

NETWORKS AND COMPLEXITY

Solution 5-6

*This is an example solution from the forthcoming book *Networks and Complexity*.*

Find more exercises at <https://github.com/NC-Book/NCB>

Ex 5.6: Local clustering coefficient [3]

In a friendship network we find $n_{--} = 45000$ three node chains and $n_{\Delta} = 1500$ three-cycles (triangles).

- a) Compute the clustering coefficient.

Solution

We compute the clustering coefficient using the formula from the chapter

$$c = \frac{3n_{\Delta}}{n_{--}} = \frac{4.500}{45.000} = 0.1 = 10\%. \quad (1)$$

- b) Ali is a node in the network, he has $k = 17$ friends. How many friendship links f do you expect to find between Ali's 17 friends?

Solution

Ali has $k = 17$ friends, that means that the number of distinct pair of friends between friendships could exist is

$$n = \frac{17 \cdot 16}{2} = 136. \quad (2)$$

This is also the number of three-node-chains that have Ali in the center.

From the clustering coefficient we know that 10% of these chains are closed triangles, so our best guess is that ca. $f = 14$ friendships exist among Ali's friends.

- c) Use the insights gained from this example to write a general formula that can be used to estimate the clustering coefficient c from k and f .

Solution

The final part asks for a general formula. We start from the observation that the clustering coefficient is the probability that two of friends of a node (ego) are also mutually friends. We can write this as

$$c = \frac{f}{n}, \quad (3)$$

where n is the number of three-node-chains centered with ego in the center or in other words the number of ways in which we can pick 2 two people from ego's set of friends. We can compute this number as

$$n = \frac{k(k-1)}{2}, \quad (4)$$

where k is the number of ego's friends. Substituting this relationship into the equation above yields the desired formula

$$c = \frac{2f}{k(k-1)}. \quad (5)$$

You can use this formula for a quick rough estimate the clustering coefficient of the human friendship network, based on your number of friends and the number of friendship links between them.