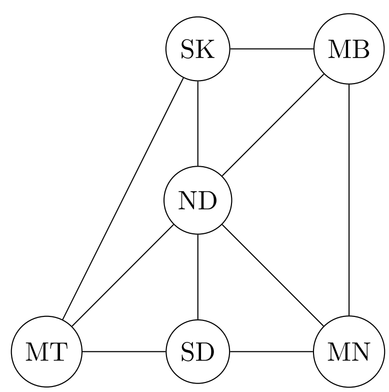
PROOF OF CAN-USA-MEX GRAPH UNABLE TO BE REPRESENTED BY THREE COLORS

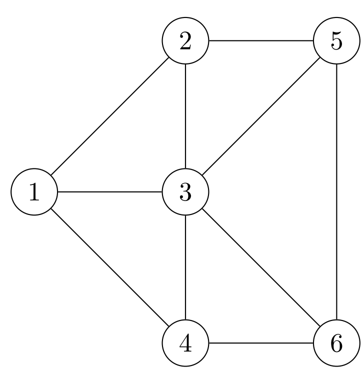
Claim

Because the Can-USA-Mex Graph contains the graph shown below, three coloring is impossible.



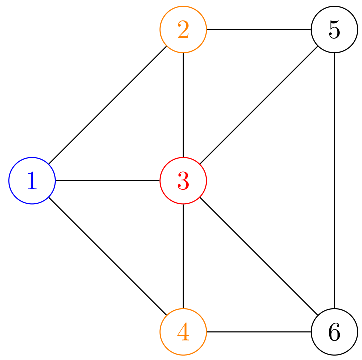
Proof

Proof. Let the graph we want to three color be made up of nodes 1 thorugh 6, topologically congruant to the graph in the claim, and connected as such:

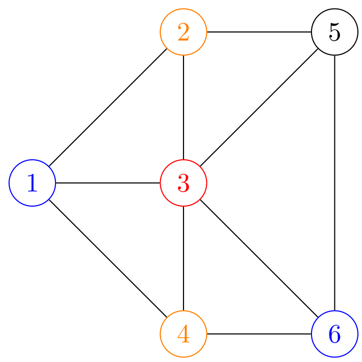


Because nodes 2 and 4 are both connected to nodes 1 and 3, regardless of the colors of nodes 1 and 3, nodes 2 and 4 must be the same color.

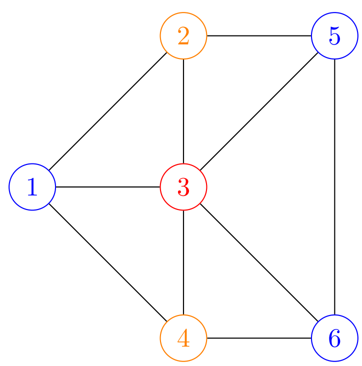
In this case, node 1 will be blue, node 3 will be red, and nodes 4 and 2 will be orange. Notice how no two adjacent nodes share the same color.



Because nodes 3 and 4 are red and orange respectively, and are also both connected to node 6, node 6 must be the third color, blue.



Because nodes 2 and 4 are the same color, orange, and nodes 2 and 3 are connected to node 5, node 5 must also be the third color, blue.



This isn't allowed in three coloring because nodes 5 and 6 share the same color, and are also adjacent to eachother.

Because of this, the graph cannot be three colored.
Therefore the Can-USA-Mex Graph can not be three colored either. □