Networks and Complexity

Solution 6-7

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 6.7: Manhattan [3]

In a city every road junction is a 4-way intersection. Write the degree distribution p_k for the network where intersections are represented by nodes and the road segments connecting them are links. Then show mathematically that $z = 4$ and $q = 3$.

Solution

Since every intersection connects to 4 roads, every node has degree 4, and hence

$$p_k = \delta_{4,k}. \tag{1}$$

To compute the mean degree mathematically we use

$$\begin{aligned} z &= \sum k p_k & (2)\\ &= \sum k \delta_{4,k} & (3)\\ &= \sum 4 \delta_{4,k} & (4)\\ &= 4. & (5) \end{aligned}$$

This shows that we should expect that a randomly chosen intersection connects to 4 roads. No big surprise, since this is true for every intersection individually.

To find the mean excess degree q we first find the excess degree distribution

$$\begin{aligned} q_k &= (k+1)p_{k+1}/z & (6)\\ &= (k+1)\delta_{4,k+1}/4 & (7)\\ &= \delta_{4,k+1} & (8)\\ &= \delta_{3,k}. & (9) \end{aligned}$$

We can now compute the mean excess degree

$$\begin{aligned} q &= \sum k q_k & (10)\\ &= \sum k \delta_{3,k} & (11)\\ &= 3. & (12) \end{aligned}$$

So our expectation is that if we follow a random link then we will find three further links at the next intersection (i.e. go straight, turn right, turn left). Again this isn't a big surprise, but it is good to see that the maths works out correctly.