Networks and Complexity

Solution 6-11

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 6.11: Normalization proof [4]

Suppose we start with a correctly normalized degree distribution p_k, such that $\sum p_k = 1$. If we calculate the corresponding excess degree distribution q_k we should get a distribution that is a proper probability distribution. Meaning it is also correctly normalized. But can you actually show that $\sum q_k = 1$, for every p_k that meets the normalization condition?

Solution

We know

$$\sum p_k = 1 \tag{1}$$

and we want to show

$$\sum q_k = 1. \tag{2}$$

Our starting point is

$$q_k = (k+1)p_{k+1}/z \tag{3}$$

If we sum the this equation over k we get

$$\sum q_k = \sum (k+1)p_{k+1}/z \tag{4}$$

So far we have the expression we want on the left-hand-side. Now we need to show the right-hand side equals 1. Because z appears our hope is that the other symbols that appear will somehow cancel the z. We know $\sum kp_k = z$, so lets try to use this. We pull the z out of the sum and shift the index in the sum on the right-hand-side such that $k+1$ becomes k. This gives us

$$\sum q_k = \frac{1}{z} \sum kp_k. \tag{5}$$

When we do the shifting we need to be careful that we don't lose or gain an edge term, but in this case we can verify that the sum started $1p_1 + \ldots$ before the shift and now it starts $0p_0 + 1p_1 + \ldots$, which is the same, so all is okay.

We now substitute $\sum kp_k = z$ which yields desired result

$$\sum q_k = \frac{1}{z} \sum kp_k = \frac{z}{z} = 1. \tag{6}$$