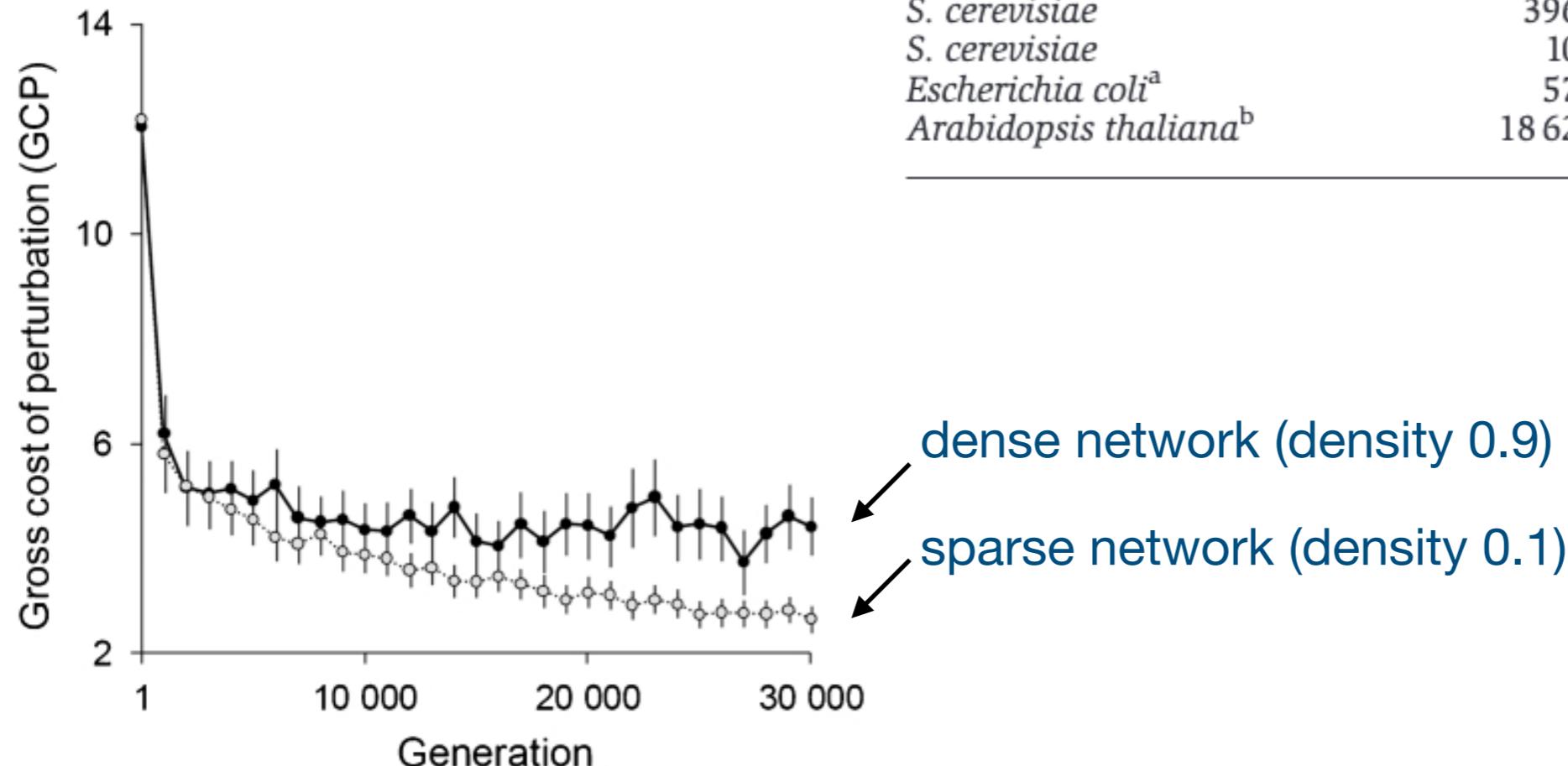
Evolutionary analysis of biological networks indicates general sparsity

Network structure must balance robustness to mutation, stochasticity and environmental queues

Sparse networks show higher robustness when accounting for costs and benefits of complexity

**Table I** Biological networks are sparsely connected

| Organism                                 | Interactions | Genes | $D$    |
|--|--------------|-------|--------|
| <i>Drosophila melanogaster</i>           | 29           | 14    | 0.148  |
| <i>D. melanogaster</i>                   | 45           | 25    | 0.072  |
| Sea urchin                               | 82           | 44    | 0.0065 |
| <i>Saccharomyces cerevisiae</i>          | 1052         | 678   | 0.0023 |
| <i>S. cerevisiae</i>                     | 3969         | 2341  | 0.0007 |
| <i>S. cerevisiae</i>                     | 106          | 56    | 0.0338 |
| <i>Escherichia coli</i> <sup>a</sup>     | 578          | 423   | 0.0032 |
| <i>Arabidopsis thaliana</i> <sup>b</sup> | 18 625       | 6760  | 0.0004 |

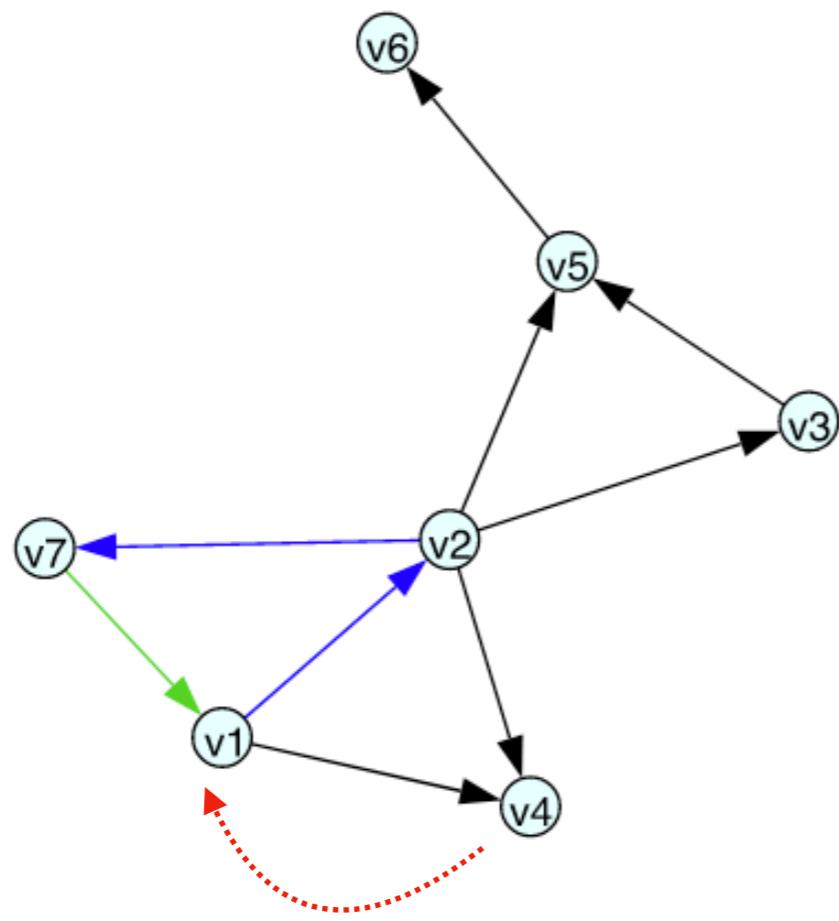


# 3. Paths

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Distance between nodes is measured in path length

In directed graphs, the shortest path between  $(a, b) \neq (b, a)$



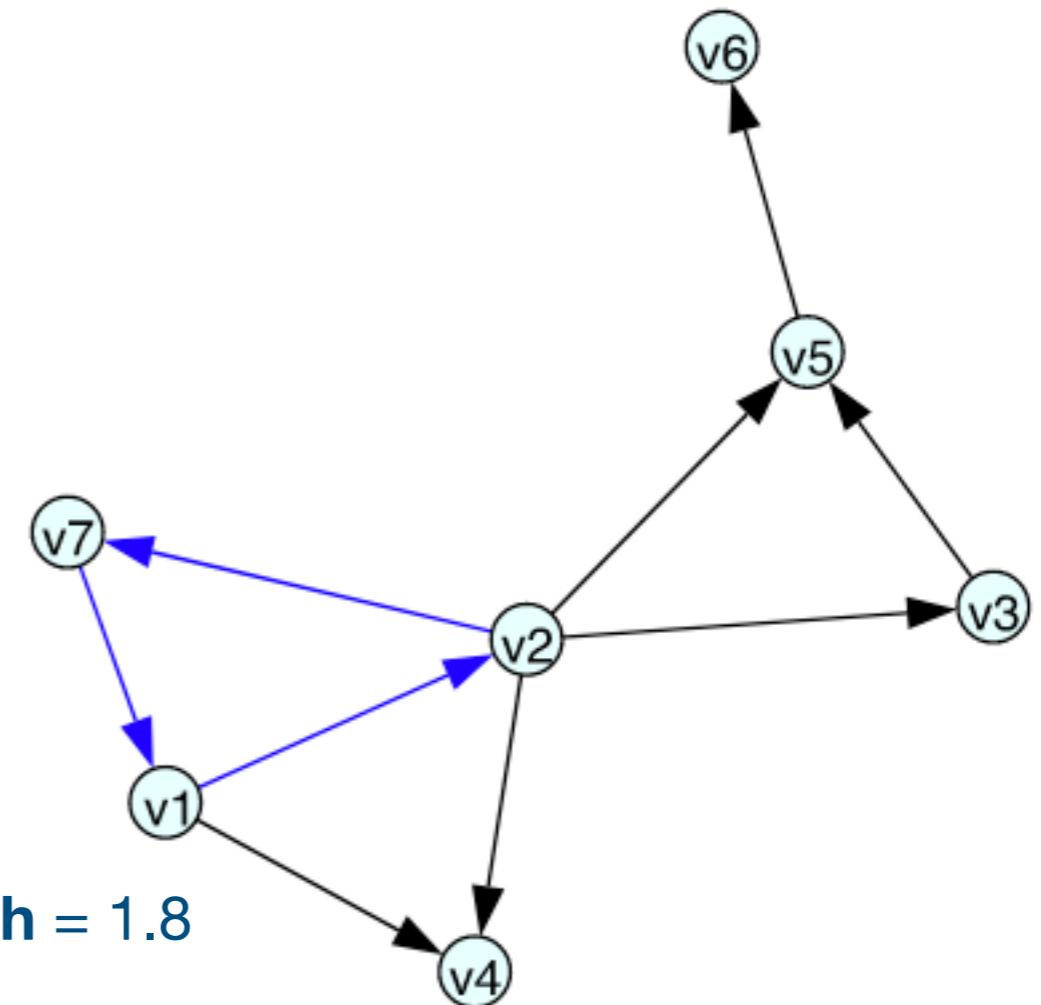
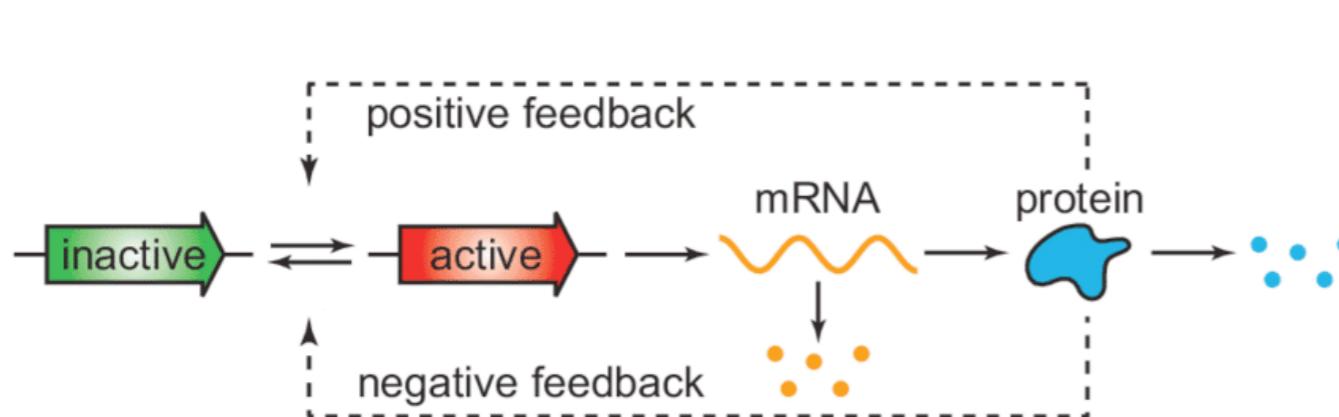
|    | v1  | v2  | v4  | v3  | v5  | v7  | v6  |
|----|-----|-----|-----|-----|-----|-----|-----|
| v1 | 0.0 | 1.0 | 1.0 | 2.0 | 2.0 | 2.0 | 3.0 |
| v2 | 2.0 | 0.0 | 1.0 | 1.0 | 1.0 | 1.0 | 2.0 |
| v4 | inf | inf | 0.0 | inf | inf | inf | inf |
| v3 | inf | inf | inf | 0.0 | 1.0 | inf | 2.0 |
| v5 | inf | inf | inf | inf | 0.0 | inf | 1.0 |
| v7 | 1.0 | 2.0 | 2.0 | 3.0 | 3.0 | 0.0 | 4.0 |
| v6 | inf | inf | inf | inf | inf | inf | 0.0 |

# 3. Paths

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## Cycles and acyclic graphs

The **average path** gives a measure of network navigability (~feature relationships)



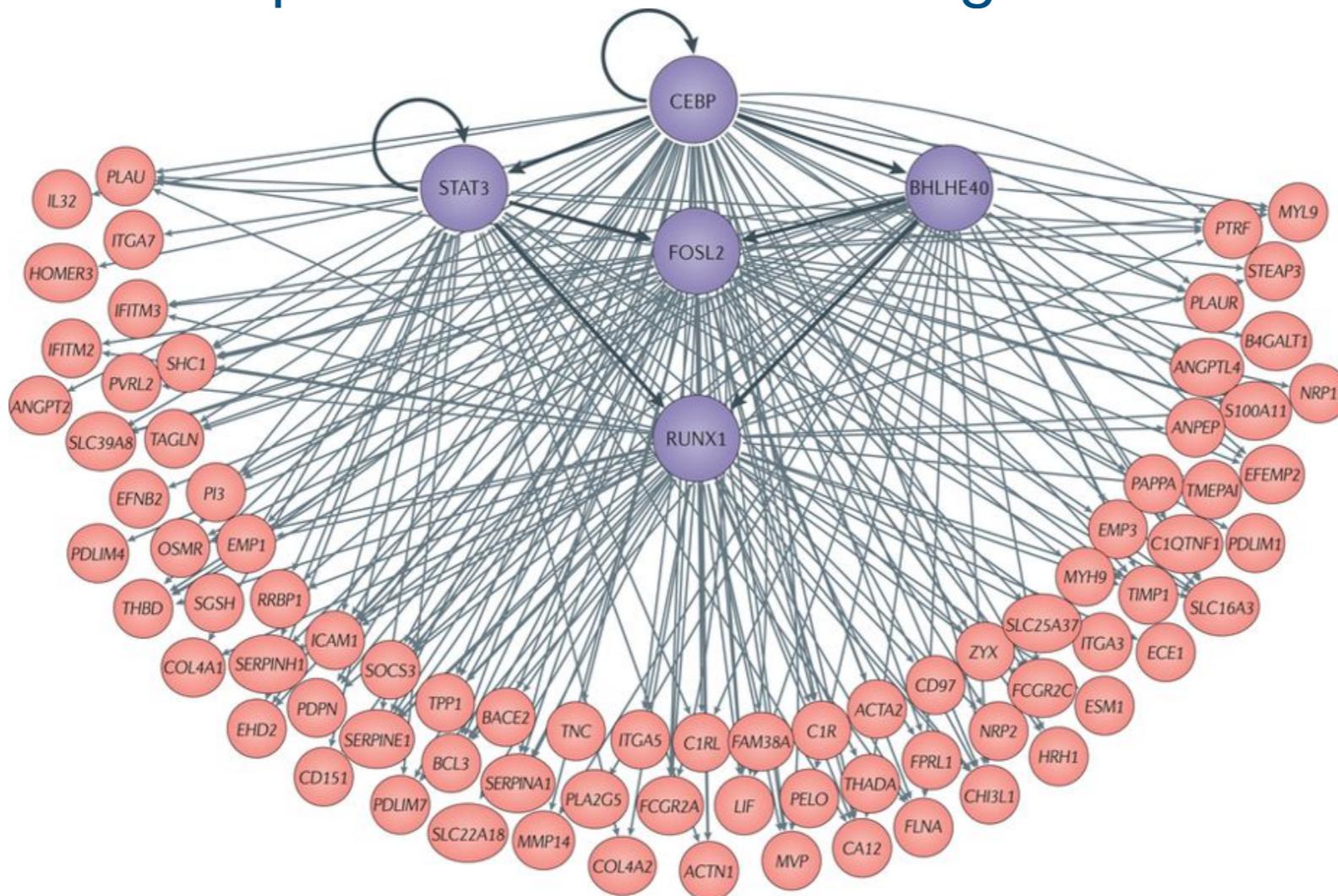
# 4. Centrality

Indicate the most central nodes in a network

Why look at the central nodes?

Hubs

Example: Transcription Factor Master Regulators



# 4. Centrality

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Indicate the most central nodes in a network

Central nodes **possibly** important in the network

There are many different measures of centrality:

- **Degree**
- **Eccentricity**
- *Closeness*
- *Betweenness*
- *Eigenvector*
- Katz
- PageRank
- Percolation
- Cross-clique

...

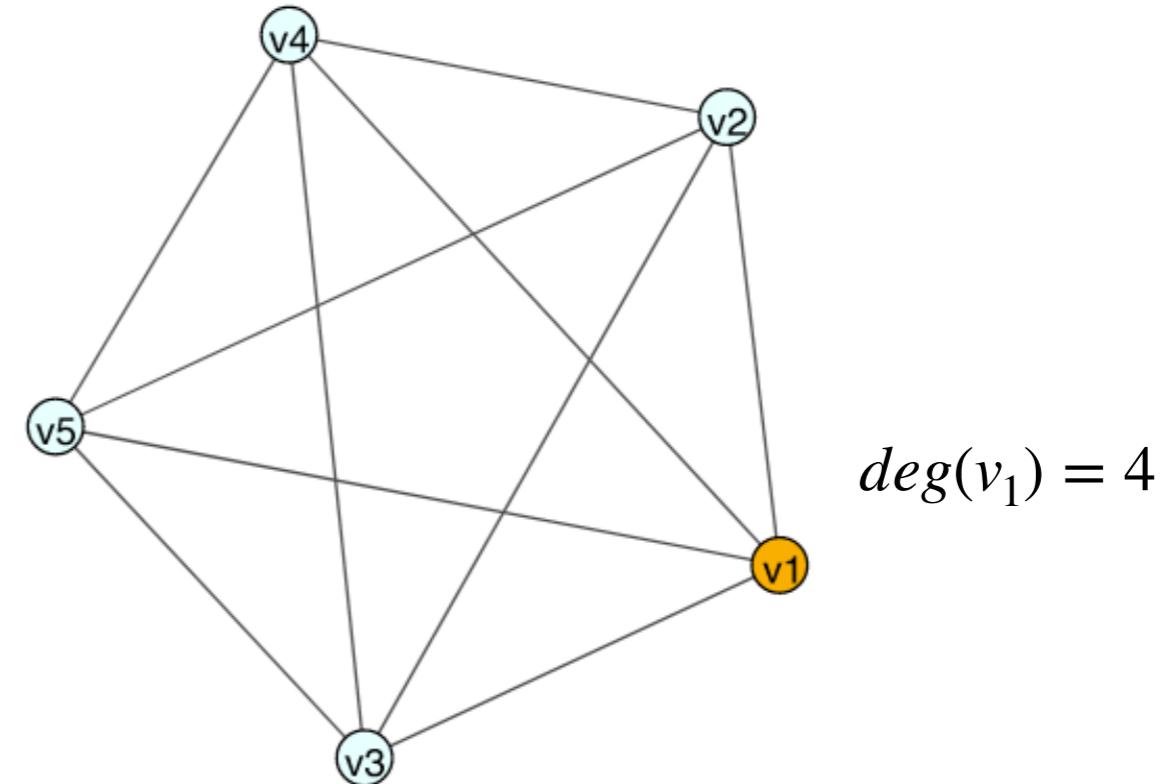
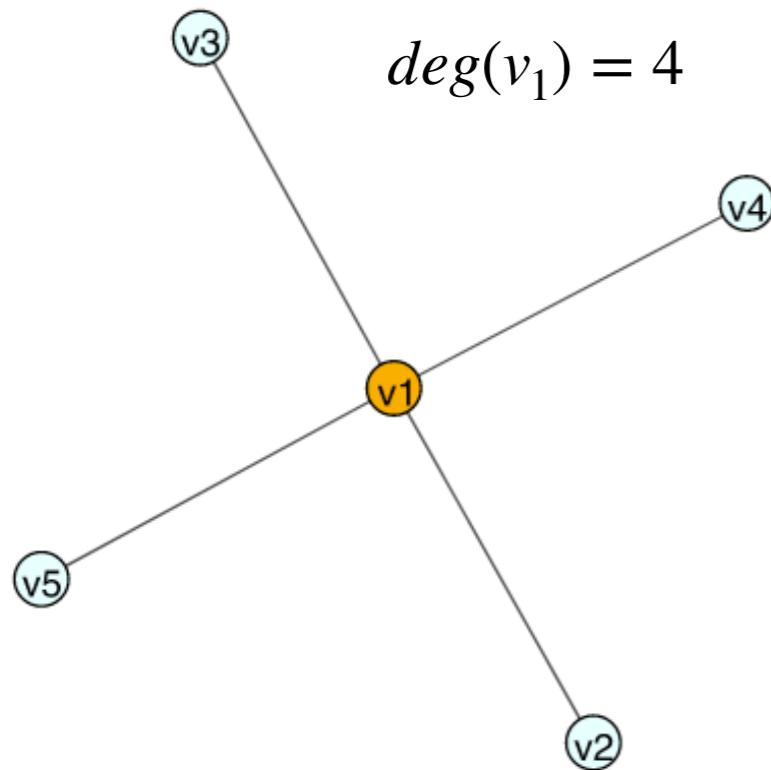
## 4. Centrality: degree centrality

---

Degree indicates the number of connections with a node

$$d(v) = |N(i)|$$

where  $N(i)$  is the number of 1st neighbours of a node.



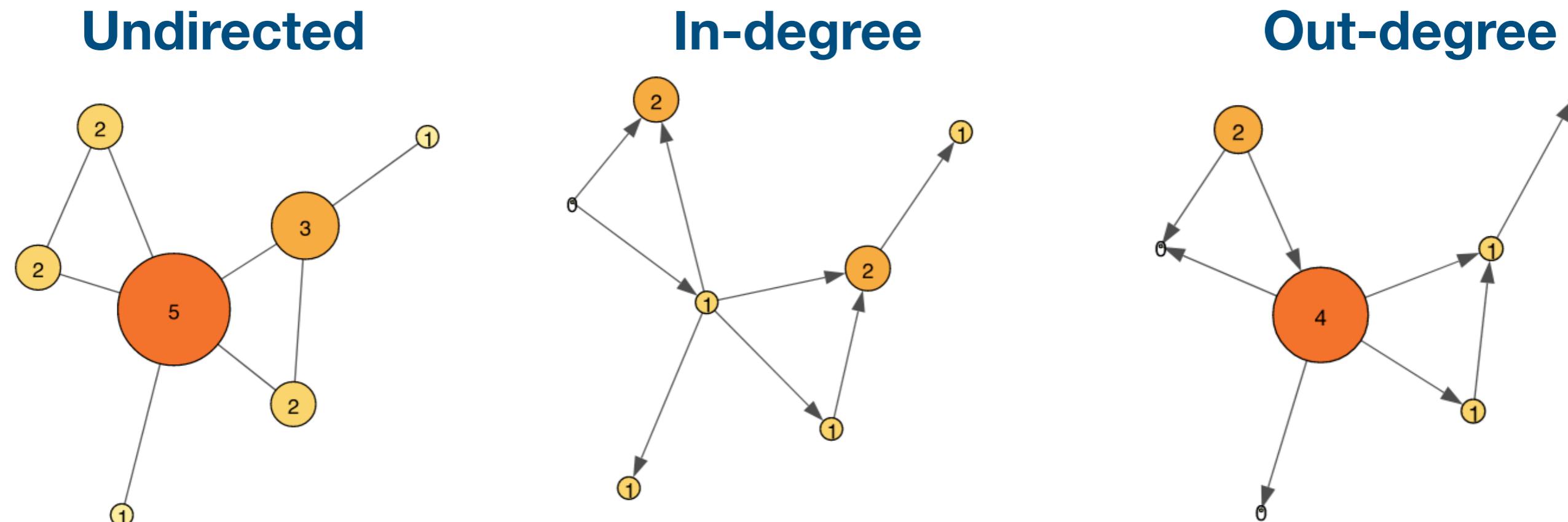
# 4. Centrality: degree centrality

Undirected networks vs directed networks

**In-degree vs Out-degree**

$$C_D(v_i) = \sum_{j=1}^N e_{ij}$$

Numbers indicate degree:



## 4. Centrality: degree centrality

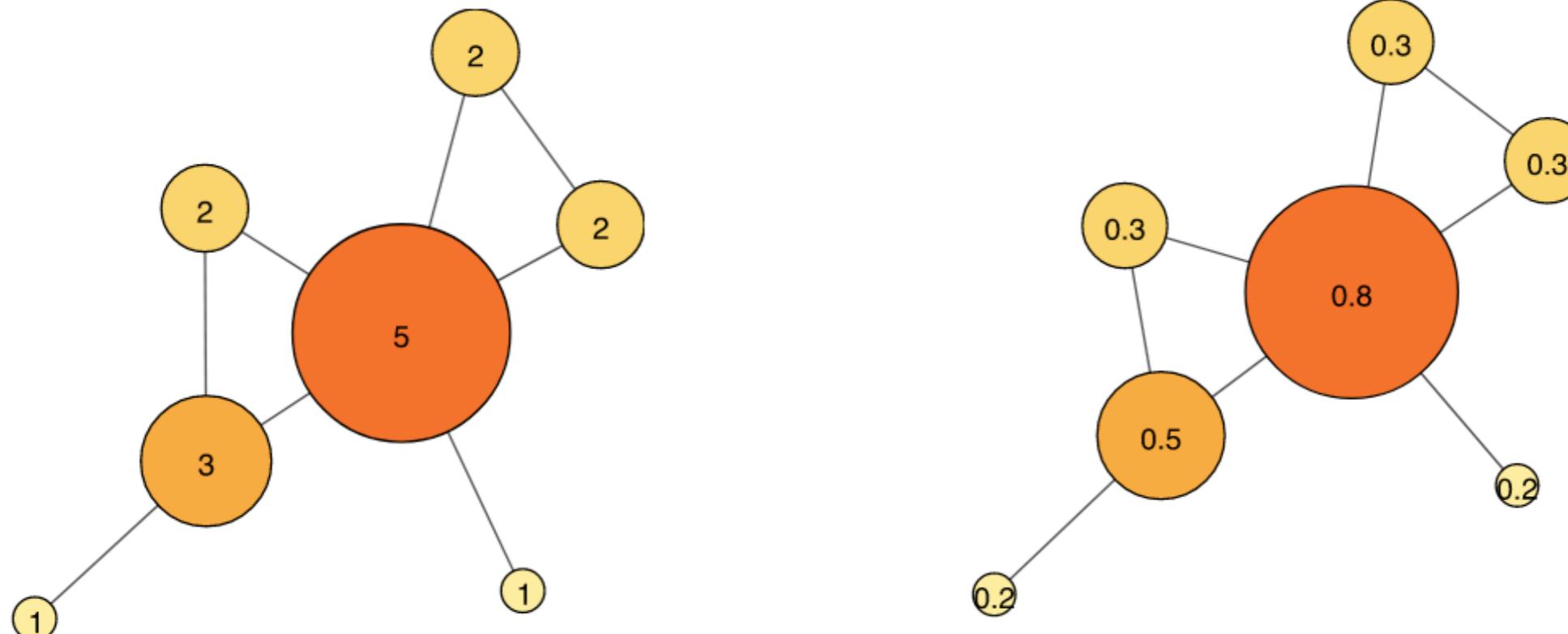
Degree centrality

$$C_D(v_i) = \sum_{j=1}^N e_{ij}$$

Normalized  
degree centrality

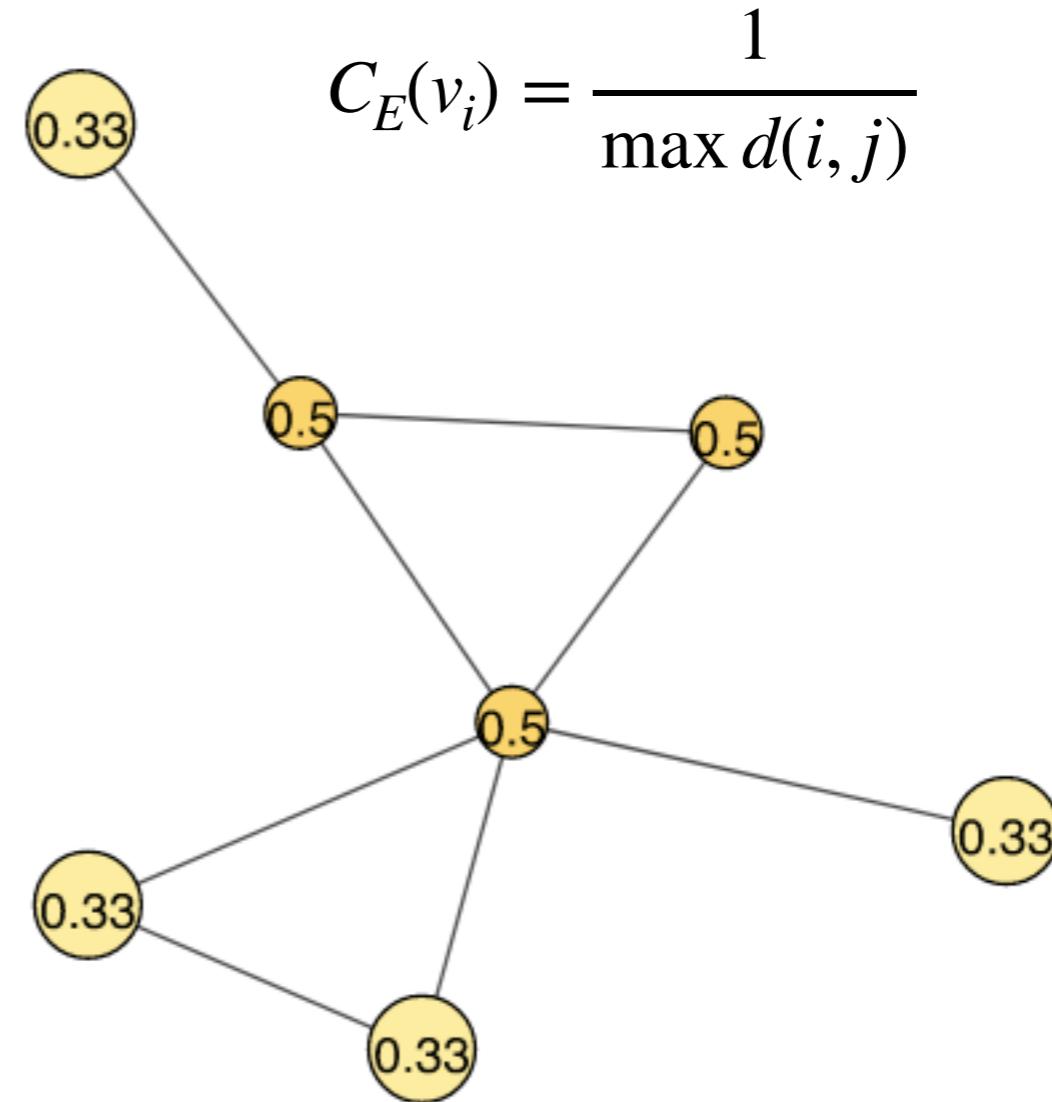
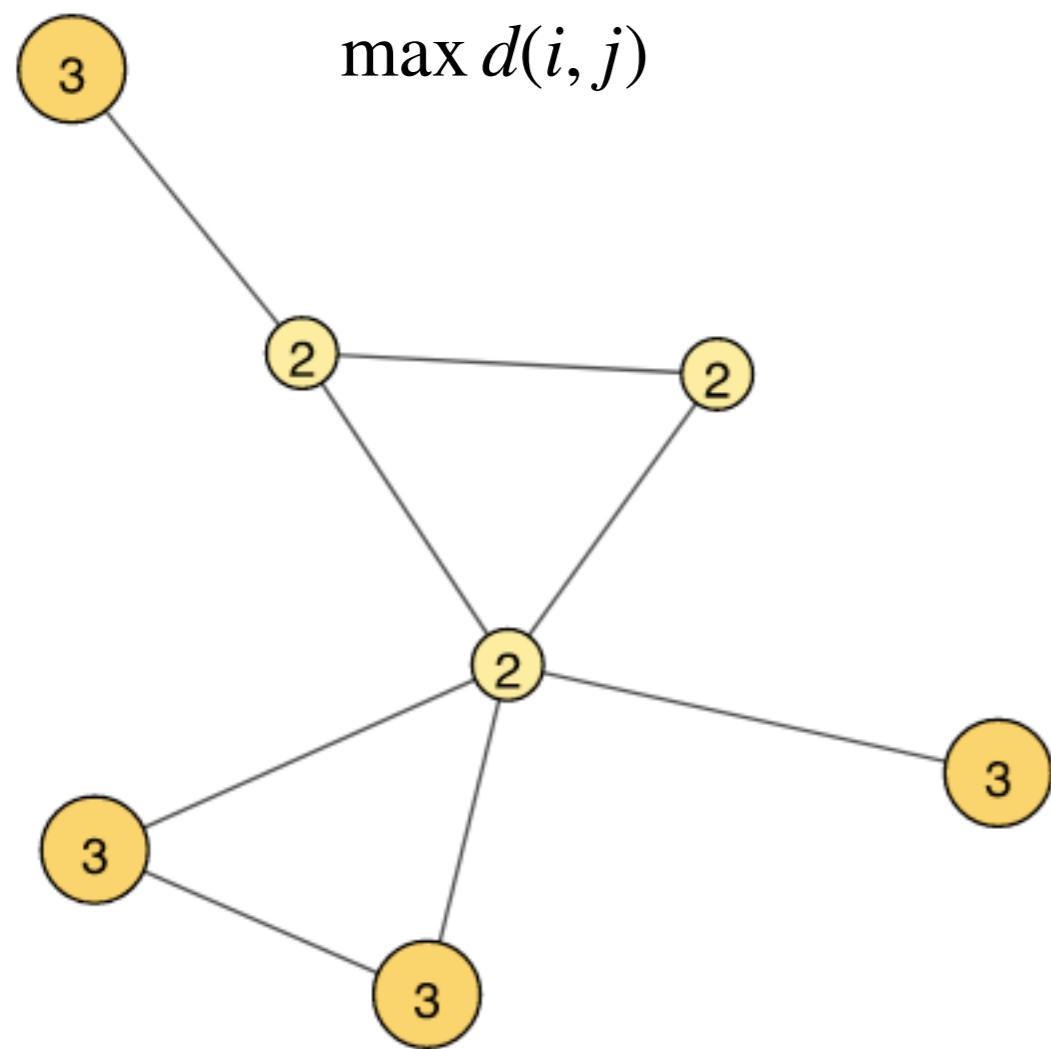
$$C_D(v_i) = \frac{\sum_{j=1}^N e_{ij}}{N - 1}$$

Centrality normalization allows for comparison between networks of different sizes



## 4. Centrality: eccentricity centrality

Eccentricity considers a node's maximum shortest path to all other nodes



# 4. Centrality: limitations & influence

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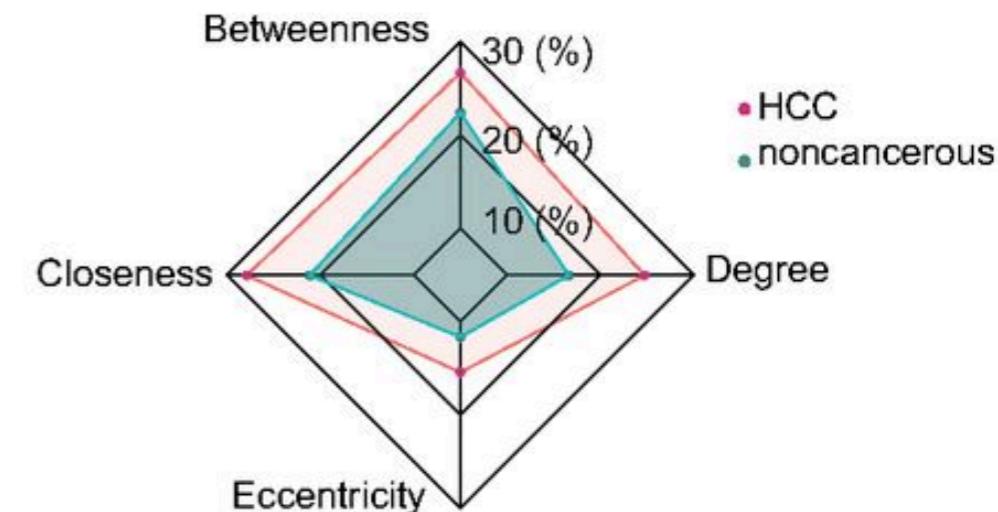
Node centrality does not necessarily imply **importance**

How to tackle this?

1. Complement with experimental observations
2. Compute multiple metrics and summarise joint observations
3. Compute node **influence**

- **Accessibility**
- **Dynamic influence**
- **Impact**
- **Expected force**

Measure **information transmission**



# Break

# Introduction to biological network analysis - part 2

Rui Benfeitas

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Science for Life Laboratory, Stockholm  
Stockholm University

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metabolic  
**ATLAS**



**NBIS**



SciLifeLab



# Key network properties to discuss

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1. Network representations
2. Network density
3. Paths
4. Centrality
- 5. Clustering coefficient**
- 6. Degree and connectivity distributions**

# 6. Clustering coefficient

How likely is it that two connected nodes are part of a highly connected group of nodes?

If node  $v_1$  is connected with  $v_2$  and  $v_3$ , it is very likely that  $v_2$  and  $v_3$  are also connected.

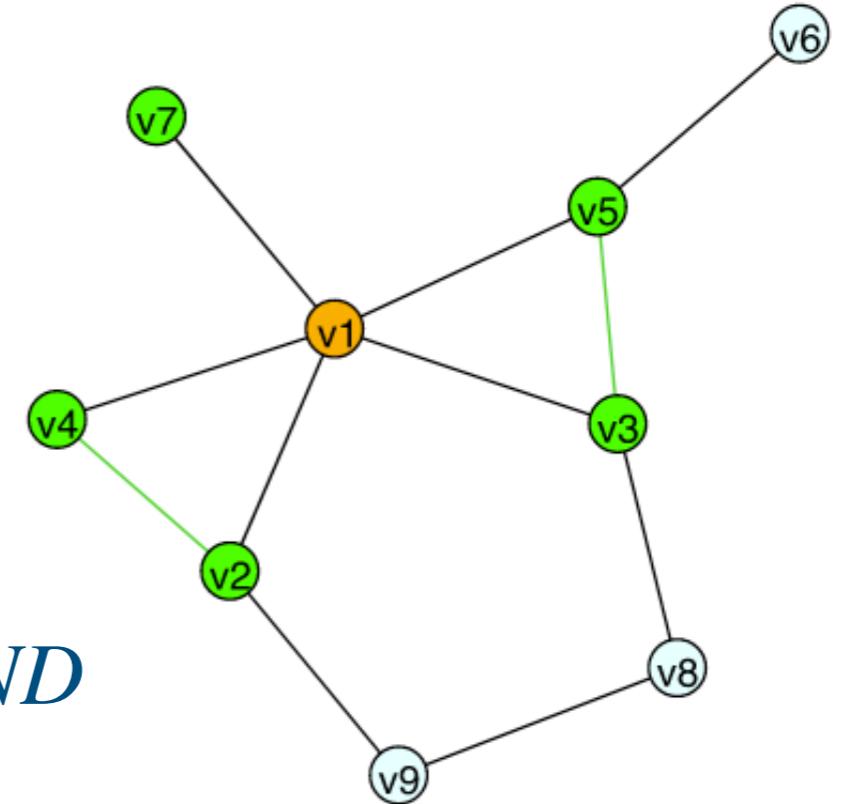
Takes into account degree of a node and the degree of its 1st neighbours

For node  $v_1$

- $\deg(v_1) = k = 5$
- $n$  connections between 1st neighbours of  $v_1 = 2$

$$C_i = \frac{2 \cdot n}{k_i \cdot (k_i - 1)}$$

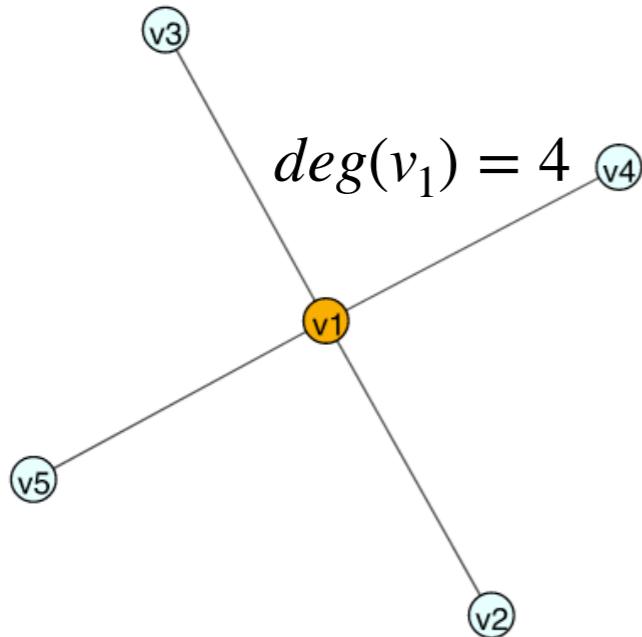
$$C(v_1) = \frac{2 \cdot 2}{5 \cdot 4} = 0.2 \quad C(v_7) = \frac{2 \cdot 0}{1 \cdot 0} = 0 \text{ or } ND$$



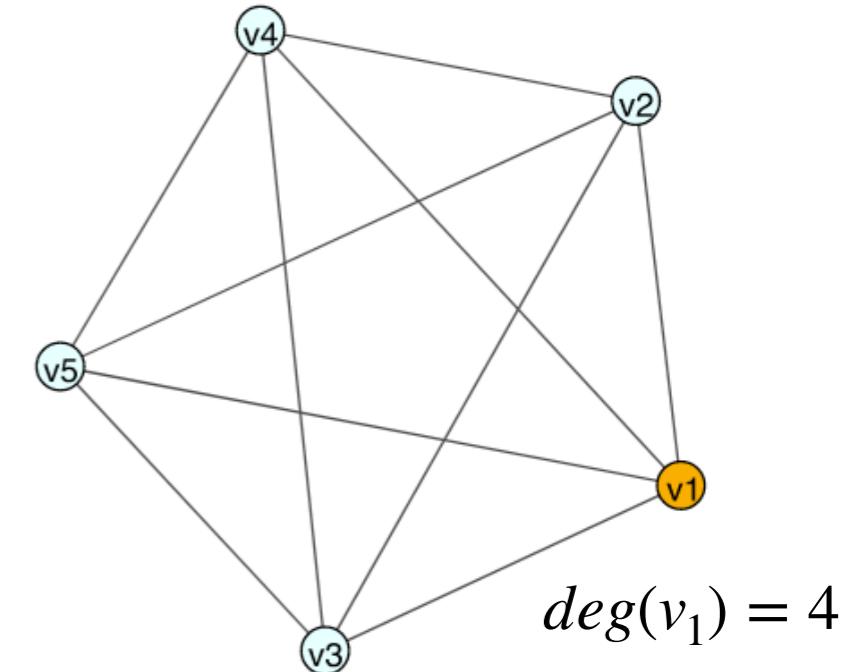
# 6. Clustering coefficient

$C_i = \frac{2 \cdot n}{k_i \cdot (k_i - 1)}$  gives the **fraction of possible interconnections** for neighbours of node  $i$

where  $\frac{k_i \cdot (k_i - 1)}{2}$  is the maximum number of triangles through a node



$$0 \leq C_i \leq 1$$



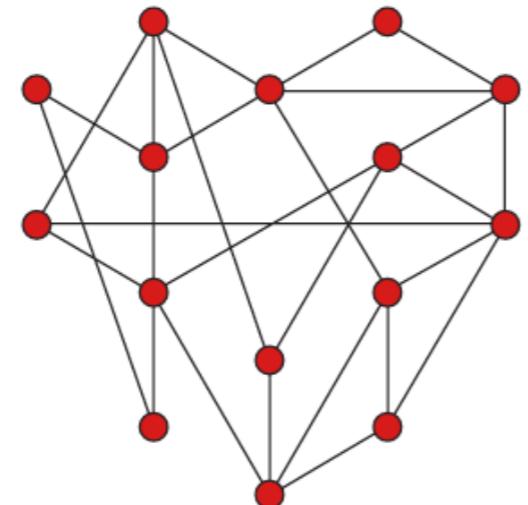
The global clustering coefficient  $C(G)$  is simply the average of its clustering coefficients

# What distinguishes biological networks from random?

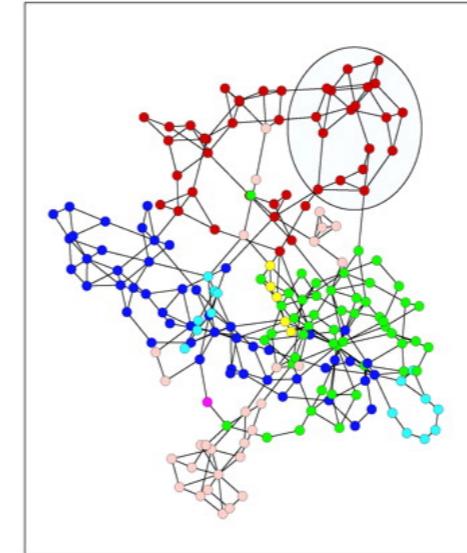
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Do metabolic networks display different network properties from random networks?

**Random network**



**Metabolic network**



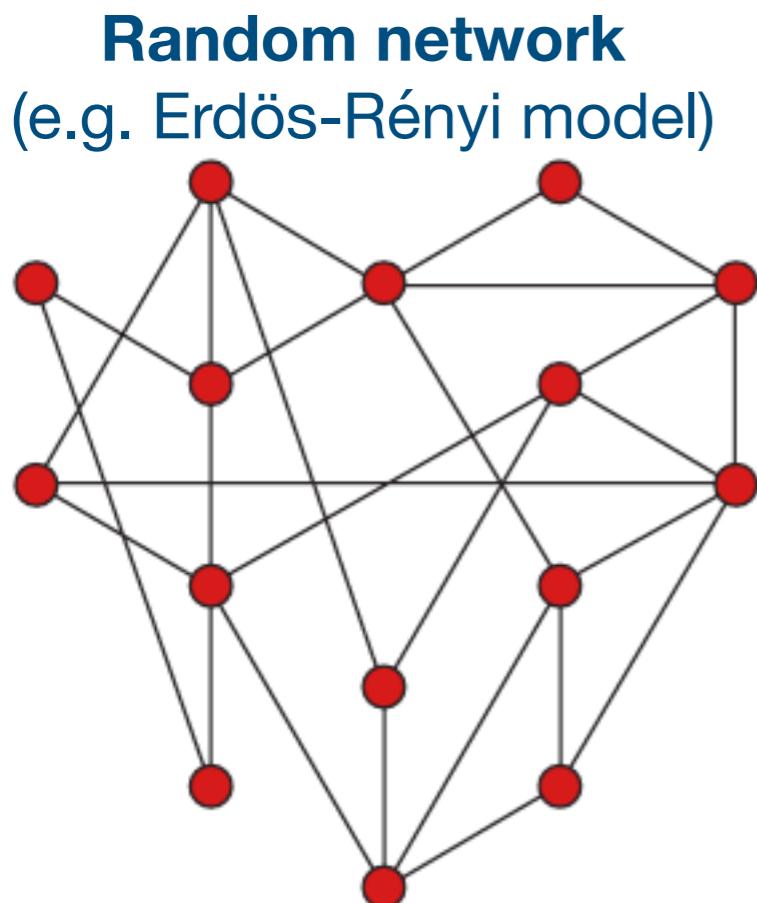
Barabasi 2004

Jeong 2000

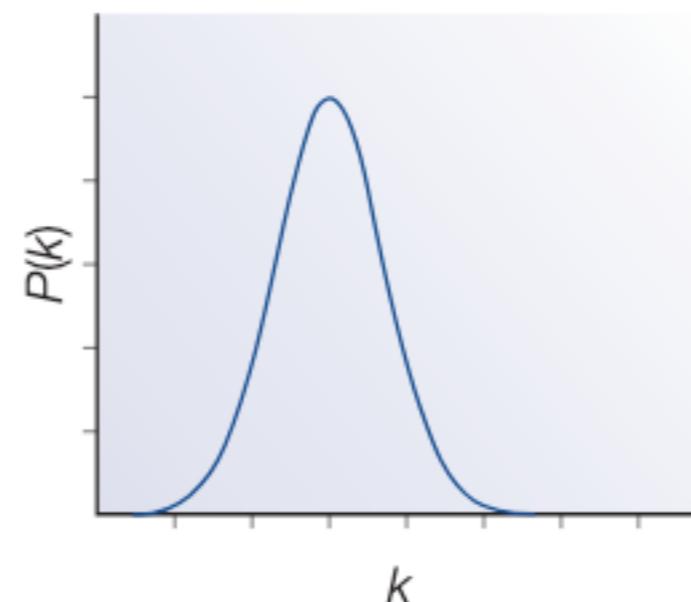
Ravasz 2002

# 7. Degree and clustering coefficient distribution

Degree distributions allow us to compare network organization



**Poisson degree distribution**  
shows no highly connected nodes

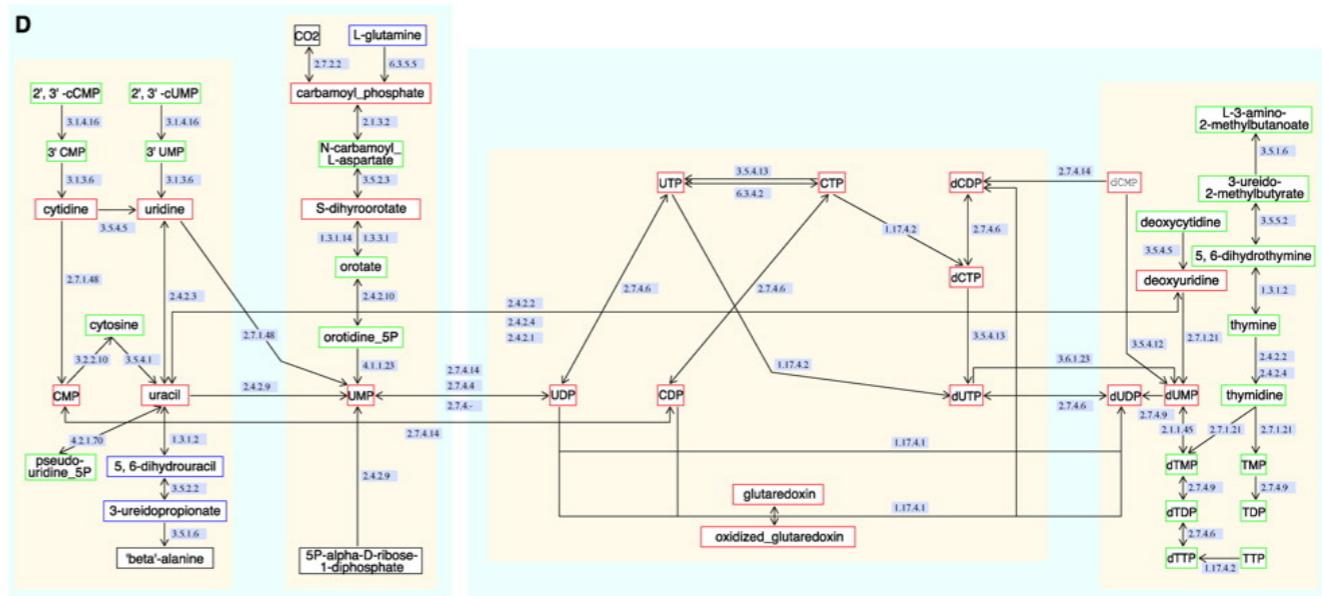
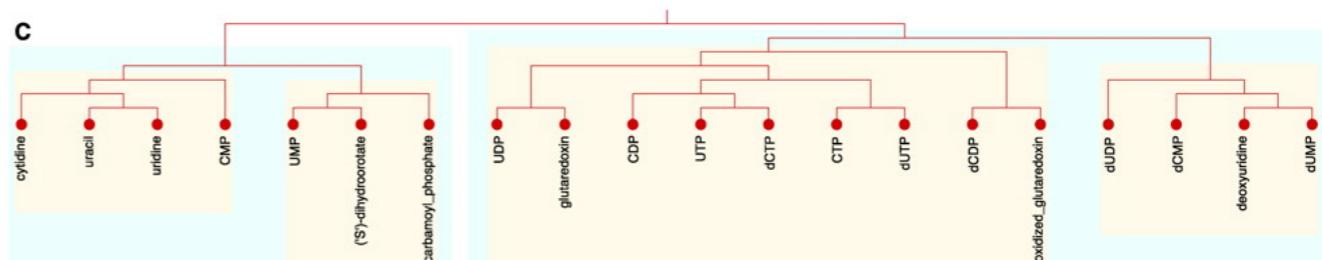
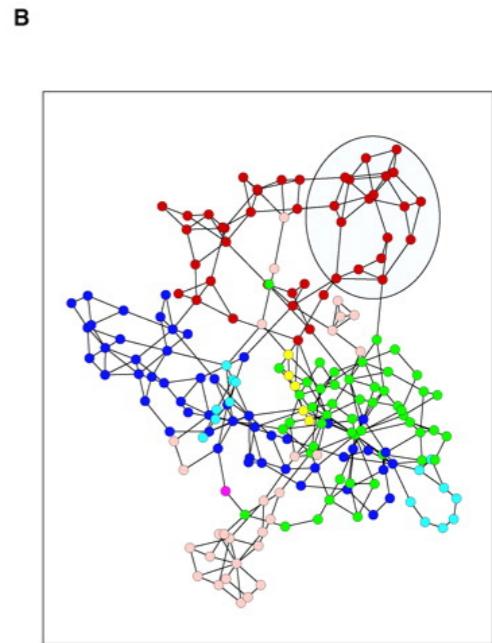


Most nodes have near  $\langle k \rangle$

# Metabolic networks show hierarchical topology

Metabolic networks of 43 organisms are organised into **small, tightly connected modules**

Their combination shows a hierarchical structure



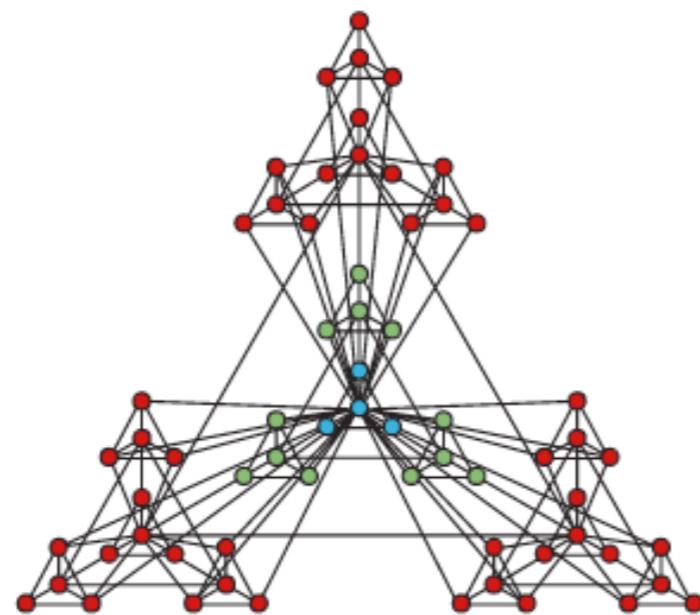
# 7. Degree distribution

Biological networks do not follow topology features of random networks.

Analysis of metabolic networks of 43 organisms shows common patterns

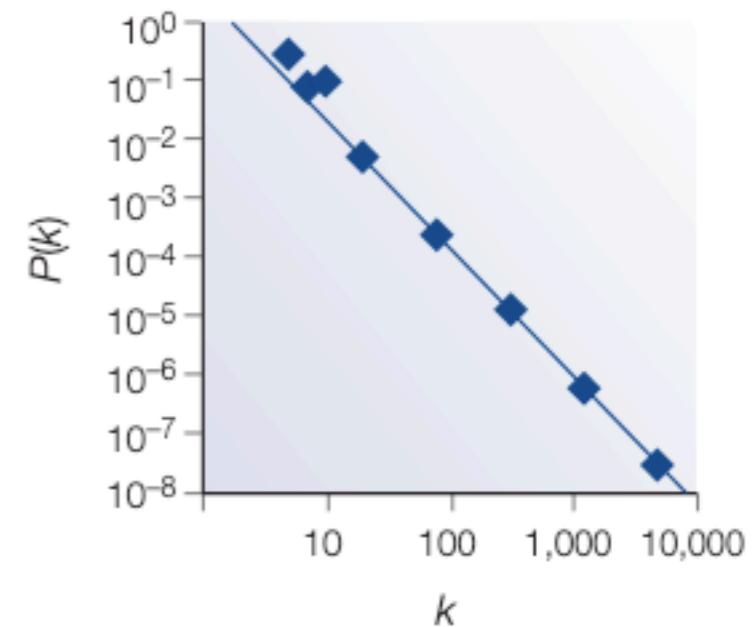
**Biological networks** tend to display high robustness to node failure:  
removal of <80% nodes still retains paths between any two nodes

Hierarchical network



Degree distribution

shows many with low degrees  
a few highly connected nodes



# 7. Degree and clustering coefficient distribution

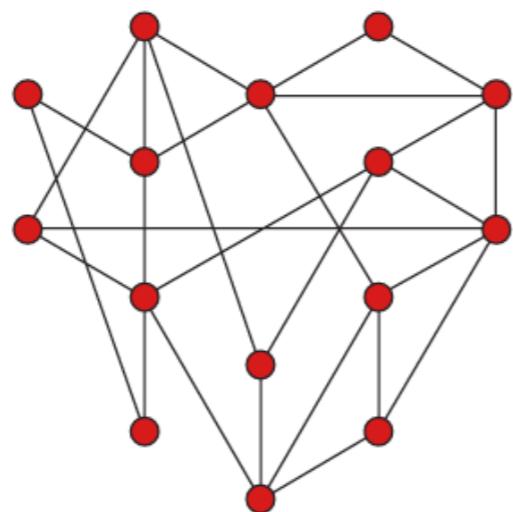
$C(k)$  shows no relationship with  $k$  in random networks: no modular organisation

$C(k) = k^{-1}$  in hierarchical networks

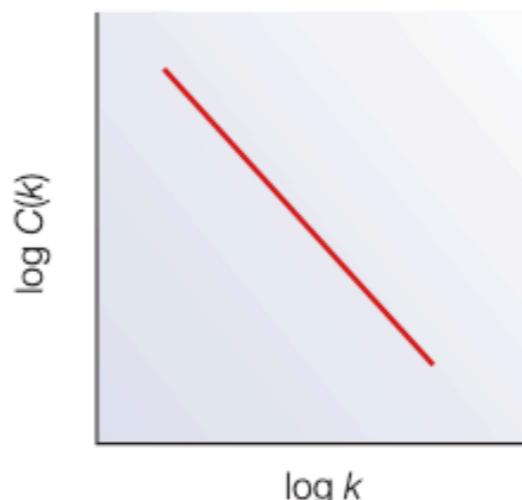
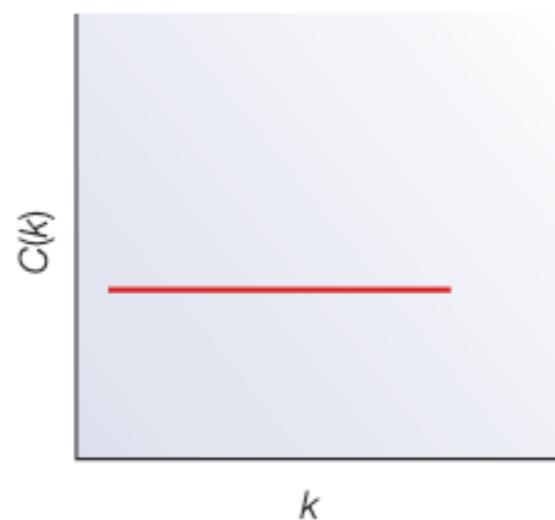
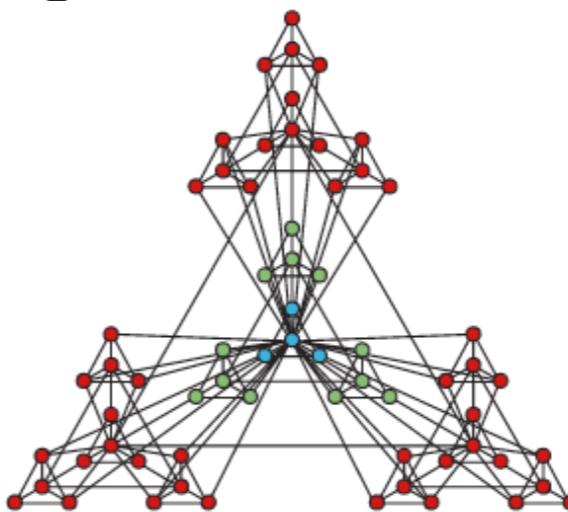
Sparingly connected nodes are part of highly modular areas

Communication between highly clustered neighbourhoods maintained by a few hubs

**Random network**



**Hierarchical network**



# 7. Small world

---

Any two nodes can be connected in a small number of steps.

This is a property seen in **random networks** where the mean path length

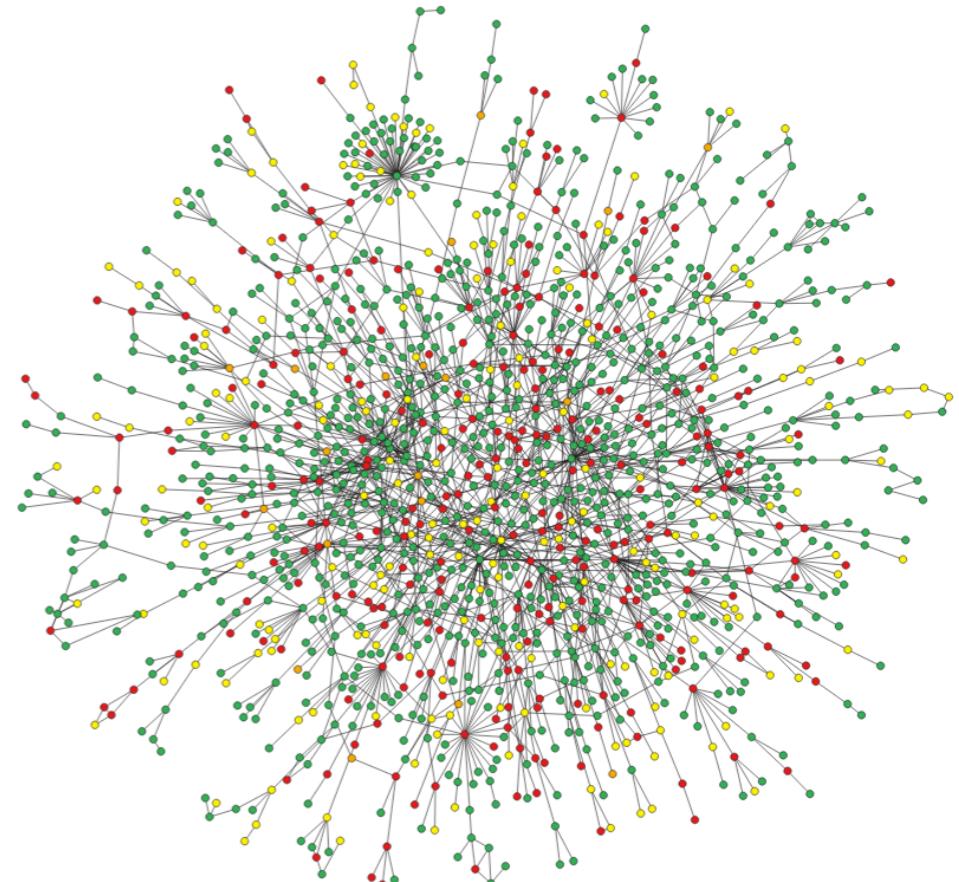
$$l(G) \approx \log N \text{ for a network of size } N$$

Scale-free networks show **ultra-small world**:

$$l(G) \approx \log(\log N)$$

Highly central hubs tend **not** to be connected in biological networks:  
they are **disassortative**

(social networks: **assortative**)



# Overview

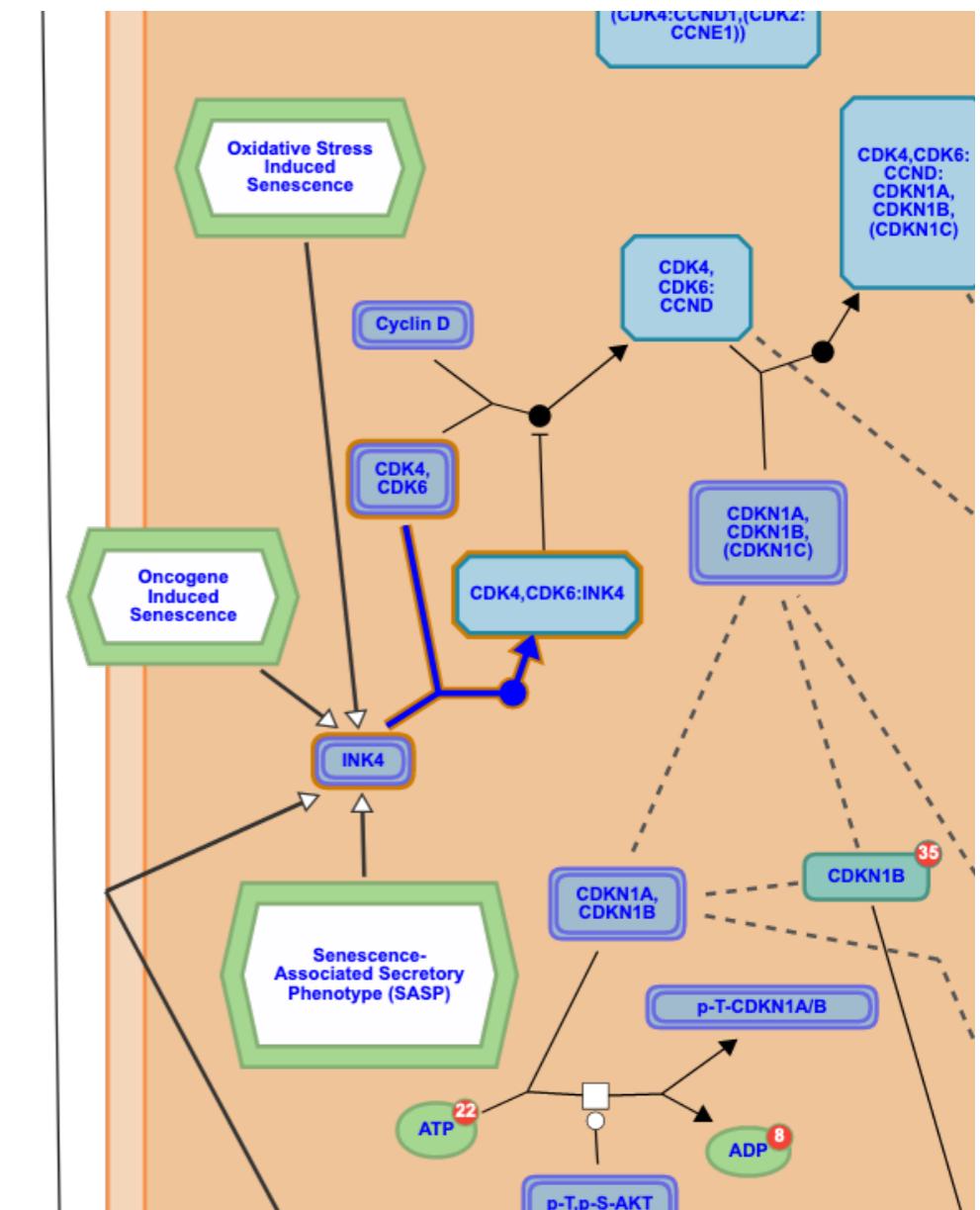
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1. Introduction to network analysis
2. Terminology
3. Network inference
4. Key network properties
- 5. Community analysis**

# What are modules?

**Modules** are physically or functionally associated nodes that work together to achieve a distinct function

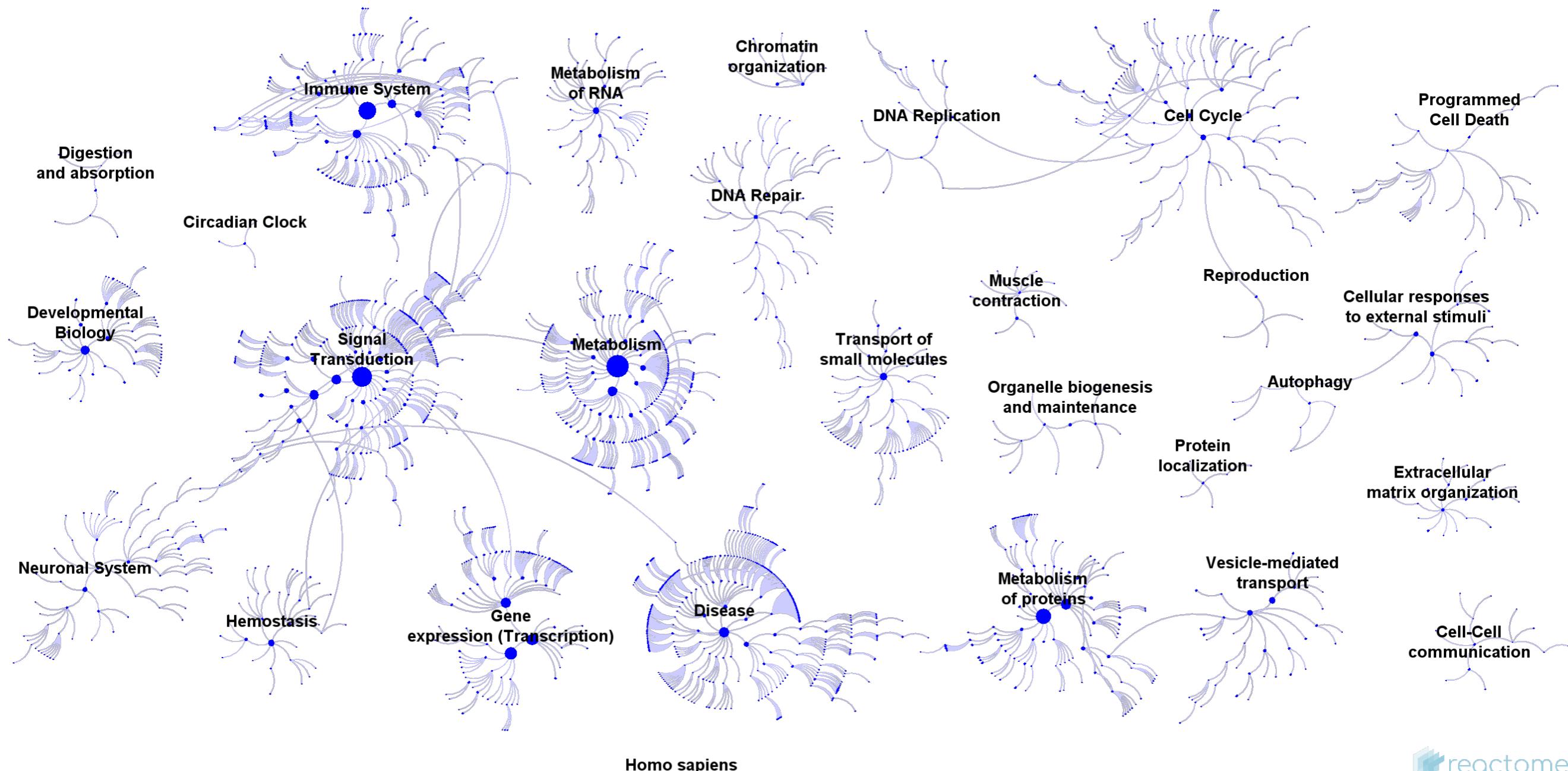
Protein complexes are physical modules



# What are modules?

Pathway-associated proteins **may** represent functional modules

## Gene Ontology



# What are modules?

In addition to physical or functional modules, one may identify other types of modules

**Topological:** derived from their high within-module degree

**Disease:** highly interconnected nodes associated with a disease response

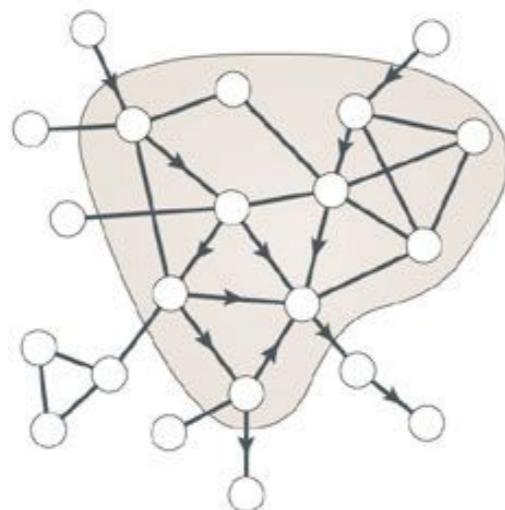
**Drug:** highly interconnected nodes associated with a drug response

**Subgroup:** highly interconnected nodes associated with a sample subgroup (e.g. cancer subtype)

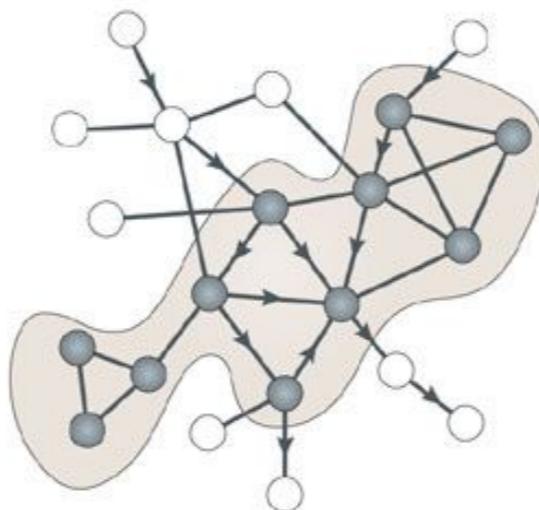
**Tissue-, cell-type-specific:** highly interconnected nodes associated with a specific tissue or cell type

Highly interlinked local regions of a network

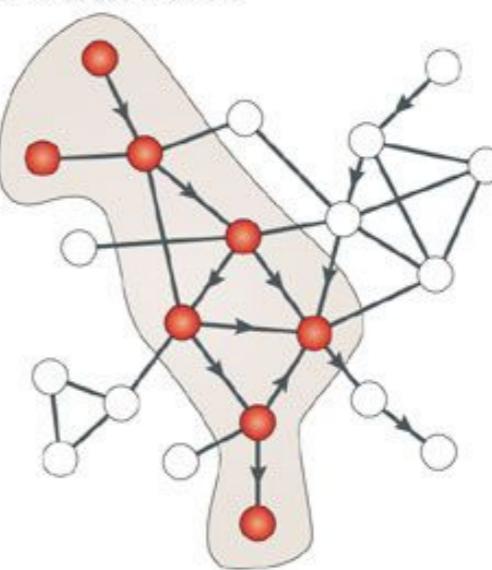
a Topological module



b Functional module



c Disease module



○ Topologically close genes (or products)

● Functionally similar genes (or products)

● Disease genes (or products)

— Bidirectional interactions

→ Directed interactions

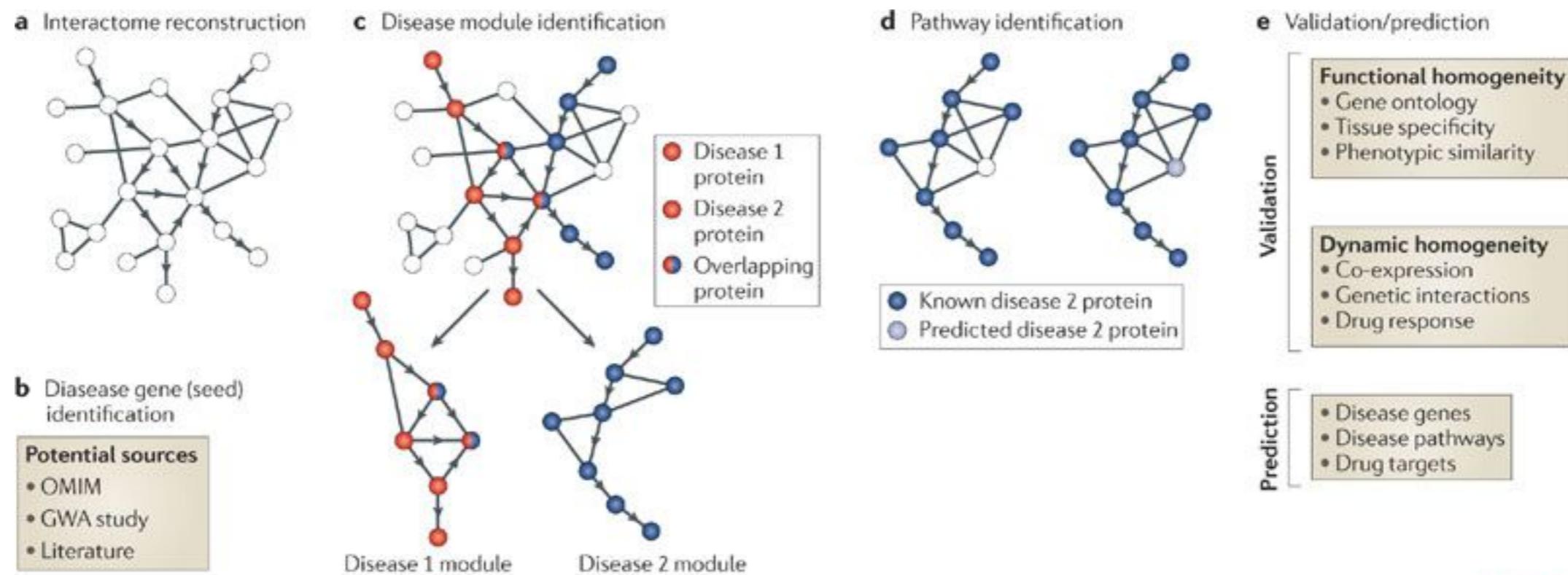
# The challenge: identify and characterise modules

Moving from full network to modular characterisation

Hypothesis: common functional properties (diseases, biological processes, etc.) are associated with the same module

Prediction: *in silico*, relies on available knowledge

Validation: experimental responses



# Modularity

**Modularity** is a property of the network

**Modularity** ( $Q$ ) measures the tendency of a graph to be organised into modules

**Modules** computed by comparing probability that an edge is in a module vs what would be expected in a random network

For a given partitioning of the network into individual groups  $s$ , compute

$$Q \propto \sum_{s \in S} [(e_s) - (\text{expected } e_s)]$$

# edges in group  $s$

Random network with same number of nodes, edges and degree per node

$Q = 1$ : much higher number of edges than expected by chance

$-1 < Q < 1$        $Q = -1$ : lower number of edges than expected by chance

$Q > 0.3 - 0.7$  means significant community structure

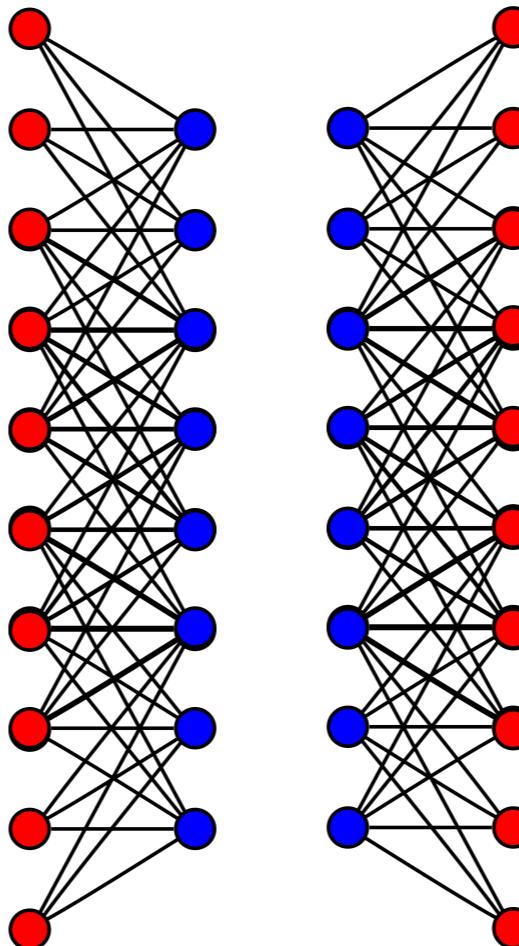
# Modularity

---

**Modularity** is different than **clustering coefficient**:

Graph composed of two bipartite complete subgraphs:

high Q but low connectivity (C)



# Modules

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A **module** (or **community**) is a set of nodes with a lot of **internal connections**, but **fewer external connections**.

How to identify modules? Maximise Q

$$Q \propto \sum_{s \in S} [(e_s) - (\text{expected } e_s)]$$

**Brute-force approach:**

1. Start with 1 node/module
2. Compute distances between nodes
3. Join closest node
4. Re-compute distances between a 2n module and each 1n module
5. Join them if Q increases

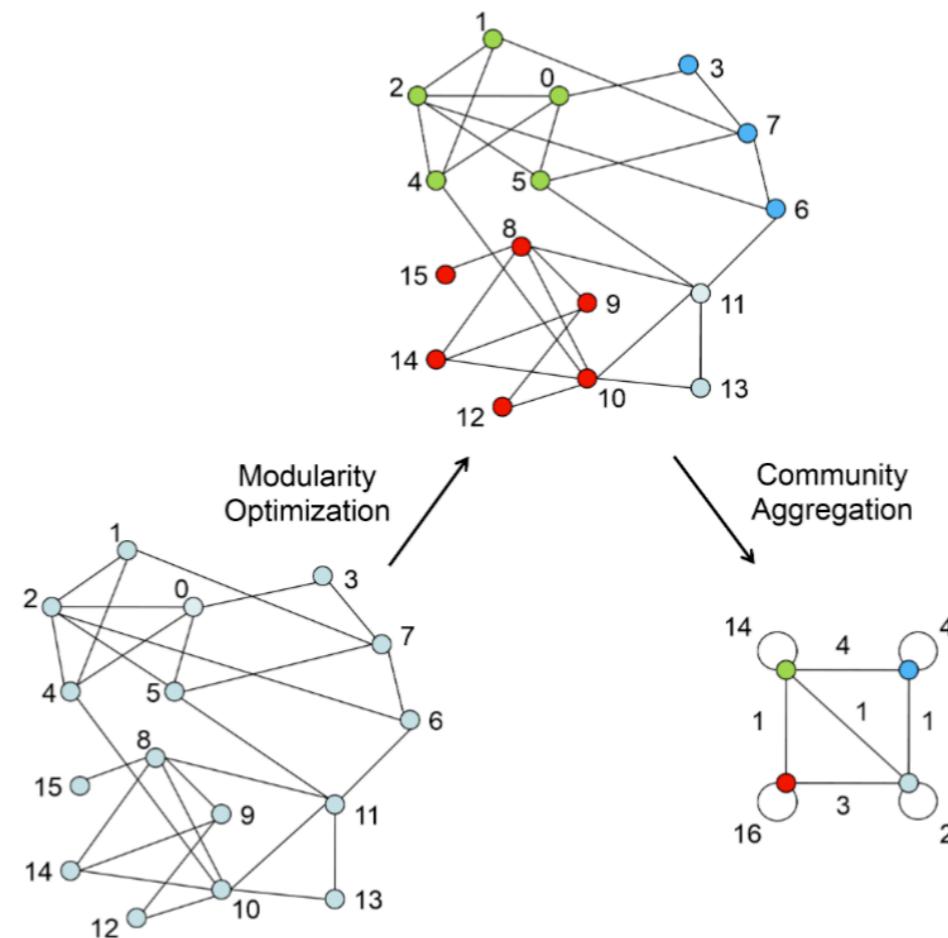
# Module detection: Louvain algorithm

## Phase 1: greedy modularity optimisation

1. Start with 1n/community
2. Compute  $Q$  by moving  $i$  to the community of  $j$
3. If  $\Delta Q > 1$ , node is placed in community
4. Repeat 1-3 until no improvement is found. Ties solved arbitrarily

## Phase 2: coarse grained community aggregation

5. Link nodes in a community into single node.
6. Self loops show intra-community associations
7. Inter-community weights kept
8. Repeat phase 1 on new network



# Community characterisation

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Clustering coefficient and degree distribution

Enrichment analysis

**Hypothesis:** community-associated features show coordinated changes associated with common biological processes

Can significantly enriched biological processes serve as “validation”?

- Mutual feature associations may reinforce data characterisations not evident by individual features
- ...or need of further network curation based on top biological terms

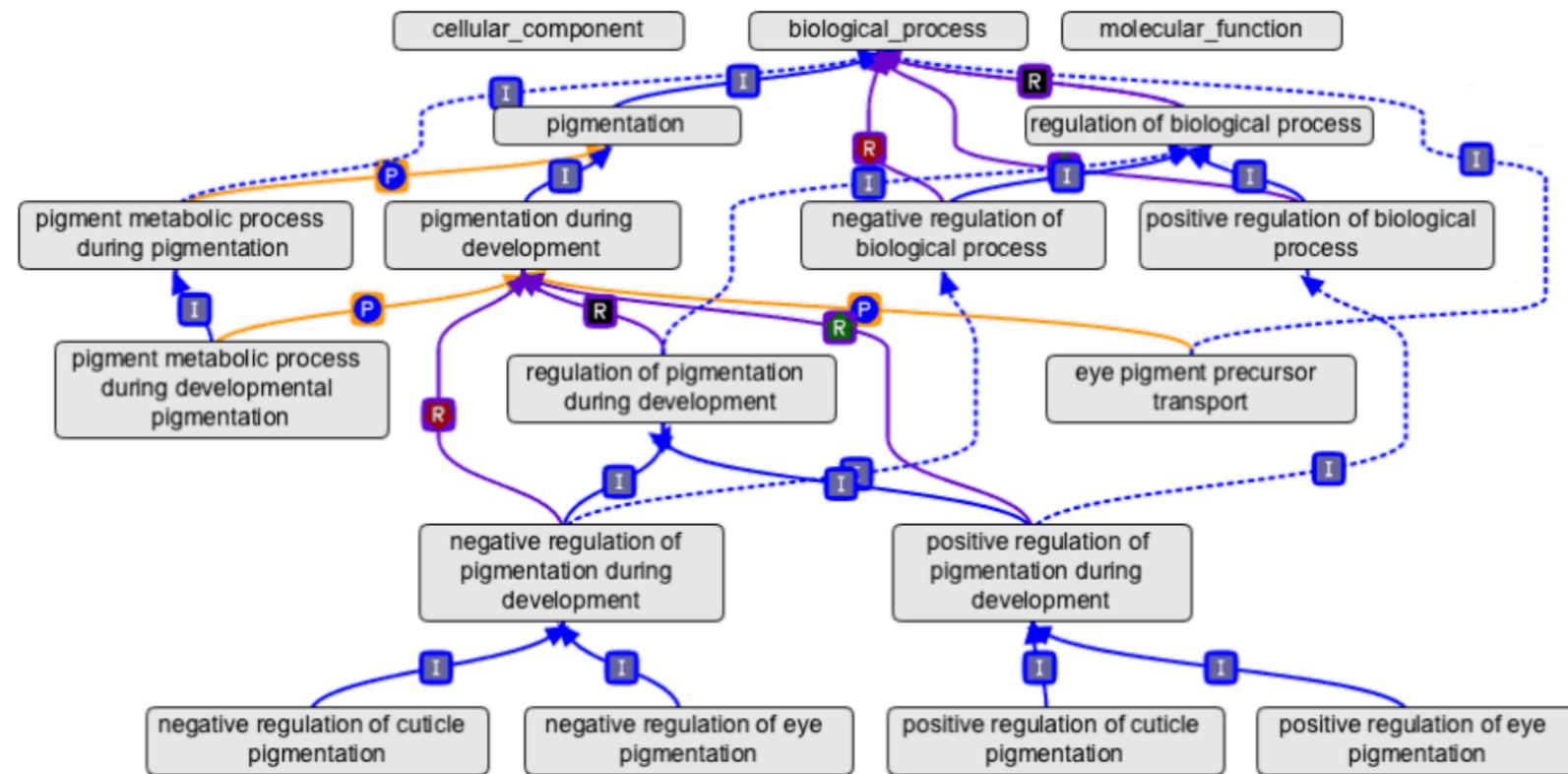
# Enrichment analysis

GO-terms, pathways, subcellular location, TF-targets, disease, drugs

Tests for significant overlap between groups

All considerations from standard enrichment analyses apply

Some biological processes may have no biological meaning in your analysis



# Enrichment analysis

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Important databases with gene-sets:

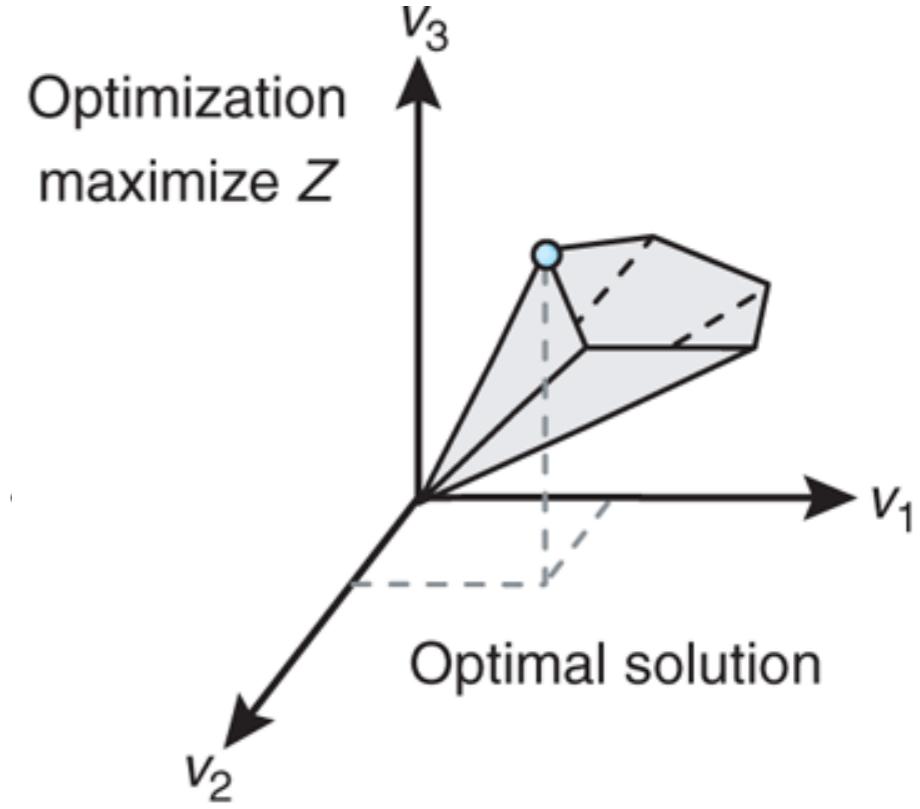
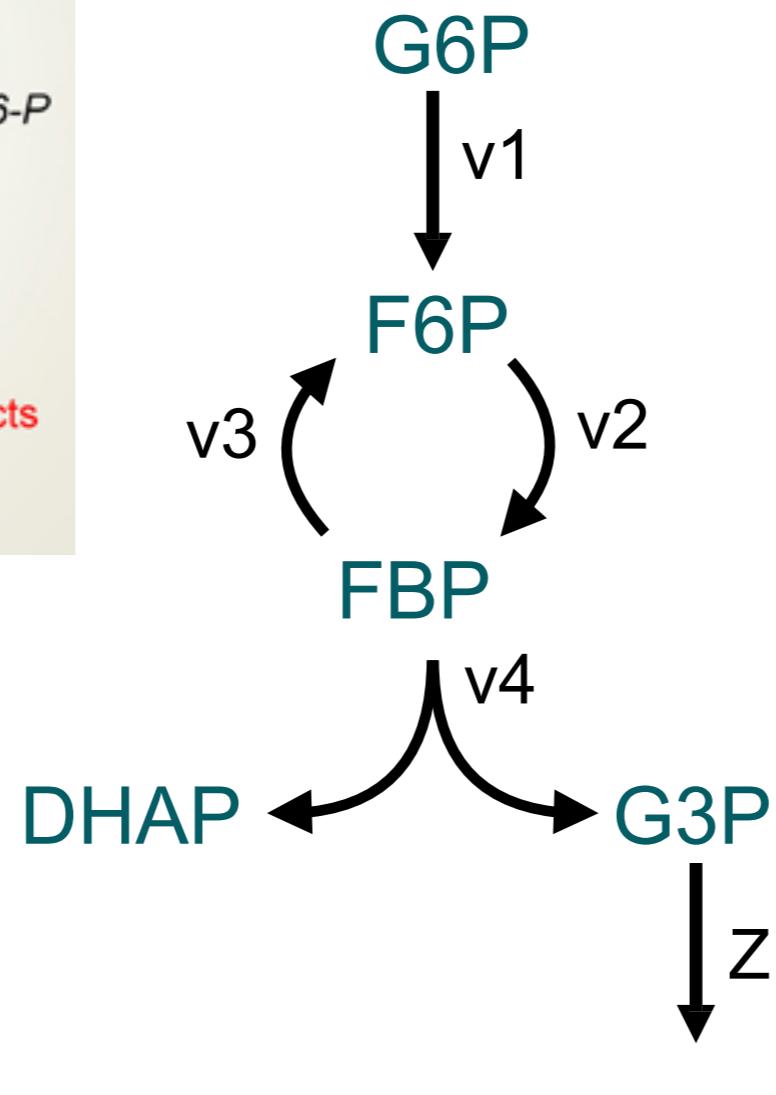
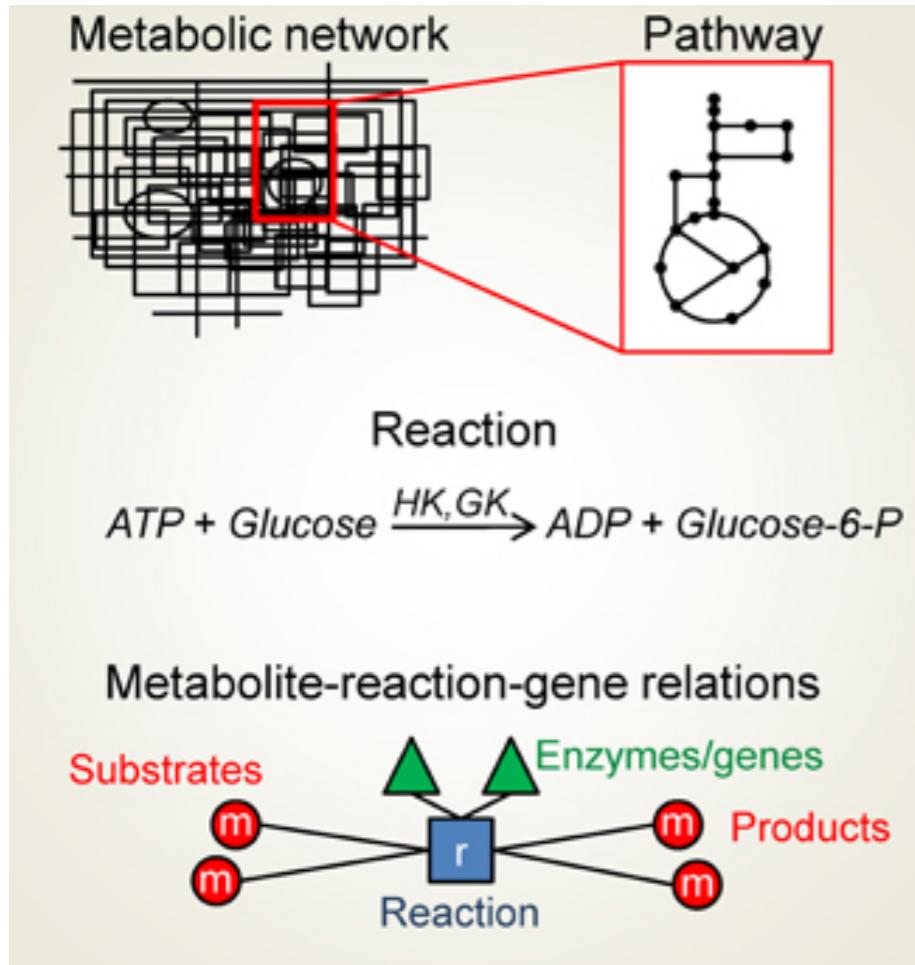
- [MSigDB](#) (gene)
- [Enrichr](#) (gene)
- [KEGG](#) (metabolite, gene)
- [DIANA](#) (miRNA)
- [MetaboAnalyst](#) (metabolite)
- [DAVID](#) (web)
- [Reactome](#) (web)

Creating custom sets and joint sets

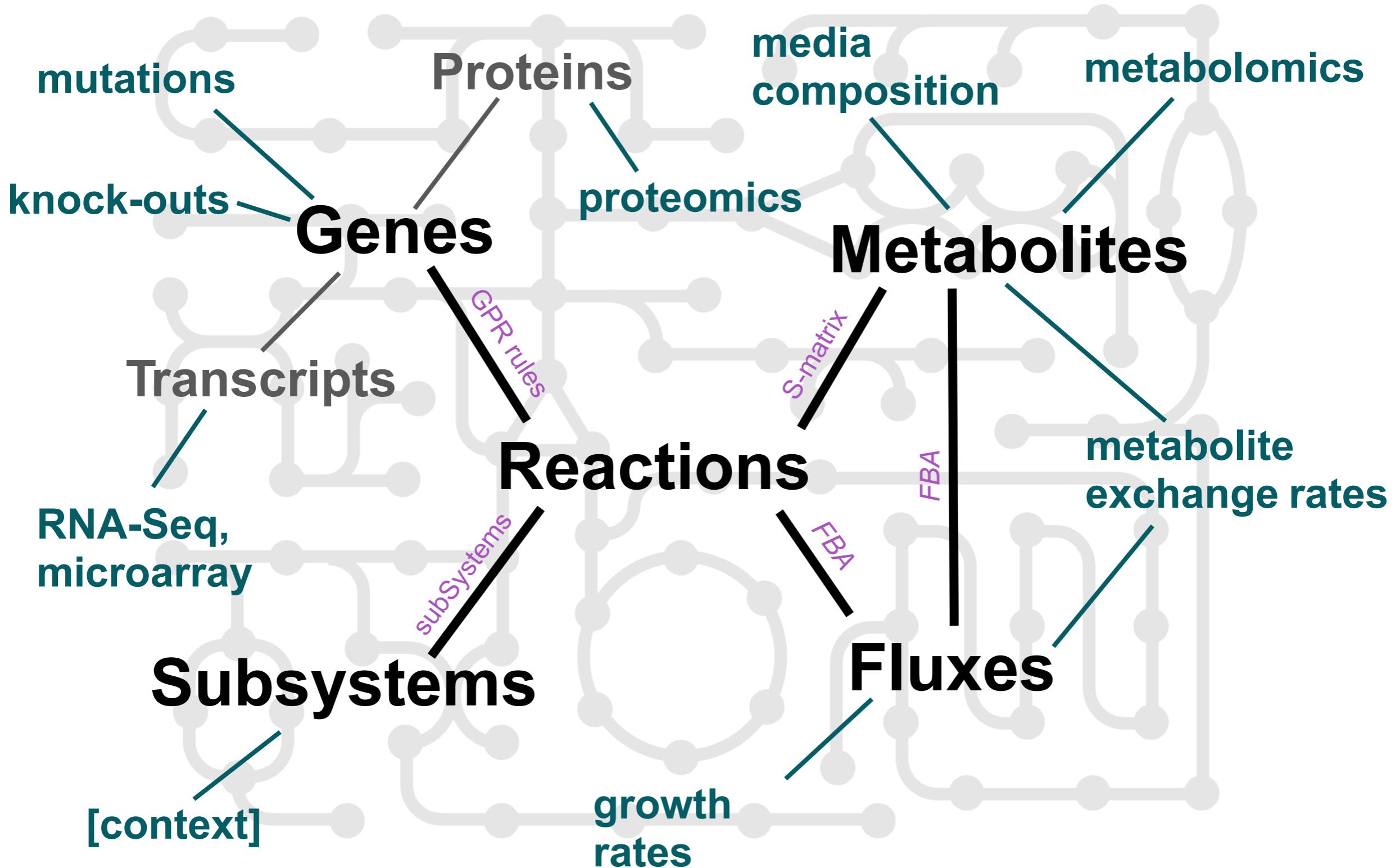
Mapping your data to common IDs

- Easy for genes and proteins: use [DAVID](#), [Biomart](#), or [MyGene](#) (in [Python](#) or [R](#))
- Hard for other data types

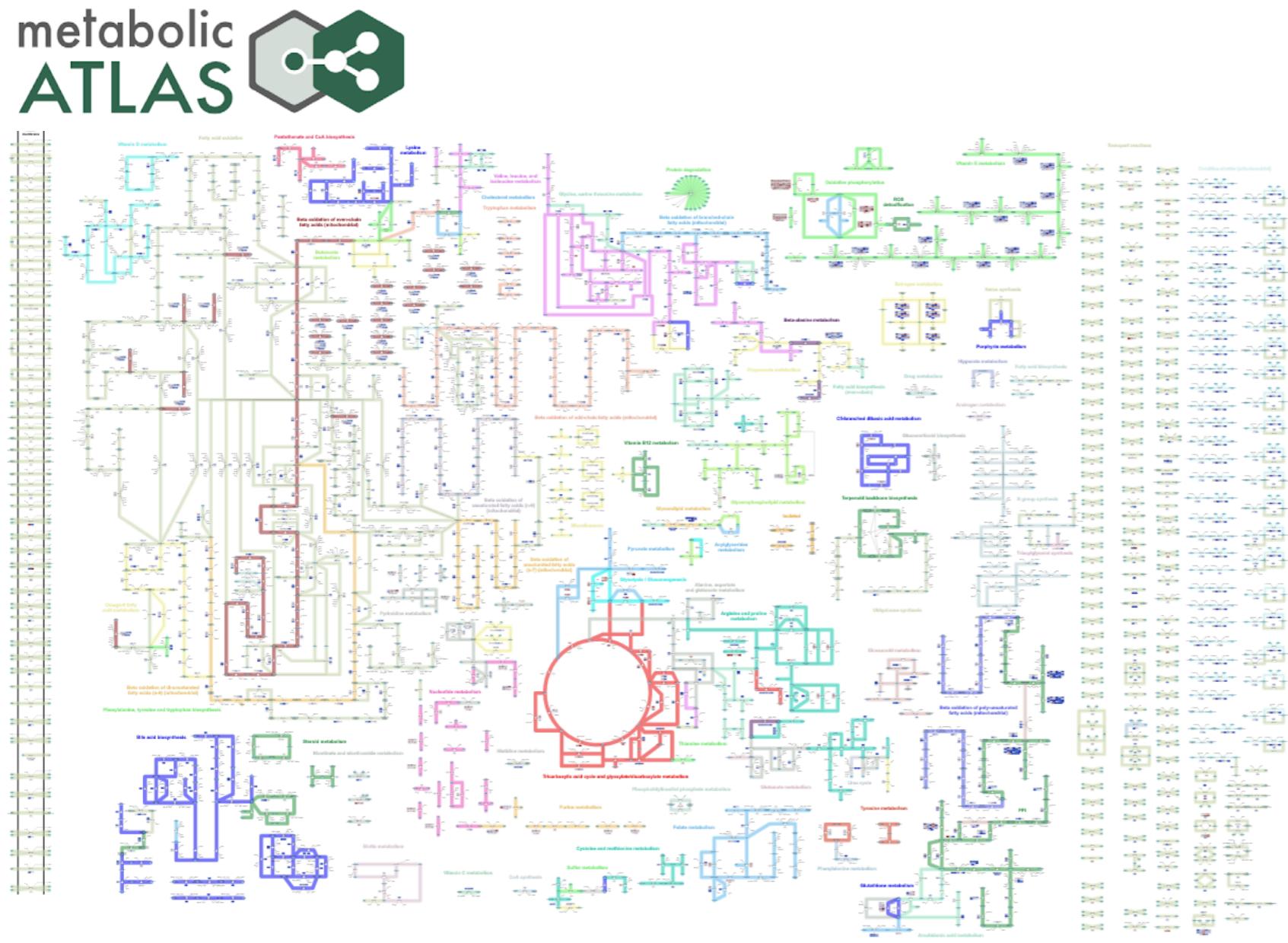
# Genome-scale metabolic models as integrative networks



# Genome-scale metabolic models as integrative networks



# Genome-scale metabolic models as integrative networks



# Simulate flux distributions

## Dysregulated pathways

## Reporter metabolites

## Essential genes

# Targetable enzymes

May be combined with standard graph analysis

# Additional reading

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- [Network Science](#) - Textbook on graph theory and network analysis.
- [Communication dynamics in complex brain networks](#) - Discussion about whether and how network topology may be applied to study the brain networks.
- [A Systematic Evaluation of Methods for Tailoring Genome-Scale Metabolic Models](#) - General review and discussion on methods to use in genome-scale metabolic models.
- [Analysis of Biological Networks](#) - General introduction into biological networks, network notation, and analysis, including graph theory.
- [Multi-omics approaches to disease](#) - Introduction to how integrative approaches may be applied in disease

Additional references displayed as hyperlinks in each slide.

# Additional reading

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- [Analysis of Biological Networks](#) - General introduction into biological networks, network notation, and analysis, including graph theory.
- [Using graph theory to analyze biological networks](#) - overview of the usage of graph theory in biological network analysis
- [Survival of the sparsest: robust gene networks are parsimonious](#) - analysis of network complexity and robustness.
- [Network biology: understanding the cell's functional organization](#) - Overview of key concepts in biological network structure
- [Graph Theory and Networks in Biology](#) - extended perspective on how graph analysis is applied in biology
- [Scale free networks are rare](#)
- [Modularity and community structure in networks](#)

Additional references displayed as hyperlinks in each figure.