

NETWORKS AND COMPLEXITY

Solution 15-2

*This is an example solution from the forthcoming book *Networks and Complexity*.*

Find more exercises at <https://github.com/NC-Book/NCB>

Ex 15.2: Voter model mean field [2]

Consider a network with N nodes and K links, where nodes represent agents and links represent social contacts. Each agent is in either of two states, say favoring one of two political parties. We call these states A and B. Further we say a link is an active link if it connects agents with different opinions, i.e. it is an AB-link. Active links get updated at rate r . In an update one of the two agents connected by the link is chosen at random and that agent's opinion is copied to the other agent. Use a mean field approximation to derive a system of differential equations for this model. (The result may be surprising)

Solution

Deriving the differential equations along the lines of the lecture yields

$$[\dot{A}] = \frac{1}{2}[AB] - \frac{1}{2}[AB] = 0 \quad (1)$$

$$[\dot{B}] = \frac{1}{2}[AB] - \frac{1}{2}[AB] = 0 \quad (2)$$

$$(3)$$

If an update occurs the number of nodes of opinion A is increased by 1 (probability 50%) or it is decreased by 1 (also probability 50%). These two processes cancel so that the net change of the number of nodes in state A (or B) is zero. The voter model is actually a very fair model.