Networks and Complexity

Solution 6-4

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 6.4: Mean degree [2]

For the networks described by the two degree distributions from Ex. 6.1 (a) and (b) compute the mean degree, $z = \sum kp_k$.

Solution

Using the results of Ex. 6.1 we can write the mean degree for the first network (from Ex. 6.1a) as

$$\begin{aligned} z &= \sum kp_k && (1)\\ &= \sum k\left(\frac{3}{4}\delta_{k,4} + \frac{1}{4}\delta_{k,8}\right) && (2)\\ &= \frac{3}{4}\left(\sum k\delta_{k,4}\right) + \frac{1}{4}\left(\sum k\delta_{k,8}\right) && (3)\\ &= \frac{3}{4}\left(\sum 4\delta_{k,4}\right) + \frac{1}{4}\left(\sum 8\delta_{k,8}\right) && (4)\\ &= \frac{12}{4} + \frac{8}{4} = \frac{20}{4} = 5. && (5) \end{aligned}$$

Similarly for the second network (from Ex. 6.1b)

$$\begin{aligned} z &= \sum kp_p && (6)\\ &= \sum k\left(\frac{1}{3}\delta_{k,3} + \frac{1}{3}\delta_{k,4} + \frac{1}{3}\delta_{k,14}\right) && (7)\\ &= \frac{1}{3}\left(\sum k\delta_{k,3}\right) + \frac{1}{3}\left(\sum k\delta_{k,4}\right) + \frac{1}{3}\left(\sum k\delta_{k,14}\right) && (8)\\ &= \frac{1}{3}\left(\sum 3\delta_{k,3}\right) + \frac{1}{3}\left(\sum 4\delta_{k,4}\right) + \frac{1}{3}\left(\sum 14\delta_{k,14}\right) && (9)\\ &= \frac{3}{3} + \frac{4}{3} + \frac{14}{3} = \frac{21}{3} = 7. && (10) \end{aligned}$$