

KEEE 2276

EXPERIMENT U4: MOMENT OF INERTIA OF FLYWHEEL

MOHD FUAD BIN SARMAN (KEE 120026)

DEMONSTRATOR:

ABSTRACT

In this experiment, the moment of inertia of flywheel is being studied by varying the point of mass of flywheel. The experiment is conducted by recording the time taken for the respective point of mass to being rotated by a fixed load until the point where the load is escaping from the flywheel and the number of rotations done after be independent from the load. The moment of inertia then is calculated by substituting the data obtained from the experiment and the experimental value is calculate and compared to the experimental one.

INTRODUCTION

The mass moment of inertia is an important concept in rotational motion. The mass moment of inertia also called as the rotational inertia of a body is a measure of how hard it is to get it rotating about some axis. Rotational inertia is one indicator of the ability of rotating body to store kinetic energy. It is also an indicator of the amount of torque that will be needed to rotational accelerate the body. Just as the mass is a measure of resistance of linear acceleration, moment of inertia is a measure of resistance to angular acceleration.

Flywheel is a solid disc of significant size and weight mounted on the shaft of machines such as steam engines, diesel engines, turbine etc. Its function is to minimize the speed fluctuations that takes place when load on such machines suddenly decreases or increases. The flywheel acquires excess kinetic energy from the machines when load on the machine is less or its running idle and supplies the stored energy to the machine when it is subjected to larger loads. The capacity of storing / shedding of kinetic energy depend on the rotational inertia of the flywheel. This rotational inertia is known as moment of inertia of rotating object namely wheels.

Formulation and calculation

Work output from fallen mass is given by the difference between lost in potential and kinetics energy during mass separation from energy wheel.

$$\begin{aligned}\text{Potential energy} &= mgh \\ &= mgh2\pi rN\end{aligned}$$

where N = the number of rotation

$$\text{Final velocity of mass } v = \omega_N r$$

$$\text{Kinetics energy} = \frac{1}{2} m (\omega_N r)^2$$

$$\text{Work produce on energy wheel} = mg (2\pi rN) - \frac{1}{2}m (\omega_N r)^2$$

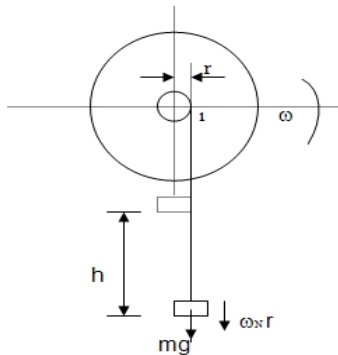


Figure 1 Free diagram of the flywheel.

Energy wheel starts from static condition. It will reach a total of N_1 rotation before stopping, when it is allowed to rotate after body separation. This means that all work was used to overcome the bearing friction which is assumed to be constant. At the time the load separates from the energy wheel, it will reach the maximum angle speed ω_N and the kinetics energy is given by $\frac{1}{2} m(\omega_N r)^2$.

Let say torsion of bearing friction is C_f . By assuming the work used to overcome the friction is equal to the output work, hence

$$mg(2\pi r N) - \frac{1}{2} m(\omega_N r)^2 = C_f(2\pi N_1) \dots\dots\dots (I)$$

Energy equivalent after N rotation is

$$mg(2\pi r N) - \frac{1}{2} m(\omega_N r)^2 = C_f(2\pi N) + \frac{1}{2} I\omega_N^2 \dots\dots\dots (II)$$

Therefore, if N , N_1 and ω_N are measured, C_f can be determined from (I) and (II)

From equation: $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

It can be shown that $\omega_N = 4\pi N/t$

OBJECTIVES

1. To compare the theoretical and experimental value of moment of inertia for flywheel.
2. To investigate the variations in moment of flywheel with different detachable parts.

RESULT AND DISCUSSION

PART 1: A COMPLETE FLYWHEEL WITH OUTER AND INNER RINGS

Readings	1	2	3	Average
Time, t (s)	9.00	9.00	9.00	9.00
No. of rotation, N ₁	52	56	54	54

Table 1 The tabulation of data for the complete energy wheel with outer and inner rings.

$$\begin{aligned}
 \omega_N &= \frac{4\pi N}{t} \\
 &= \frac{4\pi(5)}{9.00} \\
 &= 6.9813 \text{ rad/s}
 \end{aligned}$$

$$\begin{aligned}
 C_f &= \frac{mg2\pi N - \frac{1}{2}m(\omega_N r)^2}{2\pi N_1} \\
 &= \frac{(0.3)(9.81)2\pi(0.02)(5) - \frac{1}{2}(0.3)(6.9813 \times 0.02)^2}{2\pi(54)} \\
 &= 5.4414 \times 10^{-3} \text{ Nm}
 \end{aligned}$$

$$\begin{aligned}
 I_1 &= 2 \frac{mg2\pi N_0 - \frac{1}{2}m(\omega_N r)^2 - 2\pi N_0 C_f}{\omega_N^2} \\
 &= 2 \frac{0.3698(5) - 6 \times 10^{-5} (6.9813)^2 - 2\pi(5)(5.4414 \times 10^{-3})}{(6.9813)^2} \\
 &= 0.0687 \text{ kgm}^2
 \end{aligned}$$

PART 2: A COMPLETE FLYWHEEL WITH INNER RING ONLY

Readings	1	2	3	Average
Time, t (s)	8.00	8.00	8.00	8.00
No. of rotation, N ₁	73	75	67	72

Table 2 The tabulation of data for the complete energy wheel with inner ring only

$$\omega_N = \frac{4\pi N}{t}$$

$$= \frac{4\pi(5)}{8.00}$$

$$= 7.8540 \text{ rad/s}$$

$$C_f = \frac{mg2\pi rN - \frac{1}{2}m(\omega_N r)^2}{2\pi N_1}$$

$$= \frac{(0.3)(9.81)2\pi(0.02)(5) - \frac{1}{2}(0.3)(7.8540 \times 0.02)^2}{2\pi(72)}$$

$$= 4.0793 \times 10^{-3} \text{ Nm}$$

$$I_2 = 2 \frac{mg2\pi rN_0 - \frac{1}{2}m(\omega_N r)^2 - 2\pi N_0 C_f}{\omega_N^2}$$

$$= 2 \frac{0.3698(5) - 6 \times 10^{-5} (7.8540)^2 - 2\pi(5)(4.0793 \times 10^{-3})}{(7.8540)^2}$$

$$= 0.0557 \text{ kgm}^2$$

PART 3: A COMPLETE FLYHEEL WITHOUT OUTER AND INNER RINGS

Readings	1	2	3	Average
Time, t (s)	7.00	7.00	7.00	7.00
No. of rotation, N ₁	81	84	82	82

Table 3 The tabulation if data for the complete energy wheel without outer and inner rings.

$$\omega_N = \frac{4\pi N}{t}$$

$$= \frac{4\pi(5)}{7.00}$$

$$= 8.9760 \text{ rad/s}$$

$$C_f = \frac{mg2\pi N - \frac{1}{2}m(\omega_N r)^2}{2\pi N_1}$$

$$= \frac{(0.3)(9.81)2\pi(0.02)(5) - \frac{1}{2}(0.3)(8.9760 \times 0.02)^2}{2\pi(82)}$$

$$= 3.5796 \times 10^{-3} \text{ Nm}$$

$$I_3 = 2 \frac{mg2\pi N_0 - \frac{1}{2}m(\omega_N r)^2 - 2\pi N_0 C_f}{\omega_N^2}$$

$$= 2 \frac{0.3698(5) - 6 \times 10^{-5} (8.9760)^2 - 2\pi(5)(3.5796 \times 10^{-3})}{(8.9760)^2}$$

$$= 0.0430 \text{ kgm}^2$$

PART 4: A COMPLETE FLYWHEEL WITH OUTER RING ONLY

Readings	1	2	3	Average
Time, t (s)	9.00	9.00	9.00	9.00
No. of rotation, N ₁	56	58	57	57

Table 4 The tabulation of data for the complete energy wheel with outer ring only.

$$\omega_N = \frac{4\pi N}{t}$$

$$= \frac{4\pi(5)}{9.00}$$

$$= 6.9813 \text{ rad/s}$$

$$C_f = \frac{mg2\pi rN - \frac{1}{2}m(\omega_N r)^2}{2\pi N_1}$$

$$= \frac{(0.3)(9.81)2\pi(0.02)(5) - \frac{1}{2}(0.3)(6.9813 \times 0.02)^2}{2\pi(57)}$$

$$= 5.1550 \times 10^{-3} \text{ Nm}$$

$$I_4 = 2 \frac{mg2\pi rN_0 - \frac{1}{2}m(\omega_N r)^2 - 2\pi N_0 C_f}{\omega_N^2}$$

$$= 2 \frac{0.3698(5) - 6 \times 10^{-5} (6.9813)^2 - 2\pi(5)(5.1550 \times 10^{-3})}{(6.9813)^2}$$

$$= 0.0691 \text{ kgm}^2$$

Moment of inertia for components in energy wheel.

$$\text{mass} \quad m = \rho v$$

$$\text{where} \quad \rho = \text{density of steel}$$

$$= 7850 \text{ kg/m}^3$$

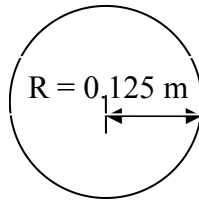
$$\text{and} \quad v = \text{volume}$$

$$= \pi r^2 t$$

Moment of inertia for solid cylindrical wheel

$$I = \frac{1}{2} MR^2$$

A. Basic Unit



$$\text{Mass} \quad m = \rho v$$

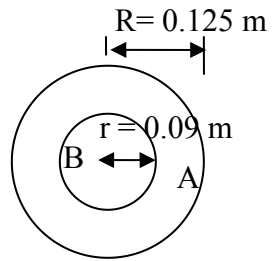
$$= 7850\pi(0.125)^2(0.030)$$

$$= 11.56 \text{ kg}$$

$$\text{Moment of inertia} \quad I = (0.5) (11.56) (0.125)^2$$

$$= 0.090 \text{ kg m}^2$$

B. Outer ring



Inertia moment of outer ring

$$I = \frac{1}{2} MR^2 - \frac{1}{2} mr^2$$

Where M = mass of A and B

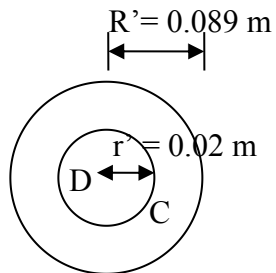
R = A's radius

m = mass of B

r = B's radius

$$\begin{aligned} I &= (0.5)[7850\pi(0.125)^2(0.02)](0.125)^2 - (0.5)[7850\pi(0.09)^2(0.02)](0.09)^2 \\ &= 0.044 \text{ kg m}^2 \end{aligned}$$

C. Inner ring



Moment of inertia,

$$I = \frac{1}{2} M'R'^2 - \frac{1}{2} m'r'^2$$

Where $M' =$ mass of C and D

$R' =$ C's radius

$m' =$ mass of D

$r' =$ D's radius

$$\begin{aligned} I &= (0.5)[7850\pi(0.089)^2(0.02)](0.089)^2 - (0.5)[7850\pi(0.02)^2(0.02)](0.02)^2 \\ &= 0.015 \text{ kg m}^2 \end{aligned}$$

Calculation of moment of inertia based on part of experiment

Part 1: Flywheel with both outer and inner rings

$$\begin{aligned} I &= I_{\text{basic unit}} + I_{\text{outer ring}} + I_{\text{inner ring}} \\ &= 0.090 + 0.044 + 0.015 \\ &= 0.149 \text{ kg m}^2 \end{aligned}$$

Part 2: Flywheel with inner ring only

$$\begin{aligned} I &= I_{\text{basic unit}} + I_{\text{inner ring}} \\ &= 0.090 + 0.015 \\ &= 0.105 \text{ kg m}^2 \end{aligned}$$

Part 3: Flywheel without either inner or outer ring

$$\begin{aligned} I &= I_{\text{basic unit}} \\ &= 0.090 \text{ kg m}^2 \end{aligned}$$

Part 4: Flywheel with outer ring only

$$\begin{aligned} I &= I_{\text{basic unit}} + I_{\text{outer ring}} \\ &= 0.090 + 0.044 \\ &= 0.134 \text{ kg m}^2 \end{aligned}$$

Part of experiment	Experimental value, I (kg m ²)	Theoretical value, I (kg m ²)	Error percentage %
1	0.0687	0.149	53.89%
2	0.0557	0.105	46.95%
3	0.0430	0.090	52.22%
4	0.0691	0.134	48.43%

Table 5 Comparison of experimental and theoretical value for moment of inertia of flywheel.

From the comparison of the theoretical and experimental value of moment of inertia for the flywheel in table 5, this experiment resulted in a very large percentage of error in which the highest occurrence is 53.89% while the smallest is 46.95%. This error might be caused from the human error in which this experiment involved almost everything was handled by human measurement instead of machine. Thus, we can detect some source of error such as the miscalculated number of rotations due to the rotation is too many and too long to be calculated. The wheel also rotates at high angular velocity. Thus, person who calculated it might be confused and miscalculation was occurred. There also might be error in recording the time taken in which it was from human mistake whether too immediate or too late to start the recording or while stopping the stopwatch and also it is depending on what type or scale of the clock being used since in this experiment, we have no specific stopwatch to be used, instead, we used our own watch. Since the experimental calculation is not in ideal case, thus, the rest assumption of source of error is pointed on

the apparatus itself whether there has the lost of energy to the surrounding. Energy is lost due to frictional force in which the friction between the flywheel with the core and the cord will cause a small fraction of energy loss in heat.

Thus, due to this error, a higher step of precaution should be taken to minimise the error. In this experiment, we done thrice reading on three parts of the experiment while the other one, we did twice repetition. This repetition is a way to minimise the error. However, it could be effective if the one who calculate the rotation is a machine since this will avoid a bit human error. Other than that, we may apply lubricant on flywheel to reduce the friction.

To explain about the theoretical concept in this experiment, we consider the theoretical one since the experimental one has a lot of inaccuracy. From the calculation of moment of inertia, we obtain that the largest value is at part one during the whole unit is on action. This is of course due to the highest mass as related in the equation of inertia for solid cylinder which is, $I = 0.5MR^2$ in which the higher the mass, the bigger the moment of inertia. This relationship also being proved by the part 3 experiment in which with the lightest mass, it gives a smaller moment of inertia. To compare on part 2 and part 4, it seems that the moment of inertia for part 2 is much smaller as compared to part 4. This is due to the relationship in which the higher the length of the radius, the higher the value of moment of inertia as described in equation $I = 0.5MR^2$.

CONCLUSION

From the experiment, the moment of inertia of flywheel had been studied in which results in the dependency of mass and radius of the wheel as stated in the equation $I = \frac{1}{2} MR^2$. From here, as the mass and the radius of then flywheel is increased, the moment of inertia is also increased. However, in the comparison of experimental value to the theoretical value, it is found that there are a huge deviation from the theoretical one. Here we conclude the error was done by human mistakes and also might be because of the energy loss due to frictional force. Thus, it is incomparable with the theoretical one because of this is not an ideal condition.

REFERENCES

1. Woodford C. (November 12, 2013). "Flywheels," retrieved at May 28, 2014, from <http://www.explainthatstuff.com/flywheels.html>
2. Bear, FP; Johnson, E.R.Jr, Vector Mech. For Eng, DYNAMICS, International Edition 1996. McGraw-Hill Co, New York 436-438.