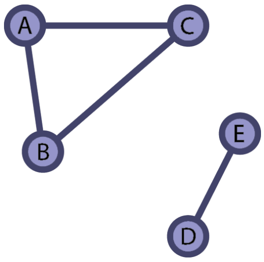
Networks and Complexity

Exercise Sheet 2: The darkest path

This is an exercise sheet from the forthcoming book Networks and Complexity.
Find more exercises and solutions at https://github.com/NC-Book/NCB

Ex 2.1: Different paths [1]

Decide which of the following is an open walk / closed walk / trail / path / circuit / cycle in the network shown below.



a) A,(A,C),C,(C,B),B;
b) A,(A,B),B,(A,C),A;
c) A,(A,B),B,(B,C),C,(A,C),A;
d) A,(A,D),D,(A,D),A;
e) A,(A,C),C,(B,C),B,(B,A),A,(A,C),C,(A,C),A;
f) B,(A,B),A,(A,C),C.

Ex 2.2: Through the mines? [2]

After leaving Rivendell and finding the Caradhras pass blocked, Frodo has to decide for an alternative route. Find the shortest pass from Rivendell to the Orodruin in a network that is described by the distance matrix

$$\mathbf{D} = \begin{pmatrix}
\oslash & 6 & \oslash & \oslash & \oslash & \oslash & 49 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash \\
6 & \oslash & \oslash & \oslash & 6 & \oslash & 48 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash \\
\oslash & \oslash & \oslash & \oslash & \oslash & 18 & 24 & \oslash & \oslash & 1 & 7 & \oslash & \oslash \\
\oslash & \oslash & \oslash & \oslash & 17 & \oslash & \oslash & 19 & 30 & \oslash & \oslash & \oslash & \oslash \\
\oslash & 7 & \oslash & 18 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash \\
\oslash & \oslash & 40 & 0 & \oslash & \oslash & 19 & 4 & \oslash & \oslash & \oslash & \oslash & \oslash \\
49 & 48 & 24 & \oslash & \oslash & 19 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash \\
\oslash & \oslash & \oslash & 20 & \oslash & 4 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash \\
\oslash & \oslash & \oslash & 30 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash \\
\oslash & \oslash & 1 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & 6 & 5 & \oslash \\
\oslash & \oslash & 6 & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & 7 & \oslash & 7 & 8 \\
\oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & 7 & 9 & \oslash & 5 \\
\oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & \oslash & 8 & 4 & \oslash
\end{pmatrix}.$$

where the nodes are 1: Hobbiton, 2: Bree, 3: Osgiliath, 4: Rivendell, 5: Weathertop, 6: Lorien, 7: Isengard, 8: Moria, 9: Erebor, 10: Cirith Ungol, 11: Morannon, 12: Orodruin, 13: Barad-Dur.

Ex 2.3: An abstract example [2]
Find the route from the first to the fifth node in a network described by the distance matrix

$$\mathbf{D} = \begin{pmatrix} 0 & 3 & 6 & 8 & 9 \\ 3 & 0 & 1 & 6 & 7 \\ 6 & 1 & 0 & 3 & 1 \\ 8 & 6 & 3 & 0 & 1 \\ 9 & 7 & 1 & 1 & 0 \end{pmatrix}. \tag{1}$$

(use Dijkstra, do not draw a map)

Ex 2.4: Another world [2]
Use Dijkstra's Algorithm to find the shortest path from Kliften to Solhaven in the network described by the following link list:

Solhaven
to Fimoria 3
to Isangel 23
to Fort Kerron 4
to Silvaster 2

Fimoria
to Solhaven 3
to Fort Kerron 5
to Isangel 2
to Kliften 4

Isangel
to Fimoria 2
to Solhaven 22
to Tewen 40
to Kliften 1

Kliften
to Isangel 1
to Fimoria 5
to Tewen 48
to Fort Kerron 9

Silvaster
to Solhaven 2
to Tewen 1

Tewen
to Silvaster 1
to Solhaven 1

Fort Kerron
to Solhaven 4
to Fimoria 5

Ex 2.5: Pathfinding [3]
In the network from the previous exercise, I also wanted to go from Fort Kerron to Tewen. I have already used Dijkstra's algorithm to work out the distances from Fort Kerron to all other place they are:

Solhaven: 4, Fimoria 5, Silvaster 6, Isangel 7, Tewen 7, Kliften 8

Unfortunately, I have lost the corresponding Dijkstra table. Let's see if we can still figure out the actual path without redoing Dijkstra's Algorithm.

a) Given the distances above and the link list from the previous exercise, is it possible that the last link in the shortest path from Fort Kerron to Tewen is Isangel → Tewen?
b) So, on the shortest path Fort Kerron to Tewen, which place do I visit directly before arriving in Tewen?
c) And how do I get to the place that is the answer to part b? Directly from Fort Kerron? Or, do I need to visit a different place on the way?
d) Formulate a general strategy to find the the shortest path given the linklist and the precomputed distances from the start point to all places.

Ex 2.6: Search and rescue [3]
A robot needs to reach a destination inside a burning office building. Fire creates a dangerous environment even for our dedicated rescue robot. Therefore the robot is complemented by a remote sensing system that assesses the risk in different parts of the building. This system has returned the risk that the robot will become dysfunctional when trying to traverse journey

segments between 7 locations: 1: Entrance, 2: Door A, 3: Door B, 4: Door C 5: Door D, 6: Door E, 7: Destination.

The survival probabilities in % for the journey segments between these destinations are given in the following matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 100 & 95 & 0 & 0 & 0 & 2 \\ 100 & 0 & 0 & 100 & 50 & 0 & 0 \\ 95 & 0 & 0 & 40 & 0 & 30 & 0 \\ 0 & 100 & 40 & 0 & 60 & 90 & 0 \\ 0 & 50 & 0 & 60 & 0 & 20 & 50 \\ 0 & 0 & 30 & 90 & 20 & 0 & 100 \\ 2 & 0 & 0 & 0 & 50 & 100 & 0 \end{pmatrix}$$

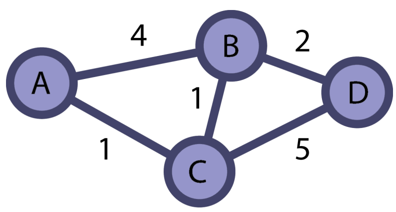
Find the optimal path for the robot from the entrance to the destination and state the probability for reaching the destination. (Do not draw a map.)

Ex 2.7: Greedy Hobbit [4]

Formulate a greedy algorithm to solve the shortest path problem. Your algorithm does not need to yield an optimal result and you can phrase it colloquially. However, make sure your algorithm terminates at some point. How long would the journey from Hobbiton to the Orodruin (from Ex. 2.2) take if we followed the algorithm.

Ex 2.8: Going by bus [4]

Suppose we want to go from A to D in the network shown below. Too boring? Well, this time we are going to take the bus, and now the weights specify the frequency of the bus connection between two places. So weight 1 means that the bus goes every hour. While for example the 4 between A and D means that the bus only goes every fourth hour. For simplicity all busses leave on the hour starting at 12:00 and each ride take slightly less than 3 hours. So if we take a bus at 2:00 we could catch a connecting bus at 5:00 (if one is leaving then). Find the optimal path from A to D, starting at 12:00, and then formulate a Dijkstra-style procedure that can solve this problem on general networks. Convince yourself that your proposed procedure will always find the optimal solution.



Ex 2.9: Writing large numbers [4]

In the previous lecture I wrote out the number 10^{317}. This is the kind of task that makes a lazy person wonder how it can be done most efficiently. Let's focus on the simpler example of writing a number with 23 zeros. Suppose I have only two operations at my disposal: a) write a zero, which takes 1 second b) copy and paste to double the number of zeros, which takes 2 seconds. Use Dijkstra's algorithm to find the fastest way of writing 23 zeros. (This may seem be a silly question but very similar optimization problems occur in Engineering.)

Ex 2.10: Component algorithm [4]

Invent an algorithm for finding all the nodes that are in the same component as a given node.

Ex 2.11: Time travel [4]

Suppose there is a magical portal in Bree. Whoever steps into the portal magically transported to Isengard. The magical travel isn't instantaneous, it's actually even faster than that. People emerge in Isengard two days before they step into the portal in Bree. Does this present a problem for Dijkstra's algorithm? Can we still use it?

Ex 2.12: Longest path problem [5]

Find an efficient method to find the longest path in a given network.