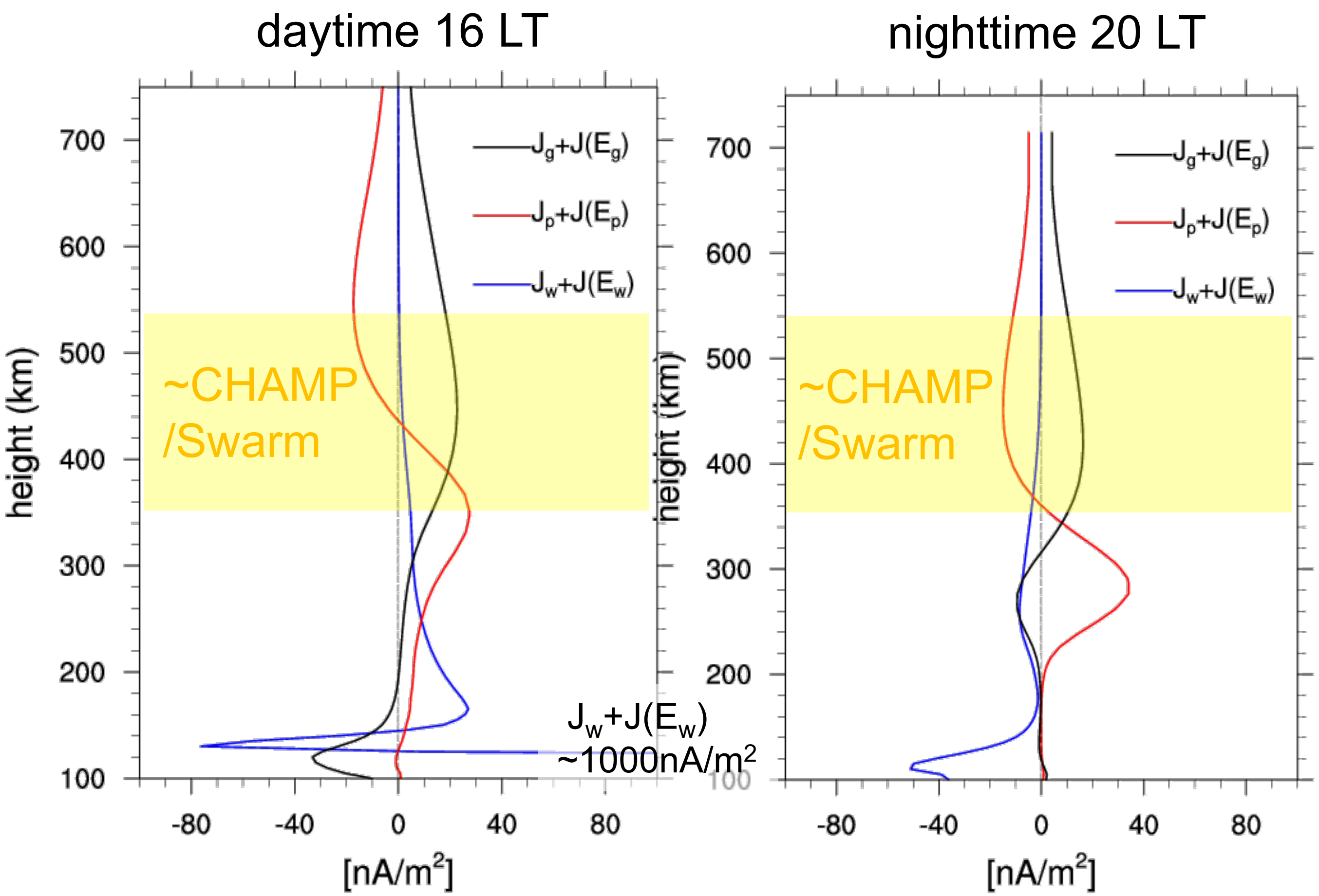
Outline

1. General ionospheric electrodynamics
2. Simulating the three-dimensional ionospheric current system
3. First steps to solve globally for the electric potential

The goal of the “new” 3Dcurrent model: 3D ionospheric current and associated ΔB



$$\boldsymbol{J_p} = -\frac{1}{B_o^{\,2}}\nabla P \times \boldsymbol{B_o}$$

$P = n_e k_B(T_i+T_e)$

$$\boldsymbol{J_g} = -\frac{n_e m_i}{B_o^{\,2}}\boldsymbol{g} \times \boldsymbol{B_o}$$

$$\boldsymbol{J_w} = \boldsymbol{\sigma_P u} \times \boldsymbol{B_o} + \\ +\boldsymbol{\sigma_H b_o}(\boldsymbol{u} \times \boldsymbol{B_o})$$

$$\boldsymbol{J}_E = \sigma_P \boldsymbol{E}_\perp \\ + \sigma_H \boldsymbol{b}_o \times \boldsymbol{E}_\perp$$

$\mathbf{E_\perp = E_w + E_p + E_g}$

σ_P, σ_H Pedersen & Hall conductivity
$\mathbf{B_o}$ geomagnetic main field
K_B Boltzmann constant
g gravitational acceleration

$\mathbf{E_\perp}$ electric field perpendicular to $\mathbf{B_o}$
$T_{i,e}$ ion/electron temperature
n_e electron density
u neutral wind

Basics of steady state ionospheric electrodynamics

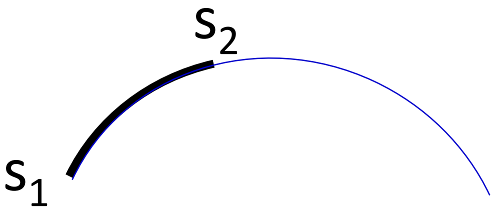
- Electrostatic $\mathbf{E} = -\nabla\Phi$
- Current continuity $\nabla \cdot \mathbf{J} = 0$
- Combining with Ohm's Law -> 3D PDE

$$\nabla \cdot \left[\sigma_P (\nabla\Phi)_\perp + \sigma_H \mathbf{b} \times (\nabla\Phi)_\perp + \sigma_\parallel (\nabla\Phi)_\parallel\right] =$$
$$\nabla \cdot \left[\sigma_P \mathbf{u_n} \times \mathbf{B} + \sigma_H \mathbf{b} \times (\mathbf{u_n} \times \mathbf{B}) + J_{pg}\right]$$

- Assume closed geomagnetic field lines are equipotential ($E_\parallel = 0$).

Integrate $\nabla \cdot \mathbf{J} = 0$ along field line from s_1 to s_2:

$$\int_{s1}^{s2} \frac{\nabla \cdot \mathbf{J}_\perp}{B} ds = \left.\frac{J_\parallel}{B}\right|_{s1} - \left.\frac{J_\parallel}{B}\right|_{s2}$$



Solving a 2D – second order PDE for the electric potential

• Φ is assumed constant during the field-line integrations, so only the conductivities, winds, and gravity/pressure-gradient currents are integrated.

• Let superscript *T* denote the sum of conjugate N and S integrals:

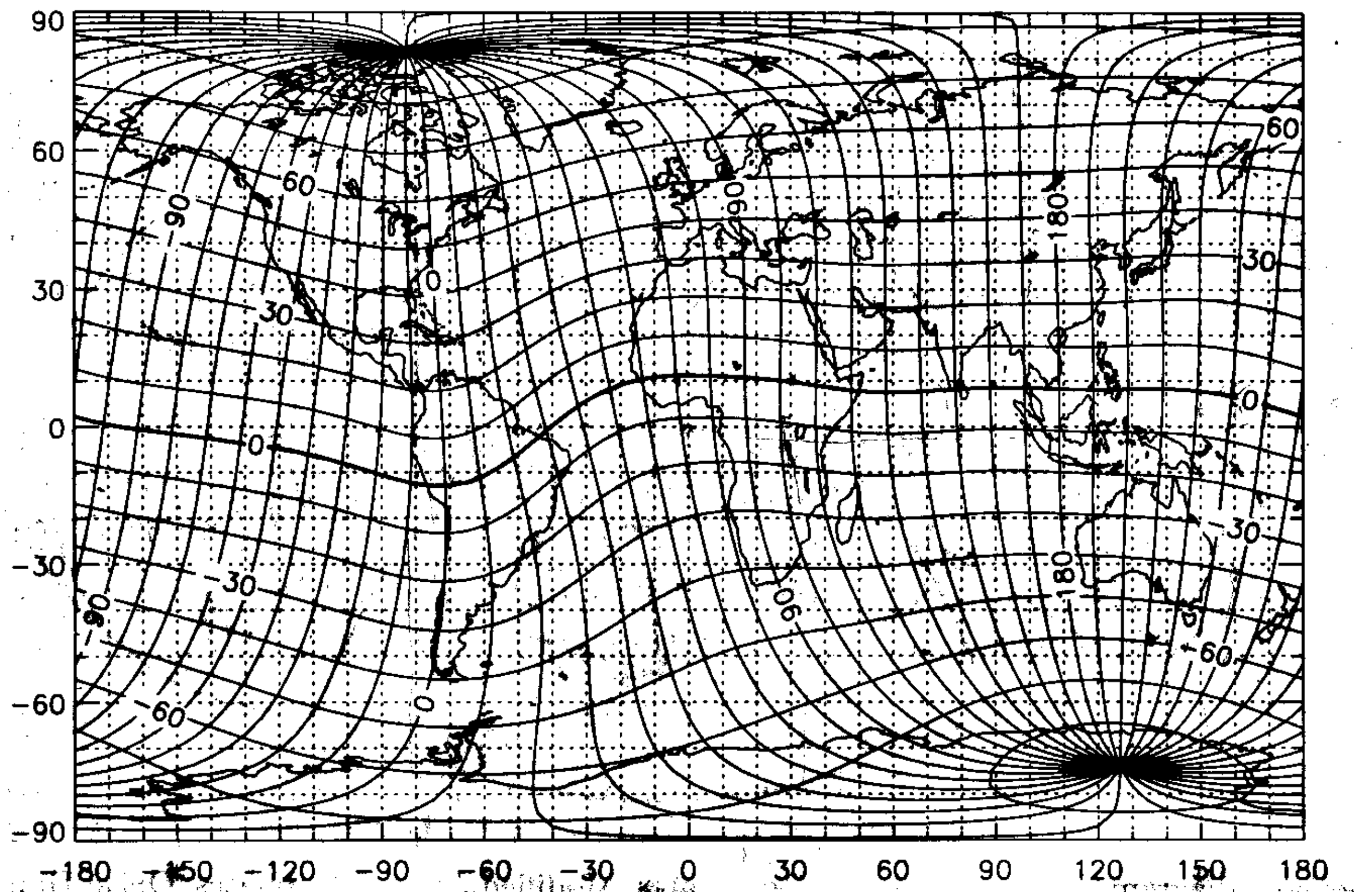
$$\frac{\partial}{\partial \phi_m}\left(\frac{\Sigma^T_{\phi\phi}}{\cos\lambda_m}\frac{\partial\Phi}{\partial\phi_m}+\Sigma^T_{\phi\lambda}\frac{\partial\Phi}{\partial|\lambda_m|}\right)+\frac{\partial}{\partial|\lambda_m|}\left(\Sigma^T_{\lambda\phi}\frac{\partial\Phi}{\partial\phi_m}+\Sigma^T_{\lambda\lambda}\cos\lambda_m\frac{\partial\Phi}{\partial|\lambda_m|}\right)=$$

$$R\frac{\partial\left(K^{DT}_{m\phi}+K^T_{I\phi}\right)}{\partial\phi_m}+R\frac{\partial\left(\cos\lambda_m K^{DT}_{m\lambda}+K^T_{I\lambda}\right)}{\partial|\lambda_m|}+R^2\cos\lambda_m J_{Mr}$$

[Richmond & Maute, 2017]

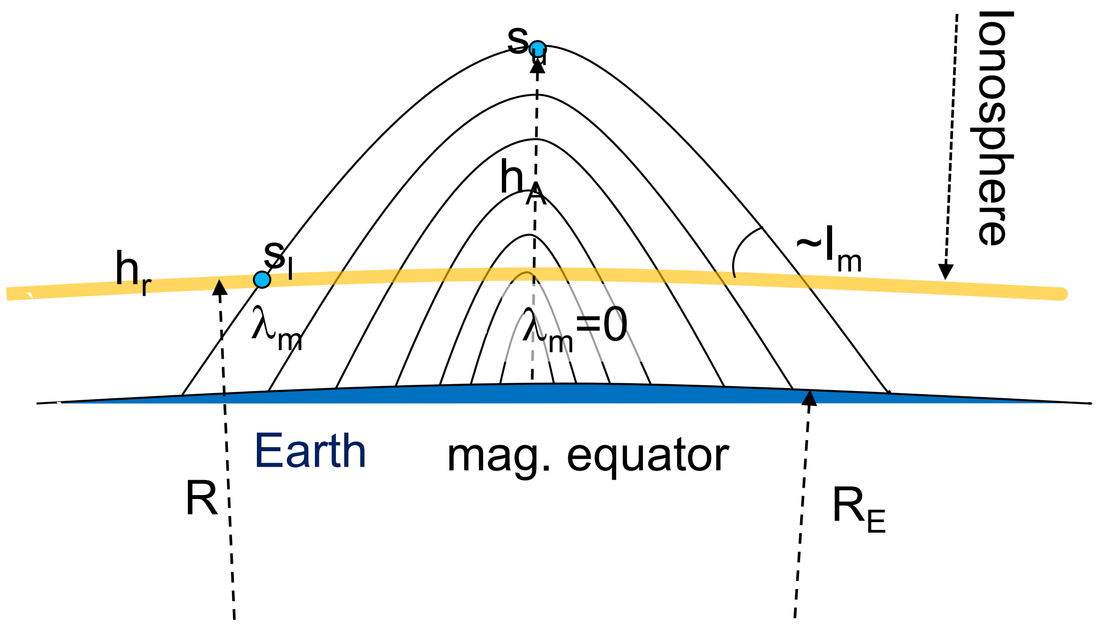
where K_I is the gravity/pressure-gradient source and J_{Mr} is the sum of conjugate upward radial current at the top of the ionosphere flowing to the magnetosphere.

Geomagnetic and geographic grid



Partly due to the dipolar tilt and the nondipolar distortion of the geomagnetic main field the neutral winds and conductivities are not symmetric about the magnetic equator. This asymmetry leads to divergence of the horizontal ionospheric current which is directly coupled to the fieldline current between the hemispheres.

Simplified schematic of geometry



- International Geomagnetic Reference Field (IGRF).
- Modified Apex coordinate system:
  - apex longitude ϕ_m,
  - apex latitude λ_m depends on apex altitude and reference height h_r
- Field lines with same apex height have same modified apex latitude.

Modified apex latitude $$\lambda_m = \pm\cos^{-1}\sqrt{\frac{R_E + h_r}{R_E + h_A}}$$

Inclination $$\sin I_m = 2\sin\lambda_m\left(4 - 3\cos^2\lambda_m\right)^{-1/2}$$

Field-aligned integration e.g. $$\Sigma_H = \int_{s_l}^{s_u} \sigma_H ds$$

Field-line integration I

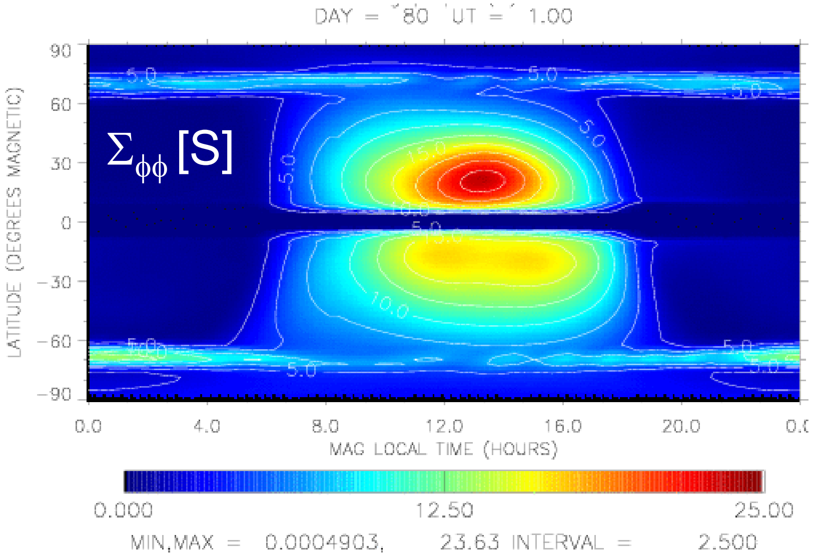
•Assume closed geomagnetic field lines are equipotential ($E_{||}= 0$).

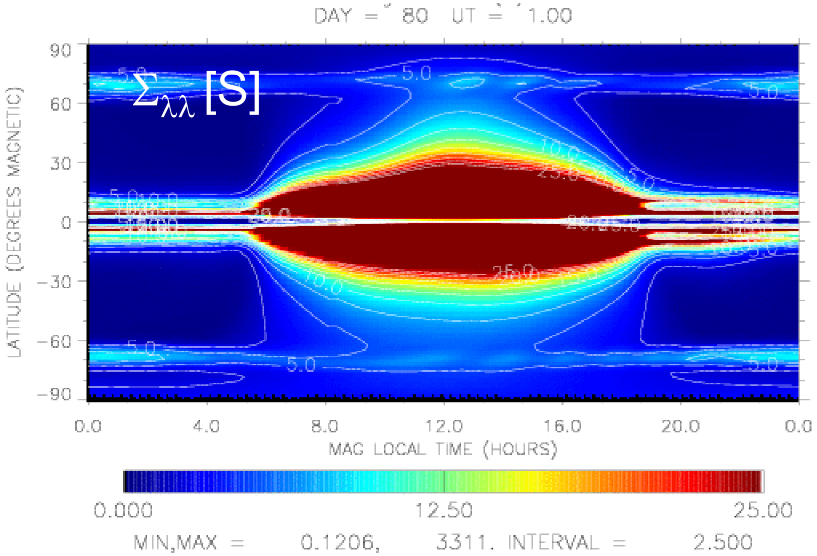
•Integrate along each field line.

$$\Sigma_{\phi\phi} = |\sin I_m| \int_{s_l}^{s_u} \frac{\sigma_p d_1^2}{D} ds$$

$$\Sigma_{\lambda\lambda} = \frac{1}{|\sin I_m|} \int_{s_l}^{s_u} \frac{\sigma_p d_2^2}{D} ds$$

[Richmond , J. Geomag. Geoelectr. ,1995]





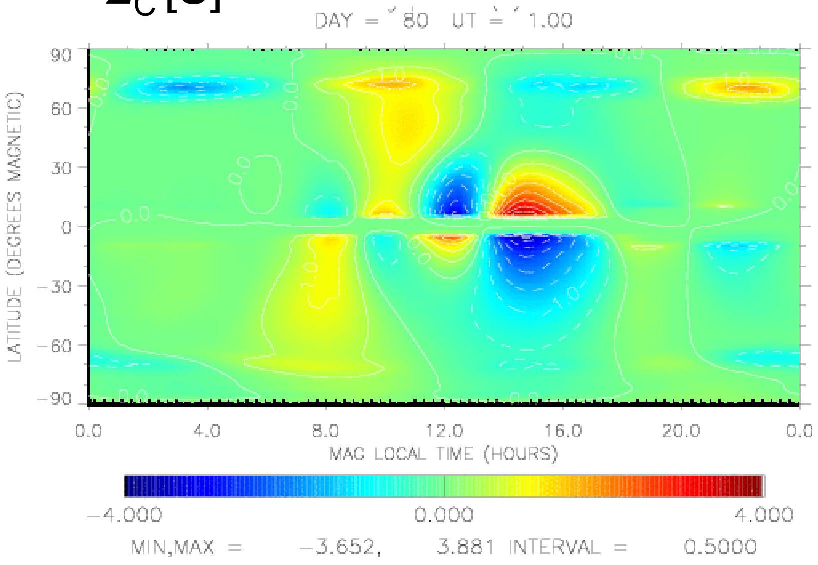
Field-line integration II

$$\Sigma_{\phi\lambda} = \pm(\Sigma_H - \Sigma_C) \qquad \Sigma_{\lambda\phi} = \mp(\Sigma_H + \Sigma_C)$$

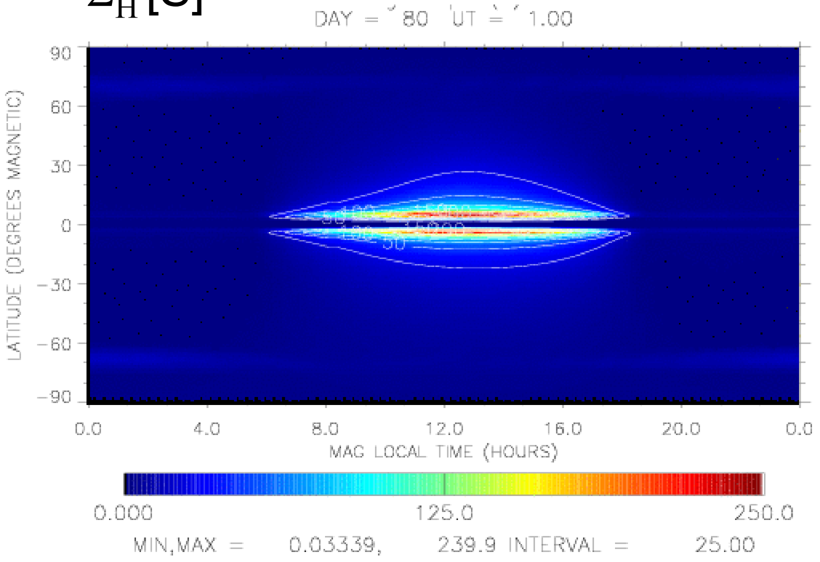
$$\Sigma_C = \int_{s_l}^{s_u} \frac{\sigma_p \mathbf{d}_1 \cdot \mathbf{d}_2}{D} ds \qquad \Sigma_H = \int_{s_l}^{s_u} \sigma_H ds$$

[Richmond , J. Geomag. Geoelectr., 1995]

Σ_C [S]



Σ_H [S]



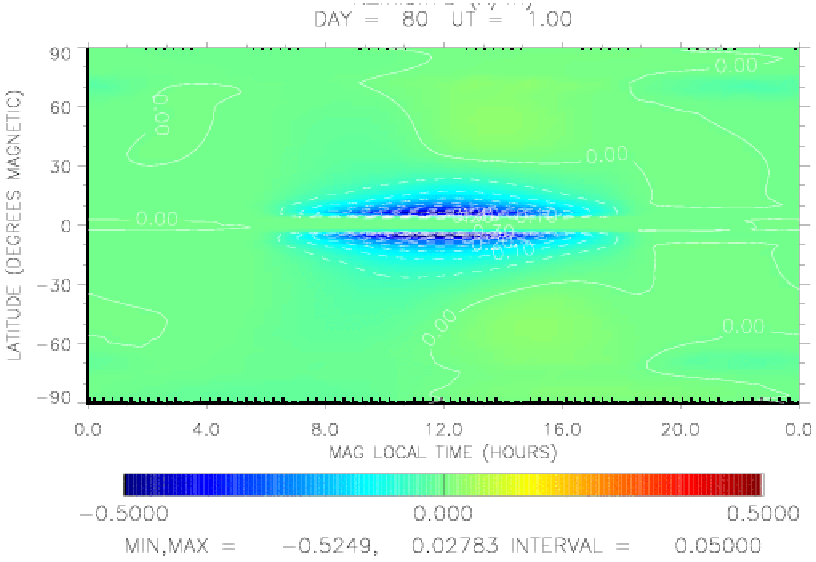
- At very low latitudes where the flux tubes are short and mainly passes through the E-region with small Pedersen conductivity it is possible that

$$\Sigma_H >> \Sigma_P$$

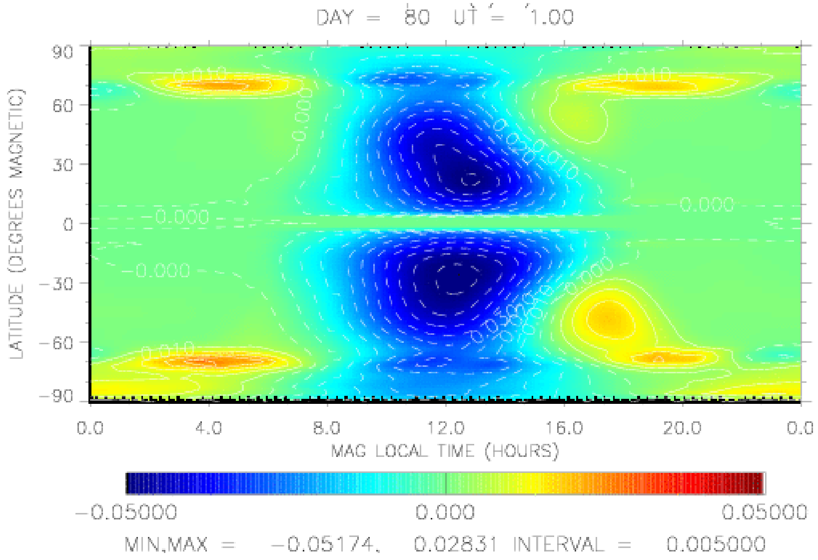
- Depending on the discretization the first order derivative of the electric potential associated with the Hall conductance can cause the diagonal terms of the matrix not to be dominant anymore. This can cause numerical problems.

Field-line integration III

$K_{m\lambda}$ [A/m]



$K_{m\phi}$ [A/m]



$$K_{m\lambda}^{D} = \mp B_{e3} \int_{s_l}^{s_u} \left[\left(\sigma_H - \frac{\sigma_p \mathbf{d}_1 \cdot \mathbf{d}_2}{D} \right) u_{e2} - \frac{\sigma_p d_2^2}{D} u_{e1} \right] ds$$

$$K_{m\phi}^{D} = B_{e3} \,|\sin I_m|\, \int_{s_l}^{s_u} \left[\frac{\sigma_p d_1^2}{D} u_{e2} + \left(\sigma_H - \frac{\sigma_p \mathbf{d}_1 \cdot \mathbf{d}_2}{D} \right) u_{e1} \right] ds$$

[Richmond , J. Geomag. Geoelectr., 1995]

Equatorial boundary condition

•Vertical/northward height-integrated current density $K_{m\lambda}$ at the equator has to go to zero

$$K_{m\lambda}^{T} = 0$$

(corresponds to Neumann BC)

$$\Sigma_{\lambda\phi}^{T} \frac{\partial \Phi}{\partial \phi_m} + \Sigma_{\lambda\lambda}^{T} \cos\lambda_m \frac{\partial \Phi}{\partial \lambda_m} = R\cos\lambda_m \left(K_{m\lambda}^{DT} + K_{I\lambda}^{T}\right)$$

•For a dipolar geomagnetic main field close to the equator it is very similar to Cowling conductivity

$$E_z = \frac{\Sigma_H^T}{\Sigma_P^T} E_\phi - \frac{1}{\Sigma_P^T} R\left(K_{m\lambda}^{DT} + K_{I\lambda}^{T}\right)$$

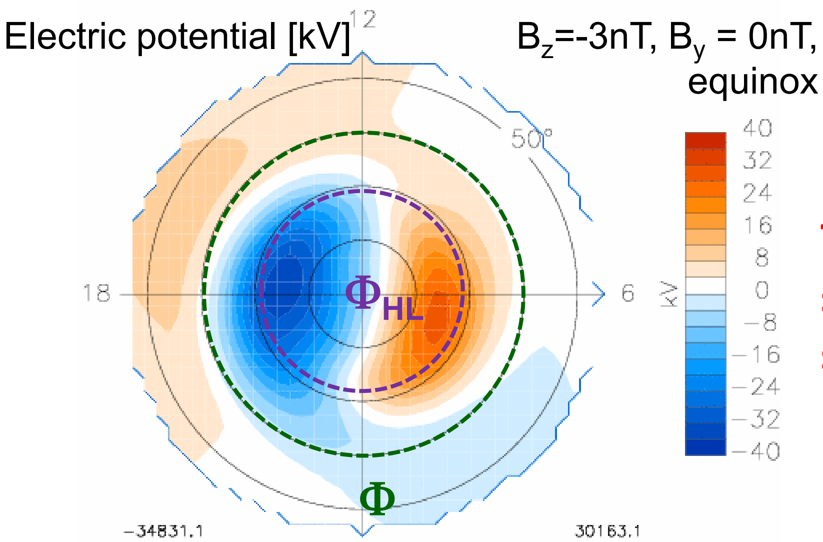
Simulate Magnetosphere-Ionosphere Coupling

- Prescribed electric potential pattern from empirical or numerical models or observations.
  - **Prescribed boundary between calculated Φ and prescribed electric potential Φ_{HL}.**
  - Boundary should be able to move depending on geophysical condition.

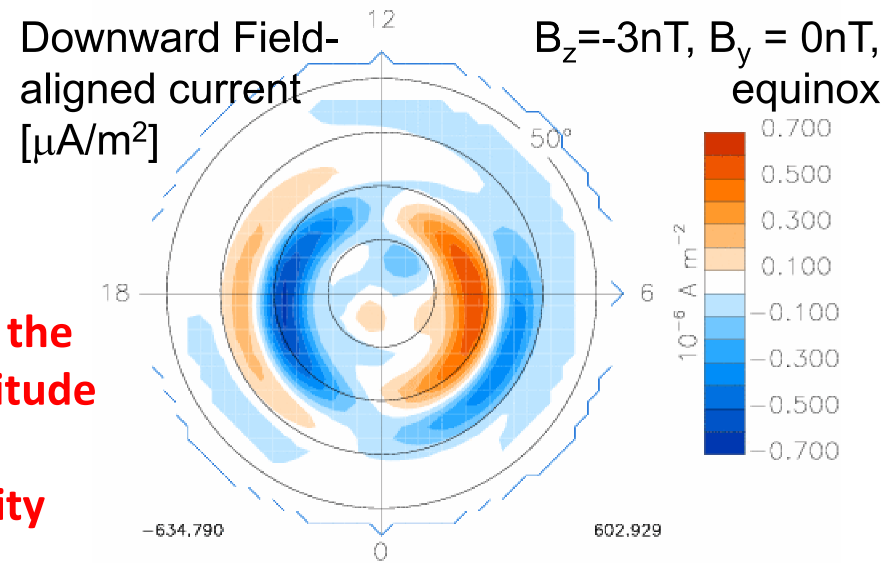
- Prescribed radial field-aligned current density J_{Mr} from empirical model, numerical model or observations.
  - Correct current density so there is no net current into or out of the ionosphere.

Size and shape of auroral oval should fit to high latitude forcing.

Boundary condition: specify potential at one point (e.g. NH pole, Dirichlet BC)



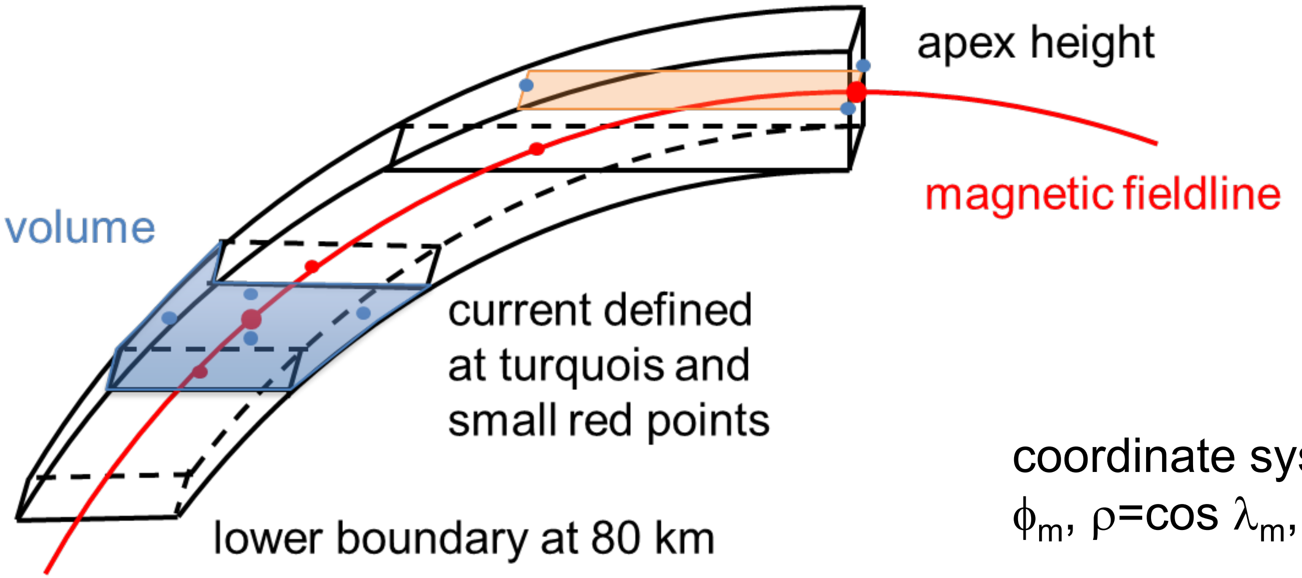
The "global potential solver" tries to avoid the sharp high to mid latitude boundary & should provide more flexibility



Outline

1. General ionospheric electrodynamics
2. **Simulating the three-dimensional ionospheric current system (3Dynamo code or “New Dynamo”)**
3. First steps to solve globally for the electric potential

“New” 3Dynamo solver



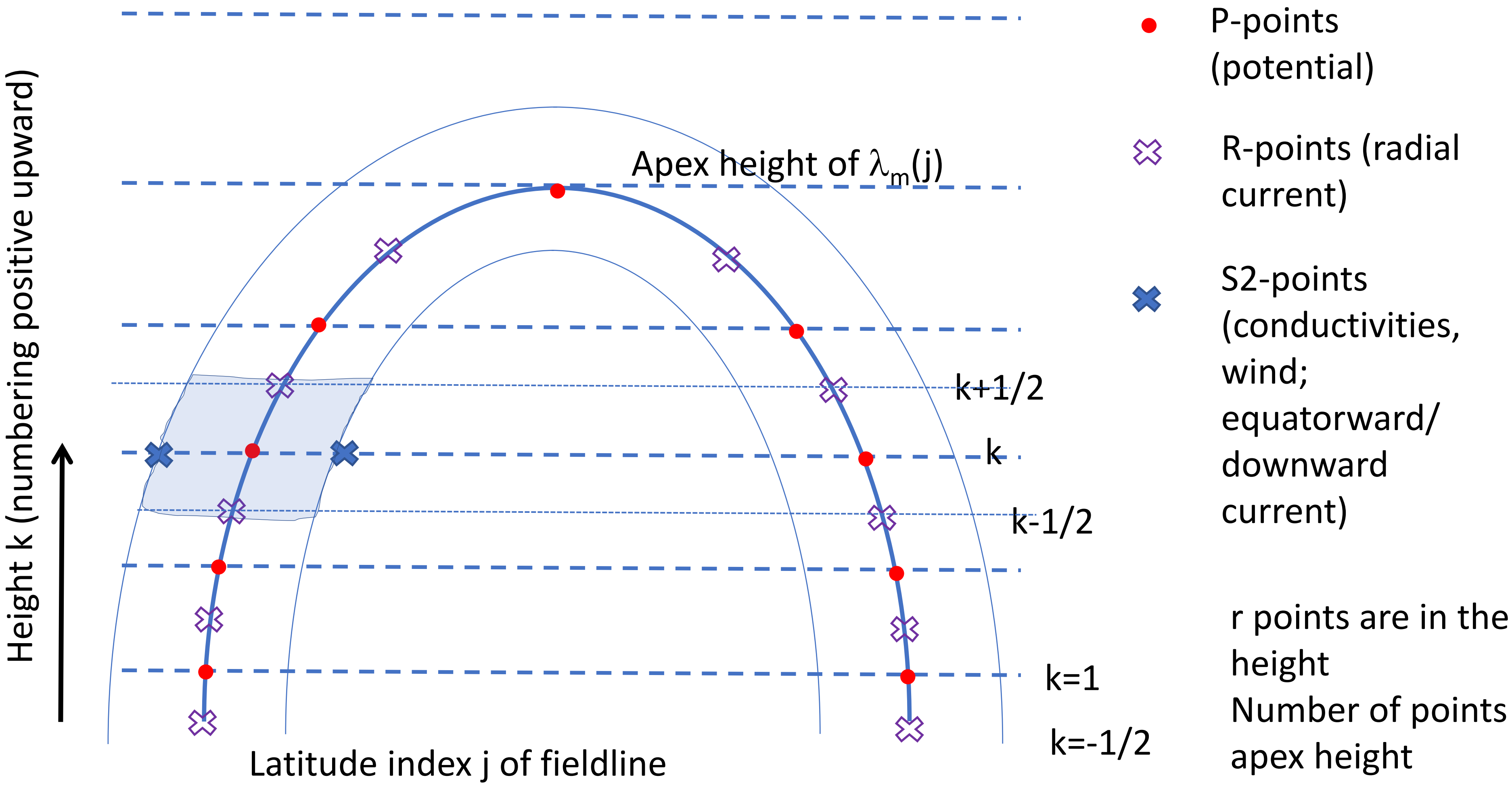
coordinate system:
ϕ_m, $\rho=\cos\lambda_m$, h

- Consistent formulation to solve for **E** and calculate **J**, and satisfies $\nabla\cdot\mathbf{J} = 0$ over each element with the following current terms

$$-\nabla\cdot[\boldsymbol{J}_E + \boldsymbol{J}_{\parallel}] = \nabla\cdot[\boldsymbol{J}_w + \boldsymbol{J}_{\boldsymbol{p}} + \boldsymbol{J}_g] \qquad (1)$$

- Discretization with constant height levels and spaced to capture height variation at the equator.
- The current is defined at the surface of an element and the electric potential in the “middle” of the element.
- Once $\mathbf{E}_\perp$ is known $J_\perp$ can be calculated, and $J_\parallel$ is determined by the integrated divergence of $J_\perp$ along a fieldline.

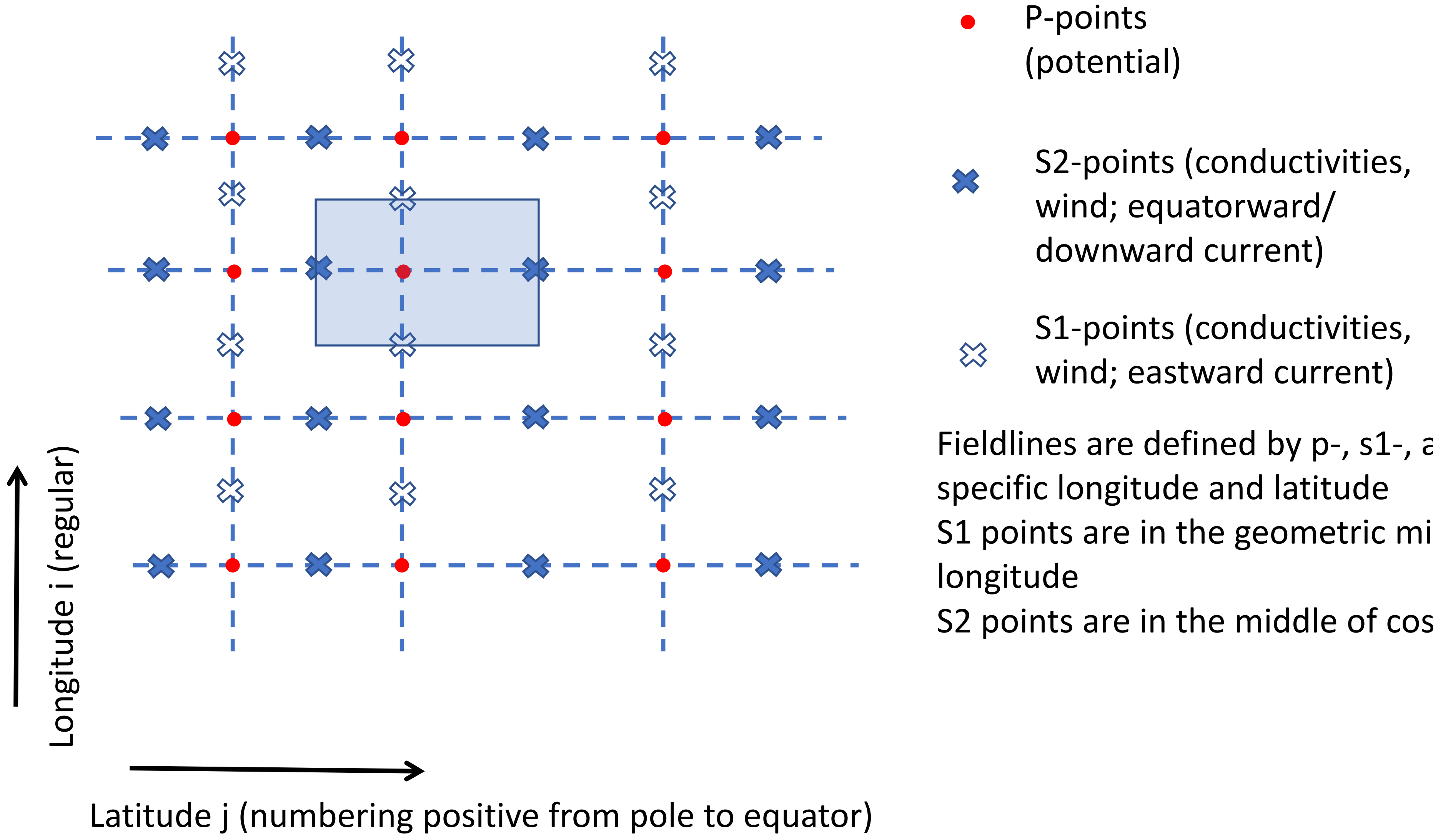
Discretization in latitude and height



r points are in the middle of p-points wrt height
Number of points on fieldline depends on apex height

Discretization in latitude and longitude

At a fixed height h_k



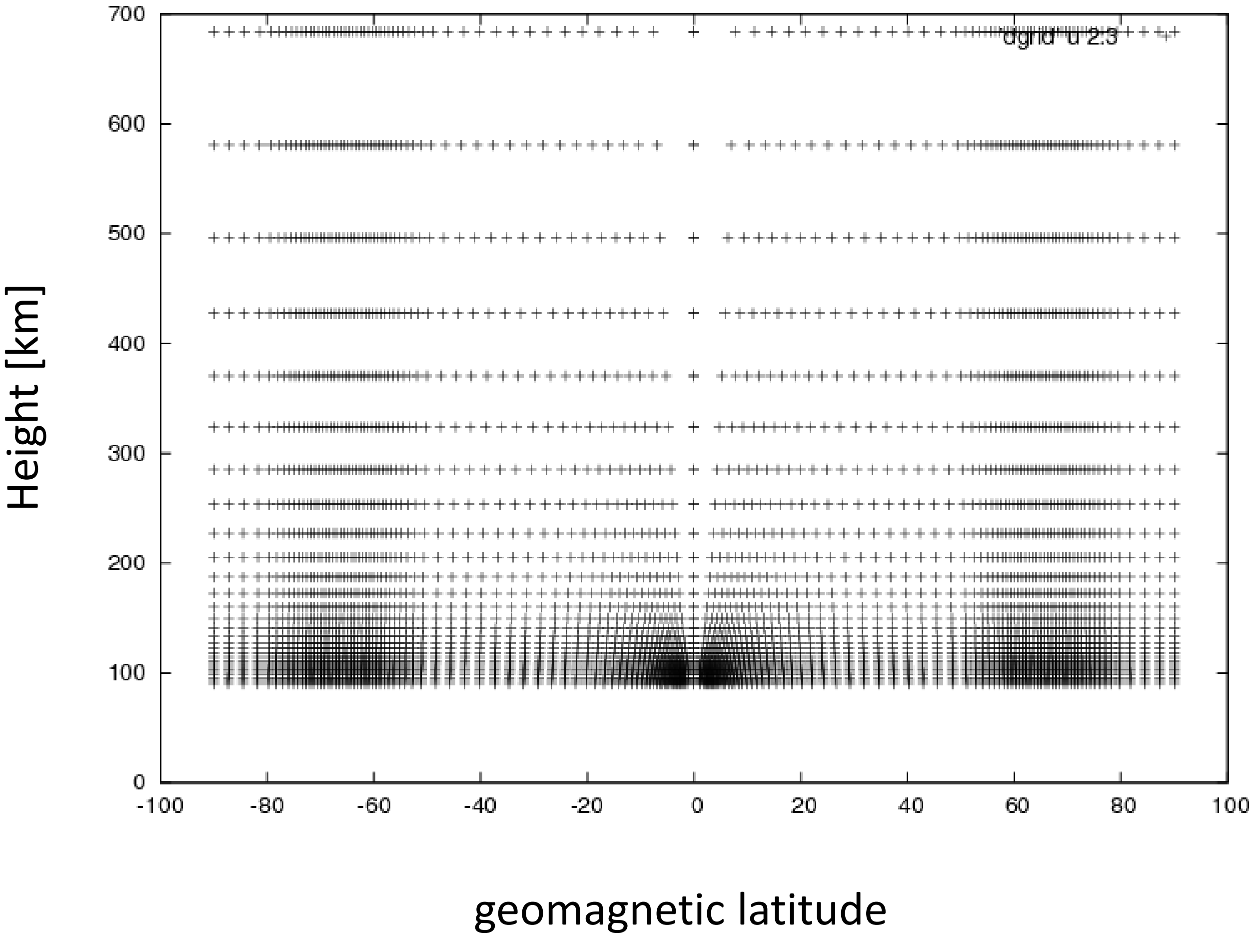
Fieldlines are defined by p-, s1-, and s2-points with a specific longitude and latitude

S1 points are in the geometric middle of p-points in longitude

S2 points are in the middle of $\cos(\lambda_m)$ of p-points

Grid example

Example discretization at a fixed magnetic longitude



The grid is flexible, and easy to change:
Only assumption is that the fixed height corresponds to an apex height (h(λm))
There are several regions in latitude to have higher resolution.

Outline

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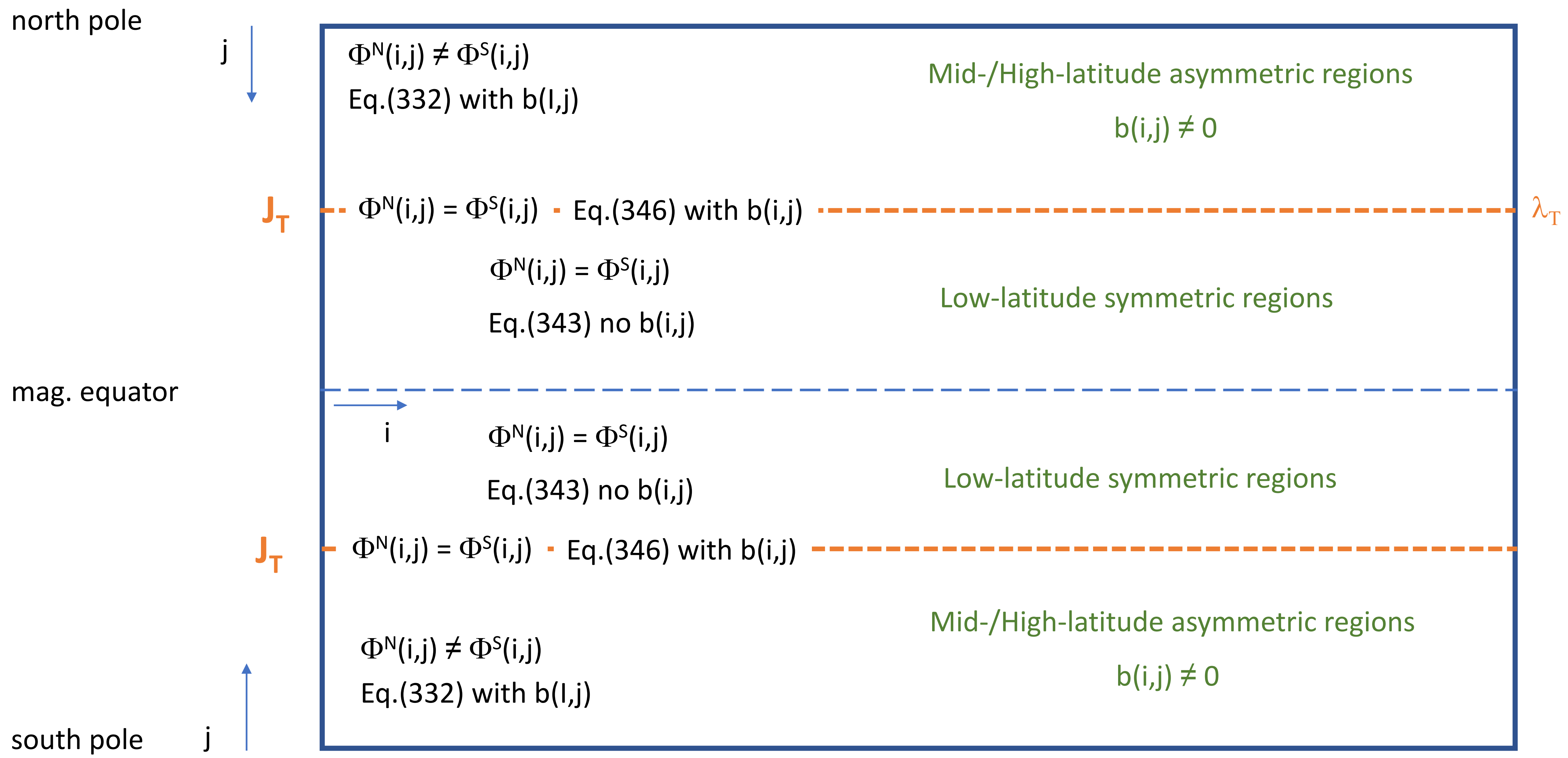
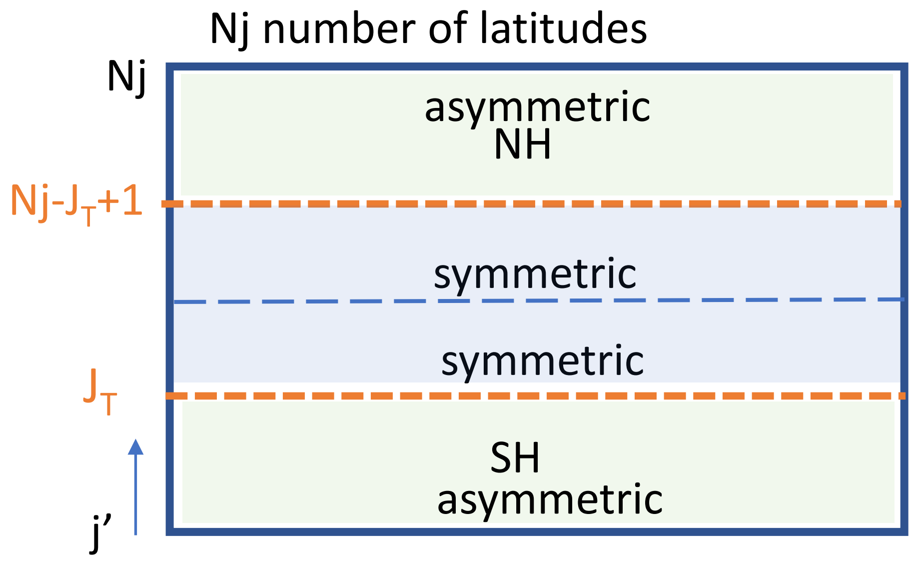
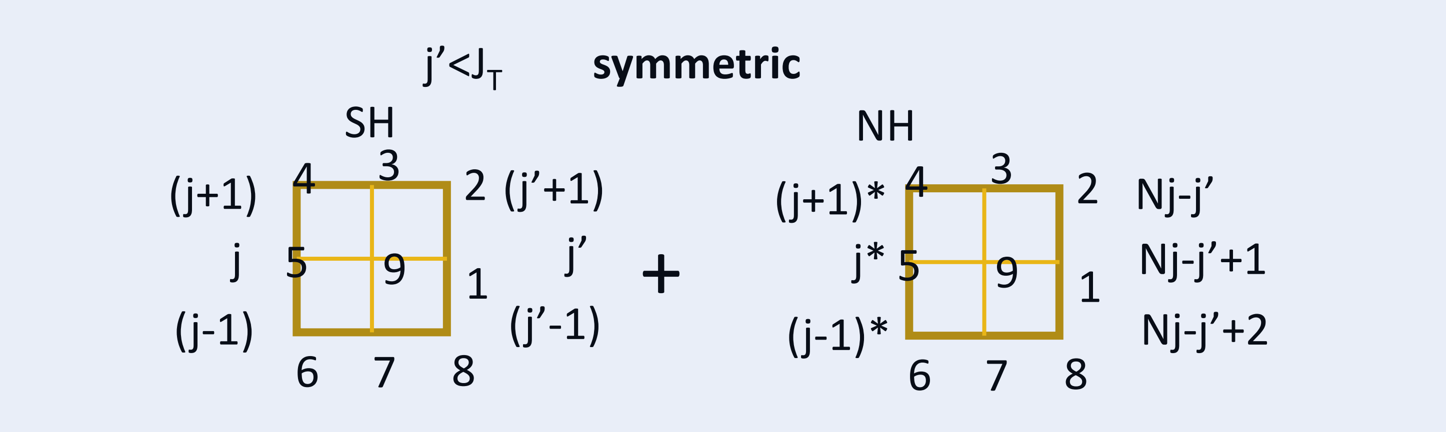
2. Simulating the three-dimensional ionospheric current system
3. **First steps to solve globally for the electric potential ("Global potential solver")**

Solving globally- schematic of different regions



Examples of the finite differencing

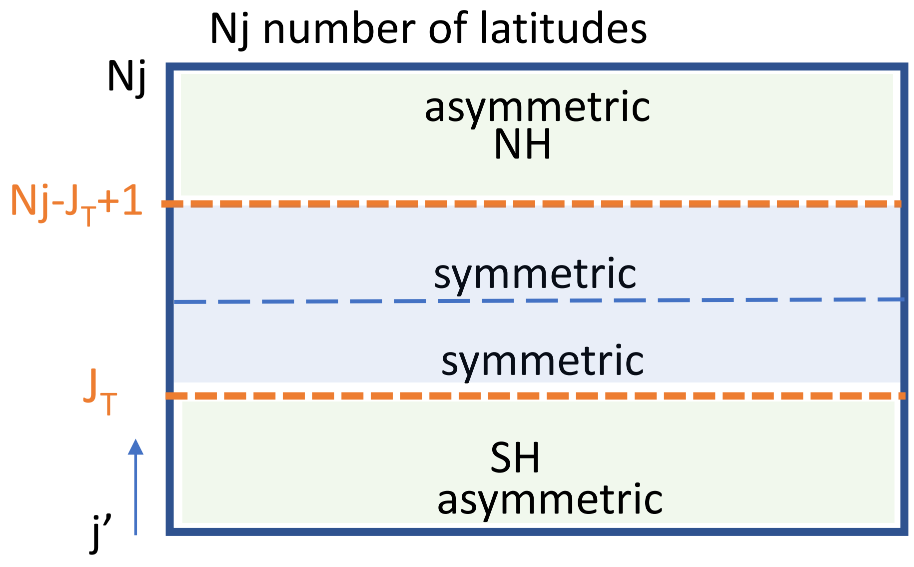


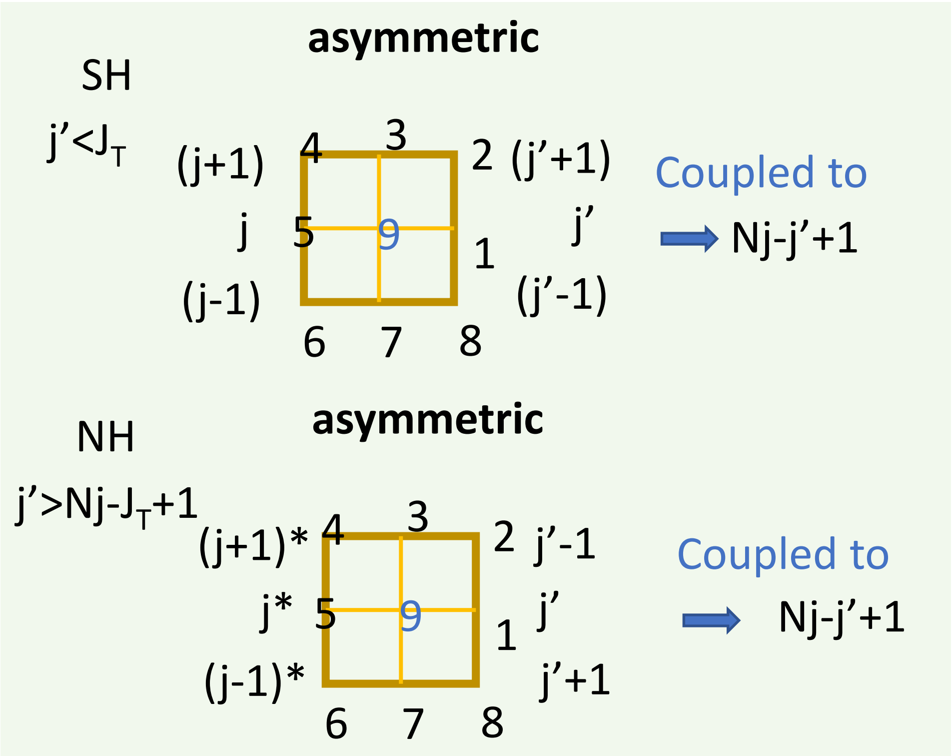


$$C_1^L(i,j)\,\Phi(i,j+1) + C_2^L(i,j)\,\Phi(i-1,j+1) + C_3^L(i,j)\,\Phi(i-1,j)$$
$$+ C_4^L(i,j)\,\Phi(i-1,j-1) + C_5^L(i,j)\,\Phi(i,j-1) + C_6^L(i,j)\,\Phi(i+1,j-1)$$
$$+ C_7^L(i,j)\,\Phi(i+1,j) + C_8^L(i,j)\,\Phi(i+1,j+1) + C_9^L(i,j)\,\Phi(i,j)$$
$$+ I_3^{r2}(i,j) = 0 \qquad (343)$$

Indexing in (343) is not like in the figure and code

Examples of the finite differencing



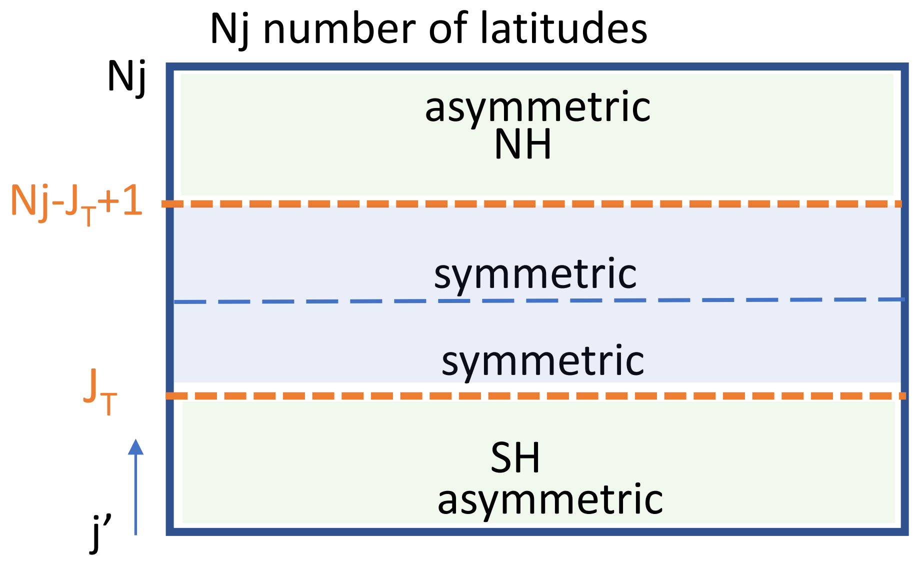


05/24/2023 stencil numbering might not be updated from code

$$
\begin{aligned}
&C_1(i,j)\,\Phi(i,j+1) + C_2(i,j)\,\Phi(i-1,j+1) + C_3(i,j)\,\Phi(i-1,j) \\
&+ C_4(i,j)\,\Phi(i-1,j-1) + C_5(i,j)\,\Phi(i,j-1) + C_6(i,j)\,\Phi(i+1,j-1) \\
&+ C_7(i,j)\,\Phi(i+1,j) + C_8(i,j)\,\Phi(i+1,j+1) + \left[C_9(i,j) - b(i,j)\right]\Phi(i,j) \\
&+ b(i,j)\,\Phi^*(i,j) + I_3^T(i,j) = 0 \qquad (332)
\end{aligned}
$$

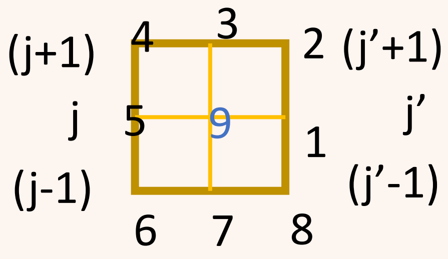
Indexing in (322) is not like in the figure and code

Examples of the finite differencing



$$
\begin{aligned}
& C_1^L(i,J_T)\,\Phi(i,J_T+1) + C_2^L(i,J_T)\,\Phi(i-1,J_T+1) + C_3^L(i,J_T)\,\Phi(i-1,J_T) \\
& + C_4^N(i,J_T)\,\Phi^N(i-1,J_T-1) + C_4^S(i,J_T)\,\Phi^S(i-1,J_T-1) \\
& + C_5^N(i,J_T)\,\Phi^N(i,J_T-1) + C_5^S(i,J_T)\,\Phi^S(i,J_T-1) \\
& + C_6^N(i,J_T)\,\Phi^N(i+1,J_T-1) + C_6^S(i,J_T)\,\Phi^S(i+1,J_T-1) \\
& + C_7^L(i,J_T)\,\Phi(i+1,J_T) + C_8^L(i,J_T)\,\Phi(i+1,J_T+1) + C_9^L(i,J_T)\,\Phi(i,J_T) \\
& + I_3^{TL}(i,J_T) = 0 \qquad (346)
\end{aligned}
$$

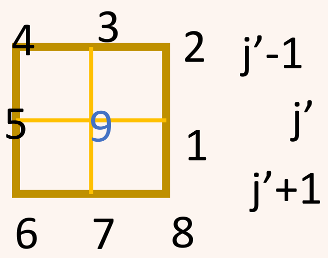
$j' = J_T$ **boundary** SH



Coupled to

Nj-j'	Add NS coefficient use $\Phi(J_T+1)$
Nj-j'+1	Add NS coefficient use $\Phi(J_T)$
Nj-j'+2	include N & S coefficients for $\Phi(J_T-1)$ and for $\Phi(Nj-J_T+2)$

$j' = Nj-J_T+1$ **boundary** NH



Coupled to

Nj-j'	Add NS coefficient use $\Phi(Nj - J_T)$
Nj-j'+1	Add NS coefficient use $\Phi(Nj-J_T+1)$
Nj-j'+2	Include N & S coefficients for $\Phi(J_T-1)$ and for $\Phi(Nj-J_T+2)$

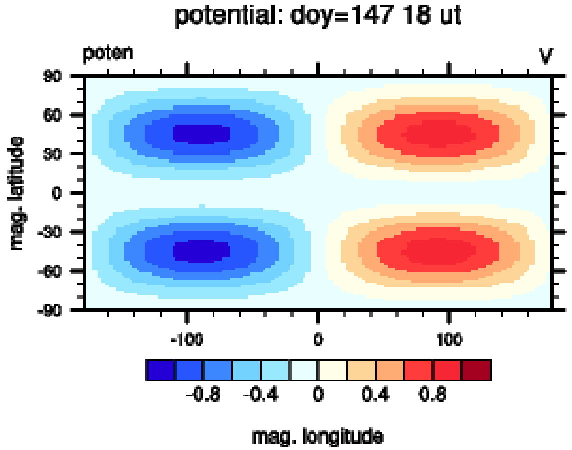
Conjugate point $(\cdot)^*$

05/24/2023 stencil numbering might not be updated from code

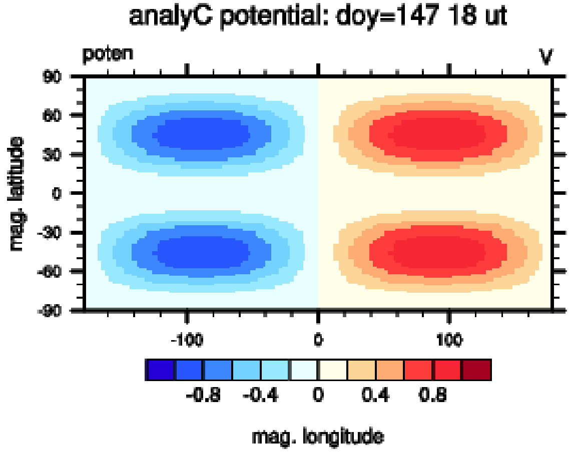
Indexing in (346) is not like in the figure and code

Test example- current status

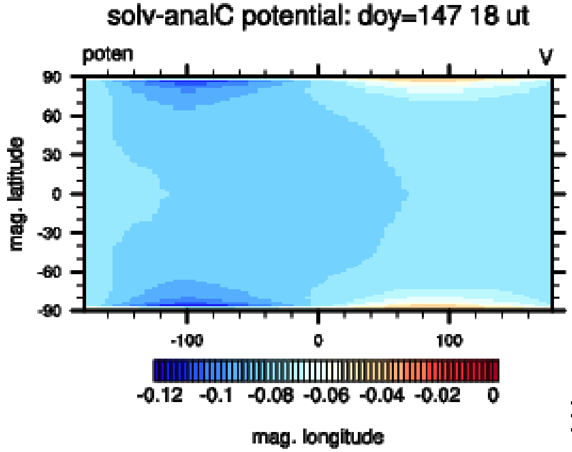
Test potential solved



analytical

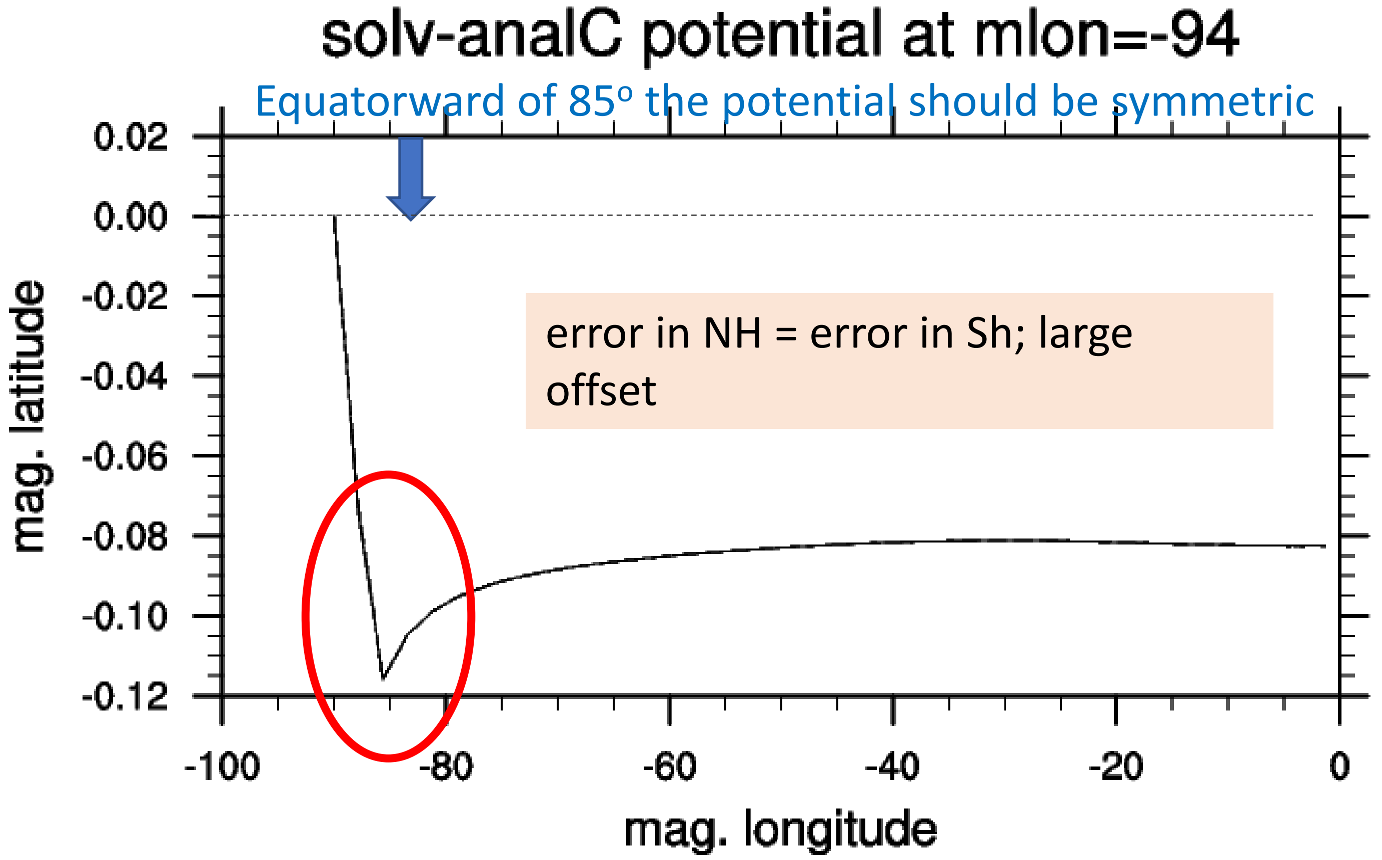


solved-analytical



real conductivities &corrected an error in the index of the conjugate point

Difference in solved & analytical potential $\Delta\Phi$



Exampling the values at JT – something wrong- check the values of the matrix at locations of JT

04/17/2023
5/26/2023

Grid example