Networks and Complexity

Solution 19-6

This is an example solution from the forthcoming book Networks and Complexity.
Find more exercises at https://github.com/NC-Book/NCB

Ex 19.6: Traffic: Closeness centrality [3]

Consider the traffic network again, but this time compute the closeness centrality. (You can consider the length of each link to be one.) Is the result reasonable?

Solution

For this one we need to find the shortest paths in the network. For this small example visual inspection is easier than Dijkstra. We find

Trip	Distance	Intermediate nodes	Trip	Distance	Intermediate nodes
A-B	1	-	A-C	1	-
A-D	2	B	A-E	2	C
A-F	3	B,G / C,E	A-G	2	B
A-H	3	B,D / C,E	A-X	1	-
B-C	1	-	B-D	1	-
B-E	2	C	B-F	2	G
B-G	1	-	B-H	2	D
B-X	2	A	C-D	2	B
C-E	1	-	C-F	2	E
C-G	2	B	C-H	2	E
C-X	2	A	D-E	2	H
D-F	2	H / G	D-G	1	-
D-H	1	-	D-X	3	A,B
E-F	1	-	E-G	2	F
E-H	1	-	E-X	3	C,A
F-G	1	-	F-H	1	-
F-X	4	E,C,A / G,B,A	G-H	2	F / D
G-X	3	A,B	H-X	4	A,B,D / A,C,E

We can now compute the centralities

$$c_{\mathrm{A}} = \frac{8}{1+1+2+2+3+2+3+1} = \frac{8}{15} \approx 0.53 \quad (1)$$

$$c_{\mathrm{B}} = \frac{8}{1+1+1+2+2+1+2+2} = \frac{8}{12} = 0.75 \quad (2)$$

$$c_{\mathrm{C}} = \frac{8}{1+1+2+1+2+2+2+2} = \frac{8}{13} \approx 0.61 \quad (3)$$

$$c_{\mathrm{D}} = \frac{8}{2+1+2+2+2+1+1+3} = \frac{8}{13} \approx 0.61 \quad (4)$$

$$c_{\mathrm{E}} = \frac{8}{2+2+1+2+1+2+1+3} = \frac{8}{14} \approx 0.57 \quad (5)$$

$$c_{\mathrm{F}} = \frac{8}{3+2+2+2+1+1+1+4} = \frac{8}{16} = 0.50 \quad (6)$$

$$c_{\mathrm{G}} = \frac{8}{2+1+2+1+2+1+2+3} = \frac{8}{14} \approx 0.57 \quad (7)$$

$$c_{\mathrm{H}} = \frac{8}{3+2+2+1+1+1+2+4} = \frac{8}{16} = 0.50 \quad (8)$$

$$c_{\mathrm{X}} = \frac{8}{1+2+2+3+3+4+3+4} = \frac{8}{22} \approx 0.36 \quad (9)$$

This was soo much more work than the spectral centrality! Note that again X lands in last place which is reasonable. In contrast to the spectral centrality B is now the most important node. This is partly because the closeness completely ignores the many double links that exist around E and F. For traffic this might be what we want, dependenig on details of the question. (Closing down B seems like it might create a headache for driver. By contrast if police is looking for somebody hanging around at F, the intersection with the highest spectral centrality might be a good idea.)