Networks and Complexity

Solution 16-1

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 16.1: A fancy derivative [1]

Consider the equation

$$a = bc + \sqrt{b}$$

where

$$b = 3c + 1$$

Compute $\mathrm{d}a/\mathrm{d}c$ at $c = 1$.

Solution

To do this we first compute

$$\frac{\mathrm{d}b}{\mathrm{d}c} = 3 \tag{1}$$

$$\frac{\partial a}{\partial c} = b \tag{2}$$

$$\frac{\partial a}{\partial b} = c - \frac{1}{\sqrt{b}} \tag{3}$$

and then write

$$\frac{\mathrm{d}a}{\mathrm{d}c} = \frac{\partial a}{\partial c} + \frac{\partial a}{\partial b}\frac{\mathrm{d}b}{\mathrm{d}c} \tag{4}$$

$$= b + 3\left(c - \frac{1}{\sqrt{b}}\right) \tag{5}$$

at $c = 1$, we find $b = 4$ and thus

$$\left.\frac{\mathrm{d}a}{\mathrm{d}c}\right|_{c=1} = 4 + 3(1 - 1/2) = 5.5 \tag{6}$$