Networks and Complexity

# Solution 17-2

*This is an example solution from the forthcoming book Networks and Complexity.*
*Find more exercises at https://github.com/NC-Book/NCB*

**Ex 17.2: Simple timescale renormalization [1]**

In the system

$$\dot{x} = k(x - x^2)$$

renormalize the timescale to make the parameter $k$ disappear. (Hint: If you are not sure how to do it, renomalise time by a factor $a$ and then decide how you need to chose this factor.)

*Solution*

We define

$$\tau = at \tag{1}$$

Hence

$$\frac{\partial t}{\partial \tau} = \frac{1}{a}. \tag{2}$$

Therefore

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = \frac{\partial t}{\partial \tau}\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{a}k(x - x^2) \tag{3}$$

This in the new units of time we can write

$$\dot{x} = \frac{k}{a}(x - x^2) \tag{4}$$

Chosing $a = k$, yields the desired equation

$$\dot{x} = (x - x^2) \tag{5}$$

Note we can always set one of the rates in the system to one by means of a timescale renormalization.