Networks and Complexity

Solution 6-1

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 6.1: From case-wise to Kronecker and back [1]
Write the following case-wise distributions using the Kronecker delta notation:

$$\text{a)} \quad p_k = \begin{cases} 3/4 & \text{if } k = 4 \\ 1/4 & \text{if } k = 8 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{b)} \quad p_k = \begin{cases} 1/3 & \text{if } k = 3 \\ 1/3 & \text{if } k = 4 \\ 1/3 & \text{if } k = 14 \\ 0 & \text{otherwise} \end{cases}$$

Then, write the following distributions using the case-wise notation:

$$\text{c)} \quad p_k = \frac{1}{6}\delta_{k,1} + \frac{1}{3}\delta_{k,2} + \frac{1}{2}\delta_{k,3} \qquad \text{d)} \quad p_k = \frac{1}{5}\delta_{k,5} + \frac{4}{5}\delta_{k,10}$$

Solution
For part (a) we find

$$p_k = \frac{3}{4}\delta_{k,4} + \frac{1}{4}\delta_{k,8}, \tag{1}$$

and similarly for (b),

$$p_k = \frac{1}{3}\delta_{k,3} + \frac{1}{3}\delta_{k,4} + \frac{1}{3}\delta_{k,14}. \tag{2}$$

Now we are going backwards and convert to case-wise notation. For (c) we get

$$p_k = \begin{cases} 1/5 & \text{if } k = 5 \\ 4/3 & \text{if } k = 10 \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

Likewise for (d),

$$p_k = \begin{cases} 1/6 & \text{if } k = 1 \\ 1/3 & \text{if } k = 2 \\ 1/2 & \text{if } k = 3 \\ 0 & \text{otherwise} \end{cases}. \tag{4}$$