Networks and Complexity

Solution 8-11

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 8.11: Hypergraph components [4]

Hypergraphs are networks, where links connect more than two nodes. (Imagine them as multi-way connectors). Compute the giant component size in a hypergraph where half of the nodes have degree 1, half the nodes have degree 3 and each link connects exactly 3 nodes.

Solution

We follow the same approach as in the chapter, however for every link the node now has two chances to connect to the giant component. So we write

$$s = 1 - \sum p_k v^{2k} v = \sum q_k v^{2k} \tag{1}$$

We also know the degree distribution

$$p_k = \frac{1}{2}\delta_{k,1} + \frac{1}{2}\delta_{k,3} \tag{2}$$

and we can therefore compute the mean degree $z = 2$ and the excess degree distribution

$$q_k = \frac{(k+1)p_{k+1}}{z} = \frac{1}{4}\delta_{k,0} + \frac{3}{4}\delta_{k,2} \tag{3}$$

Substituting into the self-consistency condition for v yields

$$v = \frac{1}{4} + \frac{3}{4}v^4 \tag{4}$$

This suggests that $v \approx 1/4$, indeed iteration reveals $v \approx 0.2530765865$. This is close enough that I am happy to continue with $v = 1/4$. We now substitute into the equation for the giant component size

$$\begin{aligned} s &= 1 - \frac{1}{2}v^2 - \frac{1}{2}v^6 && (5)\\ &= 1 - \frac{1}{2}\frac{1}{4^2} - \frac{1}{2}\frac{1}{4^6} && (6)\\ &= 1 - 2^{-5} - 2^{-13} = 0.9686 && (7) \end{aligned}$$

Using the numerical value from the iteration instead yields 0.9678.

Predictably the three-way connectors are pretty good and holding this network together, leading to a giant component that spans 96% of the network.