Networks and Complexity

Exercise Sheet 5: The Small World

This is an exercise sheet from the forthcoming book Networks and Complexity.
Find more exercises and solutions at https://github.com/NC-Book/NCB

Ex 5.1: Averages revision [1]

Compute the averages of the following sequences of numbers: a) $3, 7, 5$; b) $1, 3, 6, 1, 2, 5$; c) a sequence consisting of 13 times the number 3 and 7 times the number 17.

Ex 5.2: Logarithm revision [1]

Note that $\log_{10}(2) \approx 0.3$. Use this to solve the following equations without a calculator: a) $20 = 10^x$, b) $4000 = 10^x$ c) $2 = 1000^x$ d) $5 = 10^x$ e) $2^x = 10^9$. If you can't do it, check out the explanations in the solutions immediately.

Ex 5.3: Diameter of a specific network [2]

Consider a network consisting of a central hub of degree 9 that connects to 9 nodes of degree 1. Compute the diameter according to both definitions (do not use the approximation, it won't work well for such a small and heterogeneous network)

Ex 5.4: Large abstract network [2]

A network has $N = 20,000$ nodes, mean degree of $z = 10$ and no significant clustering coefficient ($c \approx 0$.) Estimate the diameter of this network.

Ex 5.5: Social diameter of Iceland [2]

Iceland has a population of 340,000. Let's assume that the mean degree of the acquaintance network in Iceland is $z = 120$ and the clustering coefficient is $c = 0.2$. Estimate the social diameter of Iceland.

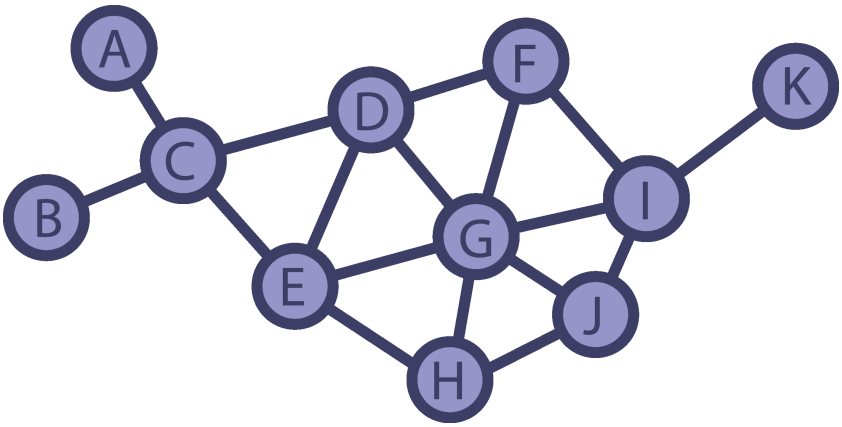
Ex 5.6: Local clustering coefficient [3]

In a friendship network we find $n_{--} = 45000$ three node chains and $n_{\Delta} = 1500$ three-cycles (triangles).

a) Compute the clustering coefficient.
b) Ali is a node in the network, he has $k = 17$ friends. How many friendship links f do you expect to find between Ali's 17 friends?
c) Use the insights gained from this example to write a general formula that can be used to estimate the clustering coefficient c from k and f.

Ex 5.7: A small example [3]

Find the clustering coefficient c and the mean degree z of the following network:



(Hint: Trying to count the three-node chains in this network is a pain. Perhaps some math can help.)

Ex 5.8: Three hop rule [3]

In the USA the Department of Homeland Security routinely collects and stores communication metadata as part of its anti-terrorism efforts. The use of this data is governed by the so-called three-hop-rule, which means that if there is a terrorism suspect investigators can access the data up to 3 links away from the suspect. We do not know how extensive the database is but it seems fair to assume that it is not very different from the facebook network. So let's assume $z = 200$, $c = 0.15$ and $N = 10^{10}$. Compute the number of records that can be accessed in one investigation. Then, suppose there are 10.000 investigations per year. How often will your records be accessed in a year?

Ex 5.9: Shapers [3]

I play an augmented reality game, that has ca. $5 \cdot 10^5$ players worldwide. Through the game I have made many friends and I regularly meet with 20 of them in the real world. Among these friends is a group of ca. 7 people, which know each other well and meet weekly. Among the rest only two know meet each other in real life. Part of the game is to pass virtual items around that behave like physical objects, i.e. to give an item to a player the two of you must meet in the real world. Often items need to reach a specific target person. It is therefore interesting to ask how many times an item has to be handed over to reach its destination. In other words, what is average path length in the network where nodes are players and links represent physical meetings.

Ex 5.10: A real problem [3]

In most western countries child labor has been illegal since the worker's rights movement in the beginning of the 20th century. Some countries also make it illegal for companies to do business with suppliers that use child labor. Suppose a new law is passed that also makes it illegal to have suppliers who have suppliers that use child labor. You know that your company does not use child labor, nor does any of your suppliers. But you are not sure about their suppliers? Assuming that there are 10^6 companies and 1000 of these use child labor. A typical company has about 600 suppliers. Given these numbers, do we have to worry that a supplier of your suppliers uses child labor?

Ex 5.11: Computer network [4]

A computer network consists of 5 servers and 100 workstations. Each server is connected to all other servers and to 20 of the workstations. The workstations have no further connections. Find the exact value of the average path length, and compare it to approximations, with and

without correction for clustering coefficient. (You will find that the approximation performs exceptionally poorly in this case. A brief explanation why this happens is given in the solution.)

Ex 5.12: A mean degree puzzle [4]

A connected network has a mean degree of $z = 1.99901234568\ldots$. How many nodes does the network contain?

[Hint: This is a puzzle! Start by drawing some small networks and calculate z. How do you have to change the network to bring z closer to the desired value, while remaining connected?]