Networks and Complexity

Solution 3-1

This is an example solution from the forthcoming book Networks and Complexity. Find more exercises at https://github.com/NC-Book/NCB

Ex 3.1: Mathematical Tools [1]

Evaluate the following expressions (If you get stuck, check out the solution, it will explain what you need to know).

a) $\sum_{i=1}^{3} i$

Solution

This represents the sum over i, where i runs from 1 to 3. So,

$$\sum_{i=1}^{3} i = 1 + 2 + 3 = 6 \tag{1}$$

b) $\sum_{i=1}^{2} 3i$

Solution

This time i takes only two values 1 and 2, but we are summing $3i$, so we have

$$\sum_{i=1}^{2} 3i = 3 \cdot 1 + 3 \cdot 2 = 3 + 6 = 9 \tag{2}$$

c) $\sum_j A_{2,j}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 1 & 0 \end{pmatrix}.$$

Solution

Here we aren't given bounds for the summation, but from the context we can conclude that we are meant to run j over all values that make sense as a column index of matrix $\mathbf{A}$. As a result the expression sums over the whole second row of the matrix

$$\sum_j A_{2,j} = A_{2,1} + A_{2,2} + A_{2,3} + A_{2,4} = 0 + 0 + 1 + 2 = 3. \tag{3}$$

d) $\prod_{i=3}^{6} i$

Solution

The product sign tells us to multiply factors. In this case it runs from 3 to 6 so we get

$$\prod_{i=3}^{6} i = 3 \cdot 4 \cdot 5 \cdot 6 = 360 \tag{4}$$

e) $\prod (A_{n,5-n} + 1)$, with $\mathbf{A}$ from above.

<u>*Solution*</u>
The first complication here is that the product sign is not annotated with the name of the index, but looking at the right-hand-side we see that a variable n appears which is otherwise undetermined. So n must be the index of the product.

Furthermore, we are not given bounds but we are using n as index in the 4×4 matrix $\mathbf{A}$ so it needs to run from 1 to 4. We can now evaluate

$$\begin{aligned}
\prod (A_{n,5-n} + 1) &= \prod_{n=1}^{4} (A_{n,5-n} + 1) && (5)\\
&= (A_{1,4} + 1) \cdot (A_{2,3} + 1) \cdot (A_{3,2} + 1) \cdot (A_{4,1} + 1) && (6)\\
&= (2 + 1) \cdot (1 + 1) \cdot (1 + 1) \cdot (2 + 1) && (7)\\
&= 3 \cdot 2 \cdot 2 \cdot 3 && (8)\\
&= 24 && (9)
\end{aligned}$$

f) 3!

<u>*Solution*</u>
The factorial $n!$ is a shorthand notation for

$$\prod_{i=1}^{n} i \tag{10}$$

So in the present case

$$3! = \prod_{i=1}^{n} i = 1 \cdot 2 \cdot 3 = 6 \tag{11}$$