Networks and Complexity

Solution 5-12

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 5.12: A mean degree puzzle [4]

A connected network has a mean degree of $z = 1.99901234568\ldots$. How many nodes does the network contain?

[Hint: This is a puzzle! Start by drawing some small networks and calculate z. How do you have to change the network to bring z closer to the desired value, while remaining connected?]

Solution

If we follow the advice we find very quickly that every small network that we draw will have a mean degree that is too high, unless it is a tree. In fact you might notice that every connected network that contains a single cycle has a mean degree of exactly 2 and every network that contains more than one cycle has a mean degree greater than two. So the network we are looking for must be a tree.

We also noticed that in a tree of N nodes the number of links is $K = N - 1$. (this can be easily proved by induction, the tree with 1 node contains no links, to add another node to the tree we need exactly one link, etc.) Hence the mean degree of a tree is

$$z = \frac{2K}{N} = \frac{2(N-1)}{N} \tag{1}$$

In the exercise we are asked to find the number of nodes for a given mean degree so let's solve for N

$$\begin{aligned} z &= 2(N-1)/N && (2)\\ Nz &= 2(N-1) && (3)\\ Nz &= 2N-2 && (4)\\ N(z-2) &= -2 && (5)\\ N &= 2/(2-z) && (6) \end{aligned}$$

Substituting the mean degree from the question we get

$$N = \frac{2}{2 - 1.99901234568} \approx 2025.0000 \tag{7}$$

so the answer is $N = 2025$.

As a little bonus, we can even compute the exact mean degree of a tree with 2025 nodes. It is the beautiful

$$z = \frac{2 \cdot 2024}{2025} = 1.999\overline{012345679}. \tag{8}$$