Solution 19-4

Ex 19.4: Traffic: Spectral Centralities [3]

Consider the following road network
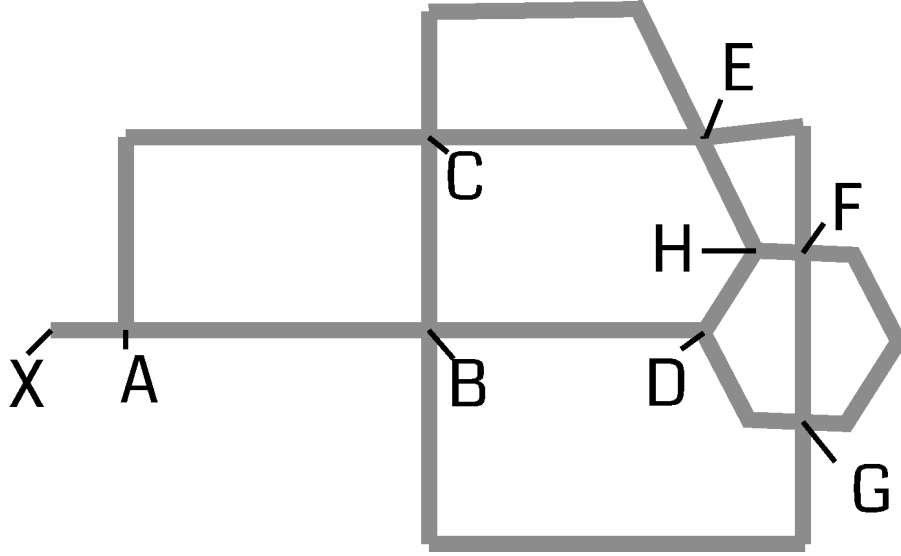
Find the relative importance of the nodes using the spectral centrality metric (Hint: mind the parallel links). Is the result reasonable?

Solution

The adjacency matrix is

$$
\mathbf{A} = \begin{pmatrix}
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 2 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\tag{1}
$$

where I used the ordering

$$A, B, C, D, E, F, G, H, X$$

I had to multiply my starting vector about 10 times until it stabilized. I then normalized the vector such that the first entry is (the one corresponding to node A) is 1. The relative importance of the nodes is then:

$$A: 1, B: 1.6, C: 1.7, D: 1.3, E: 1.8, F: 1.9, G: 1.8, H: 1.4, X: 0.2$$