Networks and Complexity

Solution 3-6

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 3.6: Handshake [3]

If a network has exactly two nodes of odd degree there is an eulerian trail that starts in one and ends in the other. And, if there are zero nodes of odd degree there is an eulerian circuit. What if there is exactly one node of odd degree?

Solution

This is not possible. Every link has two ends, and thus increases the degree of two nodes. Hence the sum of the node degrees in a network must always be even. In other words, the number of nodes of odd degree must be even. This is known as the handshake theorem.