Networks and Complexity

Solution 9-5

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 9.5: A quick test [2]

Consider a network where every node has degree 1. In this network it is not very hard to guess what the distribution of component sizes looks like.

a) Construct the generating function G of the degree distribution and use it to compute the generating function Q of the excess degree distribution.

Solution

The generating function is simply

$$G = x \tag{1}$$

We compute the excess degree generating function as

$$Q = \frac{G'}{G'(1)} = 1 \tag{2}$$

b) Use the equation from the lecture, $Y = xQ(Y)$, to determine the generating function Y that generates the number of nodes in a branch.

Solution

We substituting our $Q = 1$ into $Y = x(Q(Y))$ gives us

$$Y = x \tag{3}$$

This is the generating function's way of telling us that every branch will contain exactly one node.

c) Use $C = xG(Y)$ to find the function C that generates the component size distribution and explain the the result.

Solution

We compute

$$C = xG(Y) = x^2 \tag{4}$$

This is the generating function of a random process that returns the result 2 with 100% probability. This is the expected result: A network where every node has degree 1 must exist entirely of pairs of two nodes. SO each component has size 2.