Networks and Complexity

Solution 2-9

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 2.9: Writing large numbers [4]

In the previous lecture I wrote out the number 10^{317}. This is the kind of task that makes a lazy person wonder how it can be done most efficiently. Let's focus on the simpler example of writing a number with 23 zeros. Suppose I have only two operations at my disposal: a) write a zero, which takes 1 second b) copy and paste to double the number of zeros, which takes 2 seconds. Use Dijkstra's algorithm to find the fastest way of writing 23 zeros. (This may seem be a silly question but very similar optimization problems occur in Engineering.)

Solution

This is an example where the journey we are planning is not between different physical places but between states of a system. That means the places we visit will be states in which we have written different numbers of zeros. For example node 1 is a state where we have written written '0', while node 4 is a state where we have written '0000'. Now we can use Dijkstra's algorithm as usual. The solution is shown on the next page.

It takes 11s to write the 23 zeroes, Using the pathfinding step we can see that the number of zeros (in reverse order of time is)

23,22,11,10,5,4,3,2,1

In other words we the fastest way to write 23 zeros is

T,T,T,T,T,C,T,C,T

where T stands for typing a zero and C stands for copying the existing zeros. If you found this answer you have used Dijkstra's algorithm to navigate an infinite network. Well Done!

(One could argue that the network is finite because all the nodes beyond number 23 cannot be part of the solution, but we could have solved the related question where also deleting a zero is allowed. It is only slightly more tedious but the network in that case is truly infinite nevertheless, one can find the optimal solution with pen and paper using Dijkstra's algorithm)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1																						
	2																					
		3	4																			
			4		5																	
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