Networks and Complexity

Solution 13-1

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 13.1: Quadratic polynomials [1]

Solve the following quadratic polynomials:

a) $1 + \lambda = \lambda^2$,

Solution

We solve this as

$$\begin{aligned} \lambda^2 - \lambda &= 1 && (1)\\ \lambda^2 - \lambda + 1/4 - 1/4 &= 1 && (2)\\ (\lambda - 1/2)^2 &= 5/4 && (3)\\ \lambda - 1/2 &= \pm\sqrt{5/4} && (4) \end{aligned}$$

The result can be written as

$$\lambda = \frac{1 \pm \sqrt{5}}{2} \tag{5}$$

b) $(6 - \lambda)(2 - \lambda) - 60 = 0$,

Solution

Unfortunately we need to multiply this out

$$\begin{aligned} \lambda^2 - 8\lambda + 12 - 60 &= 0 && (6)\\ (\lambda - 4)^2 + 12 - 60 - 16 &= 0 && (7)\\ (\lambda - 4)^2 &= 64 && (8)\\ \lambda - 4 &= \pm 8 && (9) \end{aligned}$$

Hence we find the two solutions $\lambda_1 = -4$ and $\lambda_2 = 12$

c) $(2 - \lambda)^2 - 9 = 0$.

Solution

This one is convenient as we already have a quadratic expression. So straight away

$$\begin{aligned} (2 - \lambda)^2 &= 9 && (10)\\ \lambda - 2 &= \pm 3 && (11) \end{aligned}$$

and hence $\lambda_1 = 5$ and $\lambda_2 = -1$.