Networks and Complexity

Solution 14-3

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 14.3: Stability of the SIS system [2]

In the text of this chapter we studied the stability of our one-dimensional version of the SIS model

$$\dot{I} = p(N - I)I - rI$$

specifically we studied the stability of the trivial steady state $I^* = 0$ and found that this state is stable if $pN < r$ and unstable otherwise. Now consider the non-trivial steady state and determine when it is stability.

Solution

We already know that the non-trivial steady state is at

$$I^* = N - \frac{r}{p} \tag{1}$$

We nod compute the derivative

$$\lambda = \left.\frac{\partial \dot{I}}{\partial I}\right|_* = pN - 2pI^* - r. \tag{2}$$

Substituting the steady state we find

$$\begin{aligned} \lambda &= pN - 2p(N - r/p) - r & (3) \\ &= -pN + r & (4) \end{aligned}$$

So the non-trivial steady state is stable when $r > pN$ and stable otherwise. So the nontirivial state is stable when the trivial steady state is unstable and vice versa. In $r = pN$ the two states meet and exchange their stability.