Networks and Complexity

Exercise Sheet 3: Bridgewalk

This is an exercise sheet from the forthcoming book Networks and Complexity. Find more exercises and solutions at https://github.com/NC-Book/NCB

Ex 3.1: Mathematical Tools [1]

Evaluate the following expressions (If you get stuck, check out the solution, it will explain what you need to know).

a) $\sum_{i=1}^{3} i$
b) $\sum_{i=1}^{2} 3i$
c) $\sum_{j} A_{2,j}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 2 & 2 & 1 & 0 \end{pmatrix}.$$

d) $\prod_{i=3}^{6} i$
e) $\prod(A_{n,5-n} + 1)$, with $\mathbf{A}$ from above.
f) 3!

Ex 3.2: In and out [1]

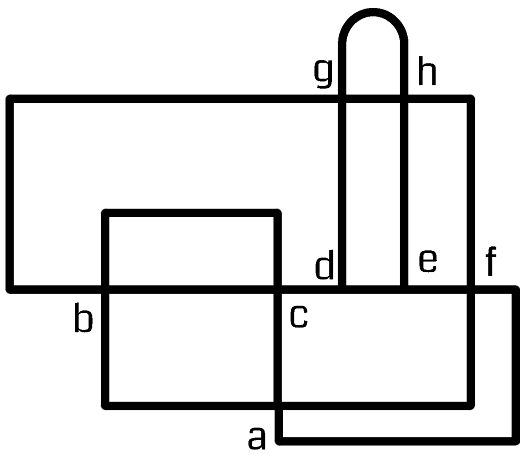
In the digraph described by the adjacency matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix},$$

compute the in- and out-degrees of the three nodes?

Ex 3.3: Cleaning an office floor [2]

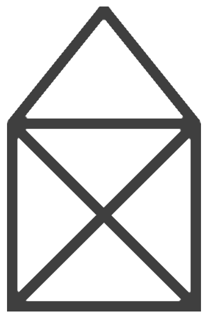
A cleaner vacuums the corridors in an office floor. The floor plan of the building is as follows:



The cleaner enters the floor via stairs at point d. Then he starts vacuuming and does not stop until all corridors are done. Is this possible without walking through a corridor twice? In which place does the cleaner finish?

Ex 3.4: St. Nic's house [3]

An old children's puzzle is to draw the following figure with an unbroken line:



Can you do it? (The rules are: once you start drawing, you can't lift the pen or draw a line twice until the house is complete.)

Ex 3.5: Security guard [3]

A security guard patrols the corridors in the office building from Exercise 3.3. The guard needs to patrol all 5 floors of the building. The floors have identical floor plans. They are connected via stairs at d and an elevator at e. The elevator can be used by the guard but otherwise does not require particular attention. However, the stairs between floors must be patrolled as well. Suppose the guard walks a circular round through the building, which bits does the guard need to walk twice on each round. (Use the notation of the form 'a3', to refer to node 'a' on floor 3).

Ex 3.6: Handshake [3]

If a network has exactly two nodes of odd degree there is an eulerian trail that starts in one and ends in the other. And, if there are zero nodes of odd degree there is an eulerian circuit. What if there is exactly one node of odd degree?

Ex 3.7: Bristol Bridgewalk [3]

The city of Bristol, like Königsberg, occupies land around two river islands. We'll call the parts of the city Clifton, Redcliffe, Spike Island, Bedminster. There are 12 bridges connecting Clifton and Redcliffe, 3 connecting Clifton and Spike Island, also 3 connecting Clifton and Bedminster, and 4 connecting Clifton to itself. There are 3 bridges, connecting Redcliffe to Spike island and 13 between Redcliffe and Bedminster. Spike Island has 4 bridges to Bedminster, 1 to itself and 2 that connected it to a tiny artificial island. Does an eulerian walk exist in Bristol?

Ex 3.8: Hierholzer's round trip [4]

In Hierholzer's algorithm, on the second attempt, we can only get stuck in the node that we started from. Explain why this is the case.

Ex 3.9: Seeing everything twice [4]

Under what conditions can we go on a walk that crosses each bridge exactly twice? What if you wanted to cross each bridge exactly n times?

Ex 3.10: Cat bridges [4]

Consider a network, where all the links are directed and can only be walked in the respective direction (i.e. they are one-way roads). Suppose that the network is at least weakly connected. Under what conditions does an eulerian circuit exist?

Ex 3.11: The best bridgewalk [5]

Formulate an efficient algorithm that finds the shortest eulerian circuit in a given network. (Of course every eulerian circuit will involve all bridges, but in a real city different eulerian circuits may cover very different distances while walking from one bridge to the next.)

Ex 3.12: Circuit census [5]

Find a formula that computes the number of distinct eulerian circuits in a network from network properties (e.g. node degrees or adjacency matrix)