Networks and Complexity

Exercise Sheet 8: A Curse of Cows

This is an exercise sheet from the forthcoming book Networks and Complexity. Find more exercises and solutions at https://github.com/NC-Book/NCB

Ex 8.1: Giant components in ER networks [1]

Derive the equation

$$v = \mathrm{e}^{(v-1)z}. \tag{1}$$

starting from

$$v = \sum q_k v^k. \tag{2}$$

Assume that the network is an ER random graph.

Ex 8.2: Abstract component sizes [2]

Compute the expected size in of the component of a randomly picked node in an ER network with

a) $z = 0.1$
b) $z = 0.9$

Ex 8.3: Abstract giant component [2]

Compute the size of the giant component in a network nodes in the following networks:

a) An ER-graph with $z = 2.5$ with 100.000 nodes.
b) A network consisting of 50.000 nodes of degree 1 and 50.000 nodes of degree 3.

Ex 8.4: Poissonian dictionary [2]

I have a Spanish-Spanish dictionary which I use to look up words, but when I look up a work there are typically a number of words in the explanation that I don't understand, so I have to look up their definitions as well. I noticed that the number of these words follows a Poisson distribution with mean 0.8. Estimate the average number of words that I have to look up to understand the meaning of one word.

Ex 8.5: Flashback to six degrees [2]

The mean degree of the human contact network is about $z = 150$. Estimate the proportion of people that are in the giant component. (Assume that the contact network is an ER random graph.)

Ex 8.6: Sexual contact network [3]

Extensive surveys found that typical people have had 5 sexual partners.

a) Does the human sexual contact network have a giant component? And, if yes what proportion of people would you expect to be in the giant component, assuming that the network is an ER random graph?
b) The real sexual contact network is more heterogeneous. How would your estimate of the giant component size change if we assumed the degree distribution $p_k = 0.8\delta_{k,1} + 0.2\delta_{k,21}$? (Bonus: What is your expectation of the number of contacts of your partner)

Ex 8.7: Astrobiochemistry [3]

Researchers want to estimate the density of the atmosphere of an extrasolar planet. They know that the atmosphere consists of 6% carbon and 94% Hydrogen. It is assumed that basically all of the atoms in the atmosphere are bound in molecules. Each carbon atom binds to four other atoms and each hydrogen atom binds to one other atom. Assume that the binding strength is irrelevant (the planet is quite hot). Use the method from the chapter to determine the size of the molecule we find typical Atoms in.

Ex 8.8: Two faces of heterogeneity [3]

Consider a network where half of the nodes have degree 1, whereas the other half follow a Poisson distribution with mean 3. Compare the mean excess degree and the size of the giant component in this network to the network from Ex. 8.3b.

Ex 8.9: Is this right? [4]

Consider a network in which every node has degree 3.

a) Estimate the size of the giant component. The result may look odd at first glance.
b) Now imagine the smallest possible component that could exist in this network. Use the results from the previous chapter to estimate how common this motif is in the network, i.e. in the limit of large network size what is the probability that a randomly picked node is part of such a motif.

Ex 8.10: Peculiar networks [4]

Our estimate of the giant component transition relies on the networks being sufficiently random. If networks are constructed in a very particular way then the transition may happen at other values of z and q.

To provide example construct networks that have...

a) $z > 4$ but no giant component.
b) $z < 0.5$ but has a giant component
c) $q > 100$ no giant component.
d) $q < 0.001$ but a giant component.

Ex 8.11: Hypergraph components [4]

Hypergraphs are networks, where links connect more than two nodes. (Imagine them as multi-way connectors). Compute the giant component size in a hypergraph where half of the nodes have degree 1, half the nodes have degree 3 and each link connects exactly 3 nodes.

Ex 8.12: Degree correlations [4]

Let us once again consider a network where 75% of the nodes have degree 1 and 25% degree 3.

a) Determine if a giant component exists, and if so, compute it's size.
b) So far we have considered the case where the nodes are connected completely randomly, but what if the network is not quite so random? Perhaps the nodes of degree 3 like to link to their own kind? Use $p_{i,j}$ to denote the probability that a link starting at a node of degree i leads to a node of degree j. Suppose we know $p_{3,3} = b$, and compute $p_{3,1}$, $p_{1,3}$ and $p_{1,1}$.
c) Find a way to compute the giant component size as a function of b. In your calculation use s_i to denote the probability that a node of degree i is part of the giant component. Use your result to compute the giant component size for $b = 3/4$.