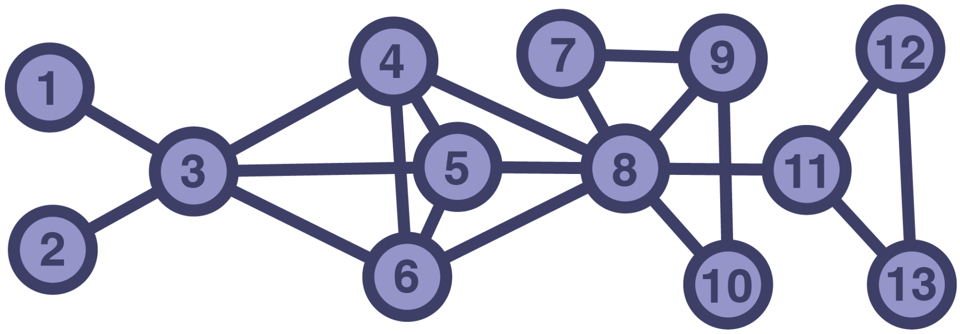
Networks and Complexity

Solution 20-1

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 20.1: Symmetry [2]

Find at least 5 eigenvalues of the adjacency matrix of the following network:



<u>*Solution*</u>

Nodes 1 and 2 form a symmetry orbit. There should be a localized eigenvector. We can guess the eigenvector

$$\boldsymbol{v_1} = (1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^{\mathrm{T}} \tag{1}$$

the corresponding eigenvalue is $\lambda_1 = 0$.

Furthermore, nodes 4, 5, 6 form a symmetric triangle. The eigenvectors can be assigned differently, but we can find two linearly independent localized eigenvectors. For example

$$\boldsymbol{v_2} = (0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0)^{\mathrm{T}} \tag{2}$$

$$\boldsymbol{v_3} = (0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0)^{\mathrm{T}} \tag{3}$$

When we multiply either of these two vectors with the adjacency matrix the -1 and the 1 change places and everything else remains zero. SO the eigenvalues are $\lambda_2 = \lambda_3 = -1$.

Also 12 and 13 are symmetric which gives us the eigenvector

$$\boldsymbol{v_4} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, -1)^{\mathrm{T}} \tag{4}$$

and another eigenvalue $\lambda_4 = -1$.

Finally, node 7 and 10 are another symmetric pair, their locvalized eigenvector is

$$\boldsymbol{v_5} = (0, 0, 0, 0, 0, 0, 1, 0, 0, -1, 0, 0, 0)^{\mathrm{T}} \tag{5}$$

and the corresponding eigenvalue is $\lambda_5 = 0$.