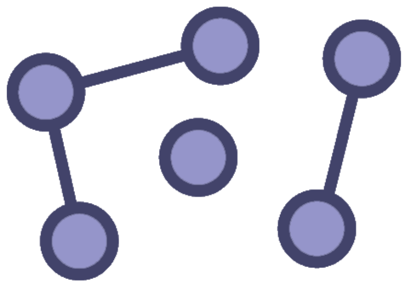
Networks and Complexity

Exercise Sheet 1: Power for Moravia

This is an exercise sheet from the forthcoming book Networks and Complexity. Find more exercises and solutions at https://github.com/NC-Book/NCB

Ex 1.1: Terminology [1]

Count the number of vertices, edges and components in following network:



Ex 1.2: Number of links [1]

Consider a fully connected network with 23 nodes. How many links are there in this networks?

Ex 1.3: Number of networks [1]

Compute the number of different networks that can be constructed between 7 labeled nodes.

Ex 1.4: Return to Moravia [2]

Solve the Moravian example with a slightly modified distance matrix that is now given by

$$\mathbf{D} = \begin{pmatrix} 0 & 137 & 100 & 74 & 77 \\ 137 & 0 & 75 & 76 & 198 \\ 100 & 75 & 0 & 51 & 121 \\ 74 & 76 & 51 & 0 & 151 \\ 77 & 198 & 121 & 151 & 0 \end{pmatrix}$$

and the nodes are

$$1 : \mathrm{B},\ 2 : \mathrm{O},\ 3 : \mathrm{L},\ 4 : \mathrm{Z},\ 5 : \mathrm{J}.$$

Ex 1.5: Another power grid [2]

Construct a power grid between 4 cities $V = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$. The distance between cities (in km) is given by

$$\mathbf{D} = \begin{pmatrix} 0 & 17 & 23 & 9 \\ 17 & 0 & 18 & 13 \\ 23 & 18 & 0 & 27 \\ 9 & 13 & 27 & 0 \end{pmatrix}.$$

Ex 1.6: Larger abstract example [2]
Construct the minimal spanning tree in a network where the weight of links is given by

$$\mathbf{D} = \begin{pmatrix} 0 & 8 & 1 & 14 & 4 & 5 \\ 8 & 0 & 7 & 12 & 9 & 10 \\ 1 & 7 & 0 & 11 & 3 & 2 \\ 14 & 12 & 11 & 0 & 15 & 13 \\ 4 & 9 & 3 & 15 & 0 & 6 \\ 5 & 10 & 2 & 13 & 6 & 0 \end{pmatrix}.$$

To reduce the tediousness, the distances in this exercise have been chosen as 1,2,3, and so on. (Hint: If you draw unknown networks like this one it is best to arrange nodes in a circle.)

Ex 1.7: Ethernet [3]
I want to install a wired network connection in my house. My internet connection is via a router that sits in the cellar, and I want to connect the bedroom, living room, and the kitchen. I don't mind installing network switches in these rooms such that a room can get network access from any other room that has network access. To connect the cellar to the kitchen I would need 6m of cable, to the living room its 8m and to the bed room its 12m. But I could connect the kitchen to the bedroom with just 3m, and to the living room with 7m. Finally, the bedroom could be connected to the living room with 4m of cable. How much cable do I need?

Ex 1.8: Another network in Moravia [3]
Let's revisit the Moravian example, described by the distance matrix

$$\mathbf{D} = \begin{pmatrix} 0 & 137 & 63 & 74 & 77 \\ 137 & 0 & 75 & 76 & 198 \\ 63 & 75 & 0 & 51 & 121 \\ 74 & 76 & 51 & 0 & 151 \\ 77 & 198 & 121 & 151 & 0 \end{pmatrix}$$

where the nodes are again

$$1: \mathrm{B},\ 2: \mathrm{O},\ 3: \mathrm{L},\ 4: \mathrm{Z},\ 5: \mathrm{J}.$$

This time we assume that when we get to work there is already a power line from B to Z. Find which additional lines need to be built such that all cities are connected and the length of additional lines built is minimal.

Ex 1.9: A tale of two cities [3]
Consider the Moravian power grid from the previous exercise for a final time. Now we start again with an empty graph, but there are two power plants, which are located in B and Z. Each of the plants has the capacity to supply the whole region. Find the lines that need to be built such that each city can get power from one of the power plants and the length of lines built is minimal. (Bonus: Can you also solve this exercise without drawing the network, you might need to invent some other form of bookkeeping.)

Ex 1.10: Pandemic rideshare [3]
During a pandemic Dave and Peter are driving home. Their friend Bob asks them if they can give him a lift. However, the two are slightly worried because such close contacts allow the disease to spread and there are many asymptomatic infections.

a) Bob argues that adding another person only increase the group size by 50%, but by what factor does it increase the number of contacts?
b) Actually Dave was planning to have his birthday party with 20 people on Friday night. Now he is wondering how many contacts that would create?

Ex 1.11: Pruning networks [4]

Develop your own algorithm for finding minimal spanning trees. In contrast to Kruskal's algorithm start with a fully-connected network and then remove links until only the minimum spanning tree is left. Make sure that your algorithm always yields the optimal result.

Ex 1.12: Counting graphs [5]

When we derived the formula for the number of network that can be constructed with a given number of nodes, we emphasized that the nodes are labelled. In an unlabelled graph the nodes are indistinguishable, so for example all networks that contain five nodes and one link are actually the same network, no matter which two nodes the link connects, To warm up find the number of networks that can be constructed between 3 unlabelled nodes (The answer is 4). Then try four nodes. Find a general formula for the number of unlabeled graphs that can be efficiently evaluated.