A Kalman Filter to Improve Measurements of Wind

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Outline

WHY DO WE USE AN INERTIAL NAVIGATION SYSTEM?

1. This is the only measurement of "attitude angles" (pitch, roll, heading)
2. Also provides aircraft-velocity vector and aircraft position.
3. Velocity and position are also available from GPS receivers.

Needed to measure wind (AC-mounted or remote sensors):

- Wind: the vector sum of air motion wrt aircraft and aircraft motion wrt ground.
- This sum requires that the aircraft attitude angles be known.
- Uncertainty in pitch and heading are the dominating sources of uncertainty in the measurement of wind.

HOW DOES THE IRU/INS WORK?

Inertial Reference Unit (IRU):

Three-axis accelerometers

Three-axis gyros

Some older systems: mounted on a stabilized platform

Strap-down systems: maintain a virtual platform

Inertial Navigation System (INS):

initialization: determines a starting state

mechanization: integrates IRU output to update the position, velocity, and attitude angles as functions of flight time.

CAN ERRORS BE REDUCED?

What causes errors:

1. Initial alignment introduces initial-state errors.
2. Errors in accelerometers/gyros: offset, sensitivity, drift.
3. Strong coupling limits error growth (horizontal velocity and pitch/roll)
4. No similar coupling for vertical velocity or heading errors – much harder to correct.

Uncertainty estimates:

- pitch and roll: specs say 0.05° for flights of several hours; resulting uncertainty in vertical wind is about 0.2 m/s.
- heading: specs say 0.2°;
  resulting uncertainty in horizontal wind is about 0.8 m/s.
- These are the dominant error sources.

REDUCE ERRORS USING MEASUREMENTS FROM A GPS RECEIVER

Very good measurements available:

Aircraft position

Aircraft velocity

- For 20 y, RAF has used a complementary-filter combination of INS and GPS, leading to very good measurement quality.
- Measurements have become better as the quality of measurements from GPS receivers has improved.
- This does not help with attitude angles.

HOW CAN ATTITUDE ANGLES BE IMPROVED?

Use feedback from GPS:

1. Pitch and roll errors lead to the gravity vector being resolved incorrectly into horizontal accelerations.
2. A heading error causes horizontal accelerations to be resolved incorrectly to give errors in aircraft velocity.

Two approaches are feasible:

1. Assume accelerometers and gyros are perfect, so errors arise only from erroneous attitude angles.
2. Incorporate the GPS information into a Kalman filter:
   (a) Use measured errors in position and velocity to find optimal estimates of all the errors.
   (b) This allows for estimation of errors in attitude angles and IRU measurements.

WHAT IS A KALMAN FILTER?

in general:

A means of combining redundant measurements, while considering their relative uncertainties, to obtain an optimal resulting measurement.

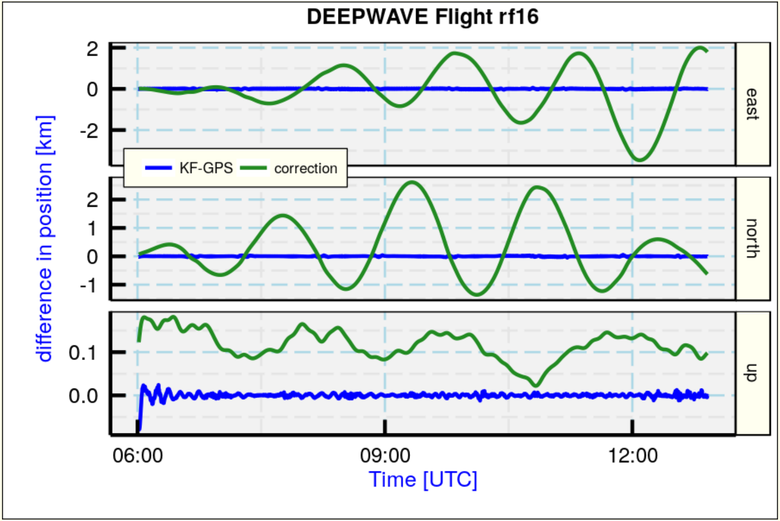
in the present case:

A means of using GPS-provided measurements to refine the INS-produced "trajectory"

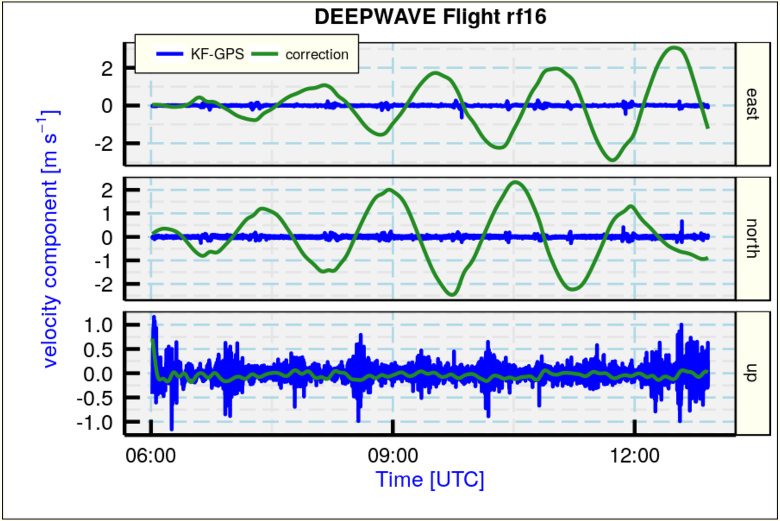
the primary value:

- *not* updating the redundant measurements:
  GPS measurements are superior and mostly could be used alone for position and velocity of the aircraft
- *updating the attitude angles*, via error-feedback:

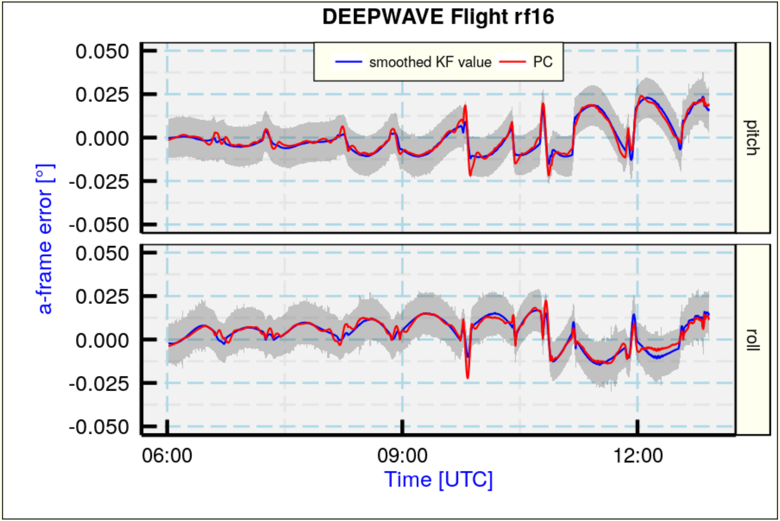
ERROR IN POSITION



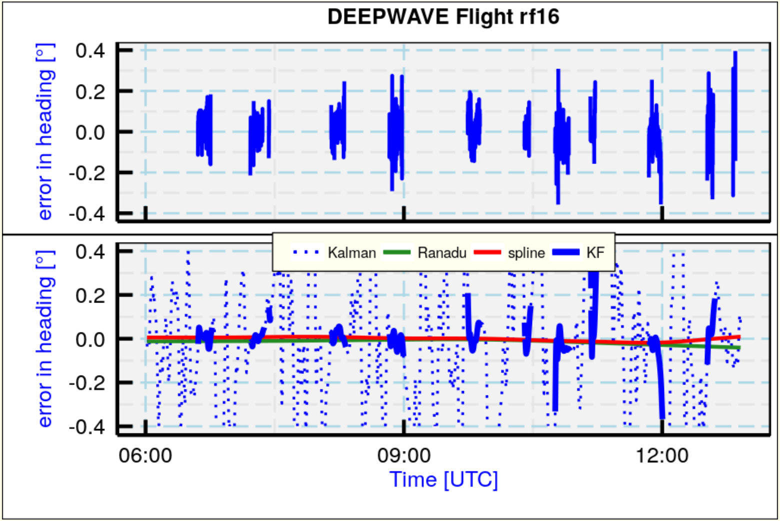
ERROR IN AIRCRAFT VELOCITY



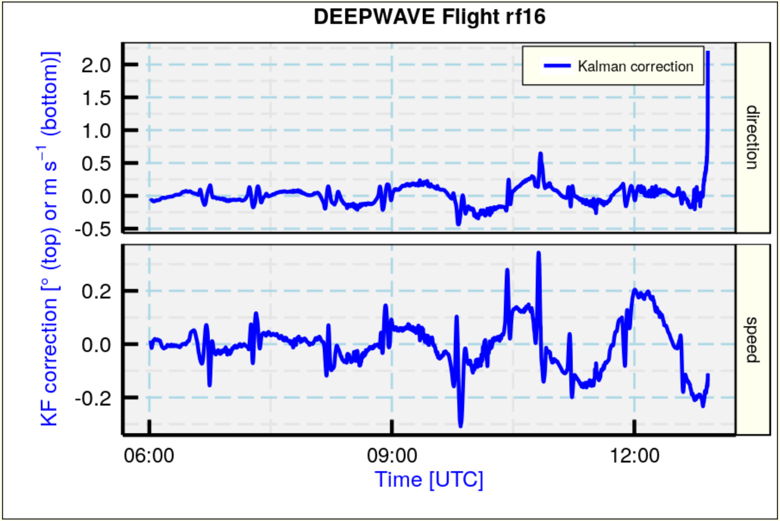
ERRORS IN PITCH AND ROLL



ERROR IN HEADING



CHANGE IN MEASURED HORIZONTAL WIND



GENERIC FLOWCHART

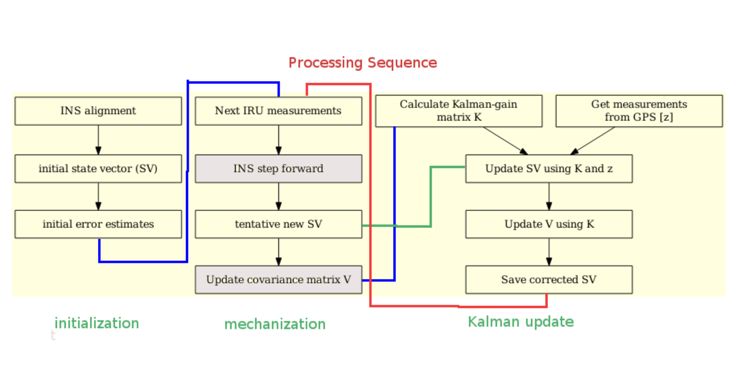


ILLUSTRATION: 1D case

Consider a single component of aircraft velocity

- INS (e.g., VEW) and GPS (e.g., GGVEW)
- Step forward uses measured acceleration
- Covariance is estimated from the variance in INS solution
- "Best" estimate is K $\times$GGVEW+(1-K)$\times$VEW*with VEW* the projected-forward value of VEW from the previous time.
- K depends on the relative magnitudes of the VEW*covariance and the covariance applicable to GGVEW.
- The loop reduces the VEW* covariance as the estimate is improved.

Extension to the multi-dimensional case

- Coupling among error terms (e.g., an error in velocity results in an error in position) leads to need for a covariance matrix.
- Also, represent noise contributions and **K** as matrices.

NEED A VALID DERIVATIVE FOR INTEGRATION

To test derivative used for integration, duplicate mechanization

Can't do this perfectly:

- INS has high-rate data internally
- build-in calibrations
- baro-loop for altitude

Project-forward 9 components of the state vector:

1. Position derivatives from velocities; velocity derivatives from accelerations
2. Accelerations are measured in the body frame, so must transform to the l-frame [local east, north, up components]
    - Involves compensation for Earth and l-frame rotation.
3. Attitude angles: Must change pitch, roll, heading as specified by gyros, also compensating for rotations vs an inertial frame.

ADVANTAGES OF AN ERROR-STATE KALMAN FILTER

State vector is the vector of error components in the INS solution

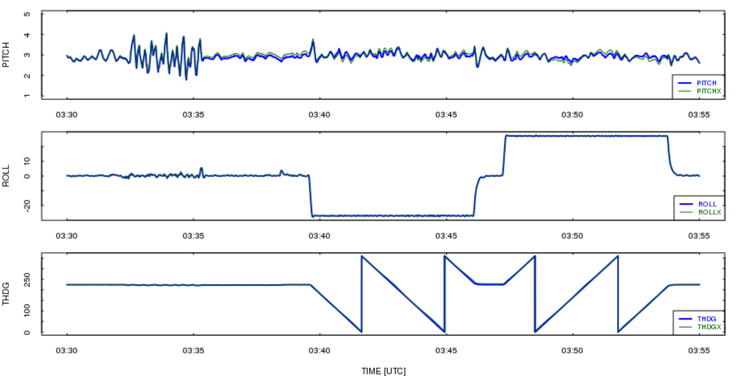
1. Don't have to repeat the original integration; instead use the original INS values as the reference solution and find errors. (Still need the derivatives of the original state vector.)
2. The error-state is small so linear assumptions re propagation are likely to be less compromising.

Key tasks:

1. Construct a function giving the rotation matrix relating the a-frame to the l-frame.
2. Construct a function that gives the first 9 derivatives of the error state, given the state vector and INU measurements of rotation rate and acceleration.
3. Validate that function by duplicating the INS-provided

VALIDATION: COMPARE TO INS SOLUTION

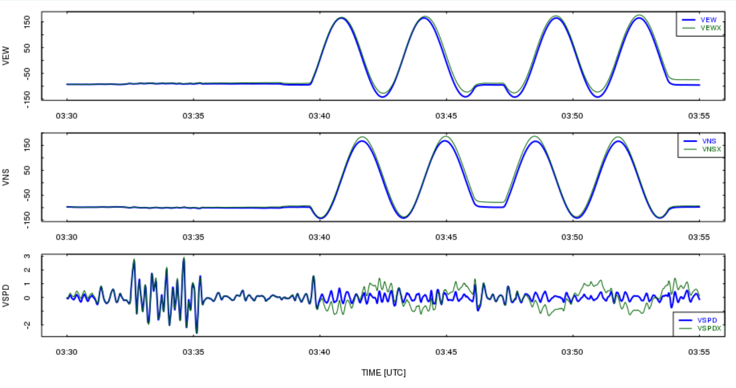
Attitude Angles [PITCHX etc are new-mechanization results]



Good match supports that angle transformations and signs are correct.

VALIDATION: COMPARE TO INS SOLUTION

GV Velocities [VEWX etc are new-mechanization results]



VSPD integration in INS uses pressure altitude and a faster update vs my slower baro-loop update to GPS velocity.

IMPLEMENTATION

Key Components

1. The preceding plots indicate that the derivative function (STMFV) is valid, at least to a good approximation.
2. The error-state transformation function can be calculated numerically from this derivative function, using the relatively fast Jacobian function in R.
3. The result also provides a valid update for the error-state covariance matrix.
4. To initiate, need estimates of the uncertainties in the initial-error-state and of noise components present in the INS measurements (from accelerometers and gyros) and in the GPS-provided measurements (position and aircraft velocity).
5. The uncertainties have been discussed in the Tech Note on uncertainty in the wind measurements, except for the

THE KALMAN-FILTER LOOP

The core R-code loop (6 lines, applied recursively)

- sv: the INS-provided state vector
- SVE: error-state vector, calculated sequentially
- STMFV: derivative function for state vector
- CV: the covariance matrix
- Q and RCV: noise matrices, INS and GPS
- H: correspondence, GPS and INS measurements
- DZ[i,]: vector of GPS measurements at time *i*
- dt: time interval between steps

```
dcm <- jacobian (STMFV, sv) * dt + diag(15)
SVE <- dcm  %*% SVE
CV <- dcm %*% (CV %*% t(dcm)) + Q
K <- CV %*% t(H) %*% solve (H %*% CV %*% t(H) + RCV)
```

THE RESULT OF THE KALMAN-FILTER LOOP

State Vector vs Time

The estimated errors at each time step, to be subtracted from the INS-produced trajectory.

Application to Recorded Variables

- The error-state vector should vary slowly, so the calculation can be done with 1-Hz or even lower-rate data.
- It may be preferable to filter inputs like VEW and GGVEW to remove high-frequency noise, but it is probably not necessary.
- For aircraft velocity and position, this solution should be a superior replacement for the complementary filter now in use.
- For higher-rate data, interpolation of the 1-Hz correction with appropriate smoothing will be valid and will avoid the need for high-rate calculation of the Kalman filter.

SUBTLETIES REGARDING PITCH, ROLL, AND HEADING

Use errors for pitch and roll in the l-frame

Why? Errors vary more smoothly than in the *a*-frame; changes in heading intermix errors in pitch and roll.

Explanation: The INS calculates orientation in an inertial frame; the l-frame is a better match to an inertial frame.

Illustration: A pitch error when flying north becomes a roll error after a turn to the west, producing an abrupt change in the *a*-frame but not in the *l*-frame.

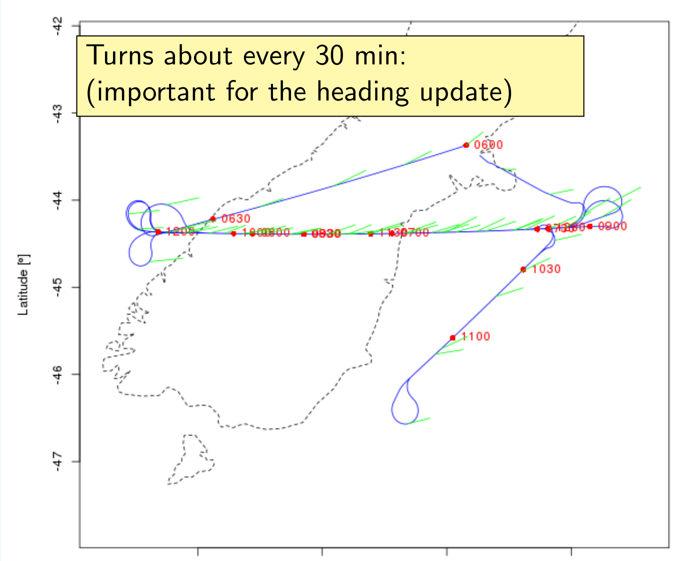
Differentiate GPS-provided velocity; use as measured acceleration

Why? Otherwise feedback to heading is slow and uncertain.

Explanation: There are many competing sources of error in velocity; this helps isolate heading errors vs others.

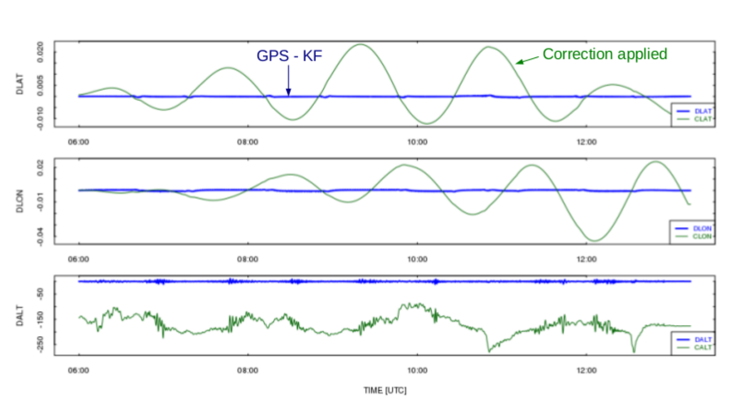
EXAMPLE: DEEPWAVE FLIGHT 16, INTENSE WAVES

Turns about every 30 min:
(important for the heading update)

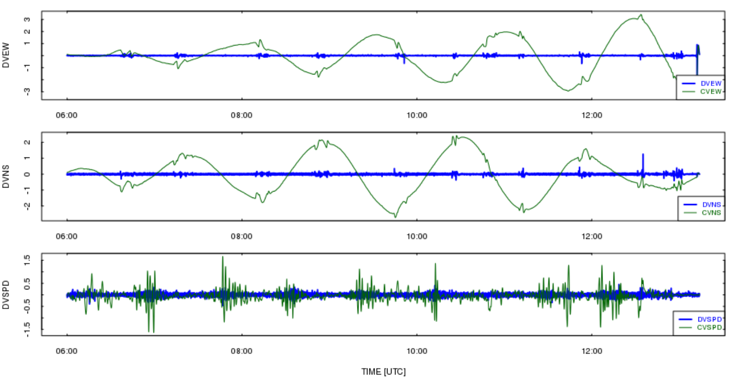


RESULTS FOR POSITION

Schuler oscillation evident in correction (good agreement with GPS)



RESULTS FOR GV VELOCITY



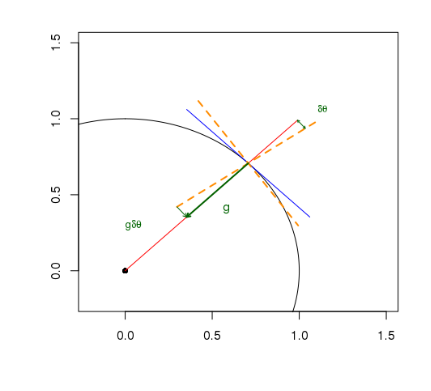
- Advantage: smooth variation
- Note Schuler-period oscillation

ALTERNATIVE FOR PITCH: "CorrectPitch ()" as in TN

Effect of a pitch error:

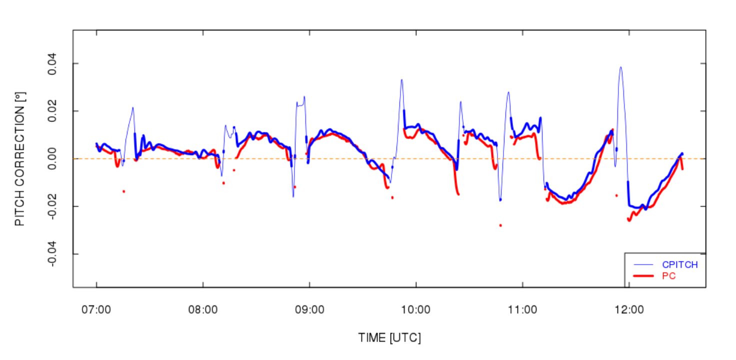
1. False platform orientation produces a false horizontal component of sensed gravit
2. Result is a false acceleration, leading to growth of an error in the ground speed:

$$\frac{d(\delta v_n)}{dt} = -g\delta\theta$$



find left side of equation by differentiating GPS-INS velocity components

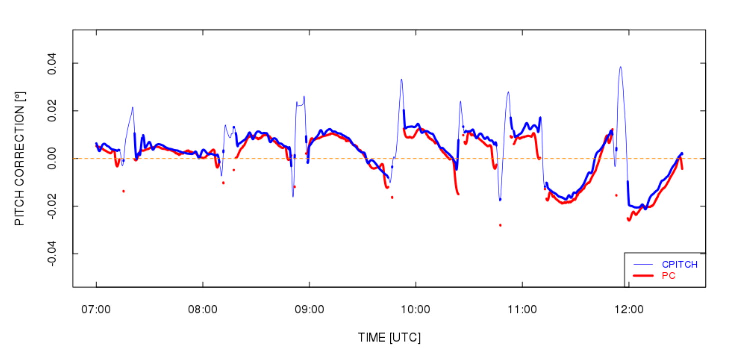
RESULTS FOR PITCH [vs PC=CorrectPitch() result]



comments:

- Thin lines are turns: possible timing differences? PC not plotted there.
- Performance is much better with errors maintained in I frame

RESULTS FOR PITCH [vs PC=CorrectPitch() result]



advantage of the Kalman-filter result:

CorrectPitch() assumes no error in measured accelerations, while the Kalman filter accounts for a possible error in that measurement

THE HEADING CORRECTION

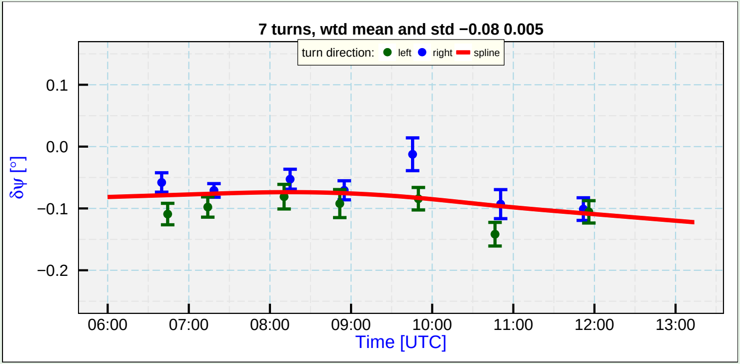
Basis for the correction:

1. Accelerations in the *a*-frame are transformed to the *l*-frame using the measured heading.
2. An incorrect heading results in velocity errors in the *l*-frame.
3. Comparison to GPS-provided velocity reveals these errors.
4. This relies on significant accelerations, best realized in turns.

A CorrectHeading() function:

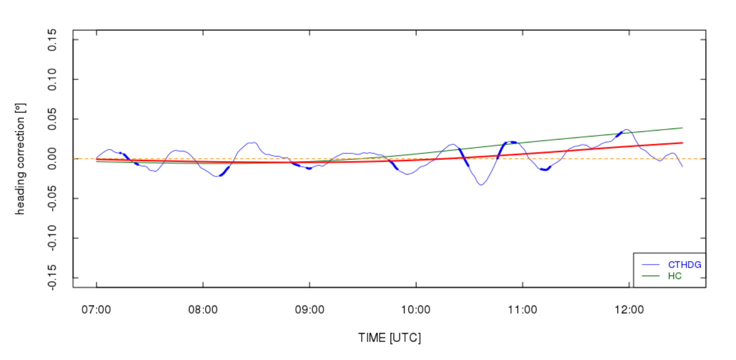
1. Compare the direction of the velocity change to that predicted from the l-frame accelerations.
2. The difference is the heading error.
3. Require significant turns; discard other periods.
4. Use a smoothing spline to extrapolate.

RESULTS FROM CorrectHeading():



Based on this and circle analyses, -0.8° offset introduced to heading.

KALMAN-FILTER RESULTS FOR HEADING:



Estimated uncertainty varies greatly

1. Thick lines: $\sigma < 0.05^{\circ}$ (only occurs in turns)
2. Red line:

CONCLUSIONS

Attitude angles are improved significantly

1. Uncertainty reduced to $<0.01^\circ$
   (for heading, only if there are regular turns)
2. Results are consistent with previous estimates but require fewer assumptions.

Aircraft-velocity components are also improved

Horizontal: Better than complementary filter, but not significantly so.

Vertical: Provides an alternative to VSPD that is even better than GGVSPD

Applicable to past projects as well as future ones.

THE GUI FOR THE PROCESSOR