Networks and Complexity

Solution 12-5

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 12.5: Birth and death [3]

A population of bacteria X grows by cell division at rate g

$$\mathrm{X} \xrightarrow{g} 2\ \mathrm{X}$$

In addition the bacteria die spontaneously at rate l

$$\mathrm{X} \xrightarrow{l} \varnothing$$

Derive an ODE for the population size x and solve it.

Solution

Both processes are proportional to X and increase / reduce the number of bacteria by 1 respectively, hence

$$\dot{x} = gx - lx = (g - l)x \tag{1}$$

We already solved such a linear equation in the chapter. The solution is

$$x(t) = x_0 \mathrm{e}^{(g-l)t} \tag{2}$$