Networks and Complexity

Solution 7-5

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 7.5: 3-chains done differently [3]

Let's try to estimate the number of 3-chains in an ER graph in a different way. Revisit the reasoning from Chap. 5 regarding the number of nodes that we find at distance 2 from a typical node. Use this type of reasoning to estimate the number of 3-chains in the network. Show that the resulting estimate is consistent with n_{--} from the present chapter.

Solution

A typical node has z neighbors. Because we reach these neighbors via a link we can expect them to have q neighbors each. So there are qz nodes at distance 2 from a typical node. In other words there are qz 3-chains starting in a typical node. In the whole network we have N nodes. So multiplying yields Nzq. But, now we are counting any 3-chain twice (once from each end). So we need to divide by two. Using that $q = z$ in the ER graph we arrive at

$$n_{--} = \frac{Nz^2}{2} \tag{1}$$

which is the same result the same result that we found in the chapter in a different way.