1. **Derive a Bezier curve controlled by points A(1,1), B(2,3), C(4,3) and D(6,4)**

Solution:

Control Points = 4

$n = 4 - 1 = 3$

A(1,1) = P0 , B(2,3) = P1 , C(4,3) = P2, D(6,4) = P3

Eq of bezier curve:

$P(t) = (1-t)^3 * A + 3(1-t)^2 * t * B + 3(1-t) * t^2 * C + t^3 * D$

on simplifying

$x(u) = (1-u)^3 + 6u(1-u)^2 + 12u^2(1-u) + 6u^3$

$y(u) = (1-u)^3 + 9u(1-u)^2 + 9u^2 (1-u) +4u^3$

u	x(u)	y(u)
0	1	1
0.2	1.712	0.984
0.4	2.616	2.632
0.6	3.664	3.088
0.8	4.808	3.496
1	6	4

2. **Using the Midpoint circle algorithm, generate a circle using center(0,0) and radius 10 units.**

Solution:

Given $(x_0,y_0) = (0,0)$, r = 10

Starting coordinate: x = 0, y = r =10

Initial decision parameter : 1-r = (1-10) = 9

Calculate the point in the 1st quadrant using the formula

If $P_k < 0$:

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k$

$P_{k+1} = P_k + 2*x_{k+1} + 1$

$P_k >= 0$:

$x_{k+1} = x_k + 1$

$y_{k+1} = y_k - 1$

$P_{k+1} = P_k + 2*x_{k+1} + 1 - 2*y_{k+1}$

P_k	P_{k+1}	x_{k+1}	y_{k+1}	Point
-	-	0	10	(0,10)
-9 < 0	-6	1	10	(1,10)
-6 < 0	-1	2	10	(2,10)
-1 < 0	6	3	10	(3,10)
6>0	-3	4	9	(4,9)
-3 < 0	8	5	9	(5,9)

Now we can calculate all the points of the quadrant of the circle, using the symmetric property of the circle.

Quad 1 : (x,y)

Quad 2 : (-x,y)

Quad 3 : (-x,-y)

Quad 4 : (x,-y)

3. **Assuming that a certain full-color (24 bit pixel) RGB raster system has a 512 by 512 frame buffer.**

a. **how many distinct color choices (intensity levels) would we have available.**

b. **How many different colors could we display at any one time?**

Solution.

In a full-color RGB raster system with 24 bits per pixel :

1. Each of the red, green and blue color channels has 256 intensity levels(2^8=256).
2. There are 16,777,216 distinct colors choices available(256 x 256 x 256)
3. The frame buffer dimensions are 512 by 512 pixels, totalling 262,144 pixels.
4. You can display 262,144 different colors once on the screen, with each pixel representing a unique color from the available choices.

   The number of distinct color choices (intensity levels) available is 16,777,216.
   The number of different colors that can be displayed at only one time is 262,144.

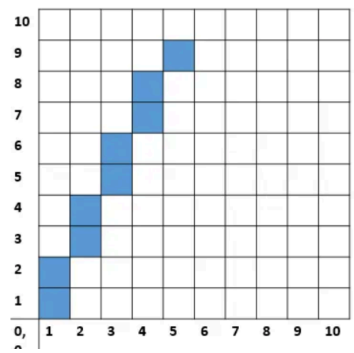
4. Draw a line from P(1,1) to Q(5,9) using DDA algorithm and find the coordinates of the points lying between them?

Solution:

P(1,1) => x1 = 1, y1= 1
Q(5,9) => x2 = 5, y2 = 9
Now lets calculate the difference:
dx = x2 — x1 = 5 −1 = 4
dy = y2 — y1 = 9 -1 = 8
Now lets calculate total number of points to plot:
noOfPointsToPlot = *max(abs(dx), abs(dy)) = max(|4|,|8|)* = 8
Now lets calculate increment for both the coordinates.
xInc = dx/noOfPointsToPlot = 4/8 = 0.5
yInc = dy/noOfPointsToPlot = 8/8 = 1

Steps	X	Y	X = X+ xInc	Y = Y + yInc	Plot(Floor(X), Floor(Y))
Initial	X = x1 =1	Y = y1 =1			Plot(1,1)
1	1	1	1 + 0.5 = 1.5	1 + 1 = 2	Plot(1,2)
2	1.5	2	1.5 + 0.5 = 2	2 + 1 = 3	Plot(2,3)
3	2	3	2 + 0.5 = 2.5	3 + 1 = 4	Plot(2,4)
4	2.5	4	2.5 + 0.5 = 3	4 + 1 = 5	Plot(3,5)
5	3	5	3 + 0.5 = 3.5	5 + 1 = 6	Plot(3,6)
6	3.5	6	3.5 + 0.5 = 4	6 + 1 = 7	Plot(4,7)
7	4	7	4 + 0.5 = 4.5	7 + 1 = 8	Plot(4,8)
8	4.5	8	4.5 + 0.5 = 5	8 + 1 = 9	Plot(5,9)

Now plotting them we get:



5. **Rotate a triangle placed at A(0,0), B(1,1) and C(5,2) by an angle 45 with respect to origin.**

Solution -> In this case since one of the edges of the triangle (A) is already at origin so after performing the transformation the values of A
should not change, which will act as a check. We will multiply the object matrix with the rotation matrix to get the solution.
As direction of the rotation is not given then we always assume it be positive.
The triangle coordinates should also be written in the matrix form, shown as follows. The rotation matrix is also given below.

$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad \begin{bmatrix} \text{Cos}(\emptyset) & -\text{Sin}(\emptyset) & 0 \\ \text{Sin}(\emptyset) & \text{Cos}(\emptyset) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Putting the value of angle in the rotation matrix (Angle given in problem statement is 45 degrees. Also put the values of cos 45 and sin 45 degrees in the matrix. The value of sin and cos 45 degrees is 1/sqrt(2).

$$\begin{array}{ccc} A & B & C \\ \end{array}$$

$$\begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying it we get final answer:

$$\begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 0 & 0 & 3/\sqrt{2} \\ 0 & 2/\sqrt{2} & 8/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Thus, New Coordinates of Triangle —

A'(0,0) , B'(0,2/sqrt(2)) and C'(3/sqrt(2), 8/sqrt(2))

6. **Rotate a triangle placed at A(0,0), B(1,1) and C(5,2) by an angle 45 with respect to point P(-1,-1).**

Solution-> The calculations available for computer graphics can be performed only at origin. It is a case of composite transformation which means this can be performed when more than one transformation is performed.
The following composite transformation matrix would be performed as follows.

a) First bring the Point P(-1,-1) to the origin => which means translation towards origin towards origin will be negative translation. So tx and ty values would be negative.
So tx =-(-1)=1 and ty =—(-1) = 1.

b) Perform the rotation of 45 degrees.

c) Send the point P(-1,-1) back => which mean translation away from origin => Away from origin would be positive translation therefore tx and ty will be positive.
So tx =-1 and ty = -1.

d) The most important point: the composite matrix should be written from right to left , So the composite rotation matrix would be :
T(x) * R(Theta) * T(-x) and NOT T(-x) * R(Theta) * T(x)

The composite rotation matrix would be = Positive translation * Rotation (45degree) * Negative translation.

$$\begin{bmatrix} 1 & 0 & -(-1) \\ 0 & 1 & -(-1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cos(45) & -Sin(45) & 0 \\ Sin(45) & Cos(45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -(1) \\ 0 & 1 & -(1) \\ 0 & 0 & 1 \end{bmatrix}$$

Translation -ve　　Rotation Matrix　　Translation +ve

Substitute of values of translation and rotation angle.

$$\begin{bmatrix} 1 & 0 & -(-1) \\ 0 & 1 & -(-1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -(1) \\ 0 & 1 & -(1) \\ 0 & 0 & 1 \end{bmatrix}$$

Translation -ve　　Rotation Matrix　　Translation +ve

Multiply the resultant rotation matrix with the triangle matrix.

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & (2/\sqrt{2}) - 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Resultant Component Matrix　　Triangle Matrix

The final resultant matrix will be as follows.

$$\begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} -1 & -1 & (3/\sqrt{2})-1 \\ (2/\sqrt{2})-1 & (4/\sqrt{2})-1 & (9/\sqrt{2})-1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

A'(-1, (2/√2)-1) , B' (-1, (4/√2)-1) and C' ((3/√2)-1, (9/√2)-1)

7. **Given a 3D triangle with coordinate points A(3, 4, 1), B(6, 4, 2), C(5, 6, 3). Apply the reflection on the XY plane and find out the new coordinates of the object.**

Solution:

Given-

- Old corner coordinates of the triangle = A (3, 4, 1), B(6, 4, 2), C(5, 6, 3)
- Reflection has to be taken on the XY plane

<u>For Coordinates A(3, 4, 1)</u>

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = Y_{old} = 4$
- $Z_{new} = -Z_{old} = -1$

Thus, New coordinates of corner A after reflection = (3, 4, -1).

<u>For Coordinates B(6, 4, 2)</u>

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = Y_{old} = 4$
- $Z_{new} = -Z_{old} = -2$

Thus, New coordinates of corner B after reflection = (6, 4, -2).

<u>For Coordinates C(5, 6, 3)</u>

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}, Z_{new}).

Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = Y_{old} = 6$
- $Z_{new} = -Z_{old} = -3$

Thus, New coordinates of corner C after reflection = (5, 6, -3).

Thus, New coordinates of the triangle after reflection = A (3, 4, -1), B(6, 4, -2), C(5, 6, -3)

8. **For the given matrix, first apply rotation of 45 degree about y axis followed by rotation of 45 degree about xaxis and determine resultant matrix?**

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 6 & 1 \end{bmatrix}$$

Solution :

→ There are two rotations R_y & R_x.

$$R_y = \begin{bmatrix} \cos 45^\circ & 0 & \sin 45^\circ & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45^\circ & 0 & \cos 45^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ & 0 \\ 0 & \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, Composite transformation.

$$R = R_x \cdot R_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The resultant matrix will be -

$$RM = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 6 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & 3/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. Given a bezier curve with 4 control points- B0[1,0] , B1[3,3] ,B2[6,3] , B3[8,1]. Determine 5 points lying on the curve.

We have-

The given curve is defined by 4 control points.
So, the given curve is a cubic bezier curve.
The parametric equation for a cubic bezier curve isP(t) = B0(1-t)3 + B13t(1-t)2 + B23t2(1-t) + B3t3
Substituting the control points B0, B1, B2 and B3, we getP(t) = [1 0](1-t)3 + [3 3]3t(1-t)2 + [6 3]3t2(1-t) + [8 1]t3 ……..(1)
Now,
To get 5 points lying on the curve, assume any 5 values of t lying in the range 0 <= t <= 1.
Let 5 values of t are 0, 0.2, 0.5, 0.7, 1
For t = 0:
Substituting t=0 in (1), we getP(0) = [1 0](1-0)3 + [3 3]3(0)(1-t)2 + [6 3]3(0)2(1-0) + [8 1](0)3
178
P(0) = [1 0] + 0 + 0 + 0
P(0) = [1 0]

For t = 0.2:

Substituting t=0.2 in (1), we getP(0.2) = [1 0](1-0.2)3 + [3 3]3(0.2)(1-0.2)2 + [6 3]3(0.2)2(1-0.2) + [8
1](0.2)3
P(0.2) = [1 0](0.8)3 + [3 3]3(0.2)(0.8)2 + [6 3]3(0.2)2(0.8) + [8 1](0.2)3
P(0.2) = [1 0] x 0.512 + [3 3] x 3 x 0.2 x 0.64 + [6 3] x 3 x 0.04 x 0.8 + [8 1] x 0.008
P(0.2) = [1 0] x 0.512 + [3 3] x 0.384 + [6 3] x 0.096 + [8 1] x 0.008
P(0.2) = [0.512 0] + [1.152 1.152] + [0.576 0.288] + [0.064 0.008]
P(0.2) = [2.304 1.448]

For t = 0.5:

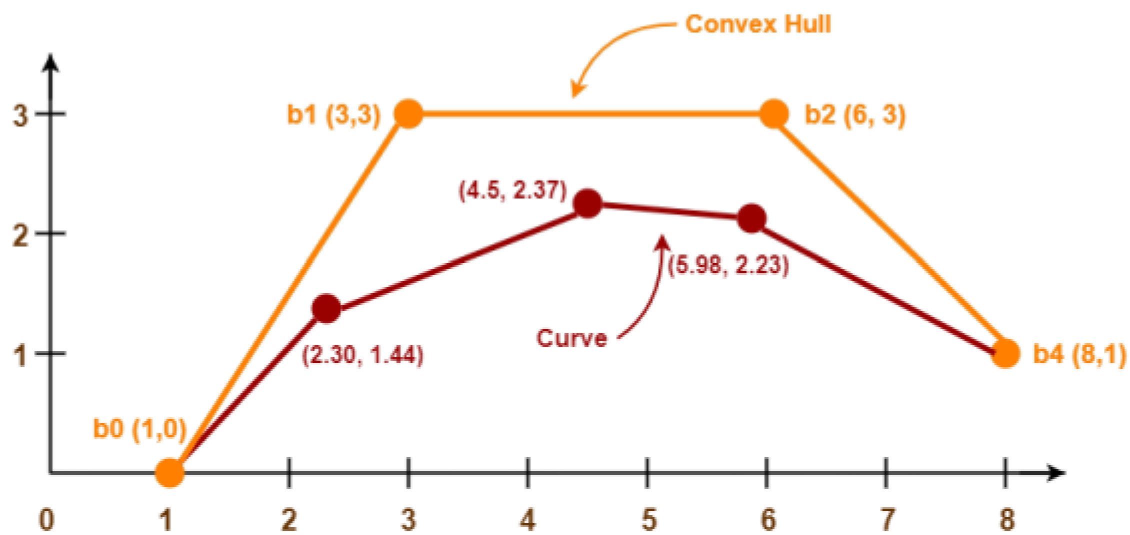
Substituting t=0.5 in (1), we getP(0.5) = [1 0](1-0.5)3 + [3 3]3(0.5)(1-0.5)2 + [6 3]3(0.5)2(1-0.5) + [8
1](0.5)3
P(0.5) = [1 0](0.5)3 + [3 3]3(0.5)(0.5)2 + [6 3]3(0.5)2(0.5) + [8 1](0.5)3
P(0.5) = [1 0] x 0.125 + [3 3] x 3 x 0.5 x 0.25 + [6 3] x 3 x 0.25 x 0.5 + [8 1] x 0.125
P(0.5) = [1 0] x 0.125 + [3 3] x 0.375 + [6 3] x 0.375 + [8 1] x 0.125
P(0.5) = [0.125 0] + [1.125 1.125] + [2.25 1.125] + [1 0.125]
P(0.5) = [4.5 2.375]

For t = 0.7:

Substituting t=0.7 in (1), we getP(t) = [1 0](1-t)3 + [3 3]3t(1-t)2 + [6 3]3t2(1-t) + [8 1]t3
P(0.7) = [1 0](1-0.7)3 + [3 3]3(0.7)(1-0.7)2 + [6 3]3(0.7)2(1-0.7) + [8

1](0.7)3
P(0.7) = [1 0](0.3)3 + [3 3]3(0.7)(0.3)2 + [6 3]3(0.7)2(0.3) + [8 1](0.7)3
P(0.7) = [1 0] x 0.027 + [3 3] x 3 x 0.7 x 0.09 + [6 3] x 3 x 0.49 x 0.3 + [8
1] x 0.343
P(0.7) = [1 0] x 0.027 + [3 3] x 0.189 + [6 3] x 0.441 + [8 1] x 0.343
P(0.7) = [0.027 0] + [0.567 0.567] + [2.646 1.323] + [2.744 0.343]
P(0.7) = [5.984 2.233]

For t = 1:
Substituting t=1 in (1), we getP(1) = [1 0](1-1)3 + [3 3]3(1)(1-1)2 + [6 3]3(1)2(1-1) + [8 1](1)3
P(1) = [1 0] x 0 + [3 3] x 3 x 1 x 0 + [6 3] x 3 x 1 x 0 + [8 1] x 1
P(1) = 0 + 0 + 0 + [8 1]
P(1) = [8 1]



10. A CPU is driven by 2 GHz clock.
(a) Compute the duration of one clock cycle.
(b) Assume that on average the execution of an instruction takes 4
clock cycles. Compute the performance of the CPU in terms of
MIPS (millions of instructions per second).
(c) Assume that executing a specific program of 400 million instructions takes 2
seconds. How many clock cycles does it take on
average to execute an instruction of this program?

Solution:

(a) *There are* $2 \cdot 10^9$ *cycles per second, thus one cycle lasts* $1/(2 \cdot 10^9) = 0.5 \cdot 10^{-9}$ *second, i.e., half nanosecond.*

(b) *The number of instructions executed per second is* $2 \cdot 10^9/4 = 500 \cdot 10^6 = 500$ *million, i.e.,* 500 *MIPS.*

(c) *Executing* $400 \cdot 10^6$ *instructions takes* $4 \cdot 10^9$ *cycles. Thus* $(4 \cdot 10^9)/(400 \cdot 10^6) = (4/400) \cdot 10^3 = 10$ *cycles are needed for one instruction.*

11.

a. Describe the concept of homogeneous coordinates and they are used in computer graphics transformations.

b. Find out the homogeneous coordinate matrix for scaling and translation matrix. (in X, Y direction both)

Solution:

a. Homogeneous coordinate systems mean expressing each coordinate as a homogeneous coordinate to represent all geometric transformation equations as matrix multiplication. The transformed matrix can be expressed in general matrix form.

b. Scaling:

```
| sx 0  0  0 |
| 0  sy 0  0 |
| 0  0  sz 0 |
| 0  0  0  1 |
```

Translation:

```
| 1  0  0  tx |
| 0  1  0  ty |
| 0  0  1  tz |
| 0  0  0   1 |
```

12. The coordinates of four control points relative to a curve are given by p1(2, 2, 0), p2(2, 3, 0), p3(3, 3, 0), p4(3, 2, 0).

a. **Write the equations of bezier curve.**
b. **Also find the curve for u = 0, ¼, ½, ¾, 1.**
c. **Also plot the bezier curve.**

Solution:

Control Points = 4

$n = 4 - 1 = 3$

P0 = (2,2,0) , P1 = (2,3,0) , P2 = (3,3,0) , P3 = (3,2,0)

Eq of bezier curve:

$$P(t) = (1-t)^3 * P0 + 3(1-t)^2 * t * P1 + 3(1-t) * t^2 * P2 + t^3 * P3$$

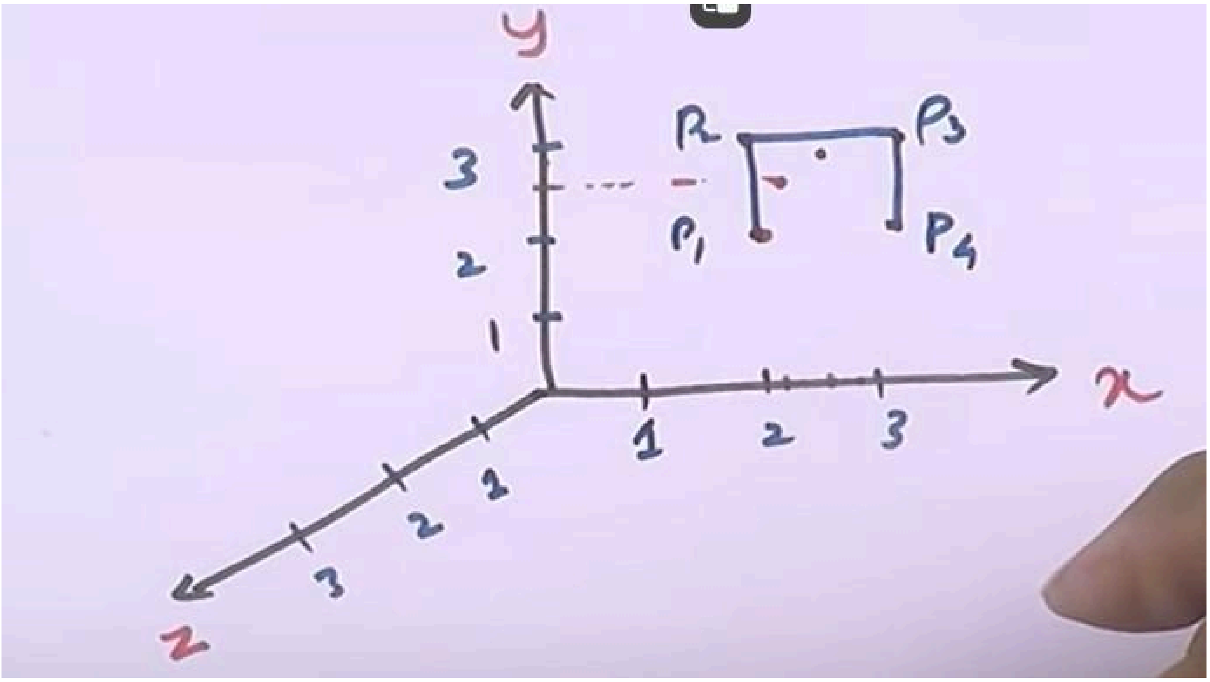
$$x(u) = 2*(1-u)^3 + 6u(1-u)^2 + 9u^2(1-u) + 3u^3$$

$$y(u) = 2*(1-u)^3 + 9u(1-u)^2 + 9u^2(1-u) + 2u^3$$

$$z(u) = 0$$

Putting the value of u in the above equation:

u	x	y	z
0	2	2	0
0.25	2.156	2.56	0
0.5	2.5	2.75	0
0.75	2.84	2.56	0
1	3	2	0



13. Suppose RGB raster system is to be designed using an 8 inch x 10 inch screen resolution of 100 pixels per inch in each direction. If we want to store 6 bits per pixel in the frame buffer, how much storage (in bytes) do we need for the frame buffer?

Solution:

Here, resolution = 8 inch X 10 inch

First, we convert it in pixel then

Now resolution = 8 X 100 by 10 X 100 pixel = 800 X 1000 pixel

1 pixel can store 6 bits

So, frame buffer size required = 800 X 1000 X 6 bits

Bytes = 6 x 10^5 bytes.

14. Show the composition of two rotation is additive by concatenating the matrix representation for R(θ1)*R(θ2) = R(θ1 + θ2)

Solution:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Eqn 1a: 2D Rotation Matrix; Source Link

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \sin\alpha\cos\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}.$$

Replace the value of alpha with **θ1** and Beta with **θ2.**

15.

a. **Write the steps involved in the reflection transformation about y = mx + c and**

b. **Find the transformation matrix for y = mx**
c. **Further extend the transformation matrix for y = mx + c**

Solution

a.

1. Translate by $(0, -b)$ so that the line $y = mx + b$ maps to $y = mx$.
2. Reflect through the line $y = mx$ using the known formula.
3. Translate by $(0, b)$ to undo the earlier translation.

b.

The translation matrices are, respectively,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

and the matrix of the reflection about $y = mx$ is

$$\frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m & 0 \\ 2m & m^2-1 & 0 \\ 0 & 0 & 1+m^2 \end{pmatrix}.$$

c.

Applying these in the correct sequence, the transformation is

$$\frac{1}{1+m^2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1-m^2 & 2m & 0 \\ 2m & m^2-1 & 0 \\ 0 & 0 & 1+m^2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m & -2mb \\ 2m & m^2-1 & 2b \\ 0 & 0 & 1+m^2 \end{pmatrix}.$$

16. Obtain the mirror reflection of the triangle formed by the vertices A(0,3),B(2,0) and C(3,2) about the line passing through the points (1,3) and (–1, –1). (solve using matrix transformation only).

Solution:

The equation of line passing through point (1, 3) and (-1, -1) is given by:

$y = 2x + 1$

Thus $m = 2$ and $b = 1$

$$= \frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m & -2mb \\ 2m & m^2-1 & 2b \\ 0 & 0 & 1+m^2 \end{pmatrix}.$$

Transformation matrix X=

A’ = XA,

B’ = XB,

C’ = XC

Thus new coordinate A’ = (8/5, 11/5), B’ = (-2, 2), C’ = (-1, 4)

17. Derive the transformation matrix for scaling an object by a scaling factor in the direction defined by the angle alpha, beta, gama.

Solution:

Scaling factor given is s.

Effective scaling in X-axis = s * cos(alpha), in Y-axis = s * cos(beta), in Z-axis = s * cos(gama).

Therefore the effective matrix is given by:

$s * \cos(\alpha)$	0	0	0
0	$s * \cos(\beta)$	0	0
0	0	$s * \cos(\gamma)$	0
0	0	0	1

18. How much time is spent scanning across each row of pixels during screen refresh on a raster system with resolution of 1280 X 1024 and a refresh rate of 60 frames per second?

Solution:

Resolution = 1280 X 1024

That means the system contains 1024 scan lines and each scan line contains 128 pixels.

Refresh rate = 60 frame/sec.

1 frame takes = 1/60 sec = 0.01666sec

1 frame buffer consist of 1024 scan lines (It means then 1024 scan lines takes 0.01666 sec)

1 scan line takes = 0.01666/1024 = 10.6 micro sec.

19. Find the transformation matrix that transforms the square ABCD whose center is at (2, 2) is reduced to half of its size, with center still remaining at (2, 2). The coordinates of square ABCD are A (0, 0), B (0, 4), C (4, 4) and D (4, 0). Find the coordinate of the new square.

Solution

After scaling by ½ , the new coordinate will be A'(0,0), B'(0, 2), C'(2,2), D'(2, 0).

Since we need to fix the center of the square, thus translate the new coordinate with (1, 1) point.

Final coordinate will be

A"(1, 1), B"(1, 3), C"(3, 3), D"(3, 1)

20. Consider two different raster systems with resolutions 640 x 480 and 1280 x 1024. What size frame buffer(in bytes) is needed for each of these systems to store 12 bits per pixel ?

Sol.

- Screen 1:
  - Total pixels = 640*480
  - Each pixel requires 12 bits
  - Frame buffer size = (640*480*12)/8
  = 460800 bytes
- Screen 2:
  - Total pixels = 1280*1024
  - Each pixel requires 12 bits
  - Frame buffer size = (1280*1024*12)/8
  = 1966080 bytes

21. Develop an equation of resulting Bezier curve for the coordinates of the four control points relative to a current WCS(World Coordinate System) are given by A(1,1) B(2,3) C(4,3) and D(6,4), find the points on the curve and draw the plot for u = 0, 0.2, 0.4, 0.6, 0.8 and 1

Sol.

control points = 4, $N = 4$

Degree $= N-1 = 4-1 = 3$

$A(1,1) = P_0(x_0, y_0)$

$B(2,3) = P_1(x_1, y_1)$

$C(4,3) = P_2(x_2, y_2)$

$D(6,4) = P_3(x_3, y_3)$

Equation of Bezier curve, $n=3$

$$P(u) = \sum_{i=0}^{n} P_i B_{i,n}(u)$$

$$B_{i,n}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

$$P(u) = P_0 B_{0,3}(u) + P_1 B_{1,3}(u) + P_2 B_{2,3}(u) + P_3 B_{3,3}(u) \quad \text{— (1)}$$

$$B_{0,3}(u) = \frac{3!}{0!(3)!} u^0 \cdot (1-u)^3 = (1-u)^3 \quad \text{— (a)}$$

$$B_{1,3}(u) = \frac{3!}{1!(2)!} u^1 (1-u)^2 = 3u(1-u)^2 \quad \text{— (b)}$$

$$B_{2,3}(u) = \frac{3!}{2!(1)!} u^2 (1-u)^1 = 3u^2(1-u) \quad \text{— (c)}$$

$$B_{3,3}(u) = \frac{3!}{3!0!} u^3 (1-u)^0 = u^3 \quad \text{— (d)}$$

$$P(u) = P_0(1-u)^3 + P_1 3u(1-u)^2 + P_2 3u^2(1-u) + P_3 u^3$$

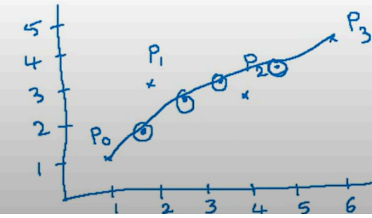
To find $x(u)$ & $y(u)$

$$x(u) = P_{0x}(1-u)^3 + P_{1x}3u(1-u)^2 + P_{2x}3u^2(1-u) + P_{3x}u^3$$
$$= 1(1-u)^3 + 6u(1-u)^2 + 12u^2(1-u) + 6u^3 \quad —(2)$$

$$y(u) = P_{0y}(1-u)^3 + P_{1y}3u(1-u)^2 + P_{2y}3u^2(1-u) + P_{3y}u^3$$
$$= 1(1-u)^3 + 9u(1-u)^2 + 9u^2(1-u) + 4u^3 \quad —(3)$$

u	$x(u)$	$y(u)$
0	1	1
0.2	1.712	1.984
0.4	2.616	2.632
0.6	3.664	3.088
0.8	4.808	3.496
1	6	4

$0 \leq u \leq 1$



22. Using Bresenham’s algorithm, generate the coordinates of the pixels that lie on a line segment having the endpoints (2,3) and (5, 8)

Sol.

```
Case: When slope (m) > 1
```

Now let’s solve the same numerical using BLA Algorithm.

S-1: x1=2; y1=3; x2=5; y2=8.

S-2: dy=y2-y1 8-3= 5 and dx = x2-x1 = 5-2 = 3

dy-dx = 5-3 = 2; and 2 * dy = 10; m(slope) = dy/dx => 5/3

Slope is more than 1 so we will follow the following method.

S-3: Calculate d = 2*dx-dy , so d=2*3 – 5 = 1.

S-4: Always remember the rule for any line algorithm, If m is less than 1 then always increment x and calculate y. If m is more than 1 then do opposite , which is, always increment y and calculate x.

In this case, we will increase y by 1 every step as m (Slope) is more than 1 and calculate y as follows.

a) If d >=0 then x1 = x1 + 1 and y1 = y1 +1 with new d = d + 2*(dx-dy)

b) If d<0 then x1 = x1 (remains same) and y1=y1+1 with new d = d + 2*dx

```
Note: y is always increasing ->Why?, its becasue for m>1, always increase y.
```

S.No.	X1	Y1	d	Pixel Plotted
1	2	3	d = 2*dx – dy = 1	**2,3**
2	**3**	**4**	From step 4 (a) d =1+ 2*(3-5) = -3	**3,4**
3	**3**	**5**	From step 4 (b) d =-3+ 2* 3= 3	**3,5**
4	**4**	**6**	From step 4 (a) d=3+ 2*(3-5)=-1	**4,6**
5	**4**	**7**	From step 4 (b) d = -1 + 2*3 = 5	**4,7**
6	**5**	**8**	From step 4 (a) d = 5+ 2*(3-5) = 1	**5,8**
	Cant increase x as x has reached final	Cant increase y as y has reached final	So algo will stop here.	

Draw a line from (1,1) to (8,7) using Bresenham's Line Algorithm.

```
Case - When Slope (m) <1
```

Draw a line from (1,1) to (8,7) using Bresenham's Line Algorithm.

```
Case - When Slope (m) <1
```

Now let's solve the same numerical using BLA Algorithm.

S-1: x1=1; y1=1; x2=8; y2=7.

S-2: dy=7-1 = 6 and dx = 8-1 = 7

dy-dx = 6-7 = -1; and 2 * dy = 12;

S-3: Calculate d = 2*dy-dx , so d=2*6 – 7 = 5 (Note the change here for m<1)

S-4: We will increase x by 1 every step as m is less than 1 and calculate y as follows

```
Rule: If slope (m) is less than 1 (m<1) then always increase x and calculate y.
```

a) If d >=0 then x1 = x1 + 1 and y1 = y1 + 1 with new d = d + 2*(dy-dx)

b) If d<0 then x1 = x1 + 1 and y1 will not change with new d = d + 2*dy

S.No.	X1	Y1	d	Pixel Plotted
1	1	1	**d = 2*dy – dx = 5**	**1,1**
2	**2**	**2**	**From step 4 (a) d = 5 + 2*(6-7)= 3**	**2,2**
3	**3**	**3**	**From step 4 (a) d = 3 +2*(6-7) = 1**	**3,3**
4	**4**	**4**	**From step 4 (a) d = 1 + 2*(6-7) = -1**	**4,4**
5	**5**	**4**	**From step 4 (b) d = -1 + 2*6 = 11**	**5,4**
6	**6**	**5**	**From step 4 (a) d = 11 +2(6-7) = 9**	**6,5**
7	**7**	6	**d=9+2*(6-7)=7**	**7,6**
8	8	7	Algorithm will stop here after plotting final pixel(8,7).	(8,7)