Networks and Complexity

Solution 1-8

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 1.8: Another network in Moravia [3]

Let's revisit the Moravian example, described by the distance matrix

$$\mathbf{D} = \begin{pmatrix} 0 & 137 & 63 & 74 & 77 \\ 137 & 0 & 75 & 76 & 198 \\ 63 & 75 & 0 & 51 & 121 \\ 74 & 76 & 51 & 0 & 151 \\ 77 & 198 & 121 & 151 & 0 \end{pmatrix}$$

where the nodes are again

$$1 : \mathrm{B},\ 2 : \mathrm{O},\ 3 : \mathrm{L},\ 4 : \mathrm{Z},\ 5 : \mathrm{J}.$$

This time we assume that when we get to work there is already a power line from B to Z. Find which additional lines need to be built such that all cities are connected and the length of additional lines built is minimal.

Solution

We can apply Kruskal's algorithm, but line (B,Z) is already there (or alternatively, we could say it has cost 0). So what we do is

1. Place (B,Z)
2. Try (L,Z) [52km] – accept
3. Try (L,B) [63km] – reject
4. Try (L,O) [75km] – accept
5. Try (O,Z) [76km] – reject
6. Try (B,J) [77km] – accept

So, the final edge set is

$$E = \{(\mathrm{B},\mathrm{Z}), (\mathrm{L},\mathrm{Z}), (\mathrm{L},\mathrm{O}), (\mathrm{B},\mathrm{J})\}. \tag{1}$$