Networks and Complexity

Solution 15-3

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 15.3: Rock-paper-scissors [2]

Consider a network of agents playing the rock-paper-scissors game. Each agent has a state which can be either rock (R), paper (P), or scissors (S). At some rate we pick a random link and the game is played by the two agents that the link connects: If both players are in the same state the game is a draw and nothing happens. Otherwise R wins against S, S wins against P, and P wins against R. Whoever is the loser then changes it's state to the strategy that would have won the game. For example if R plays against P, R loses and switches to S. Write mean field equations for the proportion of players with a given strategy, $[R]$, $[P]$, $[S]$, and find the steady states.

Solution

The mean field equations before closure are

$$\begin{aligned} [\dot{R}] &= [SP] - [RP] && (1)\\ [\dot{S}] &= [PR] - [SR] && (2)\\ [\dot{P}] &= [RS] - [PS] && (3) \end{aligned}$$

We close the system using a mean field approximation of the form $[AB] = z[A][B]$ which yields

$$\begin{aligned} [\dot{R}] &= z[P]([S] - [R]) && (4)\\ [\dot{S}] &= z[R]([P] - [S]) && (5)\\ [\dot{P}] &= z[S]([R] - [P]) && (6) \end{aligned}$$

There are a number of different steady states:

a. $[R] = [P] = [S] = 0$

b. $[R] = c$, $[P] = [S] = 0$ and related states where the types are exchanged

c. $[R] = [P] = [S]$