Networks and Complexity

Solution 12-1

This is an example solution from the forthcoming book Networks and Complexity. Find more exercises at https://github.com/NC-Book/NCB

Ex 12.1: Integration [1]

Find the general solutions for the following differential equations:

a.
$$\dot{x} = 4x$$

b.
$$\dot{x} = \frac{1}{2nx}$$

c.
$$\dot{x} = \frac{x}{a+t}$$

d.
$$\frac{\mathrm{d}c_k}{\mathrm{d}k} = -\frac{2c_k}{k}$$

Solution

All of these can be integrated by separation of variables

a.
$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= 4x && (1)\\ \frac{1}{x}\mathrm{d}x &= 4\mathrm{d}t && (2)\\ \int \frac{1}{x}\mathrm{d}x &= \int 4\mathrm{d}t && (3)\\ \log x &= 4t + C && (4)\\ x &= x_0 \mathrm{e}^{4t} && (5) \end{aligned}$$

where $x_0 = \mathrm{e}^C$.

b.
$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \frac{1}{2x} && (6)\\ \int 2x\mathrm{d}x &= \int \mathrm{d}t && (7)\\ x^2 &= t + C && (8)\\ x &= \pm\sqrt{t+C} && (9) \end{aligned}$$

c.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{x}{a+t} \tag{10}$$

$$\int \frac{1}{x}\mathrm{d}x = \int \frac{1}{a+t}\mathrm{d}t \tag{11}$$

$$\log(x) = \log(a+t) + C \tag{12}$$

$$x = \mathrm{e}^{\log(a+t)+C} \tag{13}$$

$$x = A(a+t) \tag{14}$$