Networks and Complexity

Solution 17-1

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 17.1: Simple change of variables [1]
Consider the system

$$\dot{x} = 2x - 3y \tag{1}$$
$$\dot{y} = xy \tag{2}$$

Rewrite the system in terms of x and the new variable $s = x + y$

Solution
We start with

$$s = x + y \tag{3}$$

Differentiating with respect to time yields

$$\dot{s} = \dot{x} + \dot{y} \tag{4}$$

Substituting the equations for the time derivatives we find

$$\dot{s} = 2x - 3y + xy \tag{5}$$

Do the system is now

$$\dot{x} = 2x - 3y \tag{6}$$
$$\dot{s} = 2x - 3y + xy \tag{7}$$

To replace the instances of y we solve $s = x + y$ for

$$y = s - x \tag{8}$$

Substituting into the system we find

$$\dot{x} = 2x - 3(s - x) \tag{9}$$
$$\dot{s} = 2x - 3(s - x) + x(s - x) \tag{10}$$

which we can simplify to

$$\dot{x} = 5x - 3s \tag{11}$$
$$\dot{s} = 5x - 3s + xs - x^2 \tag{12}$$