Networks and Complexity

Solution 6-2

This is an example solution from the forthcoming book Networks and Complexity. Find more exercises at https://github.com/NC-Book/NCB

Ex 6.2: Kroneckerisms [1]

Compute the following sums:

$$\text{a)} \sum k\delta_{k,3} \qquad \text{b)} \sum k\delta_{k+1,2} \qquad \text{c)} \sum 3\delta_{k,5} \qquad \text{d)} \sum 4k(\delta_{k,2}-\delta_{k,1})$$

Solution

For part (a), we just sum (starting at $k = 0$),

$$\sum k\delta_{k,3} = 0 + 0 + 0 + 3 + 0 + \ldots = 3 \tag{1}$$

Alternatively we could have used the value substitution and vanishing trick rules, which yields the same result directly.

For part (b) we compute

$$\sum k\delta_{k+1,2} = \sum k\delta_{k,1} = 1 \tag{2}$$

We used maths-in-the-index to subtract 1 from both sides of the index, then value substitution and vanishing act to get the result.

For part (c),

$$\sum 3\delta_{k,5} = 3 \tag{3}$$

This was a straight forward vanishing act. Still many people get this one wrong because it is 'too simple,' If you fell for it, verify the result is right by writing the relevant terms of the sum explicitly.

For part (d),

$$\sum 4k(\delta_{k,2}-\delta_{k,1}) = \left(\sum 8\delta_{k,2}\right) - \left(\sum 4\delta_{k,1}\right) = 4 \tag{4}$$

Splitting the sum and value substitution in the first step, then and the vanishing trick in the second step.