# Networks and Complexity

## Solution 16-2

*This is an example solution from the forthcoming book Networks and Complexity. Find more exercises at https://github.com/NC-Book/NCB*

**Ex 16.2: Transcritical bifurcation [2]**

We can write the normal form of the transcritical bifurcation as

$$\dot{x} = apx + bx^2 \tag{1}$$

Find the steady states of this dynamical system, analyze their stability and draw the bifurcation diagram.

*Solution*

To compute the steady states we start by demanding

$$0 = apx + bx^2 \tag{2}$$

we can immediately see that $x^* = 0$ is a steady state. Dividing the equation above by $x$ we find

$$0 = ap + bx \tag{3}$$

which gives us a second steady state

$$x^* = -\frac{a}{b}p \tag{4}$$

To determine the stability of the steady states we compute

$$f_{\mathrm{x}} = ap + 2bx \tag{5}$$

Substituting $x^* = 0$, yields

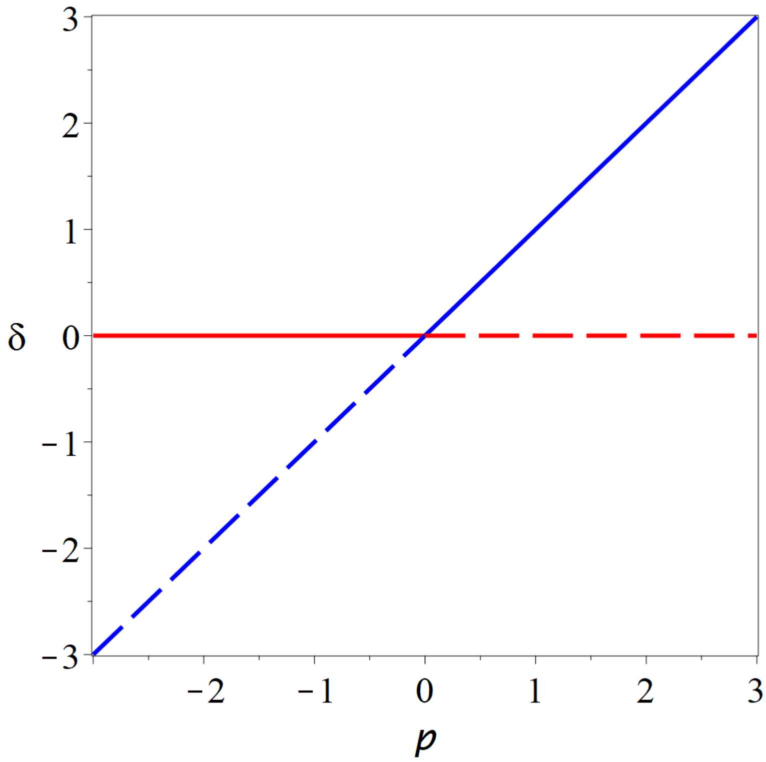
$$f_{\mathrm{x}} = ap \tag{6}$$

This shows that the steady state at 0 is stable if $ap < 0$. Substituting the second steady state $x^* = -ap/b$ gives us

$$f_{\mathrm{x}} = ap - 2b\frac{a}{b}p = -ap \tag{7}$$

which shows that the second steady state is stable if $ap > 0$.

We can now draw the bifurcation diagram




This diagram corresponds to the case $a > 0$ (second steady state is stable for positive $p$) and $b > 0$ (second steady state is positive for positive $p$)