Networks and Complexity

Solution 12-11

This is an example solution from the forthcoming book Networks and Complexity. Find more exercises at https://github.com/NC-Book/NCB

Ex 12.11: Box of bolts [4]

In a factory there is a big box of bolts, some of which are broken. At a rate r a worker takes a bolt that is not broken out of the box.

a) Write a differential equation for the number of bolts N that are in the box.

Solution

By now we can write the equation for N straight forwardly

$$\dot{N} = -r \tag{1}$$

b) Express the proportion of broken bolts x as a function of N and the number of broken bolts B.

Solution

$$x = \frac{B}{N} \tag{2}$$

c) Differentiate the equation from part (b) with respect to time to find a differential equation for the number of broken bolts. (Note that B is constant in time). Then use substitution to express the right hand side of the differential equation in terms of x (There can be a B on the right hand side because B is a constant but we want to replace the dynamical variable N)

Solution

Differentiating with respect to time yield

$$\dot{x} = -\frac{B}{N^2}\dot{N} \tag{3}$$

We substitute $\dot{N} = -r$ and $N = B/x$ to find

$$\dot{x} = r\frac{x^2}{B} \tag{4}$$

d) Solve the differential equation for x.

Solution

We use separation of variables to write

$$\int \frac{1}{x^2}\mathrm{d}x = \int \frac{r}{B}\mathrm{d}t \tag{5}$$

Integrating we find

$$-\frac{1}{x} = \frac{rt}{B} + C \tag{6}$$

which we can solve to find

$$x = \frac{1}{-C - \frac{rt}{B}} \tag{7}$$

By considering $t = 0$ we find that $-C = 1/x_0$, hence

$$x(t) = \frac{1}{\frac{1}{x_0} - \frac{rt}{B}} \tag{8}$$

e) Bonus: Check your result by computing $x(t)$ in a different way.

<u>*Solution*</u>
We can define the number of functional (non-broken bolts as F). Straightforwardly the number of functional bolts is

$$F = F_0 - rt \tag{9}$$

Now we write x as

$$x = \frac{B}{N} = \frac{B}{B + F} \tag{10}$$

and substituting F we find

$$x(t) = \frac{B}{B + F_0 - rt} \tag{11}$$

We can write this as

$$x(t) = \frac{1}{\frac{B+F_0}{B} - \frac{rt}{B}} = \frac{1}{\frac{1}{x_0} - \frac{rt}{B}} \tag{12}$$