Networks and Complexity

Solution 3-2

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 3.2: In and out [1]

In the digraph described by the adjacency matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix},$$

compute the in- and out-degrees of the three nodes?

Solution

We defined the in-degree

$$k_i^{\text{in}} = \sum_j A_{ij} \tag{1}$$

which means

$$k_1^{\text{in}} = \sum_j A_{1j} = A_{11} + A_{12} + A_{13} = 0 + 1 + 2 = 3 \tag{2}$$

$$k_2^{\text{in}} = \sum_j A_{2j} = A_{21} + A_{22} + A_{23} = 1 + 0 + 1 = 2 \tag{3}$$

$$k_3^{\text{in}} = \sum_j A_{3j} = A_{31} + A_{32} + A_{33} = 0 + 0 + 2 = 2 \tag{4}$$

Similarly the out-degree is

$$k_i^{\text{out}} = \sum_j A_{ji} \tag{5}$$

which means

$$k_1^{\text{out}} = \sum_j A_{j1} = A_{11} + A_{21} + A_{31} = 0 + 1 + 0 = 1 \tag{6}$$

$$k_2^{\text{out}} = \sum_j A_{j2} = A_{12} + A_{22} + A_{32} = 1 + 0 + 0 = 1 \tag{7}$$

$$k_3^{\text{out}} = \sum_j A_{j3} = A_{13} + A_{23} + A_{33} = 2 + 1 + 2 = 5 \tag{8}$$