Networks and Complexity

Solution 8-2

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 8.2: Abstract component sizes [2]

Compute the expected size in of the component of a randomly picked node in an ER network with

a) $z = 0.1$

Solution

For (a) and (b) use the formula for the size of small components

$$S = 1 + \frac{z}{1-q} \tag{1}$$

Because the network is an ER-graph, $z = q$. And hence in the case (a) we get

$$S = 1 + \frac{0.1}{0.9} \approx 1.1 \tag{2}$$

This makes sense, every component contains at least the randomly picked node. If the mean degree is 0.1, then we expect less than every 10th node to have a degree of 1. So only about every tenth randomly picked node will be in a component of size 2. The chances of picking a node that is in a component containing more than two nodes are very small.

b) $z = 0.9$

Solution

Now $z = q = 0.9$. Substituting into the same equation as in (a) yields

$$S = 1 + \frac{0.1}{0.9} \approx 1 \tag{3}$$

and in the case (b)

$$S = 1 + \frac{0.9}{0.1} = 10 \tag{4}$$

As the mean degree approaches 1 the components are becoming larger quickly.