Apex Dynamo Notes

Arthur D. Richmond

February 7, 2023

Chapter 14

Apex Dynamo Notes

14.1 Ideas for constructing the array $b(i,j)$

2023 February 7

$b(i,j)$ has the units of conductance. It must have the same value at conjugate points in the northern (N) and southern (S) hemispheres.

In (328) the term $b(i,j)[\Phi(i,j) - \Phi^*(i,j)]$ is antisymmetric between the N and S hemispheres, and so to evaluate how to choose values for $b(i,j)$ we look at the antisymmetric components of this equation. Although $I_3^R(i,j)$ may have an antisymmetric component, this antisymmetric component is generally poorly determined from observations, especially at middle latitudes, and its uncertainty will be comparable in magnitude to $b(i,j)[\Phi(i,j) - \Phi^*(i,j)]$, which should therefore be comparable to the magnitude of the antisymmetric component of $I_3(I,j,K+\frac{1}{2})$.

In fact, at middle and low latitudes $I_3^R(i,j)$ is usually ignored, so that $I_3(I,j,K+\frac{1}{2})$ equals $b(i,j)[\Phi(i,j)-\Phi^*(i,j)]$. At midlatitudes $I_3(i,j,K+\frac{1}{2})$ is basically the amount of field-aligned current flowing above the ionosphere through the (i,j) flux tube, which intersects the surface $r = r_{K+\frac{1}{2}}$ over a horizontal area given by $M_3(i,j,K+\frac{1}{2})$. That is,

$$I_3(i,j,K+\frac{1}{2}) = M_3(i,j,K+\frac{1}{2})J_r(i,j,K+\frac{1}{2}). \tag{14.1}$$

J_r is the divergence of the height-integrated horizontal current density below. A very rough estimate of J_r can be obtained by considering only the Pedersen current driven by the electric field, ignoring the wind-driven current and the electric-field-driven Hall currrent. In a region where the Pedersen conductance

$$\Sigma_P(i,j) = \sum_{k=1}^{K} \left(h_{k+\frac{1}{2}} - h_{k-\frac{1}{2}}\right) \sigma_P(i,j,k) \tag{14.2}$$

is roughly constant, $J_r(i,j,K+\frac{1}{2})$ roughly has the magnitude of $\Sigma_P(i,j)$ times the magnitude of the Laplacian of the electric potential, which is very roughly $|\Phi|/L^2$, where $|\Phi|$ is a characteristic magnitude of Φ and L is a characteristic scale length of Φ. It will usually be the characteristic scale length in the north-south direction that has the greater influence on the magnitude of the Laplacian of Φ, as compared with that in the east-west direction, because L in the north-south direction is usually the smaller of the two scale lengths.

If Σ_P differs between the northern (N) and southern (S) hemispheres, the hemisphere with the smaller Σ_P will tend to limit J_r in both hemispheres. That is, it tends to be the average of the resistances $1/\Sigma_P$ in the two hemispheres that controls the field-aligned current. The effective conductance is therefore $2/(1/\Sigma_P^N + 1/\Sigma_P^S)$. A very rough measure of the magnitude of the antisymmetric component of I_3 is thus

$$|I_3^{\text{antisymmetric}}(i,j,K+\frac{1}{2})| \approx \frac{|\Phi|\left[M_3^N(i,j,K+\frac{1}{2}) + M_3^S(i,j,K+\frac{1}{2})\right]}{[1/\Sigma_P^N(i,j) + 1/\Sigma_P^S(i,j)]L^2} \tag{14.3}$$

where the average of M_3 for the N and S hemispheres is taken to make the estimate of $I_3^{\text{antisymmetric}}$ independent of hemisphere. Let a characteristic magnitude of $[\Phi(i,j) - \Phi*(i,j)]$ be $\Delta\Phi$. If (14.3) is equated to $b(i,j)\Delta\Phi$, then

$$b(i,j) \approx \frac{|\Phi|}{\Delta\Phi L^2} \times \frac{M_3^N(i,j,K+\frac{1}{2}) + M_3^S(i,j,K+\frac{1}{2})}{1/\Sigma_P^N(i,j) + 1/\Sigma_P^S(i,j)} \tag{14.4}$$

The quantity $|\Phi|/(\Delta\Phi L^2)$ is now to be specified as a function of magnetic latitude, and perhaps also longitude, in a way that prevents interhemispheric potential differences on field lines from becoming too large at low, middle, and auroral latitudes, while allowing potential differences for a given absolute magnetic latitude and magnetic longitude in the polar caps to be unconstrained. In the polar caps $b(i,j)$ should be essentially 0.

At midlatitudes, large-scale electric potentials have a characteristic scale length L on the order of 2×10^6 m. If we want to limit $\Delta\Phi/|\Phi|$ to, say, 1%, then $|\Phi|/\Delta\Phi$ might be roughly 100, which would give $|\Phi|/(\Delta\Phi L^2)$ the value 25×10^{-12} m^{-2}.

At auroral latitudes L for large-scale features is on the order of 3×10^5 m. Because of field-line potential drops and displacement of field-line footpoints owing to magnetospheric currents that distort field lines, we expect $|\Phi|/\Delta\Phi$ to be considerably smaller than at mid-latitudes, maybe roughly 3. For these values, $|\Phi|/(\Delta\Phi L^2)$ would be about 33×10^{-12} m^{-2}.

It may be reasonable to use the same value of $|\Phi|/(\Delta\Phi L^2)$ for auroral latitudes as for midlatitudes, although testing will be necessary to determine what gives satisfactory, stable solutions.

Concerning the transition of b between polar and auroral regions, there are a number of possibilities.

A. A simple option would be to pre-define a fixed transition region (say, 65-75° magnetic latitude) over which b drops from the value (14.4) at the auroral edge to 0 at the polar-cap edge.

B. If the upper-boundary reference current J_r^R comes from an MHD magnetospheric model, that model may be able to determine the open-closed field-line boundary, and b could have a sharp transition to 0 everywhere poleward of that boundary.

C. A more complicated option than A might use a narrower, shifting transition region that takes into account polar-cap expansion and contraction, as defined either by geomagnetic indices or by an algorithm that finds the latitude of Region-1 currents.

14.2 [Optional] Getting a rough estimate of Σ_P from C_1 and C_7

To get an approximate value of Σ_P from quantities already calculated in the model, we can use Equations (53'), (86), (88), and (334). Since we are concerned here mainly with latitudes above about 45°, where ρ is less than than $\sqrt{0.5}$, we can simplify the square-root terms within the square brackets of (53') as follows. First write $1 - \frac{r}{R}\rho^2$ as $(1-\rho^2) - \frac{h}{R}\rho^2$ and note that the second term is much smaller than the first, because $h \ll R$. Then

$$\sqrt{1 - \frac{r}{R}\rho^2} = \sqrt{1-\rho^2}\sqrt{1 - \frac{h}{R}\frac{\rho^2}{(1-\rho^2)}} \tag{14.5}$$

which is approximately

$$\sqrt{1 - \frac{r}{R}\rho^2} \approx \sqrt{1-\rho^2}\left[1 - \frac{h}{2R}\frac{\rho^2}{(1-\rho^2)}\right]. \tag{14.6}$$

Applying this to (53') we find, to a good approximation when $\rho < \sqrt{0.5}$,

$$M_2(i, j-\frac{1}{2}, k) \approx \frac{R\left(\frac{r_k}{R}\right)^3 \sqrt{1 - \frac{3}{4}\rho^2_{j-\frac{1}{2}}}\left(h_{k+\frac{1}{2}} - h_{k-\frac{1}{2}}\right)\rho_{j-\frac{1}{2}}\left(\phi_{i+\frac{1}{2}} - \phi_{i-\frac{1}{2}}\right)}{\sqrt{1-\rho^2_{j-\frac{1}{2}}}F(i, j-\frac{1}{2}, k)} \qquad (351) \quad (14.7)$$

Using this in (86) gives

$$N_2(i,j-\frac{1}{2},k) = \frac{\left(\frac{r_k}{R}\right)^3 \left(1-\frac{3}{4}\rho^2_{j-\frac{1}{2}}\right)\left(h_{k+\frac{1}{2}}-h_{k-\frac{1}{2}}\right)\rho_{j-\frac{1}{2}}\left(\phi_{i+\frac{1}{2}}-\phi_{i-\frac{1}{2}}\right)\sigma_P(i,j-\frac{1}{2},k)d_2^2(i,j-\frac{1}{2},k)}{\sqrt{1-\rho^2_{j-\frac{1}{2}}}F(i,j-\frac{1}{2},k)(\rho_j-\rho_{j-1})} \tag{14.8}$$

Considering that (r_k/R), d_2, and F have magnitudes not too different from 1, and that ρ^2 can roughly be neglected in comparison with 1 at higher latitudes where $\rho < 0.71$, (14.8) has the rough approximation

$$N_2(i,j-\frac{1}{2},k) \approx \frac{\left(h_{k+\frac{1}{2}}-h_{k-\frac{1}{2}}\right)\rho_{j-\frac{1}{2}}\left(\phi_{i+\frac{1}{2}}-\phi_{i-\frac{1}{2}}\right)\sigma_P(i,j-\frac{1}{2},k)}{(\rho_j-\rho_{j-1})} \tag{14.9}$$

Notice in (88) that

$$c_1(i,j,k)+c_7(i,j,k) = N_2(i,j-\frac{1}{2},k)+N_2(i,j+\frac{1}{2},k) \tag{14.10}$$

Assuming that, approximately,

$$\sigma_P(i,j-\frac{1}{2},k) \approx \sigma_P(i,j,k) \approx \sigma_P(i,j+\frac{1}{2},k) \tag{14.11}$$

$$\rho_j-\rho_{j-1} = \rho_{j+1}-\rho_j \tag{14.12}$$

then using (14.9) to estimate the right-hand side of (14.10) gives roughly

$$c_1(i,j,k)+c_7(i,j,k) \approx \frac{(h_{k+\frac{1}{2}}-h_{k-\frac{1}{2}})\rho_j\left(\phi_{i+\frac{1}{2}}-\phi_{i-\frac{1}{2}}\right)\sigma_P(i,j,k)}{\rho_{j+1}-\rho_{j-1}} \tag{14.13}$$

Summing over k gives

$$C_1(i,j) + C_7(i,j) \approx \frac{\rho_j \left(\phi_{i+\frac{1}{2}} - \phi_{i-\frac{1}{2}}\right) \Sigma_P(i,j)}{\rho_{j+1} - \rho_{j-1}} \tag{14.14}$$

From (14.14) a rough estimate of Σ_P is

$$\Sigma_P(i,j) \approx \frac{\left[\left(C_1(i,j) + C_7(i,j)\right]\left(\rho_{j+1} - \rho_{j-1}\right)}{\rho_j \left(\phi_{i+\frac{1}{2}} - \phi_{i-\frac{1}{2}}\right)} \tag{14.15}$$