Networks and Complexity

Solution 5-2

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 5.2: Logarithm revision [1]

Note that $\log_{10}(2) \approx 0.3$. Use this to solve the following equations without a calculator: a) $20 = 10^x$, b) $4000 = 10^x$ c) $2 = 1000^x$ d) $5 = 10^x$ e) $2^x = 10^9$. If you can't do it, check out the explanations in the solutions immediately.

<u>*Solution*</u>

The logarithm $x = \log_a y$ is (by definition) the solution to

$$y = a^x \tag{1}$$

. Some useful consequences of this definition are

$$\begin{aligned} &\text{i)} && a^{\log_a(x)} = x = \log_a(a^x) && (2)\\ &\text{ii)} && \log_a(xy) = \log_a(x) + \log_a(y) && (3)\\ &\text{iii)} && \log_a(x^y) = y\log_a(x) && (4)\\ &\text{iv)} && \log_a(x) = \log_b(x)/\log_b(a) && (5)\\ &\text{v)} && \log_a(1/x) = -\log_a(x) && (6)\\ &\text{vi)} && \log_x(x) = 1 && (7) \end{aligned}$$

To solve (a) we use

$$x = \log_{10}(20) = \log_{10}(2) + \log_{10}(10) \approx 0.3 + 1 = 1.3 \tag{8}$$

Note that the second term, where we used rule ii, and then rule vi.

For part (b) we can do the following:

$$x = \log_{10}(8000) = \log_{10}(8) + \log_{10}(1000) = \log_{10}(2^3) + \log_{10}(10^3) \approx 3 \cdot 0.3 + 3 \approx 3.9 \tag{9}$$

where we used rule ii, then wrote the numbers as powers to apply rule iii.

To solve (c), we need rule iv

$$x = \log_{1000}(2) = \frac{\log_{10}(2)}{\log_{10}(1000)} \approx \frac{0.3}{3} = 0.1 \tag{10}$$

We do part (d) almost in the same way

$$x = \log_{10}(5) = \log_{10}(10/2) = \log_{10}(10) - \log_{10}(2) \approx 1 - 0.3 = 0.7 \tag{11}$$

The key idea here is to write 5 as 10/2 and then use rules ii and v.

Finally for (e) we start with a change of basis (rule iv)

$$x = \log_2(10^9) = \frac{\log_{10}(10^9)}{\log_{10}(2)} \approx \frac{9}{0.3} = \frac{90}{3} = 30 \tag{12}$$